Covariance, Dynamics and Symmetries, and Hadron Physics

Craig D. Roberts

cdroberts@anl.gov

Physics Division, Argonne National Laboratory
Universal Truths

- Spectrum of excited states and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron’s characterising properties amongst its QCD constituents.
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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
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Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.

Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.
Dichotomy of Pion
– Goldstone Mode and Bound state
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How does one make an almost massless particle from two massive constituent-quarks?
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Must exhibit $m^2_\pi \propto m_q$

Current Algebra … 1968
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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
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**Highly Nontrivial**
Minimal requirements

- detailed understanding of connection between Current-quark and Constituent-quark masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.
What’s the Problem?

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  – Can’t be done using perturbation theory
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Differences!
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Relativistic QFT!

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- Differences!
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  - Interaction between quarks – the Interquark “Potential” – unknown throughout > 98% of a hadron’s volume
Intranucleon Interaction
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.

98% of the volume
Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
  - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=−$
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  Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour arises from apparently simple rules
Dyson-Schwinger Equations
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Well suited to Relativistic Quantum Field Theory
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- Simplest level: Generating Tool for Perturbation Theory
  Materially Reduces Model Dependence
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- Qualitative and Quantitative Importance of:
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    - Generation of fermion mass from nothing
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    - Coloured objects not detected, not detectable?
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⇒ Understanding InfraRed (long-range)

\[ \text{behaviour of } \alpha_s(Q^2) \]
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- Method yields Schwinger Functions ≡ Propagators
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Cross-Sections built from Schwinger Functions
**Perturbative**

*Dressed-quark Propagator*
Perturbative Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation
Perturbative Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma \]

\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]
Perturbative Dressed-quark Propagator

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- dressed-quark propagator

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\[ \Sigma = \quad \]

Weak Coupling Expansion

Reproduces Every Diagram in Perturbation Theory
Perturbative

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\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \rightarrow 0 \]
Perturbative

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\[ \begin{align*}
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B(p^2) &= m \left(1 - \frac{\alpha}{\pi} \ln \left[\frac{p^2}{m^2}\right] + \ldots\right) \\
m &\to 0^+ \quad \text{No DCSB Here!}
\end{align*} \]

Weak Coupling Expansion
Reproduces Every Diagram in Perturbation Theory

But in Perturbation Theory

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UW Madison NPAC Seminar: 12 February 2009...
Kernel of Gap Equation: \( D_{\mu \nu} (p - q) \Gamma_\nu(q) \)

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

Kernel of Gap Equation: $D_{\mu \nu}(p - q) \Gamma_\nu(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:


Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:


Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
\[ D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

- Suppression means \( \exists \) IR gluon mass-scale \( \approx 1 \text{ GeV} \)

- Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)
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[Graph showing data points and curves for lattice, DSE, and fit to DSE]
Dyson-Schwinger Equations
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Dressed-Quark Propagator
Dyson-Schwinger Equations
Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma = \frac{D \gamma \Gamma}{\gamma S \Gamma} \]

Gap Equation
Dyson-Schwinger Equations

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- Gap Equation’s Kernel Enhanced on IR domain

\[ \implies \text{IR Enhancement of } M(p^2) \]
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  \[ \Rightarrow \] IR Enhancement of \( M(p^2) \)

Euclidean Constituent–Quark Mass: \( M^E_f : p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
<th>( u/d )</th>
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<th>( c )</th>
<th>( b )</th>
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<tbody>
<tr>
<td>( M^E ) / ( m_\xi )</td>
<td>( \sim 10^2 )</td>
<td>( \sim 10 )</td>
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UW Madison NPAC Seminar: 12 February 2009
Dyson-Schwinger Equations

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Predictions confirmed in numerical simulations of **lattice-QCD**
\[ \Sigma = D \gamma \Gamma \]

**Gap Equation**
Frontiers of Nuclear Science: Theoretical Advances

\[ \Sigma = \bar{D} \Gamma \]

Gap Equation

\begin{figure}
\centering
\includegraphics[width=\textwidth]{gap_equation}
\caption{Rapid acquisition of mass is the effect of the gluon cloud.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{mass_acquisition}
\caption{Mass acquisition curves for different values of \( m \): \( m = 0 \) (Chiral limit), \( m = 30 \) MeV, and \( m = 70 \) MeV.}
\end{figure}
Mass from nothing.

