A Supernova Equation of State with Light and Heavy Clusters

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Motivation I

- **equation of state** (EoS) of dense matter:

  essential ingredient in many astrophysical models
  (supernovae calculations, compact star models, . . .)

  ⇒ static properties and dynamical evolution

  ⇒ energetics, temperature, composition, . . .

- properties required for large range of **conditions**
  - density: $10^{-8} \lesssim n/n_{\rm sat} \lesssim 10$
  - temperature: $0 \leq T \lesssim 100$ MeV
  - proton fraction: $0 \leq Y_p \lesssim 0.6$

    (or isospin asymmetry $\beta = 1 - 2Y_p$)

  ⇒ practical global approach needed

Motivation II

- many EoS developed in the past:

  from simple parametizations to sophisticated models

- many investigations of detailed aspects:

  often restricted to particular conditions

- only few EoS in practical use: e.g.
  - J.M. Lattimer, F.D. Swesty (Nucl. Phys. A 535 (1991) 331)
  - H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi (Prog. Theor. Phys. 100 (1998) 1013)

- several deficiencies in applied EoS

Motivation III

aim: development of improved EoS for astrophysics

- more microscopic, self-consistent description
- well adjusted model parameters, better constrained
- relativistic approach
- additional particle species/clusters, respecting Bose/Fermi statistics
- cover widest possible range of n, T, Y_p

with special attention to

- consistency
- constraints

Outline

- General Considerations
- Relativistic Mean-Field (RMF) Model
- Constraints
- Light Clusters
- Generalized RMF Model
- Phase Transition
- Heavy Clusters
- Summary and Outlook

General Considerations I

- **ingredients**:

standard

- neutrons
- protons

partly optional

- electrons
  (inhomogeneous systems with Coulomb interaction)

optional

- muons
- photons
- neutrinos
- light nuclei/clusters
  (^{2}H, ^{3}H, ^{3}He, ^{4}He, . . .)
- heavy nuclei/clusters
- hyperons
- other exotica?
  (pions, kaons, quarks, . . .)

General Considerations II

- **calculation**:

input quantities

- total baryon number density n
- baryonic charge fraction Y_q
- temperature T

output quantities

- (free) energy density e (f),
- entropy density s,
- pressure p,
- chemical potentials μ_i,
- particle abundancies X_i,
- . . .

two-step procedure

- local minimization of free energy density $f(n, Y_q, T)$ by varying individual particle densities for given conditions n, Y_q, T (e.g. charge neutrality, chemical equilibrium)
- construction of global convexity of free energy density f $\Rightarrow$ phase transition(s)

General Considerations III

- **non-hadronic** contributions
  - homogeneous spatial distribution for uncharged particles
  - only Coulomb interaction for charged particles
  - simple thermodynamics
- **hadronic** contributions
  - possible general strategy:

    start with given nucleon-nucleon interaction,
    apply machinery of many-body theory . . . (cf. Dirac-Brueckner calculations)
    . . . too complicated, too limited
  - here: more phenomenological approach, combination
    - **relativistic mean-field model** with density-dependent nucleon-meson couplings
    - **generalized Beth-Uhlenbeck approach** (⇒ light clusters)
    - modified **Wigner-Seitz cell** in relativistic **Thomas-Fermi approximation** (⇒ heavy clusters)

Relativistic Mean-Field Model I - Lagrangian

- standard **Lagrangian density** of Walecka type
  with nucleons (ψ), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$) and photons ($A_\mu$) as degrees of freedom

$$\mathcal{L} = \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - \Gamma_\omega \omega_\mu - \Gamma_\rho \vec{\tau}\cdot\vec{\rho}_\mu - \Gamma_\gamma \frac{1+\tau_3}{2} A_\mu\right) - (m - \Gamma_\sigma \sigma)\right]\psi + \mathcal{L}_M$$

contribution of free bosons

$$\mathcal{L}_M = \frac{1}{2}\left[\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} + m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{2} \vec{H}_{\mu\nu}\cdot\vec{H}^{\mu\nu} + m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{2} F_{\mu\nu} F^{\mu\nu}\right]$$

field tensors $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad G_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \quad \vec{H}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$

  - mesons: convenient auxiliary fields, same quantum numbers as "real" mesons
  - only minimal (linear) meson-nucleon couplings
  - density-dependent couplings Γ_i, no non-linear meson self-interactions

$\Rightarrow$ nucleon/meson/photon field equations, solved in mean-field approximation
(Hartree approximation, no-sea approximation, classical meson/photon fields)

Relativistic Mean-Field Model II - Couplings

- **nucleon-meson couplings** $\Gamma_i = \Gamma_i(\varrho)$

  functionals of vector density $\varrho = \sqrt{j_\mu j^\mu}$ with $j_\mu = \bar{\psi}\gamma_\mu\psi$

  ◦ functional form as suggested by Dirac-Brueckner calculations of nuclear matter

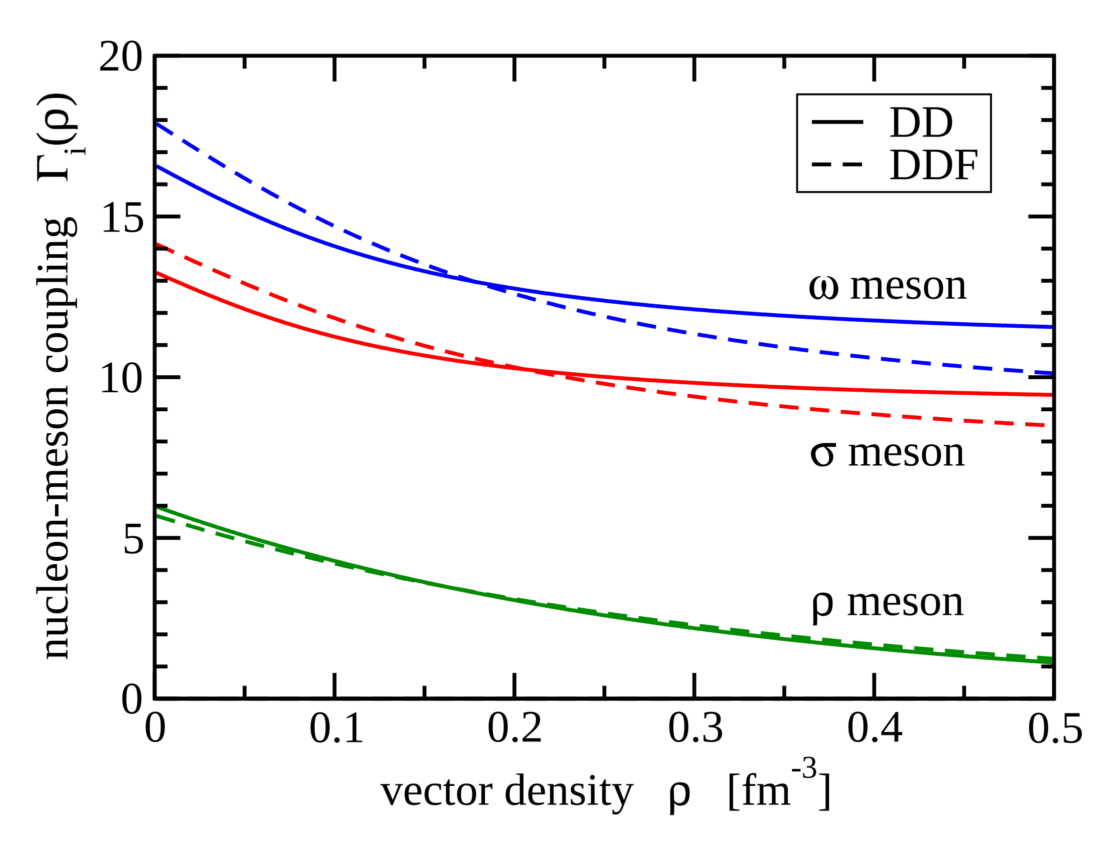
  ◦ very flexible approach

  ◦ well controlled asymptotics

  ⇒ rearrangement contributions in nucleon self-energies

  ⇒ thermodynamical consistency

$$p_{\text{field}} = \frac{1}{3}\sum_{m=1}^{3}\langle T^{mm}\rangle = p_{\text{thermo}} = n^2 \left.\frac{\partial(\varepsilon/n)}{\partial n}\right|_{T,V}$$



S. Typel, H.H. Wolter, Nucl. Phys. A 656 (1999) 331;

C. Fuchs, H. Lenske, H.H. Wolter, Phys. Rev. C 52 (1995) 3043

Relativistic Mean-Field Model III - Parameters

parameters of the model

- coupling functions $\Gamma_i(\varrho) = \Gamma_i(\varrho_{\text{sat}}) f_i(\varrho/\varrho_{\text{sat}})$
  - coupling strengths at saturation $\Gamma_\omega(\varrho_{\text{sat}})$, $\Gamma_\sigma(\varrho_{\text{sat}})$, $\Gamma_\rho(\varrho_{\text{sat}})$
  - functional dependence on density: 3 or 4 parameters for ω and σ mesons, 1 parameter for ρ meson

  $\Rightarrow$ 5 or 6 isoscalar and 2 isovector parameters

- masses of nucleons m_p, m_n and mesons m_i
  - experimental values for masses of nucleons, ω and ρ meson
  - mass of σ meson variable

  $\Rightarrow$ 1 parameter

- $i = \omega, \sigma$ $\quad f_i(x) = a\frac{1+b(x+d)^2}{1+c(x+d)^2}$ with conditions $f_i(1) = 1$, $f_i''(0) = 0$
- $i = \rho$ $\quad f_i(x) = \exp[-a(x-1)]$

(S. Typel and H.H. Wolter, Nucl. Phys. A 656 (1999) 331)

$\Rightarrow$ in total **8 or 9 free parameters** (highly correlated)
fit, constraints?

Constraints I - Nuclei and NN scattering

fit of model parameters to

- properties of spherical nuclei
  - binding energies
  - spin-orbit splittings
  - charge form factor (charge and diffraction radius, surface thickness)
  - neutron skin thickness

  ⇒ couplings $\Gamma_i(\rho)$ close to saturation density and below

  - density dependence essential
  - very successful

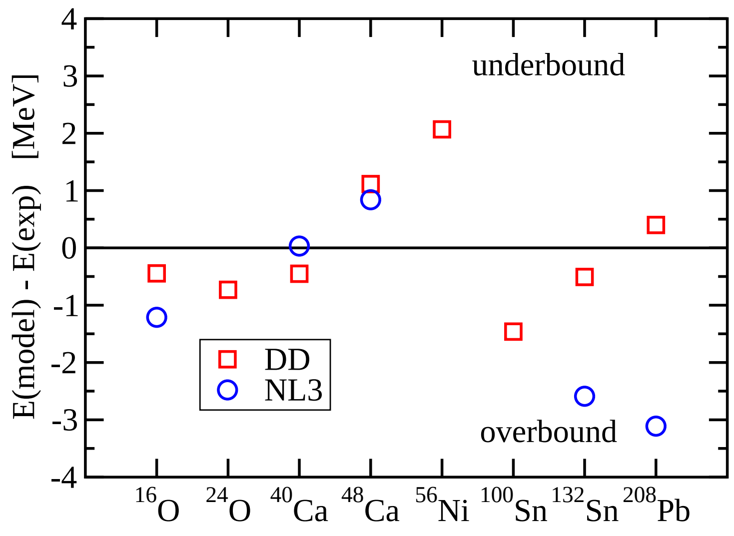
  ⇒ nuclear matter parameters

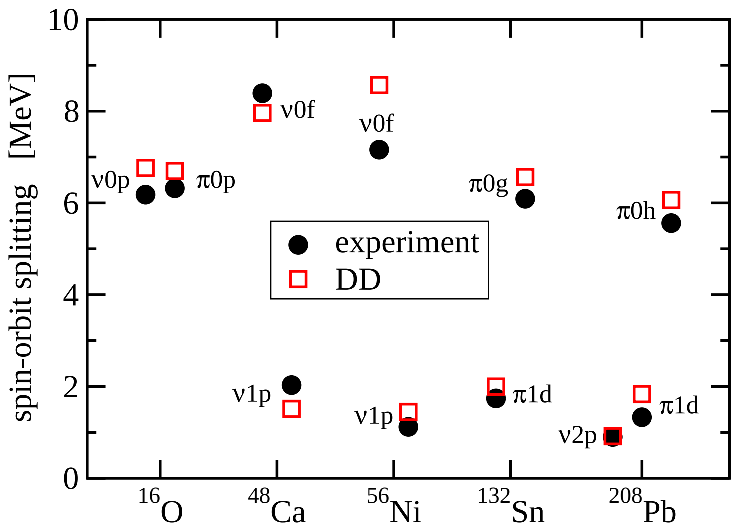
- low-energy nucleon-nucleon scattering
  - scattering lengths
  - effective ranges
  - phase shifts
  - deuteron binding energy

