Universal Truths

- Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.
Universal Truths

- Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1 \text{GeV}^2$ require a Poincaré-covariant approach.
Universal Truths

Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.

Calculations at $Q^2 > 1 \text{ GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.
Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents. Calculations at $Q^2 > 1 \text{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function. DCSB is most important mass generating mechanism for matter in the Universe.
Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.

Calculations at $Q^2 > 1 \text{ GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.

DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
Universal Truths

- Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.

- Calculations at $Q^2 > 1\text{ GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.

- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
Universal Truths

- Form factors give information about distribution of hadron’s characterising properties amongst its QCD constituents.

- Calculations at $Q^2 > 1 \text{ GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.

- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.
QCD’s Challenges
Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
  - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=−$
QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=-$

- Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=-$

- Neither of these phenomena is apparent in QCD’s Lagrangian *yet* they are the dominant determining characteristics of real-world QCD.

- QCD – Complex behaviour arises from apparently simple rules
Why?
The nucleon and pion hold special places in non-perturbative studies of QCD.
The nucleon and pion hold special places in non-perturbative studies of QCD.

An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.
The nucleon and pion hold special places in non-perturbative studies of QCD.

An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.

Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.
The nucleon and pion hold special places in non-perturbative studies of QCD.

An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.

Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.

Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.
The nucleon and pion hold special places in non-perturbative studies of QCD.

An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.

Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.

Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.

Despite this, many urgent questions remain unanswered.
Some Questions

- What is the role of pion cloud in nucleon electromagnetic structure?
- Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?
Some Questions

Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?
Some Questions

Do we understand the high $Q^2$ behavior of the proton form factor ratio in the space-like region?

Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?
Some Questions

- Can we understand the rich structure of the time-like proton form factors in terms of resonances?
- What do we expect for the proton form factor ratio in the time-like region?
- What is the relation between proton and neutron form factor in the time-like region?
- How do we understand the ratio between time-like and space-like form factors?
Some Questions

What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?

What is the impact of these contributions on other form factor measurements?
How accurately can the pion form factor be extracted from the $ep \rightarrow e'n\pi^+$ reaction?
Current status is described in

- J. Arrington, C. D. Roberts and J. M. Zanotti
  “Nucleon electromagnetic form factors,”

- C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
  “Nucleon electromagnetic form factors,”
  Prog. Part. Nucl. Phys. 59, 694 (2007);
Current status is described in

- J. Arrington, C. D. Roberts and J. M. Zanotti
  “Nucleon electromagnetic form factors,”

- C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
  “Nucleon electromagnetic form factors,”
  Prog. Part. Nucl. Phys. 59, 694 (2007);

Most recently:
“ECT* Workshop on Hadron Electromagnetic Form Factors”
Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts
Presentations, etc., available on-line
http://ect08.phy.anl.gov/
Dichotomy of Pion
– Goldstone Mode and Bound state
Dichotomy of Pion
– Goldstone Mode and Bound state

How does one make an **almost massless** particle
............. from two **massive** constituent-quarks?
Dichotomy of Pion
– Goldstone Mode and Bound state

How does one make an almost massless particle
from two massive constituent-quarks?

Not Allowed to do it by fine-tuning a potential
Must exhibit $m_\pi^2 \propto m_q$

Current Algebra … 1968
How does one make an almost massless particle from two massive constituent-quarks?

Not Allowed to do it by fine-tuning a potential

Must exhibit \[ m_\pi^2 \propto m_q \]

Current Algebra ... 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

well-defined and valid chiral limit;

and an accurate realisation of dynamical chiral symmetry breaking.
**Dichotomy of Pion**

– **Goldstone Mode and Bound state**

- How does one make an almost massless particle from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

\[ m^2_\pi \propto m_q \]

Current Algebra \ldots 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.

**Highly Nontrivial**
There is a sense in which it is easy to fabricate a model that can reproduce the elastic electromagnetic pion form factor.
There is a sense in which it is easy to fabricate a model that can reproduce the elastic electromagnetic pion form factor.

However, a veracious description of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^* \pi \rightarrow \gamma$ transition form factor.
There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor

However, a *veracious description* of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^* \pi \rightarrow \gamma$ transition form factor

The latter is connected with the *Abelian anomaly* – therefore fundamentally connected with chiral symmetry and its dynamical breaking – no mere model can successfully describe this without fine tuning
There is a sense in which it is easy to fabricate a model that can reproduce the elastic electromagnetic pion form factor.

