Self-Consistent Green's Functions approach to nuclear matter at finite temperature

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Outline

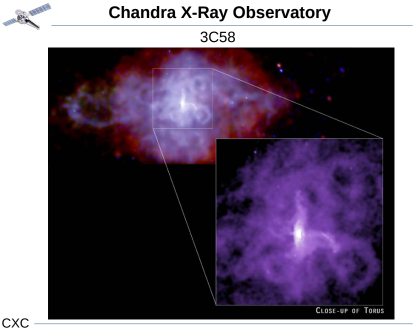
1. Self-Consistent Green's Functions at Finite Temperature
2. Symmetric Nuclear Matter
3. Neutron Matter
4. Asymmetric Nuclear Matter
5. Conclusions and outline

Outline

"Hot" nuclear systems

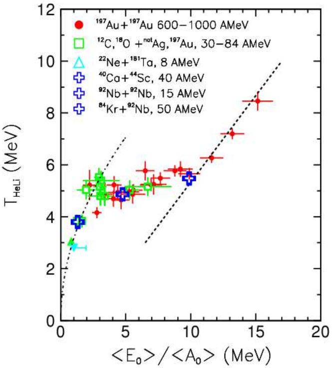
$$E \sim 1 \text{ MeV} \Rightarrow T \sim 10^{10} \text{ K}$$

Proto-neutron stars



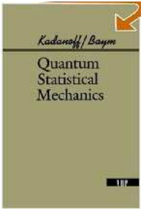
SN 1181 remnant (SNR3C58) and Pulsar PSRJ0205+6449

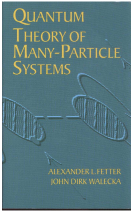
AA collisions

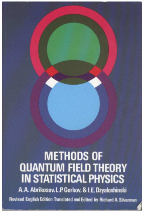


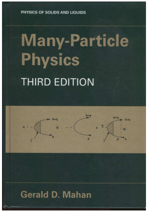
Nuclear caloric curve

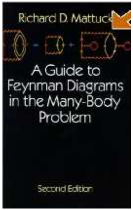
Ingredients











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- Main approximation: ladder decoupling at the level of $\mathcal{G}_{III}$
- Accounts for short-range and tensor correlations
- Ladder approximation includes hole-hole propagation
- Finite temperature $\Rightarrow$ no pairing instability!
- Full off-shell energy dependence is considered

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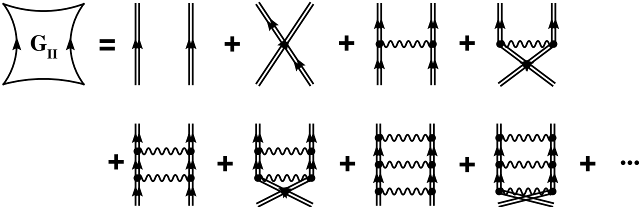

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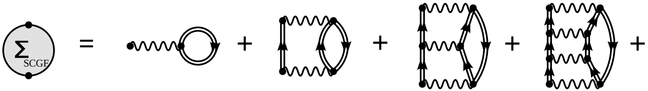

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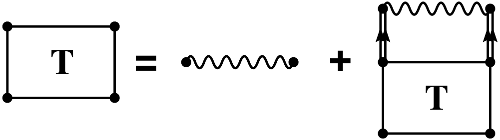

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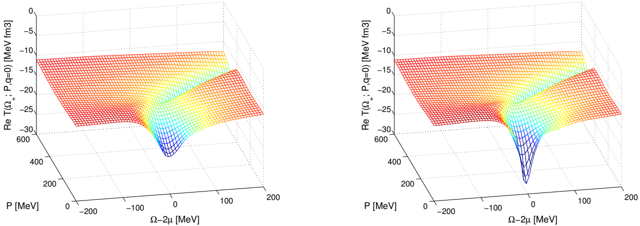

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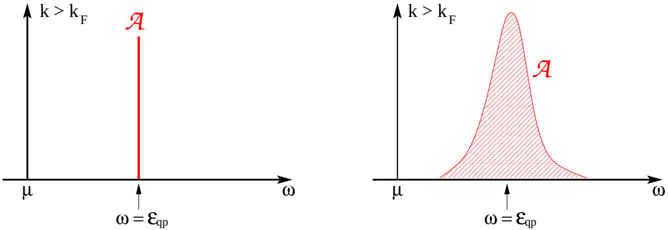

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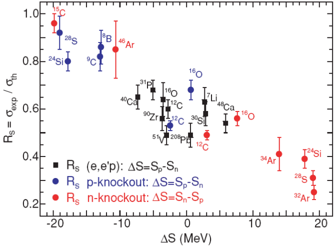
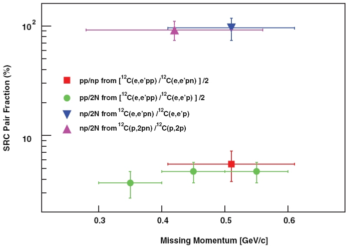
Yet another method?