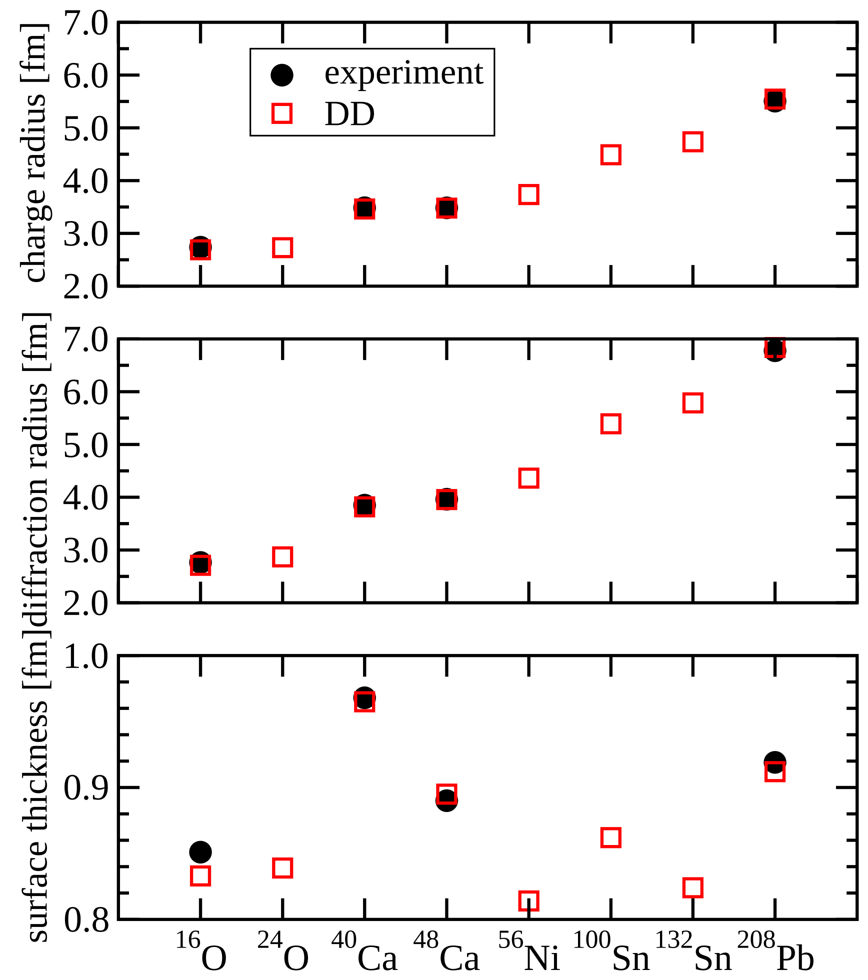
  ⇒ couplings $\Gamma_i(\rho)$ at zero density

  - "effective one-boson exchange potential"
  - only reproduction of observables needed, no "realistic" potential

Constraints II - Properties of Nuclei







$\Rightarrow$ high-quality description

Constraints III - Nuclear Matter

- **binding energy per nucleon** near saturation:

$$\frac{E}{A}(n,\beta) = \frac{\varepsilon}{n} = a_V + \frac{K}{18}x^2 - \frac{K'}{162}x^3 + \beta^2 \left(J + \frac{L}{3}x + \dots \right) + \dots$$

with $x = (n - n_{\mathrm{sat}})/n_{\mathrm{sat}}$, asymmetry $\beta = 1 - 2Y_p$ and

nuclear matter parameters

◦ n_{sat}, a_V, J ⇒ general trend of binding energies

◦ K ⇒ giant resonances

K and K' ⇒ ratio surface tension/surface thickness

K' small, even negative in parameter fits to surface properties ⇒ stiffer EoS

◦ L: neutron skin thickness ⇒ density dependence of symmetry energy

⇒ fit to surface properties of atomic nuclei important

Constraints IV - Nuclear Matter

- **effective Dirac mass** at saturation $m_D = m - \Sigma$ with scalar self-energy Σ

  $\Rightarrow$ spin-orbit splitting in nuclei

  ($m_D \neq$ effective Landau mass $\Rightarrow$ density of states)

- **7 parameters** $(n_{\text{sat}}, a_V, K, K', J, L, m_D)$ quite well **constrained**

model	n_{sat} [fm^{-3}]	a_V [MeV]	K [MeV]	K' [MeV]	J [MeV]	L [MeV]	m_D/m
DD	0.1487	-16.021	240.0	-134.6	32.0	56.0	0.565
DDF	0.1469	-16.024	223.1	757.8	31.6	56.0	0.556
NL3	0.1482	-16.240	271.5	-203.0	37.4	100.9	0.596

DD: S. Typel, Phys. Rev. C 71 (2005) 064301

DDF: T. Klähn et al., Phys. Rev. C 74 (2006) 035802

NL3: G.A. Lalazissis, J. König, P. Ring, Phys. Rev. C 55 (1997) 540

Constraints V - Astronomy and Heavy-Ion Collisions

fit of model parameters to

- properties of neutron stars

  (T. Klähn et al., Phys. Rev. C 74 (2006) 035802)

  - maximum mass $\Leftrightarrow$
    stiffness of symmetric matter EoS
    (K, K')
  - cooling by direct URCA process $\Leftrightarrow$
    stiffness of symmetry energy
    (J, L)
  - mass - radius relation
  - gravitational mass - baryon number relation

- heavy-ion collisions
  - elliptic flow $\Rightarrow$
    high-density EoS not too stiff

    (P. Danielewicz et al., Science 298 (2002) 1592)
  - sub-threshold kaon production $\Rightarrow$
    soft EoS of symmetric nuclear matter

    (C. Fuchs, Pro. Part. Nucl. Phys. 56 (2006) 1)
  - isoscaling of fragment yields $\Rightarrow$
    soft symmetry energy ($L \approx 60 - 80$ MeV)

    (D. V. Shetty et al., Phys. Rev. C 75 (2007) 034603)

$\Rightarrow$ couplings $\Gamma_i(\rho)$ above saturation density
modification of density dependence?

Light Clusters I - Theoretical Methods

low densities:

two-, three-, . . . many-body correlations due to NN interaction ⇒

- ∘ modification of thermodynamical properties
- ∘ bound states appear as new particle species

theoretical approaches:

- virial expansion in classical description (with potentials)
  - ∘ expansion of grand-canonical partition function $\mathcal{Z}(V,T,\mu_i)$ in powers of fugacities $z_i = \exp(\mu_i/T) \ll 1$ with chemical potentials $\mu_i \ll 0$
- quantum mechanical generalization (with density of states)
  (G. E. Beth and E. Uhlenbeck Physica 3 (1936) 729, Physica 4 (1937) 915)
- generalization with medium effects in thermodynamical Green's function appoach
  (M. Schmidt, G. Röpke and H. Schulz, Ann. Phys. 202 (1990) 57)

Light Clusters II - Virial Expansion

- virial expansion of grand-canonical potential

$$\Omega(V,T,\mu_i) = -VT\left(\sum_i b_i \frac{z_i}{\lambda_i^3} + \sum_{ij} b_{ij} \frac{z_i z_j}{\lambda_i^{3/2}\lambda_j^{3/2}} + \dots\right) = -pV$$

  with thermal wave lengths $\lambda_i = \hbar\sqrt{2\pi/(m_i T)}$ and virial coefficients b_i, $b_{ij}, \dots$

  ◦ first virial coefficient $b_i = g_i$ statistical degeneracy factor

  ◦ second virial coefficient b_{ij} $\Leftrightarrow$ correlation due to interaction

- particle densities

$$n_i = \left.\frac{\partial p}{\partial \mu_i}\right|_{V,T} = b_i \frac{z_i}{\lambda_i^3} + 2\sum_j b_{ij} \frac{z_i z_j}{\lambda_i^{3/2}\lambda_j^{3/2}} + \dots = n_i^{\mathrm{free}} + n_i^{\mathrm{corr}}$$

  with free and correlated densities

- natural upper limit for virial expansion: $n_i\lambda_i^3 < 1$

Light Clusters III - Second Virial Coefficient

- second quantummechanical virial coefficient

$$b_{ij}(T) = \frac{1+\delta_{ij}}{2} \frac{\lambda_i^{3/2}\lambda_j^{3/2}}{\lambda_{ij}^3} \int dE \, D_{ij}(E) \exp\left(-\frac{E}{T}\right)$$

with density of states $D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ij)}) + \sum_l g_l^{(ij)} \frac{1}{\pi} \frac{d\delta_l^{(ij)}}{dE}$

  – contributions from bound states at energies $E_k^{(ij)} < 0$
  – contributions from continuum states in channels l with phase shifts $\delta_l^{(ij)}(E)$
  – correction $(1 + \delta_{ij})$ for identical particles
  – further corrections for Fermi or Bose statistics

- if bound state energies $E_k^{(ij)} < 0$ and phase shifts $\delta_l^{(ij)}$ are known experimentally

  $\Rightarrow$ low-density behaviour of EoS established model-independently

  (e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

Light Clusters IV - Deuteron Formation

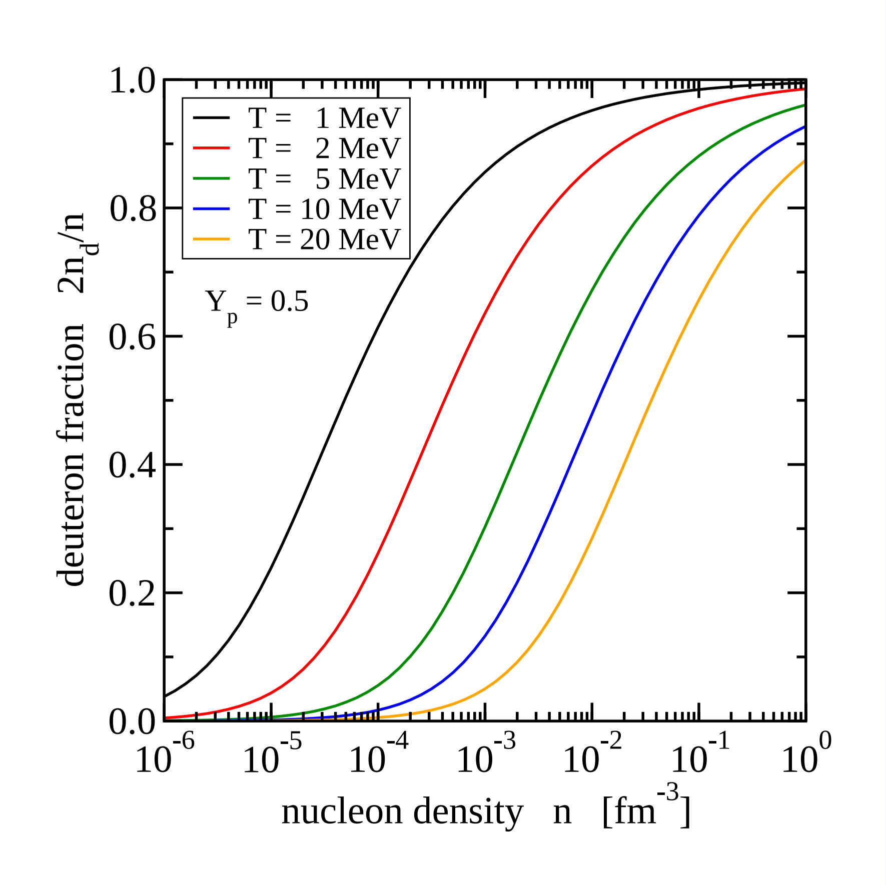
example

- deuteron formation in symmetric nuclear matter:

  total nucleon density $n = n_{\text{free}} + 2n_d$ with

  $$n_d = 3 \cdot 2^{-\frac{5}{2}} \lambda^3 n_{\text{free}}^2 \exp(-E_d/T)$$

  $\Rightarrow$ law of mass action $n + p \Leftrightarrow d$



- high densities: unphysical behaviour,
  no clusters but homogeneous matter expected

$\Rightarrow$ generalization necessary:

- ◦ in-medium interaction
- ◦ Pauli blocking of states
- ◦ neutron-proton asymmetric systems
- ◦ three-body (^{3}H, ^{3}He), four-body (^{4}He) correlations and beyond
- ◦ Boson/Fermion statistics

Light Clusters V - Generalized BU Approach

- thermodynamic Green's function approach
  (M. Schmidt, G. Röpke, H. Schulz, Ann. Phys. 202 (1990) 57)
  ⇒ go beyond quasi-particle approximation of spectral function

$$\Rightarrow \quad n_{\mathrm{corr}}(\mu, T) = \int \frac{d^3P}{(2\pi\hbar)^3} \int dE \, g\left[E + E_{\mathrm{cont}}(P, \mu, T), \mu, T\right] \, \varrho(E, P, \mu, T)$$

with c.m. momentum $\vec{P}$, relative energy E, Bose-Einstein distribution function g, continuum edge E_{cont} (energy of continuum state with relative energy $E = 0$) and generalized density of states

$$\varrho(E, P, \mu, T) = \sum_{\alpha} c_{\alpha} \left[\delta(E - E_{\alpha}) + \frac{2}{\pi} \sin^2 \delta_{\alpha} \, \frac{d}{dE} \delta_{\alpha} \right]$$

  - medium-dependent shift of binding energies $E_{\alpha} = E_{\alpha}(P, \mu, T)$, no bound state contributions for c.m. momenta $P < P_{\mathrm{Mott}}$
  - generalized scattering phase shifts $\delta_{\alpha} = \delta_{\alpha}(E, P, \mu, T)$ from in-medium T-matrix

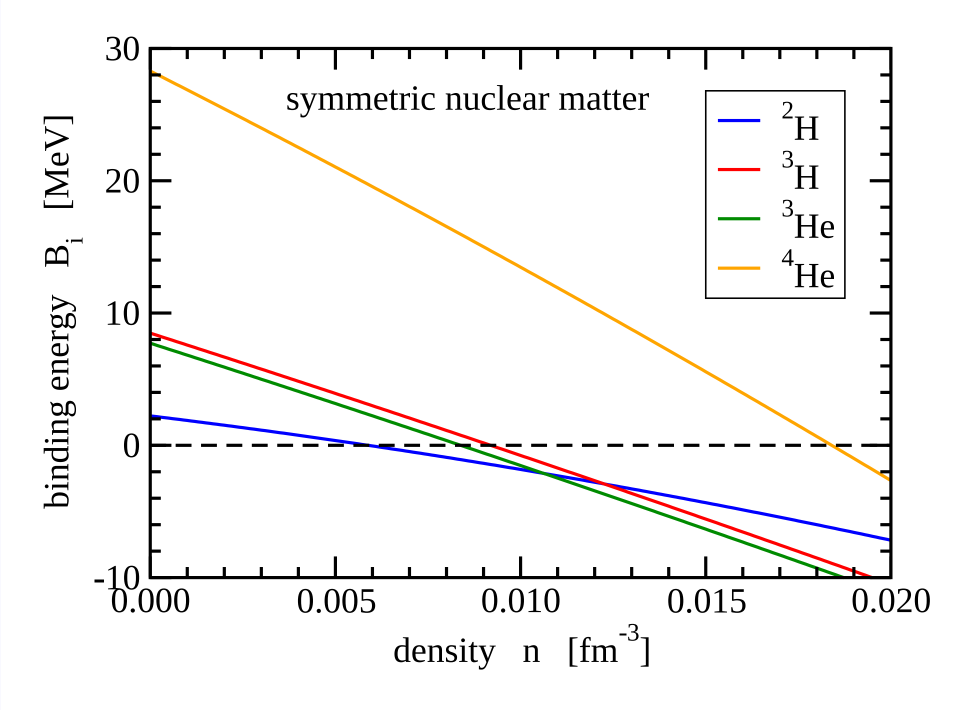
Light Clusters VI - BU Approach and RMF

- **model calculations** needed
  ⇒ parametrization of
    - effective binding energies
    - phase shifts

- combine generalized Beth-Uhlenbeck approach with relativistic mean-field model
  ⇒ treat cluster states (bound & continuum) as explicit degrees of freedom in RMF Lagrangian

- **density and temperature dependence** of effective cluster binding energies
  ⇒ 'rearrangement' contributions to nucleon self-energies and entropy density
  ⇒ essential for thermodynamical consistency

- consistency of interactions?



