

EoS in Astrophysics, Argonne, 26. 08. 08

# Mott Dissociation of Bound States in Nuclear Matter

Gerd Röpke, Rostock

ESF RNP CompStar



# Supernova

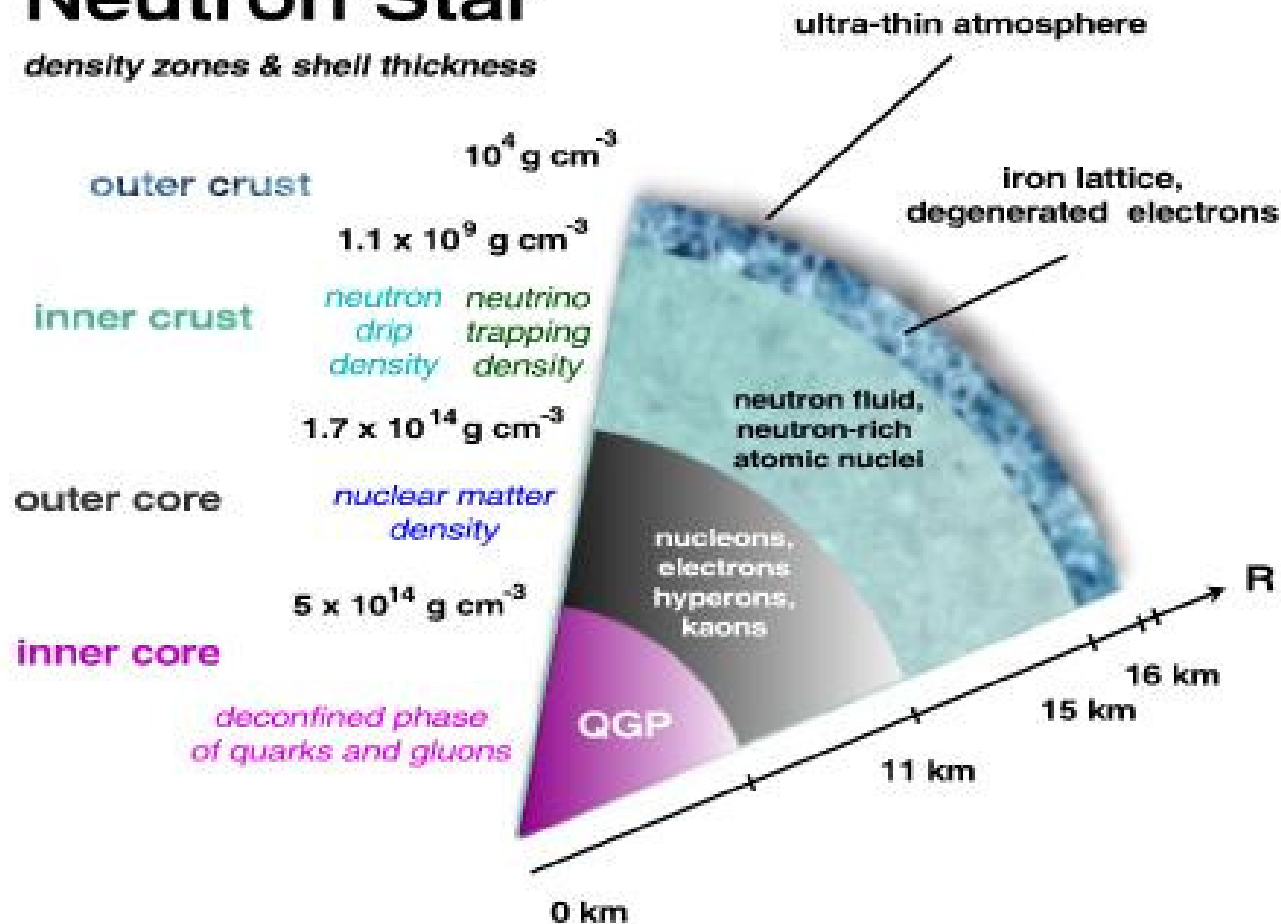
Crab nebula, 1054 China, PSR 0531+21



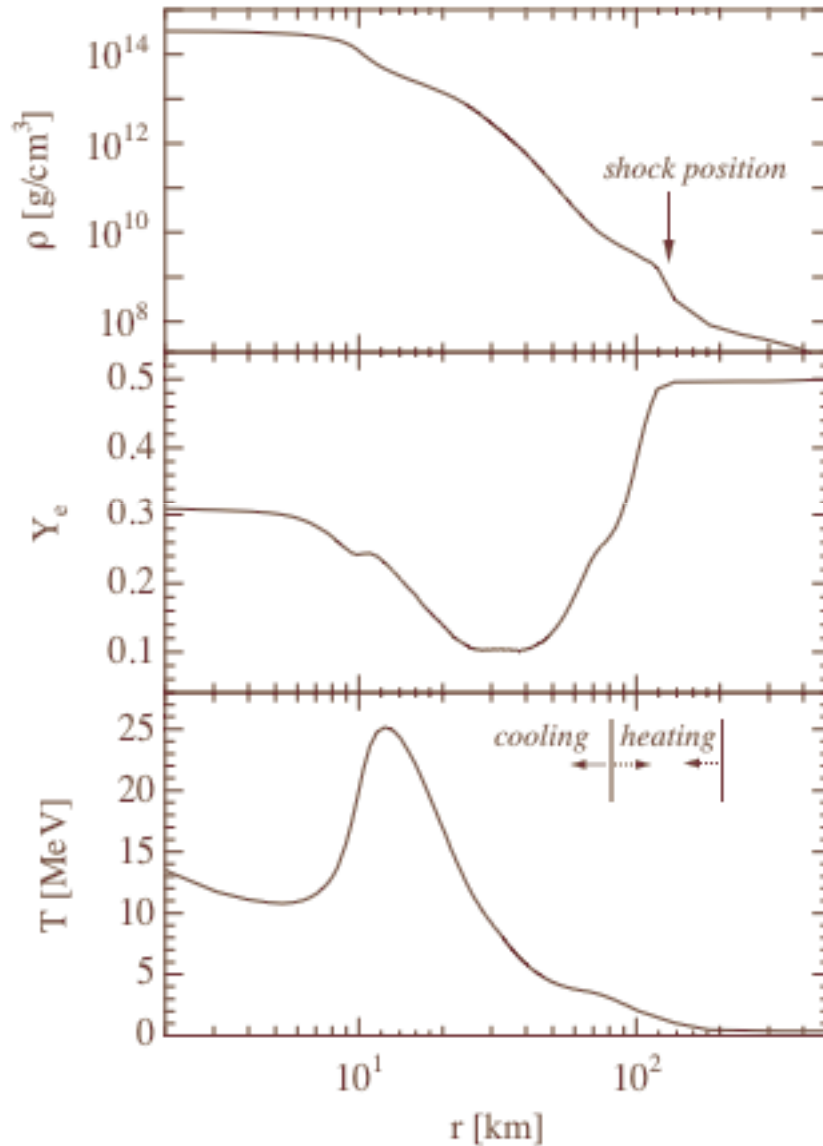
# Structure of a Neutron star

## Neutron Star

*density zones & shell thickness*



# Core-collapse supernovae



Density.

electron fraction, and

temperature profile

of a 15 solar mass supernova  
at 150 ms after core bounce  
as function of the radius.

Influence of cluster formation  
on neutrino emission  
in the cooling region and  
on neutrino absorption  
in the heating region ?

K.Sumiyoshi et al.,  
*Astrophys.J.* **629**, 922 (2005)

# Supernova collapse: spherically symmetric simulations

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

A. Arcones et al.  
Neutrino driven winds,  
Talk 25. 2. 08 Ladek;  
PRC **78**, 015806 (08)

# Parameter range: Explosion

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

T. Fischer, On the possible fate of massive progenitor stars, Talk 25.2.08 Ladek

# Problems:

- Warm Dilute Matter: Nuclear matter at subsaturation densities ( $T, n_p, n_n$ ):  
Temperature  $T \leq 16 \text{ MeV} = E_s/A$ , baryon density  $n_B \leq 0.17 \text{ fm}^{-3} = n_s$ , asymmetry
- Formation of clusters (nuclei in matter):  
 $A = 1,2,3,4$ : free neutrons, free protons, deuterons ( ${}^2\text{H}$ ), tritons ( ${}^3\text{H}$ ), helions ( ${}^3\text{He}$ ), alphas ( ${}^4\text{He}$ )
- Low-density, low-temperature limit:  
Virial expansion, non-interacting nuclides, quantum condensates
- Transition to higher densities:  
Medium effects, quasiparticles. Interpolation between Beth-Uhlenbeck and DBHF / RMF
- Cluster formation (correlations) vs. mean field:  
Consistent quantum-statistical approach

# Outline

- Schrödinger equation with medium corrections:  
Self-energy and Pauli blocking
- Composition of the nuclear gas:  
Generalized Beth-Uhlenbeck equation
- Quantum condensates:  
Pairing and quartetting
- Composition and the EoS of nuclear matter  
(astrophysics: supernovae explosions)
- Symmetry energy in the low-density region  
(heavy ion collisions: cluster abundances)
- Cluster formation in dilute nuclei  
(Hoyle state and THSR wave function)



# Ideal mixture of reacting nuclides

$$n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A \{ E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

$$n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A \{ E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

(statistical multifragmentation)

mass number  $A$ ,

charge  $Z_A$ ,

energy  $E_{A,\nu,K}$ ,

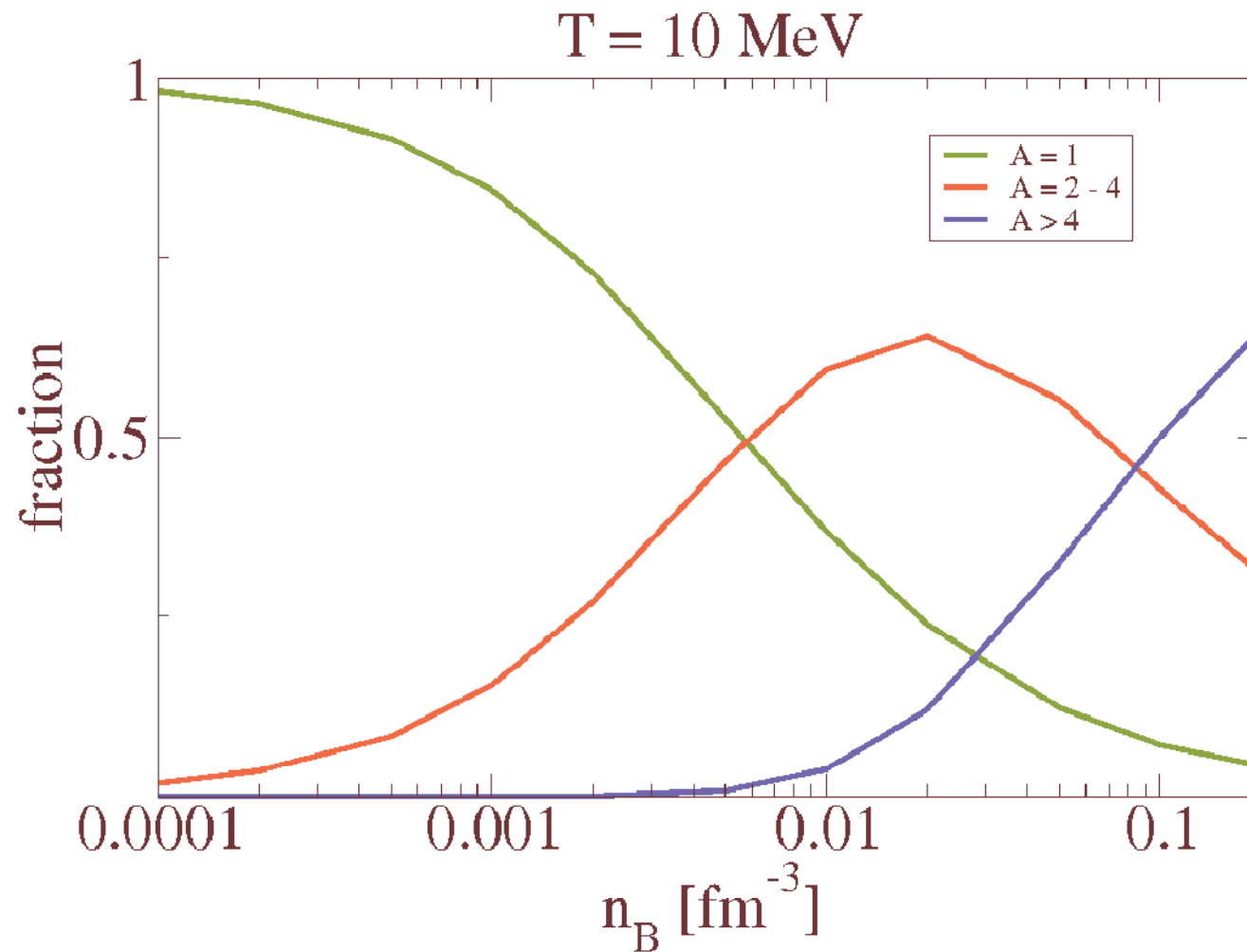
$\nu$ : internal quantum number,

$K$ : center of mass momentum

$$f_A(z) = \frac{1}{\exp(z/T) - (-1)^A}$$

# Composition of symmetric matter

## Ideal mixture of nuclides



# Virial expansion

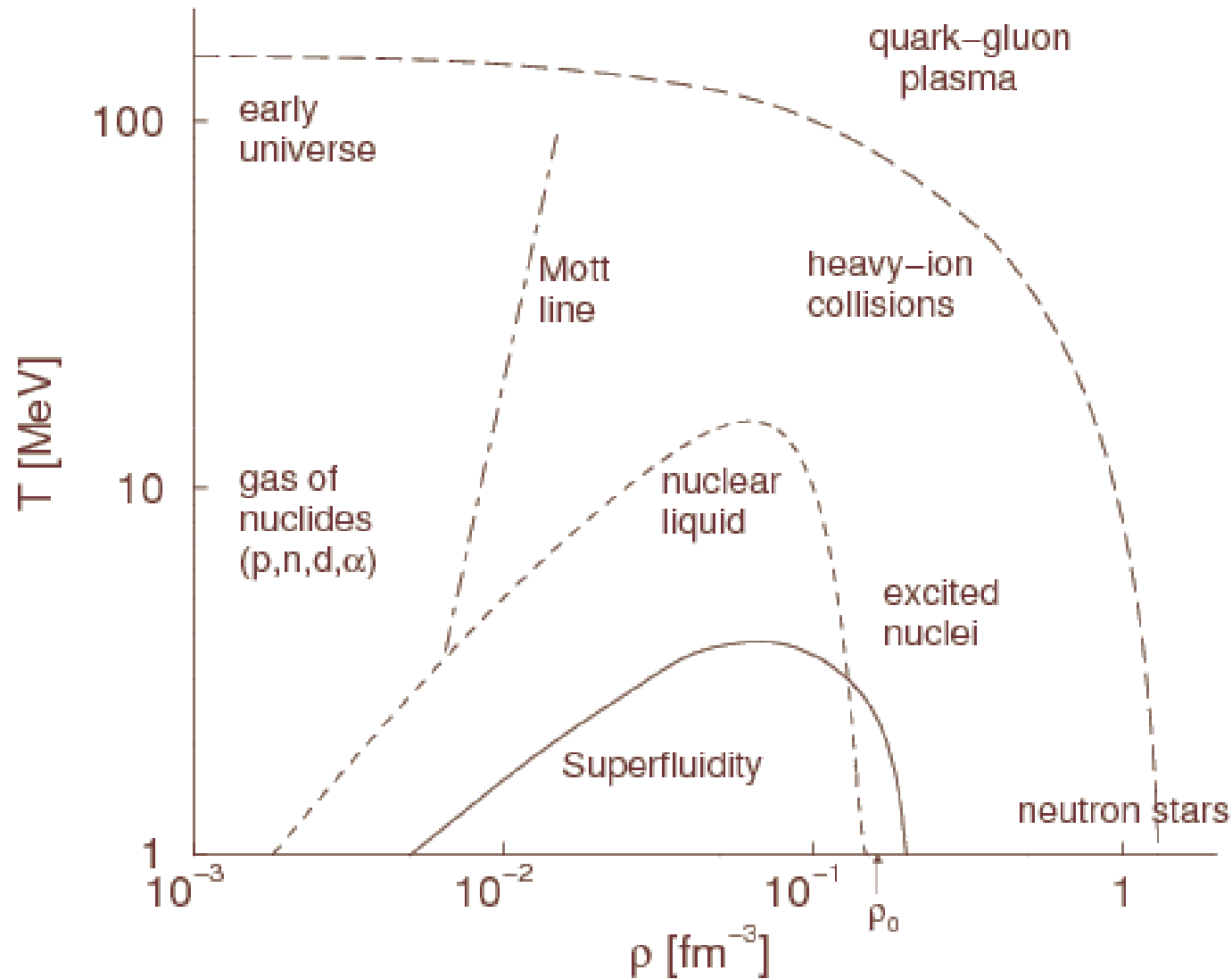
- excited nuclei
- resonances
- scattering phase shifts (no double counting)
- virial expansions
- quantum statistical approach

*Particle clustering and Mott transition in nuclear matter at finite temperatures,*  
G. Röpke, M. Schmidt, L. Münchow, H. Schulz: NPA **399**, 587-602 (1983).