However, a veracious description of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^* \pi \rightarrow \gamma$ transition form factor.

The latter is connected with the Abelian anomaly – therefore fundamentally connected with chiral symmetry and its dynamical breaking – no mere model can successfully describe this without fine tuning.

Must similarly require prediction of $\gamma^* \pi \rightarrow \pi\pi$ and all other anomalous processes.
What’s the Problem?
Minimal requirements

- detailed understanding of connection between \textit{Current-quark} and \textit{Constituent-quark} masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.
What’s the Problem?

- Minimal requirements
  - detailed understanding of connection between \textit{Current-quark} and \textit{Constituent-quark} masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

- Means ... must calculate hadron \textit{wave functions}
  - Can’t be done using perturbation theory
What’s the Problem?

Minimal requirements

- detailed understanding of connection between Current-quark and Constituent-quark masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.

Means . . . must calculate hadron *wave functions*
– Can’t be done using perturbation theory

Why problematic? Isn’t same true in quantum mechanics?
What’s the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron wave functions
  - Can’t be done using perturbation theory
- Why problematic? Isn’t same true in quantum mechanics?
- Differences!
What's the Problem?

Relativistic QFT!

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

- Differences!
  - Here relativistic effects are crucial – virtual particles, quintessence of Relativistic Quantum Field Theory – must be included
What’s the Problem? Relativistic QFT!

Minimal requirements
- detailed understanding of connection between Current-quark and Constituent-quark masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.

Differences!
- Here relativistic effects are crucial – virtual particles, quintessence of Relativistic Quantum Field Theory – must be included
- Interaction between quarks – the Interquark “Potential” – unknown throughout > 98% of a hadron’s volume
Intranucleon Interaction
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.

98% of the volume
Dyson-Schwinger Equations
Dyson-Schwinger Equations
Dressed-Quark Propagator
Dyson-Schwinger Equations

Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation
Dyson-Schwinger Equations

Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

- Gap Equation’s Kernel Enhanced on IR domain
  \[ \Rightarrow \text{IR Enhancement of } M(p^2) \]

![Graph showing IR enhancement of M(p^2) for different quarks including b-, c-, s-, u, d-quarks and chiral limit.](image)
Dyson-Schwinger Equations
Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

- Gap Equation’s Kernel Enhanced on IR domain
  \[ \Rightarrow \text{IR Enhancement of } M(p^2) \]

Euclidean Constituent–Quark Mass: \( M_f^E: p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
<th>( u/d )</th>
<th>( s )</th>
<th>( c )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{M^E}{m_\zeta} )</td>
<td>( \sim 10^2 )</td>
<td>( \sim 10 )</td>
<td>( \sim 1.5 )</td>
<td>( \sim 1.1 )</td>
</tr>
</tbody>
</table>
Dyson-Schwinger Equations
Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation’s Kernel Enhanced on IR domain
⇒ IR Enhancement of \( M(p^2) \)

Euclidean Constituent–Quark Mass: \( M_f^E : p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
<th>( u/d )</th>
<th>( s )</th>
<th>( c )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{M^E}{m_\zeta} )</td>
<td>( \sim 10^2 )</td>
<td>( \sim 10 )</td>
<td>( \sim 1.5 )</td>
<td>( \sim 1.1 )</td>
</tr>
</tbody>
</table>

Predictions confirmed in numerical simulations of \textbf{lattice-QCD}
Hadrons

- Established understanding of two- and three-point functions
- Established understanding of two- and three-point functions
- What about bound states?
Without bound states, Comparison with experiment is impossible
• Without bound states, Comparison with experiment is impossible

• They appear as pole contributions to \( n \geq 3 \)-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
Hadrons

• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$?
Hadrons

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, $K$?
What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD is not related in any simple way to the light-quark interaction.
Bethe-Salpeter Kernel
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{5\mu}(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f i\gamma_5 + \frac{1}{2} \lambda_f i\gamma_5 \mathcal{S}^{-1}(k_-) \]

\[ -M_\zeta i\Gamma^l_5(k; P) - i\Gamma^l_5(k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 S^{-1}(k_-) \]

\[ -M_\zeta i \Gamma_5^l (k; P) - i \Gamma_5^l (k; P) M_\zeta \]