Generalized RMF Model I - Lagrangian

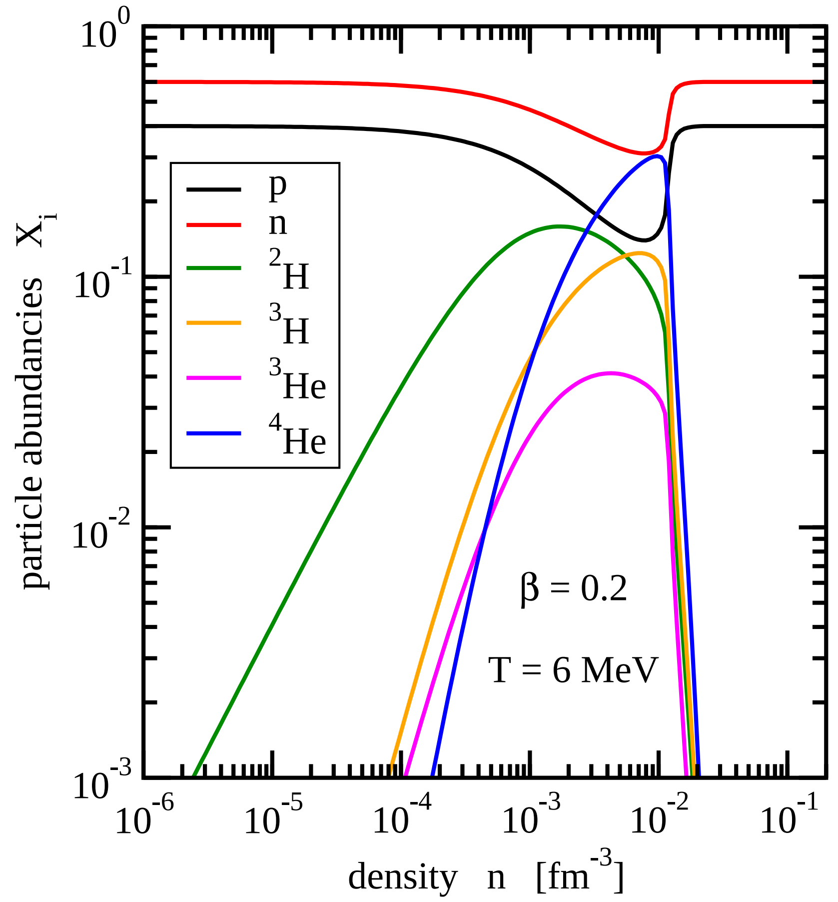
- **Lagrangian density** $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_B + \mathcal{L}_M$ with contributions of
  - fermions $(p, n, \Lambda, \Sigma^+, \dots, {}^3\mathrm{H}, {}^3\mathrm{He}, \dots, e, \mu, \nu_e, \dots)$ $\mathcal{L}_F = \sum_{i \in F} \bar{\psi}_i \left[\gamma_\mu i D_i^\mu - M_i\right] \psi$
  - bosons $({}^2\mathrm{H}, {}^4\mathrm{He})$ $\mathcal{L}_B = \frac{1}{4}\left(i D_\alpha^\mu \varphi_\alpha\right)^* \left(i D_{\alpha\mu} \varphi_\alpha\right) - \frac{1}{2}\varphi_\alpha^* M_\alpha^2 \varphi_\alpha$
    $+\frac{1}{2}\left(i D_d^\mu \varphi_d^\nu - i D_d^\nu \varphi_d^\mu\right)^* \left(i D_{d\mu} \varphi_{d\nu} - i D_{d\nu} \varphi_{d\mu}\right) - \frac{1}{2}\varphi_d^{\mu *} M_d^2 \varphi_{d\mu}$
  - mesons $(\sigma, \omega_\mu, \vec{\rho}_\mu)$ und photons (A_μ) $\mathcal{L}_M = \dots$
  - ∘ interaction via
    - covariant derivatives $i D_i^\mu = i\partial^\mu - \Gamma_{i\omega}\omega^\mu - \Gamma_{i\rho}\vec{\tau} \cdot \vec{\rho}^\mu - \Gamma_{i\gamma} A^\mu$
    - effective masses $M_i = m_i - \Gamma_{i\sigma}\sigma - B_i$

    with density dependent couplings Γ_{ij} and effective binding energies B_i of clusters with masses $m_i = Z_i m_p + N_i m_n$

⇒ field equations with additional source terms and modified "rearrangement" contributions in self-energies

Generalized RMF Model II - Cluster Formation

- consider 2-, 3-, and 4-body correlations
  - ◦ only bound-state contributions
  - ◦ energy shifts in quadratic approximation (function of free nucleon densities)
  - ⇒ deuterons, tritons, helions, and alphas
  - ◦ continuum contributions (to be included)
- Mott effect: suppression of cluster formation at high densities
- correct limits for low and high densities
- no heavy clusters/phase transition included here

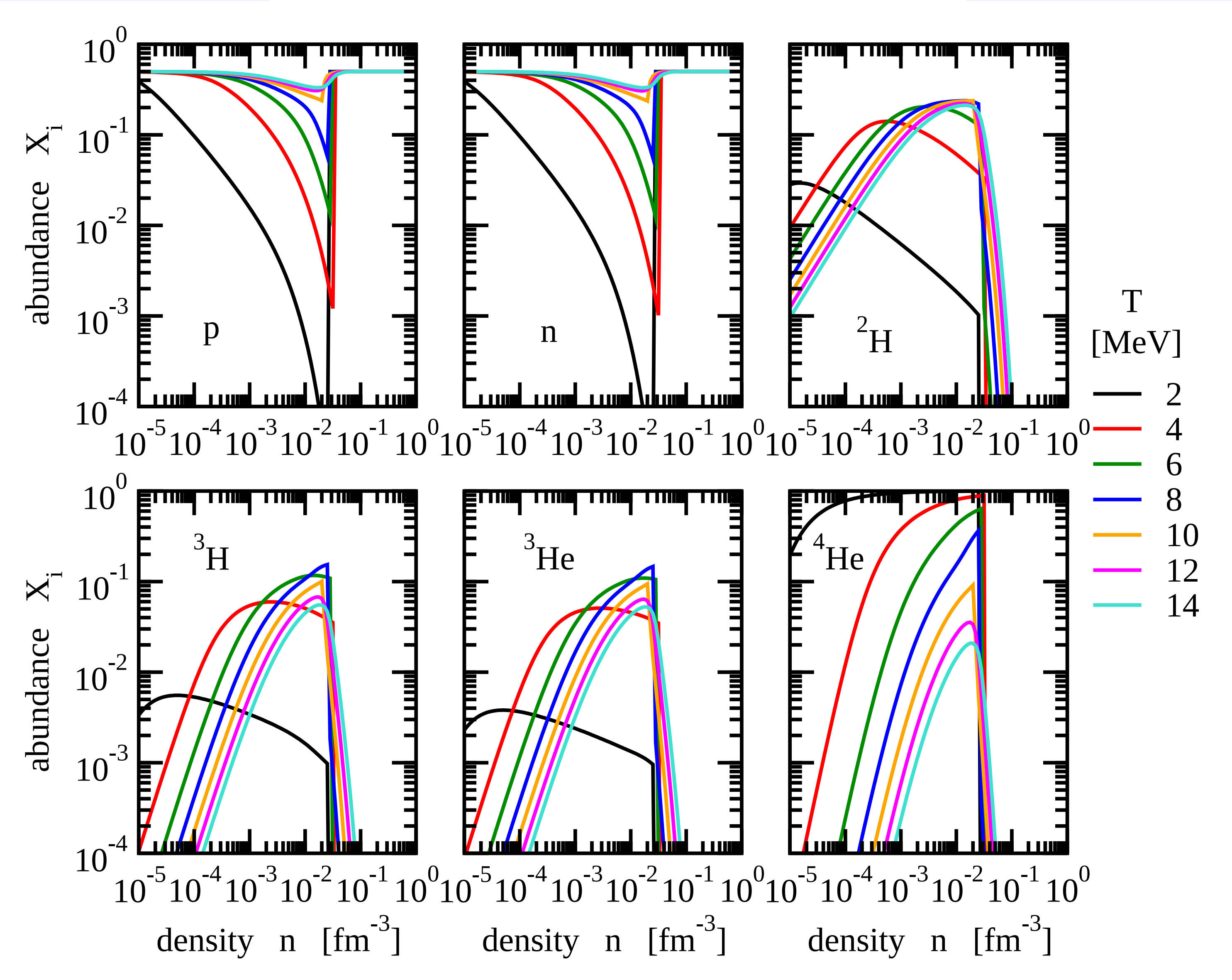


⇒ correlations in the medium

- ◦ many-body bound states treated as individual particles (in chemical equilibrium)
- ◦ two-particle correlations in scattering states (to be included)

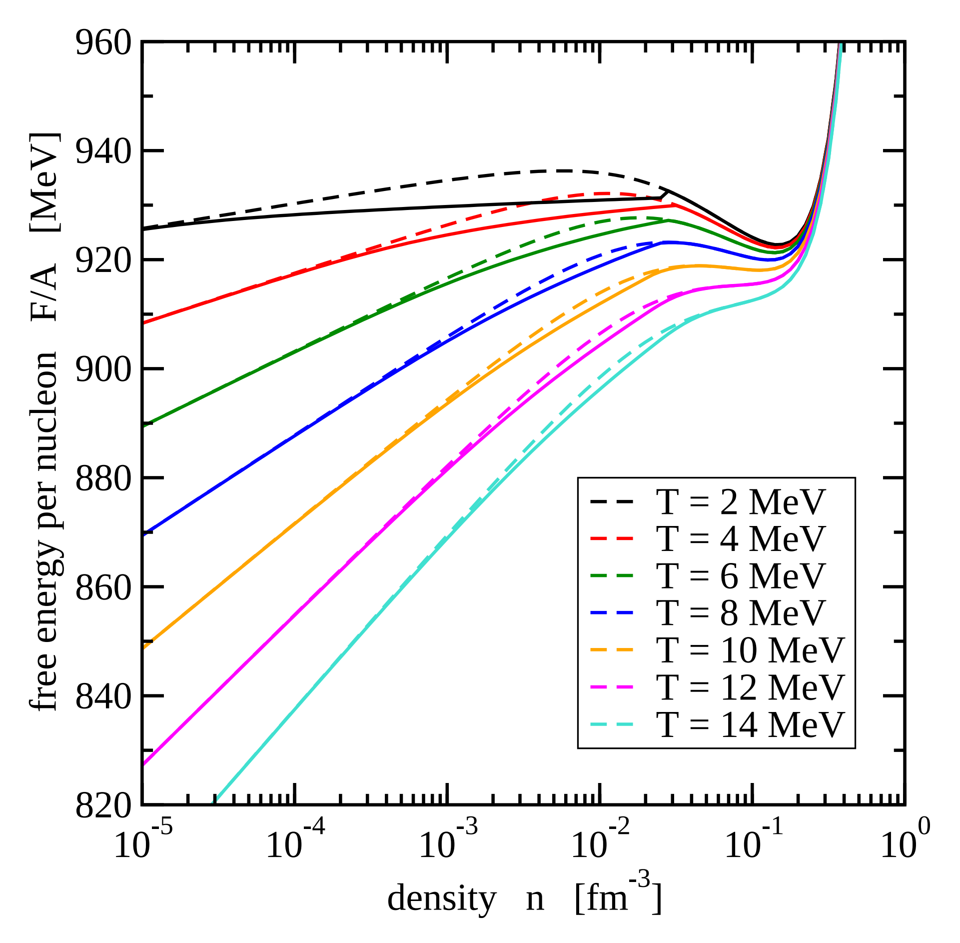
Generalized RMF Model III - Particle Abundancies