Advantages

- Connection to experimental results
- Thermodynamical consistency
- Large density-temperature coverage
- SRC effects fully included
- Local and non-local two-body interactions

Disadvantages

- No three-body forces*
- $T = 0$ results elusive
- Elaborate numerics

*Soma and Bozek, arxiv:0808.2929

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1. Chemical potential
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   - Imaginary part
   - Real part
3. Angle average
4. In-medium T-matrix
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6. Spectral function

$$\rho = \nu \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k,\omega) f(\omega,\mu)$$

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Self-Consistent Green's Function scheme

1. Chemical potential
2. Non-correlated two-body propagator
   - Imaginary part
   - Real part
3. Angle average
4. In-medium T-matrix
5. Ladder self-energy
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   - Real part
6. Spectral function

$$\rho = \nu \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k,\omega) f(\omega,\mu)$$

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Self-Consistent Green's Function scheme

$$\rho = \nu \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k,\omega) f(\omega, \mu)$$

1. Chemical potential

$$\mathrm{Im}\, \mathcal{G}^0_{II}(\Omega_+; k_1, k_2) = -\frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k_1,\omega)\mathcal{A}(k_1, \Omega-\omega) \times [1 - f(\omega,\mu) - f(\Omega-\omega,\mu)]$$

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2. Non-correlated two-body propagator
   - Imaginary part
   - Real part

$$\overline{\mathcal{G}^0_{II}}(\Omega; K, q_r) = \frac{1}{2} \int_{-1}^{1} d(\cos\theta)\, \mathcal{G}^0_{II}(\Omega; |\mathbf{K}/2 + \mathbf{q}_r|, |\mathbf{K}/2 - \mathbf{q}_r|)$$

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$$\langle \mathbf{k}_r | T(\Omega, K) | \mathbf{p}_r \rangle = \langle \mathbf{k}_r | V | \mathbf{p}_r \rangle + \int d^3q_r\, \langle \mathbf{k}_r | V | \mathbf{q}_r \rangle\, \overline{\mathcal{G}^0_{II}}(\Omega; K, q_r)\, \langle \mathbf{q}_r | T(\Omega_+; K) | \mathbf{p}_r \rangle$$

4. In-medium T-matrix

$$\mathrm{Im}\, \Sigma_L(k, \omega_+) = \int \frac{d^3k'}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \langle \boldsymbol{kk}' | \mathrm{Im}\, T(\omega + \omega'_+, K) | \boldsymbol{kk}' \rangle_A \times \mathcal{A}(k', \omega') [f(\omega') + b(\omega + \omega')]$$

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   - Imaginary part
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6. Spectral function

Self-Consistent Green's Function scheme

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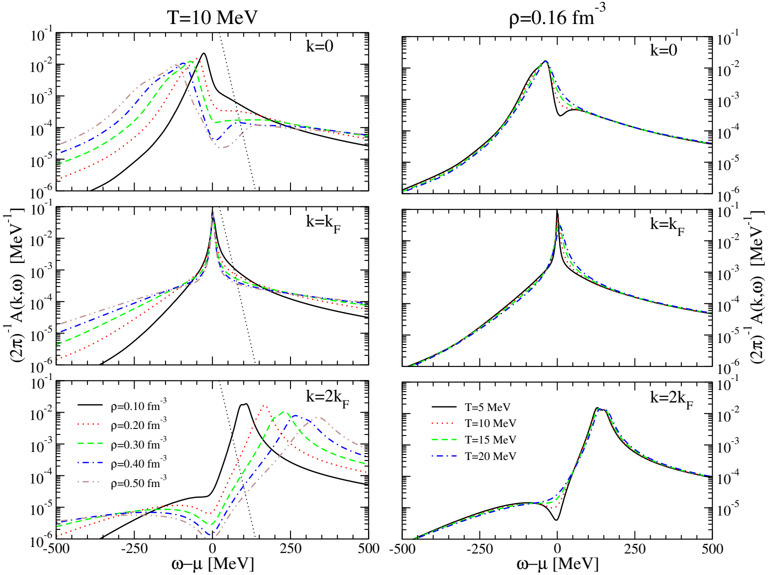
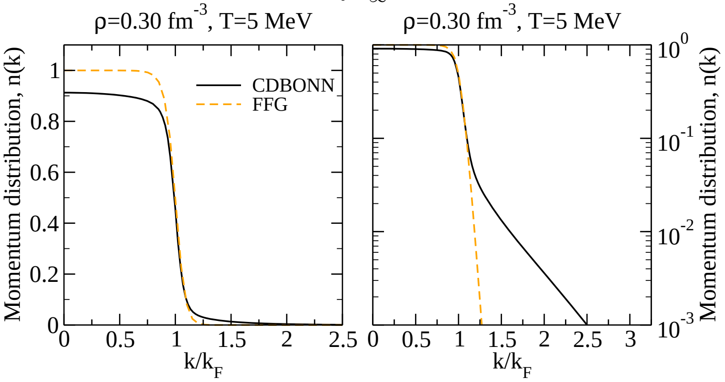
Outline

