Hadron Physics and Continuum Strong QCD

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http://www.phy.anl.gov/theory/staff/cdr.html
Form Factors: Why?
The nucleon and pion hold special places in non-perturbative studies of QCD.
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An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.
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- Despite this, many urgent questions remain unanswered.
Thomas Jefferson National Accelerator Facility
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- Electrons accelerated by repeated journeys along linacs
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- Current Peak Electron Beam Energy
  Nearly 6 GeV
JLab Hall-A
Measured Ratio of Proton’s Electric and Magnetic Form Factors


If JLab Correct, then

Completely Unexpected Result:

In the Proton
– On Relativistic Domain
– Distribution of Quark-Charge Not Equal

Distribution of Quark-Current!

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XII Mexican Workshop on Particles and Fields: Mini-courses, 4-8 Nov. 2009... 48 – p. 4/48
Some Questions

What is the role of pion cloud in nucleon electromagnetic structure?

Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?
Some Questions

Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?
Some Questions

- Do we understand the high $Q^2$ behavior of the proton form factor ratio in the space-like region?
- Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?
Some Questions

- Can we understand the rich structure of the time-like proton form factors in terms of resonances?
- What do we expect for the proton form factor ratio in the time-like region?
- What is the relation between proton and neutron form factor in the time-like region?
- How do we understand the ratio between time-like and space-like form factors?
Some Questions

- What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?

- What is the impact of these contributions on other form factor measurements?
Some Questions

How accurately can the pion form factor be extracted from the $e p \rightarrow e' n \pi^+$ reaction?
Current status is described in

- J. Arrington, C. D. Roberts and J. M. Zanotti
  “Nucleon electromagnetic form factors,”

- C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
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Most recently:
“ECT* Workshop on Hadron Electromagnetic Form Factors”
Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts
Presentations, etc., available on-line
http://ect08.phy.anl.gov/
Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
QCD’s Challenges

- **Quark and Gluon Confinement**
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- **Dynamical Chiral Symmetry Breaking**
  - Very unnatural pattern of bound state masses
  - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=$−
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Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

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QCD – Complex behaviour arises from apparently simple rules
Infinitely Heavy Quarks . . . Picture in Quantum Mechanics

\[ V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r} \]

\[ \sqrt{\sigma} \sim 470 \text{ MeV} \]

Necco & Sommer
he-la/0108008
Illustrate this in terms of the action density . . . analogous to plotting the Force
\[ F_{QQ}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2} \]
What happens in the real world; namely, in the presence of light-quarks?
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Bali, et al.
he-la/0512018
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"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

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Confinement

What happens in the real world; namely, in the presence of light-quarks? No one knows ... but $\bar{Q}Q + 2 \times \bar{s}s$

The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.

Energy stored in string at instant before disappearance:

$$E_c \sim 1.25 \text{ GeV}$$
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Flux tube collapses instantly and entirely when the energy it contains exceeds that required to produce the lightest constituent quark-antiquark pair.
Therefore . . . No information on potential between light-quarks. Confinement

What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but \( \bar{Q}Q + 2 \times \bar{s}s \)

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Euler-Lagrange equations for quantum field theory

Well suited to Relativistic Quantum Field Theory
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- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?
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- Method yields Schwinger Functions $\equiv$ Propagators

**Cross-Sections built from Schwinger Functions**
Schwinger Functions
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  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
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- Proving fruitful.
World

DSE Perspective
Persistent Challenge
Persistent Challenge

Infinitely Many Coupled Equations

\[ \Sigma = \gamma \rightarrow S \rightarrow \Gamma \]
Persistent Challenge

- Infinitely Many Coupled Equations

- Coupling between equations necessitates truncation
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  - Coupling between equations *necessitates* truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
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- Weak coupling expansion $\Rightarrow$ Perturbation Theory
  Not useful for the nonperturbative problems in which we’re interested
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  *Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
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- Examples:
  MIT – *The Net Advance of Physics*
  Review Articles and Tutorials in an Encyclopædic Format
  web.mit.edu/redingtn/www/netadv/Xdysonschw.html
The perturbative dressed-quark propagator is given by:

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma = D \gamma \Gamma S \]

This is the gap equation.
Perturbative

**Dressed-quark Propagator**

\[
S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}
\]

\[\Sigma\]

*Gap Equation*

\[
S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}
\]
Perturbative

Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gapped Quark Propagator

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Gap Equation

Weak Coupling Expansion

Reproduces Every Diagram in Perturbation Theory
Perturbative Dressed-quark Propagator

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dressed-quark propagator

Weak Coupling Expansion
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"But in Perturbation Theory"