$X_i = A_i \frac{n_i}{n}$ for asymmetry $\beta = 0.0$

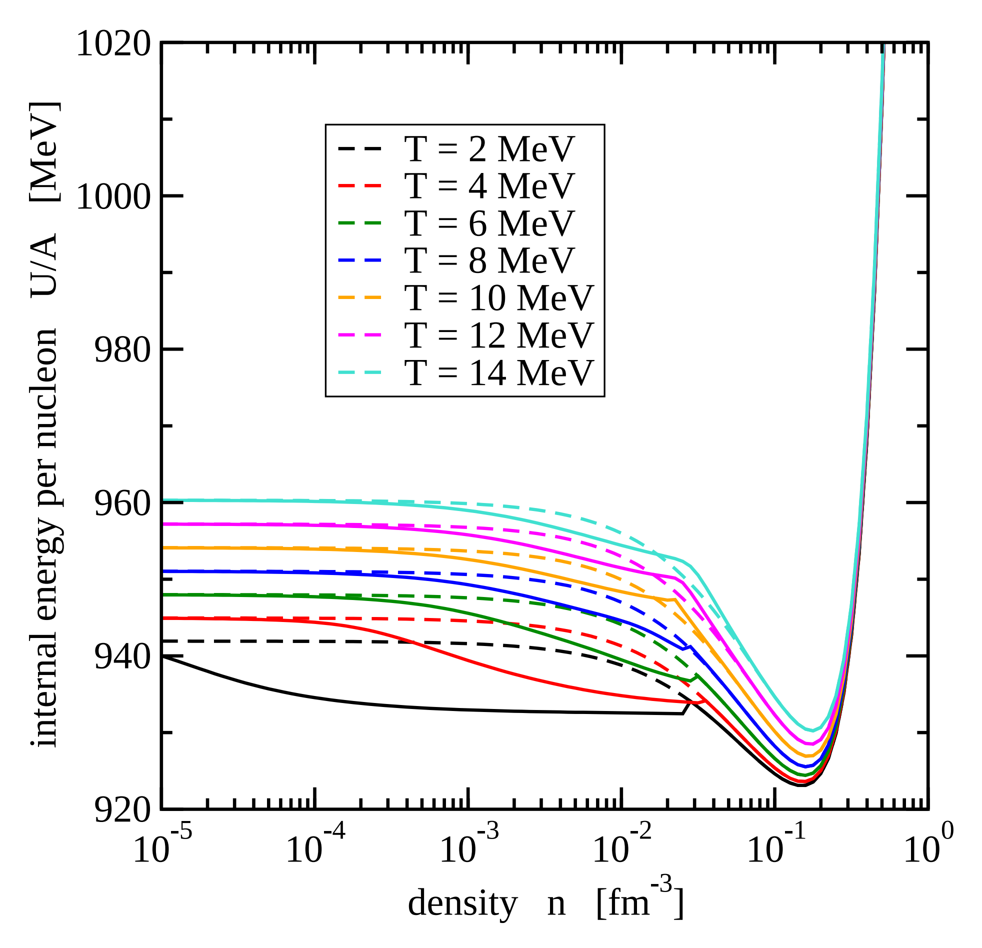


without (dashed) and with (solid) clusters for asymmetry $\beta = 0.0$

free energy per nucleon $F/A = f/n$



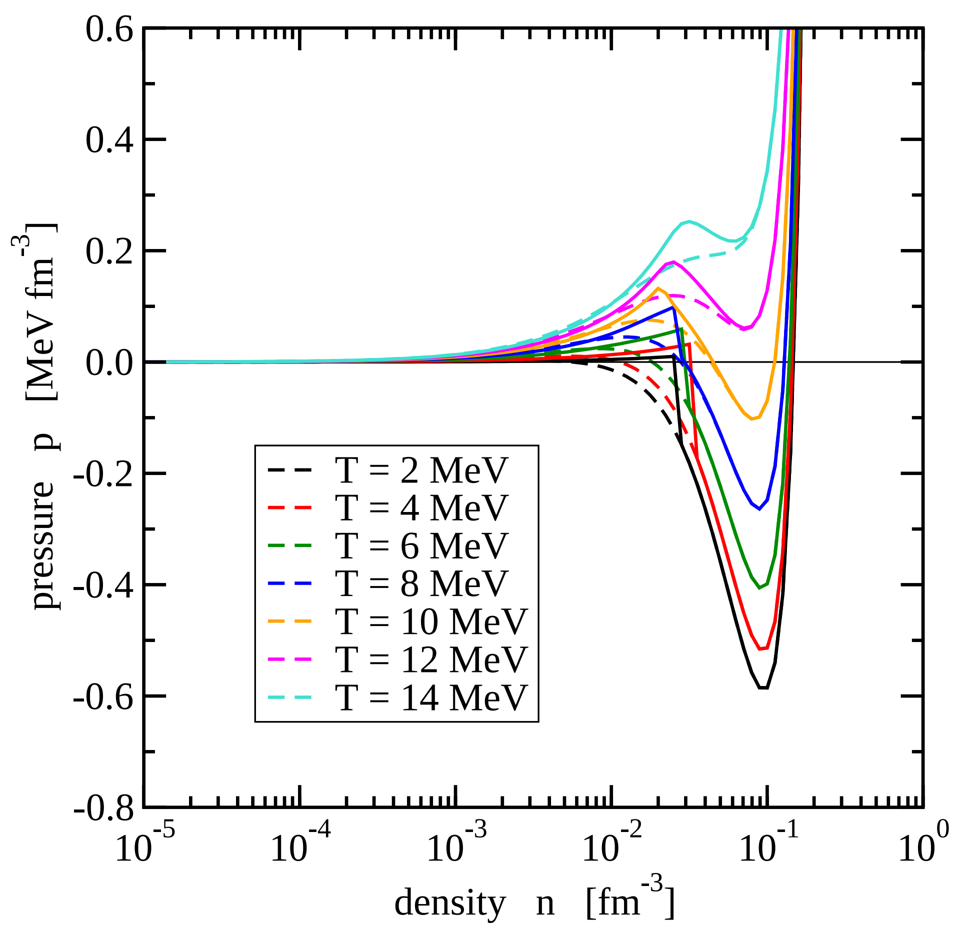
internal energy per nucleon $U/A = u/n$



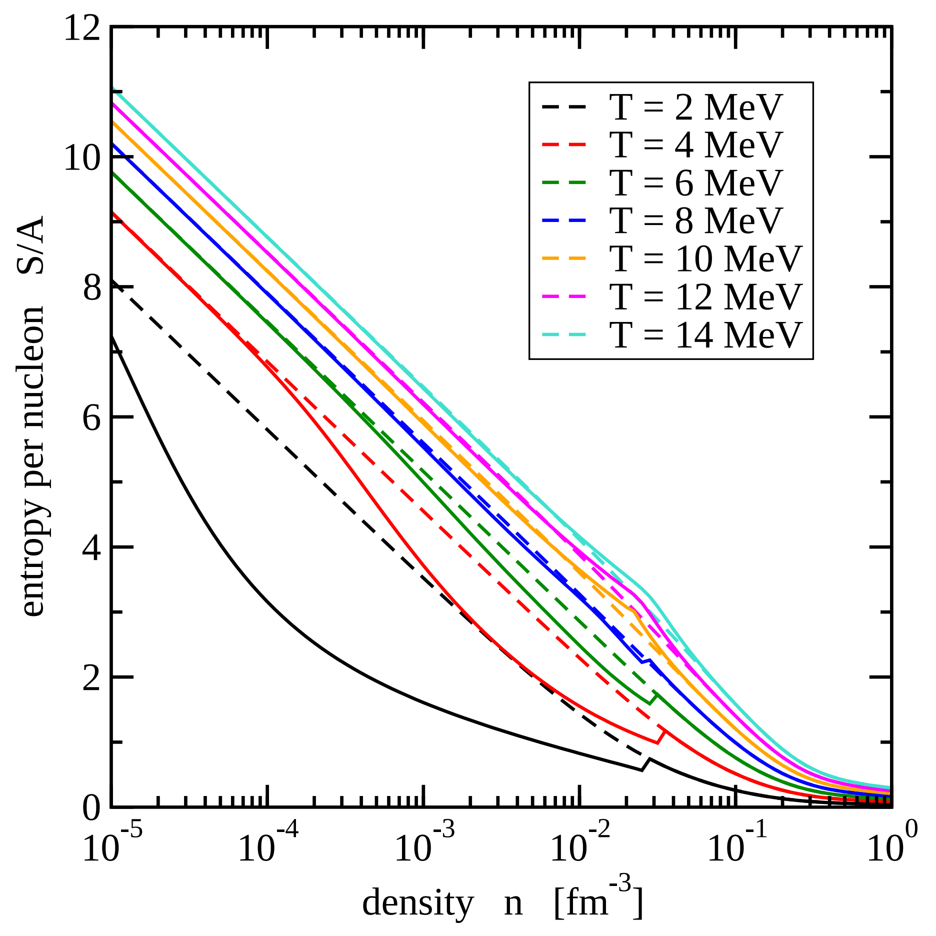
Generalized RMF Model V - Pressure and Entropy

without (dashed) and with (solid) clusters for asymmetry $\beta = 0.0$

pressure p



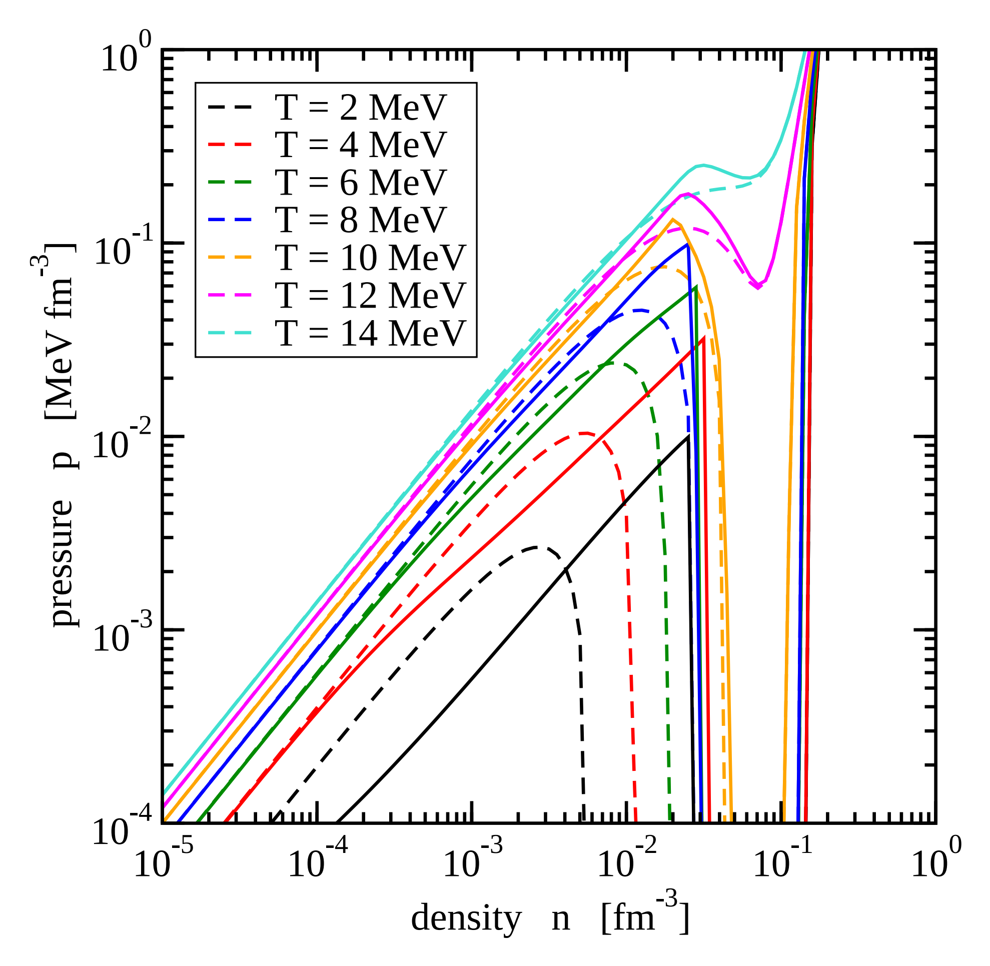
entropy per nucleon $S/A = s/n$



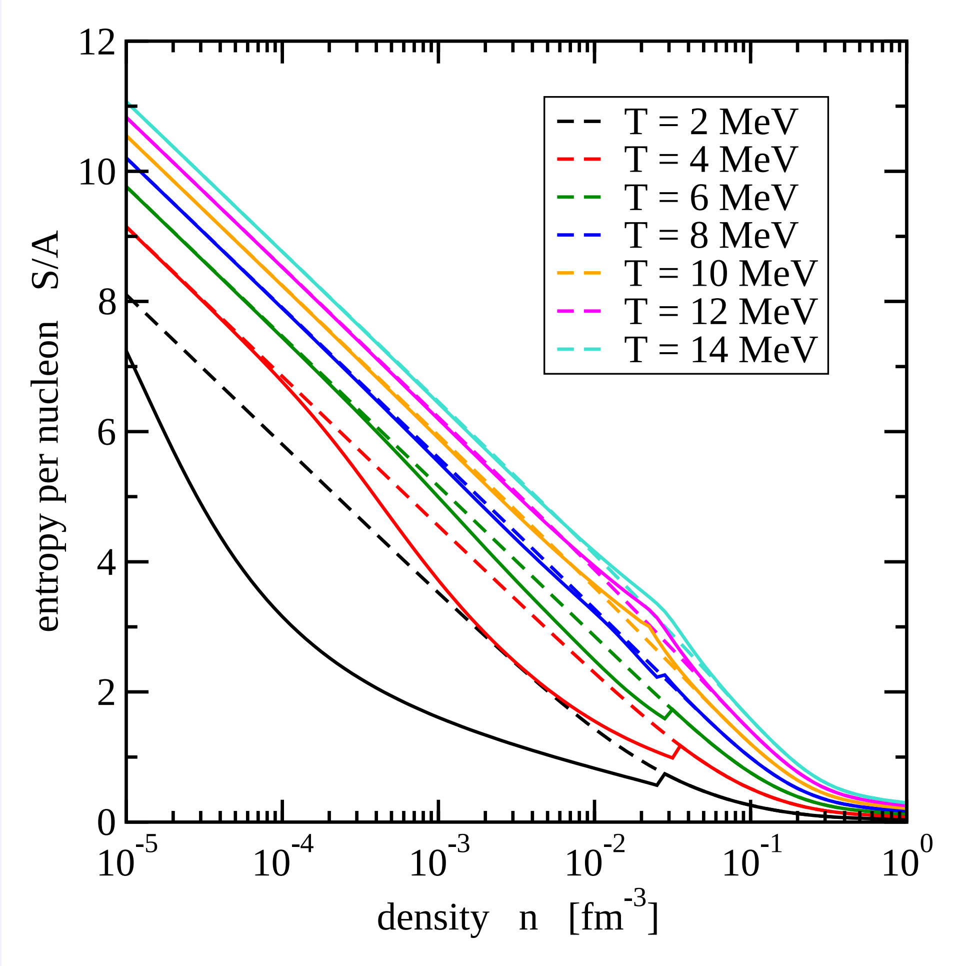
Generalized RMF Model V - Pressure and Entropy

