Impact of DCSB on Meson Structure and Interactions

Dressed–quark Mass Function

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http://www.phy.anl.gov/theory/staff/cdr.html
Universal Truths
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- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron’s characterising properties amongst its QCD constituents.
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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.

- Running of quark mass entails that calculations at even modest $Q^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.
Universal Truths

Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
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E.g., one problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.
Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.

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Resolution

- *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
- DCSB obtained via coherent contribution from countable infinity of higher Fock-state components in LF-wavefunction.

QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
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  - Very unnatural pattern of bound state masses
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Dynamical Chiral Symmetry Breaking

Very unnatural pattern of bound state masses

e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^P=+$ and $J^P=-$

Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

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- Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.

- QCD – Complex behaviour arises from apparently simple rules
Confinement can be related to the analytic properties of QCD's Schwinger functions.
Charting the Interaction between light-quarks

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- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD’s universal $\beta$-function.
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Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD’s universal $\beta$-function.

This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
Charting the Interaction between light-quarks

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Of course, the behaviour of the $\beta$-function on the perturbative domain is well known.
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This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.

Of course, the behaviour of the $\beta$-function on the perturbative domain is well known.

This is a well-posed problem whose solution is an elemental goal of modern hadron physics.
What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD is not related in any known way to the light-quark interaction.
Through QCD’s Dyson-Schwinger equations (DSEs) the pointwise behaviour of the $\beta$-function determines pattern of chiral symmetry breaking.
Charting the Interaction between light-quarks

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DSEs connect $\beta$-function to experimental observables. Hence, comparison between computations and observations of, e.g.,
- hadron mass spectrum;
- elastic and transition form factors

can be used to chart $\beta$-function’s long-range behaviour.
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E.g.: Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of $\beta$-function than those of the ground state.
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(See nucl-th/9602012 and references thereto)
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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary.
- Steady quantitative progress is being made with a scheme that is systematically improvable (See nucl-th/9602012 and references thereto).
- Enabled proof or exact results in QCD: e.g., BRST – arXiv:1005.4610 [nucl-th]; and . . .
Radial Excitations & Chiral Symmetry

\[ f_H \, m_H^2 = - \, \rho_H^2 \, \mathcal{M}_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho_\zeta^H \ \mathcal{M}_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \rho_H \ \mathcal{M}_H \]

\[ \mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M(\mu) \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \)

Craig Roberts – Impact of DCSB on meson structure and interactions
11th International Workshop on Mesons – Kraków, Poland, 10-15 June 2010
Radial Excitations & Chiral Symmetry

\(-i \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle\)

\[ f_H m_H^2 = - \rho_\zeta^H M_H \]

\[ f_H p_\mu = Z_2 \int_q \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu S(q+) \Gamma_H(q; P) S(q-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant

\[ \pi \rightarrow -f_\pi k^\mu \quad \vec{A}_5^\mu \quad \Gamma_5 \quad \pi \]

Craig Roberts – Impact of DCSB on meson structure and interactions
11th International Workshop on Mesons – Kraków, Poland, 10-15 June 2010… 29 – p. 9/30
Radial Excitations & Chiral Symmetry

\[
\langle 0 | \bar{q} \gamma_5 q | \pi \rangle
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f_H \ m_H^2 = - \rho_\zeta^H \ M_H
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- **Pseudoscalar** projection of BS wave function at \( x = 0 \)

\[
\vec{\pi} - \rho_\pi \ L^5 \ \vec{P}_5 = i \Gamma_5 \ i(\tau/2) \ \gamma_5
\]
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \rho_H^\zeta \ M_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)

- \( f_H \rightarrow f_H^0 \) & \( \rho_H^\zeta \rightarrow -\langle \bar{q}q \rangle_0^\zeta \frac{f_H^0}{f_H} \), Independent of \( m_q \)

Hence \[ m_H^2 = \frac{-\langle \bar{q}q \rangle_0^\zeta}{(f_H^0)^2} m_q \] ...GMOR relation, a corollary
Radial Excitations & Chiral Symmetry

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- Heavy-quark + light-quark
  \[ f_H \propto \frac{1}{\sqrt{m_H}} \quad \text{and} \quad \rho_H^H \propto \sqrt{m_H} \]

