Dressed–quark Mass Function

\[ M(p) \]

DCSB & Confinement

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http://www.phy.anl.gov/theory/staff/cdr.html
Universal Truths
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- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron’s characterising properties amongst its QCD constituents.
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- Running of quark mass entails that calculations at even modest $Q^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.
Intranucleon Interaction
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
The question must be rigorously defined, and the answer mapped out using experiment and theory.

98% of the volume
What is the light-quark Long-Range Potential?

NO NEED TO REPENT.
THE END OF THE WORLD IS NOT POSSIBLE
AND WE'RE NOT GOING TO BURN IN HELL.

THE CRANK.
What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD is not related in any simple way to the light-quark interaction.
Dyson-Schwinger Equations

Euler-Lagrange equations for quantum field theory

Well suited to Relativistic Quantum Field Theory
Dyson-Schwinger Equations

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Simplest level: Generating Tool for Perturbation Theory

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Dyson-Schwinger Equations

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- NonPerturbative, Continuum approach to QCD
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- Hadrons as Composites of Quarks and Gluons
  - Qualitative and Quantitative Importance of:
    - Dynamical Chiral Symmetry Breaking
      - Generation of fermion mass from nothing
    - Quark & Gluon Confinement
      - Coloured objects not detected, not detectable?
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- Understanding ⇒ InfraRed behaviour of $\alpha_s(Q^2)$
**Dyson-Schwinger Equations**

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- Understanding ⇒ **InfraRed** behaviour of \( \alpha_s(Q^2) \)
- Method yields Schwinger Functions \( \equiv \) Propagators

**Cross-Sections built from Schwinger Functions**

Craig Roberts – *Empirically charting dynamical chiral symmetry breaking*

Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010
Charting the Interaction between light-quarks
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Confinement can be related to the analytic properties of QCD’s Schwinger functions
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- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD’s universal $\beta$-function.
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Of course, the behaviour of the $\beta$-function on the perturbative domain is well known.

- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.
Through DSEs the pointwise behaviour of the $\beta$-function determines pattern of chiral symmetry breaking
Charting the Interaction between light-quarks

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DSEs connect $\beta$-function to experimental observables. Hence, comparison between computations and observations of, e.g.,
- hadron mass spectrum;
- transition form factors
can be used to chart $\beta$-function’s long-range behaviour.
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Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of $\beta$-function than those of the ground state.
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To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary.
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To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary.

Steady quantitative progress is being made with a scheme that is systematically improvable.
Charting the Interaction between light-quarks

Through DSEs the pointwise behaviour of the $\beta$-function determines pattern of chiral symmetry breaking. DSEs connect $\beta$-function to experimental observables. Hence, comparison between computations and observations can be used to chart $\beta$-function’s long-range behaviour.

To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary. On other hand, at present significant qualitative advances possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions.
Empirically charting dynamical chiral symmetry breaking

Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010
\[
\Sigma = D \Gamma
\]

Gap Equation

Frontiers of Nuclear Science: Theoretical Advances
Gap Equation

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

- Rapid acquisition of mass is effect of gluon cloud
- m = 0 (Chiral limit)
- m = 30 MeV
- m = 70 MeV
Mass from nothing

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{5\mu}(k; P) = S^{-1}(k_+) \left( \frac{1}{2} \lambda^l_i \gamma_5 + \frac{1}{2} \lambda^l_f i \gamma_5 \right) S^{-1}(k_-) \]

\[ -M_\zeta i \Gamma^l_5(k; P) - i \Gamma^l_5(k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
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Satisfies BSE  Satisfies DSE
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- Relation must be preserved by truncation
- Nontrivial constraint
Bethe-Salpeter Kernel

Axial-vector Ward-Takahashi identity

\[ P_\mu \, \Gamma_l^{5\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i\gamma_5 + \frac{1}{2} \lambda_f i\gamma_5 \, S^{-1}(k_-) \]

\[ -M_\zeta \, i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) \, M_\zeta \]