without (dashed) and with (solid) clusters for asymmetry $\beta = 0.0$

pressure p

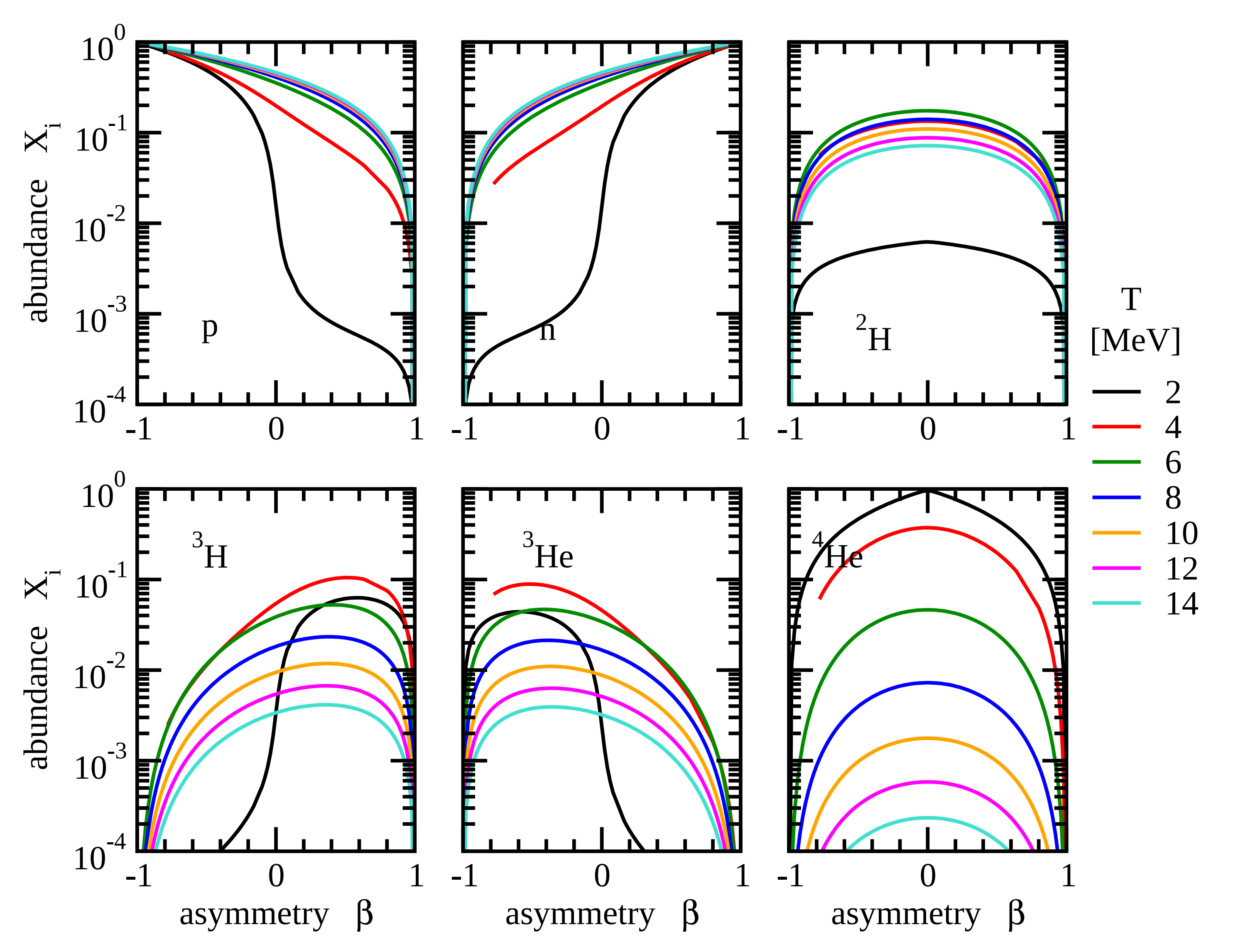


entropy per nucleon $S/A = s/n$



Generalized RMF Model VI - Particle Abundancies

$X_i = A_i \frac{n_i}{n}$ for density $n = 10^{-3}$ fm^{-3}

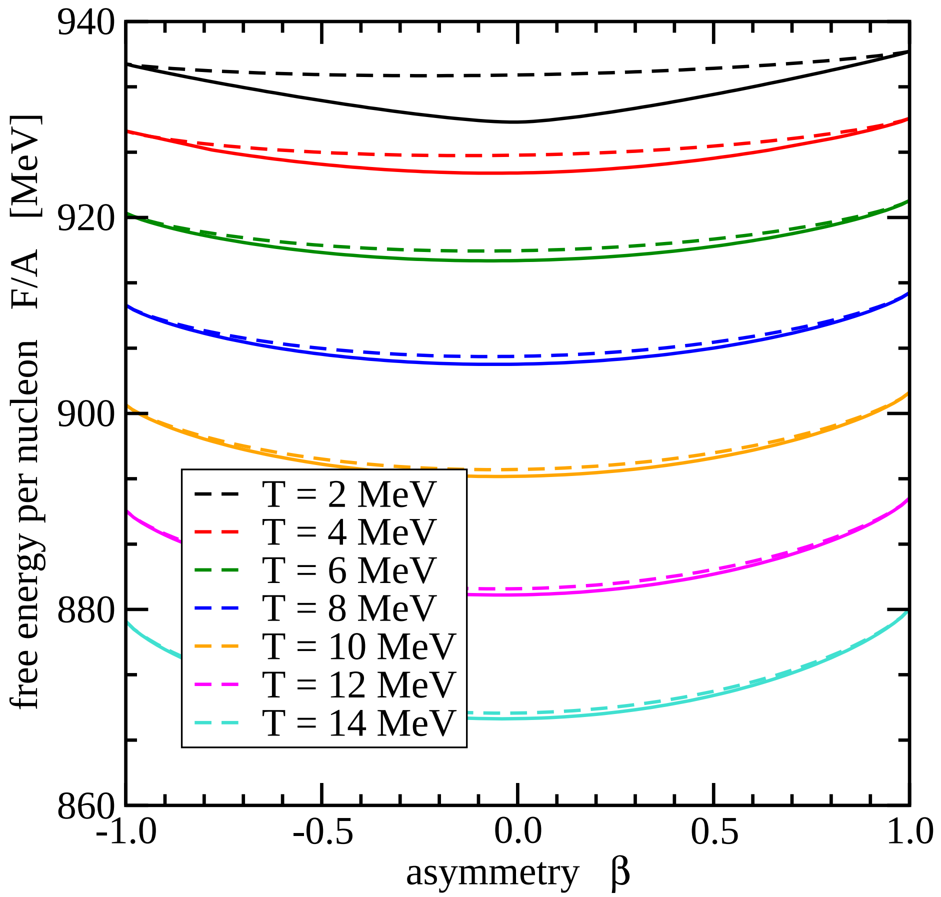


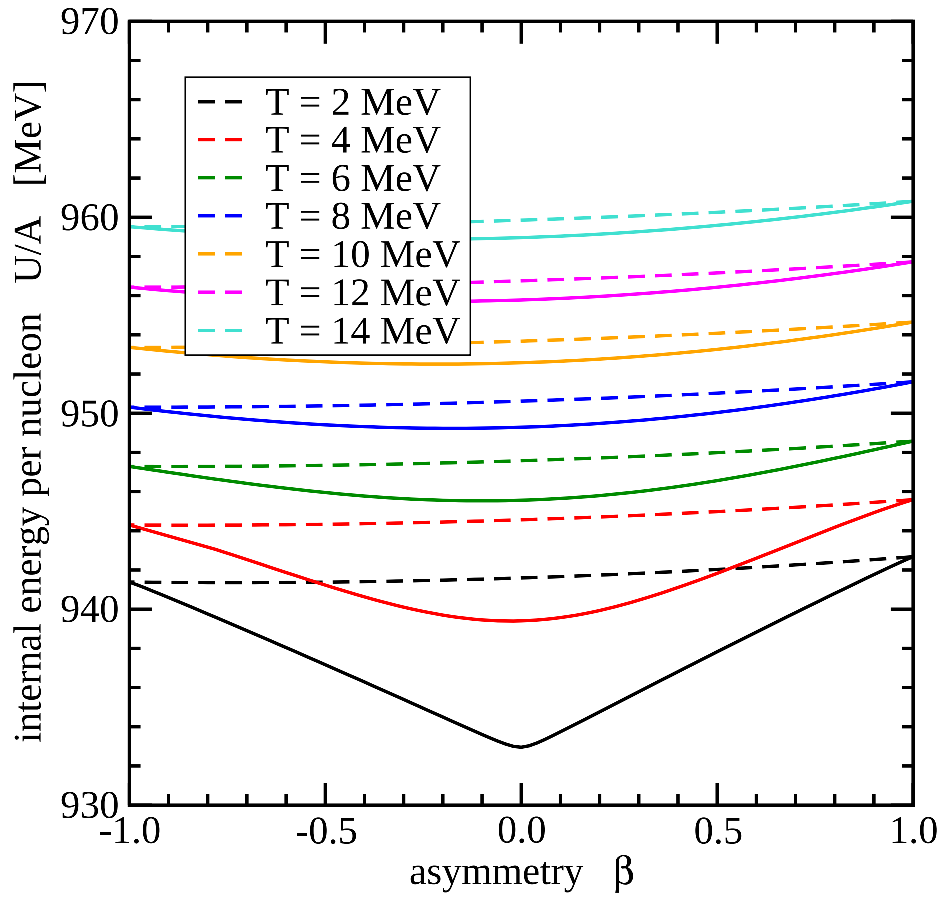
Generalized RMF Model VII - Energies

without (dashed) and with (solid) clusters for density $n = 10^{-3}$ fm^{-3}

free energy per nucleon $F/A = f/n$

internal energy per nucleon $U/A = u/n$





Generalized RMF Model VIII - Symmetry Energy

- general definition for zero temperature:

$$E_s(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\Big|_{\beta=0}$$

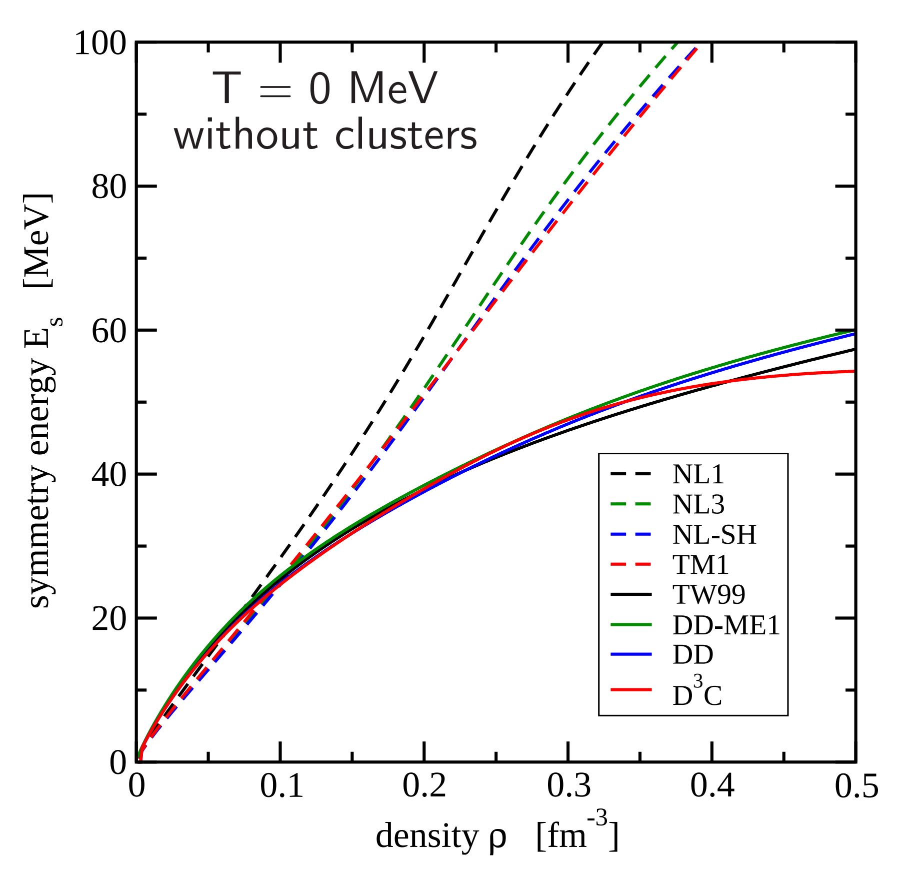
$\Rightarrow$ nuclear matter parameters

$$J = E_s(n_{\rm sat}) \qquad L = 3n\frac{d}{dn}E_s\Big|_{n=n_{\rm sat}}$$

- with clusters and at finite temperatures:
  - use approximation

$$E_s(n) = \frac{1}{2}\left[\frac{E}{A}(n,1) - 2\frac{E}{A}(n,0) + \frac{E}{A}(n,-1)\right]$$