Satisfies BSE  Satisfies DSE
Axial-vector Ward-Takahashi identity

\[ P_\mu \, \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 \ S^{-1}(k_-) \]

\[ -M_\zeta \ i \Gamma_{5}^l (k; P) - i \Gamma_{5}^l (k; P) \ M_\zeta \]

Satisfies BSE

Kernels very different

but must be \textit{intimately} related

Satisfies DSE
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_5^{\mu \ell} (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 S^{-1}(k_-) \]

\[ -M_\zeta i \Gamma_5^{\ell} (k; P) - i \Gamma_5^{\ell} (k; P) M_\zeta \]

Satisfies BSE

Satisfies DSE

Kernels very different

but must be \textit{intimately} related

\textbullet \ Relation must be preserved by truncation
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 S^{-1}(k_-) \]

\[ -M_\zeta i \Gamma_5^l (k; P) - i \Gamma_5^l (k; P) M_\zeta \]

Satisfies BSE

\[ \text{Kernels very different} \]

but must be \textit{intimately} related

• Relation \textit{must} be preserved by truncation

• Nontrivial constraint

Satisfies DSE
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+ \frac{1}{2} \lambda_f i \gamma_5) + S^{-1}(k_- \frac{1}{2} \lambda_f i \gamma_5) \]

\[ -M_\zeta i \Gamma_{5}^l (k; P) - i \Gamma_{5}^l (k; P) M_\zeta \]

Satisfies BSE

Satisfies DSE

Kernels very different but must be intimately related

- Relation must be preserved by truncation
- Failure \(\Rightarrow\) Explicit Violation of QCD’s Chiral Symmetry
Procedure Now Straightforward
Solve Gap Equation

⇒ Dressed-Quark Propagator, $S(p)$
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$

\[
\begin{align*}
    i\Gamma & = i\Gamma \\
    \text{---} & \quad \text{---}
\end{align*}
\]
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$
- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, $\Gamma_\mu$
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

\[ \Gamma_\pi(k; P) \]

\[ \Gamma_\mu(k; P) \]

\[ S(p) \]
Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

\[ \Gamma_\pi(k; P) \]  

\[ \Gamma_\mu(k; P) \]  

\[ S(p) \]  

Evaluate this final, three-dimensional integral
Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied
Numerical method improved in 2005

\[ Q^2 F_\pi(Q^2) \text{ [GeV}^2\text{]} \]

- Amendolia et al.
- Ackermann et al.
- Brauel et al.
- Tadevosyan et al.
- Horn et al.
- Maris and Tandy, 2005

Craig Roberts: Hadron Form Factors
JLab User Group, 16-18 June 08... 26 – p. 16/45
Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied
Numerical method improved in 2005

Data published in 2001. Subsequently revised
Timelike Pion Form Factor
**Timelike Pion Form Factor**

*Ab initio* calculation into timelike region
Deeper than ground-state $\rho$-meson pole
**Timelike Pion Form Factor**

*Ab initio* calculation into timelike region
Deeper than ground-state $\rho$-meson pole
**Timelike Pion Form Factor**

*Ab initio* calculation into timelike region

Deeper than ground-state $\rho$-meson pole

$\rho$-meson not put in “by hand”—generated dynamically as a bound-state of dressed-quark and dressed-antiquark
Nucleon Challenge
Another Direction . . . Also want/need information about three-quark systems
Another Direction . . . Also want/need information about three-quark systems

With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
Another Direction . . . Also want/need information about three-quark systems

With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
Nucleon EM Form Factors: A Précis

Nucleon EM Form Factors: A Précis
Nucleon EM Form Factors: A Précis
Nucleon EM Form Factors: A Précis

Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
⇒ Covariant dressed-quark Faddeev Equation
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  ⇒ Covariant dressed-quark Faddeev Equation

- Excellent mass spectrum (octet and decuplet)

  Easily obtained:

  \[
  \left( \frac{1}{N_H} \sum_{H} \left[ M_{H}^{\text{exp}} - M_{H}^{\text{calc}} \right]^2 \right)^{1/2} = 2\% 
  \]
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

**Easily** obtained:

\[
\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%
\]

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)
Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
\[ \Rightarrow \text{Covariant dressed-quark Faddeev Equation} \]

- Excellent mass spectrum (octet and decuplet)

Easily obtained:
\[
\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%
\]

- But is that good?
Nucleon EM Form Factors: A Précis

Cloët, et al.: 

• Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons 
⇒ Covariant dressed-quark Faddeev Equation

• Excellent mass spectrum (octet and decuplet)

  Easily obtained:

  \[
  \left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\% 
  \]

• But is that good?