\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \xrightarrow{m \to 0} 0 \]
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Explanation?
Kernel of Gap Equation: $D_{\mu\nu}(p - q) \Gamma_\nu(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

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Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:


Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:


Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
Dressed-gluon Propagator

\[ D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

Suppression means \( \exists \) IR gluon mass-scale \( \approx 1 \text{ GeV} \)

Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)
Dressed-gluon Propagator

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Gap Equation
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- Gap Equation’s Kernel Enhanced on IR domain

\[ \Rightarrow \text{IR Enhancement of } M(p^2) \]
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Gap Equation’s Kernel Enhanced on IR domain

⇒ IR Enhancement of \( M(p^2) \)

Euclidean Constituent–Quark Mass: \( M^E_f : p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
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<tr>
<td>( \frac{M^E}{m_\xi} )</td>
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Predictions confirmed in numerical simulations of lattice-QCD
Dressed-Quark Propagator

Longstanding Prediction of Dyson-Schwinger Equation Studies
Dressed-Quark Propagator

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Dressed-Quark Propagator

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  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*,
  - C. D. Roberts and A. G. Williams,
  - Prog. Part. Nucl. Phys. 33 (1994) 477
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\[ \Sigma = D \gamma S \Gamma \]
Frontiers of Nuclear Science:
Theoretical Advances

\[ S(p) = \frac{Z(p^2)}{i \gamma \cdot p + M(p^2)} \]

Gap Equation

\[ \Sigma \rightarrow = \gamma \rightarrow \gamma \rightarrow S \rightarrow \Gamma \]

Rapid acquisition of mass is
effect of gluon cloud

\[ m = 0 \text{ (Chiral limit)} \]
\[ m = 30 \text{ MeV} \]
\[ m = 70 \text{ MeV} \]
Mass from nothing

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.
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\[
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In QCD
a quark’s mass must depend on its momentum
• Established understanding of two- and three-point functions
- Established understanding of two- and three-point functions
- What about bound states?
• Without bound states, Comparison with experiment is **impossible**
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• They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is **impossible**

• Bethe-Salpeter **Equation**

QFT Generalisation of Lippmann-Schwinger Equation.
• Without bound states, Comparison with experiment is **impossible**

• Bethe-Salpeter **Equation**

QFT Generalisation of Lippmann-Schwinger Equation.

• **What is the kernel,** $K$?

or **What is the long-range potential in QCD?**
What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD is not related in any simple way to the light-quark interaction.
Bethe-Salpeter Kernel
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{5\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 S^{-1}(k_-) \]

\[ -M_\zeta i\Gamma^l_{5}(k; P) - i\Gamma^l_{5}(k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
Axial-vector Ward-Takahashi identity

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Satisfies BSE

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Kernels very different
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Satisfies BSE

Satisfies DSE

Kernels very different

but must be \textit{intimately} related

- Relation \textbf{must} be preserved by truncation
- Nontrivial constraint
Axial-vector Ward-Takahashi identity

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Satisfies DSE

Kernels very different but must be \textit{intimately} related

- Relation \textbf{must} be preserved by truncation
- \textbf{Failure} \implies Explicit Violation of QCD’s Chiral Symmetry
Goldstone’s Theorem

- In the chiral limit the QCD Action possesses chiral symmetry
- The chiral limit is a good approximation in QCD for $u$- and $d$-quarks
- If this $SU(N_f = 2)$ chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons
- These three Goldstone Bosons have long been identified with the pions: $\pi^+, \pi^0, \pi^-$
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- These three Goldstone Bosons have long been identified with the pions: $\pi^+, \pi^0, \pi^-$
  - E.g., $V(x, y) = (\sigma^2 + \pi^2 - 1)^2$
  - Hamiltonian: $T + V$, is Rotationally Invariant
  - Ground State
    - Ball at any $(\sigma, \pi)$ for which $\sigma^2 + \pi^2 = 1$
    - All Positions have Same (Minimum) Energy
    - But not invariant under rotations
Goldstone’s Theorem

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If this \( SU(N_f = 2) \) chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons

These three Goldstone Bosons have long been identified with the pions: \( \pi^+ \), \( \pi^0 \), \( \pi^- \)

If one assumes the \( s \)-quark is also light; namely, assumes that \( SU(N_f = 3) \) chiral symmetry is a good approximation, then the kaons are four more Goldstone Bosons
Pion and ... Pseudoscalar Mesons?
Can a bound-state of massive constituents truly be massless ... without fine-tuning?
Dichotomy of Pion
– Goldstone Mode and Bound state
Dichotomy of Pion  
– Goldstone Mode and Bound state

How does one make an *almost massless* particle 
............... from two *massive* constituent-quarks?
How does one make an almost massless particle from two massive constituent-quarks?