  - distinguish free symmetry energy F_s and internal symmetry energy U_s



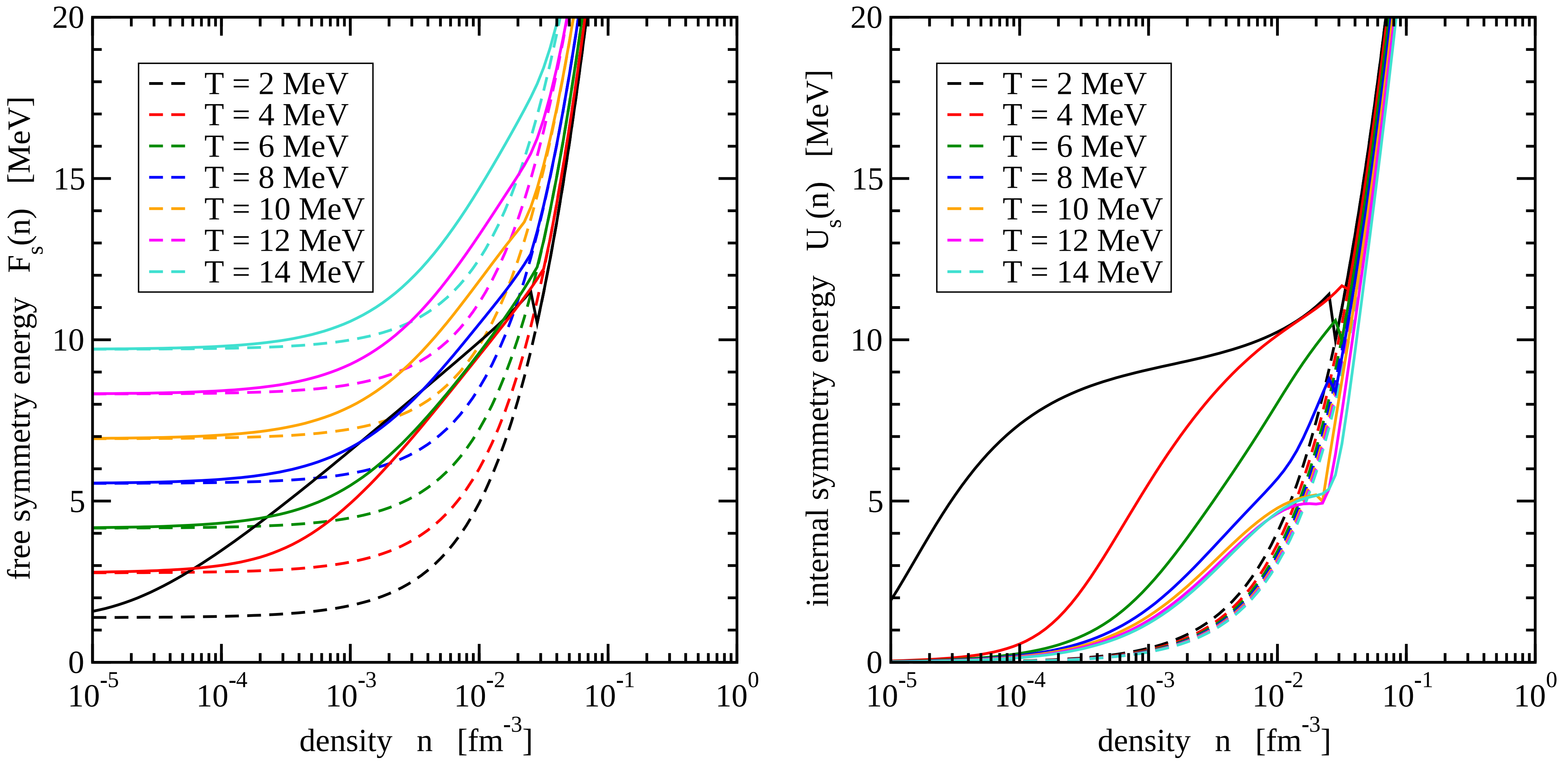
- in early RMF models: constant ρ meson coupling Γ_ρ
  - almost linear increase of E_s with n
  - too large values of J and L
- models with density-dependent couplings: realistic values for J and L

Generalized RMF Model IX - Symmetry Energies

without (dashed) and with (solid) clusters

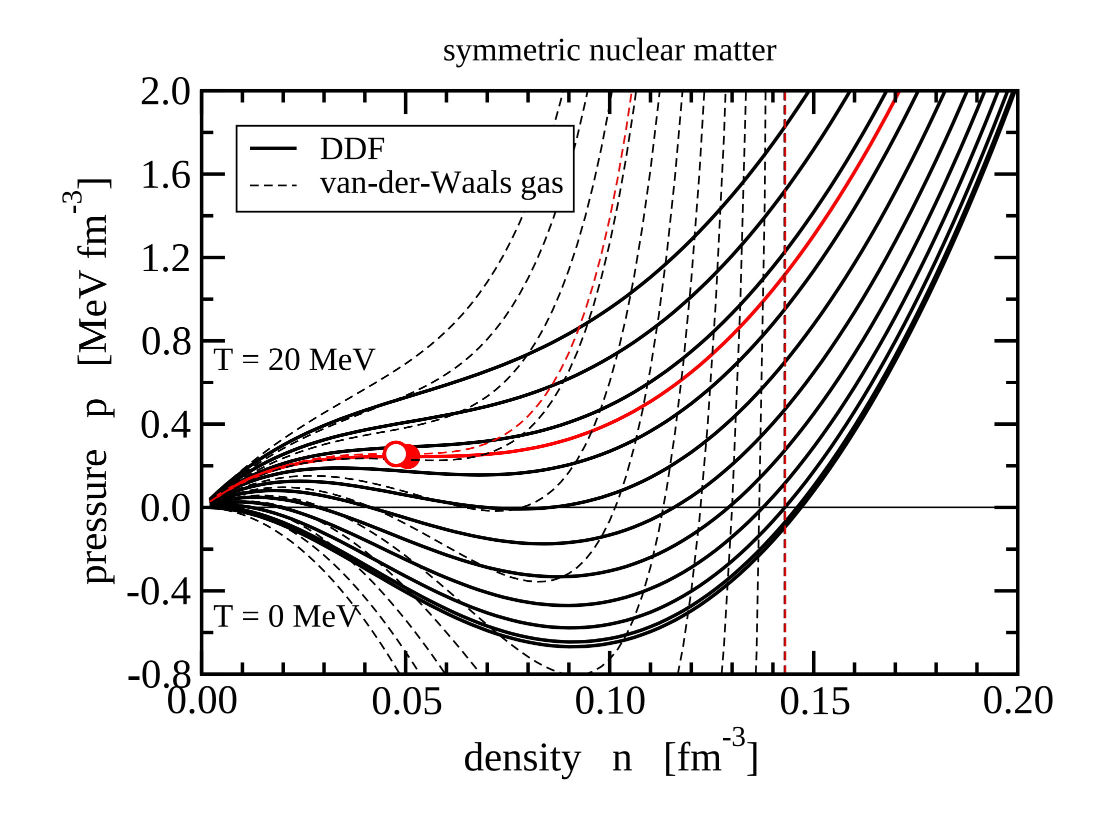
free symmetry energy

internal symmetry energy



Phase Transition I - Example

- homogeneous symmetric nuclear matter without clusters
- consider isothermes in pressure-density diagram
  $\Rightarrow$ critical point
  example: parametrization DDF
  $T_c = 15.2$ MeV, $n_c = 0.0505$ fm^{-3},
  $p_c = 0.244$ MeV fm^{-3}
  $\Rightarrow p_c/(n_c T_c) = 0.318$
  cf. van-der-Waals gas:
  $p_c/(n_c T_c) = 0.375$



- $T < T_c$: homogeneous matter
  mechanically instable in certain range of densities $\left(\left.\frac{\partial p}{\partial n}\right|_{T,V} < 0\right)$

$\Rightarrow$ appearance of low-density and high-density phases with the same asymmetry,
Maxwell construction: pressure and chemical potential constant

- asymmetric nuclear matter: more complicated situation

Phase Transition II - Asymmetric Nuclear Matter

- phase transition with two conserved charges:
  neutron and proton number or baryon and baryonic charge number
  (see, e.g., H. Müller and B. D. Serot, Phys. Rev. C 52 (1995) 2072)
  relation of chemical potentials: $\mu_b = \mu_n$, $\mu_q = \mu_p - \mu_n$

- distinction necessary:
  - spinodals (instability boundaries) $\Leftrightarrow$ local criterion on free energy density f:
    stable if matrix $\left(\frac{\partial^2 f}{\partial n_i \partial n_j}\right)\Big|_{T,V}$ positive (e.g. mechanical and diffusive stability)
  - binodals (phase separation boundaries) $\Leftrightarrow$ global criterion on free energy density f:
    convexity of free energy density (two phases I and II)
    $f(T, n_i) \leq \lambda f(T, n_i^I) + (1-\lambda) f(T, n_i^{II})$ with
    $n_i = \lambda n_i^I + (1-\lambda) n_i^{II} \qquad 0 \leq \lambda \leq 1$

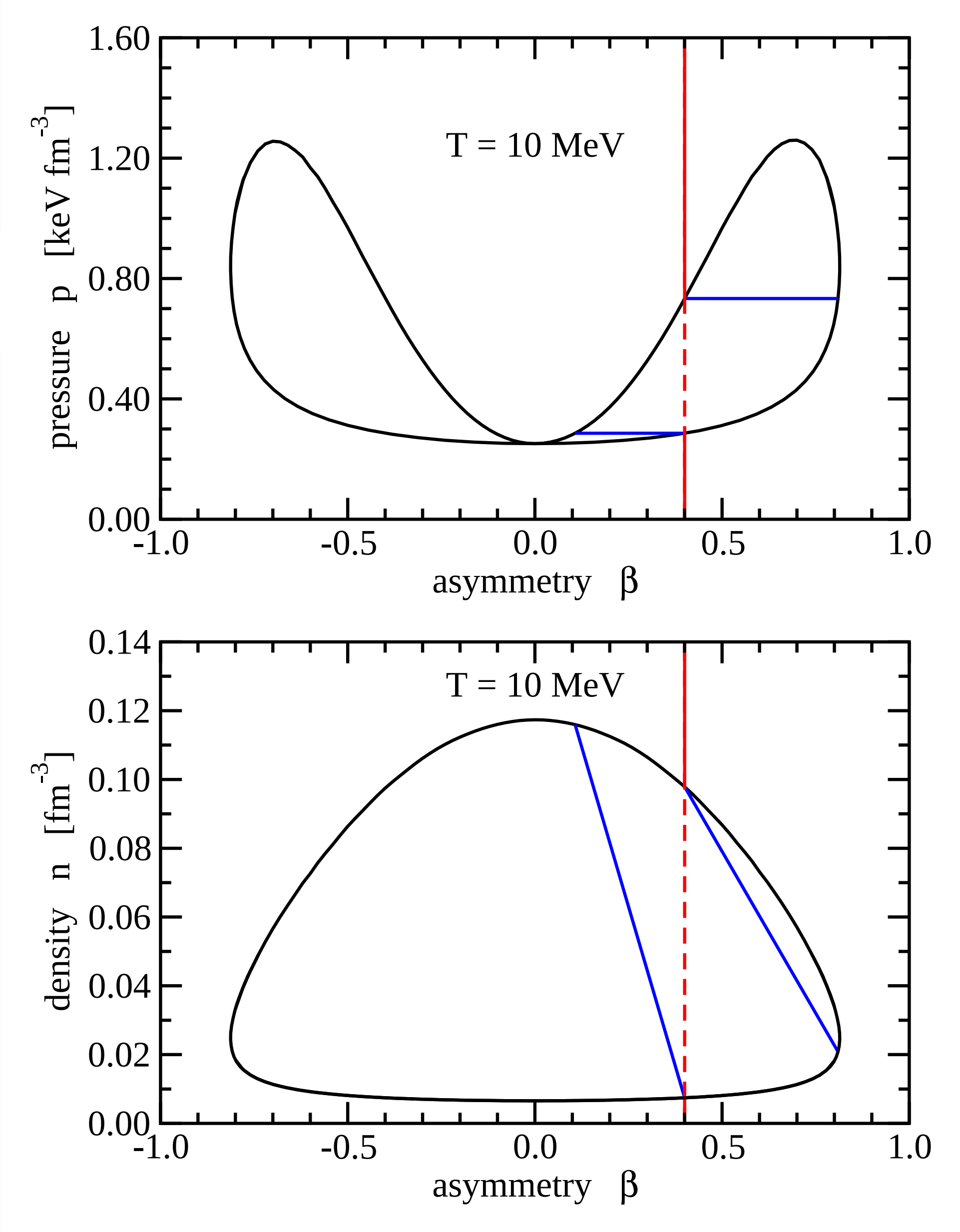
- spinodals enclosed by binodals $\Rightarrow$ binodals relevant for system in equilibrium

Phase Transition III - Construction

- construction of coexisting phases I, II
  in equilibrium with Gibbs conditions:
  equal intensive variables, i.e.

$$T^{I} = T^{II} \quad p^{I} = p^{II} \quad \mu_b^{I} = \mu_b^{II} \quad \mu_q^{I} = \mu_q^{II}$$

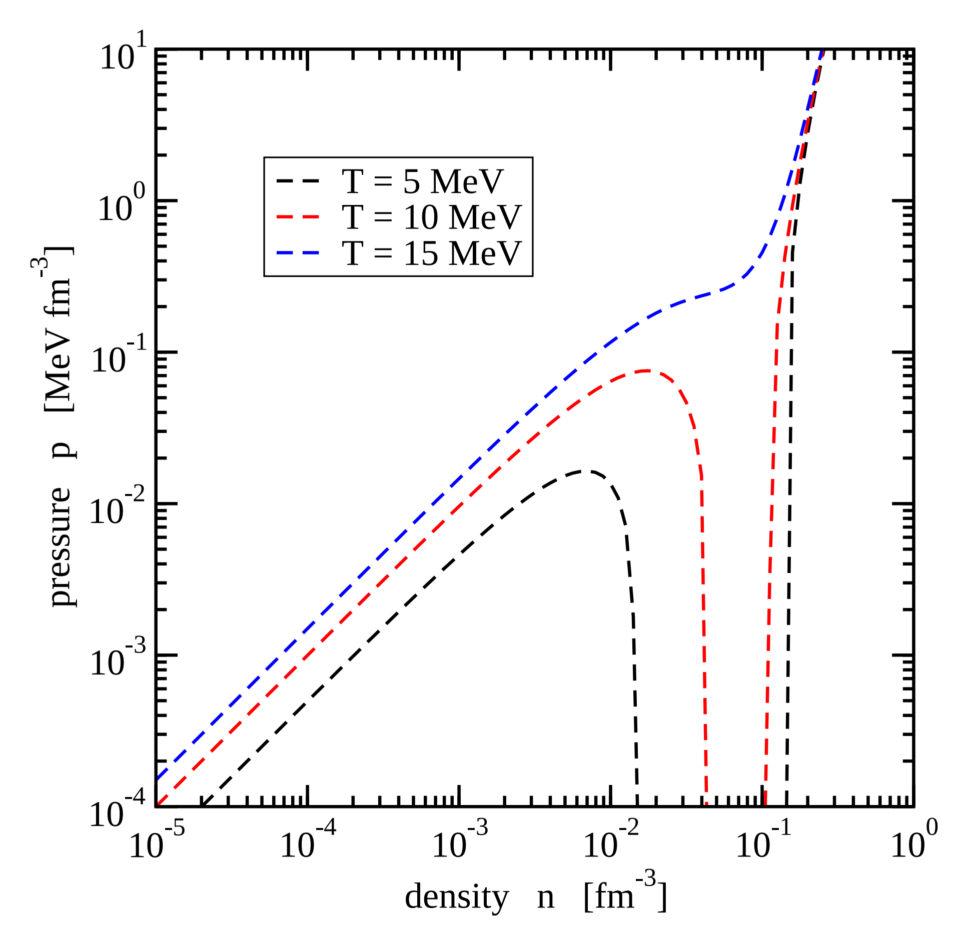
- pressure and chemical potentials
  not necessarily constant
  during phase transition
- two phases with different densities
  and different asymmetries
- description of system inside binodal
  not needed
- generalization for system with electrons
  possible (charge neutrality)
- no surface effects, no Coulomb interaction
  ⇒ inhomogeneities, heavy clusters