*Generalized Beth-Uhlenbeck Approach for Hot Nuclear Matter,*  
M. Schmidt, G. Röpke, H. Schulz: Annals of Physics **202**, 57 - 99 (1990).

*Cluster formation and the virial equation of state of low-density nuclear matter,*  
C. J. Horowitz and A. Schwenk, Nucl. Phys. **A 776**, 55 (2006).

# Symmetric nuclear matter: Phase diagram



# Nucleon-nucleon interaction

- general form:

$$V_{\alpha}(p, p') = \sum_{i,j=1}^N w_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}(p') \quad \text{uncoupled}$$

and

$$V_{\alpha}^{LL'}(p, p') = \sum_{i,j=1}^N w_{\alpha i}^L(p) \lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \quad \text{coupled}$$

$p, p'$  in- and outgoing relative momentum

$\alpha \dots$  channel

$N \dots$  rank

$\lambda_{\alpha ij}$  coupling parameter

$L, L'$  orbital angular momentum

# Many-particle theory

- equilibrium correlation functions

e.g. equation of state  $n(\beta, \mu) = \frac{1}{\Omega_0} \sum_1 \langle a_1^\dagger a_1 \rangle$

density matrix  $\langle a_1^\dagger a_1^\dagger \rangle = \int \frac{d\omega}{2\pi} e^{-i\omega t} f_1(\omega) A(1, 1', \omega)$

- Spectral function

$$A(1, 1', \omega) = \text{Im} [G(1, 1', \omega + i\eta) - G(1, 1', \omega - i\eta)]$$

- Matsubara Green function

$$G(1, 1', iz_\nu), \quad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \dots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega-\mu)} + 1}, \quad \Omega_0 - \text{volume}$$

# Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of  $\Sigma(1, iz_\nu)$ :  
perturbation expansion, diagram representation

$$A(1, \omega) = \frac{2\text{Im} \Sigma(1, \omega + i0)}{[\omega - E(1) - \text{Re} \Sigma(1, \omega)]^2 + [\text{Im} \Sigma(1, \omega + i0)]^2}$$

approximation for  
self energy



approximation for  
equilibrium correlation functions

alternatively: simulations, path integral methods

# Different approximations

- Expansion for small  $\text{Im } \Sigma(1, \omega + i\eta)$

$$A(1, \omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{d}{dz} \text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}} - \mu_1}} - 2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \frac{P}{\omega + \mu_1 - E^{\text{quasi}}(1)}$$

quasiparticle energy  $E^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states  $\hat{=}$  new species

summation of ladder diagrams, Bethe-Salpeter equation





# Medium effects: Quasiparticle approximation

- Skyrme
- relativistic mean field (RMF)
- Lagrangian: non-linear sigma
- TM1 parameters
- Single particle modifications
- energy shift, effective mass
- DD-RMF [S.Tyepel, Phys. Rev. C 71, 064301 (2007)]:  
expansion of the scalar field and the vector fields  
in powers of proton/neutron densities
- Dirac-Brueckner Hartree Fock (DBHF)

# Quasiparticle energy shifts

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

Comparison of different approximations, BonnA separable interaction potential.

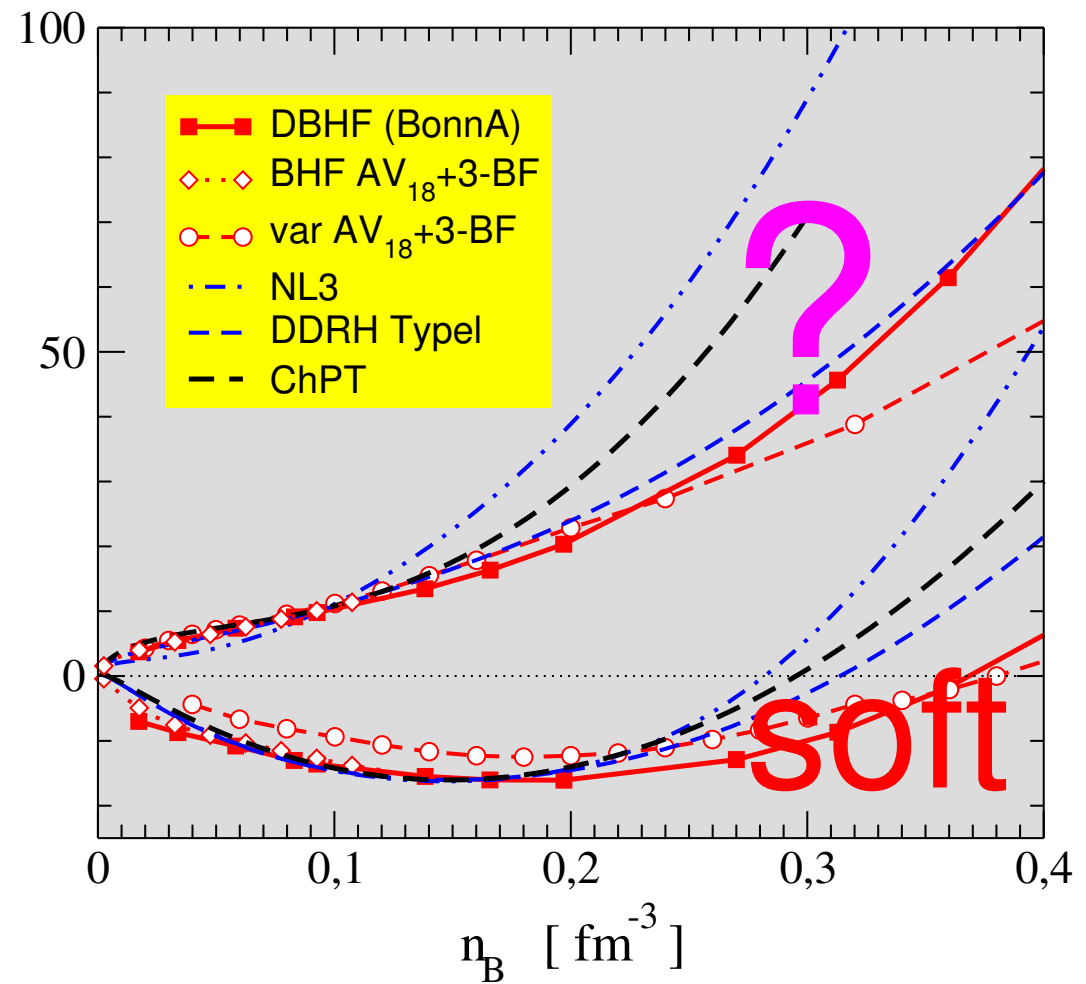
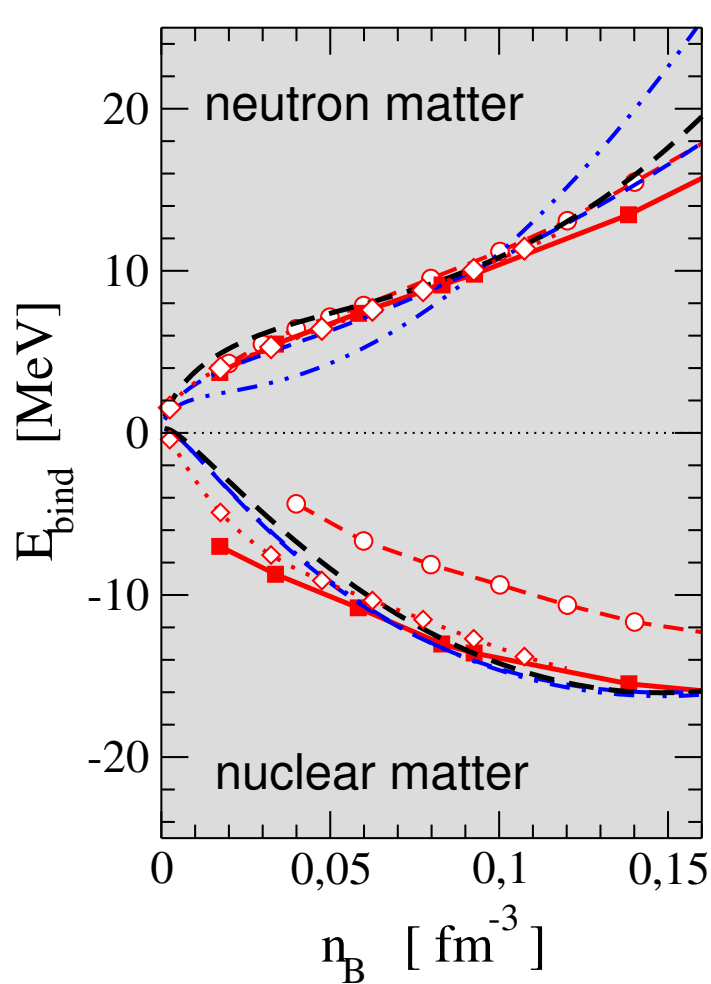
Full line - generalized Beth-Uhlenbeck approach,

dotted line - the same but the Pauli operator

$(1 - f_1)(1 - f_1)$  instead of  $(1 - f_1 - f_1)$ ,

dashed line - Brueckner-Bethe-Goldstone calculation with the Pauli operator  $(1 - f_1)(1 - f_1)$ .

# Quasiparticle picture: RMF and DBHF



# Different approximations

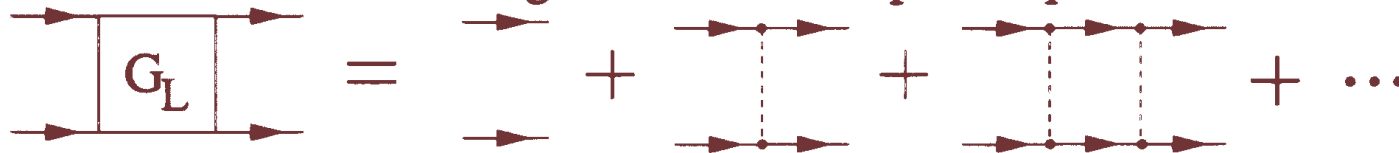
- Expansion for small  $\text{Im } \Sigma(1, \omega + i\eta)$

$$A(1, \omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{d}{dz} \text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}} - \mu_1}} - 2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \frac{P}{\omega + \mu_1 - E^{\text{quasi}}(1)}$$

quasiparticle energy  $E^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states  $\hat{=}$  new species

summation of ladder diagrams, Bethe-Salpeter equation



# Different approximations

low density limit:

$$G_2^L(12, 1'2', i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi_{n\mathbf{P}}^*(12)$$

$$\Sigma = \text{Diagram: a box labeled } T_2^L \text{ with a loop on top.}$$

$$n(\beta, \mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2, n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}}) + \sum_{2, n\mathbf{P}} \int_0^\infty dk \delta_{\mathbf{k}, \mathbf{p}_1 - \mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2 \sin^2 \delta_n(k) \frac{1}{\pi} \frac{d}{dk} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula  
correct low density/low temperature limit:  
mixture of free particles and bound clusters

# Effective wave equation for the deuteron in matter

$$\left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}\right)\Psi_{n,P}(p_1, p_2) + \sum_{p_1', p_2'} (1 - f_{p_1} - f_{p_2}) V(p_1, p_2; p_1', p_2') \Psi_{n,P}(p_1', p_2')$$

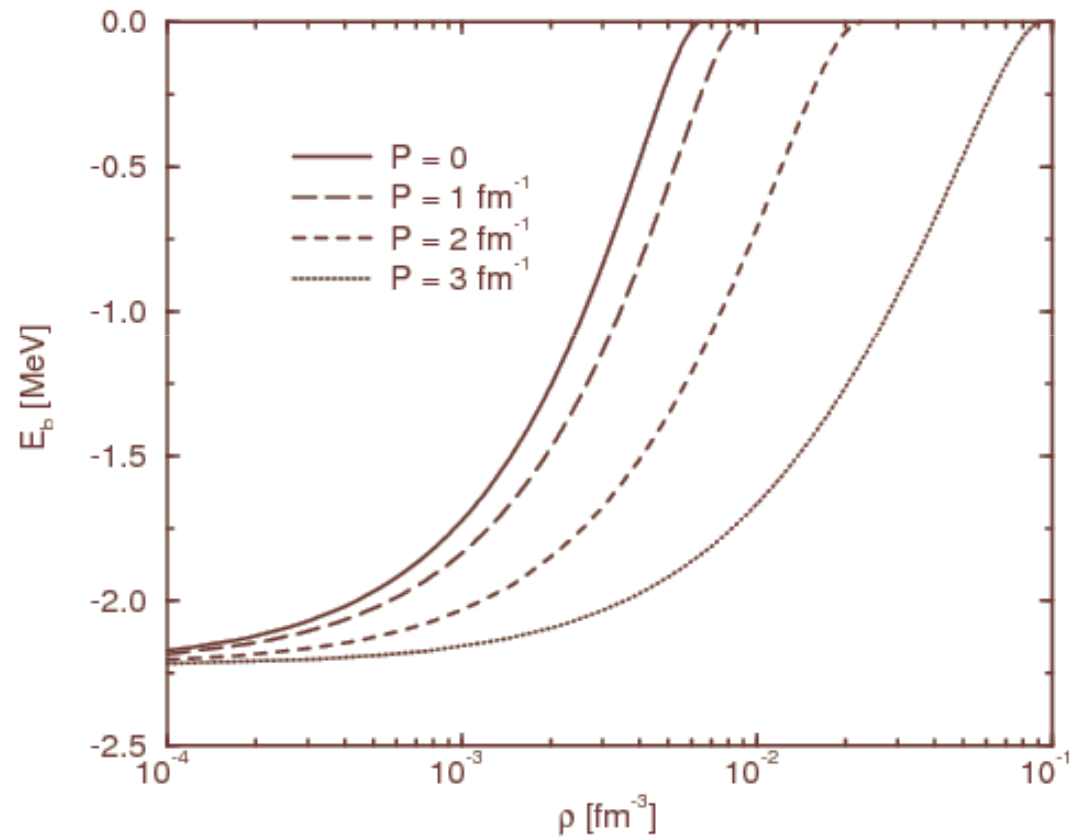
Add self-energy                      Pauli-blocking                       $= E_{n,P} \Psi_{n,P}(p_1, p_2)$

