Mott Dissociation of Bound States in Nuclear Matter

Gerd Röpke, Rostock

ESF RNP CompStar
Supernova

Crab nebula, 1054 China, PSR 0531+21
Structure of a Neutron star

Neutron Star

density zones & shell thickness

outer crust

inner crust

outer core

inner core

10^4 g cm^-3

1.1 \times 10^9 g cm^-3

1.7 \times 10^{14} g cm^-3

5 \times 10^{14} g cm^-3

neutron drip density

neutrino trapping density

nuclear matter density

deconfined phase of quarks and gluons

neutron fluid, neutron-rich atomic nuclei

nucleons, electrons, hyperons, kaons

QGP

ultra-thin atmosphere

iron lattice, degenerated electrons

R

16 km

15 km

11 km

0 km
Core-collapse supernovae

Density, electron fraction, and temperature profile of a 15 solar mass supernova at 150 ms after core bounce as function of the radius.

Influence of cluster formation on neutrino emission in the cooling region and on neutrino absorption in the heating region?

Supernova collapse: spherically symmetric simulations

A. Arcones et al.
Neutrino driven winds,
Talk 25. 2. 08 Ladek;
PRC 78, 015806 (08)
Parameter range: Explosion

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

T. Fischer, On the possible fate of massive progenitor stars, Talk 25.2.08 Ladek
Problems:

• Warm Dilute Matter: Nuclear matter at subsaturation densities \((T, n_p, n_n)\):
  
  Temperature \(T \leq 16\) MeV = \(E_s/A\), baryon density \(n_B \leq 0.17\) fm\(^{-3}\) = \(n_s\), asymmetry

• Formation of clusters (nuclei in matter):
  
  \(A = 1,2,3,4\): free neutrons, free protons, deuterons (\(^2\)H), tritons (\(^3\)H), helions (\(^3\)He), alphas (\(^4\)He)

• Low-density, low-temperature limit:
  
  Virial expansion, non-interacting nuclides, quantum condensates

• Transition to higher densities:
  
  Medium effects, quasiparticles. Interpolation between Beth-Uhlenbeck and DBHF / RMF

• Cluster formation (correlations) vs. mean field:
  
  Consistent quantum-statistical approach
Outline

• Schrödinger equation with medium corrections:
  Self-energy and Pauli blocking

• Composition of the nuclear gas:
  Generalized Beth-Uhlenbeck equation

• Quantum condensates:
  Pairing and quartetting

• Composition and the EoS of nuclear matter
  (astrophysics: supernovae explosions)

• Symmetry energy in the low-density region
  (heavy ion collisions: cluster abundances)

• Cluster formation in dilute nuclei
  (Hoyle state and THSR wave function)
Ideal mixture of reacting nuclides

\[ n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A \{ E_{A,\nu,K} - Z_A \mu_p - (A - Z_A) \mu_n \} \]

\[ n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A \{ E_{A,\nu,K} - Z_A \mu_p - (A - Z_A) \mu_n \} \]

(statistical multifragmentation)

mass number \( A \),
charge \( Z_A \),
energy \( E_{A,\nu,K} \),
\( \nu \): internal quantum number,
\( K \): center of mass momentum

\[
f_A(z) = \frac{1}{\exp(z/T) - (-1)^A}\
\]
Composition of symmetric matter
Ideal mixture of nuclides

\[ T = 10 \text{ MeV} \]

\[ n_B \text{ [fm}^{-3}\text{]} \]

Fraction vs. \( n_B \text{ [fm}^{-3}\text{]} \):
- Green line: \( A = 1 \)
- Red line: \( A = 2 - 4 \)
- Blue line: \( A > 4 \)
Virial expansion

• excited nuclei
• resonances
• scattering phase shifts (no double counting)
• virial expansions
• quantum statistical approach

*Particle clustering and Mott transition in nuclear matter at finite temperatures,*

*Generalized Beth-Uhlenbeck Approach for Hot Nuclear Matter,*

*Cluster formation and the virial equation of state of low-density nuclear matter,*
Symmetric nuclear matter: Phase diagram