Satisfies BSE

Kernels very different

but must be intimately related

• Relation must be preserved by truncation

• Failure \(\Rightarrow\) Explicit Violation of QCD’s Chiral Symmetry
Bound-state DSE
Bound-state DSE
Bethe-Salpeter Equation

Standard form, familiar from textbooks

\[ \left[ \Gamma_j^\pi(k; P) \right]_{tu} = \int_q^\Lambda \left[ S(q + P/2) \Gamma_j^\pi(q; P) S(q - P/2) \right]_{sr} K_{r^s}^{t^u}(q, k; P) \]

\[ K(q, k; P) : \text{Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel} \]
**Bound-state DSE**

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- Blocked progress for more than 60 years.
Gap Equation
General Form
\[ S_f(p)^{-1} = Z_2 \left( i \gamma \cdot p + m_f^{\text{bm}} \right) + \Sigma_f(p), \]

\[ \Sigma_f(p) = Z_1 \int_q \Lambda \ g^2 D_{\mu \nu} (p - q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_\nu (q, p), \]
\[ S_f(p)^{-1} = Z_2 (i \gamma \cdot p + m_{f}^{bm}) + \Sigma_f(p), \]
\[ \Sigma_f(p) = Z_1 \int_{q}^{\Lambda} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_{\nu}(q, p), \]

- \( Z_1, Z_2 (\zeta^2, \Lambda^2) \) are respectively the vertex and quark wave function renormalisation constants, with \( \zeta \) the renormalisation point.
- \( m^{bm}(\Lambda) \) is the Lagrangian current-quark bare mass.
- \( D_{\mu\nu}(k) \) is the dressed-gluon propagator.
- \( \Gamma^f_{\nu}(q, p) \) is the dressed-quark-gluon vertex.
\[ S_f(p)^{-1} = Z_2 \left( i\gamma \cdot p + m_f^{\text{bm}} \right) + \Sigma_f(p), \]
\[ \Sigma_f(p) = Z_1 \int_q^{\Lambda} g^2 D_{\mu\nu}(p - q) \frac{\lambda^a}{2} \gamma_{\mu} S_f(q) \frac{\lambda^a}{2} \Gamma_f^{\nu}(q, p), \]

- \( Z_{1,2}(\zeta^2, \Lambda^2) \) are respectively the vertex and quark wave function renormalisation constants, with \( \zeta \) the renormalisation point
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- \( \Gamma_f^{\nu}(q, p) \) is the dressed-quark-gluon vertex

Suppose one has in-hand the exact form of \( \Gamma_f^{\nu}(q, p) \)

What is the associated Symmetry-preserving Bethe-Salpeter Kernel?
Bethe-Salpeter Equation
General Form

Bender, Detmold, Roberts, Thomas:
“Bethe-Salpeter equation and a nonperturbative quark gluon vertex;”

\[ \Gamma_{f g}^{5\mu}(k; P) = Z_2 \gamma_5 \gamma_{\mu} \]

\[ - \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q+) \Gamma_{f g}^{5\mu}(q; P) S_g(q-) \frac{\lambda^a}{2} \Gamma_{g\beta}^g(q-, k-) \]

\[ + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P), \]

(Poincaré covariance, hence \( q_\pm = q \pm P/2 \), etc., without loss of generality.)

First exploration of effects arising from complete resummation of a diagrammatic subclass: ladder-gluon planar vertex corrections
In fact, this is $K \rightarrow \Lambda$-equivalent exact form:

$$\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

$$- \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q^+) \Gamma_{5\mu}^{fg}(q; P) S_g(q^-) \frac{\lambda^a}{2} \Gamma_{\beta}^{g}(q^-, k^-)$$

$$+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q^+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)
Bethe-Salpeter Equation

General Form

Equivalent exact form:

\[ \Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu \]

\[ - \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q^+) \Gamma_{5\mu}^{fg}(q; P) S_g(q^-) \frac{\lambda^a}{2} \Gamma_{\beta}^{g}(q^-, k^-) \]

\[ + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q^+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P), \]

(Poincaré covariance, hence \( q_\pm = q \pm P/2 \), etc., without loss of generality.)