  • Cloudy Bag: \( \delta M_{+}^{\pi-\text{loop}} = -300 \) to \(-400\) MeV!
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:
  \[
  \left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%
  \]
- But is that good?
  - Cloudy Bag: \( \delta M_+^{\pi-\text{loop}} = -300 \) to \(-400\) MeV!
  - Critical to anticipate pion cloud effects

Roberts, Tandy, Thomas, et al., nu-th/02010084
Faddeev equation
The Faddeev equation is given by:

\[ \Psi^a \rightarrow \Psi^b = \Gamma^a \psi_p \Gamma^b \psi_p \]
Faddeev equation

\[ \Psi^a_{p q} \Gamma^a \Psi^b_{p d} = \Psi^a_{p q} \Gamma^b_{q p} \Psi^b_{p d} \]

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . \( s- \), \( p- \) & \( d- \)wave correlations
Diquark correlations
Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red.

Confined ... Does not escape from within baryon.

Scalar is isosinglet, Axial-vector is isotriplet.

DSE and lattice-QCD

\[
\begin{align*}
    m_{[ud]}_{0^+} &= 0.74 - 0.82 \\
    m_{(uu)}_{1^+} &= m_{(ud)}_{1^+} = m_{(dd)}_{1^+} = 0.95 - 1.02
\end{align*}
\]
Nucleon-Photon Vertex

Craig Roberts: Hadron Form Factors
JLab User Group, 16-18 June 08
M. Oettel, M. Pichowsky 
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude
6 terms . . .

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude

Nucleon-Photon Vertex
Form Factor Ratio: GE/GM

\[ \mu_p G_E^p / G_M^p \]

- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer

\[ Q^2 \text{ [GeV}^2] \]

Craig Roberts: Hadron Form Factors
JLab User Group, 16-18 June 08
Combine these elements . . .
Combine these elements . . .

Dressed-Quark Core

Form Factor Ratio:

GE/GM

\[
\frac{\mu P E}{G M} \quad Q^2 \text{[GeV}^2]\]

Craig Roberts: Hadron Form Factors
JLab User Group, 16-18 June 08 . . . 26 – p. 24/45
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi**
  Identity preserving current

![Graph showing the form factor ratio GE/GM with data points and error bars.](image)
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud's Contribution
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud's Contribution

Form Factor Ratio: GE/GM
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . Not varied.
- Combine these elements...
  - Dressed-Quark Core
  - *Ward-Takahashi*
    - Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution
- All parameters fixed in other applications... **Not** varied.
- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
Combine these elements ...

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current
- Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications ... Not varied.

- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi**
  Identity preserving current
- **Anticipate and Estimate Pion Cloud’s Contribution**

All parameters fixed in other applications . . . **Not** varied.

- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
- Predict Zero at $Q^2 \approx 6.5 \text{ GeV}^2$
Improved current
Improved current

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large-$Q^2$ behaviour
Improved current

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large-$Q^2$ behaviour
  - Improved performance of code
  - Implemented corrections so that large-$Q^2$ behaviour of form factors could be reliably calculated
  - Exposed two weaknesses in rudimentary Ansatz
Improved current

- Composite axial-vector diquark correlation
  - Improved performance of code
  - Implemented corrections so that large-$Q^2$ behaviour of form factors could be reliably calculated
  - Exposed two weaknesses in rudimentary Ansatz
    - Diquark effectively pointlike to hard probe
    - Didn’t account for diquark being off-shell in recoil
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

![Graph showing $G_E/G_M$ Ratios vs. $Q^2$ (GeV$^2$)]
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

Proton – zero shifted:

