Hadron Physics & DSE Perspective

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Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
QCD’s Challenges

- **Quark and Gluon Confinement**
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- **Dynamical Chiral Symmetry Breaking**
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=−$
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- Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
QCD’s Challenges

Understand Emergent Phenomena

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Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.

- QCD – Complex behaviour
  
  arises from apparently simple rules
Dichotomy of Pion
– Goldstone Mode and Bound state
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How does one make an almost massless particle from two massive constituent-quarks?
Dichotomy of Pion – Goldstone Mode and Bound state

How does one make an almost massless particle from two massive constituent-quarks?

Not Allowed to do it by fine-tuning a potential

Must exhibit $m^2_\pi \propto m_q$

Current Algebra ... 1968
Dichotomy of Pion
– Goldstone Mode and Bound state

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  ............. from two **massive** constituent-quarks?

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*Current Algebra ... 1968*

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.
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dynamical chiral symmetry breaking.

Highly Nontrivial
What’s the Problem?
Minimal requirements

- detailed understanding of connection between Current-quark and Constituent-quark masses;
- and systematic, symmetry preserving means of realising this connection in bound-states.
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Why problematic? Isn’t same true in quantum mechanics?
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- Differences!
What’s the Problem?

Relativistic QFT!

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Relativistic QFT!

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- Differences!
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  - Interaction between quarks – the Interquark “Potential” – unknown throughout > 98% of a hadron’s volume
Intranucleon Interaction
Intranucleon Interaction
Intranucleon Interaction

98% of the volume
What is the Intraneucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.

98% of the volume
Dyson-Schwinger Equations
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Well suited to Relativistic Quantum Field Theory
Dyson-Schwinger Equations

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- Simplest level: Generating Tool for Perturbation Theory
  Materially Reduces Model Dependence
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- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    – Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    – Coloured objects not detected, not detectable?
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- Understanding InfraRed (long-range) behaviour of $\alpha_s(Q^2)$
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- Method yields Schwinger Functions $\equiv$ Propagators
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Cross-Sections built from Schwinger Functions
Schwinger Functions

Solutions are Schwinger Functions (Euclidean Green Functions)
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Not all are Schwinger functions are experimentally observable but...

- all are same VEVs measured in numerical simulations of lattice-regularised QCD

opportunity for comparisons at pre-experimental level... cross-fertilisation
Schwinger Functions

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- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.
Persistent Challenge
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Infinitely Many Coupled Equations
Persistent Challenge

- Infinitely Many Coupled Equations

- Coupling between equations necessitates truncation

\[
\Sigma = D \gamma S \Gamma
\]
Persistent Challenge

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  Weak coupling expansion $\Rightarrow$ Perturbation Theory
Persistent Challenge

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- Weak coupling expansion ⇒ Perturbation Theory
  Not useful for the nonperturbative problems in which we’re interested
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme


*Dynamic chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*


*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
Persistent Challenge

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- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors
Perturbative

Dressed-quark Propagator
Perturbative

Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma = \Gamma \gamma S \]

Gap Equation
Perturbative Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma \]

\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]

\[ \Gamma \]
**Perturbative**

**Dressed-quark Propagator**

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

**Gap Equation**

\[ \Sigma \]

**Weak Coupling Expansion**

Reproduces Every Diagram in Perturbation Theory
Perturbative Dressed-quark Propagator

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dressed-quark propagator

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Weak Coupling Expansion

Reproduces Every Diagram in Perturbation Theory

But in Perturbation Theory

\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \quad m \to 0 \]
Perturbative Dressed-quark Propagator

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Dressed-quark propagator

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- Gap Equation’s Kernel Enhanced on IR domain
  \( \Rightarrow \) IR Enhancement of \( M(p^2) \)
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**Euclidean Constituent–Quark Mass:** \( M_f^E: p^2 = M(p^2)^2 \)

<table>
<thead>
<tr>
<th>flavour</th>
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<th>( s )</th>
<th>( c )</th>
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<tbody>
<tr>
<td>( M_f^E / m_\zeta )</td>
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<td>( \sim 10 )</td>
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Craig Roberts: Hadron Physics & DSE Perspective
"XI Mexican Workshop on Particles and fields" Tuxtla Gutierrez, Mexico: 7-12/11/07...
**Dressed-Quark Propagator**

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Predictions confirmed in numerical simulations of **lattice-QCD**

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Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies
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Dressed-Quark Propagator

Longstanding Prediction of Dyson-Schwinger Equation Studies

E.g., *Dyson-Schwinger equations and their application to hadronic physics*,

C. D. Roberts and A. G. Williams,

*Prog. Part. Nucl. Phys.*

**33** (1994) 477
Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

Long used as basis for efficacious hadron physics phenomenology

Quenched-QCD

Dressed-Quark Propagator

\[ M(p) \]

\[ Z(p) \]
“data:” Quenched Lattice Meas.
– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](http://arxiv.org/abs/he-lat/0209129)
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*Curves*: Quenched DSE Cal.
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Current-quark masses: 30 MeV, 50 MeV, 100 MeV

Curves: Quenched DSE Cal.
– Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003