Spectral functions



Momentum distribution

$$n(k) = \langle \hat{a}_k^\dagger \hat{a}_k \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k,\omega) f(\omega)$$



- Depletion at low k, population at high k
- Competition between thermal and dynamical correlations
- Dependence on NN interaction

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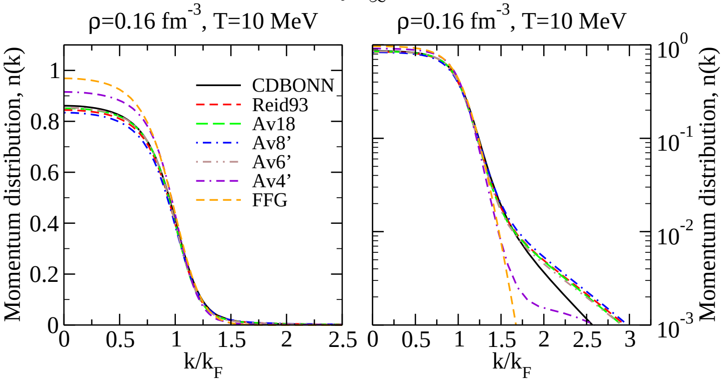

- Depletion at low k, population at high k
- Competition between thermal and dynamical correlations
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Thermodynamics of correlated nucleons

Free energy: $F = E - TS$

- Energy (GMK sum rule)

$$E^{GMK} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{2} \left\{ \frac{k^2}{2m} + \omega \right\} \mathcal{A}(k,\omega) f(\omega)$$

- Entropy

$$S = ???$$

  - Can one compute S from the one-body propagator?
  - Does fragmentation affect the TD properties?
  - Calculation of the correlated entropy?

- Luttinger-Ward formalism:

$$\ln Z\{\mathcal{G}\} = \widetilde{\mathrm{Tr}} \ln \left[-\mathcal{G}^{-1} \right] + \widetilde{\mathrm{Tr}}\, \Sigma \mathcal{G} - \Phi\{\mathcal{G}\}$$

Luttinger and Ward, PR **118**,1417 (1960)

Baym, PR **127**,1391 (1962)

Entropy within the LW formalism

$$S = \left. \frac{\partial T \ln Z}{\partial T} \right|_{\mu} = S^{DQ} + S'$$

- Dynamical quasi-particle entropy

$$S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) \mathcal{B}(k,\omega)$$

with the statistical factor σ and the $\mathcal{B}$ spectral function:

$$\sigma(\omega) = -\left\{ f(\omega) \ln\left[f(\omega)\right] + \left[1 - f(\omega)\right] \ln\left[1 - f(\omega)\right] \right\}$$

$$\mathcal{B}(k,\omega) = \mathcal{A}(k,\omega)\left[1 - \frac{\partial \mathrm{Re}\,\Sigma(k,\omega)}{\partial\omega}\right] + \frac{\partial \mathrm{Re}\,\mathcal{G}(k,\omega)}{\partial\omega}\Gamma(k,\omega)$$

- Higher order entropy $\Rightarrow$ neglected at low T's, Φ-dependence
  Carneiro and Pethick, PR **11**,1106 (1975)

$$S' = -\frac{\partial}{\partial T} T\Phi\{\mathcal{G}\} + \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\partial f(\omega)}{\partial T} \mathcal{A}(k,\omega) \mathrm{Re}\,\Sigma(k,\omega)$$

