QCD’s Challenges

- Quark and Gluon Confinement
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Very unnatural pattern of bound state masses

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QCD’s Challenges

Understand Emergent Phenomena

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Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.

- QCD – Complex behaviour arises from apparently simple rules
Dichotomy of the Pion
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How does one make an almost massless particle from two massive constituent-quarks?
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Current Algebra ... 1968
Dichotomy of the Pion

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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

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Dichotomy of the Pion

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Highly Nontrivial
Minimal requirements

- detailed understanding of connection between Current-quark and Constituent-quark masses;
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Why problematic? Isn’t same true in quantum mechanics?
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- Why problematic? Isn't same true in quantum mechanics?
- Differences!
What’s the Problem?

**Relativistic QFT!**

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- **Differences!**
  - Here relativistic effects are crucial – *virtual particles*, quintessence of **Relativistic Quantum Field Theory** – must be included.
What’s the Problem?

Relativistic QFT!

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Differences!

- Here relativistic effects are crucial – virtual particles, quintessence of Relativistic Quantum Field Theory – must be included
- Interaction between quarks – the Interquark Potential – unknown throughout > 98% of an hadron’s volume
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
The question must be rigorously defined, and the answer mapped out using experiment and theory.
Dyson-Schwinger Equations
Dyson-Schwinger Equations

Well suited to Relativistic Quantum Field Theory
Dyson-Schwinger Equations

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- Simplest level: Generating Tool for Perturbation Theory

.......................... Materially Reduces Model Dependence
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  Qualitative and Quantitative Importance of:
  
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    - Generation of fermion mass from *nothing*
  - **Quark** & **Gluon** Confinement
    - Coloured objects not detected, not detectable?
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  ⇒ Understanding InfraRed (long-range)

  ................................................... behaviour of $\alpha_s(Q^2)$
**Dyson-Schwinger Equations**

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- Method yields Schwinger Functions $\equiv$ Propagators
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Cross-Sections built from Schwinger Functions
Schwinger Functions
Solutions are Schwinger Functions (Euclidean Green Functions)
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- Proving fruitful.
World...

DSE Perspective
Persistent Challenge
Persistent Challenge

- Infinitely Many Coupled Equations

\[
\Sigma = \begin{array}{c}
\gamma \\
S \\
\Gamma
\end{array}
\]
Persistent Challenge

- Infinitely Many Coupled Equations

- Coupling between equations necessitates truncation
Persistent Challenge

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- Weak coupling expansion ⇒ Perturbation Theory
Persistent Challenge

- **Infinitely Many Coupled Equations**

- Coupling between equations *necessitates* truncation

- Weak coupling expansion $\Rightarrow$ Perturbation Theory
  
  *Not useful* for the nonperturbative problems in which we’re interested
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  *Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
Persistent Challenge

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- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors
Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i \gamma \cdot p + M(p^2)}$$

Gap Equation
Dressed-Quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

- Gap Equation’s Kernel Enhanced on IR domain

\[ \Rightarrow \text{IR Enhancement of } M(p^2) \]
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Euclidean Constituent–Quark Mass: \( M_f^E: p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
<th>( \frac{M_f^E}{m_\zeta} )</th>
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Predictions confirmed in numerical simulations of lattice-QCD
• Established understanding of two- and three-point functions
Hadrons

- Established understanding of two- and three-point functions
- What about bound states?
• Without bound states, Comparison with experiment is impossible
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• They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
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QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$?
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QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$?
What is the light-quark Long-Range Potential?
Bethe-Salpeter Kernel
**Bethe-Salpeter Kernel**

**Axial-vector Ward-Takahashi identity**

\[
P_{\mu} \Gamma_{5\mu}^{l}(k; P) = S^{-1}(k_{+}) \frac{1}{2} \lambda_{f}^{l} i\gamma_{5} + \frac{1}{2} \lambda_{f}^{l} i\gamma_{5} S^{-1}(k_{-})
\]

\[-M_{\zeta} i\Gamma_{5}^{l}(k; P) - i\Gamma_{5}^{l}(k; P) M_{\zeta}\]

