Superfluid Heat Conduction in the Neutron Star Crust

Sanjay Reddy
Los Alamos National Lab

Collaborators :
Deborah Aguilera
Vincenzo Cirigliano
Jose Pons
Rishi Sharma

arXiv:0807.4754

Superfluidity in the Crust Enhances Heat Conduction:

$$\kappa_{\rm sPh} = 1.5\times 10^{22} \left(\frac{T}{10^8\ {\rm K}}\right)^3 \left(\frac{0.1}{v_s}\right)^2 \left(\frac{\lambda_{\rm sPh}}{\rm cm}\right) \frac{\rm erg}{\rm cm\ s\ K}$$

Conventional Wisdom: Electrons dominate conduction

At neutron drip and T=10^8 K $\qquad \kappa_{\rm e} \simeq 10^{18} \frac{\rm ergs}{\rm cm\ s\ K}$

Superfluidity in the Crust Enhances Heat Conduction:

$$\kappa_{\rm sPh} = 1.5{\times}10^{22} \left(\frac{T}{10^8 \ {\rm K}}\right)^3 \left(\frac{0.1}{v_s}\right)^2 \left(\frac{\lambda_{\rm sPh}}{\rm cm}\right) \frac{\rm erg}{\rm cm \ s \ K}$$

Conventional Wisdom: Electrons dominate conduction

At neutron drip and T=10^8 K $\qquad \kappa_{\rm e} \simeq 10^{18} \frac{\rm ergs}{\rm cm \ s \ K}$

At B = 10^{14} G $\qquad \kappa_{\rm e}^{\perp} \simeq 10^{16} \frac{\rm ergs}{\rm cm \ s \ K}$

Superfluidity in the Crust Enhances Heat Conduction:

$$\kappa_{\rm sPh} = 1.5\times10^{22} \left(\frac{T}{10^8\ {\rm K}}\right)^3 \left(\frac{0.1}{v_s}\right)^2 \left(\frac{\lambda_{\rm sPh}}{\rm cm}\right) \frac{\rm erg}{\rm cm\ s\ K}$$

Conventional Wisdom: Electrons dominate conduction

At neutron drip and T=10^8 K $\qquad \kappa_{\rm e} \simeq 10^{18} \frac{\rm ergs}{\rm cm\ s\ K}$

At B = 10^{14} G $\qquad \kappa_{\rm e}^{\perp} \simeq 10^{16} \frac{\rm ergs}{\rm cm\ s\ K}$

Thermal Conductivity - Some Data

material	κ (ergs/cm·s·K)
air	0.00025
bronze	1.10
copper	4.01
diamond	8.95
graphite	19.5
helium (II)	>1000
ice cream (powder)	0.0005