Not Allowed to do it by fine-tuning a potential

Must exhibit \( m_\pi^2 \propto m_q \)

Current Algebra ... 1968
Dichotomy of Pion – Goldstone Mode and Bound state

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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.
**Dichotomy of Pion**

– **Goldstone Mode and Bound state**

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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

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**Highly Nontrivial**
Resolving the Dichotomy

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
Resolving the Dichotomy

Minimal requirements

- detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.

Satisfying these requirements enables

- Proof of numerous exact results for pseudoscalar mesons
- Formulation of reliable models
  - To illustrate those results
  - Make predictions of observables with quantifiable errors
Goldberger-Treiman for pion
Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

\[
\Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ i E_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) \\
+ \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_{\pi}(k; P) \right]
\]
Goldberger-Treiman for pion

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- Dressed-quark Propagator: \( S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \)
Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

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\[ f_{\pi} E_{\pi}(k; P = 0) = B(p^2) \]
Goldberger-Treiman for pion

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\Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) \\
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S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}
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\[
\Rightarrow f_{\pi} E_{\pi}(k; P = 0) = B(p^2) \\
F_R(k; 0) + 2 f_{\pi} F_{\pi}(k; 0) = A(k^2) \\
G_R(k; 0) + 2 f_{\pi} G_{\pi}(k; 0) = 2A'(k^2) \\
H_R(k; 0) + 2 f_{\pi} H_{\pi}(k; 0) = 0
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\[ \Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ i E_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) + \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right] \]

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Exact in Chiral QCD
Radial Excitations & Chiral Symmetry
\[ f_H \ m_H^2 = - \ \rho_H^{\zeta} \ M_H \]
Radial Excitations & Chiral Symmetry

\[
f_H m_H^2 = - \rho^H \zeta M_H
\]

- Mass\(^2\) of pseudoscalar hadron

(Maris, Roberts, Tandy nu-th/9707003)
Radial Excitations & Chiral Symmetry

\( f_H \quad m_H^2 = - \rho_H^H \quad \mathcal{M}_H \)

\[ \mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M(\mu) \left\{ T^H, (T^H)^t \right\} \right] = m_{q1} + m_{q2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T_{K^+}^K = \frac{1}{2} (\lambda^4 + i\lambda^5) \)
Radial Excitations & Chiral Symmetry

\[ (\pi, \bar{q}\gamma_5\gamma_\mu q)|\pi\rangle \]

\[ f_H p_\mu = Z_2 \int_0^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)_5 \gamma_\mu \hat{S}(q_+)(\Gamma_H(q; P)\hat{S}(q_-)) \right\} \]

- **Pseudovector** projection of BS wave function at \( x = 0 \)
- **Pseudoscalar** meson’s leptonic decay constant

\[ \pi \rightarrow -f_\pi k^\mu A^\mu_5 \quad \Gamma_5 \]

Craig Roberts: Hadron Physics and Continuum Strong QCD
XII Mexican Workshop on Particles and Fields: Mini-courses, 4-8 Nov. 2009... 48 – p. 30/48
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = -\rho^H \ \mathcal{M}_H \]

\[ i\rho^H_\zeta = Z_4 \int_q^{\Lambda} \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 S(q_+) \Gamma_H(q; P) S(q_-) \right\} \]

- Pseudoscalar projection of BS wave function at \( x = 0 \)

\[ \langle 0 | \bar{q} \gamma_5 q | \pi \rangle \]

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Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ \mathcal{M}_H \]

- **Light**-quarks; i.e., \( m_q \sim 0 \)

\[ f_H \to f_H^0 \quad \& \quad \rho_H^\zeta \to -\langle \bar{q}q \rangle_0^\zeta \frac{f_H^0}{f_H^2}, \text{ Independent of } m_q \]

Hence \( m_H^2 = \frac{-\langle \bar{q}q \rangle_0^\zeta}{(f_H^0)^2} m_q \) \ldots \text{GMOR relation, a corollary}
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[
f_H \quad m_H^2 = - \frac{\rho_\zeta^H}{\zeta} \mathcal{M}_H
\]

- Light-quarks; i.e., \( m_q \sim 0 \)

\[
f_H \to f_0^H \quad \text{and} \quad \rho_\zeta^H \to \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_0^H}, \text{ Independent of } m_q
\]

Hence

\[
m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_0^H)^2} m_q \quad \ldots \text{GMOR relation, a corollary}
\]