Hence\( , \quad m_H \propto m_q \)\( \ldots \) QCD Proof of Potential Model result
Valid for ALL Pseudoscalar mesons
Radial Excitations
& Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

\[ f_H \ m_H^2 = - \ \rho^H_\zeta \ \mathcal{M}_H \]

- Valid for **ALL** Pseudoscalar mesons
- \( \rho_H \rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
Valid for ALL Pseudoscalar mesons

$\rho_H \Rightarrow \text{finite, nonzero value in chiral limit, } \mathcal{M}_H \rightarrow 0$

“radial” excitation of $\pi$-meson, not the ground state, so

$m_{\pi_n \neq 0}^2 > m_{\pi_n = 0}^2 = 0$, in chiral limit
Valid for **ALL** Pseudoscalar mesons

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- \( \Rightarrow f_H = 0 \)

**ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit
Valid for **ALL** Pseudoscalar mesons

\[ f_H \quad m_H^2 = - \quad \rho_H^H \quad M_H \]

- \( \rho_H \Rightarrow \) finite, nonzero value in chiral limit, \( M_H \to 0 \)
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- \( \Rightarrow f_H = 0 \)

**ALL** pseudoscalar mesons except \( \pi(140) \) in **chiral limit**

- **Dynamical Chiral Symmetry Breaking**

- – **Goldstone’s Theorem** –

impacts upon **every** pseudoscalar meson
When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
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CLEO: $\tau \rightarrow \pi(1300) + \nu_\tau$

$\Rightarrow f_{\pi_1} < 8.4 \text{ MeV}$

*Diehl & Hiller*

*he-ph/0105194*
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Lattice-QCD check:
$16^3 \times 32,$
$a \sim 0.1 \text{ fm},$
two-flavour, unquenched
$\Rightarrow \frac{f_{\pi_1}}{f_{\pi}} = 0.078 (93)$
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Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)
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The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
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Enabled proof or exact results in QCD, as we’ve just seen.
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On other hand, at present significant qualitative advances possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions
Gap Equation
General Form
Gap Equation
General Form

\[ S_f(p)^{-1} = Z_2 (i \gamma \cdot p + m^{bm}_f) + \Sigma_f(p), \]
\[ \Sigma_f(p) = Z_1 \int_q^A g^2 D_{\mu \nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_\nu(q, p), \]
\[ S_f(p)^{-1} = Z_2 \left( i \gamma \cdot p + m_{f}^{bm} \right) + \Sigma_f(p), \]

\[ \Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_{\nu}(q, p), \]

- \( Z_{1,2}(\zeta^2, \Lambda^2) \) are respectively the vertex and quark wave function renormalisation constants, with \( \zeta \) the renormalisation point
- \( m_{f}^{bm}(\Lambda) \) is the Lagrangian current-quark bare mass
- \( D_{\mu\nu}(k) \) is the dressed-gluon propagator
- \( \Gamma^f_{\nu}(q, p) \) is the dressed-quark-gluon vertex
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\( m^{\text{bm}}(\Lambda) \) is the Lagrangian current-quark bare mass.

\( D_{\mu\nu}(k) \) is the dressed-gluon propagator.

\( \Gamma^f_{\nu}(q, p) \) is the dressed-quark-gluon vertex.

Suppose one has in-hand the exact form of \( \Gamma^f_{\nu}(q, p) \) What is the associated Symmetry-preserving Bethe-Salpeter Kernel?
Bound-state DSE

Bethe-Salpeter Equation

Standard form, familiar from textbooks

\[
[\Gamma^{j}_{\pi}(k; P)]_{tu} = \int_{q}^{\Lambda} [S(q + P/2)\Gamma^{j}_{\pi}(q; P)S(q - P/2)]_{sr} K^{rs}_{tu}(q, k; P)
\]

\(K(q, k; P)\): Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel
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Blocked progress for more than 60 years.
Bethe-Salpeter Equation

General Form

L. Chang and C. D. Roberts
Equivalent exact form:

\[ \Gamma^{fg}_{5\mu}(k; P) = Z_2 \gamma_5 \gamma_{\mu} \]

\[ - \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^\alpha}{2} \gamma_\alpha S_f(q+) \Gamma^{fg}_{5\mu}(q; P) S_g(q-) \frac{\lambda^\alpha}{2} \Gamma^{g}_{\beta}(q-, k-) \]

\[ + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^\alpha}{2} \gamma_\alpha S_f(q+) \frac{\lambda^\alpha}{2} \Lambda^{fg}_{5\mu\beta}(k, q; P), \]

(Poincaré covariance, hence \( q_\pm = q \pm P/2 \), etc., without loss of generality.)
Equivalent exact form:

\[ \Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu \]

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\[ + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^\alpha}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^\alpha}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P), \]

(Poincaré covariance, hence \( q_\pm = q \pm P/2 \), etc., without loss of generality.)