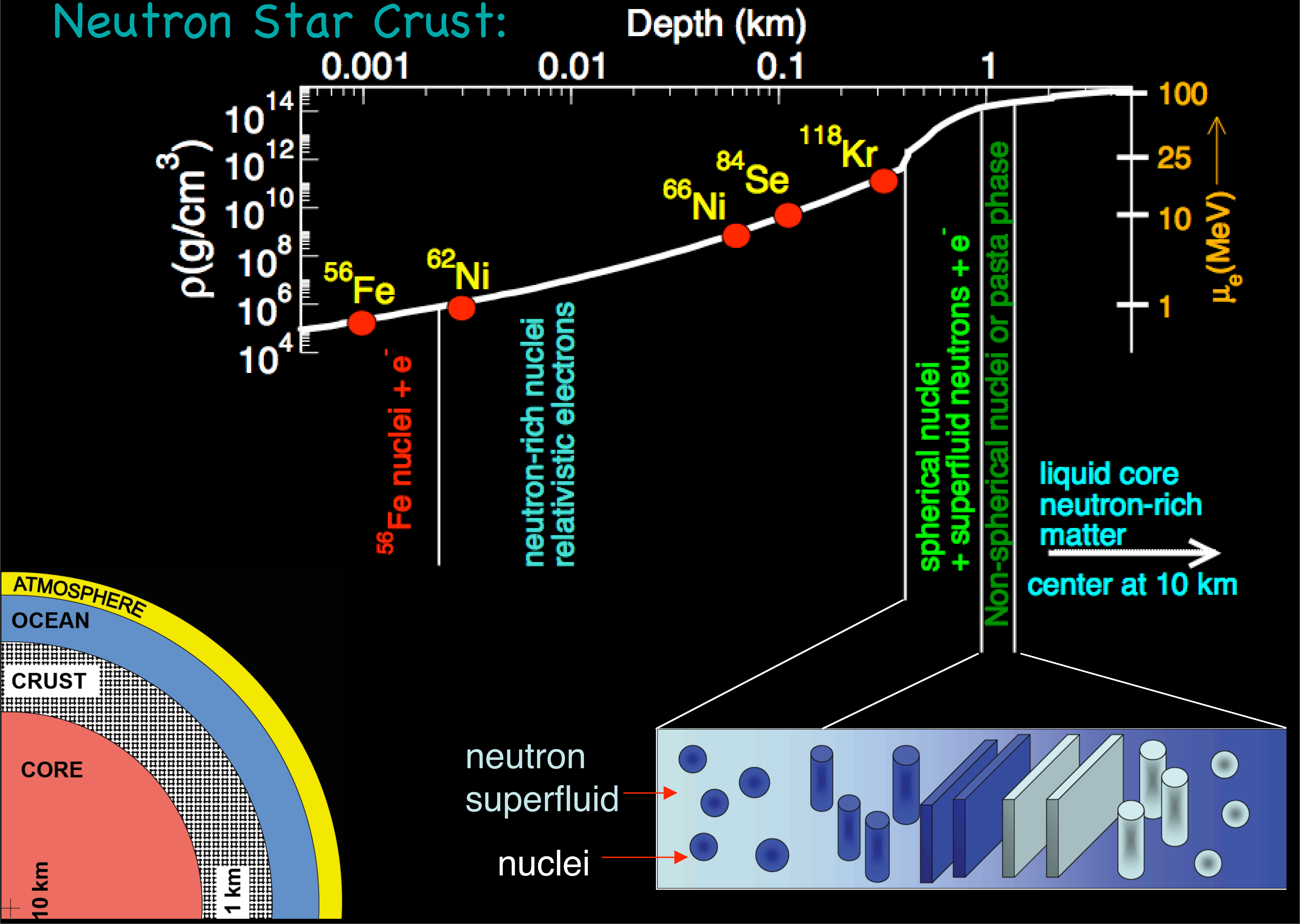
Thermal Conductivity - Some Data

material	κ (ergs/cm·s·K)
air	0.00025
bronze	1.10
copper	4.01
diamond	8.95
graphite	19.5
helium (II)	>1000
ice cream (powder)	0.0005

Thermal Conductivity - Some Data

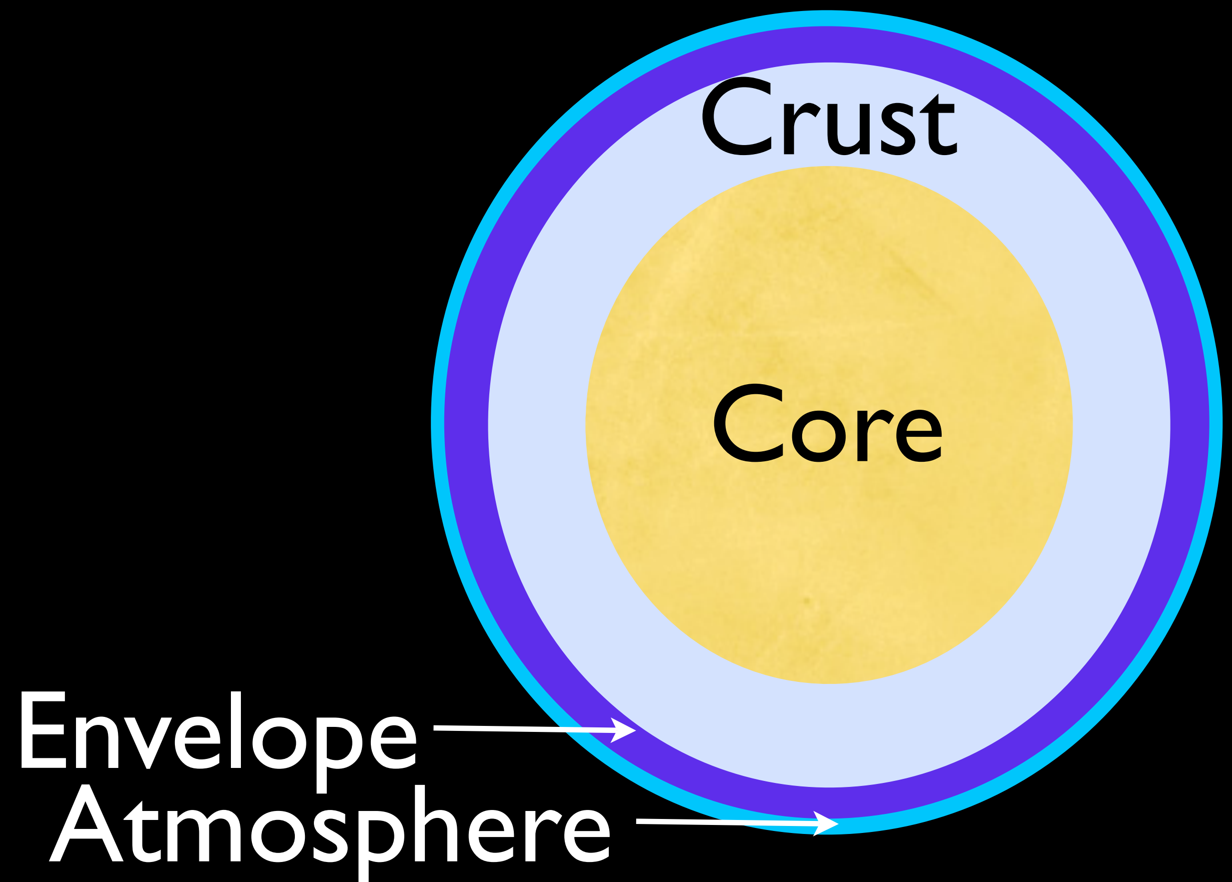
material	κ (ergs/cm·s·K)
air	0.00025
bronze	1.10
copper	4.01
diamond	8.95
graphite	19.5
helium (II)	>1000
ice cream (powder)	0.0005