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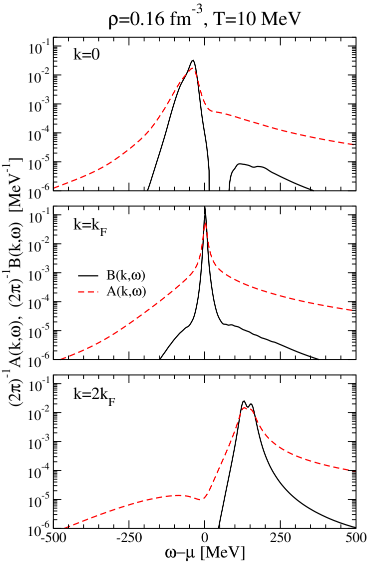
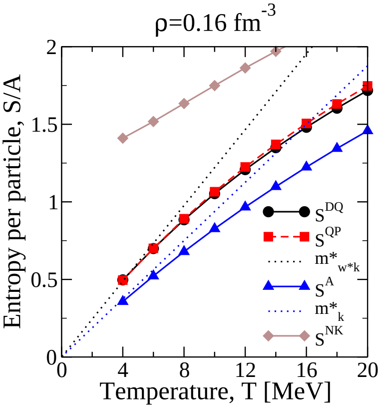
$\mathcal{B}$ spectral function



- $\mathcal{B}$ has a larger quasi-particle peak
- $\mathcal{B}$ has less strength at large energies
- Fragmentation of the qp peak plays a small role

A. Rios *et al.*, PRC 74, 054317 (2006)

Different approximations



$$S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) \mathcal{B}(k,\omega)$$

$$S^{QP} = \sum_k \int_{-\infty}^{\infty} d\omega \, \sigma(\omega) \, \delta\left[\omega - \varepsilon_{SCGF}(k)\right]$$

$$S^{A} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) \mathcal{A}(k,\omega)$$

$$S^{NK} = -\sum_k \{n(k)\ln n(k) + [1-n(k)]\ln[1-n(k)]\}$$

$$S^{FLT} = \frac{\pi^2}{3} N(0) T$$

- $S^{DQ} \sim S^{QP} \Rightarrow$ width effects unimportant, S^{NK} too large
- Different lineal slopes $\Rightarrow$ different density of states, $N(0)$'s

$$N(0) = \sum_k \mathcal{B}(k,\omega=\mu) \rightarrow \frac{\nu k_F m^*_\omega m^*_k}{2\pi^2} \quad \text{vs.} \quad N_A(0) = \sum_k \mathcal{A}(k,\omega=\mu) \rightarrow \frac{\nu k_F m^*_k}{2\pi^2}$$

A. Rios *et al.*, PRC 74, 054317 (2006)

Thermodynamics of correlated nucleons

Free energy "recipe": $F = E^{GMK} - TS^{DQ}$

- Energy (GMK sum rule)

$$E^{GMK} = \sum_k \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{1}{2} \left\{ \frac{k^2}{2m} + \omega \right\} \mathcal{A}(k,\omega) f(\omega)$$

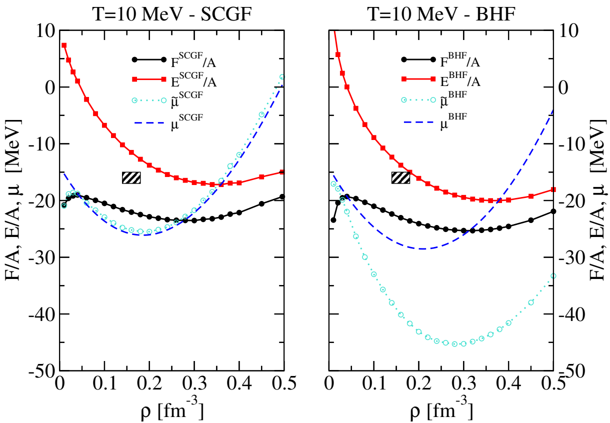
- Entropy (LW formalism)

$$S^{DQ} = \sum_k \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega) \mathcal{B}(k,\omega)$$

- TD consistency

$$\mu = \frac{\partial F/\mathcal{V}}{\partial \rho} \quad \text{vs.} \quad \rho = \nu \int \frac{\mathrm{d}^3 k}{(2\pi)^3} n(k,\tilde{\mu})$$