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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Always look on the bright side.
Established understanding of two- and three-point functions
Hadrons

- Established understanding of two- and three-point functions
- What about bound states?
• Without bound states, Comparison with experiment is **impossible**
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• They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

\[ i\Gamma = i\Gamma \quad K \]

QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$? 
• or What is the long-range potential in QCD?
Confinement

Infinitely Heavy Quarks . . . Picture in Quantum Mechanics

\[ V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r} \]

\( \sigma \sim 470 \text{ MeV} \)

Necco & Sommer
he-la/0108008
Illustrate this in terms of the action density . . . analogous to plotting the Force $F_{QQ}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$

Bali, et al.
he-la/0512018
Confinement

What happens in the real world; namely, in the presence of light-quarks?
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What happens in the real world; namely, in the presence of light-quarks? No one knows ... but $\bar{Q}Q + 2 \times \bar{q}q$

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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

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What happens in the real world; namely, in the presence of light-quarks? No one knows ... but $\bar{Q}Q + 2 \times \bar{q}q$

"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Therefore ... No information on potential between light-quarks.
What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.
Bethe-Salpeter Kernel
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i\gamma_5 + \frac{1}{2} \lambda_f i\gamma_5 \ S^{-1}(k_-) \]

\[ -M_\zeta i\Gamma_5^l (k; P) - i\Gamma_5^l (k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
Axial-vector Ward-Takahashi identity

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Kernels very different but must be intimately related
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Kernels very different

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• Relation must be preserved by truncation

Satisfies DSE
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{\bar{5} \mu}(k; P) = S^{-1}(k_+ \bar{S} + k) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 \bar{S}^{-1}(k_-) \]

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- Nontrivial constraint

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Satisfies BSE

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Kernels very different but must be intimately related

- Relation must be preserved by truncation
- Failure \(\Rightarrow\) Explicit Violation of QCD’s Chiral Symmetry
Persistent Challenge
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Infinitely Many Coupled Equations

\[ \Sigma = \Gamma \]

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Persistent Challenge

- Infinitely Many Coupled Equations

- Coupling between equations necessitates truncation
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  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
Persistent Challenge

- Infinitely Many Coupled Equations
- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    - Not useful for the nonperturbative problems in which we’re interested
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme


_Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations_


_Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation_
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- Has Enabled Proof of EXACT Results in QCD
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  - Make Predictions with Readily Quantifiable Errors
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \ \rho^H_\zeta \ \mathcal{M}_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho^H \ \mathcal{M}_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \rho_H^\zeta \quad M_H \]

\[ M_H := \text{tr}_{\text{flavour}} \left[ M_\mu \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} \left( \lambda^4 + i\lambda^5 \right) \)
Radial Excitations & Chiral Symmetry

\[ f_H m_H^2 = - \rho^H \mathcal{M}_H \]

\[ f_H p_\mu = Z_2 \int_\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu S(q+) \Gamma_H(q; P) S(q-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
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\[ f_H \ m_H^2 = - \rho^H_{\zeta} \ M_H \]

\[ i\rho^H_{\zeta} = Z_4 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 S(q_+ \Gamma_H(q; P) S(q_-) \right\} \]

- Pseudoscalar projection of BS wave function at \( x = 0 \)

\[ \pi \ 
\]

\[ -\rho_\pi \]

\[ P_5 \ 
\]

\[ i \Gamma_5 \]

\[ i(\tau/2) \gamma_5 \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

\[ f_H^2 m_H^2 = - \rho_\zeta^H M_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)

- \( f_H \rightarrow f_H^0 \) & \( \rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0} \), Independent of \( m_q \)

Hence

\[ m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \]

...GMOR relation, a corollary
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = -\rho^H_\zeta \quad \mathcal{M}_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)
  
  \[ f_H \rightarrow f_H^0 \quad \text{and} \quad \rho^H_\zeta \rightarrow -\frac{\langle \bar{q}q \rangle^0_\zeta}{f_H^0}, \quad \text{Independent of} \quad m_q \]