- Early universe
- Quark-gluon plasma
- Gas of nuclides (p, n, d, α)
- Excited nuclei
- Heavy-ion collisions
- Nuclear liquid
- Superfluidity
- Neutron stars

Temperature (T [MeV]) vs. density (ρ [fm^-3])
Nucleon-nucleon interaction

- general form:

\[ V_\alpha(p, p') = \sum_{i,j=1}^{N} w_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}(p') \quad \text{uncoupled} \]

and

\[ V_\alpha^{LL'}(p, p') = \sum_{i,j=1}^{N} w^L_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \quad \text{coupled} \]

\[ p, p' \quad \text{in- and outgoing relative momentum} \]
\[ \alpha \quad \text{channel} \]
\[ N \quad \text{rank} \]
\[ \lambda_{\alpha ij} \quad \text{coupling parameter} \]
\[ L, L' \quad \text{orbital angular momentum} \]
Many-particle theory

- equilibrium correlation functions
e.g. equation of state $n(\beta, \mu) = \frac{1}{\Omega_0} \sum_1 \langle a_1^\dagger a_1 \rangle$

  density matrix $\langle a_1^\dagger a_1 \rangle = \int \frac{d\omega}{2\pi} e^{-i\omega t} f_1(\omega) A(1, 1', \omega)$

- Spectral function

  $A(1, 1', \omega) = \text{Im} \left[ G(1, 1', \omega + i\eta) - G(1, 1', \omega - i\eta) \right]$  

- Matsubara Green function

  $G(1, 1', i\nu), \quad z_\nu = \frac{\pi \nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \cdots$

  $1 \equiv \{ p_1, \sigma_1, c_1 \}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega - \mu)} + 1}, \quad \Omega_0 - \text{volume}$
Many-particle theory

- Dyson equation and self energy (homogeneous system)

\[ G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)} \]

- Evaluation of \( \Sigma(1, iz_\nu) \):
  
  perturbation expansion, diagram representation

\[ A(1, \omega) = \frac{2\text{Im} \Sigma(1, \omega + i0)}{[\omega - E(1) - \text{Re} \Sigma(1, \omega)]^2 + [\text{Im} \Sigma(1, \omega + i0)]^2} \]

approximation for self energy \( \rightarrow \) approximation for equilibrium correlation functions

alternatively: simulations, path integral methods
Different approximations

- Expansion for small $\text{Im } \Sigma(1, \omega + i\eta)$

$$A(1, \omega) \approx \frac{2\pi\delta(\omega - \text{E}^{\text{quasi}}(1))}{1 - \frac{d}{dz}\text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}} - \mu_1}}$$

$$-2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \frac{P}{\omega + \mu_1 - \text{E}^{\text{quasi}}(1)}$$

quasiparticle energy $\text{E}^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states $\approx$ new species

summation of ladder diagrams, Bethe-Salpeter equation

\[ G_L = \cdots + \text{ } + \text{ } + \text{ } + \cdots \]
Medium effects:
Quasiparticle approximation

- Skyrme
- relativistic mean field (RMF)
- Lagrangian: non-linear sigma
- TM1 parameters
- Single particle modifications
- energy shift, effective mass

- Dirac-Brueckner Hartree Fock (DBHF)
Quasiparticle energy shifts

Comparison of different approximations, BonnA separable interaction potential.

Full line - generalized Beth-Uhlenbeck approach,
dotted line - the same but the Pauli operator $(1 - f_1)(1 - f_1)$ instead of $(1 - f_1 - f_1)$,
dashed line - Brueckner-Bethe-Goldstone calculation with the Pauli operator $(1 - f_1)(1 - f_1)$. 
Quasiparticle picture: RMF and DBHF

J. Margueron et al., PRC 76, 034309 (2007)
Different approximations

- Expansion for small \( \text{Im } \Sigma(1, \omega + i\eta) \)

\[
A(1, \omega) \approx \frac{2\pi \delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{d}{dz} \text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}} - \mu_1} - 2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \frac{P}{\omega + \mu_1 - E^{\text{quasi}}(1)}
\]

quasiparticle energy \( E^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}(1)} \)