Thermodynamical consistency

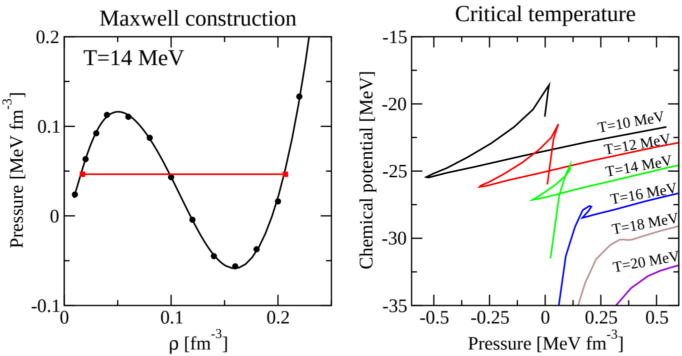


- SCGF + LW yields $\mu \sim \tilde{\mu}$
- BHF violates HvH theorem by 20 MeV
- Incorrect saturation (no TBF)

$$\mu = \frac{\partial F/\mathcal{V}}{\partial \rho} \Leftrightarrow \rho = \nu \int \frac{d^3k}{(2\pi)^3} n(k, \tilde{\mu})$$

A. Rios *et al.*, PRC 74, 054317 (2006)

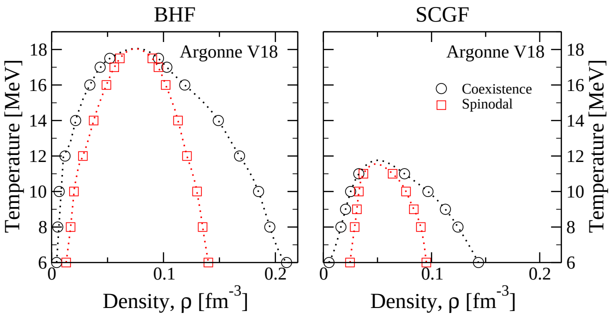
Liquid-gas phase transition



- Liquid-gas phase transition at low densities
- Spinodal zone related to mechanical instability
- Maxwell construction sets phase coexistence
- T_c can be determined from $p - \mu$ curves

A. Rios *et al.*, arxiv:0805.2318

Liquid-gas phase transition



Potential	Approach	T_f (MeV)	T_c (MeV)	ρ_c (fm^{-3})	p_c (MeV fm^{-3})	$\frac{p_c}{T_c \rho_c}$
Argonne V18	SCGF	9.5	11.6	0.05	0.08	0.14
	BHF	13.1	18.1	0.08	0.40	0.28
CDBONN	SCGF	14.4	18.5	0.11	0.40	0.20
	BHF	17.2	23.3	0.11	0.73	0.28

- $T_c^{BHF} > T_c^{SCGF} \Rightarrow$ different critical behaviour!
- No clusterization or three-body forces! $\Rightarrow$ upper estimate