- Heavy-quark + light-quark

\[
\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}} \quad \text{and} \quad \rho_\zeta^H \propto \sqrt{m_H}
\]

Hence,

\[
m_H \propto m_q \quad \ldots \text{QCD Proof of Potential Model result}
\]
Radial Excitations

- Spectrum contains 3 pseudoscalars \([I^G(J^P) L = 1^- (0^-) S]\)

  masses below 2 GeV: \(\pi(140)\); \(\pi(1300)\); and \(\pi(1800)\)
Radial Excitations

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  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

- The Pion

- Consituent-Q Model: 1st three members of $n^{1S_0}$ trajectory; i.e., ground state plus radial excitations?
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- The Pion

- Consituent-Q Model: 1st three members of $n\,^1S_0$ trajectory; i.e., ground state plus radial excitations?

- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some “flux tube angular momentum” content:
  
  \[ S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0 \]
  
  & $L_F = 1 \Rightarrow ^3S_1 \oplus ^3S_1$ ($\bar{Q}Q$) decays suppressed?
Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$
  
  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

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- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
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- Radial excitations & Hybrids & Exotics \(\Rightarrow\) Long-range radial wave functions \(\Rightarrow\) sensitive to confinement

- NSAC Long-Range Plan, 2002: . . . an understanding of confinement “remains one of the greatest intellectual challenges in physics”
\[ f_H \quad m_H^2 = - \quad \rho_H^\zeta \quad M_H \]

- Valid for **ALL** Pseudoscalar mesons
Valid for **ALL** Pseudoscalar mesons

\[ f_H m_H^2 = - \rho_H^H \mathcal{M}_H \]

- \( \rho_H \rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
Radial Excitations & Chiral Symmetry

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- \( \rho_H \Rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
- “radial” excitation of \( \pi \)-meson, not the ground state, so \( m^2_{\pi_n \neq 0} > m^2_{\pi_n = 0} = 0 \), in **chiral limit**
Radial Excitations & Chiral Symmetry

$$f_H \ m_H^2 = - \ \rho_H^{\zeta} \ \mathcal{M}_H$$

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- \(\rho_H\) ⇒ finite, nonzero value in chiral limit, \(\mathcal{M}_H \rightarrow 0\)
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- \(\Rightarrow f_H = 0\)

**ALL pseudoscalar mesons except \(\pi(140)\) in chiral limit**
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \rho_H^H \quad M_H \]

- Valid for **ALL** Pseudoscalar mesons
- \( \rho_H \Rightarrow \) finite, nonzero value in chiral limit, \( M_H \rightarrow 0 \)
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  \[ \Rightarrow f_H = 0 \]
- **ALL** pseudoscalar mesons except \( \pi(140) \) in **chiral limit**
- **Dynamical Chiral Symmetry Breaking**
  \[ \text{– Goldstone’s Theorem –} \]
- impacts upon **every** pseudoscalar meson
Charge Neutral Pseudoscalar Mesons
non-Abelian Anomaly and $\eta$-$\eta'$ mixing
non-Abelian Anomaly and $\eta-\eta'$ mixing

- Mesons containing $\bar{s}-s$ are special: $\eta$ & $\eta'$
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

Mesons containing $\bar{s}$-$s$ are special: $\eta$ & $\eta'$

**Problem:** $\eta'$ is a pseudoscalar meson but it’s much more massive than the other eight constituted from light-quarks.
non-Abelian Anomaly and $\eta-\eta'$ mixing

Mesons containing $\bar{s}-s$ are special: $\eta$ & $\eta'$

Problem: $\eta'$ is a pseudoscalar meson but it’s much more massive than the other eight constituted from light-quarks.

Origin: While the classical action associated with QCD is invariant under $U_A(1)$ (Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

- Mesons containing $\bar{s}$-$s$ are special: $\eta$ & $\eta'$
- Flavour mixing takes place in singlet channel: $\lambda^0 \leftrightarrow \lambda^8$
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

- Mesons containing $\bar{s}$-$s$ are special: $\eta$ & $\eta'$
- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$

\[ u,d,s \quad \text{and} \quad u,d,s \]
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

Mesons containing $s\bar{s}$ are special: $\eta$ & $\eta'$

This is a perturbative diagram.
It has almost nothing to do with $\eta \Leftrightarrow \eta'$ mixing.
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

- Mesons containing $\bar{s}$-$s$ are special: $\eta$ & $\eta'$
- Driver is the non-Abelian anomaly
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