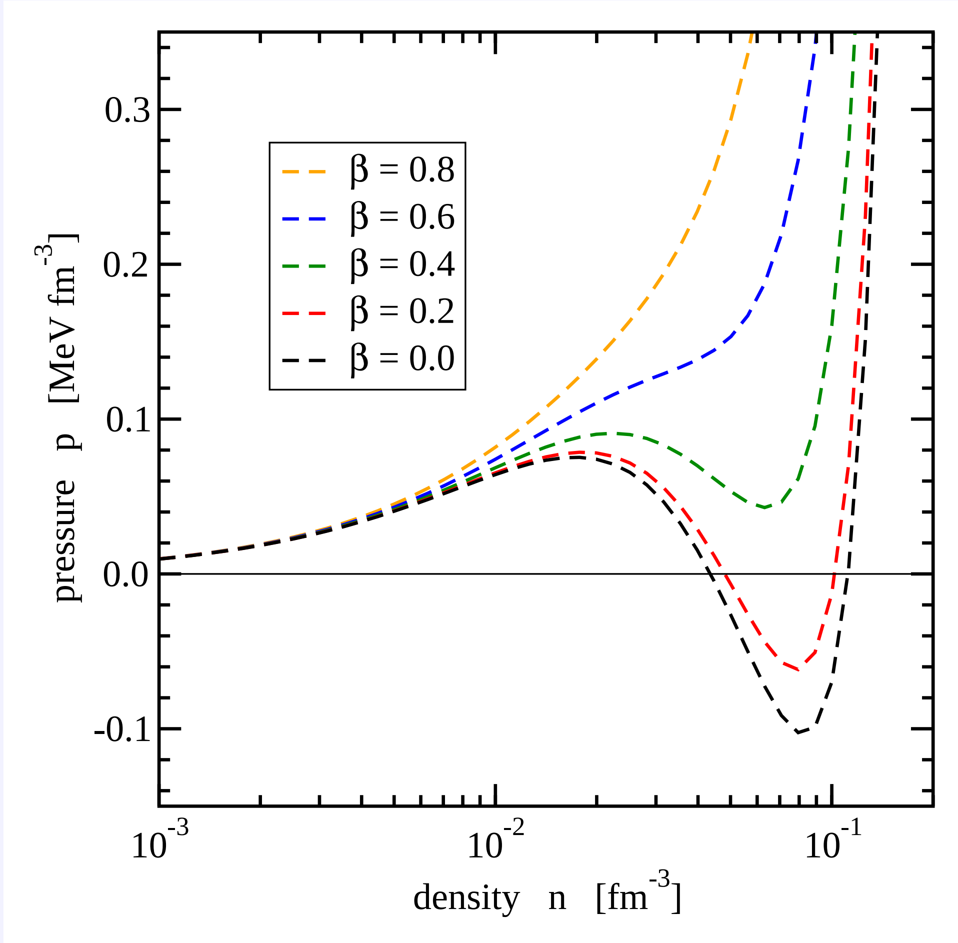
Phase Transition IV - Pressure

without phase transition

constant asymmetry $\beta = 0.0$



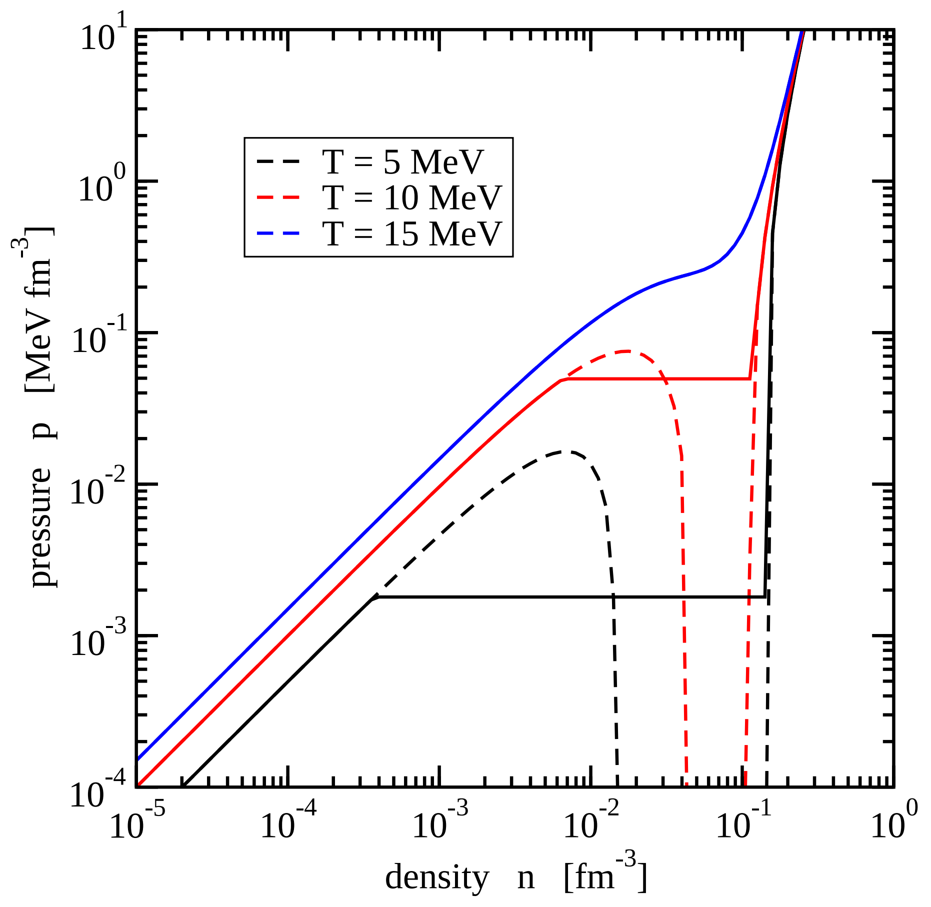
constant temperature $T = 10$ MeV



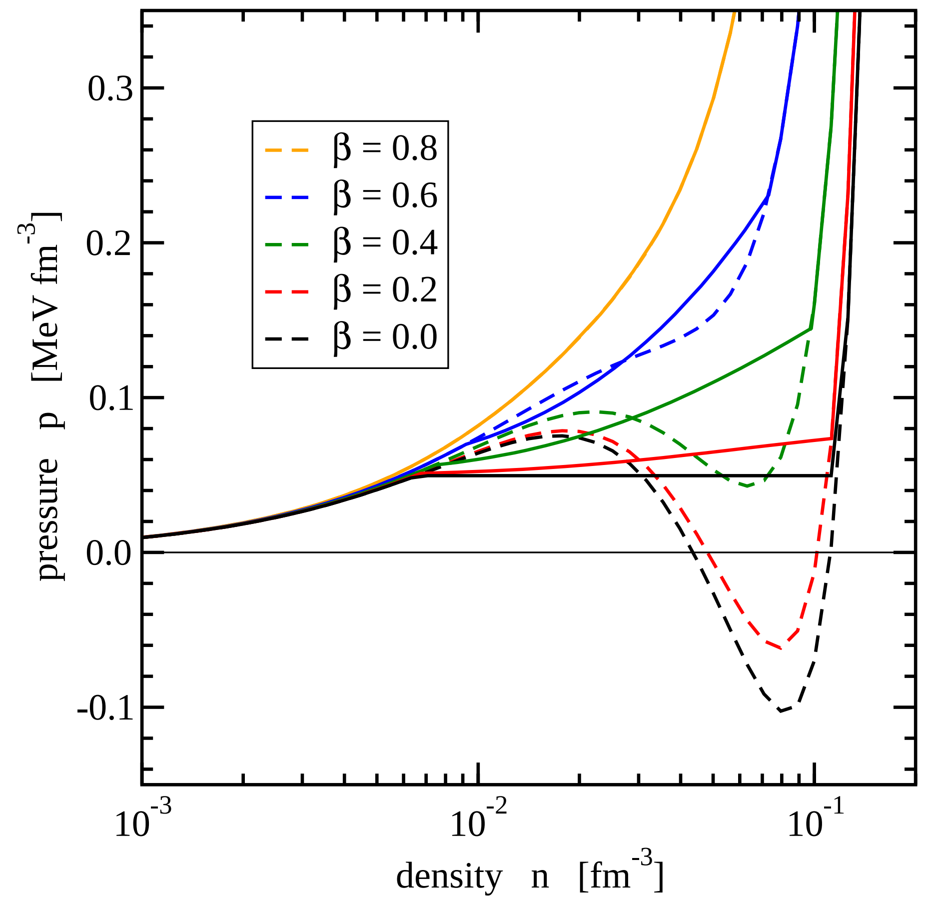
Phase Transition IV - Pressure

without (dashed) and with (solid) phase transition

constant asymmetry $\beta = 0.5$



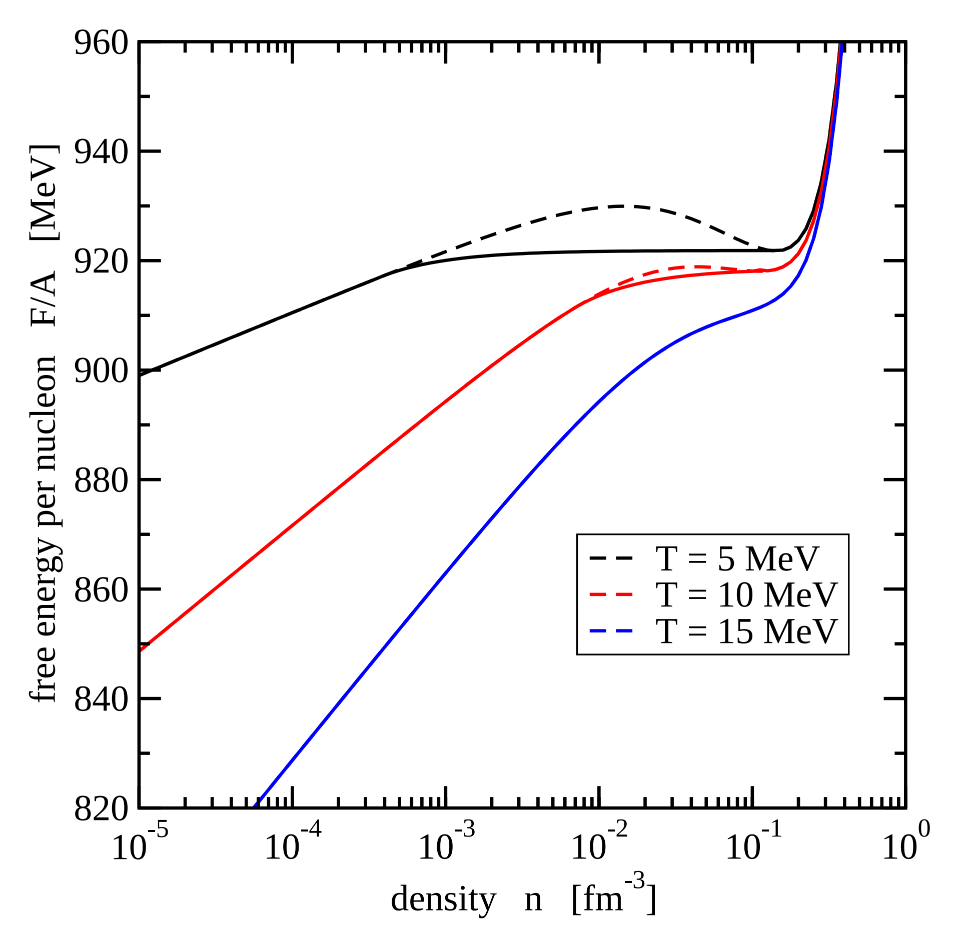
constant temperature $T = 10$ MeV



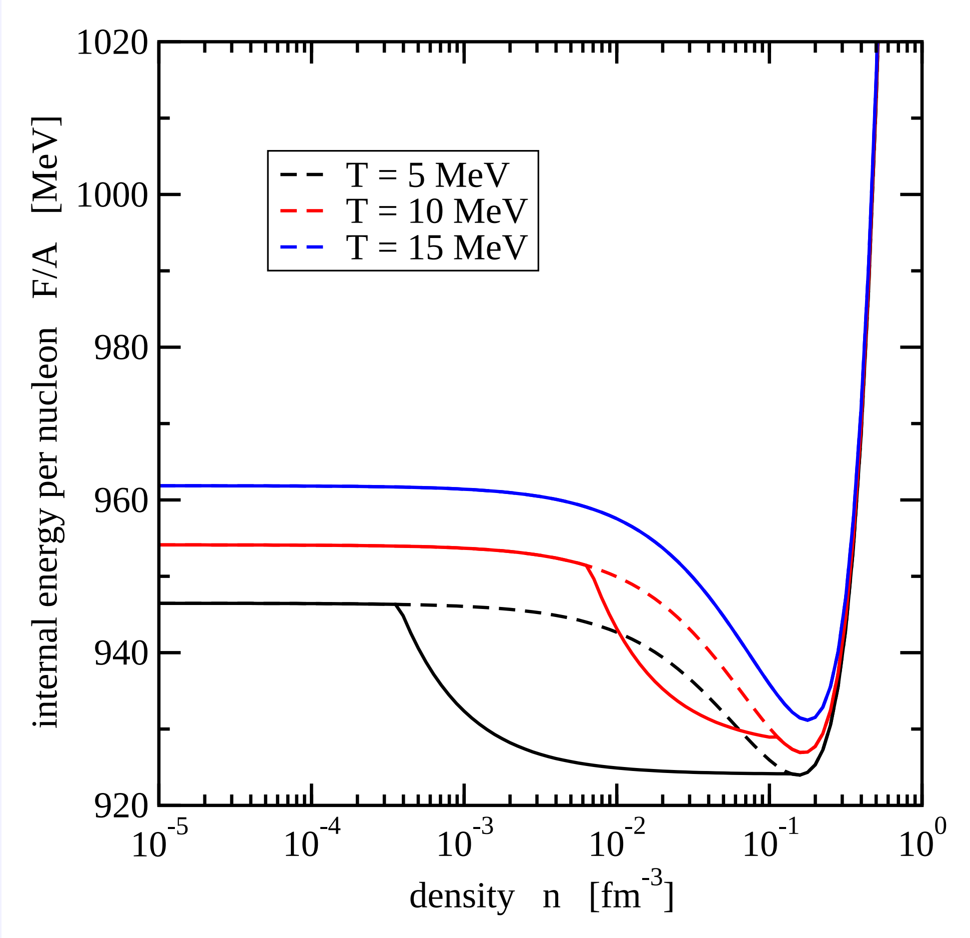
Phase Transition V - Energies

without (dashed) and with (solid) phase transition for asymmetry $\beta = 0$

free energy per nucleon $F/A = f/n$



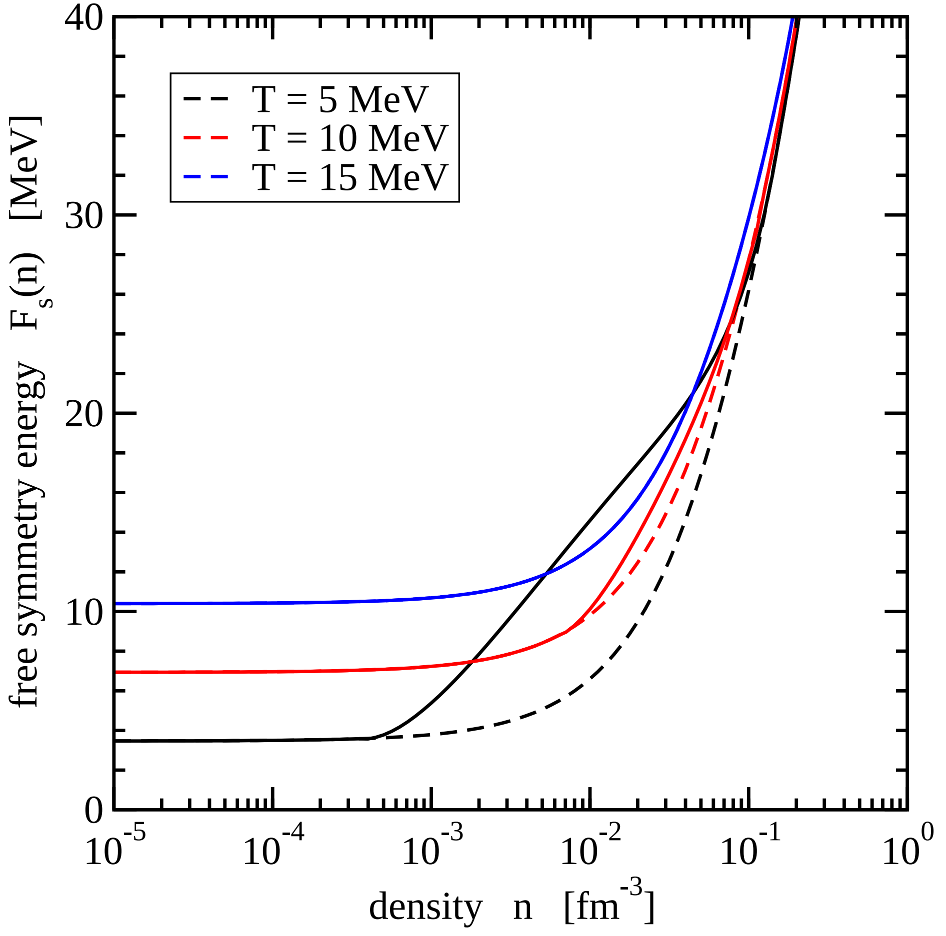
internal energy per nucleon $U/A = u/n$



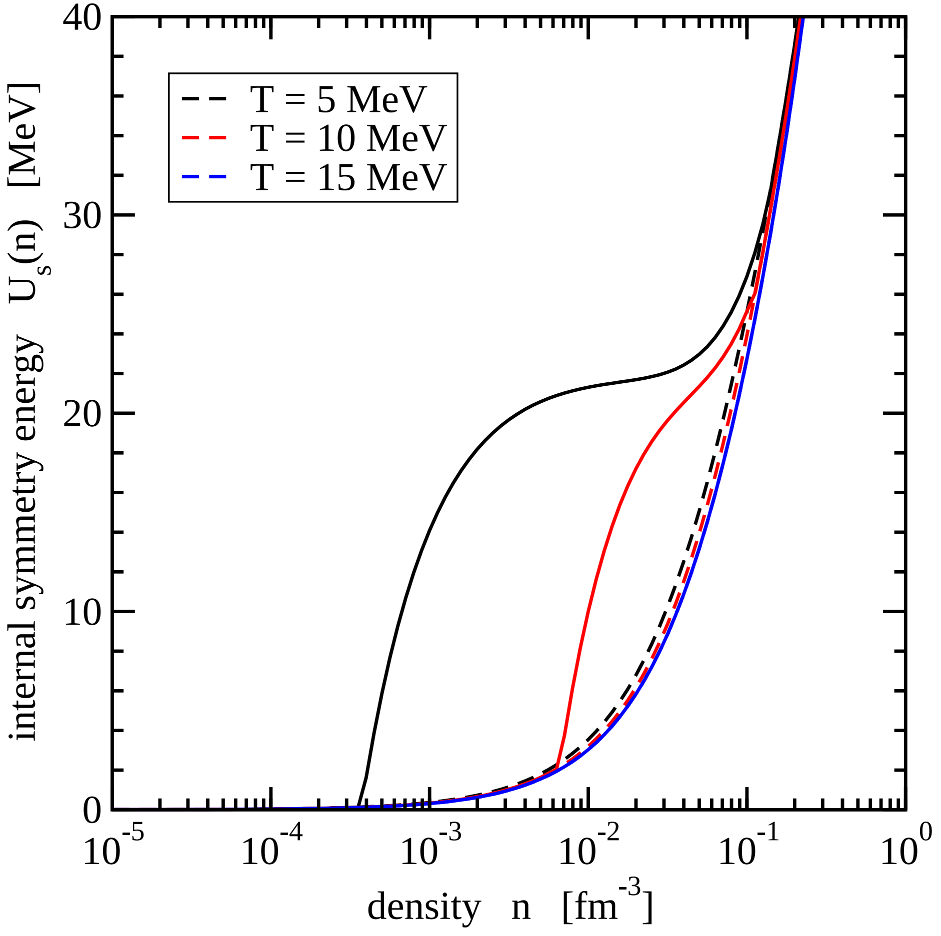
Phase Transition VI - Symmetry Energies

without (dashed) and with (solid) phase transition

free symmetry energy



internal symmetry energy



Heavy Clusters I - Inhomogeneous Matter

low densities and low temperatures

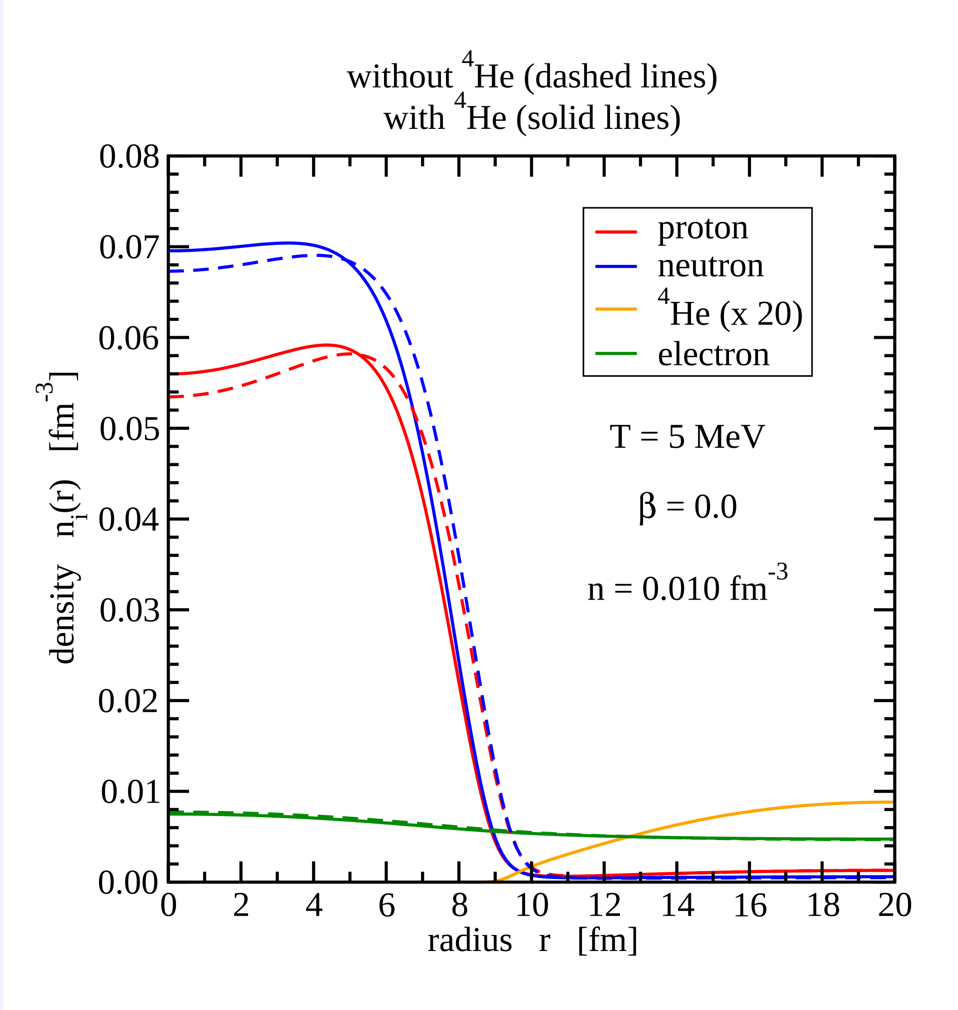
- homogeneous system not stable ⇒ inhomogeneities develop
- single phase transition (with low and high density phases) replaced by sequence of transitions with various shapes of density distributions ("pasta" phases)
- surface effects and Coulomb interaction important
- global charge neutrality ⇒ compensation of proton charge by electron charge

first approximation

- lattice periodic distribution of heavy nuclei;
  nucleons, electrons, light clusters in between
- Wigner-Seitz approximation: replace (cubic) lattice cell by spherical WS cell
- relativistic Thomas-Fermi approximation with RMF energy density functional (beyond local density approximation)
  ⇒ selfconsistent density distributions with smooth transition to homogeneous phase
- lattice correlation energy by long-range Coulomb interaction important
- construction of phase transition still required

Heavy Clusters II - Example

- inhomogeneous matter with protons, neutrons, electrons and α particles
- self-consistent density distributions in spherical Wigner-Seitz cell