6.5 → 8.0 GeV$^2$
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

Proton – zero shifted:
6.5 → 8.0 GeV^2

\[
\begin{align*}
\frac{G_E}{G_M} & \approx 1.0 \\
\mu_p \times \frac{G_E^p}{G_M^p} & \approx 0.5 \\
\mu_n \times \frac{G_E^n}{G_M^n} & \approx 0.0
\end{align*}
\]
**Improved current**

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

**Proton** – zero shifted:
6.5 → 8.0 GeV

**Neutron** – peak shifted:
7.5 → 5.0 GeV

& now predict zero a little above 11 GeV

---

![Graph showing the ratio of electric to magnetic form factors for proton and neutron in Q^2 space.](graph.png)
**Improved current**

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

- Proton – zero shifted: $6.5 \rightarrow 8.0 \text{ GeV}^2$
- Neutron – peak shifted: $7.5 \rightarrow 5.0 \text{ GeV}^2$
  & now predict zero a little above $11 \text{ GeV}^2$
Comparison between Faddeev equation result and Kelly’s parametrisation.

Graph showing the comparison of $F_1^\text{n}$ with $Q^2/M_N^2$.

- Red line: Kelly Parametrisation
- Green line: DSE
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core

Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4 M_N^2$
Epilogue
Epilogue
DCSB exists in QCD.
Epilogue

- **DCSB** exists in QCD.
- It is manifest in dressed propagators and vertices
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks
  - pseudoscalar mesons are unnaturally light
  - pseudoscalar mesons couple unnaturally strongly to light-quarks
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that:
  - light current-quarks become heavy constituent-quarks
  - pseudoscalar mesons are unnaturally light
  - pseudoscalar mesons couple unnaturally strongly to light-quarks
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
- It impacts dramatically upon observables.
Epilogue

- Form Factors - progress anticipated in near- to medium-term
- Quantifying pseudoscalar meson “cloud” effects
Epilogue

Form Factors - progress anticipated in near- to medium-term

- Quantifying pseudoscalar meson “cloud” effects
- Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
Epilogue

Form Factors - progress anticipated in near- to medium-term
- Quantifying pseudoscalar meson “cloud” effects
- Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
- Explaining the high $Q^2$ behavior of the proton form factor ratio in the space-like region
Epilogue

Form Factors - progress anticipated in near- to medium-term
- Quantifying pseudoscalar meson “cloud” effects
- Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
- Explaining the high $Q^2$ behavior of the proton form factor ratio in the space-like region
- Detailing broadly the role of two-photon exchange contributions
Epilogue

Form Factors - progress anticipated in near- to medium-term
- Quantifying pseudoscalar meson “cloud” effects
- Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
- Explaining the high $Q^2$ behavior of the proton form factor ratio in the space-like region
- Detailing broadly the role of two-photon exchange contributions
- Explaining relationship between parton properties on the light-front and rest frame structure of hadrons
Contents

1. Universal Truths
2. QCD’s Challenges
3. Why?
4. Some Questions
5. Status
6. Dichotomy of the Pion
7. Pion Form Factors
8. Dressed-Quark Propagator
9. Hadrons
10. Bethe-Salpeter Kernel
11. Pion FF
12. Calculated Pion FF
13. Nucleon Challenge
14. Nucleon EM Form Factors
15. Faddeev equation
16. Diquark correlations
17. Nucleon-Photon Vertex
18. Form Factor Ratio
19. Improved current
20. Pion Cloud
21. Dyson-Schwinger Equations
22. Schwinger Functions
23. Persistent Challenge
24. Quenched-QCD
25. Frontiers of Nuclear Science
26. $r_\pi f_\pi$
27. Two-photon Couplings
28. Two-photon Results
29. Pions and Form Factors
30. Masses: Nucleon and $\Delta$
31. DIS Pion
Dyson-Schwinger Equations
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  
  Materially Reduces Model Dependence
Dyson-Schwinger Equations

Well suited to Relativistic Quantum Field Theory

Simplest level: Generating Tool for Perturbation Theory

Materially Reduces Model Dependence

NonPerturbative, Continuum approach to QCD
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  Hadrons as Composites of Quarks and Gluons
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  - Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?
**Dyson-Schwinger Equations**

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  
  Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  
  Hadrons as Composites of Quarks and Gluons
  
  Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    
    - Coloured objects not detected, not detectable?
  