Fermi distribution function

$$f_p = \left[ e^{(p^2/2m - \mu)/k_B T} + 1 \right]^{-1}$$

BEC-BCS crossover:  
Alm et al., 1993

# Deuterons in nuclear matter



$T=10$  MeV,  $P$ : center of mass momentum

# Deuteron quasiparticle properties

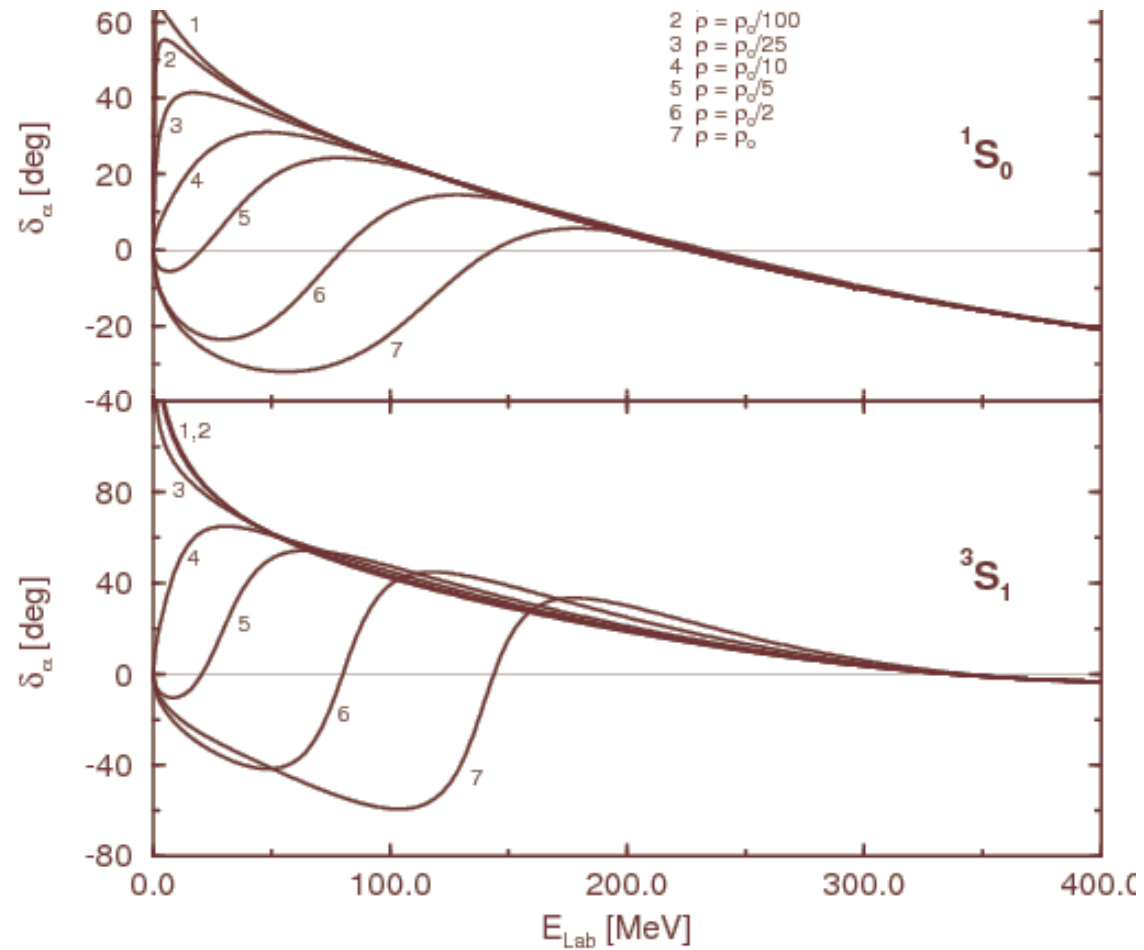
$$E_d^{\text{qu}}(P) = E_d^{\text{free}} + \Delta E_d + \frac{\hbar}{2m_d^*} P^2 + O(P^4)$$
$$E_d^{\text{free}} = -2.225\text{MeV}$$

$$\Delta E_d^{\text{Pauli}}(T, n_B, \alpha) = \delta E_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$
$$\frac{m_d^*}{m_d}(T, n_B, \alpha) = 1 + \delta m_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$

T [MeV]	delta E [MeV fm <sup>3</sup> ]	delta m <sup>*</sup> [fm <sup>3</sup> ]
10	364.3	21.3
4	712.9	87.1

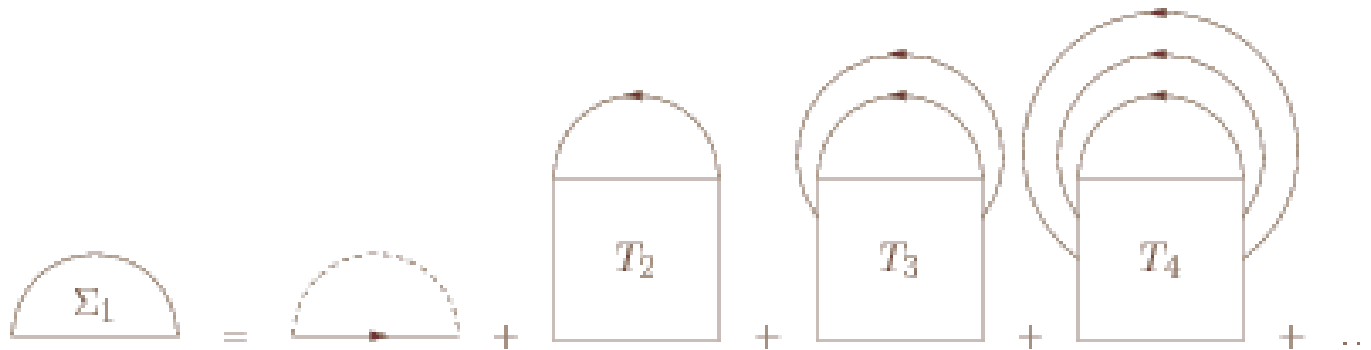


# Scattering phase shifts in matter





# Cluster decomposition of the self-energy



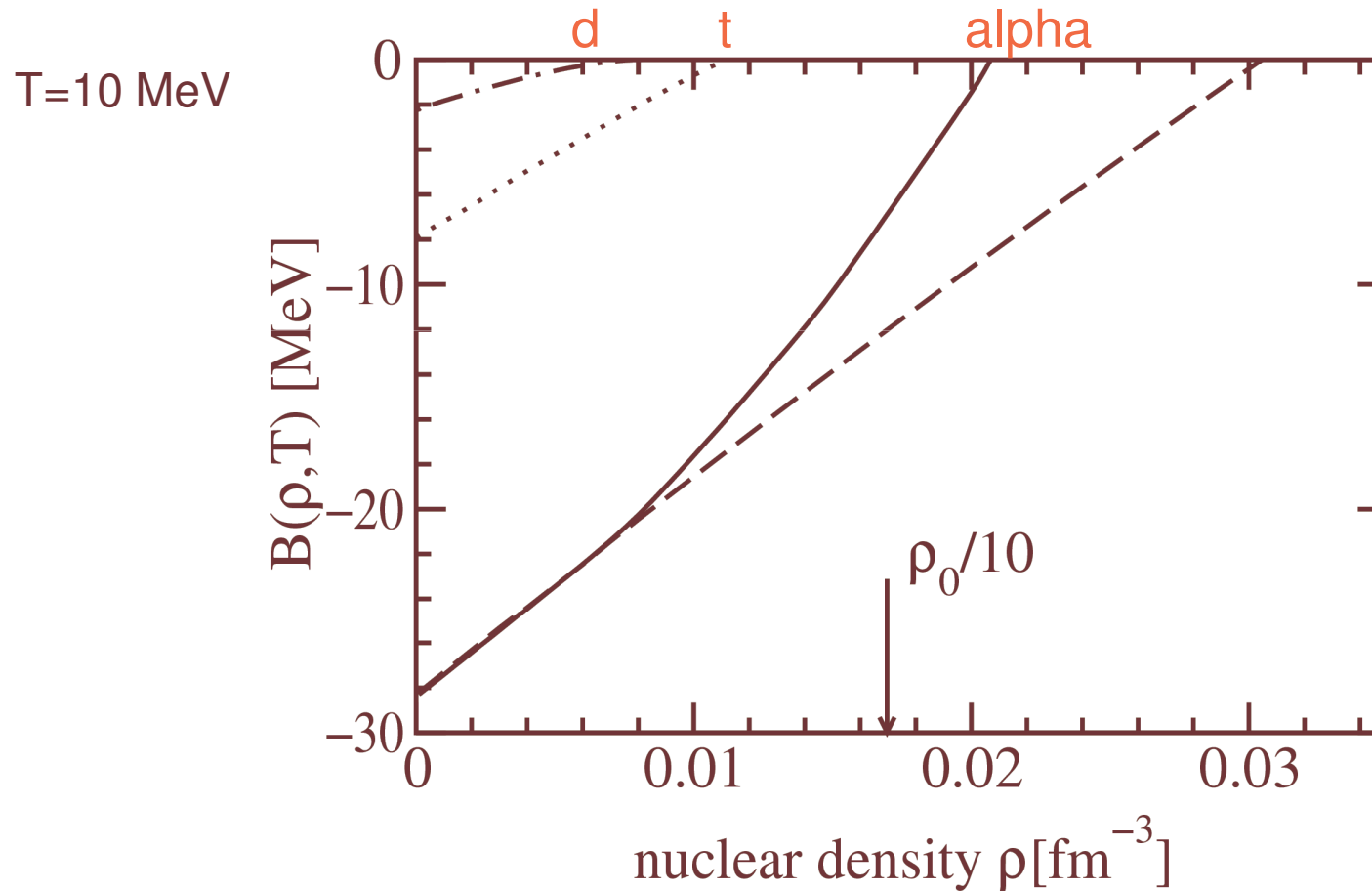
# Few-particle Schoedinger equation in a dense medium

Four-particle Schrödinger equation with medium effects

$$\begin{aligned}
 & [E^{\text{HF}}(p_1) + E^{\text{HF}}(p_2) + E^{\text{HF}}(p_3) + E^{\text{HF}}(p_4)] \psi_{nP}(p_1, p_2, p_3, p_4) \\
 & + \sum_{p'_1 p'_2 p'_3 p'_4} \left\{ [1 - \underline{f(p_1)} - \underline{f(p_2)}] V(p_1 p_2, p'_1 p'_2) \delta_{p_3 p'_3} \delta_{p_4 p'_4} \right. \\
 & \quad + [1 - \underline{f(p_1)} - \underline{f(p_3)}] V(p_1 p_3, p'_1 p'_3) \delta_{p_2 p'_2} \delta_{p_4 p'_4} \\
 & \quad \left. + \text{permutations} \right\} \psi_{nP}(p'_1, p'_2, p'_3, p'_4) \\
 & = E_{nP} \psi_{nP}(p_1, p_2, p_3, p_4)
 \end{aligned}$$

# In-medium shift of binding energies of clusters

Solution of the Faddeev-Yakubovskii equation with Pauli blocking



# Composition of dense nuclear matter

$$n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A \{ E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

$$n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A \{ E_{A,\nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

mass number  $A$ ,

charge  $Z_A$ ,

energy  $E_{A,\nu,K}$ ,

$\nu$ : internal quantum number,

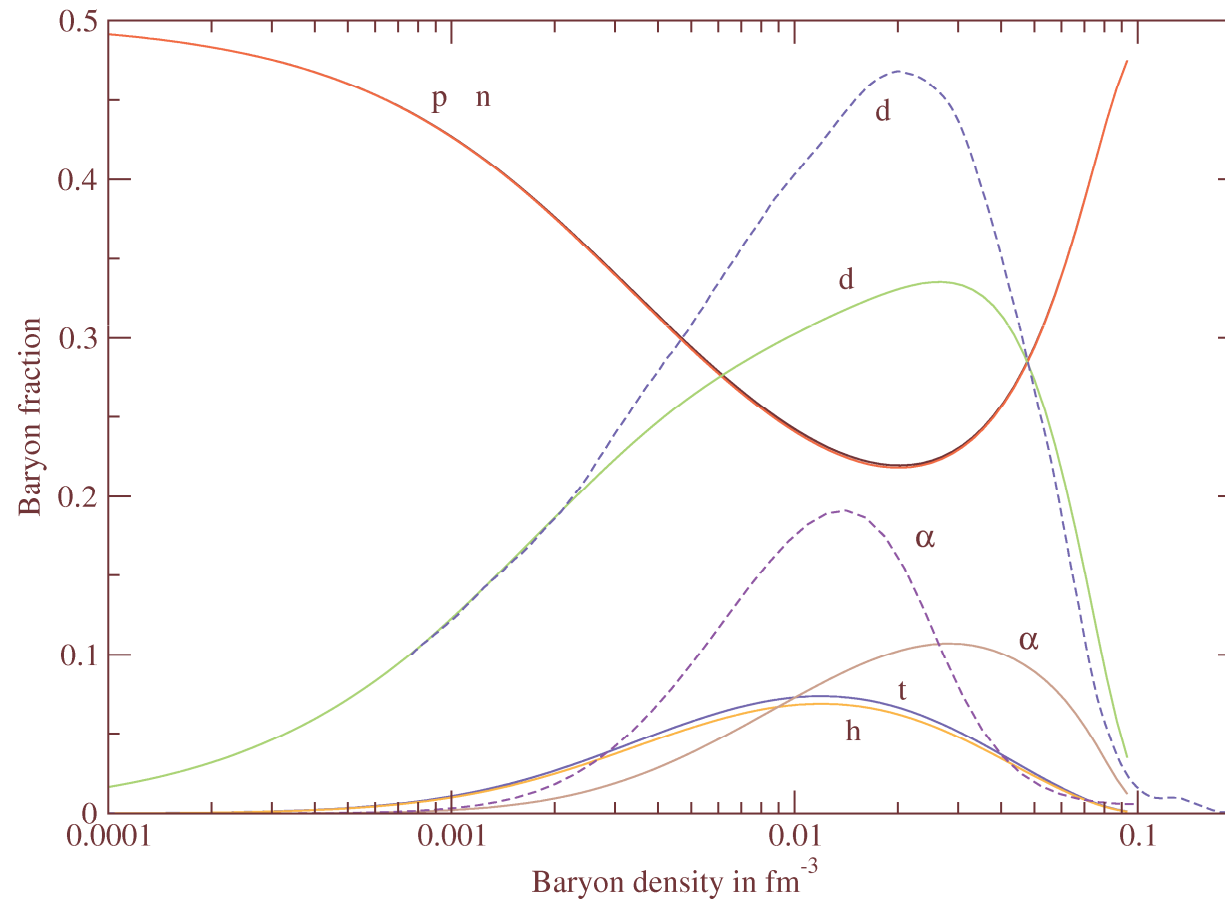
$$f_A(z) = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations
- Medium effects:
  - self-energy and **Pauli blocking shifts** of binding energies,
  - Coulomb corrections due to screening (Wigner-Seitz, Debye)