- formation of heavy nucleus
- shell/bubble solutions possible
- distribution of α particles around nucleus (no excluded volume mechanism needed)
- effects of Coulomb interaction
- inhomogeneous electron distribution (charge screening)



Heavy Clusters III - Lattice Energy

correction of Coulomb field energy

- in spherical WS cell of radius R: $E_C^{WS} = \frac{\Gamma_\gamma}{2} \int_{V_{WS}} d^3r \; A_0(r) \varrho_q(r)$ $\quad -\Delta A_0 = \Gamma_\gamma \varrho_q$

  with spherical field $A_0(r)$ and charge density $\varrho_q(r)$ $\quad \int_{V_{WS}} d^3r \; \varrho_q(r) = 0$

  boundary conditions: $A_0(R) = 0$, $A_0'(R) = 0$

- place WS cells on lattice sites with lattice constant a
  $\Rightarrow$ lattice periodic fields and densities

  replace E_C^{WS} by $E_C^{\mathrm{lat}} = \frac{\Gamma_\gamma^2 a}{(2\pi)^2} \sum'_{h,k,l} \left(\frac{N_{hkl} I_{hkl}}{h^2+k^2+l^2} \right)^2$ with $N_{hkl} = 1 + (-1)^{h+k+l}$ (bcc cell)

  $I_{hkl} = \int_0^R dr \; r \; [\varrho_q(r) - \varrho_q(R)] \sin(q_{hkl} r)$ $\quad q_{hkl} = \frac{2\pi}{a}\sqrt{h^2+k^2+l^2}$

$\Rightarrow$ violation of thermodynamical consistency $p = n^2 \left. \frac{\partial (f/n)}{\partial n} \right|_{T,V,\beta} \neq \sum_i \mu_i n_i - f$

- use lattice field energy (photons and mesons) in energy density functional
  for derivation of field equations, self-energies, etc.

Summary and Outlook

- construction of improved equation of state of dense nuclear matter
  - relativistic mean-field model with density-dependent couplings
  - generalized Beth-Uhlenbeck approach (light clusters)
  - relativistic Thomas-Fermi calculations in modified Wigner-Seitz cell (heavy clusters)

- various constraints of model parameters
- improved consistency

- work in progress
  - parametrization of medium effects for clusters/correlations to be investigated
  - improvement of RMF parametrization
  - only preliminary results so far, still numerical difficulties

- application (future)
  - detailed investigation of new EoS
  - effects on astrophysical models for supernovae?