  ⇒ Understanding InfraRed (long-range)

  .......................................................... behaviour of $\alpha_s(Q^2)$
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
- Materially Reduces Model Dependence

NonPerturbative, Continuum approach to QCD

- Hadrons as Composites of Quarks and Gluons

  Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?

- Method yields Schwinger Functions $\equiv$ Propagators
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions
Solutions are Schwinger Functions
(Euclidean Green Functions)
Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable
Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.
Persistent Challenge

Infinitely Many Coupled Equations
Persistent Challenge

- Infinitely Many Coupled Equations

Coupling between equations necessitates truncation
Persistent Challenge

- Infinitely Many Coupled Equations

- Coupling between equations necessitates truncation

- Weak coupling expansion $\Rightarrow$ Perturbation Theory
Infinitely Many Coupled Equations

Coupling between equations necessitates truncation

Weak coupling expansion $\Rightarrow$ Perturbation Theory
Not useful for the nonperturbative problems in which we’re interested
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme


*Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*


*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors
Quenched-QCD

Dressed-Quark Propagator

\( M(p) \)

\( Z(p) \)
"data:” Quenched Lattice Meas.
- Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
"data:" Quenched Lattice Meas.
– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)
current-quark masses: 30 MeV, 50 MeV, 100 MeV
“data:” Quenched Lattice Meas.
- Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
current-quark masses: 30 MeV, 50 MeV, 100 MeV

Curves: Quenched DSE Cal.
“data:” Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)
current-quark masses: 30 MeV, 50 MeV, 100 MeV

Curves: Quenched DSE Cal.


Linear extrapolation of lattice data to chiral limit is inaccurate
\[ \Sigma = D \gamma \Gamma \]

Gap Equation
Frontiers of Nuclear Science: Theoretical Advances

Gap Equation

\[ \Sigma = \Gamma \]

\[ \gamma \quad S \]

Rapid acquisition of mass is the effect of gluon cloud

- \( m = 0 \) (Chiral limit)
- \( m = 30 \text{ MeV} \)
- \( m = 70 \text{ MeV} \)
Mass from nothing.

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.
Mass from nothing.

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies \((m = 0\), red curve) acquires a large constituent mass at low energies.
Dimensionless product: $r_\pi f_\pi$
Dimensionless product: $r_\pi f_\pi$
Dimensionless product: $r_\pi f_\pi$

- Improved rainbow-ladder interaction
Dimensionless product: $r_\pi f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
Dimensionless product: $r_\pi f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Great strides towards placing nucleon studies on same footing as mesons
Dimensionless product: $r_\pi f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$
**Dimensionless product:** \( r_\pi f_\pi \)

- Improved rainbow-ladder interaction
- Repeating \( F_\pi(Q^2) \) calculation
- Experimentally: \( r_\pi f_\pi = 0.315 \pm 0.005 \)

![Graph showing \( r_\pi f_\pi \) vs \( m_\pi^2 [GeV^2] \)]
Dimensionless product: $r_\pi f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

- DSE prediction
- Lattice results
  – James Zanotti [UK QCD]

![Graph showing $r_\pi f_\pi$ versus $m_\pi^2[GeV^2]$]
Dimensionless product: \( r_\pi f_\pi \)

- Improved rainbow-ladder interaction
- Repeating \( F_\pi(Q^2) \) calculation
- Experimentally: \( r_\pi f_\pi = 0.315 \pm 0.005 \)

DSE prediction

Lattice results
- James Zanotti [UK QCD]

Fascinating result:
DSE and Lattice
- Experimental value obtains independent of current-quark mass.

![Graph showing \( r_\pi f_\pi \) vs. \( m_\pi^2 [GeV^2] \)]
**Dimensionless product**: $r_{\pi} f_{\pi}$

- Improved rainbow-ladder interaction
- Repeating $F_{\pi}(Q^2)$ calculation
- Experimentally: $r_{\pi} f_{\pi} = 0.315 \pm 0.005$

**DSE prediction**

- Fascinating result: DSE and Lattice – Experimental value obtains independent of current-quark mass.
- Potentially useful but must first be understood.