- chemical picture: bound states \( \hat{=} \) new species

summation of ladder diagrams, Bethe-Salpeter equation

\[
\begin{array}{c}
G_L \\
\hline \hline \\
\end{array}
= \quad + \\
+ \\
+ \\
+ \cdots
\]
Different approximations

low density limit:

\[
G_2^L(12, 1'2', i\lambda) = \sum_{n_P} \frac{\Psi_{n_P}(12)}{i\omega - E_{n_P}} \frac{1}{\Psi^*_{n_P}(12)}
\]

\[
\Sigma = \begin{array}{c}
T_2^L
\end{array}
\]

\[
n(\beta, \mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n_P} g_{12}(E_{n_P})
\]

\[
+ \sum_{2,n_P} \int_0^{\infty} dk \delta_{k,-p_1-p_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2\sin^2\delta_n(k) \frac{1}{\pi} \frac{d}{dk} \delta_n(k)
\]

- generalized Beth-Uhlenbeck formula
- correct low density/low temperature limit:
- mixture of free particles and bound clusters
Effective wave equation for the deuteron in matter

\[
\left( \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \right) \Psi_{n,P}(p_1, p_2) + \sum_{p_1', p_2'} (1 - f_{p_1} - f_{p_2}) V(p_1, p_2; p_1', p_2') \Psi_{n,P}(p_1', p_2') = E_{n,P} \Psi_{n,P}(p_1, p_2)
\]

Add self-energy \hspace{1cm} \text{Pauli-blocking}

Fermi distribution function

\[
f_p = \left[ e^{(\frac{p^2}{2m-\mu})/k_B T} + 1 \right]^{-1}
\]

BEC-BCS crossover: Alm et al., 1993
Deuterons in nuclear matter

\[ T = 10 \text{ MeV}, \ P: \text{center of mass momentum} \]
Deuteron quasiparticle properties

\[ E_d^{\text{qu}}(P) = E_d^{\text{free}} + \Delta E_d + \frac{\hbar}{2m_d^*} P^2 + O(P^4) \]

\[ E_d^{\text{free}} = -2.225\text{MeV} \]

\[ \Delta E_d^{\text{Pauli}}(T, n_B, \alpha) = \delta E_d^{(0)}(T, \alpha)n_B + O(n_B^2) \]

\[ \frac{m_d}{m_d^*}(T, n_B, \alpha) = 1 + \delta m_d^{(0)}(T, \alpha)n_B + O(n_B^2) \]

<table>
<thead>
<tr>
<th>T [MeV]</th>
<th>delta E [MeV fm^3]</th>
<th>delta m^* [fm^3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>364.3</td>
<td>21.3</td>
</tr>
<tr>
<td>4</td>
<td>712.9</td>
<td>87.1</td>
</tr>
</tbody>
</table>
Scattering phase shifts in matter
Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

Cluster decomposition of the self-energy
Few-particle Schrödinger equation in a dense medium

Four-particle Schrödinger equation with medium effects

\[
\begin{align*}
&[E_{\text{HF}}^{\text{HF}}(p_1) + E_{\text{HF}}^{\text{HF}}(p_2) + E_{\text{HF}}^{\text{HF}}(p_3) + E_{\text{HF}}^{\text{HF}}(p_4)] \psi_{nP}(p_1, p_2, p_3, p_4) \\
+ & \sum_{p_1'p_2'p_3'p_4'} \left\{ [1 - f(p_1) - f(p_2)] V(p_1p_2, p_1'p_2') \delta_{p_3p_3'} \delta_{p_4p_4'} \\
& + [1 - f(p_1) - f(p_3)] V(p_1p_3, p_1'p_3') \delta_{p_2p_2'} \delta_{p_4p_4'} \\
& + \text{permutations} \right\} \psi_{nP}(p_1', p_2', p_3', p_4') \\
= & E_{nP} \psi_{nP}(p_1, p_2, p_3, p_4)
\end{align*}
\]
In-medium shift of binding energies of clusters