  Hence \[ m_H^2 = -\frac{\langle \bar{q}q \rangle^0_\zeta}{(f_H^0)^2} \quad m_q \] \quad \ldots \text{GMOR relation, a corollary}

- Heavy-quark + light-quark
  
  \[ f_H \propto \frac{1}{\sqrt{m_H}} \quad \text{and} \quad \rho^H_\zeta \propto \sqrt{m_H} \]

  Hence, \[ m_H \propto m_q \]

\ldots \text{QCD Proof of Potential Model result}
\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ \mathcal{M}_H \]

- Valid for **ALL** Pseudoscalar mesons
\[ f_H \quad m_H^2 = - \quad \rho_H^H \quad M_H \]

- Valid for **ALL** Pseudoscalar mesons
- \( \rho_H \rightarrow \) finite, nonzero value in chiral limit, \( M_H \rightarrow 0 \)
Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts

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\( \Rightarrow f_H = 0 \)

ALL pseudoscalar mesons except \( \pi(140) \) in chiral limit
Radial Excitations & Chiral Symmetry

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- \( \Rightarrow f_H = 0 \)
- **ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit

Dynamical Chiral Symmetry Breaking

- Goldstone’s Theorem – impacts upon **every** pseudoscalar meson
When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
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CLEO: $\tau \rightarrow \pi(1300) + \nu_\tau$
$\Rightarrow f_{\pi_1} < 8.4$ MeV

Diehl & Hiller
he-ph/0105194
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Lattice-QCD check:

$16^3 \times 32,$

$a \sim 0.1 \text{ fm},$

two-flavour, unquenched

$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$
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Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)
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The suppression of \( f_{\pi_1} \) is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
Procedure Now Straightforward
Solve Gap Equation
⇒ Dressed-Quark Propagator, $S(p)$
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$
- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, $\Gamma_\mu$
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

\[ \Gamma_\pi(k; P) \]

\[ \Gamma_\mu(k; P) \]

\[ S(p) \]

Evaluate this final, three-dimensional integral
Ab initio calculation into timelike region. Deeper than ground-state $\rho$-meson pole.
Ab initio calculation into timelike region. Deeper than ground-state $\rho$-meson pole
Ab initio calculation into timelike region. Deeper than ground-state $\rho$-meson pole. **Timelike Pion Form Factor**

$\rho$-meson not put in “by hand” – generated dynamically as a bound-state of dressed-quark and dressed-antiquark.
Answer for the pion
Answer for the pion

Two $\rightarrow$ Infinitely many . . .
Two $\rightarrow$ Infinitely many ... Handle that properly in quantum field theory
Two $\rightarrow$ Infinitely many . . .

Handle that properly in quantum field theory.

. . .
momentum-dependent dressing
Answer for the pion

Two $\rightarrow$ Infinitely many . . .
Handle that properly in quantum field theory . . .
momentum-dependent dressing . . .
perceived distribution of mass depends on the resolving scale
Are we there yet?
New Challenges
Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
New Challenges

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Move on to the problem of a symmetry preserving treatment of hybrids and exotics.
● Another Direction ... Also want/need information about three-quark systems
New Challenges

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- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ∼ 1995.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ∼ 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

- However, that is beginning to change . . .
Nucleon . . .

Three-body Problem?
What is the picture in quantum field theory?
Nucleon . . .
Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!
Unifying Study of Mesons and Baryons
How does one incorporate dressed-quark mass function, \( M(p^2) \), in study of baryons?
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Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks.

Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel.
Faddeev equation
Faddeev equation

\[ \Psi^a_{p_d} \rightarrow \Gamma^a \rightarrow \Psi^b_{p_d} = \Psi^a_{p_q} \rightarrow \Gamma^b \rightarrow \Psi^b_{p_q} \]
Faddeev equation

\[
p_q \quad \Psi^a \quad P \quad \Psi^b \quad P_q \quad \Gamma^a \quad q \quad \Gamma^b \quad p_d
\]

- Linear, Homogeneous Matrix equation
- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon’s Rest Frame Amplitude has ... s—, p— & d—wave correlations
Diquark correlations
Diquark correlations

Same interaction that describes mesons also generates three coloured quark-quark correlations:
- blue–red,
- blue–green,
- green–red

Confined ... Does not escape from within baryon.