![Graph showing $r_{\pi} f_{\pi}$ vs. $m_{\pi}^2$]
Two-photon Couplings of Pseudoscalar Mesons


\[ T_{\pi_0^0}(k_1, k_2) = \frac{\alpha}{\pi} i \epsilon_{\mu \nu \rho \sigma} k_1^\rho k_2^\sigma G_{\pi_0^0}(k_1, k_2) \]

Define: \( T_{\pi_0^0}(P^2, Q^2) = G_{\pi_0^0}(k_1, k_2) \bigg|_{k_1^2 = Q^2 = k_2^2} \)

This is a transition form factor.

Physical Processes described by couplings:

\( g_{\pi_0^0 \gamma \gamma} := T_{\pi_0^0}(-m_{\pi_0^0}^2, 0) \)

Width: \( \Gamma_{\pi_0^0 \gamma \gamma} = \alpha_{em}^2 \frac{m_{\pi_0^0}^3}{16\pi^3} g_{\pi_0^0 \gamma \gamma}^2 \)
Two-photon Couplings: Goldstone Mode

Höll, Krassnigg, Maris, et al.,
“Electromagnetic properties of ground and excited state pseudoscalar mesons,” nu-th/0503043

\[
\pi_0^0(P) \rightarrow \gamma(k_1) \rightarrow \gamma(k_2)
\]

\[
T_{\mu\nu}^{\pi_0^0}(k_1, k_2) = \frac{\alpha}{\pi} i \varepsilon_{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma G^{\pi_0^0}(k_1, k_2)
\]

Chiral limit, model-independent and algebraic result

\[
g^{\pi_0^0\gamma\gamma} := T^{\pi_0^0}(-m_{\pi_0^0}^2 = 0, 0) = \frac{1}{2} \frac{1}{f_{\pi_0}}
\]

So long as truncation veraciously preserves chiral symmetry and the pattern of its dynamical breakdown

The most widely known consequence of the Abelian anomaly

Two-photon Couplings:
Transition Form Factor

\[ \pi_0^0 (P) \rightarrow \gamma (k_2), k_1^2 = Q^2 \]

\[ \gamma (k_1), k_1^2 = Q^2 \]

\[ T_{\mu \nu}^{\pi_0^0} (k_1, k_2) = \frac{\alpha}{\pi} i \varepsilon_{\mu \nu \rho \sigma} k_1^\rho k_2^\sigma G^{\pi_0^0} (k_1, k_2) \]

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – \( \forall n \):

\[ T_{\pi_0^0} (P^2, Q^2) = G^{\pi_0^0} (k_1, k_2) \bigg|_{k_1^2 = Q^2 = k_2^2} \]

\[ Q^2 \gg \Lambda_{QCD}^2 \quad \Rightarrow \quad \frac{4 \pi^2}{3} \frac{f_{\pi n}}{Q^2} \]
Two-photon Couplings: Transition Form Factor

Maris and Tandy, “Electromagnetic transition form-factors of light mesons,” nucl-th/0201017

![Graph showing the transition form factor $F(Q^2)$ versus $Q^2$ in GeV$^2$, with data points for CELLO and CLEO, and a DSE calculation line.](image-url)
Two-photon Couplings: Transition Form Factor

Maris and Tandy, “Electromagnetic transition form-factors of light mesons,” nucl-th/0201017

DSE result:
- normalisation calculated
- $\rho$-meson generated dynamically
- pQCD result accurate to $\sim 20\%$ or better for $Q^2 \geq 3 \text{ GeV}^2$
Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


Pion cloud effects are large in the low $Q^2$ region.

Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor $G_D$. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.
Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


- Pion cloud effects are large in the low $Q^2$ region.

**Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor $G_D$.**

Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.

- Quark Core
  - Responsible for only 2/3 of result at small $Q^2$
  - Dominant for $Q^2 > 2 – 3$ GeV$^2$
Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for “$\pi$-cloud” contributions

<table>
<thead>
<tr>
<th>set</th>
<th>$M_N$</th>
<th>$M_\Delta$</th>
<th>$m_0^+$</th>
<th>$m_1^+$</th>
<th>$\omega_{0^+}$</th>
<th>$\omega_{1^+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.94</td>
<td>1.23</td>
<td>0.63</td>
<td>0.84</td>
<td>0.44=1/(0.45 fm)</td>
<td>0.59=1/(0.33 fm)</td>
</tr>
<tr>
<td>B</td>
<td>1.18</td>
<td>1.33</td>
<td>0.80</td>
<td>0.89</td>
<td>0.56=1/(0.35 fm)</td>
<td>0.63=1/(0.31 fm)</td>
</tr>
</tbody>
</table>