In this form ... \( \Lambda_{5\mu\beta}^{fg} \)

is completely defined via the dressed-quark self-energy
E.g., in any reliable study of light-quark hadrons, axial-vector vertex must satisfy Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i_5 + i_5 S_g^{-1}(k_-) \]

\[ - i [m_f(\zeta) + m_g(\zeta)] \Gamma_5^{fg}(k; P), \]

Expresses chiral symmetry & pattern by which it’s broken
E.g., in any reliable study of light-quark hadrons, axial-vector vertex must satisfy Ward-Takahashi identity

\[ P_\mu \Gamma^{fg}_{5\mu}(k; P) = S_f^{-1}(k_+)i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) \]

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Expresses chiral symmetry & pattern by which it’s broken

The condition (\( \Lambda^{fg}_{5\beta} \) pseudoscalar analogue of \( \Lambda^{fg}_{5\mu\beta} \))

\[ P_\mu \Lambda^{fg}_{5\mu\beta}(k, q; P) = \Gamma^{f}_{\beta}(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma^{g}_{\beta}(q_-, k_-) \]

\[ - i \left[ m_f(\zeta) + m_g(\zeta) \right] \Lambda^{fg}_{5\beta}(k, q; P), \]

a new Ward-Takahashi identity, is Necessary & Sufficient to ensure \( \Gamma^{fg}_{5\mu}(k; P) \) Ward-Takahashi identity satisfied.
The condition \((\Lambda_{5\mu\beta}^{fg} \text{ pseudoscalar analogue of } \Lambda_{5\mu\beta}^{fg})\)

\[
P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_{\beta\mu}^{f}(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_{\beta\mu}^{g}(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\mu\beta}^{fg}(k, q; P),
\]

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Rainbow-ladder...

\[
\Gamma_{\beta\mu}^{f}(q, k) = \gamma_\mu
\]

\[
\Rightarrow \Lambda_{5\mu\beta}^{fg}(k, q; P) = 0 = \Lambda_{5\beta}^{fg}(k, q; P)
\]
Bethe-Salpeter equation introduced in 1951
Bethe-Salpeter Kernel
60 year problem

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity

\[ P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_{\beta}^{f}(q_+, k_+) i \gamma_5 + i \gamma_5 \Gamma_{\beta}^{g}(q_-, k_-) \]

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For first time: can construct Ansatz for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex

Consistent means - all symmetries preserved!
Bethe-Salpeter equation introduced in 1951

Newly-derived Ward-Takahashi identity

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For first time: can construct Ansatz for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex

Exemplified the procedure and results to expect . . .
Numerical Illustration

L. Chang and C. D. Roberts

π cf. σ

\[ M_\pi \] vs. \[ m \] for 0−+

\[ M_\sigma \] vs. \[ m \] for 0++

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Numerical Illustration

\[ \pi \text{ cf. } \sigma \]

L. Chang and C. D. Roberts

**Vertex:**

- leading-order rainbow-ladder truncation
- cf. Ball-Chiu–consistent Ansatz – Essentially nonperturbative content;
  Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown

Same interaction. One mass-scale in both truncations: \( 1/\omega = 0.4 \text{ fm} \), defining border between IR & UV.
Numerical Illustration

L. Chang and C. D. Roberts

\[ \pi \text{ cf. } \sigma \]

\[ \begin{align*}
0^- + & \quad 0^{++} \\
\begin{array}{c}
\text{Vertex:} \\
\text{leading-order rainbow-ladder truncation} \\
\text{cf. Ball-Chiu–consistent Ansatz} & \quad \text{Essentially nonperturbative content; Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown}
\end{array}
\end{align*} \]

Same interaction. One mass-scale in both truncations: \( 1/\omega = 0.4 \text{ fm} \), defining border between IR & UV.

\[ \begin{align*}
\text{GMOR . . . plainly satisfied by both truncations} \\
\text{A little attraction introduced in pseudoscalar channel} \\
\text{Enormous repulsion introduced in scalar channel}
\end{align*} \]
Rainbow-ladder DSE truncation, $\varepsilon_{RL}^{\sigma} := \frac{2M(0) - m_\sigma}{2M(0)} = (0.3 \pm 0.1)$.

BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_{BC}^{\sigma} \lesssim 0.1$. 

Spin-orbit Interaction
Rainbow-ladder DSE truncation, \( \varepsilon_{RL}^\sigma := \frac{2M(0) - m_\sigma}{2M(0)} \) \( RL \) = (0.3 ± 0.1).

BC-consistent Bethe-Salpeter kernel; viz., \( \varepsilon_{\sigma}^{BC} \lesssim 0.1 \).

Scalar mesons = \( ^3P_0 \) states: Constituents’ spins aligned and one unit of constituent orbital angular momentum.

From this viewpoint, scalar is a spin and orbital excitation of a pseudoscalar meson.
Spin-orbit Interaction

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Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting in the absence of orbital angular momentum

Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit interaction.

Effect owes to influence of quark’s dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

Impossible to demonstrate effect without our new procedure
Rainbow-ladder DSE truncation, \( \varepsilon^{\text{RL}}_\sigma := \frac{2M(0) - m\sigma}{2M(0)} \) \( \text{RL} \) = (0.3 ± 0.1).

BC-consistent Bethe-Salpeter kernel; viz., \( \varepsilon^{\text{BC}}_\sigma \lesssim 0.1 \).

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Effect owes to influence of quark’s dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

Impossible to demonstrate effect without our new procedure

Expect this feature to have material impact

Especially on mesons with mass greater than 1 GeV.

\textit{prima facie} . . . can overcome longstanding shortcoming of systematic, symmetry-preserving truncations; viz., splitting between vector & axial-vector mesons is too small
Spin-orbit Interaction

Rainbow-ladder DSE truncation, \( \varepsilon_{\sigma}^{RL} := \frac{2M(0) - m_{\sigma}}{2M(0)} \) \( \varepsilon_{RL}^{BC} = (0.3 \pm 0.1) \).

BC-consistent Bethe-Salpeter kernel; viz., \( \varepsilon_{\sigma}^{BC} \lesssim 0.1 \).

Scalar mesons = \( ^3P_0 \) states: Constituents’ spins aligned and one unit of constituent orbital angular momentum

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Promise of realistic meson spectroscopy . . . First time, also for mass > 1 GeV.
That was where things stood in March/09
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Now, we’ve solved inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum

\[ \Gamma_{qq}(k = 0, P^2) \]

Exhibits a zero at ground-state mass-squared

Padé approximant extrapolation to locate zero
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Padé approximant extrapolation to locate zero

Almost precisely method used for ground-state masses in lattice-QCD

Intelligent use gives dependable results

“Schwinger functions and light-quark bound states”

Rainbow-Ladder

$$\Gamma_{\mu}(q, k) = \gamma_{\mu}$$
\( m_{a_1} (759 \text{ MeV}) - m_\rho (644 \text{ MeV}) = 115 \text{ MeV} \) \( \) \( \Gamma_{qq}(k=0, P^2) = \frac{1}{P^2} \)
Chang Lei & CDR, in-preparation

\[ m_{a_1} \ (1066 \text{ MeV}) - m_{\rho} \ (924 \text{ MeV}) = 142 \text{ MeV} \ldots \text{expt.} = 455 \text{ MeV} \]

**Ball-Chiu**

\[ \Gamma_\mu(q, k) = \frac{1}{\Gamma_{qq}(k=0, P^2)} \]

\[
\begin{aligned}
&\Gamma_\mu(q, k) = \frac{1}{2 \gamma_\mu} \\
&\quad \left[ i \gamma \cdot k \left( A(q^2) - A(k^2) \right) \right. \\
&\quad \left. + 2k_\mu \left( B(q^2) - B(k^2) \right) \right]
\end{aligned}
\]