Linear extrapolation of lattice data to chiral limit is inaccurate
Kernel of Gap Equation: \( D_{\mu \nu}(p - q) \Gamma_\nu(q) \)

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:

Kernel of Gap Equation: $D_{\mu\nu}(p-q)\Gamma_{\nu}(q)$

Dressed-gluon propagator and dressed-quark-gluon vertex

Reliable DSE studies of Dressed-gluon propagator:


Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:


Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex
Dressed-gluon Propagator

\[ D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \]

- Suppression means \( \exists \) IR gluon mass-scale
  \( \approx 1 \text{ GeV} \)

- Naturally, this scale has the same origin as \( \Lambda_{\text{QCD}} \)

\[
\begin{align*}
Z(p^2) & = & & \begin{cases} 
  & \text{lattice, } N_f=0 \\
  & \text{DSE, } N_f=0 \\
  & \text{DSE, } N_f=3 \\
  & \text{Fit to DSE, } N_f=3
\end{cases} \\
\text{p}^2 [\text{GeV}^2] & = & & \begin{cases} 
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  & \text{DSE, } N_f=0 \\
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Dynamical chiral symmetry breaking and a critical mass

Lei Chang, et al., nucl-th/0605058
Dynamical chiral symmetry breaking and a critical mass

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Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$M(p^2; m = 0) \neq 0.$$
Critical Mass for Chiral Expansion

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Chiral symmetry realised in Nambu-Goldstone mode; i.e.,

**Dynamical Chiral Symmetry Breaking** – characterised by nonzero dressed-quark mass function in the chiral limit:

\[ M(p^2; m = 0) \neq 0. \]

Does this mass function have a **convergent** expansion in current-quark mass about its nonzero chiral-limit value:

\[ M(0; m) = M(0, 0) + m \left. \frac{\partial}{\partial m} M(0; m) \right|_{m=0} + \ldots ? \]
Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

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\[ M(0; m) = M(0, 0) + \sum_{n=1}^{\infty} m^n a_n \]

Radius of convergence: \( m_{rc} = \lim_{n \to \infty} \left( \frac{1}{|a_n|} \right)^{1/n} \)
Critical Mass for Chiral Expansion

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\[ m_{rc} = 0.034 \pm 0.001 \]
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$$M(p^2; m = 0) \neq 0.$$ 

- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass

$$m_{0-}^{cr} \sim 0.45 \text{ GeV}, \ [m_{0-}^{cr}]^2 \sim 0.2 \text{ GeV}^2.$$
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$$m_{0-}^{cr} \sim 0.45 \text{ GeV}, [m_{0-}^{cr}]^2 \sim 0.2 \text{ GeV}^2.$$  

Entails, e.g., lattice-QCD simulations must have results at

$$m_\pi^2 < [m_{0-}^{cr}]^2 \sim 0.2 \text{ GeV}^2$$ for reasonable extrapolation via EFT.
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$
\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)} , \ (M^E)^2 := s \mid s = M(s)^2 .
$$
Consituent-quark $\sigma$-term

- Impact of Dynamical chiral symmetry breaking ... exhibited via constituent-quark $\sigma$-term

\[ \sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2. \]

- Renormalisation-group-invariant and determined from solutions of the gap equation
**Consituent-quark $\sigma$-term**

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

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$$

Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \ (M^E)^2 := s \mid s = M(s)^2.$$ 

Ratio

$$\frac{\sigma_f}{M^E_f} = \frac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}$$

measures effect of EXPLICIT chiral symmetry breaking on dressed-quark mass-function

cf. SUM of effects of EXPLICIT AND DYNAMICAL CHIRAL SYMMETRY BREAKING
Consituent-quark $\sigma$-term

Impact of Dynamical chiral symmetry breaking \ldots exhibited via constituent-quark $\sigma$-term

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$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$ 

Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.
**Consituent-quark \( \sigma \)-term**

Impact of Dynamical chiral symmetry breaking ... exhibited via constituent-quark \( \sigma \)-term

\[
\sigma_f := m_f(\zeta) \frac{\partial M^E_f}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.
\]

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass.
Established understanding of two- and three-point functions
Hadrons

- Established understanding of two- and three-point functions
- What about bound states?
Without bound states, Comparison with experiment is impossible
Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
Hadrons

• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
Hadrons

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• What is the kernel, $K$?
Hadrons

• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.

• What is the kernel, $K$?

or
What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in numerical simulations of lattice-QCD is not related in any simple way to the light-quark interaction.
Bethe-Salpeter Kernel
Axial-vector Ward-Takahashi identity

\[ P_\mu \, \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \, \frac{1}{2} \lambda^l \gamma_5 + \frac{1}{2} \lambda^l \gamma_5 \, S^{-1}(k_-) \]

\[ -M_\zeta \, i\Gamma_{5}^l (k; P) - i\Gamma_{5}^l (k; P) \, M_\zeta \]

QFT Statement of Chiral Symmetry
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma^l_{5\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda^l_f i \gamma_5 + \frac{1}{2} \lambda^l_f i \gamma_5 S^{-1}(k_-) \]

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Satisfies BSE

Satisfies DSE
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Satisfies DSE
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Satisfies BSE  \quad \text{Satisfies DSE}