Scalar is isosinglet,
Axial-vector is isotriplet

DSE and lattice-QCD

\[ m_{[ud]}^{0+} = 0.74 - 0.82 \]
\[ m_{(uu)}^{1+} = m_{(ud)}^{1+} = m_{(dd)}^{1+} = 0.95 - 1.02 \]
Ab-initio study of mesons & nucleons
Ab-initio study of mesons & nucleons

Eichmann et al.
Eichmann et al.

Leading-order truncation of DSEs – rainbow-ladder
Eichmann et al.

- Leading-order truncation of DSEs – rainbow-ladder
- Corrections vanish with increasing current-quark mass
  $\Rightarrow$ rainbow-ladder exact in heavy-quark limit

Ab-initio study
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- However, at physical light-quark mass, corrections to observables not protected by symmetries: uniformly \( \approx 35\% \)
  - Roughly 50/50-split between nonresonant and resonant (pseudoscalar meson loop) contributions
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- arXiv:0810.1222 [nucl-th]

**Ab-initio study of mesons & nucleons**

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- Symmetry preserving and systematic approach can elucidate and account for these effects
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- Symmetry preserving and systematic approach can elucidate and account for these effects
  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs
Eichmann et al.

Ab-initio study
of mesons & nucleons

\[ m_\rho \text{ [GeV]} \]

\[ m_\pi^2 \text{ [GeV}^2\]}

- ETMC
- RBC/UKQCD
- MILC
- CP-PACS + Adelaide
- Experiment
Ab-initio study of mesons & nucleons

Eichmann et al.

Rainbow-Ladder DSE result

one parameter for IR – “confinement radius”

Results insensitive to value on material domain
Ab-initio study of mesons & nucleons

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Numerical simulations of lattice-QCD
Ab-initio study of mesons & nucleons

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Rainbow-Ladder DSE result
one parameter for IR – “confinement radius”
Results insensitive to value on material domain

Numerical simulations of lattice-QCD

FRR extrapolation of lattice CP-PACS result
Eichmann et al.

Precisely the same interaction
Eichmann et al.

- Precisely the same interaction
- Same \( \rho \)-meson curve
Eichmann et al.

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- Same $\rho$-meson curve
- $m_\pi^2$-dependence of $0^+$ and $1^+$ diquark masses
- “unobservable” – show marked sensitivity to single model parameter; viz., confinement radius
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- Precisely the same interaction
- Same $\rho$-meson curve
- $m^2_{\pi}$-dependence of $0^+$ and $1^+$ diquark masses
- “unobservable” – show marked sensitivity to single model parameter; viz., confinement radius

But . . . $[m_{av} - m_{sc}]$, $m_{\rho}$ & $M_N$ . . . are independent of that parameter
Ab-initio study of mesons & nucleons

Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode

Eichmann et al.
Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode
- DSE and lattice agree on heavy-quark domain

Eichmann et al.
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Ab-initio study of mesons & nucleons

Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode

DSE and lattice agree on heavy-quark domain

Prediction: at physical $m^2_\pi$, $M^\text{quark-core}_N = 1.26(2)$ GeV

cf. FRR+lattice-QCD, $M^\text{quark-core}_N = 1.27(2)$ GeV

$\Rightarrow$ subleading corrections, including $0^-$-meson loops,

$\delta M_N = -320$ MeV,

$\delta m_\rho = -220$ MeV
Eichmann et al.

Ab-initio study of mesons & nucleons

Bethe-Salpeter & Faddeev equations built from same RG-improved rainbow-ladder interaction
Eichmann et al.

