Indirect Technique in Nuclear Astrophysics: ANC

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Techniques to obtain reaction rates in Nuclear Astrophysics

- **Direct Measurements** (LUNA, LENA, DRAGON, . . .)
- **Radiative widths** for resonance rates
  - populate resonance state and measure decay
- **Coulomb dissociation**
  - applications with radioactive beams
- **Trojan Horse**
  - unique way to understand screening
- **Asymptotic Normalization Coefficients**
  - stable and radioactive beams
Direct radiative capture

Transition Amplitude: 
\[ M = \left\langle I^A_{Bp}(r_{Bp}) \right| O(r_{Bp}) \left| \psi_i^{(+)}(r_{Bp}) \right\rangle \]

Direct Radiative proton capture \[ \sigma \propto \left| M \right|^2 \]

Low B.E.:
\[ I^A_{Bp}(r_{Bp}) \stackrel{r_{Bp} > R_N}{\approx} C^A_{Bp} \frac{W_{-n_{l+\frac{1}{2}},l+\frac{1}{2}}(2\kappa_{Bp}r_{Bp})}{r_{Bp}} \]

ANC \Rightarrow \text{amplitude for tail of overlap function}

Find:
\[ \sigma_{\text{capture}} \propto (C^A_{Bp})^2 \]
* Major sources of γ-ray lines:
  1- Following β-decays
  2- electron-positron annihilation.

* \( ^{18}\text{F} \) emits positron \( (T_{1/2} = 158 \text{ min}) \).

* \( ^{18}\text{F} \) production may be influenced by:

\( ^{17}\text{F}(p,\gamma)^{18}\text{Ne} \)?
The nuclear structure for $^{18}$O & $^{18}$Ne are similar.

The ANCs for $^{18}$O will be obtained from $^{13}$C($^{17}$O,$^{18}$O)$^{12}$C reaction.

Stable beam ($^{17}$O) enables the ability to separate between interesting levels in $^{18}$O.
Experimental Setup

# Elastic Scatterings:

Beams
\[ ^{17}\text{O} + ^{13}\text{C} \]
\[ ^{18}\text{O} + ^{12}\text{C} \]

Targets
100 μg/cm²

# Transfer Reactions:

\[ ^{13}\text{C} (^{17}\text{O}, ^{18}\text{O}) ^{12}\text{C} \]

MDM Spectrometer

Oxford Detector
$J^\pi = (0^+_1, 2^+_1, 4^+_1, 2^+_2)$

- $2^+$ states are combinations of $(d_{5/2})^2$ & $(d_{5/2}s_{1/2})$, T. Dehnhard, et al, PRC 13 (1976) 55.
- $4^+$ & $0^+$ have pure $(d_{5/2})^2$ configuration.

\begin{tabular}{|c|c|}
\hline
4.45 & 1^- \\
3.92 & 2^+ \\
3.63 & 0^+ \\
3.56 & 4^+ \\
1.98 & 2^+ \\
0 & 0^+ \\
\hline
\end{tabular}
* Comparison ANC vs S: The reaction is *peripheral*

* The ANCs are obtained using:

\[
\frac{C_{ij}^2}{b_{ij}^2} (^{18}O) = \frac{C_{ij}^2}{b_{ij}^2} (^{18}\text{Ne})
\]

* Charge Symmetry implies:

<table>
<thead>
<tr>
<th>J$^\pi$</th>
<th>Proton Orbital</th>
<th>$^{18}$O B.E. [MeV]</th>
<th>$C_{ij}^2$ [fm$^{-1}$]</th>
<th>$^{18}$Ne B.E. [MeV]</th>
<th>$C_{ij}^2$ [fm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>d$_{5/2}$</td>
<td>8.04</td>
<td>7.33 ± 0.73</td>
<td>3.92</td>
<td>10.76 ± 0.97</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>d$_{5/2}$</td>
<td>6.06</td>
<td>2.06 ± 0.21</td>
<td>2.04</td>
<td>2.17 ± 0.24</td>
</tr>
<tr>
<td></td>
<td>s$_{1/2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>d$_{5/2}$</td>
<td>4.48</td>
<td>1.05 ± 0.11</td>
<td>0.54</td>
<td>2.17 ± 0.22</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>d$_{5/2}$</td>
<td>4.12</td>
<td>0.49 ± 0.06</td>
<td>0.31</td>
<td>2.69 ± 0.32</td>
</tr>
<tr>
<td></td>
<td>s$_{1/2}$</td>
<td></td>
<td></td>
<td></td>
<td>127 ± 17</td>
</tr>
</tbody>
</table>
* **Direct Capture Reaction Rate (RADCAP)**:

\[ N_A \langle \sigma v \rangle = 5.1 \tau^2 S_{\text{eff}} (T_9) e^{-\tau} \left[ \frac{\text{cm}^3}{\text{mole.s}} \right] \]

* **Resonant Capture reaction rate:**


* DC >> RS for \( T_9 \leq 0.4 \)
  \(\Rightarrow\) DC dominates in ONe novae

* The uncertainty of DC rate is ±20%

* **Astrophysical Implications:**

  - Our rate is comparable with Bardayan & Garcia. *(Considered SLOW)*
  - If nova \( M = 1.25 \, M_\odot \), more \(^{18}F\) & \(^{18}O\).
  - If \( M \geq 1.35 \, M_\odot \), less \(^{18}F\).

Collaborators:

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Asymptotic Normalization Coefficients (ANC)

**A. Mukhamedzhanov et al., PRC 56, 1302 (1997)**

Direct Capture Reactions for charged particles:
- The binding energy of the captured particle is low.
- The capture occurs through the tail of the overlap function.
- The Amplitude of the tail is given by the ANCs.

For a Transfer reaction \((X+A \rightarrow Y+B)\):
- The DWBA amplitude:
  \[
  M (E) = \langle \chi_f^{(-)} I_{A,p}^B (r_{A,p}) \mid \Delta V \mid I_{Y,p}^X (r_{Y,p}) \chi_i^{(+)} \rangle 
  \]
- The reaction cross section is related to the DWBA by:
  \[
  \frac{d \sigma}{d \Omega} = \sum_{l_B j_B l_X j_X} S_{A a l_B j_B} S_{Y a l_X j_X} \sigma_{l_B j_B l_X j_X}^{DWBA} 
  \]
- For a peripheral reaction \(r > R_N\),
  \[
  I_{A,p}^B (r) = \sqrt{S_{A,p}} \varphi (r) \quad \Rightarrow \quad S = \frac{C^2}{b^2}
  \]