Kernels very different  \quad \text{but must be \textit{intimately} related}

- Relation \textit{must} be preserved by truncation
Axial-vector Ward-Takahashi identity

\[
P_{\mu} \Gamma_5^{l\mu}(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f i \gamma_5 + \frac{1}{2} \lambda_f i \gamma_5 S^{-1}(k_-) \]

\[\quad - M_\zeta i \Gamma_5^l(k; P) - i \Gamma_5^l(k; P) M_\zeta\]

Satisfies BSE
Satisfies DSE

Kernels very different
but must be \textit{intimately} related

- Relation \textbf{\textit{must}} be preserved by truncation
- Nontrivial constraint
Axial-vector Ward-Takahashi identity

\[ P_\mu \Gamma_5^l (k; P) = S^{-1}(k_+) \frac{1}{2} \lambda_f^i \gamma_5 + \frac{1}{2} \lambda_f^i \gamma_5 S^{-1}(k_-) \]

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Satisfies BSE  \quad \text{Satisfies DSE}

Kernels very different but must be **intimately** related

- Relation **must** be preserved by truncation
- **Failure** ⇒ Explicit Violation of QCD’s Chiral Symmetry
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \quad \rho_H \zeta \quad M_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho_H^H \ \mathcal{M}_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

\[ f_H \quad m_H^2 = - \rho_{\zeta} \quad \mathcal{M}_H \]

\[ \mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M(\mu) \left\{ T^H, (T^H)^t \right\} \right] = m_{q1} + m_{q2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \)
Radial Excitations & Chiral Symmetry

\[ f_H m_H^2 = - \rho_H^2 M_H \]

\[ f_H p_\mu = Z_2 \int_0^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant

\[ \pi^- \rightarrow f_\pi k^\mu \quad \bar{A}_5^\mu \]

Craig Roberts: Hadron Physics & DSE Perspective
"XI Mexican Workshop on Particles and fields" Tuxtla Gutierrez, Mexico: 7-12/11/07...
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy, nu-th/9707003)

\[ f_H \quad m_H^2 = -\frac{\rho_H^H}{\zeta} M_H \]

\[ i\rho_H^H = Z_4 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 S(q_+)\Gamma_H(q; P)S(q_-) \right\} \]

- **Pseudoscalar** projection of BS wave function at \( x = 0 \)
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H m_H^2 = - \rho^H_\zeta \mathcal{M}_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)

- \( f_H \to f_H^0 \) & \( \rho^H_\zeta \to \frac{-\langle \bar{q}q \rangle^0_\zeta}{f_H^0} \), Independent of \( m_q \)

Hence \[ m_H^2 = \frac{-\langle \bar{q}q \rangle^0_\zeta}{(f_H^0)^2} m_q \] \dots GMOR relation, a corollary
Radial Excitations & Chiral Symmetry

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Hence \( m_H^2 = \frac{-\langle \bar{q}q\rangle_0}{(f_0^H)^2} m_q \) \ldots GMOR relation, a corollary

- Heavy-quark + light-quark

\[ \Rightarrow f_H \propto \frac{1}{\sqrt{m_H}} \quad \text{and} \quad \rho^H_\zeta \propto \sqrt{m_H} \]

Hence, \( m_H \propto m_q \)

\ldots \text{QCD Proof of Potential Model result}
Valid for ALL Pseudoscalar mesons

\[ f_H \ m_H^2 = - \rho_\zeta^H \ M_H \]
Valid for **ALL** Pseudoscalar mesons

\[ f_H \ m_H^2 = - \ \rho_H^H \ M_H \]

- \( \rho_H \to \) finite, nonzero value in chiral limit, \( M_H \to 0 \)
Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

\[ f_H \ m_H^2 = - \ \rho_H^H \ \mathcal{M}_H \]

- Valid for ALL Pseudoscalar mesons
- \( \rho_H \rightarrow \) finite, nonzero value in chiral limit, \( \mathcal{M}_H \rightarrow 0 \)
- “radial” excitation of \( \pi \)-meson, not the ground state, so
  \[ m_{\pi \neq 0}^2 > m_{\pi = 0}^2 = 0, \] in chiral limit
Valid for ALL Pseudoscalar mesons

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- \( \Rightarrow f_H = 0 \)

ALL pseudoscalar mesons except \( \pi(140) \) in chiral limit
Höll, Krassnigg, Roberts

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- \( \Rightarrow f_H = 0 \)
- **ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit
- **Dynamical Chiral Symmetry Breaking**
  - Goldstone’s Theorem – impacts upon **every** pseudoscalar meson
When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.

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$\Rightarrow f_{\pi_1} < 8.4\text{ MeV}$

Diehl & Hiller

he-ph/0105194
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Lattice-QCD check:
$16^3 \times 32$,
$a \sim 0.1\text{ fm}$,
two-flavour, unquenched
$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$
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Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators).
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The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.
Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.
Pion \ldots J = 0

but \ldots

Pseudoscalar meson Bethe-Salpeter amplitude

\[ \chi_\pi(k; P) = \gamma_5 \left[ i E_\pi_n(k; P) + \gamma \cdot P F_\pi_n(k; P) \right. \]
\[ \left. \gamma \cdot k k \cdot P G_\pi_n(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi_n(k; P) \right] \]
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\[ \left. \gamma \cdot k \cdot P \mathcal{G}_{\pi n}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} \mathcal{H}_{\pi n}(k; P) \right] \]

\[ J = 0 \ldots \ but \] while \( \mathcal{E} \) and \( \mathcal{F} \) are purely \( L = 0 \) in the rest frame, the \( \mathcal{G} \) and \( \mathcal{H} \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function always contains both \( S \)- and \( P \)-wave components.
Pion \( J = 0 \) but ... 