**QFT Statement of Chiral Symmetry**
Bethe-Salpeter Kernel

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Satisfies BSE

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Kernels must be intimately related
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Satisfies BSE  Satisfies DSE

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- Nontrivial constraint
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Kernels must be intimately related

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- Failure \implies Explicit Violation of QCD’s Chiral Symmetry
Radial Excitations & Chiral Symmetry
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\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ \mathcal{M}_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \ m_H^2 = - \rho^H \ M_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \rho^H \zeta \quad \mathcal{M}_H \]

\[ \mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M_\mu \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \)
Radial Excitations & Chiral Symmetry

\[ f_H m_H^2 = - \rho_H^H M_H \]

\[ f_H p_\mu = Z_2 \int_\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu S(q_+) \Gamma_H(q; P) S(q_-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant
Radial Excitations
& Chiral Symmetry

\[ f_H \quad m_H^2 = -\rho_H^H \mathcal{M}_H \]

\[ i\rho_H^H = Z_4 \int_q ^\Lambda \frac{1}{2} \text{tr} \left\{ (T_H^H)^t \gamma_5 S(q+) \Gamma_H(q; P) S(q-) \right\} \]

- **Pseudoscalar** projection of BS wave function at \( x = 0 \)

\[ \pi \quad -\rho_\pi \quad \vec{P}_5 \quad \equiv \quad \vec{i} \Gamma_5 \quad i(\tau/2) \gamma_5 \]
Light-quarks; i.e., $m_q \sim 0$

$f_H \rightarrow f_H^0$ and $\rho^H_\zeta \rightarrow -\frac{\langle \bar{q}q \rangle^0_\zeta}{f_H^0}$, Independent of $m_q$

Hence

$$m_H^2 = -\frac{\langle \bar{q}q \rangle^0_\zeta}{(f_H^0)^2} m_q \ldots \text{GMOR relation, a corollary}$$
\[ f_H \ m_H^2 = - \ \rho_\zeta^H \ \mathcal{M}_H \]

- Valid for ALL Pseudoscalar mesons
Radial Excitations & Chiral Symmetry

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- Valid for **ALL** Pseudoscalar mesons
- \( \rho_H \to \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \to 0 \)
Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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  - \( \Rightarrow f_H = 0 \)
- **ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit
Valid for ALL Pseudoscalar mesons

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Dynamical Chiral Symmetry Breaking

– Goldstone’s Theorem –

impacts upon every pseudoscalar meson
Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Craig Roberts: Dynamics, Symmetries, and Hadron Properties
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$\Rightarrow f_{\pi_1} < 8.4 \text{ MeV}$

Diehl & Hiller

he-ph/0105194
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Lattice-QCD check:
\[ 16^3 \times 32, \]
\[ a \sim 0.1 \text{ fm}, \]
two-flavour, unquenched

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Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators).
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$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$

The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.
Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.
**Pion ... \( J = 0 \)**

but ...

**Pseudoscalar meson Bethe-Salpeter amplitude**

\[
\chi_\pi(k; P) = \gamma_5 \left[ i\mathcal{E}_\pi(k; P) + \gamma \cdot P \mathcal{F}_\pi(k; P) \right] + \gamma \cdot k \cdot P \mathcal{G}_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_\pi(k; P)
\]
Pion \( J = 0 \) but...

Pseudoscalar meson Bethe-Salpeter amplitude

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\left. \gamma \cdot k k \cdot P G_{\pi n}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi n}(k; P) \right]
\]

\( J = 0 \) but while \( E \) and \( F \) are purely \( L = 0 \) in the rest frame, the \( G \) and \( H \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function always contains both \( S \)- and \( P \)-wave components.
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Introduce mixing angle $\theta_\pi$ such that

$$
\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
$$
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$L$ is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.
\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+)i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
\[ -2i M^{ab} \Gamma^b_5(k; P) - A^a(k; P) \]
Charge Neutral Pseudoscalar Mesons

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\(\{F^a | a = 0, \ldots, N_f^2 - 1\}\) are the generators of \(U(N_f)\)
Charge Neutral
Pseudoscalar Mesons

\[ \begin{align*}
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&\quad - 2i M^{ab} \Gamma_5 (k; P) - A^a (k; P)
\end{align*} \]