# Composition of symmetric nuclear matter

T=10 MeV

G.Ropke, A.Grigo, K. Sumiyoshi, Hong Shen,  
Phys.Part.Nucl.Lett. **2**, 275 (2005)



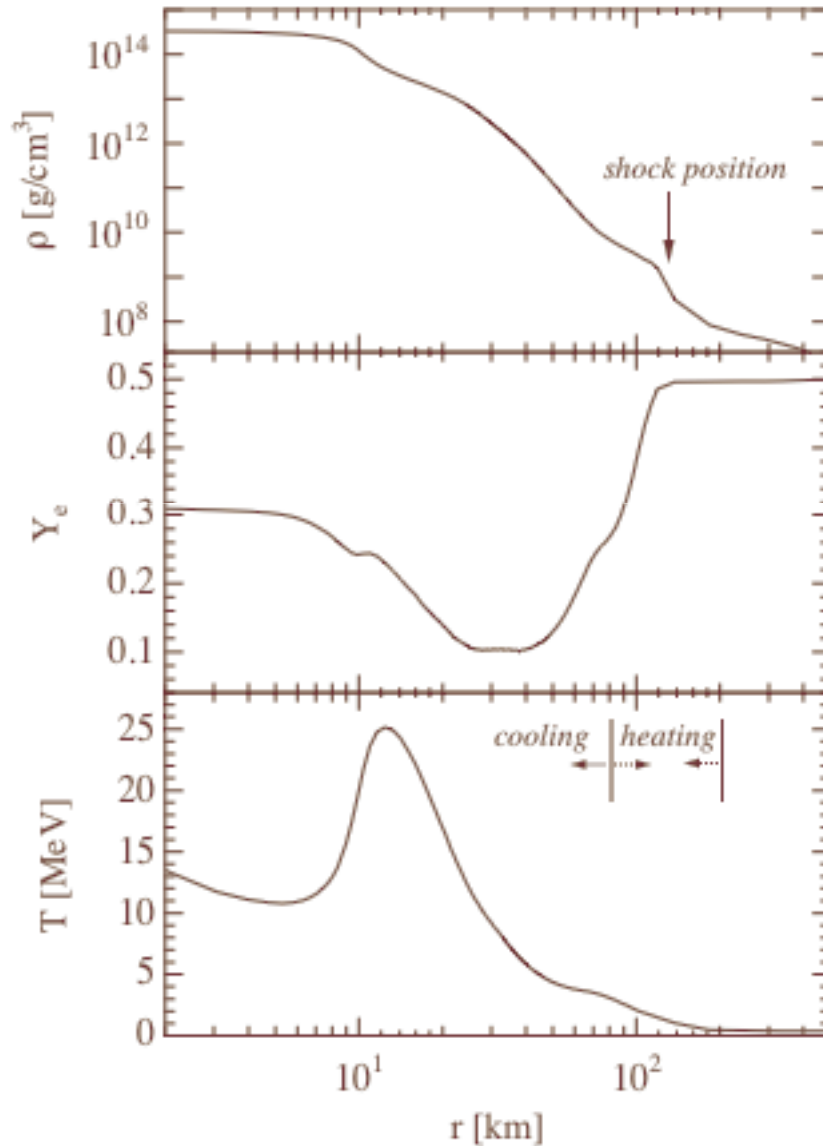
# Light Cluster Abundances

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

S. Typel, 2007



# Core-collapse supernovae



Density.

electron fraction, and

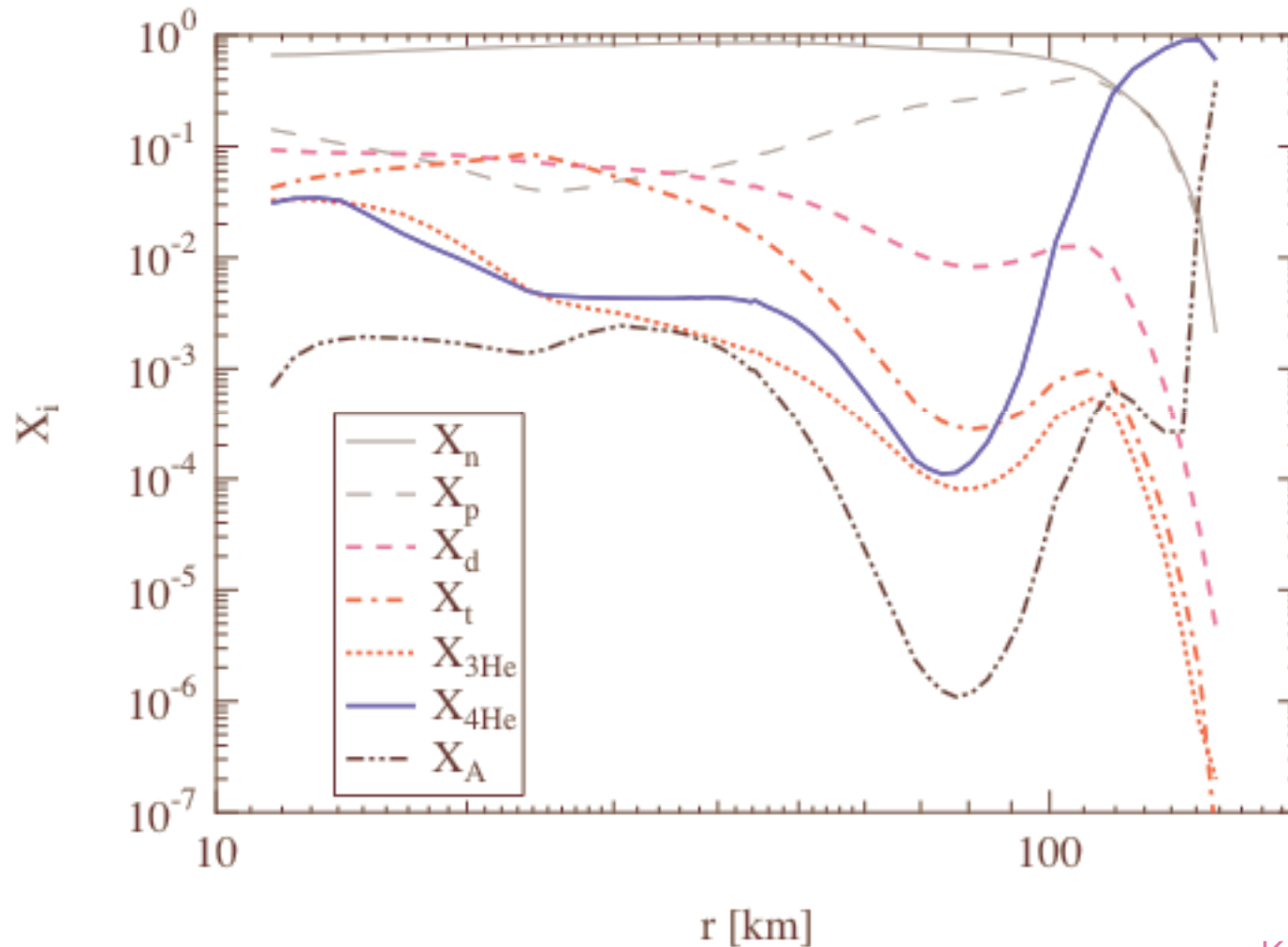
temperature profile

of a 15 solar mass supernova  
at 150 ms after core bounce  
as function of the radius.

Influence of cluster formation  
on neutrino emission  
in the cooling region and  
on neutrino absorption  
in the heating region ?

K.Sumiyoshi et al.,  
*Astrophys.J.* **629**, 922 (2005)

# Composition of supernova core



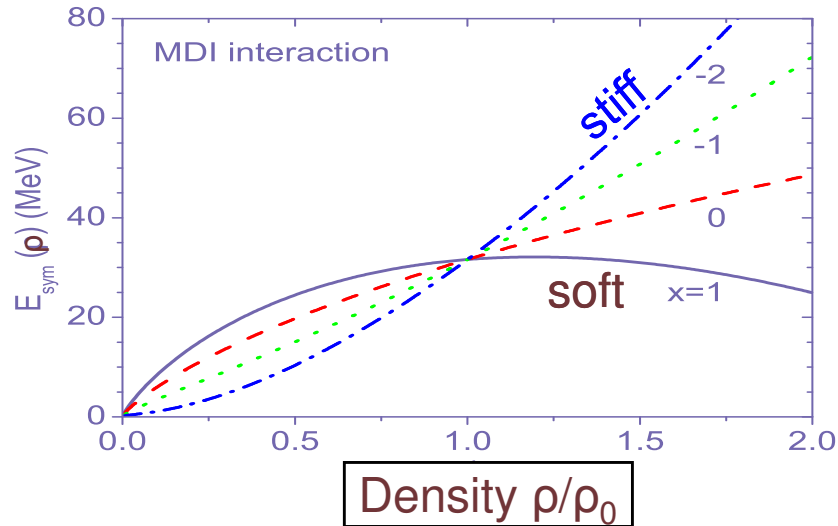
Mass fraction  $X$  of light clusters for a post-bounce supernova core

K. Sumiyoshi, G. R.,  
PRC 77, 055804 (08)

# Symmetry energy of a low density nuclear gas

- L. W. Chen, C. M. Ko, and B. A. Li,  
Phys. Rev. Lett. **94**, 032701 (2005).
- T. Klähn *et al.*,  
Phys. Rev. C **74**, 035802 (2006).
- C. J. Horowitz and A. Schwenk,  
Nucl. Phys. **A 776**, 55 (2006).
- S. Kowalski *et al.*,  
Phys. Rev. C **75**, 014601 (2007).

## Symmetry energy and single nucleon potential used in the IBUU04 transport model



The  $x$  parameter is introduced to mimic various predictions by different microscopic Nuclear many-body theories using different Effective interactions

Single nucleon potential within the HF approach using a modified Gogny force:

$$U(\rho, \delta, \bar{p}, \tau, x) = A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^{\sigma} (1 - x\delta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \delta \rho_{\tau}$$

$$+ \frac{2C_{\tau, \tau}}{\rho_0} \int d^3 p' \frac{f_{\tau}(r, p')}{1 + (p - p')^2 / \Lambda^2} + \frac{2C_{\tau, \tau'}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(r, p')}{1 + (p - p')^2 / \Lambda^2}$$

$$\tau, \tau' = \pm \frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma + 1}, A_u(x) = -96 - \frac{2Bx}{\sigma + 1}, K_0 = 211 \text{ MeV}$$

The momentum dependence of the nucleon potential is a result of the non-locality of nuclear effective interactions and the Pauli exclusion principle

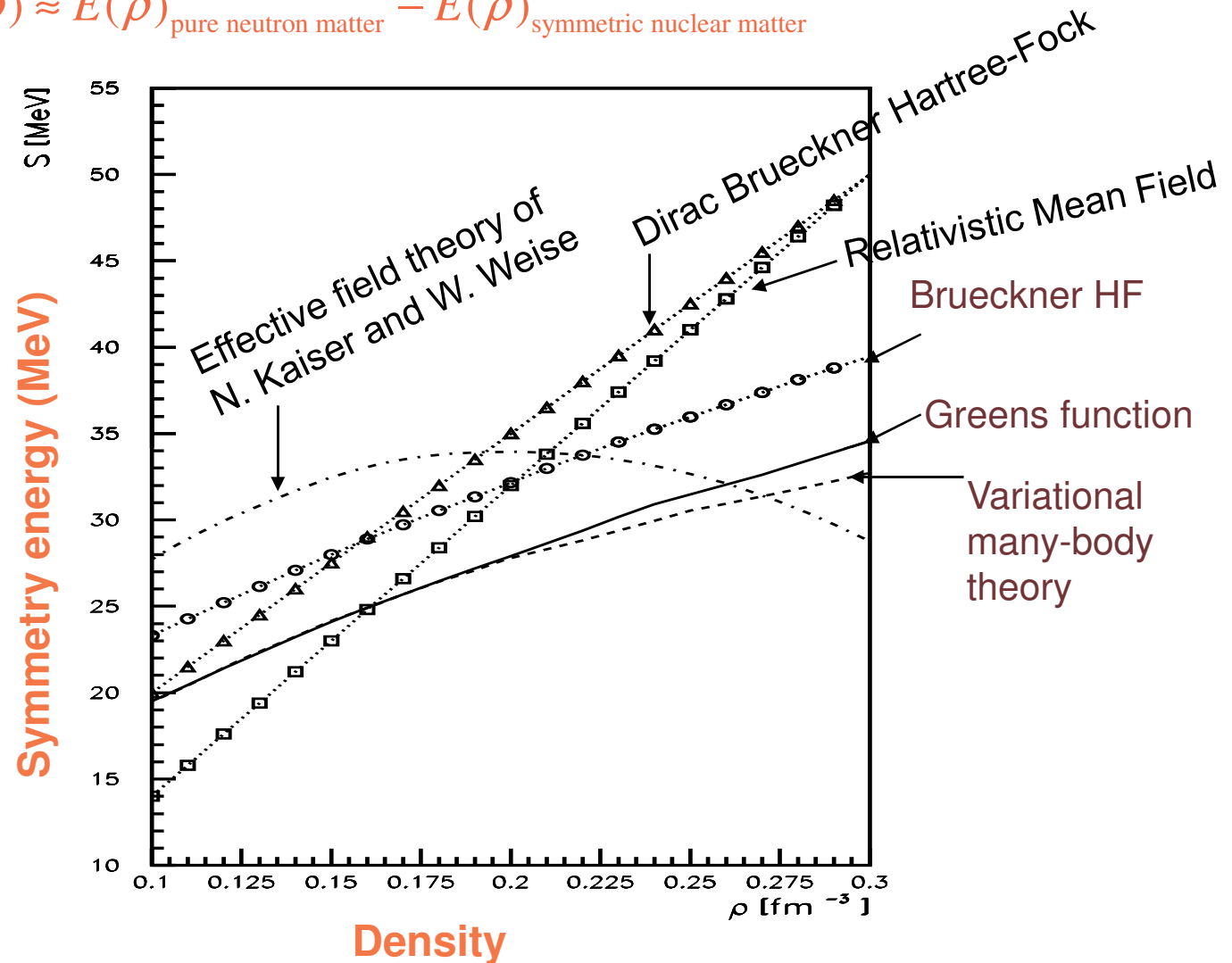
C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC 69, 034614; NPA 735, 563 (2004).