Bethe-Salpeter & Faddeev equations built from same
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Simultaneous calculation of
baryon & meson properties,
& prediction of their correlation
Ab-initio study
of mesons & nucleons

- Eichmann et al.
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Systematically improvable
Nucleon-Photon Vertex
Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude
Nucleon-Photon Vertex

constructed systematically ... current conserved automatically for on-shell nucleons described by Faddeev Amplitude

\[
\begin{align*}
\Psi_f & \rightarrow \Gamma & \Psi_i \\
\Psi_i & \rightarrow \Gamma & \Psi_f
\end{align*}
\]

axial vector \quad scalar

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DSE-based Faddeev Equation

Cloët et al.
- arXiv:0710.2059 [nucl-th]
- arXiv:0710.5746 [nucl-th]
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Faddeev equation input –
algebraic parametrisations of
DSE results, constrained by $\pi$
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- Two parameters
  - $M_{0^+} = 0.8$ GeV,
  - $M_{1^+} = 0.9$ GeV

- chosen to give
  - $M_N = 1.18$, $M_\Delta = 1.33$

- allow for pseudoscalar meson contributions
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- Sensitivity to details of the current – expressed through diquark radius

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UW Madison NPAC Seminar: 12 February 2009... – p. 37/43
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**Sensitivity to details of the current**
- Expressed through diquark radius

**On $Q^2 \lesssim 4 \text{ GeV}^2$**
- Result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions

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Always a zero but position depends on details of current
ab initio

Faddeev Equation
Eichmann et al.
Eichmann et al.
- arXiv:0810.1222 [nucl-th]

Parameter-free rainbow-ladder Faddeev equation – result qualitatively identical and in semiquantitative agreement

\[ \mu_p \frac{G_E^p}{G_M^p} \]

\[ Q^2[GeV^2] \]
Eichmann *et al.*

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Improved numerical algorithm needed to extend calculation to larger $Q^2$
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Improved numerical algorithm needed to extend calculation to larger $Q^2$

Calculation unifies $\pi$, $\rho$ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD’s Goldstone mode
Ratio of Neutron Pauli & Dirac Form Factors

\[ \frac{\hat{Q}^2}{(\ln \hat{Q}^2 / \hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)} \]

\[ \hat{\Lambda} = \Lambda / M_N = 0.44 \]

Ensures proton ratio constant for \( \hat{Q}^2 \geq 4 \)
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Brown band

– \textit{ab initio} RL result
Pion Cloud

F2 – neutron
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core

Pseudoscalar contribution
20% of peak value
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core

Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4 M_N^2$
Epilogue
Epilogue
DCSB exists in QCD.
Epilogue

- DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices

- It predicts, amongst other things, that light current-quarks become heavy constituent-quarks: $4 \rightarrow 400$ MeV

- Pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV

- Pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q}q} \approx 4.3$

- Pseudoscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi \bar{N}N} \approx 12.8 \approx 3g_{\pi \bar{q}q}$
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- Pseudoscalar mesons couple unnaturally strongly to the lightest baryons
  \[ g_\pi \bar{N}N \approx 12.8 \approx 3g_\pi \bar{q}q \]
- It impacts dramatically upon observables.
Epilogue

- Dyson-Schwinger Equations
  - Poincaré covariant unification of meson and baryon observables
Epilogue

Dyson-Schwinger Equations

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- All global and pointwise corollaries of DCSB are manifested naturally without fine-tuning
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  - Mesons already being studied
  - Baryons are within practical reach
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Ab-initio study of $N \rightarrow \Delta$ transition underway

Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks
1. Universal Truths
2. Dichotomy of the Pion
3. QCD’s Challenges
4. Dyson-Schwinger Equations
5. Perturbative Propagator
6. QCD & Interaction Between Light-Quarks
7. Dressed-gluon Propagator
8. Dressed-Quark Propagator
9. Frontiers of Nuclear Science
10. Hadrons
11. Confinement
12. Bethe-Salpeter Kernel
13. Persistent Challenge
14. Radial Excitations
15. Radial Excitations & Lattice-QCD
16. Pion FF
17. Timelike Pion Form Factor
18. Nucleon Challenge
19. Unifying Meson & Nucleon
20. Faddeev equation
21. Diquark correlations
22. Ab-initio study of mesons & nucleons
23. Nucleon-Photon Vertex
24. DSE-based Faddeev Equation
25. Ratio of Neutron FFs
26. Pion Cloud