- \{ F^a | a = 0, \ldots, N_f^2 - 1 \} are the generators of \( U(N_f) \)
- \( S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \)
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_5^a(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 \mathcal{F}^a S^{-1}(k_-) \]

\[ -2i M^{ab} \Gamma_5^b(k; P) - A^a(k; P) \]

\{ \mathcal{F}^a | a = 0, \ldots, N_f^2 - 1 \} are the generators of \( U(N_f) \)

\( S = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \)

\( M^{ab} = \text{tr}_F \left[ \{ \mathcal{F}^a, \mathcal{M} \} \mathcal{F}^b \right] \),

\( \mathcal{M} = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \) matrix of current-quark bare masses
\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+)i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
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- \( \mathcal{M}^{ab} = \text{tr}_F \left[ \{F^a, \mathcal{M}\} F^b \right] \)
- \( \mathcal{M} = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses} \)
- The final term in the second line expresses the non-Abelian axial anomaly.
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_5^a (k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) - 2i M^{ab} \Gamma_5^b (k; P) - A^a (k; P) \]

\[ A^a (k; P) = S^{-1}(k_+) \delta^{a0} A_U (k; P) S^{-1}(k_-) \]

\[ A_U (k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle F_0 q(x) Q(0) \bar{q}(y) \rangle \]
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\[ Q(x) = i\frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x) \]

... The topological charge density operator.
(Trace is over colour indices & \( F_{\mu\nu} = \frac{1}{2} \lambda^a F^a_{\mu\nu} \).)
Charge Neutral

Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]

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... The topological charge density operator.

Important that only \( A^{a=0} \) is nonzero.
Charge Neutral 
Pseudoscalar Mesons

\[ P_\mu \Gamma_{5\mu}^a (k; P) = S^{-1} (k_+ ) i \gamma_5 F^a + i \gamma_5 F^a S^{-1} (k_-) \]
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\[ \ldots \text{ The topological charge density operator.} \]

\[ \text{NB. While } Q (x) \text{ is gauge invariant, the associated Chern-Simons current, } K_\mu, \text{ is not } \Rightarrow \text{ in QCD no physical} \]
\[ \text{boson can couple to } K_\mu \text{ and hence no physical states can} \]
\[ \text{contribute to resolution of } U_A (1) \text{ problem.} \]
Only $\mathcal{A}^0 \neq 0$ is interesting
Only $A^0 \neq 0$ is interesting . . . otherwise all pseudoscalar mesons are Goldstone Modes!
Anomaly term has structure

\[ A^0(k; P) = F^0 \gamma_5 [iE_A(k; P) + \gamma \cdot P F_A(k; P) + \gamma \cdot k k \cdot P G_A(k; P) + \sigma_{\mu \nu} k_\mu P_\nu H_A(k; P)] \]
AVWTI gives generalised Goldberger-Treiman relations

\[ 2 f_{\eta'}^0 E_{BS}(k; 0) = 2 B_0(k^2) - \mathcal{E}_A(k; 0), \]
\[ F_R^0(k; 0) + 2 f_{\eta'}^0 F_{BS}(k; 0) = A_0(k^2) - \mathcal{F}_A(k; 0), \]
\[ G_R^0(k; 0) + 2 f_{\eta'}^0 G_{BS}(k; 0) = 2 A'_0(k^2) - \mathcal{G}_A(k; 0), \]
\[ H_R^0(k; 0) + 2 f_{\eta'}^0 H_{BS}(k; 0) = -\mathcal{H}_A(k; 0), \]

\( A_0, \ B_0 \) characterise gap equation’s chiral limit solution.
Charge Neutral Pseudoscalar Mesons

AVWTI gives generalised Goldberger-Treiman relations

\[ 2f_{\eta'} E_{BS}(k; 0) = 2B_0(k^2) - \mathcal{E}_A(k; 0), \]
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\( A_0, B_0 \) characterise gap equation’s chiral limit solution.