\( J = 0 \) ... but while \( \mathcal{E} \) and \( \mathcal{F} \) are purely \( L = 0 \) in the rest frame, the \( \mathcal{G} \) and \( \mathcal{H} \) terms are associated with \( L = 1 \). Thus a pseudoscalar meson Bethe-Salpeter wave function always contains both \( S \)- and \( P \)-wave components.

Introduce mixing angle \( \theta_\pi \) such that

\[
\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
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Introduce mixing angle \( \theta_\pi \) such that
\[
\chi_\pi \sim \cos \theta_\pi |L = 0\rangle + \sin \theta_\pi |L = 1\rangle
\]

\( L \) is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.
\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+)i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
\[ -2iM^{ab}\Gamma^b_{5}(k; P) - A^a(k; P) \]
\[
P_\mu \Gamma_5^{\alpha \mu} (k; P) = \ S^{-1}(k_{+}) i\gamma_5 \mathcal{F}^{\alpha} + i\gamma_5 \mathcal{F}^{\alpha} S^{-1}(k_{-}) \nonumber
\]
\[\quad - 2 i \mathcal{M}^{ab} \Gamma_5^{\beta} (k; P) - A^a (k; P) \nonumber\]

\[
\{ \mathcal{F}^{\alpha} \mid a = 0, \ldots, N_f^2 - 1 \} \text{ are the generators of } U(N_f) \nonumber
\]

\[
\mathcal{S} = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots] \nonumber
\]

\[
\mathcal{M}^{ab} = \text{tr}_F \left[ \{ \mathcal{F}^{\alpha}, \mathcal{M} \} \mathcal{F}^{\beta} \right] \nonumber
\]

\[
\mathcal{M} = \text{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses} \nonumber
\]
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_{5\mu}^a(k; P) = S^{-1}(k_+) i\gamma_5 F^a + i\gamma_5 F^a S^{-1}(k_-) \]
\[ - 2i M^{ab} \Gamma_5^b(k; P) - A^a(k; P) \]

- \{F^a | a = 0, \ldots, N_f^2 - 1\} are the generators of \( U(N_f) \)
- \( S = \text{diag} [S_u, S_d, S_s, S_c, S_b, \ldots]\)
- \( M^{ab} = \text{tr}_F \left[ \{F^a, M\} F^b \right], \)
  \( M = \text{diag} [m_u, m_d, m_s, m_c, m_b, \ldots] = \text{matrix of current-quark bare masses}\)

- The final term in the second line expresses the non-Abelian axial anomaly.
Charge Neutral Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]

\[ -2i M^{ab} \Gamma^b_{5}(k; P) - A^a(k; P) \]

\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

\[ A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle F^0 q(x) Q(0) \bar{q}(y) \rangle \]
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_5^{\alpha\mu} (k; P) = S^{-1}(k_+) \gamma_5 F^\alpha + i\gamma_5 F^\alpha S^{-1}(k_-) \]
\[ -2i M^{ab}\Gamma_5^b (k; P) - A^a (k; P) \]
\[ \mathcal{A}^a (k; P) = S^{-1}(k_+) \delta^{a0} \mathcal{A}_U (k; P) S^{-1}(k_-) \]
\[ \mathcal{A}_U (k; P) = \int d^4x d^4y \ e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle \mathcal{F}^0 q (x) \ Q(0) \ \bar{q} (y) \rangle \]
\[ Q(x) = i \frac{\alpha_s}{4\pi} \operatorname{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu (x) \]

\[ \ldots \text{The topological charge density operator.} \]
\[
P_\mu \Gamma_{5\mu}^a(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \\
-2i \mathcal{M}^{ab} \Gamma_5^b(k; P) - A^a(k; P)
\]

\[
A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-)
\]

\[
A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle \mathcal{F}_0 q(x) Q(0) \bar{q}(y) \rangle
\]

\[
Q(x) = i \frac{\alpha_s}{4\pi} \text{tr}_C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x)
\]

\[
\text{... The topological charge density operator.}
\]

(Trace is over colour indices & \(F_{\mu\nu} = \frac{1}{2} \lambda^a F^a_{\mu\nu}\).)
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma_5^a (k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]
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\[ Q(x) = i \frac{\alpha_s}{4\pi} \text{tr} C [ \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} (x)] = \partial_\mu K_\mu (x) \]

\[ \ldots \text{ The topological charge density operator.} \]

Important that only \( A^{a=0} \) is nonzero.
Charge Neutral
Pseudoscalar Mesons

\[ P_\mu \Gamma^a_{5\mu}(k; P) = S^{-1}(k_+) i \gamma_5 F^a + i \gamma_5 F^a S^{-1}(k_-) \]