## $E_{sym}(\rho)$ predicted by microscopic many-body theories

EOS:  $E(\rho, \delta) = E_0(\rho, 0) + E_{sym}(\rho)\delta^2 + o(\delta^4)$ , where  $\delta \equiv (\rho_n - \rho_p)/\rho$

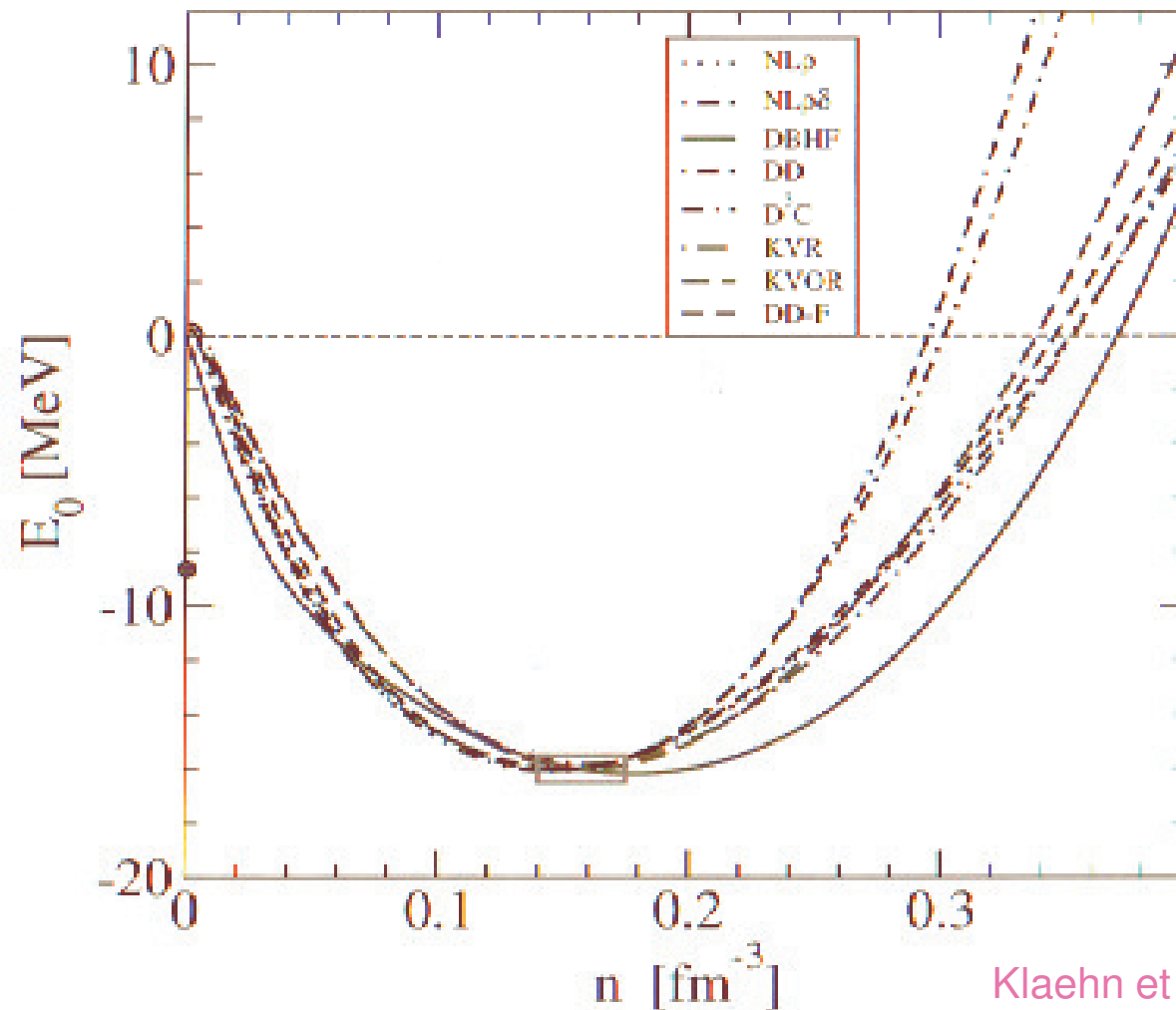
$$E_{sym}(\rho) \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$



A.E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier and V. Rodin, Phys. Rev. C68 (2003) 064307

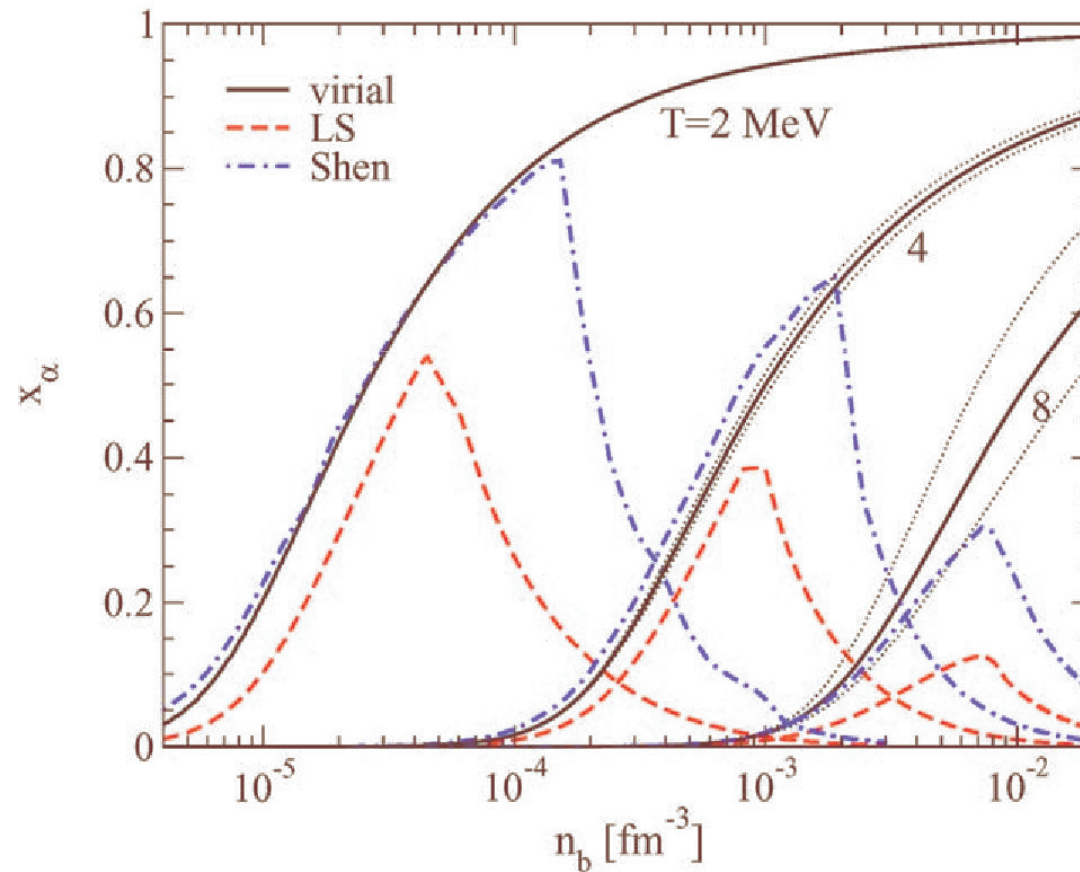
## Quasiparticle approximation for nuclear matter

### Equation of state for symmetric matter



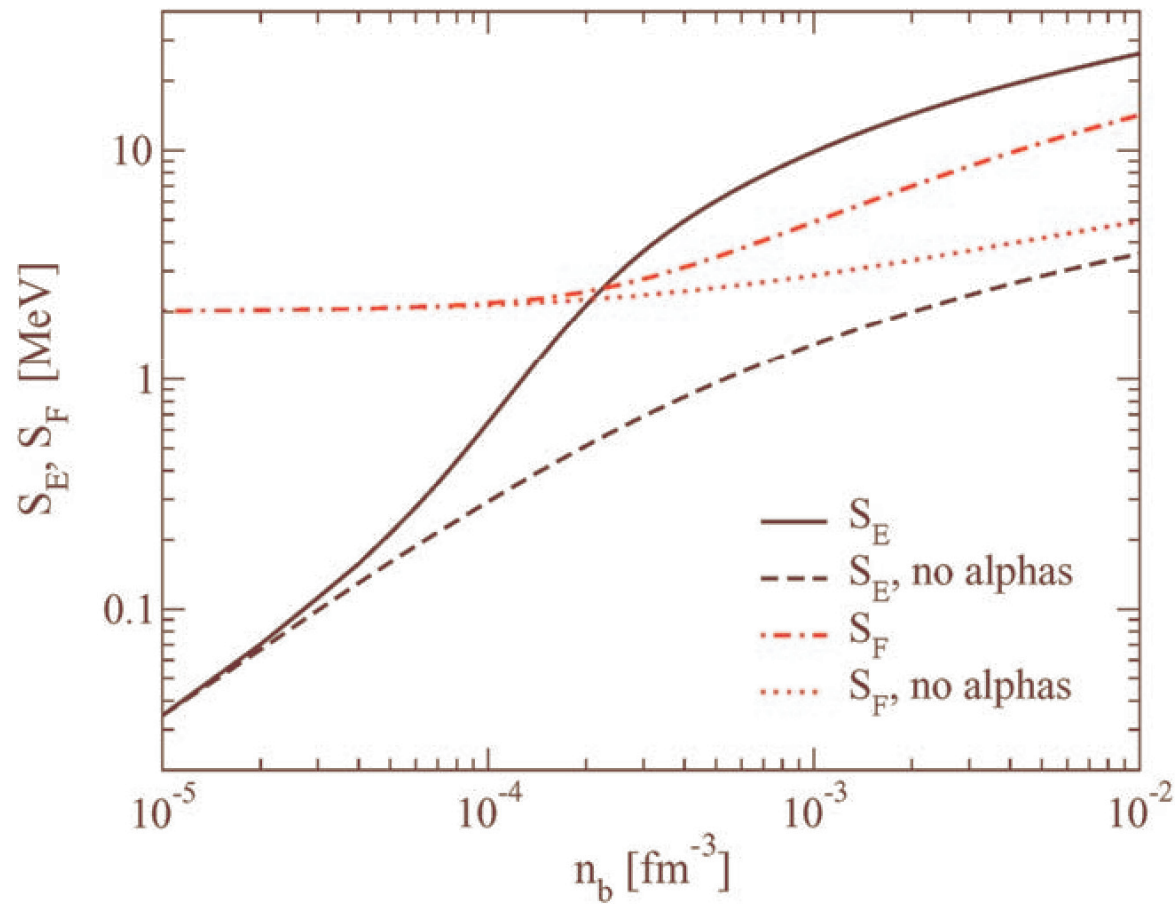
# Alpha-particle fraction in the low-density limit

symmetric matter,  $T=2, 4, 8$  MeV



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

# Symmetry energy and symmetry free energy



$T=4\text{MeV}$

Horowitz & Schwenk,  
NPA (2006)

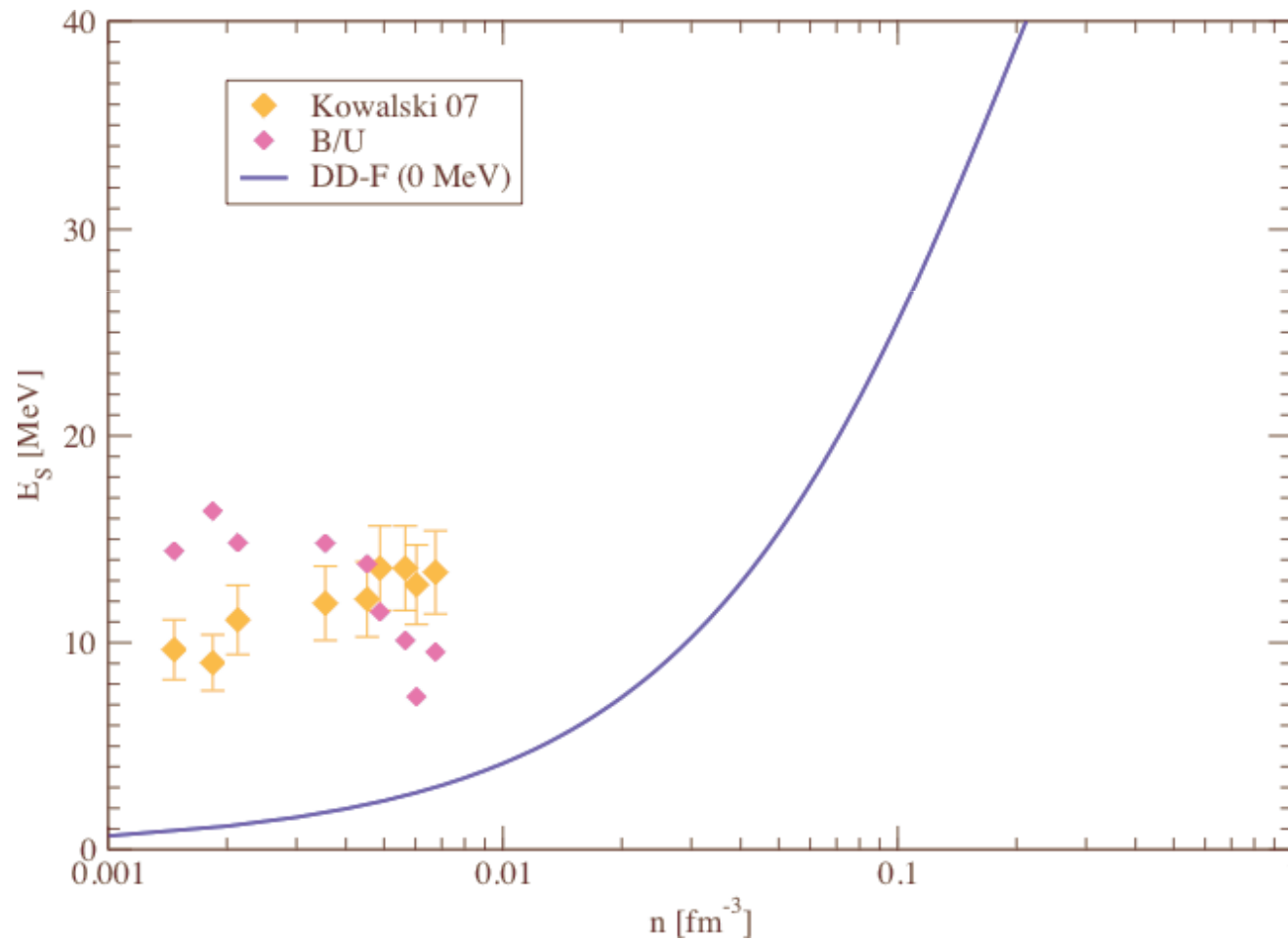


# Free and Internal Symmetry Energy

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

# Symmetry energy

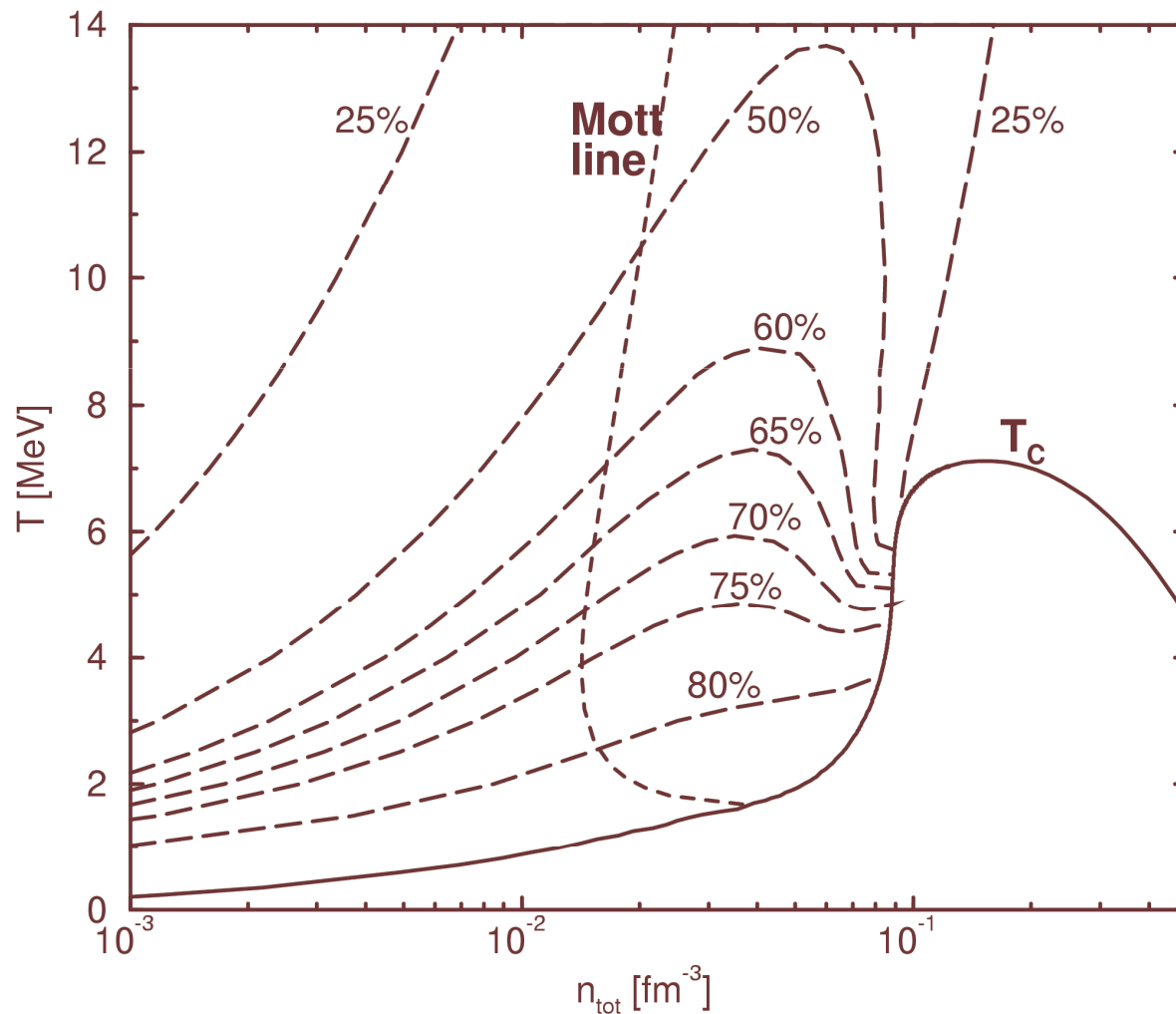
Heavy-ion collisions, spectra of emitted clusters, temperature (3 - 10 MeV), free energy



# Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

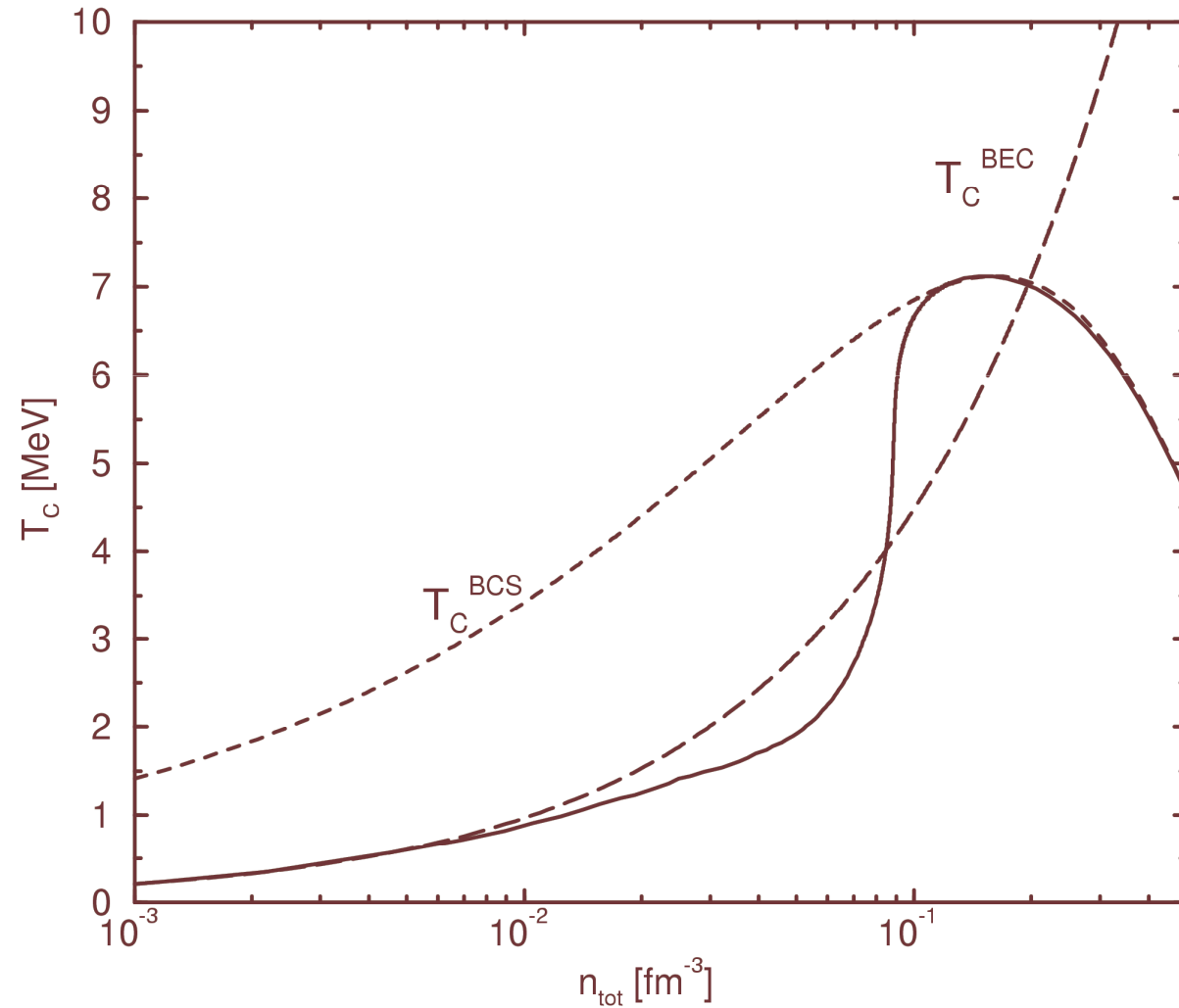
H. Stein et al.,  
Z. Phys. **A351**, 259 (1995)



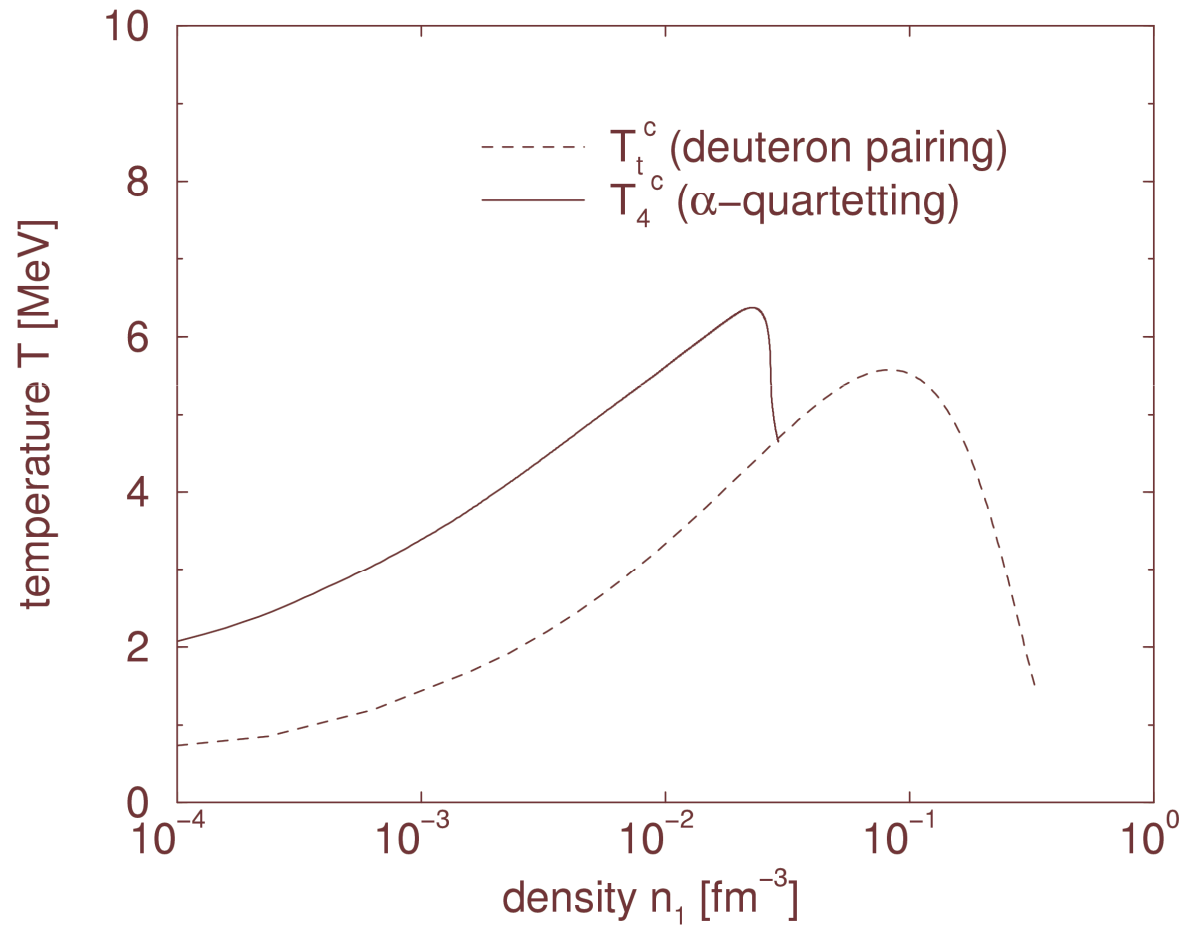
# Quantum condensate

Bose-Einstein-  
Condensation  
of deuterons  
(BEC)

Bardeen-Cooper  
Schrieffer  
pairing  
(BCS)

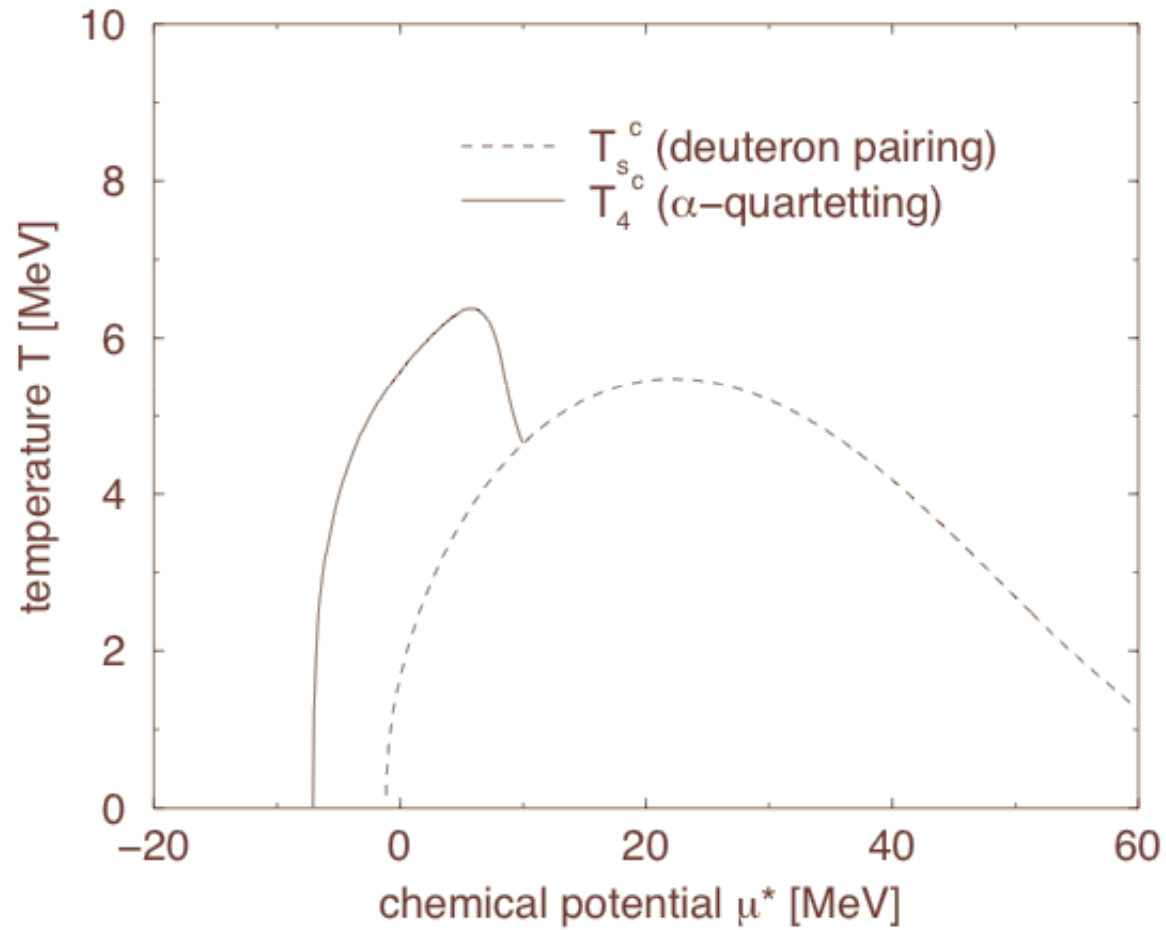


# $\alpha$ -cluster-condensation (quartetting)

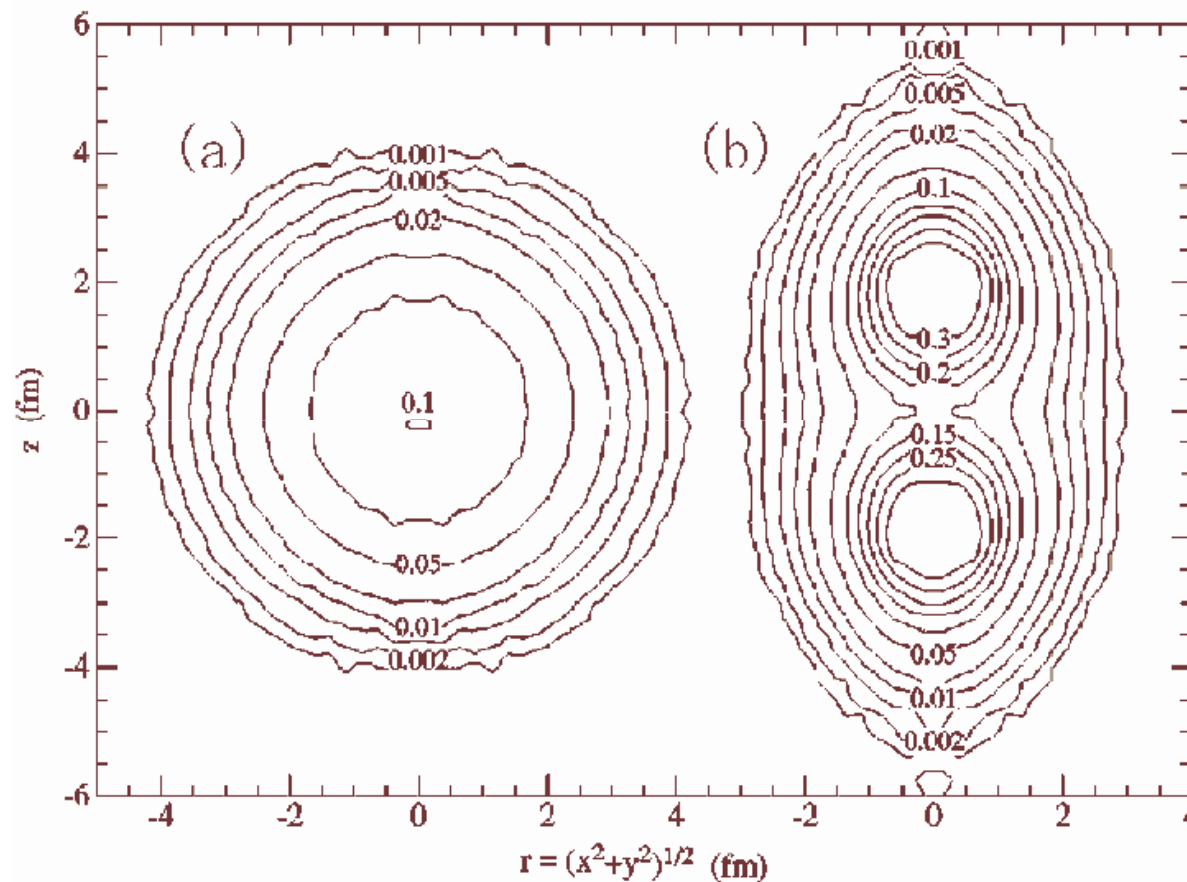


G. Röpke, A. Schnell, P. Schuck, and P. Nozieres, PRL **80**, 3177 (98)

# $\alpha$ -cluster-condensation (quartetting)



# Alpha cluster structure of Be 8



R.B. Wiringa et al.,  
PRC 63, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for  $^8\text{Be}(0^+)$ .  
The left side is in the laboratory frame while the right side is in the intrinsic frame.