Follows that \( \mathcal{E}_A(k; 0) = 2B_0(k^2) \) is necessary and sufficient condition for absence of massless \( \eta' \) bound-state.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

\[ B_0(k^2) \neq 0 \text{ if, and only if, chiral symmetry is dynamically broken.} \]

Hence, absence of massless $\eta'$ bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
\[ \varepsilon_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

- \( B_0(k^2) \neq 0 \) if, and only if, chiral symmetry is dynamically broken.
- Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Further highlighted . . . proved

\[ \langle \bar{q}q \rangle^0_\zeta = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \text{tr}_{CD} \int_q^\Lambda S^0(q, \zeta) \]

\[ = N_f \int d^4x \langle \bar{q}(x)i\gamma_5q(x)Q(0) \rangle^0. \]
AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Employed in an analysis of pseudoscalar- and vector-meson bound-states
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts

- \( \eta - \eta' \) mixing angles of \( \sim -15^\circ \) (Expt.: \( -13.3^\circ \pm 1.0^\circ \))
- \( \pi^0 - \eta \) angles of \( \sim 1.2^\circ \) (Expt. \( p d \rightarrow ^3\text{He} \pi^0: 0.6^\circ \pm 0.3^\circ \))
- Strong neutron-proton mass difference . . .
  \( \lesssim 75\% \) current-quark mass-difference
New Challenges
New Challenges

Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
New Challenges

- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

- Move on to the problem of a *symmetry preserving* treatment of hybrids and exotics.
Another Direction . . . Also want/need information about three-quark systems
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
Nucleon EM Form Factors: A Précis

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• Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
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- Excellent mass spectrum (octet and decuplet)

**Easily obtained:**

\[
\left( \frac{1}{N_H} \sum_H \left[ \frac{M_{H}^{\text{exp}} - M_{H}^{\text{calc}}}{M_{H}^{\text{exp}}} \right]^2 \right)^{1/2} = 2\% 
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  (Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)
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- But is that good?

  - Cloudy Bag: \( \delta M_+^{\text{π-loop}} = -300 \) to \(-400\) MeV!

- **Critical** to anticipate pion cloud effects

Roberts, Tandy, Thomas, et al., nu-th/02010084
Faddeev equation
Faddeev equation

\[ \Psi^a \Gamma^a \rightarrow \Psi^b \Gamma^b = \]

\[ p_q \]
\[ p_d \]
\[ P \]
\[ = \]
\[ p_q \]
\[ p_d \]
\[ P \]
Faddeev equation

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks ... In Nucleon’s Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations
Diquark correlations
Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red

Confined ... Does not escape from within baryon.

Scalar is isosinglet, Axial-vector is isotriplet

DSE and lattice-QCD

\[ m_{[ud]}^{0+} = 0.74 - 0.82 \]

\[ m_{(uu)}^{1+} = m_{(ud)}^{1+} = m_{(dd)}^{1+} = 0.95 - 1.02 \]
Results: Nucleon and $\Delta$ Masses
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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for “$\pi$-cloud” contributions

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$m_{1+} \to \infty$: $M_{N}^{A} = 1.15$ GeV; $M_{N}^{B} = 1.46$ GeV
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Axial-vector diquark provides significant attraction
Results: Nucleon and Δ Masses

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Constructive Interference: $1^{++}$-diquark + $\partial_\mu \pi$
Nucleon-Photon Vertex
Nucleon-Photon Vertex

M. Oettel, M. Pichowsky and L. von Smekal, nu-th/9909082

6 terms . . . constructed systematically . . . current conserved automatically for on-shell nucleons described by Faddeev Amplitude
Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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Form Factor Ratio: $\frac{G_E}{G_M}$

\[ \mu_p \frac{G_E^p}{G_M^p} \]

- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer

\[ Q^2 \text{ [GeV}^2\text{]} \]
Form Factor Ratio: GE/GM

Combine these elements . . .

![Graph showing data points and error bars for the ratio of muon polarization transfer and Rosenbluth precision. The x-axis represents Q^2 in [GeV^2] and the y-axis represents \( \mu_p G_E^P / G_M^P \).]
Combine these elements . . .