Solution of the Faddeev-Yakubovski equation with Pauli blocking

M. Beyer et al., PLB 488, 247 (00), A. Sedrakian et al., PRC 73, 035803 (06)
Composition of dense nuclear matter

\[
n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A \{E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n\}
\]

\[
n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A \{E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n\}
\]

mass number \(A\),
charge \(Z_A\),
energy \(E_{A,\nu,K}\),
\(\nu\): internal quantum number,

- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)
Composition of symmetric nuclear matter

T=10 MeV

Light Cluster Abundances

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

S. Typel, 2007
Core-collapse supernovae

Density, electron fraction, and temperature profile of a 15 solar mass supernova at 150 ms after core bounce as function of the radius.

Influence of cluster formation on neutrino emission in the cooling region and on neutrino absorption in the heating region?

Composition of supernova core

Mass fraction $X$ of light clusters for a post-bounce supernova core

K. Sumiyoshi, G. R., PRC 77, 055804 (08)
Symmetry energy of a low density nuclear gas


Symmetry energy and single nucleon potential used in the IBUU04 transport model

The x parameter is introduced to mimic various predictions by different microscopic Nuclear many-body theories using different Effective interactions.

Single nucleon potential within the HF approach using a modified Gogny force:

\[
U(\rho, \delta, \bar{\rho}, \tau, x) = A_u(x)\frac{\rho_{\tau}}{\rho_0} + A_l(x)\frac{\rho_{\bar{\tau}}}{\rho_0} + B(\frac{\rho}{\rho_0})^\sigma (1 - x\delta^2) - 8\tau x\frac{B}{\sigma + 1}\frac{\rho_{\sigma - 1}}{\rho_0}\delta \rho_{\tau}.
\]

\[
+ \frac{2C_{\tau,\bar{\tau}}}{\rho_0} \int d^3 p' \frac{f_{\tau}(r, p')}{1 + (p - p')^2 / \Lambda^2} + \frac{2C_{\tau,\bar{\tau}}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(r, p')}{1 + (p - p')^2 / \Lambda^2}
\]

\[
\tau, \bar{\tau}' = \pm \frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma + 1}, A_u(x) = -96 - \frac{2Bx}{\sigma + 1}, K_0 = 211 M eV
\]

The momentum dependence of the nucleon potential is a result of the non-locality of nuclear effective interactions and the Pauli exclusion principle.


$E_{\text{sym}}(\rho)$ predicted by microscopic many-body theories

**EOS:** $E(\rho, \delta) = E_0(\rho, 0) + E_{\text{sym}}(\rho)\delta^2 + o(\delta^4)$, where $\delta \equiv (\rho_n - \rho_p) / \rho$

$E_{\text{sym}}(\rho) \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$

---

Quasiparticle approximation for nuclear matter

Equation of state for symmetric matter

Klaehn et al., PRC 74, 035802 (06)
Alpha-particle fraction in the low-density limit

symmetric matter, $T=2, 4, 8$ MeV

Symmetry energy and symmetry free energy

T=4MeV

Horowitz & Schwenk, NPA (2006)
Free and Internal Symmetry Energy

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters, temperature (3 - 10 MeV), free energy
Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

Quantum condensate

Bose-Einstein-Condensation of deuterons (BEC)

Bardeen-Cooper-Schrieffer pairing (BCS)
α-cluster-condensation
(quartetting)

G. Röpke, A. Schnell, P. Schuck, and P. Nozieres, PRL 80, 3177 (98)
\( \alpha \)-cluster-condensation

(quartetting)

\[ T_s^c \text{ (deuteron pairing)} \]

\[ T_4^c \text{ (\( \alpha \)-quartetting)} \]

\[ \text{temperature } T \text{ [MeV]} \]

\[ \text{chemical potential } \mu^* \text{ [MeV]} \]
Alpha cluster structure of Be 8

Contours of constant density, plotted in cylindrical coordinates, for 8Be(0+) . The left side is in the laboratory frame while the right side is in the intrinsic frame.