# Self-conjugate $4n$ nuclei

$^{12}\text{C}$ :

$0^+$  state at 0.39 MeV above the  $3\alpha$  threshold energy:  
 $\alpha$  cluster interact predominantly in relative  $S$  waves,  
gaslike structure

$\alpha$ -particle condensation in low-density nuclear matter  
( $\rho \leq \rho_0/5$ )

$n\alpha$  cluster condensed states  
— a general feature in  $N = Z$  nuclei?



# Self-conjugate 4n nuclei

$n\alpha$  nuclei:  ${}^8\text{Be}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ ,  ${}^{20}\text{Ne}$ ,  ${}^{24}\text{Mg}$ , ...

Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$  break up at the threshold energy  $E_{n\alpha}^{\text{thr}} = nE_{\alpha}$

# Variational ansatz

$$|\Phi_{n\alpha}\rangle = (C_{\alpha}^{\dagger})^n |\text{vac}\rangle$$

$\alpha$ - particle creation operator

$$C_{\alpha}^{\dagger} = \int d^3R e^{-\vec{R}^2/R_0^2} \\ \times \int d^3r_1 \dots d^3r_4 \phi_{0s}(\vec{r}_1 - \vec{R}) a_{\sigma_1\tau_1}^{\dagger}(\vec{r}_1) \dots \phi_{0s}(\vec{r}_4 - \vec{R}) a_{\sigma_4\tau_4}^{\dagger}(\vec{r}_4)$$

with

$$\phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)}$$

# Variational ansatz

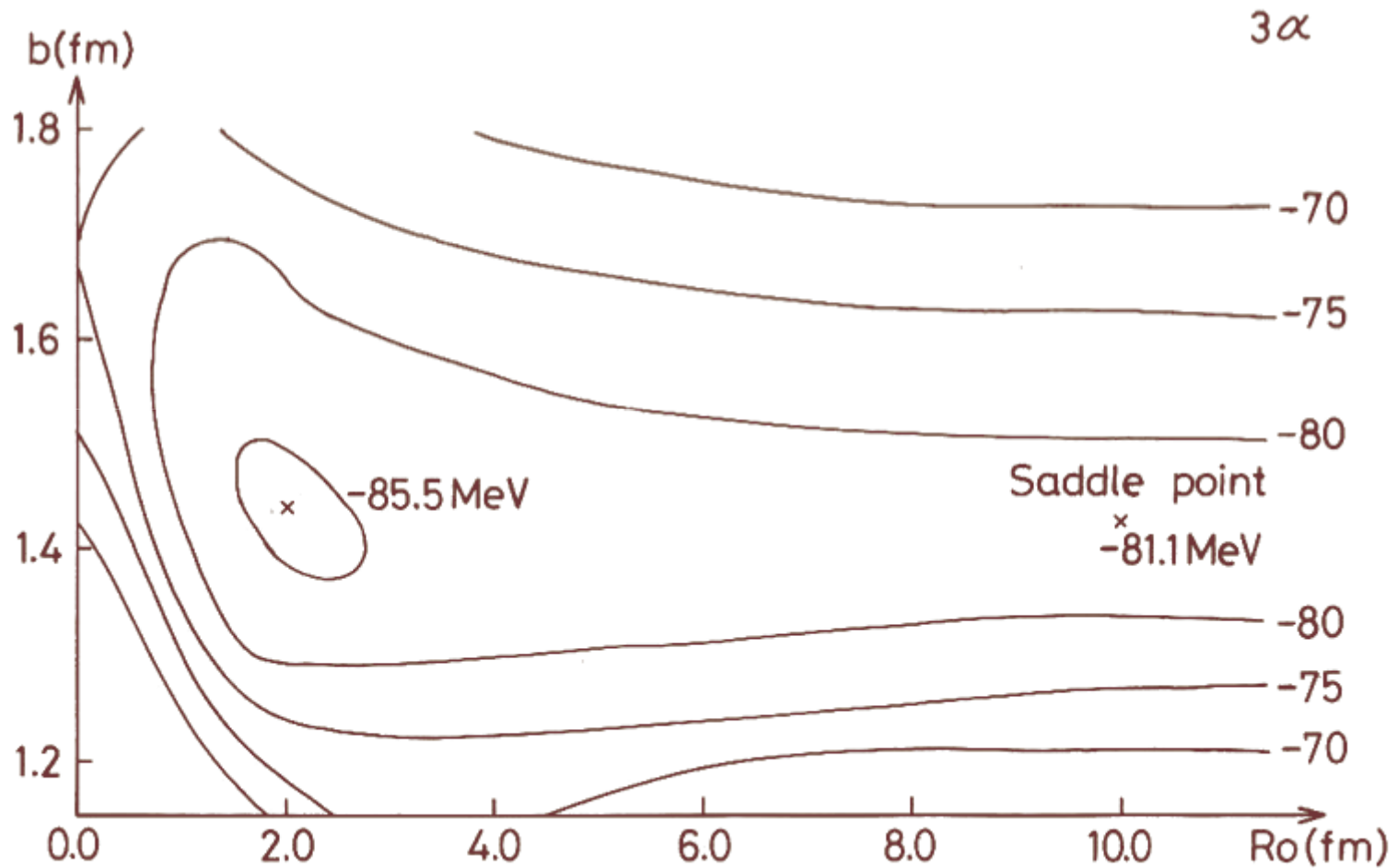
total  $n\alpha$  wave function

$$\langle \vec{r}_1 \sigma_1 \tau_1 \dots \vec{r}_{4n} \sigma_{4n} \tau_{4n} | \Phi_{n\alpha} \rangle \\ \propto \mathcal{A} \left\{ e^{-\frac{2}{B^2} (\vec{X}_1^2 + \dots + \vec{X}_n^2)} \phi(\alpha_1) \dots \phi(\alpha_n) \right\}$$

where  $B^2 = (b^2 + 2R_0^2)$ ,  $\vec{X}_i = \frac{1}{4} \sum_n \vec{r}_{in}$ ,  
 $\phi(\alpha_i) = e^{-\frac{1}{8b^2} \sum_{m>n} (\vec{r}_{im} - \vec{r}_{in})^2}$  - internal  $\alpha$  wave function

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL **87**,  
192501 (2001)

# 3 alpha variational energy



# Results

	$E_k$ (MeV)	$E_{\text{exp}}$ (MeV)	$E_k - E_{n\alpha}^{\text{thr}}$ (MeV)	$(E - E_{n\alpha}^{\text{thr}})_{\text{exp}}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{\text{exp}}$ (fm)
$^{12}\text{C}$ $k = 1$	-85.9	-92.16 ( $0_1^+$ )	-3.4	-7.27	2.97	2.65
$k = 2$	-82.0	-84.51 ( $0_2^+$ )	+0.5	0.38	4.29	
$E_{3\alpha}^{\text{thr}}$	-82.5	-84.89				
$^{16}\text{O}$ $k = 1$	-124.8 (-128.0)*	-127.62 ( $0_1^+$ )	-14.8 (-18.0)*	-14.44	2.59	2.73
$k = 2$	-116.0	-116.36 ( $0_3^+$ )	-6.0	-3.18	3.16	
$k = 3$	-110.7	-113.62 ( $0_5^+$ )	-0.7	-0.44	3.97	
$E_{4\alpha}^{\text{thr}}$	-110.0	-113.18				
$^8\text{Be}$			-0.17	+0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values.  $E_{n\alpha}^{\text{thr}} = nE_\alpha$  denotes the threshold energy for the decay into  $\alpha$ -clusters, the values marked by \* correspond to a refined mesh.

## Estimation of condensate fraction in zero temperature $\alpha$ -matter

$$n_0 = \frac{\langle \Psi | a_0^\dagger a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

destruction of the BEC of the ideal Bose gas:  
thermal excitation, but also correlations

“excluded” volume for  $\alpha$ -particles  $\approx 20 \text{ fm}^3$  Shimizu  
at nucleon density  $\rho = 0.048 \text{ fm}^{-3}$  filling factor  $\approx 28 \%$   
(liquid  $^4\text{He}$ : 8 % condensate),  
destruction of the condensate at  $\approx \rho_0/3$

## Estimation of condensate fraction in zero temperature $\alpha$ -matter

$\alpha$ -cluster condensate in  $^{12}\text{C}$ ,  $^{16}\text{O}$ :

resonating group method → Yamada et al. (1994)

occupation numbers of  $\alpha$ -orbits in  $^{12}\text{C}$

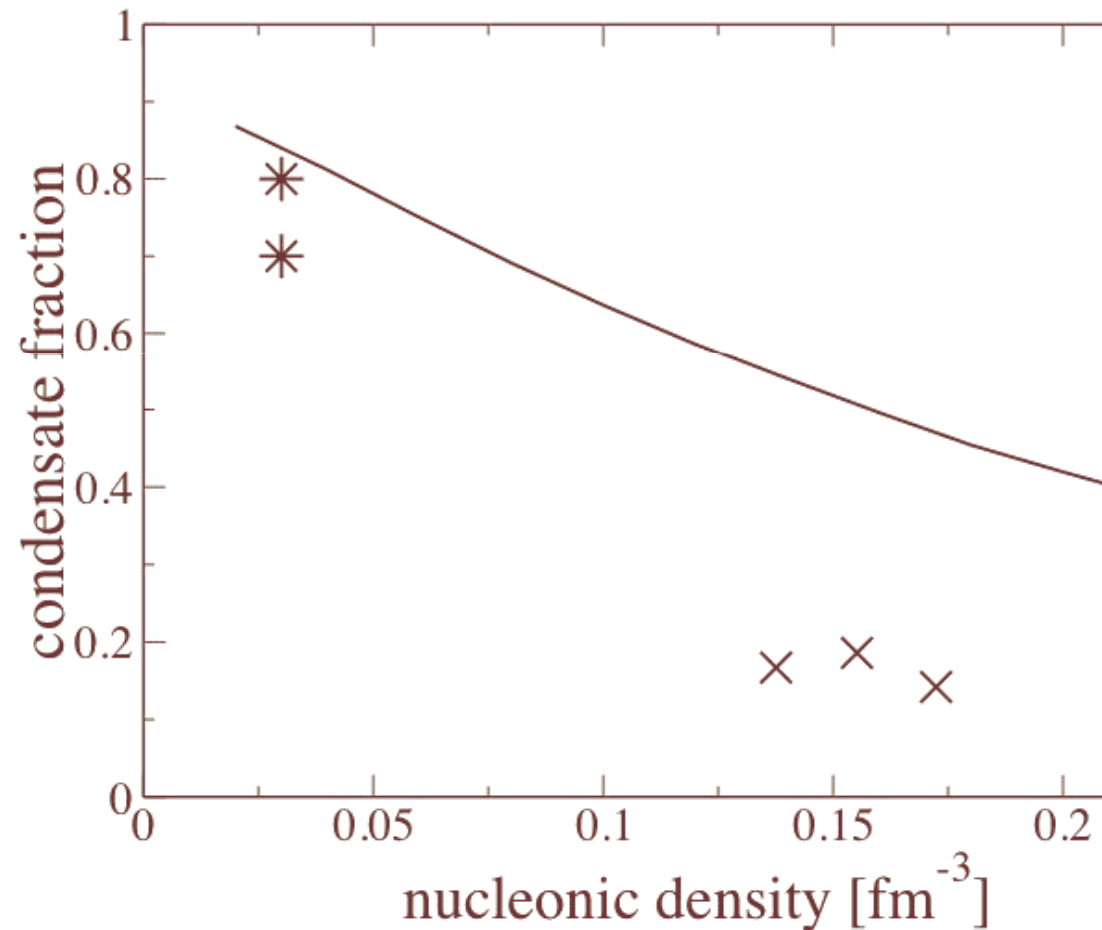
	RMS radii	S-orbit	D-orbit	G-orbit
$\text{O}_1^+$ (g.s.)	2.44 fm	1.07	1.07	0.82
$\text{O}_2^+$	4.31 fm	2.38	0.29	0.16