DCSB enhanced spin-orbit interaction
\[ m_{a_1} (1066 \text{ MeV}) - m_\rho (924 \text{ MeV}) = 142 \text{ MeV} \ldots \text{expt.} = 455 \text{ MeV} \]

Ball-Chiu

\[
\Gamma_\mu(q, k) = i \left[ \frac{A(q^2) + A(k^2)}{2\gamma_\mu} \right] + 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right]
\]

DCSB enhanced spin-orbit interaction

What’s missing?
\[ m_{a_1} - m_{\rho} \]
Chang Lei & CDR, in-preparation

$m_{a_1} (1230 \text{ MeV}) = m_\rho (745 \text{ MeV})$

\[m_{a_1} - m_\rho = 485 \text{ MeV} \ldots \text{ expt.} = 455 \text{ MeV}\]

**Ball-Chiu**

\[
\Gamma_\mu(q, k) = i[A(q^2) + A(k^2)]/2 \gamma_\mu \\
+ 2k_\mu \left[ i \gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} \\
+ \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right]
\]

DCSB enhanced spin-orbit interaction

Craig Roberts – *Empirically charting dynamical chiral symmetry breaking*
Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010 ... 29 – p. 21/31
$m_{a_1} (1230 \text{ MeV}) - m_\rho (745 \text{ MeV})$
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Ball-Chiu + an. mag. mom.

$$\Gamma_\mu(q, k) = i\left[ A(q^2) + A(k^2) \right] / 2 \gamma_\mu$$
$$+ 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} \right]$$
$$\left[ + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \sigma_{\mu\nu}(q - k)_{\nu} \right]$$

DCSB enhanced anomalous chromomagnetic moment
$m_{a_1} (1230 \text{ MeV}) - m_\rho (745 \text{ MeV}) = 485 \text{ MeV} \ldots \text{ expt.} = 455 \text{ MeV}$

**Ball-Chiu + an. mag. mom.**

$$\Gamma_\mu(q, k) = \frac{i[A(q^2) + A(k^2)]}{2\gamma_\mu} + 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right] + \sigma_{\mu\nu}(q - k)_{\nu} \frac{B(q^2) - B(k^2)}{q^2 - k^2}$$

Inextricably connected with DCSB.

DCSB enhanced anomalous chromomagnetic moment

Can’t appear in chirally symmetric theory

Paves way for truly reliable light-quark meson spectrum
Ratio – Kaon/Pion
u-valence distribution
Ratio – Kaon/Pion

$u$-valence distribution

\[
u_K(x)/\nu_\pi(x)
\]

Craig Roberts – Empirically charting dynamical chiral symmetry breaking

Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010
Hard-cutoff NJL model (constant mass)
cf. QCD-DSE-based result \([M(p^2) + \Gamma_\pi(p; P)]\)

Influence of Mass-function felt strongly for \(x > 0.5\)

Accessible at Upgraded JLab & Electron-Ion Collider
How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.
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In quantum field theory a nucleon appears as a pole in a six-point quark Green function.

Residue is proportional to nucleon’s Faddeev amplitude.

Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks.
How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.

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Residue is proportional to nucleon’s Faddeev amplitude

Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks

Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel.
Faddeev equation

Faddeev equation

\[ \Psi^a \quad P \quad \Psi^b \]

\[ p_d \rightarrow \Psi^a \quad p_q \rightarrow P \quad p_d \rightarrow \Psi^b \]

Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010
Faddeev equation

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . s—, p— & d—wave correlations
Nucleon-Photon Vertex
Nucleon-Photon Vertex

6 terms . . . constructed systematically . . . current conserved automatically for on-shell nucleons described by Faddeev Amplitude
Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude

\[ \Psi_f \rightarrow \Psi_i \]

\[ P_f \rightarrow P_i \]

\[ \Gamma \rightarrow \Gamma \]

\[ Q \rightarrow Q \]

\[ \mu \rightarrow \mu \]