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\[ A^a(k; P) = S^{-1}(k_+) \delta^{a0} A_U(k; P) S^{-1}(k_-) \]

\[ A_U(k; P) = \int d^4x d^4y e^{i(k_+ \cdot x - k_- \cdot y)} N_f \langle F^0_\mu q(x) Q(0) \bar{q}(y) \rangle \]

\[ Q(x) = i \frac{\alpha_s}{4\pi} tr C [\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)] = \partial_\mu K_\mu(x) \]

\[ \cdots \text{The topological charge density operator.} \]

\[ \text{NB. While } Q(x) \text{ is gauge invariant, the associated Chern-Simons current, } K_\mu, \text{ is not } \Rightarrow \text{ in QCD no physical boson can couple to } K_\mu \text{ and hence no physical states can contribute to resolution of } \text{U}_A(1) \text{ problem.} \]
Charge Neutral
Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118
Only $A^0 \not\equiv 0$ is interesting
Only $A^0 \neq 0$ is interesting . . . otherwise all pseudoscalar mesons are Goldstone Modes!
Anomaly term has structure

\[ A^0(k; P) = \mathcal{F}_0 \gamma_5 \left[ i\mathcal{E}_A(k; P) + \gamma \cdot P \mathcal{F}_A(k; P) \right] + \gamma \cdot k \cdot P \mathcal{G}_A(k; P) + \sigma_{\mu\nu} k_\mu P_\nu \mathcal{H}_A(k; P) \]
AVWTI gives generalised Goldberger-Treiman relations

\[
2f^0_{\eta'} E_{BS}(k; 0) = 2B_0(k^2) - \mathcal{E}_A(k; 0),
\]

\[
F^0_R(k; 0) + 2f^0_{\eta'} F_{BS}(k; 0) = A_0(k^2) - F_A(k; 0),
\]

\[
G^0_R(k; 0) + 2f^0_{\eta'} G_{BS}(k; 0) = 2A'_0(k^2) - G_A(k; 0),
\]

\[
H^0_R(k; 0) + 2f^0_{\eta'} H_{BS}(k; 0) = -\mathcal{H}_A(k; 0),
\]

\(A_0, B_0\) characterise gap equation’s chiral limit solution.
AVWTI gives generalised Goldberger-Treiman relations

\[ 2 f_{\eta'}^0 E_{BS}(k; 0) = 2B_0(k^2) - \mathcal{E}_A(k; 0), \]
\[ F^0_R(k; 0) + 2 f_{\eta'}^0 F_{BS}(k; 0) = A_0(k^2) - \mathcal{F}_A(k; 0), \]
\[ G^0_R(k; 0) + 2 f_{\eta'}^0 G_{BS}(k; 0) = 2A'_0(k^2) - \mathcal{G}_A(k; 0), \]
\[ H^0_R(k; 0) + 2 f_{\eta'}^0 H_{BS}(k; 0) = -\mathcal{H}_A(k; 0), \]

\( A_0, B_0 \) characterise gap equation’s chiral limit solution.

Follows that \( \mathcal{E}_A(k; 0) = 2B_0(k^2) \) is necessary and sufficient condition for absence of massless \( \eta' \) bound-state.
\[ \mathcal{E}_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

\[ B_0(k^2) \neq 0 \text{ if, and only if, chiral symmetry is dynamically broken.} \]

Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
\[ E_A(k; 0) = 2B_0(k^2) \]

Discussing the chiral limit

\[ B_0(k^2) \neq 0 \text{ if, and only if, chiral symmetry is dynamically broken.} \]

Hence, absence of massless \( \eta' \) bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Further highlighted . . . proved

\[
\langle \bar{q}q \rangle_\zeta^0 = - \lim_{\Lambda \to \infty} Z_4(\zeta^2, \Lambda^2) \text{tr}_{CD} \int_q^\Lambda S^0(q, \zeta)
\]

\[
= N_f \int d^4x \langle \bar{q}(x) i\gamma_5 q(x) Q(0) \rangle^0.
\]
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Employed in an analysis of pseudoscalar- and vector-meson bound-states
AVWTI ⇒ QCD mass formulae for neutral pseudoscalar mesons

Implications of mass formulae illustrated using elementary dynamical model, which includes Ansatz for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts

- $\eta-\eta'$ mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
- $\pi^0-\eta$ angles of $\sim 1.2^\circ$ (Expt. $p d \rightarrow ^3$He $\pi^0$: $0.6^\circ \pm 0.3^\circ$)
- Strong neutron-proton mass difference . . .
  \[ \lesssim 75\% \text{ current-quark mass-difference} \]
New Challenges
**New Challenges**

**Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
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Move on to the problem of a symmetry preserving treatment of hybrids and exotics.
Another Direction . . . Also want/need information about three-quark systems.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
Nucleon EM Form Factors: A Précis

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  ⇒ Covariant dressed-quark Faddeev Equation
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

**Easily obtained:**

\[
\left( \frac{1}{N_H} \sum_{H} \left[ M_H^{\text{exp}} - M_H^{\text{calc}} \right]^2 \right)^{1/2} = 2\%
\]
Nucleon EM Form Factors: A Précis


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\]