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- Driver is the non-Abelian anomaly

\[ K_A \sim \sum_{IS} \]

\[ e.g. \quad IS = \]
non-Abelian Anomaly and $\eta$-$\eta'$ mixing

- Mesons containing $\bar{s}$-$s$ are special: $\eta$ & $\eta'$
- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly.
All terms have the “hairpin” structure.
No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.
Charge Neutral

Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
\[ -2iM^{ab}\Gamma^b_5(k; P) - A^a(k; P) \]
\[ P_{\mu} \Gamma_{5\mu}^a (k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]

\[-2i M^{ab} \Gamma_5^b (k; P) - A^a(k; P) \]

\[ \{ F^a \mid a = 0, \ldots, N_f^2 - 1 \} \text{ are the generators of } U(N_f) \]

\[ S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \]

\[ M^{ab} = \text{tr}_F \left[ \{ F^a, M \} F^b \right], \]

\[ M = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses} \]
Charge Neutral Pseudoscalar Mesons

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\( M = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses} \)

The final term in the second line expresses the non-Abelian axial anomaly.
Charge Neutral Pseudoscalar Mesons

\[ P_\mu \Gamma_5^{a\mu}(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]

\[ -2i M^{ab} \Gamma_5^{b}(k; P) - A^a(k; P) \]

\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

\[ A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle \mathcal{F}^0 q(x) Q(0) \bar{q}(y) \rangle \]
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_5^{\alpha} (k; P) = S^{-1}(k_+) i \gamma_5 F^\alpha + i \gamma_5 F^\alpha S^{-1}(k_-) \]
\[-2i M^{ab} \Gamma_5^{b} (k; P) - A^a (k; P) \]

\[ A^a (k; P) = S^{-1}(k_+) \delta^{a0} A_U (k; P) S^{-1}(k_-) \]
\[ A_U (k; P) = \int d^4x d^4y e^{i(k_+ x - k_- y)} N_f \langle F^0 q(x) Q(0) \bar{q}(y) \rangle \]

\[ Q(x) = i \frac{\alpha_s}{4\pi} tr_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu (x) \]

... The topological charge density operator.
\[ P_\mu \Gamma_{\mu 5}^a (k; P) = S^{-1}(k_+) \delta_0^a A_U(k; P) S^{-1}(k_-) \]

\[ -2i \mathcal{M}^{ab} \Gamma_5^b (k; P) \]

\[ A^a(k; P) = S^{-1}(k_+) \delta_0^a A_U(k; P) S^{-1}(k_-) \]

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\[ \ldots \text{ The topological charge density operator.} \]

(Trace is over colour indices & \( F_{\mu\nu} = \frac{1}{2} \lambda^a F_{\mu\nu}^a \).)
Charge Neutral

Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]

\[ -2i M^{ab} \Gamma^b_{5}(k; P) - A^a(k; P) \]

\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

\[ A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle F^0 q(x) Q(0) \bar{q}(y) \rangle \]

\[ Q(x) = i \frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x) \]

\[ \ldots \text{ The topological charge density operator.} \]

\[ \text{Important that only } A^{a=0} \text{ is nonzero.} \]
\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]
\[ -2i M^{a b} \Gamma^{b}_{5}(k; P) - A^a(k; P) \]
\[ A^a(k; P) = S^{-1}(k_+) \delta^{a 0} A_U(k; P) S^{-1}(k_-) \]
\[ A_U(k; P) = \int d^4 x d^4 y e^{i(k_+ x - k_- y)} N_f \langle \mathcal{F}^0 q(x) Q(0) \bar{q}(y) \rangle \]
\[ Q(x) = i \frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x) \]

\dots The topological charge density operator.

NB. While \( Q(x) \) is gauge invariant, the associated Chern-Simons current, \( K_\mu \), is not \( \Rightarrow \) in QCD no physical boson can couple to \( K_\mu \) and hence no physical states can contribute to resolution of \( U(1) \) problem.
Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy

nucl-th/arXiv:0708.1118
Only $A^0 \neq 0$ is interesting
Only $A^0 \neq 0$ is interesting . . . otherwise all pseudoscalar mesons are Goldstone Modes!
Anomaly term has structure

\[ \mathcal{A}^0(k; P) = \mathcal{F}^0 \gamma_5 \left[ i\mathcal{E}_A(k; P) + \gamma \cdot P \mathcal{F}_A(k; P) + \gamma \cdot k k \cdot P \mathcal{G}_A(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_A(k; P) \right] \]
AVWTI gives generalised Goldberger-Treiman relations

\[ 2f_{\eta'} E_{BS}(k; 0) = 2B_0(k^2) - e_A(k; 0), \]
\[ F_R^0(k; 0) + 2f_{\eta'} F_{BS}(k; 0) = A_0(k^2) - F_A(k; 0), \]
\[ G_R^0(k; 0) + 2f_{\eta'} G_{BS}(k; 0) = 2A'_0(k^2) - G_A(k; 0), \]
\[ H_R^0(k; 0) + 2f_{\eta'} H_{BS}(k; 0) = -H_A(k; 0), \]