A. Rios *et al.*, arxiv:0805.2318

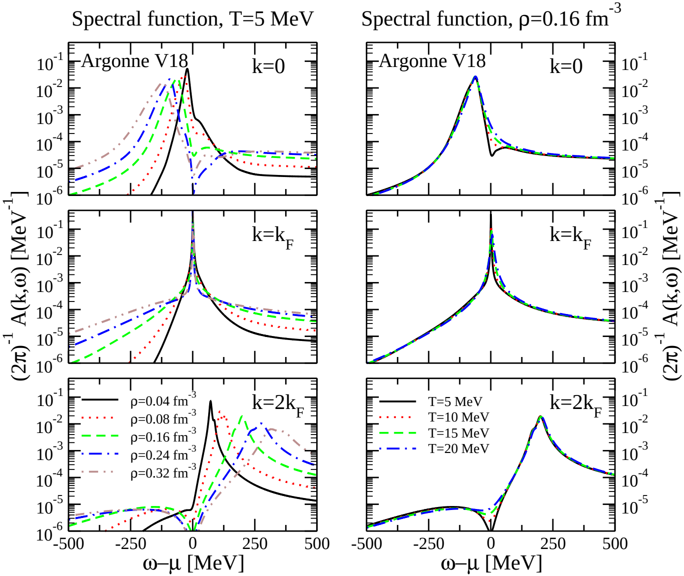
Outline

Neutron matter and the unitary limit

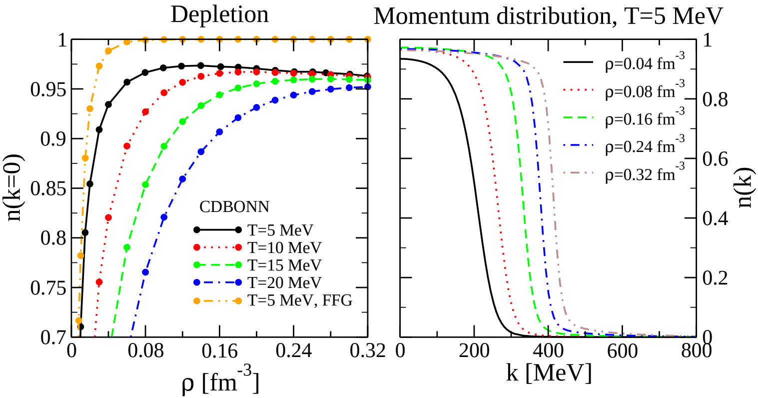
- $a_{nn} = -18$ fm $>> \frac{1}{k_F} \sim$ unitary regime, $a \to -\infty$
- $T = 0 \Rightarrow E = \gamma E_{FG}$, $\gamma \sim 0.44$
- $T \neq 0 \Rightarrow$ Universal thermodynamics, $x = T/\varepsilon_F$
- Connection to ultracold gases & Feshbach resonances
- SCGF good tool to study such systems:
  - Can handle strong interactions
  - Low densities
  - Smaller 3-body effects
  - No tensor force

Ho, PRL 92, 090402 (2004).

Spectral functions

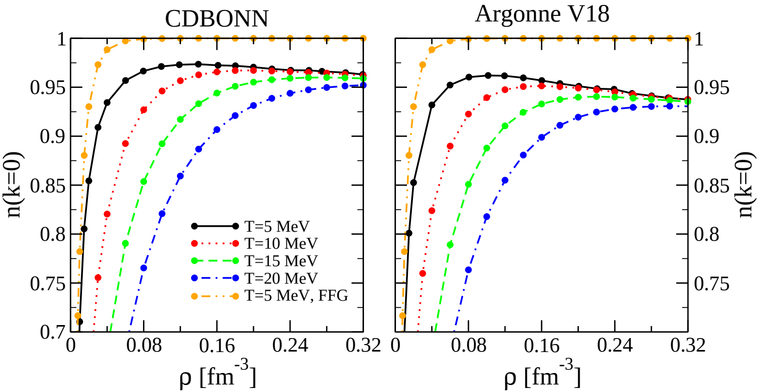


Depletion



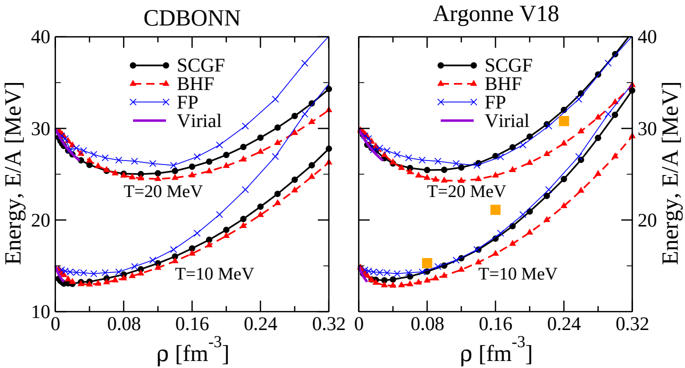
- Low density ⇒ classical limit ⇒ thermal correlations
- High density ⇒ degenerate limit ⇒ dynamical correlations

Depletion



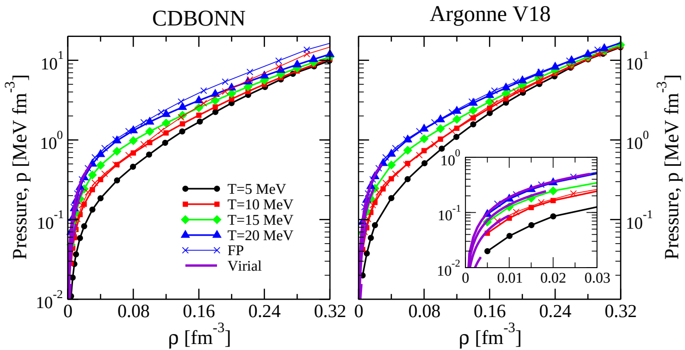
- Low density ⇒ classical limit ⇒ thermal correlations
- High density ⇒ degenerate limit ⇒ dynamical correlations

Energy per particle of neutron matter



- Strong potential dependence for $\rho > \rho_0$
- Deviations from BHF
- Agreement with variational calculations (FP)
- Agreement with virial