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  \[ \Rightarrow \text{Covariant dressed-quark Faddeev Equation} \]

- Excellent mass spectrum (octet and decuplet)
  
  \[
  \text{Easily obtained:} \quad \left( \frac{1}{N_H} \sum_H \left[ M_H^{\text{exp}} - M_H^{\text{calc}} \right]^2 \right)^{1/2} = 2\%
  \]

- But is that good?
Nucleon EM Form Factors: A Précis


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  ⇒ Covariant dressed-quark Faddeev Equation

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  \]

- But is that good?
  - Cloudy Bag: \( \delta M_+^{\pi-\text{loop}} = -300 \) to \(-400\) MeV!
Nucleon EM Form Factors: A Précis


- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  ⇒ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:
  \[
  \left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\% 
  \]
- But is that good?
  - Cloudy Bag: \( \delta M_+^{\pi-\text{loop}} = -300 \) to \(-400\) MeV!
- Critical to anticipate pion cloud effects

Roberts, Tandy, Thomas, et al., nu-th/02010084
Faddeev equation
Faddeev equation

\[ \Psi^a_p \rightarrow P_p q \rightarrow \Psi^b_{p_d} = \Gamma^a_q \rightarrow \Gamma^b_{p_d} \rightarrow P_p \]
Faddeev equation

\[ \Psi^a \rightarrow p_q \Psi^b \]

\[ \Psi^b \rightarrow p_d \Psi^a \]

\[ \Gamma^a \rightarrow p_q \Gamma^b \]

\[ \Gamma^b \rightarrow p_d \Gamma^a \]

Linear, Homogeneous Matrix equation

- Yields *wave function* *(Poincaré Covariant Faddeev Amplitude)* that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . \(s\)–, \(p\)– & \(d\)–wave correlations
Diquark correlations
Diquark correlations

Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red

Confined ... Does not escape from within baryon.

Scalar is isosinglet, Axial-vector is isotriplet

DSE and lattice-QCD

\[
\begin{align*}
    m_{[ud]}^{0+} &= 0.74 - 0.82 \\
    m_{(uu)}^{1+} &= m_{(ud)}^{1+} = m_{(dd)}^{1+} = 0.95 - 1.02
\end{align*}
\]
Harry Lee

Pions and Form Factors

Craig Roberts: Hadron Physics & DSE Perspective

"XI Mexican Workshop on Particles and fields" Tuxtla Gutierrez, Mexico: 7-12/11/07

38
Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the \( \Delta(1236) \)


Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$


Pion cloud effects are large in the low $Q^2$ region.

*Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor $G_D$. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.*
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**Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor $G_D$.**

Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.

Quark Core

- Responsible for only 2/3 of result at small $Q^2$
- Dominant for $Q^2 > 2 - 3$ GeV$^2$
Results: Nucleon and $\Delta$ Masses
Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses.

Set A – fit to the actual masses was required; whereas for Set B – fitted mass was offset to allow for “$\pi$-cloud” contributions.

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$m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
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\[ m_{1+} \rightarrow \infty: M^A_N = 1.15 \text{ GeV}; M^B_N = 1.46 \text{ GeV} \]

Axial-vector diquark provides significant attraction
## Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

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Constructive Interference: $1^{++}$-diquark $+ \partial_\mu \pi$
6 terms . . . constructed systematically . . . current conserved automatically for on-shell nucleons described by Faddeev Amplitude
6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude
Form Factor Ratio: GE/GM

\[ \mu_p \frac{G_E^P}{G_M^P} \]

- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer

\[ Q^2 \text{ [GeV}^2\text{]} \]
Combine these elements...
Combine these elements . . .

Dressed-Quark Core

Form Factor Ratio:
GE/GM

\[ \frac{G^E_p}{G^M_p} \]

Rosenbluth precision Rosenbluth polarization transfer polarization transfer
Combine these elements . . .

- Dressed-Quark Core
- Ward-Takahashi Identity preserving current

Form Factor Ratio: GE/GM

![Graph showing the ratio of G_E to G_M against Q^2 in GeV^2.](image-url)
Combine these elements . . .

- **Dressed-Quark Core**
- **Ward-Takahashi Identity preserving current**
- **Anticipate and Estimate Pion Cloud’s Contribution**

**Form Factor Ratio:**

\[ \frac{G_E}{G_M} \]

\[ \mu \]

\[ Q^2 \text{ [GeV}^2]\]
Combine these elements . . .

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- *Ward-Takahashi*
  Identity preserving current

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\[ \mu_p G_E^P / G_M^P \]

![Graph showing the form factor ratio \( \mu_p G_E^P / G_M^P \) vs. \( Q^2 \) in GeV^2 with data points and error bars.](chart.png)
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All parameters fixed in other applications . . . Not varied.
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Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\, \text{GeV}^2$
Combine these elements . . .

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- \textit{Ward-Takahashi} Identity preserving current

Anticipate and Estimate Pion Cloud’s Contribution

All parameters fixed in other applications . . . \textbf{Not} varied.

- Agreement with Pol. Trans. data at \( Q^2 \gtrsim 2 \text{ GeV}^2 \)
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
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All parameters fixed in other applications . . . **Not** varied.