80 % condensate at 1/8 nuclear matter density

T. Yamada et al., P. Schuck:  $(2.16 - \text{normal})/3 \approx 60\%$

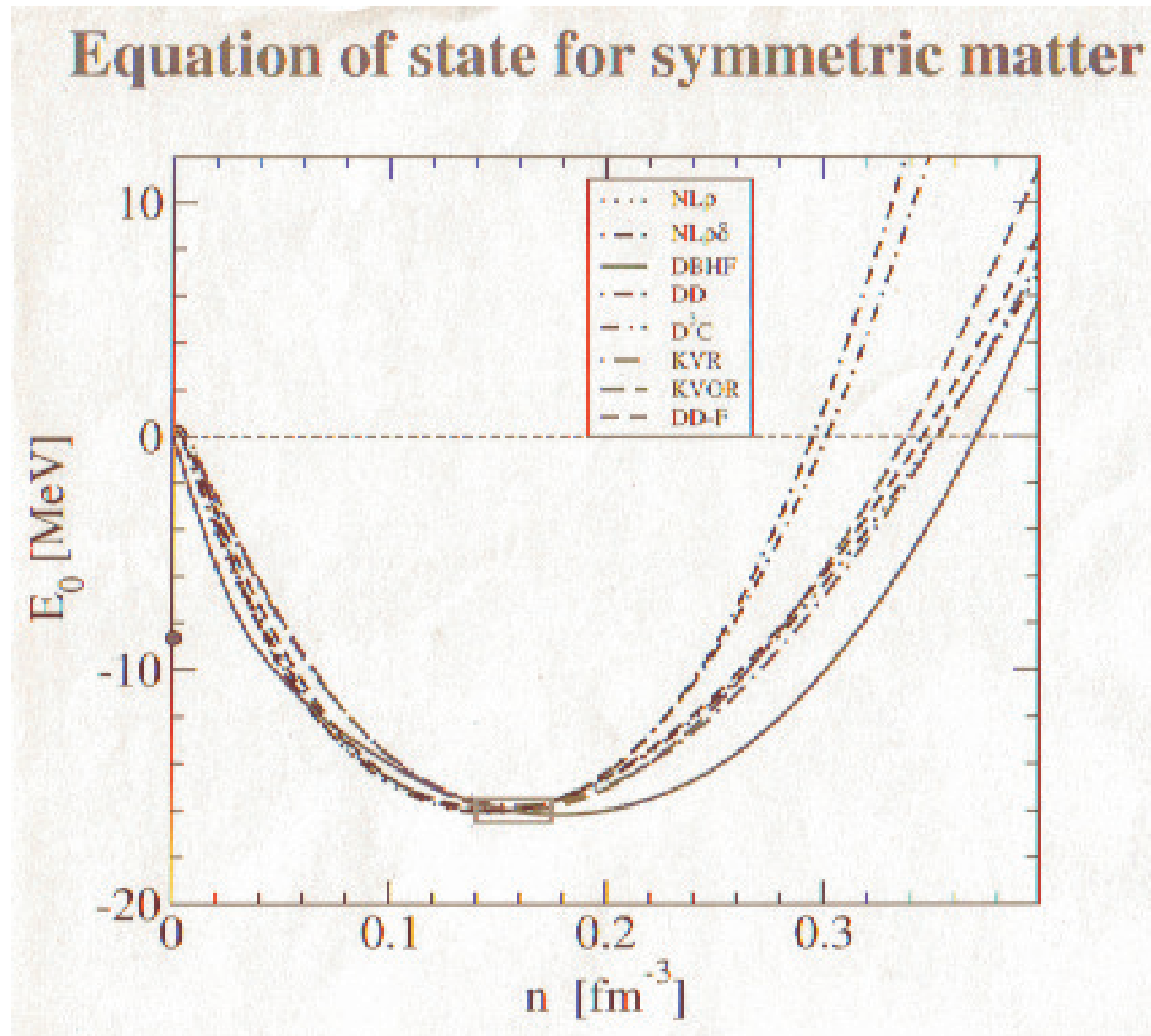
# Suppression of condensate fraction

- Alpha-alpha interaction (Ali/Bodmer), no Pauli blocking:
- Variational calculation (Clark/Jastrow approach to the alpha-particle condensate amplitude) (crosses)
- First order approximation (full line)
- Yamada/Schuck's result for condensate in C12 - O2+ (stars)





## Quasiparticle approximation for nuclear matter



# Summary

- The low-density limit of the nuclear matter EoS can be rigorously treated. The Beth-Uhlenbeck virial expansion is a benchmark.
- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self-energy and Pauli blocking is included. An interpolation between low and high densities is possible.
- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation (quartetting), and the behavior of the symmetry energy.

# Thanks

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S. Shlomo, P. Schuck,  
A. Sedrakian, K. Sumiyoshi, S. Typel, H. Wolter  
for collaboration

to you

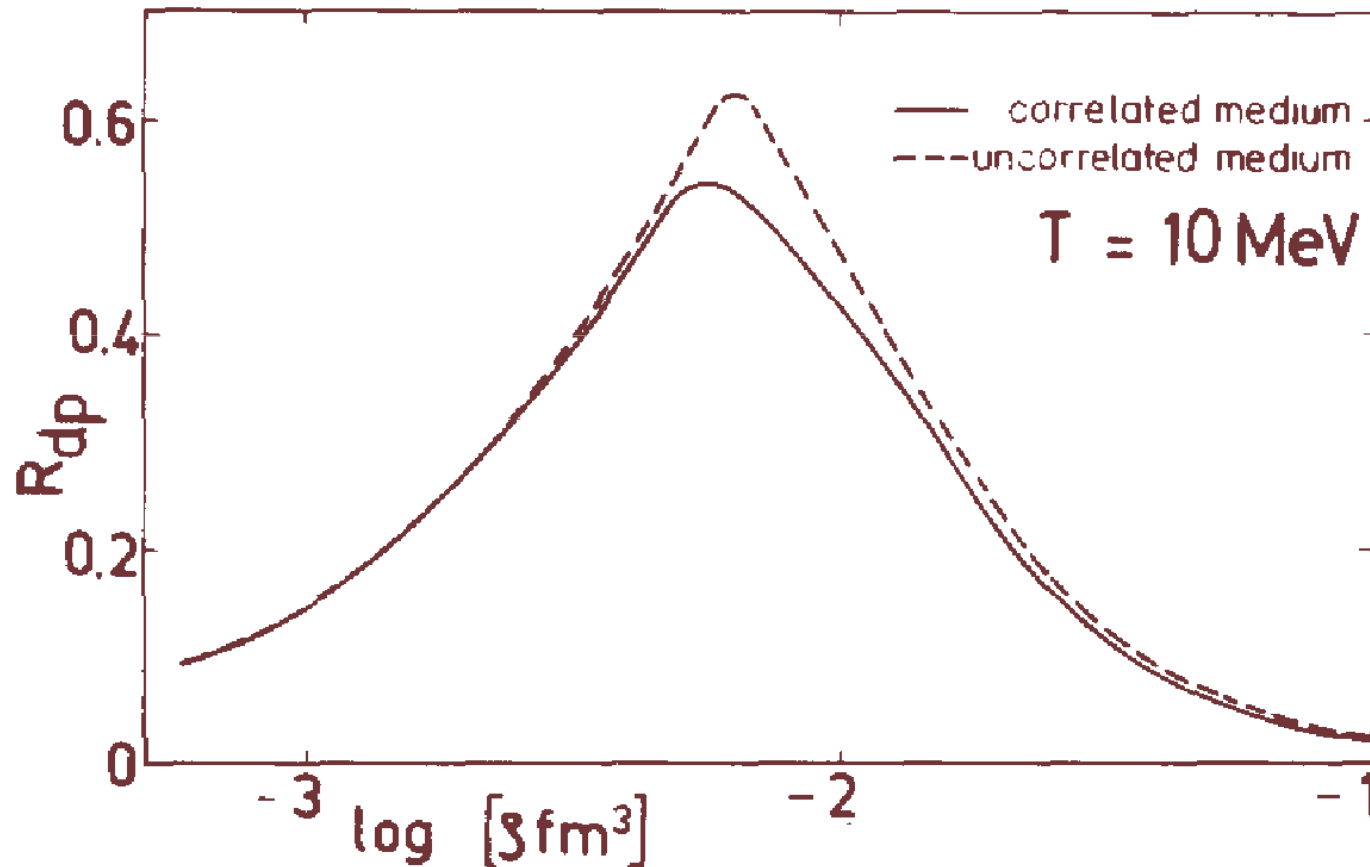
for attention

# Correlations in the medium

$$\Sigma_2 =$$

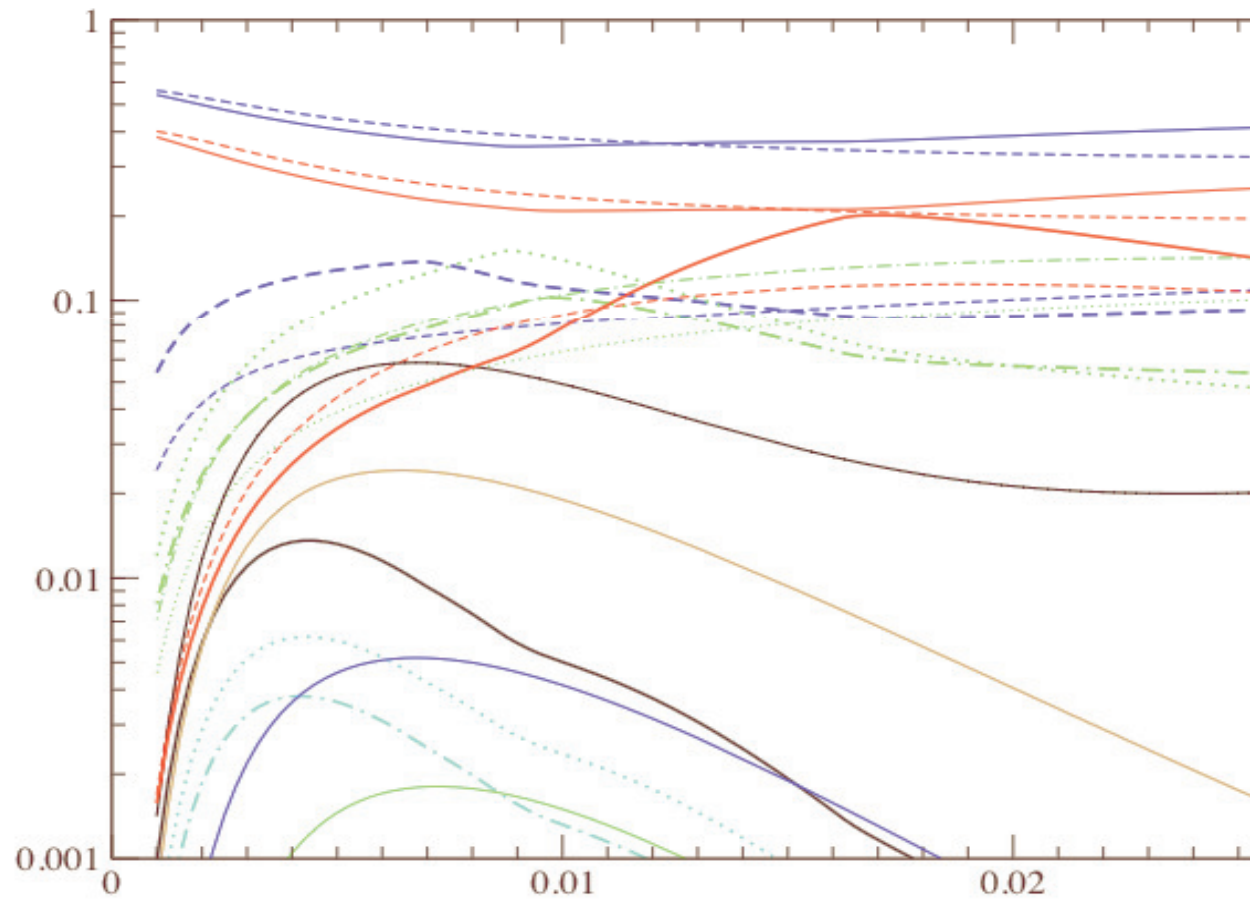
The equation shows the second-order self-energy  $\Sigma_2$  as a sum of six diagrams, each labeled with a weight of  $(2x)$ . The diagrams are arranged in two rows of three. Each diagram features a fermion propagator (represented by a line with an arrow) and a ghost propagator (represented by a dashed line). The diagrams illustrate various ways in which a fermion loop and a ghost loop can be inserted into the propagator lines, representing different topologies of self-energy corrections.

# Account of two-particle correlations in the medium



# Heavy nuclei abundances in nuclear matter

$T=10$  MeV, asymmetry 0.42, as function of baryon density



n, p, d, t,  $\text{He}_3$ ,  $\text{He}_4$ ,  $\text{Li}_5$ ,...

# Alpha-condensate (quartetting) in $4n$ symmetric nuclei

- A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,  
Phys. Rev. Lett. **87**, 192501 (2001).
- G. Röpke, A. Schnell, P. Schuck, and P. Nozieres,  
Phys. Rev. Lett. **80**, 3177 (1998).
- Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke,  
Phys. Rev. C **67**, 051306(R) (2003); Eur. Phys. J. **A 28**, 259 (2006).
- T. Yamada, P. Schuck,  
Phys. Rev. C **69**, 024309 (2004).

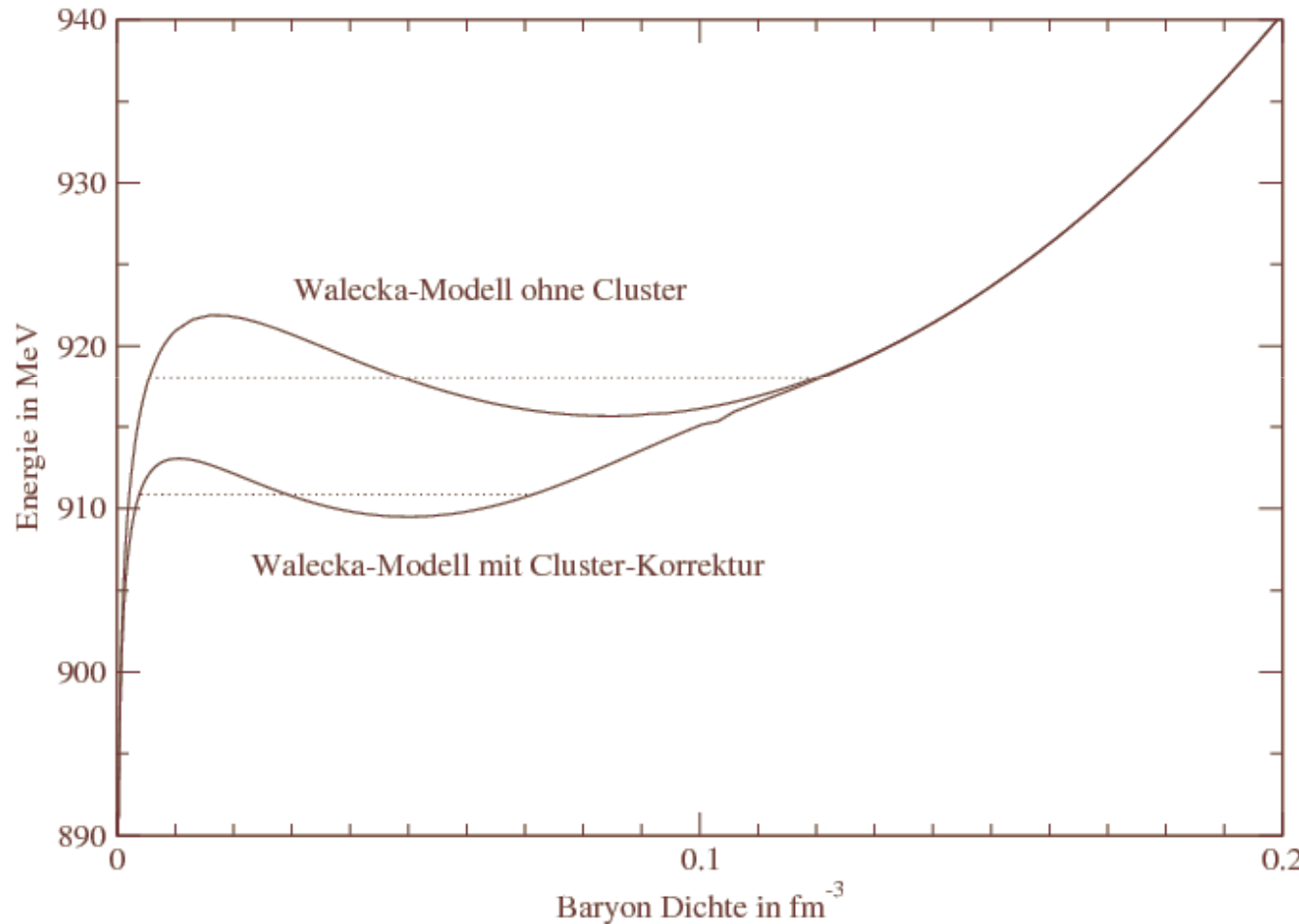
# Approximations to the symmetry energy

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

S. Kubis,  
Neutron stars with  
non-homogeneous  
core,  
Talk 26.2.08, Ladek



# Influence of cluster formation on the equation of state



T=10 MeV

Chemical  
potential of  
symmetric  
nuclear matter.

Inclusion of  
cluster formation  
shifts down the  
chemical potential.

The region of  
thermodynamical  
instability is reduced.

G.R., A. Grigo (2003)