\( A_0, B_0 \) characterise gap equation’s chiral limit solution.
AVWTI gives generalised Goldberger-Treiman relations

\[ 2 f_{\eta'} E_{BS}(k; 0) = 2 B_0(k^2) - E_A(k; 0), \]
\[ F^0_R(k; 0) + 2 f_{\eta'} F_{BS}(k; 0) = A_0(k^2) - F_A(k; 0), \]
\[ G^0_R(k; 0) + 2 f_{\eta'} G_{BS}(k; 0) = 2 A'_0(k^2) - G_A(k; 0), \]
\[ H^0_R(k; 0) + 2 f_{\eta'} H_{BS}(k; 0) = -H_A(k; 0), \]

\( A_0, B_0 \) characterise gap equation’s chiral limit solution.

Follows that \( E_A(k; 0) = 2 B_0(k^2) \) is necessary and sufficient condition for absence of massless \( \eta' \) bound-state.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

- \( B_0(k^2) \neq 0 \) if, and only if, chiral symmetry is dynamically broken.

Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

- \( B_0(k^2) \neq 0 \) if, and only if, chiral symmetry is dynamically broken.

- Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Further highlighted \ldots proved

\[
\langle \bar{q}q \rangle_\zeta^0 = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \text{tr}_{\text{CD}} \int_q^\Lambda S^0(q, \zeta) = N_f \int d^4x \langle \bar{q}(x)i\gamma_5q(x)Q(0) \rangle^0.
\]
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
Charge Neutral
Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Employed in an analysis of pseudoscalar- and vector-meson bound-states
AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts

- $\eta - \eta'$ mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
- $\pi^0 - \eta$ angles of $\sim 1.2^\circ$ (Expt. $pd \to ^3\text{He} \pi^0$: $0.6^\circ \pm 0.3^\circ$)
- Strong neutron-proton mass difference $\ldots$
  $\lesssim 75\%$ current-quark mass-difference
Ab-Initio Calculations
Ab-Initio Calculations

Pieter Maris

Peter Tandy
Maris & Tandy, Series of *Five Articles*: 1999 – Present

Perfected a *Renormalisation-Group Improved Rainbow-Ladder Model* of *Quark-Quark Interaction*
Ab-Initio Calculations

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- Rainbow-Ladder = First Order in Truncation Described Above

- Anticipate Accurate for $0^-$ & $1^-$ Mesons
Ab-Initio Calculations

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- One Parameter = Interaction Energy: \( E \approx 700 \text{ MeV} \)
- Dressed-Glue Mass scale: Characterises DCSB and light-quark Confinement
- Both Phenomena Disappear for \( E \lesssim 200 \text{ MeV} \)
Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

- One Parameter = Interaction Energy: \( \epsilon \approx 700 \text{ MeV} \)

- Dressed-Glue Mass scale: Characterises DCSB and light-quark Confinement

- Both Phenomena Disappear for \( \epsilon \lesssim 200 \text{ MeV} \)

- Dyson-Schwinger equations: A Tool for Hadron Physics

  P. Maris and C.D. Roberts, nu-th/0301049
Kernel of Bethe-Salpeter Equation

\[ K(p, k; P) \approx \frac{\alpha^{\text{eff}}((p - k)^2)}{(p - k)^2} \]
Kernel of Bethe-Salpeter Equation

\[ K(p, k; P) \approx \alpha^{\text{eff}}\left(\frac{(p - k)^2}{(p - k)^2}\right) \]

Prescribes Gap Equation’s Kernel
Kernel of Bethe-Salpeter Equation

\[ K(p, k; P) \approx \frac{\alpha_{\text{eff}}((p - k)^2)}{(p - k)^2} \]

Prescribes Gap Equation’s Kernel

Connects Ansatz for long-range part of QCD’s interaction with Observables.