In this form \( \Lambda_{5\mu\beta}^{fg} \) is completely defined via the dressed-quark self-energy.
Bethe-Salpeter Kernel

L. Chang and C. D. Roberts

- Bethe-Salpeter equation introduced in 1951
Bethe-Salpeter equation introduced in 1951

Newly-derived Ward-Takahashi identity

\[
P_{\mu} \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_{\beta}^{f}(q_+, k_+) i \gamma_5 + i \gamma_5 \Gamma_{\beta}^{g}(q_-, k_-) - i [m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),
\]
Bethe-Salpeter equation introduced in 1951

Newly-derived Ward-Takahashi identity

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\]

For first time: can construct \textit{Ansatz} for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex

Consistent means - all symmetries preserved!
Bethe-Salpeter Kernel

60 year problem

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity

\[ P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_{\beta}^{f}(q_{+}, k_{+}) \gamma_{5} + i \gamma_{5} \Gamma_{\beta}^{g}(q_{-}, k_{-}) \]

\[ - i [m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P), \]

- For first time: can construct Ansatz for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Procedure & results to expect . . .

see arXiv:1003.5006 [nucl-th]
<table>
<thead>
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<td>mass $\rho$</td>
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<td>mass-splitting</td>
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- Splitting known experimentally for more than 35 years.
- Hitherto, no explanation.
### Mass Splitting

\[ a_1 - \rho \]

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- Systematic, symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.
- Never better than \( \sim \frac{1}{4} \) of splitting.
- Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed: \( M(p^2) \) is absent!
Mass Splitting

\( \alpha_1 - \rho \)

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New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of \( \text{arXiv:0903.5461 [nucl-th]} \)

Ball-Chiu \textit{Ansatz} for quark-gluon vertex

\[
\Gamma^{BC}_\mu(k, p) = \ldots + (k + p)_\mu \frac{B(k) - B(p)}{k^2 - p^2}
\]

Some effects of DCSB built into vertex

Explains \( \pi - \sigma \) splitting but not this problem

Ball-Chiu augmented by *quark anomalous chromomagnetic moment* term:  
\[
\Gamma_\mu (k, p) = \Gamma^\text{BC}_\mu + \sigma_{\mu\nu} (k - p)_\nu \frac{B(k) - B(p)}{k^2 - p^2}
\]

DCSB is the answer. Subtle interplay between competing effects, which can only now be explicated

Promise of first reliable prediction of light-quark meson spectrum, including the so-called hybrid and exotic states.
Massless fermion can’t possess an anomalous magnetic moment.

Interaction term

\[ \int d^4x \frac{1}{2} g \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x) \]

explicitly breaks chiral symmetry.
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explicitly breaks chiral symmetry

However, DCSB can generate a large anomalous chromomagnetic moment even in chiral limit
– This explains the \( a_1 - \rho \) mass-splitting
Massless fermion can’t possess an anomalous magnetic moment.

Interaction term

\[
\int d^4x \, \frac{1}{2} g \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)
\]

explicitly breaks chiral symmetry.


\[ M(p^2) \Rightarrow \kappa(p^2) \]

Preliminary result for \( \mu \) distributions.
Massless fermion can’t possess an anomalous magnetic moment

Interaction term

$$\int d^4 x \frac{1}{2} g \bar{\psi}(x) \sigma_{\mu \nu} \psi(x) F_{\mu \nu}(x)$$

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$$M(p^2) \Rightarrow \kappa(p^2)$$

Preliminary result for $\mu$ distributions

Cloët & Roberts

Effect on hadron form factors?
Craig Roberts – Impact of DCSB on meson structure and interactions
11th International Workshop on Mesons – Kraków, Poland, 10-15 June 2010
Gap Equation
Frontiers of Nuclear Science: Theoretical Advances