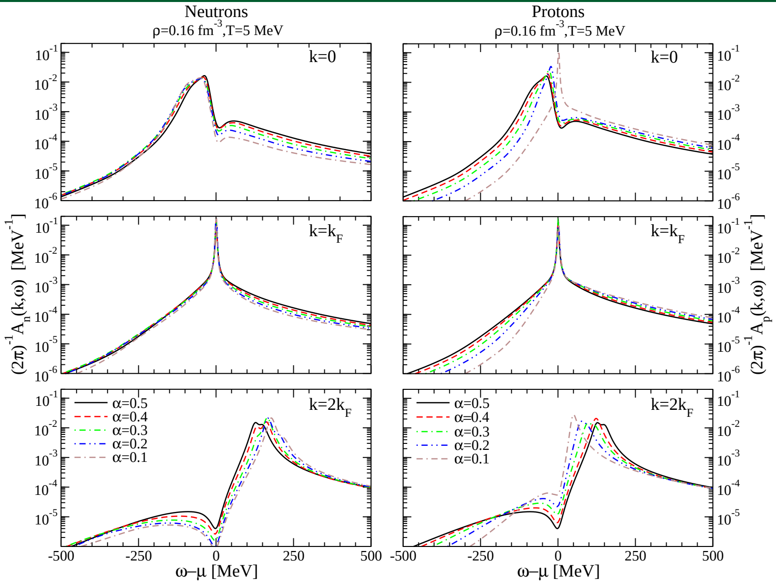
EoS for neutron matter



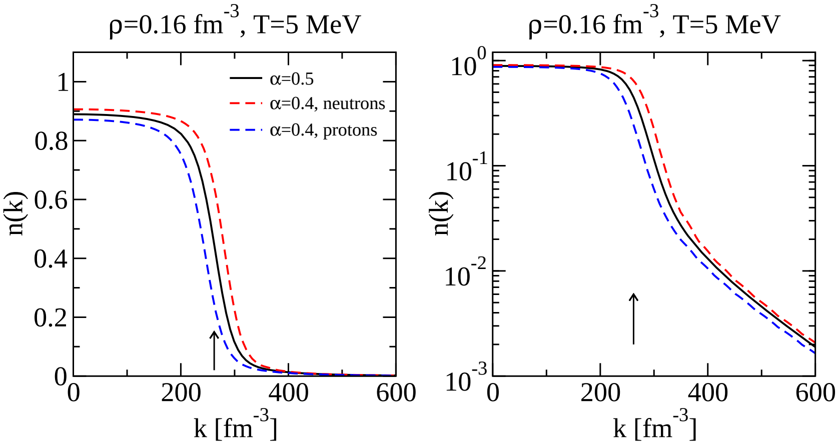
- Strong potential dependence for $\rho > \rho_0$
- Soft due to lack of TBF
- Agreement with variational calculations (FP)
- Agreement with virial