- Agreement with Pol. Trans. data at $Q^2 \gtrsim 2$ GeV$^2$
- Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
- **Predict Zero at** $Q^2 \approx 6.5$ GeV$^2$
Neutron Form Factors

\[ \mu_n G_E^n / G_M^n \]

\[ Q^2 \text{ [GeV}^2\text{]} \]

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Neutron Form Factors

Expt. Madey, et al. nu-ex/0308007
Neutron Form Factors

- Expt. Madey, et al. nu-ex/0308007
- Calc. Bhagwat, et al. nu-th/0610080

\[
\mu_p \frac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\frac{r_n^2}{\frac{6}{Q^2}} \quad \text{Valid for } r_n^2 Q^2 \lesssim 1
\]
Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007

\[ \mu_p \frac{G^n_E(Q^2)}{G^n_M(Q^2)} = -\frac{r_n^2}{6} Q^2 \]

Valid for \( r_n^2 Q^2 \lesssim 1 \)

- No sign yet of a zero in \( G^n_E(Q^2) \), even though calculation predicts \( G^p_E(Q^2 \approx 6.5 \text{ GeV}^2) = 0 \)

- Data to \( Q^2 = 3.4 \text{ GeV}^2 \) is being analysed (JLab E02-013)
Epilogue
DCSB exists in QCD.
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
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Confinement

- Expressed and realised in dressed propagators and vertices associated with elementary excitations
- Observables can be used to explore model realisations
DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It impacts dramatically upon observables.

Confinement

- Expressed and realised in dressed propagators and vertices associated with elementary excitations
- Observables can be used to explore model realisations
- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables
1. QCD's Challenges
2. Dichotomy of the Pion
3. What's the Problem?
4. Dyson-Schwinger Equations
5. Schwinger Functions
6. Persistent Challenge
7. Truncation
8. Dressed-Quark Propagator
9. Quenched-QCD cf. Lattice
10. QCD & Interaction
11. Dressed-gluon
12. Critical Mass & Chiral Expansion
13. C-quark $\sigma$-term
14. Hadrons
15. Bethe-Salpeter Kernel
16. Radial Excitations
17. Radial Excitations (cont.)
18. Radial Excitations & Lattice-QCD
19. Pion OAM
20. Neutral Pseudo-scalar
21. New Challenges
22. Nucleon EM Form Factors
23. Faddeev equation
24. Diquark correlations
25. Pions and Form Factors
26. Baryon Masses
27. Nucleon-Photon Vertex
28. Form Factor Ratio
29. Neutron Form Factors
30. Contemporary Reviews
31. Colour-singlet Kernel
32. $\pi$ and $\rho$
33. Angular Momentum
34. Extant DIS $\pi$
35. Distribution function
Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. 45 (2000) S1

- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,

- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,

- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,

- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
Colour-singlet

Bethe-Salpeter equation

Detmold et al., nu-th/0202082

Bhagwat, et al., nu-th/0403012
Colour-singlet

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Detmold et al., nu-th/0202082
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- Coupling-modified dressed-ladder vertex

\[ \Gamma_{\mu}(k,p) = C + C^2 + \ldots \]
Detmold et al., nu-th/0202082

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- Coupling-modified dressed-ladder vertex
  \[ \Gamma_\mu(k, p) = C + C^2 + \ldots \]

- BSE consistent with vertex
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Bethe-Salpeter equation

• Coupling-modified dressed-ladder vertex

\[ \Gamma_\nu(k, p) = \Gamma_\nu + C + C^2 + \ldots \]

• BSE consistent with vertex

\[ \Gamma_M = \sum_n \left[ \Gamma^n_M + \Lambda^{\alpha;\tau}_{\nu} \right] \]

• Bethe-Salpeter kernel ... recursion relation

\[ -\frac{1}{8C} \]
Colour-singlet

Bethe-Salpeter equation

- Coupling-modified dressed-ladder vertex

\[ \Gamma_\mu(k, p) = \begin{array}{c}
\text{vertex term} \\
\text{BSE consistent with vertex} \\
\text{Bethe-Salpeter kernel . . . recursion relation} \\
\text{Kernel necessarily non-planar, even with planar vertex}
\end{array} \]

Detmold et al., nu-th/0202082
Bhagwat, et al., nu-th/0403012
π and ρ mesons
### Mesons

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- $\pi$ massless in chiral limit ... **NO** Fine Tuning.
\[ \begin{array}{cccc}
\pi, m = 0 & M_H^{n=0} & M_H^{n=1} & M_H^{n=2} \\
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- **\( \pi \)** massless in chiral limit \( \ldots \) **NO** Fine Tuning
- **ALL** \( \pi-\rho \) mass splitting present in chiral limit
## π and ρ mesons

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- \( \pi \) massless in chiral limit . . . **NO** Fine Tuning
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**π and ρ mesons**

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- **Extending kernel**
\( \pi \) and \( \rho \) mesons

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- Extending kernel: **NO** effect on \( m_{\pi} \)
\( \pi \) and \( \rho \) mesons

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<td>0.798</td>
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</tr>
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- \( \pi \) massless in chiral limit . . . **NO** Fine Tuning
- \( \pi - \rho \) mass splitting **driven** by \( D_\chi \)SB mechanism
  - Not constituent-quark-model-like hyperfine splitting
- Extending kernel: **NO** effect on \( m_\pi \)
  - For \( m_\rho \) – zeroth order, accurate to **20%**
### $\pi$ and $\rho$ mesons

<table>
<thead>
<tr>
<th></th>
<th>$M^{n=0}_H$</th>
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### \( \pi \) and \( \rho \) mesons

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    - one loop, accurate to 13%
    - two loop, accurate to 4%
Crude estimate based on magnitudes ⇒ probability for a $u$-quark to carry the proton’s spin is $P_{u\uparrow} \sim 80\%$, with $P_{u\downarrow} \sim 5\%$, $P_{d\uparrow} \sim 5\%$, $P_{d\downarrow} \sim 10\%$.