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation

Rapid acquisition of mass is
effect of gluon cloud

\[ \Sigma = \Gamma \]

\[ \gamma \]

\[ S \]

\[ \Gamma \]

\[ 0 \] \[ 0.1 \] \[ 0.2 \] \[ 0.3 \] \[ 0.4 \]

\[ M(p) \text{ [GeV]} \]

\[ p \text{ [GeV]} \]

- \( m = 0 \) (Chiral limit)
- \( m = 30 \text{ MeV} \)
- \( m = 70 \text{ MeV} \)
Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

\[ S(p) = \frac{Z(p^2)}{i \gamma \cdot p + M(p^2)} \]

Lattice QCD confirms DSE prediction of DCSB
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$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$
Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion
Pseudoscalar Bethe-Salpeter amplitude

\[ \Gamma_{j\pi}(k; P) = \tau^{j\pi} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) \right. \\
+ \left. \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right] \]
• Pseudoscalar Bethe-Salpeter amplitude

\[ \Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) + \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right] \]

• Dressed-quark Propagator: 
\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]
Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

\[
\Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ iE_\pi(k; P) + \gamma \cdot PF_\pi(k; P) \right.
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- Dressed-quark Propagator: \( S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \)

- Axial-vector Ward-Takahashi identity

\[ f_\pi E_\pi(k; P = 0) = B(p^2) \]
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\[ F_R(k; 0) + 2 f_\pi F_\pi(k; 0) = A(k^2) \]
\[ G_R(k; 0) + 2 f_\pi G_\pi(k; 0) = 2A'(k^2) \]
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What does this mean for observables?
What does this mean for observables?

\[ q^2 F_\pi(q^2) \ (\text{GeV}^2) \]

- **Including** \( F_\pi \to 1/Q^2 \)
- **Only** \( E_\pi \to 1/Q^4 \)
What does this mean for observables?

\[ \left( \frac{Q}{2} \right)^2 = 2 \text{ GeV}^2 \]
\[ \Rightarrow Q^2 = 8 \text{ GeV}^2 \]

Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor.
GT for pion – Contact Interaction


Craig Roberts – Impact of DCSB on meson structure and interactions
11th International Workshop on Mesons – Kraków, Poland, 10-15 June 2010
Bethe-Salpeter amplitude can’t depend on relative momentum

⇒ General Form

\[ \Gamma_\pi(P) = \gamma_5 [iE_\pi(P) + \frac{1}{M_Q} \gamma \cdot PF_\pi(P)] \]
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Solve chiral-limit gap and Bethe-Salpeter equations

$$P^2 = 0 : M_Q = 0.40, \ E_\pi = 0.98, \ \frac{F_\pi}{M_Q} = 0.50$$
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Origin of pseudovector component: \( E_\pi \) drives \( F_\pi \)

RHS Bethe-Salpeter equation:

\[ \gamma_\mu S(k + P/2) i\gamma_5 E_\pi S(k - P/2) \gamma_\mu \]
Bethe-Salpeter amplitude can’t depend on relative momentum

⇒ General Form

\[ \Gamma_\pi(P) = \gamma_5 \left[ i E_\pi(P) + \frac{1}{M_Q} \gamma \cdot P F_\pi(P) \right] \]

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\[ \gamma_\mu S(k + P/2) i \gamma_5 E_\pi S(k - P/2) \gamma_\mu \]

Has pseudovector component

\[ \sim E_\pi \left[ \sigma S(k_+) \sigma V(k_-) + \sigma S(k_-) \sigma V(k_+) \right] \]
Bethe-Salpeter amplitude can’t depend on relative momentum

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RHS Bethe-Salpeter equation:

\[ \gamma_\mu S(k + P/2)i\gamma_5 E_\pi S(k - P/2)\gamma_\mu \]

Hence \( F_\pi \) on LHS is forced to be nonzero because \( E_\pi \) on RHS is nonzero owing to DCSB
Bethe-Salpeter amplitude: General Form

\[ \Gamma_\pi(P) = \gamma_5 \left[ iE_\pi(P) + \frac{1}{M_Q} \gamma \cdot PF_\pi(P) \right] \]
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Asymptotic form of electromagnetic pion form factor
Bethe-Salpeter amplitude: General Form

\[ \Gamma_\pi(P) = \gamma_5 \left[ iE_\pi(P) + \frac{1}{M_Q} \gamma \cdot P \Gamma_\pi(P) \right] \]