Dressed-Quark Core

Form Factor Ratio: GE/GM

\[ \mu_{p} \frac{G_{E}^{p}}{G_{M}^{p}} \]

Rosenbluth
precision Rosenbluth
polarization transfer
polarization transfer

Craig Roberts: Dynamics, Symmetries, and Hadron Properties
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi**
  Identity preserving current

![Graph](image)

- **Form Factor Ratio:**
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Craig Roberts: Dynamics, Symmetries, and Hadron Properties
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All parameters fixed in other applications . . . Not varied.
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- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2 \text{ GeV}^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
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- Predict Zero at $Q^2 \approx 6.5$ GeV$^2$
**Quark Distribution Functions**

**DIS**

\[ \ell' \quad k', s' \]

\[ \ell \quad k, s \]

**SI–DIS**

\[ \ell' \quad k', s' \]

\[ \ell \quad k, s \]

\[ q \quad \theta \]

\[ X \]

\[ A \quad P, S \]

\[ P_X \]

\[ P_h \]
Three twist-2 parton distributions ($k_\perp = 0$):

- Spin-Independent: $q(x)$
- Helicity: $\Delta q(x)$
- Transversity: $\Delta_T q(x)$

All distributions have probability interpretation.

By definition, contain essentially non-perturbative information about a given process.
Light-cone Fourier transforms:

\[ \Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i x p^+ \xi^-} \langle p, s | \bar{\psi}_q(0) \gamma^+ \gamma^1 \gamma_5 \psi_q(\xi^-) | p, s \rangle_c \]

\[ q(x) = \langle \gamma^+ \rangle, \quad \Delta q(x) = \langle \gamma^+ \gamma_5 \rangle \]

Related to the nucleon axial & tensor charges via

\[ g_A = \int dx [\Delta u(x) - \Delta d(x)], \quad g_T = \int dx [\Delta_T u(x) - \Delta_T d(x)], \]

Must satisfy: positivity constraints and Soffer bound

\[ \Delta q(x), \Delta_T q(x) \leq q(x), \quad q(x) + \Delta q(x) \geq 2 |\Delta_T q(x)| \]
Once more on the one that got away.
Model predictions

Cloët, Bentz, Thomas

Model predictions

- Simplified Faddeev equation

Satisfy: Soffer bound, baryon & momentum SRs.
**Model predictions**

- **Simplified Faddeev equation**

- **Satisfy: Soffer bound, baryon & momentum SRs.**

- **Moments at** $Q^2 = 0.16$ GeV$^2$:
  
  \[
  \Delta u = 0.97, \quad \Delta d = -0.30 \quad \implies \quad g_A = 1.267 \\
  \Delta_T u = 1.04, \quad \Delta_T d = -0.24 \quad \implies \quad g_T = 1.28
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  Model constraint
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  \[ \Delta q(x) \sim \Delta_T q(x) \text{ in valence region for } Q^2 \lesssim 10 \text{ GeV}^2 \]
Epilogue
DCSB exists in QCD.
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
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- Confinement
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations
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- **DCSB exists in QCD.**
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.

- **Confinement**
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations
  - DSEs ... contemporary tool that describes and explains these phenomena, and connects them with prediction of observables
Quenched-QCD

Dressed-Quark Propagator

$M(p)$

$Z(p)$
Quenched-QCD

Dressed-Quark Propagator

2002

$M(p)$

$Z(p)$

“data”: Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
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Curves: Quenched DSE Cal.
- Bhagwat, Pichowsky, Roberts, Tandy [nu-th/0304003](nu-th/0304003)
"data:" Quenched Lattice Meas.
- Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)

current-quark masses: 30 MeV, 50 MeV, 100 MeV

Curves: Quenched DSE Cal.