Hence, by this reckoning, $\sim 30\%$ of proton’s rest-frame spin is located in dressed-quark angular momentum.
Deep-inelastic scattering
Deep-inelastic scattering

Looking for Quarks
Deep-inelastic scattering

- Looking for Quarks
Deep-inelastic scattering

Signature Experiment for QCD:
Discovery of Quarks at SLAC

Looking for Quarks
Deep-inelastic scattering

Looking for Quarks

Signature Experiment for QCD:
Discovery of Quarks at SLAC

Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x), g(x)$
Pion’s valence quark distn
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- $\pi$ is Two-Body System: “Easiest” Bound State in QCD
- However, NO $\pi$ Targets!
Pion’s valence quark distn

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Pion’s valence quark distn

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- However, NO $\pi$ Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \rightarrow \mu^+ \mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^{-5\text{GeV}} - p_{25\text{GeV}}$ Collider $\rightarrow$ Accurate “Measurement”
\[ W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T^{+\mu}_{\nu}(q; P) + T^{-\mu\nu}(q; P) \right] \]

\[ T^{+\mu}_{\nu}(q, P) = \text{tr} \int \frac{d^4k}{(2\pi)^4} \tau_- \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) i e Q \Gamma_\nu(k_{-0}, k) \]

\[ \times S(k) i e Q \Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{-0}) \]
Bjorken Limit: $q^2 \to \infty$, $P \cdot q \to -\infty$

but $x := -\frac{q^2}{2P \cdot q}$ fixed.

Numerous algebraic simplifications

\[
W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} \left[ T^+_{\mu\nu}(q; P) + T^-_{\mu\nu}(q; P) \right]
\]

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\]

\[
\times S(k) ieQ \Gamma_{\mu}(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{--})
\]
Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)

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"XI Mexican Workshop on Particles and fields" Tuxtla Gutierrez, Mexico: 7-12/11/07...
Quark Distribution Functions

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Quark

Distribution Functions

DIS

SI–DIS

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Three twist-2 parton distributions \((k_{\perp} = 0)\):

- **Spin-Independent:** \(q(x)\)
- **Helicity:** \(\Delta q(x)\)
- **Transversity:** \(\Delta_T q(x)\)

All distributions have probability interpretation.

By definition, contain essentially non-perturbative information about a given process.
Definition and Sum Rules
Definition and Sum Rules

- **Light-cone Fourier transforms:**
  \[
  \Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s | \bar{\psi}_q(0) \gamma^+ \gamma^5 \psi_q(\xi^-) | p, s \rangle_c 
  \]
  \[
  q(x) = \langle \gamma^+ \rangle, \quad \Delta q(x) = \langle \gamma^+ \gamma^5 \rangle 
  \]

- Related to the nucleon axial & tensor charges via
  \[
  g_A = \int dx [\Delta u(x) - \Delta d(x)], \quad g_T = \int dx [\Delta_T u(x) - \Delta_T d(x)], 
  \]

- Must satisfy: positivity constraints and Soffer bound
  \[
  \Delta q(x), \Delta_T q(x) \leq q(x), \quad q(x) + \Delta q(x) \geq 2 |\Delta_T q(x)| 
  \]
Once more on the one that got away.
Model predictions

Cloët, Bentz, Thomas
Simplified Faddeev equation

Satisfy: Soffer bound, baryon & momentum SRs.
**Model predictions**

### Simplified Faddeev equation

![Graph showing up quark distributions and transversity distributions.]

- **Satisfy:** Soffer bound, baryon & momentum SRs.
- **Moments at** \( Q^2 = 0.16 \text{ GeV}^2 \):
  
  \[
  \Delta u = 0.97, \quad \Delta d = -0.30 \quad \Rightarrow \quad g_A = 1.267 \\
  \Delta_T u = 1.04, \quad \Delta_T d = -0.24 \quad \Rightarrow \quad g_T = 1.28
  \]

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Model predictions

- **Simplified Faddeev equation**

- **Up quark distributions**

- **Satisfy: Soffer bound, baryon & momentum SRs.**

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  \[
  \Delta u = 0.97, \quad \Delta d = -0.30 \implies g_A = 1.267
  \]

  \[
  \Delta T u = 1.04, \quad \Delta T d = -0.24 \implies g_T = 1.28
  \]

\[
\Delta q(x) \sim \Delta T q(x) \text{ in valence region for } Q^2 \lesssim 10 \text{ GeV}^2
\]