Asymptotic form of electromagnetic pion form factor

\[ E_\pi^2 \text{-term} \Rightarrow F_{\pi E}^{\text{em}}(Q^2) \sim \frac{M^2}{Q^2}, \text{photon}(Q) \]
Bethe-Salpeter amplitude: General Form

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Asymptotic form of electromagnetic pion form factor

- \( E_\pi^2 \)-term \( \Rightarrow F_{\pi E}^{\text{em}}(Q^2) \sim \frac{M^2}{Q^2} \), photon(\( Q \))
- \( E_\pi F_\pi \)-term.
Bethe-Salpeter amplitude: General Form

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- \( E_\pi F_\pi \)-term. Breit Frame:
  \[ \text{pion}(P = (0, 0, -Q/2, iQ/2)) \]
Bethe-Salpeter amplitude: General Form

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  pion\((P = (0, 0, -Q/2, iQ/2))\)
  \[ F_{\pi E F}^{\text{em}}(Q^2) \sim 2 S \gamma \cdot (P + Q) F_\pi S \gamma_4 S E_\pi \]
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\[ \Rightarrow F_{\pi E F}^{\text{em}}(Q^2) \propto \frac{Q^2}{M_Q^2} \frac{F_\pi}{E_\pi} \times E_\pi^2 - \text{term} = \text{constant!} \]
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This behaviour dominates for \( Q^2 \gtrsim M_Q^2 \frac{E_\pi}{F_\pi} > 0.8 \text{ GeV}^2 \)
DSE prediction: $M(p^2)$; i.e., interaction $\frac{1}{|x - y|^2}$

cf. $M(p^2) = \text{Constant}$; i.e., interaction $\delta^4(x - y)$
DSE prediction: \( M(p^2); \) i.e., interaction \( \frac{1}{|x - y|^2} \)

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Single mass-scale parameter in both studies
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Same predictions for $Q^2 = 0$ properties
Computation: Elastic Pion Form Factor

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Single mass-scale parameter in both studies

Same predictions for \( Q^2 = 0 \) properties

Disagreement > 20% for \( Q^2 > M^2 \)
Pion's valence distribution

\[ x u^\pi_v(x) \]

[Graph showing the distribution of pion's valence quarks]
\[ \frac{\alpha(q^2)}{q^2} \gtrsim \frac{M_D^2}{\alpha} \left( \frac{1}{q^2} \right)^{1+\kappa} \]

\[ x u_v^\pi(x) \]

\[ x \]

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$$\frac{\alpha(q^2)}{q^2} \propto \left( \frac{1}{q^2} \right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \propto \left( \frac{1}{k^2} \right)^{1+\kappa}$$
Holt & Roberts: arXiv:1002.4666 [nucl-th]

\[ \frac{\alpha(q^2)}{q^2} \sim M_D^2 \left( \frac{1}{q^2} \right)^{1+\kappa} \]

\[ \Rightarrow B(k^2) \sim M_D^2 \left( \frac{1}{k^2} \right)^{1+\kappa} \]

\[ \Rightarrow q^\pi_v(x; Q_0) \sim (1-x)^2(1+\kappa) \text{ at } Q_0 \gtrsim M_D. \]
Pion's valence distribution

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\frac{\alpha(q^2)}{q^2} \propto q^2 \gtrsim M_D^2 \sim \left(\frac{1}{q^2}\right)^{1+\kappa}
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\(M_D \approx 0.4 \text{ GeV}\)

\(- p^2\)-location of DCSB inflexion point in \(M(p^2)\)
\[ \frac{\alpha(q^2)}{q^2} \propto \frac{q^2 \gtrsim M_D^2}{1} \left( \frac{1}{q^2} \right)^{1+\kappa} \]

\[ \Rightarrow B(k^2) \propto \frac{k^2 \gtrsim M_D^2}{1} \left( \frac{1}{k^2} \right)^{1+\kappa} \]

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\( M_D \approx 0.4 \text{ GeV} \)

\(- p^2\)-location of DCSB inflexion point in \( M(p^2) \)