Linear extrapolation of lattice data to chiral limit is inaccurate
Kernel of Gap Equation: $D_{\mu\nu}(p - q) \Gamma_\nu(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

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Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:


Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:


Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
Dressed-gluon Propagator

\[ D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

Suppression means \( \exists \) IR gluon mass-scale \( \approx 1 \text{ GeV} \)

Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)

- lattice, \( N_f=0 \)
- \( N_f=0 \) DSE
- \( N_f=3 \) DSE
- Fit to DSE, \( N_f=3 \)
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Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass
Lei Chang, Yu-Xin Liu, Mandar S. Bhagwat, Craig D. Roberts and Stewart V. Wright … nucl-th/0605058
Critical Mass for Chiral Expansion

Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
Critical Mass for Chiral Expansion

Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

- Wigner solution and one DCSB solution are destabilised by current-quark mass & both disappear when that mass exceeds a critical value, $m_{cr}$

The zeros of $G(M)$ characterise the different solutions of the gap equation. Solid curve: obtained with $m^{bm} = 0$, in which case $G(M)$ is odd under $M \to -M$; long-dashed curve: $m^{em} = 0.01$; short-dashed curve: $m^{bm} = m^{em} = 0.033$; dotted curve: $m^{bm} = 0.05$. 
Critical Mass for Chiral Expansion

Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

$m_{cr}$ also bounds domain on which surviving DCSB solution possesses a chiral expansion: $m_{cr} = \lim_{n \to \infty} \left( \frac{1}{a_n} \right)^{1/n}$
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For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{0-}^{cr} \sim 0.45$ GeV.
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- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass \( m_{cr}^{0-} \sim 0.45 \text{ GeV} \).

- Entails lattice-QCD simulations must have many results at \( m_\pi < m_{0-}^{cr} \sim 0.45 \text{ GeV} \) for reliable extrapolation via EFT.
Realistic models of QCD’s gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.
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The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.

The behaviour of this condensate indicates that the essentially dynamical component of chiral symmetry breaking decreases with increasing current-quark mass.
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \ (M^E)^2 := s \mid s = M(s)^2.$$
**Consituent-quark $\sigma$-term**

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

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- Renormalisation-group-invariant and determined from solutions of the gap equation
Consituent-quark $\sigma$-term

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Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function
Constituent-quark $\sigma$-term

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$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$  

Ratio $\frac{\sigma_f}{M^E_f} = \frac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}$

measures effect of EXPLICIT chiral symmetry breaking on dressed-quark mass-function
cf. SUM of effects of EXPLICIT AND DYNAMICAL CHIRAL SYMMETRY BREAKING
Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$
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$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$ 

Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}$, $(M^E)^2 := s \mid s = M(s)^2$.

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass.
Crude estimate based on magnitudes $\Rightarrow$ probability for a $u$-quark to carry the proton’s spin is $P_{u\uparrow} \sim 80\%$, with $P_{u\downarrow} \sim 5\%$, $P_{d\uparrow} \sim 5\%$, $P_{d\downarrow} \sim 10\%$.

Hence, by this reckoning $\sim 30\%$ of proton’s rest-frame spin is located in dressed-quark angular momentum.
Neutron Form Factors

\[ \frac{\mu_n G^E_n}{G^M_n} \]

\( Q^2 \) [GeV^2]

nucl-ex 0308007
Neutron Form Factors

Expt. Madey, et al. nu-ex/0308007

\[ \frac{\mu_n G_E^n}{G_M^n} \]

\[ Q^2 \text{ [GeV}^2] \]
Neutron Form Factors

- Expt. Madey, et al. nu-ex/0308007
- Calc. Bhagwat, et al. nu-th/0610080

\[
\mu_p \frac{G^n_E(Q^2)}{G^n_M(Q^2)} = -\frac{r^2_n}{6} Q^2
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Valid for \( r^2_n Q^2 \lesssim 1 \)
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\[ \mu_p \frac{G^n_E(Q^2)}{G^n_M(Q^2)} = - \frac{r^2_n}{6} Q^2 \]
Valid for \( r^2_n Q^2 \lesssim 1 \)

No sign yet of a zero in \( G^n_E(Q^2) \), even though calculation predicts \( G^p_E(Q^2 \approx 6.5 \text{ GeV}^2) = 0 \)

Data to \( Q^2 = 3.4 \text{ GeV}^2 \) is being analysed (JLab E02-013)
Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. 45 (2000) S1

- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,

- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,

- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,

- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,