$m_{1^+} \to \infty$: $M_N^A = 1.15 \text{GeV}; M_N^B = 1.46 \text{GeV}$
Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

Set A – fit to the actual masses was required; whereas for Set B – fitted mass was offset to allow for “$\pi$-cloud” contributions

<table>
<thead>
<tr>
<th>set</th>
<th>$M_N$</th>
<th>$M_\Delta$</th>
<th>$m_{0+}$</th>
<th>$m_{1+}$</th>
<th>$\omega_{0+}$</th>
<th>$\omega_{1+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.94</td>
<td>1.23</td>
<td>0.63</td>
<td>0.84</td>
<td>0.44=1/(0.45 fm)</td>
<td>0.59=1/(0.33 fm)</td>
</tr>
<tr>
<td>B</td>
<td>1.18</td>
<td>1.33</td>
<td>0.80</td>
<td>0.89</td>
<td>0.56=1/(0.35 fm)</td>
<td>0.63=1/(0.31 fm)</td>
</tr>
</tbody>
</table>

$m_{1+} \rightarrow \infty$: $M_N^A = 1.15 \text{ GeV}$; $M_N^B = 1.46 \text{ GeV}$

Axial-vector diquark provides significant attraction
Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for Set B – fitted mass was offset to allow for “π-cloud” contributions

<table>
<thead>
<tr>
<th>set</th>
<th>$M_N$</th>
<th>$M_\Delta$</th>
<th>$m_{0+}$</th>
<th>$m_{1+}$</th>
<th>$\omega_{0+}$</th>
<th>$\omega_{1+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.94</td>
<td>1.23</td>
<td>0.63</td>
<td>0.84</td>
<td>0.44 = 1/(0.45 fm)</td>
<td>0.59 = 1/(0.33 fm)</td>
</tr>
<tr>
<td>B</td>
<td>1.18</td>
<td>1.33</td>
<td>0.80</td>
<td>0.89</td>
<td>0.56 = 1/(0.35 fm)</td>
<td>0.63 = 1/(0.31 fm)</td>
</tr>
</tbody>
</table>

$m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Constructive Interference: $1^{++}$-diquark $+ \partial_\mu \pi$
Deep-inelastic scattering
Looking for Quarks
Deep-inelastic scattering

- Looking for Quarks
Deep-inelastic scattering

Looking for Quarks

Signature Experiment for QCD:
Discovery of Quarks at SLAC
Deep-inelastic scattering

- Looking for Quarks

Signature Experiment for QCD:
Discovery of Quarks at SLAC

Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x), g(x)$
Pion's valence quark distn
Pion’s valence quark distn

- $\pi$ is Two-Body System: “Easiest” Bound State in QCD
- However, NO $\pi$ Targets!
Pion’s valence quark distn

- \( \pi \) is Two-Body System: “Easiest” Bound State in QCD
- However, NO \( \pi \) Targets!
- Existing Measurement Inferred from Drell-Yan:
  \[ \pi N \rightarrow \mu^+ \mu^- X \]
Pion’s valence quark distn

- $\pi$ is Two-Body System: “Easiest” Bound State in QCD
- However, NO $\pi$ Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \rightarrow \mu^+ \mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^{-5 \text{GeV}} - p_{25 \text{GeV}}$ Collider → Accurate “Measurement”
Handbag diagrams
Handbag diagrams

\[ W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right] \]

\[ T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_\pi \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) i e Q \Gamma_\nu(k_{-0}, k) \]
\[ \times S(k) i e Q \Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_\pi \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{--}) \]
Bjorken Limit: \( q^2 \to \infty \), \( P \cdot q \to -\infty \)

but \( x := -\frac{q^2}{2P \cdot q} \) fixed.

Numerous algebraic simplifications

\[
W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P) \right]
\]

\[
T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_- \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) i e Q \Gamma_\nu(k_{-0}, k) \\
\times S(k) i e Q \Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{-0})
\]
Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)

\[ x u(x) \]

- E615 πN Drell–Yan 4GeV
- NLO Analysis of E615 ... \( \beta = 1.87 \)
- DSE ... \( \beta = 2.61 \)
- NJL ... \( \beta = 1.27 \)