\( \kappa_{QCD} = 0 \Rightarrow q_\pi^V(x; 1 \text{ GeV}^2) \propto (1 - x)^2 \)

DSE calculation shows this valid for \( \mathcal{L}_x = \{ x | x > 0.86 \} \).
Ratio – Kaon/Pion

$u$-valence distribution

Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]


- $u_K / u_\pi$ at $q = 5$ GeV (Full BSE)
- $u_K / u_\pi$ at $q_0 = 0.57$ GeV (Full BSE)
- $u_K / u_\pi$ at $q_0 = 0.57$ GeV ($E^0, F^0$)
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DSE–result obtained using interaction that predicted $F_\pi(Q^2)$

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Influence of $M(p^2)$ felt strongly for $x > 0.5$

QCD-$M(p^2) \Rightarrow$ prediction:
$u_{\pi,K}(x) \propto (1 - x)^2$

at resolving-scale
$Q_0 = 0.6 \text{ GeV}$
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$u_{\pi,K}(x = 1)$ invariant under DGLAP-evolution
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Accessible at Upgraded JLab & Electron-Ion Collider
Some projects currently underway

Elucidate effects of confinement & DCSB in

- light-quark meson spectrum, including so-called hybrids and exotics, using Poincaré-covariant symmetry-preserving Bethe-Salpeter equation (L. Chang, arXiv:0903.5461 [nucl-th])
- hadron valence-quark distribution functions (A. Bashir, P.C. Tandy)
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Following extensive study of nucleon elastic electromagnetic form factors,

Survey of nucleon electromagnetic form factors

with numerous predictions, either verified by experiment or serving to motivate new experiments, studies are underway to elucidate signals of \( M(p^2) \) in \( Q^2 \)-evolution of nucleon elastic and transition form factors; viz.,

\begin{itemize}
  \item \( N \rightarrow \Delta \)
  \item \( N \rightarrow P11(1440) \)
  \item \( \kappa(p^2) \)
\end{itemize}

\( (M. \text{ Bhagwat, L. Chang, I. Cloët, H. Roberts}) \)

e.g., \( F_1^d(Q^2) = 0 \) at \( Q^2/M^2 \approx 5 \)
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  e.g., $F_{1d}^d(Q^2) = 0$ at $Q^2/M^2 \approx 5$
(M. Bhagwat, L. Chang, I. Cloët, H. Roberts)

Incorporate “resonant contributions” (pion cloud) in kernels of bound-state equations (e.g., Eichmann, Roberts et al. – 0802.1948 [nucl-th] & Cloët, Roberts – 0811.2018 [nucl-th]; and Fischer, Williams – 0808.3372 [hep-ph])
Epilogue
DCSB exists in QCD.
DCSB exists in QCD.

It is manifest in dressed propagators and vertices.
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks: $4 \to 400 \text{ MeV}$
  - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140 \text{ MeV}$
  - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
    \[ g_{\pi \bar{N}N} \approx 12.8 \approx 3g_{\pi \bar{q}q} \]
Epilogue

- DCSB impacts dramatically upon observables
Epilogue

- DCSB impacts dramatically upon observables
- Spectrum; e.g., splittings: $\sigma-\pi \& \alpha_1-\rho$
- Elastic and Transition Form Factors
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- But $M(p^2)$ is an *essentially* quantum field theoretical effect
  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.
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- DSEs provide such a framework.
- Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe
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- Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe
DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks
Now is an exciting time . . .
Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-$Q^2$
- Parton distribution functions
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Positioned to unify phenomena as apparently disparate as

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- Elastic and transition form factors, from small- to large-$Q^2$
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Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, **Dynamical Chiral Symmetry Breaking**
1. Universal Truths
2. QCD’s Challenges
3. Charting the Interaction
4. Radial Excitations & $\chi$-Symmetry
5. Radial Excitations & $\chi$-Symmetry II
6. Radial Excitations & Lattice-QCD
7. Bound-state DSE
8. BSE – General Form
9. $a_1 - \rho$
11. Frontiers of Nuclear Science
12. Goldberger-Treiman for pion
13. GT – Contact Interaction
14. Computation: $F_\pi(Q^2)$
15. Pion’s valence distribution
16. Kaon/Pion $u$-valence distribution
17. Current Projects