Craig Roberts: Hadron Physics and Continuum Strong QCD XII Mexican Workshop on Particles and Fields: Mini-courses, 4-8 Nov. 2009...
Kernel of Bethe-Salpeter Equation

\[ K(p, k; P) \approx \frac{\alpha^\text{eff}((p - k)^2)}{(p - k)^2} \]

Prescribes Gap Equation’s Kernel

IR-Enhancement at long-range agrees semi-quantitatively with Bhagwat, et al.
Procedure Now Straightforward
Solve Gap Equation

⇒ Dressed-Quark Propagator, $S(p)$
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$
- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, $\Gamma_\mu$
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

\[ \Gamma_\pi(k; P) \]

\[ \Gamma_\mu(k; P) \]

\[ S(p) \]
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

\[ \Gamma_\pi(k; P) \]

\[ \Gamma_\mu(k; P) \]

\[ S(p) \]

Evaluate this final, three-dimensional integral
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

![Graph showing calculated pion form factors](image)

- **Amendolia**
- **Brauel, re-analyzed**
- **Volmer**
- **DSE calculation**
- **VMD ρ monopole, m_ρ=770 MeV**
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied
Data published in 2001

Craig Roberts: Hadron Physics and Continuum Strong QCD
XII Mexican Workshop on Particles and Fields: Mini-courses, 4-8 Nov. 2009
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001

Many subsequent successful applications. Again, parameters Fixed.

Notably $\pi\pi$ Scattering


Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 [i\mathcal{E}_{\pi n}(k; P) + \gamma \cdot P\mathcal{F}_{\pi n}(k; P) \]

\[ \gamma \cdot k \cdot P\mathcal{G}_{\pi n}(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_{\pi n}(k; P) ] \]
Pion \ldots \, J = 0

but \ldots

Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma^5 \left[ i\mathcal{E}_\pi(k; P) + \gamma \cdot \mathcal{F}_\pi(k; P) \right] \]

\[ \gamma \cdot k \cdot P \mathcal{G}_\pi(k; P) + \sigma_{\mu \nu} k_\mu P_\nu \mathcal{H}_\pi(k; P) \]

Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.
Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 \left[ i \xi_\pi_n(k; P) + \gamma \cdot P \mathcal{F}_\pi_n(k; P) \right. \]
\[ \left. + \gamma \cdot k \cdot P \mathcal{G}_\pi_n(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_\pi_n(k; P) \right] \]

Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.
Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 \left[ i\mathcal{E}_\pi(k; P) + \gamma \cdot P \mathcal{F}_\pi(k; P) \right. \\
\left. \gamma \cdot k \cdot P \mathcal{G}_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_\pi(k; P) \right] \]

In QCD, a Poincaré invariant theory with interactions, \( \mathcal{E} \neq 0 \), forces nonzero results for \( \mathcal{F} \), \( \mathcal{G} \) and \( \mathcal{H} \)
Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 [i\mathcal{E}_\pi(n)(k; P) + \gamma \cdot P\mathcal{F}_\pi(n)(k; P) \]

\[ \gamma \cdot k \cdot P \mathcal{G}_\pi(n)(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} \mathcal{H}_\pi(n)(k; P) ] \]

- In QCD, a Poincaré invariant theory with interactions, \( \mathcal{E} \neq 0 \) forces nonzero results for \( \mathcal{F}, \mathcal{G} \) and \( \mathcal{H} \).

- \( J = 0 \ldots \) but while \( \mathcal{E} \) and \( \mathcal{F} \) are purely \( L = 0 \) in the rest frame, the \( \mathcal{G} \) and \( \mathcal{H} \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function always contains both \( S \)- and \( P \)-wave components.
\( Pion \ldots J = 0 \)

\( \textbf{but \ldots} \)

\( J = 0 \ldots \textit{but} \) while \( E \) and \( F \) are purely \( L = 0 \) in the rest frame, the \( G \) and \( H \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function \textit{always} contains both \( S \)- and \( P \)-wave components.

Introduce mixing angle \( \theta_\pi \) such that

\[
\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
\]
$J = 0 \ldots \text{but}$ while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both $S$- and $P$-wave components. Introduce mixing angle $\theta_\pi$ such that

$$\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle$$
\( J = 0 \ldots \text{but} \) while \( E \) and \( F \) are purely \( L = 0 \) in the rest frame, the \( G \) and \( H \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function always contains both \( S \)- and \( P \)-wave components.

Introduce mixing angle \( \theta_\pi \) such that

\[
\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
\]

\( \theta_\pi \) is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.
Deep-inelastic scattering
Deep-inelastic scattering

Looking for Quarks
Deep-inelastic scattering

Looking for Quarks
Deep-inelastic scattering

Looking for Quarks

Signature Experiment for QCD:

Discovery of Quarks at SLAC
Deep-inelastic scattering

- Looking for Quarks

**Signature Experiment** for QCD:

**Discovery of Quarks at SLAC**

Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x), g(x)$
Pion’s valence quark distn
Pion's valence quark distn

π is Two-Body System: “Easiest” Bound State in QCD

However, NO π Targets!
\( \pi \) is Two-Body System: “Easiest” Bound State in QCD

However, NO \( \pi \) Targets!