R.B. Wiringa et al., PRC 63, 034605 (01)
Self-conjugate 4n nuclei

$^{12}\text{C}$:
$0^+$ state at 0.39 MeV above the $3\alpha$ threshold energy:
$\alpha$ cluster interact predominantly in relative $S$ waves,
gaslike structure

$\alpha$-particle condensation in low-density nuclear matter
($\rho \leq \rho_0/5$)

$n\alpha$ cluster condensed states
-- a general feature in $N = Z$ nuclei?
Self-conjugate 4n nuclei

\( n\alpha \) nuclei: \(^8\text{Be}\), \(^{12}\text{C}\), \(^{16}\text{O}\), \(^{20}\text{Ne}\), \(^{24}\text{Mg}\), ...  

Single-particle shell model, or

Cluster type structures

ground state, excited states

\( n\alpha \) break up at the threshold energy \( E_{n\alpha}^{\text{thr}} = nE_\alpha \)
Variational ansatz

\[ |\Phi_{n\alpha}\rangle = (C^\dagger_\alpha)^n |\text{vac}\rangle \]

\( \alpha \)- particle creation operator

\[ C^\dagger_\alpha = \int d^3R e^{-\vec{R}^2/R_0^2} \]

\[ \times \int d^3r_1 \ldots d^3r_4 \phi_{0s}(\vec{r}_1 - \vec{R}) a_{\sigma_1\tau_1}^\dagger(\vec{r}_1) \ldots \phi_{0s}(\vec{r}_4 - \vec{R}) a_{\sigma_4\tau_4}^\dagger(\vec{r}_4) \]

with

\[ \phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)} \]
Variational ansatz

total \( n\alpha \) wave function

\[
\langle \vec{r}_1 \sigma_1 \tau_1 \ldots \vec{r}_{4n} \sigma_{4n} \tau_{4n} | \Phi_{n\alpha} \rangle
\]

\[
\propto \mathcal{A} \left\{ e^{-\frac{2}{B^2} (\vec{X}_1^2 + \ldots + \vec{X}_n^2)} \phi(\alpha_1) \ldots \phi(\alpha_n) \right\}
\]

where \( B^2 = (b^2 + 2R_0^2) \), \( \vec{X}_i = \frac{1}{4} \Sigma_n \vec{r}_{in} \),

\[
\phi(\alpha_i) = e^{-\frac{1}{8b^2} \Sigma_{m>n} (\vec{r}_{im} - \vec{r}_{in})^2} - \text{internal } \alpha \text{ wave function}
\]

3 alpha variational energy
### Results

<table>
<thead>
<tr>
<th></th>
<th>$E_k$ (MeV)</th>
<th>$E_{\text{exp}}$ (MeV)</th>
<th>$E_k - E_{n\alpha}^{\text{thr}}$ (MeV)</th>
<th>$(E - E_{n\alpha}^{\text{thr}})_{\text{exp}}$ (MeV)</th>
<th>$\sqrt{\langle r^2 \rangle}$ (fm)</th>
<th>$\sqrt{\langle r^2 \rangle}_{\text{exp}}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>$k = 1$</td>
<td>-85.9</td>
<td>-92.16 ($0^+_1$)</td>
<td>-3.4</td>
<td>-7.27</td>
<td>2.97</td>
</tr>
<tr>
<td></td>
<td>$k = 2$</td>
<td>-82.0</td>
<td>-84.51 ($0^+_2$)</td>
<td>+0.5</td>
<td>0.38</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td>$E^{\text{thr}}_{3\alpha}$</td>
<td>-82.5</td>
<td>-84.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>$k = 1$</td>
<td>-124.8</td>
<td>-127.62 ($0^+_1$)</td>
<td>-14.8</td>
<td>-14.44</td>
<td>2.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-128.0)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k = 2$</td>
<td>-116.0</td>
<td>-116.36 ($0^+_3$)</td>
<td>-6.0</td>
<td>-3.18</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>$k = 3$</td>
<td>-110.7</td>
<td>-113.62 ($0^+_5$)</td>
<td>-0.7</td>
<td>-0.44</td>
<td>3.97</td>
</tr>
<tr>
<td></td>
<td>$E^{\text{thr}}_{4\alpha}$</td>
<td>-110.0</td>
<td>-113.18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $B_e$ = -0.17 + 0.1