Craig Roberts – Empirically charting dynamical chiral symmetry breaking
Achievements and New Directions in Subatomic Physics, 15-19 Feb 2010
Cloët, Roberts et al.
Cloët, Roberts et al.
- arXiv:0710.2059 [nucl-th]
- arXiv:0710.5746 [nucl-th]
- arXiv:0804.3118 [nucl-th]

DSE-Faddeev Equation prediction

\[ \frac{\mu_n G_E(Q^2)}{G_M(Q^2)} \]

Bogdan Wojtsekhowski
Cloët, Roberts et al.
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DSE-Faddeev Equation prediction

Red long-dashed curve


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\[ \mu_n G_E(Q^2) \]
\[ G_M(Q^2) \]

DSE-Faddeev Equation prediction

Red long-dashed curve

This evolution very sensitive to momentum-dependence dressed-quark propagator


Bogdan Wojtsekhowski
Epilogue
DCSB exists in QCD.
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- It is manifest in dressed propagators and vertices.
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks: $4 \rightarrow 400 \text{ MeV}$
  - pseudoscalar mesons are unnaturally light: $m_\rho = 770 \text{ cf. } m_\pi = 140 \text{ MeV}$
  - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
    $$g_{\pi \bar{N}N} \approx 12.8 \approx 3g_{\pi \bar{q}q}$$
Epilogue

- DCSB impacts dramatically upon observables
Epilogue

- DCSB impacts dramatically upon observables
- Spectrum; e.g., splittings: $\sigma - \pi$ & $a_1 - \rho$
- Elastic and Transition Form Factors
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- Elastic and Transition Form Factors
- But $M(p^2)$ is an *essentially* quantum field theoretical effect
- Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.
Epilogue

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- DSEs provide such a framework.
- Studies underway will identify observable signals of $M(p^2)$,
  the most important mass-generating mechanism for visible
  matter in the Universe
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DSEs provide such a framework.
- Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe
- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks
Now is an exciting time . . .
Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-\(Q^2\)
- Parton distribution functions
Now is an exciting time . . .

 Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-$Q^2$
- Parton distribution functions

Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, Dynamical Chiral Symmetry Breaking
Just the Basic Facts
Tony graduated from Flinders University in 1973.
Tony graduated from Flinders University in 1973. Whereafter he immediately left for British Columbia, in search of adventure.
And adventures he had ...
And adventures he had...
Just the Basic Facts

- And adventures he had . . .

- In time, even I learnt to use it.
Just the Basic Facts

- But now Tony has returned to Australia and to the University of Adelaide.
But now Tony has returned to Australia and to the University of Adelaide.

Part of the answer lies in the lead-off for the Faculty of Science at the University of Adelaide . . .
But now Tony has returned to Australia and to the University of Adelaide.

Part of the answer lies in the lead-off for the Faculty of Science at the University of Adelaide . . .

“Eating, loving, singing and digesting are, in truth, the four acts of the comic opera known as life, and they pass like bubbles of a bottle of champagne. Whoever lets them break without having enjoyed them is a complete fool.” Gioacchino Rossini
Tony:

Thankyou.

And a Belated Happy Birthday.
1. Universal Truths
2. QCD’s Challenges
3. Dichotomy of Pion
4. Confinement
5. Dyson-Schwinger Equations
6. Schwinger Functions
7. Charting Light-quark Interaction
8. Frontiers of Nuclear Science
9. Hadrons
10. Bethe-Salpeter Kernel
11. Persistent Challenge
12. Bound-state DSE
13. BSE General Form
14. $[m_{\alpha_1} - m_\rho]$
15. Unifying Meson & Nucleon
16. Faddeev equation
17. Diquark correlations
18. Nucleon-Photon Vertex
19. $\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$
20. $\frac{G^n_M(Q^2)}{\mu_n G_D(Q^2)}$
21. Goldberger-Treiman
22. Pion Form Factor
23. DSE-based Faddeev Equation
24. Pion cloud - TSH
25. Pion Cloud