Existing Measurement Inferred from Drell-Yan:

\[ \pi N \rightarrow \mu^+ \mu^- X \]
- Pion’s valence quark distn

- π is Two-Body System: “Easiest” Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  \[ \pi N \rightarrow \mu^+ \mu^- X \]
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

\[ e^{-5 \text{GeV}} - p^{25 \text{GeV}} \text{ Collider} \rightarrow \text{Accurate “Measurement”} \]
Handbag diagrams
\[ W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right] \]

\[ T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_+ \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) i\epsilon Q \Gamma_\nu(k_{-0}, k) \times S(k) i\epsilon Q \Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{-0}) \]
Bjorken Limit: \( q^2 \to \infty \), \( P \cdot q \to -\infty \) but \( x := -\frac{q^2}{2P \cdot q} \) fixed.

Numerous algebraic simplifications

\[
W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right]
\]

\[
T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_- \bar{\Gamma} \pi (k_{-\frac{1}{2}}; -P) S(k_{-0}) i\epsilon Q \Gamma_{\nu}(k_{-0}, k) \times S(k) i\epsilon Q \Gamma_{\mu}(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma \pi (k_{-\frac{1}{2}}; P) S(k_{-\frac{1}{2}})
\]
Calc. $u_V(x) \; \text{cf. Drell-Yan data}$
Calc. $u_V(x)$ cf. Drell-Yan data

Hecht, Roberts, Schmidt
nucl-th/0008049

Craig Roberts: Hadron Physics and Continuum Strong QCD
XII Mexican Workshop on Particles and Fields: Mini-courses, 4-8 Nov. 2009... 48 – p. 45/48
Calc. $u_V(x)$ cf. Drell-Yan data

Resolving Scale: $q_0 = 0.54 \text{ GeV} = 1/(0.37 \text{ fm})$
Calc. \( u_V(x) \) cf. Drell-Yan data

\[
x u_v(x; q_0 = 0.54 \text{GeV}) \quad x u_v(x; q = 2 \text{GeV}) \\
\text{E615 \pi N Drell-Yan 4GeV} \quad \text{SMRS 92 Fit} \\
x u_v(x; q = 4 \text{GeV}) \quad \text{Fit } x^\alpha (1-x)^\beta
\]

\[
\langle x \rangle_{q_0} = 0.24 \quad 0.24 \pm 0.01 \quad 0.27 \pm 0.01 \\
\langle x^2 \rangle_q = 0.10 \quad 0.10 \pm 0.01 \quad 0.11 \pm 0.03 \\
\langle x^3 \rangle_q = 0.050 \quad 0.058 \pm 0.004 \quad 0.048 \pm 0.020
\]
Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)

\[ x u_v(x) \]

- E615 πN Drell–Yan 4GeV
- NLO Analysis of E615 ... \( \beta = 1.87 \)
- DSE ... \( \beta = 2.61 \)
- NJL ... \( \beta = 1.27 \)
Answer for the pion
Answer for the pion

Two → Infinitely many ...
Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory.
Two $\rightarrow$ Infinitely many . . .
Handle that properly in quantum field theory
. . . momentum-dependent dressing
Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory . . .
momentum-dependent dressing . . .
perceived distribution of mass depends on the resolving scale
Hadron Physics is $\sim$ $300$-million/year effort in USA alone
Hadron Physics is \(\sim\) $300$-million/year effort in USA alone

- Subject is QCD \ldots in the *nonperturbative* domain
Hadron Physics is \(~\) $300\text{-}\text{million/year}$ effort in USA alone

Subject is QCD . . . in the *nonperturbative* domain

Keystones are the **Emergent Phenomena**

- **Confinement**
  - quarks and gluons never alone reach a detector

- **Dynamical Chiral Symmetry Breaking**
  - counter-intuitive pattern of bound state masses and interactions
Hadron Physics is $\sim$ $300$-million/year effort in USA alone

Subject is QCD . . . in the *nonperturbative* domain

Keystones are the Emergent Phenomena

- Confinement
  - quarks and gluons never alone reach a detector

- Dynamical Chiral Symmetry Breaking
  - counter-intuitive pattern of bound state masses and interactions

Review presentation in Mazatlan:

- Elastic electromagnetic pion form factor
- Nature of Baryons