Outline

Spectral functions: asymmetric matter

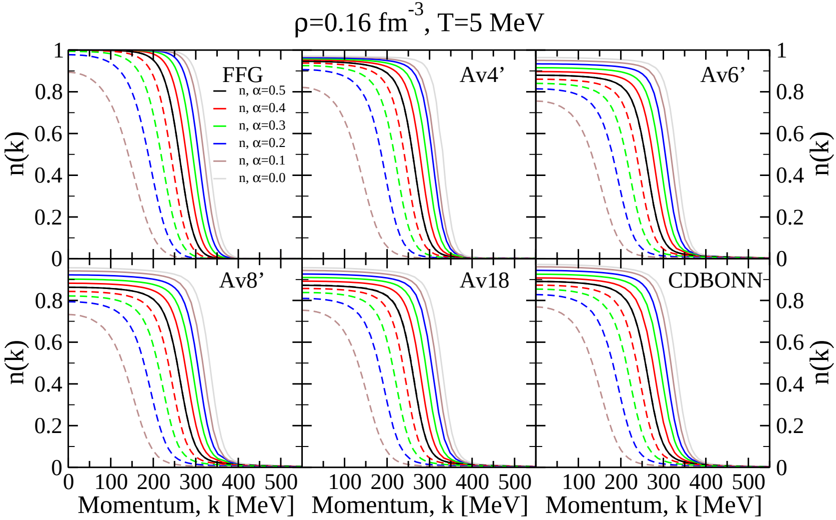


Momentum distribution: asymmetric matter



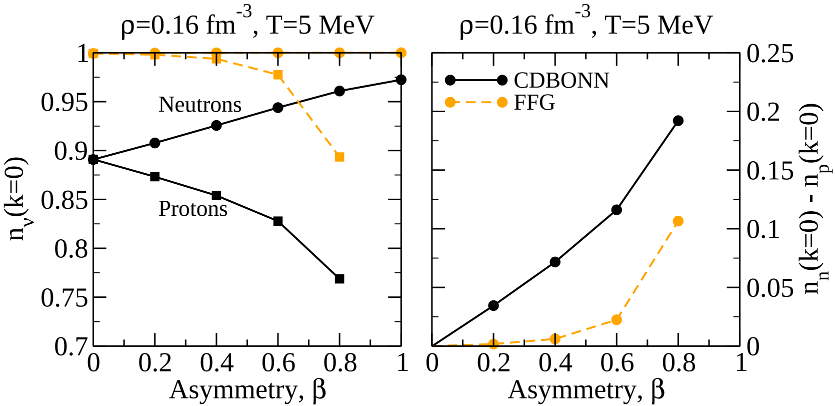
- Protons $\sim 5\%$ more depleted
- Important splitting already for finite nuclei!

Momentum distribution: potentials



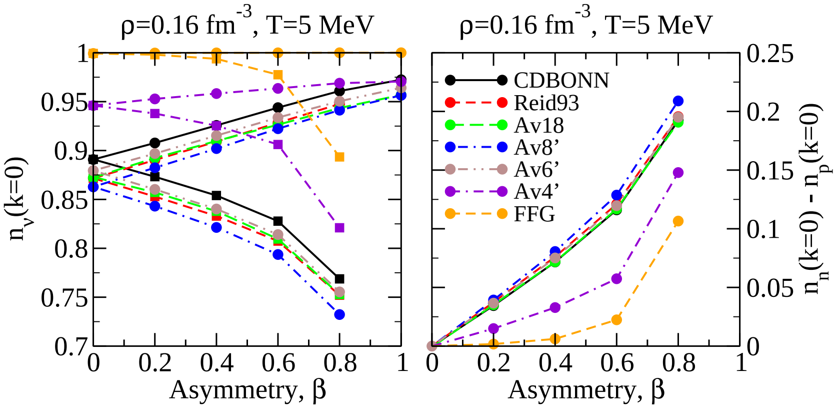
- Importance of tensor for isospin splitting!

Depletion and width at $k = 0$



- Protons are more "correlated" than neutrons
- Proton depletion has a thermal component
- Realistic NN potentials lie in a narrow band!

Depletion and width at $k = 0$

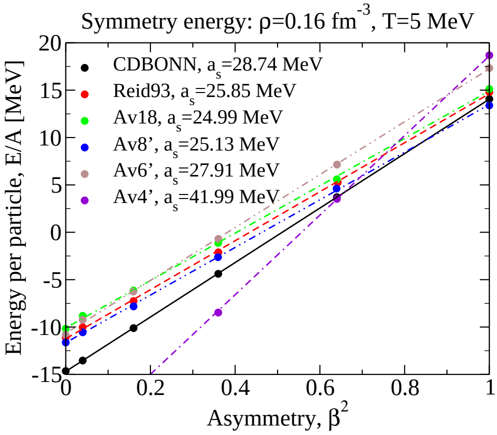


- Protons are more "correlated" than neutrons
- Proton depletion has a thermal component
- Realistic NN potentials lie in a narrow band!

Symmetry energy

$$e = \frac{\nu}{2} \sum_{\tau} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[\frac{k^2}{2m_\tau} + \omega \right] \mathcal{A}_\tau(k,\omega) f_\tau(\omega)$$

$$e(\rho,\beta) \sim e(\rho,\beta=0) + a_s(\rho)\beta^2$$



- Energy from GMK sum rule
- Low symmetry energy
- Corresponds to SRC component
- Off-shell structure?

Outline

Conclusions

- SCGF method is a consistent many-body framework to study nuclear many-body systems at finite temperature
- Microscopic one-body properties can be obtained from SCGF
- Thermodynamic properties can be computed consistently
- Interplay between thermal and dynamic correlations
- Symmetric matter leads to more repulsive results than BHF
- Neutron matter calculations show agreement with other methods
- Isospin asymmetry and tensor correlations are related in a non-trivial way
- Interaction dependent results!

Outlook

- Inclusion of three-body effects
- Calculation of two-body properties: $g(r), \mathcal{G}_{II}$ ...
- In-medium phase-shifts and cross sections
- Transport properties from Kubo relations
- ρ, T, α dependences of microscopic and bulk properties
- Pairing phase transition beyond quasi-particle approach
- Extension to time-dependent systems (nuclear reactions)

Thank you!