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{\text{thr}} = nE_\alpha$ denotes the threshold energy for the decay into $\alpha$-clusters, the values marked by * correspond to a refined mesh.
Estimation of condensate fraction in zero temperature $\alpha$-matter

$$n_0 = \frac{\langle \Psi | a^+_0 a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

destruction of the BEC of the ideal Bose gas: thermal excitation, but also correlations

“excluded” volume for $\alpha$-particles $\approx 20 \text{ fm}^3$ 

at nucleon density $\rho = 0.048 \text{ fm}^{-3}$ filling factor $\approx 28\%$  
(liquid $^4\text{He}$: 8% condensate),

destruction of the condensate at $\approx \rho_0/3$
Estimation of condensate fraction in zero temperature $\alpha$-matter

$\alpha$-cluster condensate in $^{12}\text{C}$, $^{16}\text{O}$:
resonating group method
occupation numbers of $\alpha$-orbits in $^{12}\text{C}$

<table>
<thead>
<tr>
<th></th>
<th>RMS radii</th>
<th>S-orbit</th>
<th>D-orbit</th>
<th>G-orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1^+$</td>
<td>2.44 fm</td>
<td>1.07</td>
<td>1.07</td>
<td>0.82</td>
</tr>
<tr>
<td>$O_2^+$</td>
<td>4.31 fm</td>
<td>2.38</td>
<td>0.29</td>
<td>0.16</td>
</tr>
</tbody>
</table>

80% condensate at 1/8 nuclear matter density

T. Yamada & P. Schuck: $(2.16 - \text{normal})/3 \approx 60\%$
Suppression of condensate fraction

- Alpha-alpha interaction (Ali/Bodmer), no Pauli blocking:
- Variational calculation (Clark/Jastrow approach to the alpha-particle condensate amplitude) (crosses)
- First order approximation (full line)
- Yamada/Schuck’s result for condensate in C12 - O2+ (stars)

Y. Funaki et al., PRC 77, 064312 (08)
Quasiparticle approximation for nuclear matter
Summary

• The low-density limit of the nuclear matter EoS can be rigorously treated. The Beth-Uhlenbeck virial expansion is a benchmark.

• An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self-energy and Pauli blocking is included. An interpolation between low and high densities is possible.

• Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation (quartetting), and the behavior of the symmetry energy.
Thanks
to D. Blaschke, C. Fuchs, J. Natowitz, T. Klaehn,
S. Shlomo, P. Schuck,
A. Sedrakian, K. Sumiyoshi, S. Typel, H. Wolter
for collaboration

to you
for attention
Correlations in the medium

\[ \sum_2 = \begin{array}{c} \includegraphics[width=0.3\textwidth]{diagram1} \\
\includegraphics[width=0.3\textwidth]{diagram2} \\
\includegraphics[width=0.3\textwidth]{diagram3} \\
\includegraphics[width=0.3\textwidth]{diagram4} \\
\includegraphics[width=0.3\textwidth]{diagram5} \\
\end{array} \]
Account of two-particle correlations in the medium

\[ R_{dp} \]

- correlated medium
- uncorrelated medium

\[ T = 10 \text{ MeV} \]

\[ \log [\text{fm}^3] \]

-3 -2 -1
Heavy nuclei abundances in nuclear matter

$T=10$ MeV, asymmetry 0.42, as function of baryon density

$n, p, d, t, \text{He}3, \text{He}4, \text{Li}5, \ldots$
Alpha-condensate (quartetting) in 4n symmetric nuclei

• A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,

• G. Röpke, A. Schnell, P. Schuck, and P. Nozieres,

• Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,

• T. Yamada, P. Schuck,
Approximations to the symmetry energy

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

S. Kubis, Neutron stars with non-homogeneous core, Talk 26.2.08, Ladek
Influence of cluster formation on the equation of state

T = 10 MeV

Chemical potential of symmetric nuclear matter.

Inclusion of cluster formation shifts down the chemical potential.

The region of thermodynamical instability is reduced.