Neutron Star Crust:



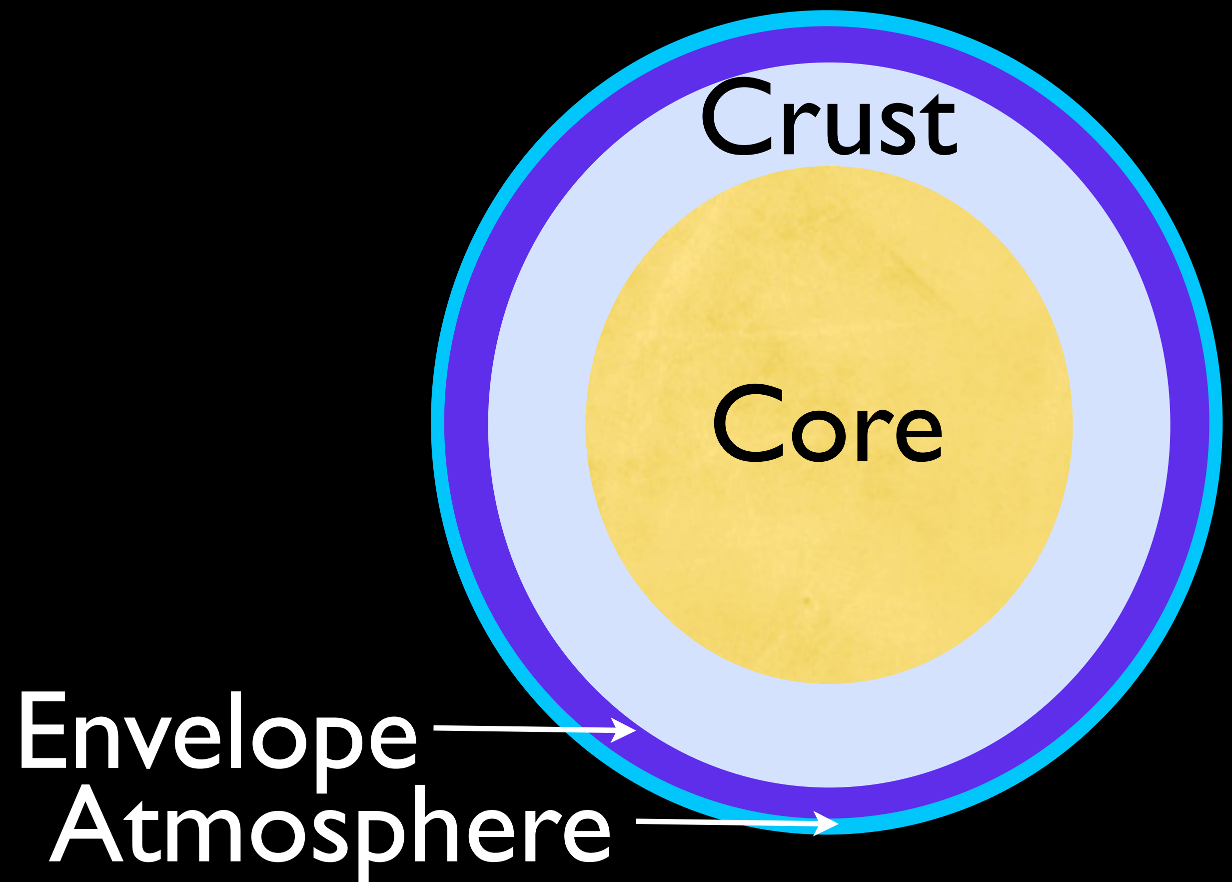
Neutron Star Thermal Evolution

- Long term cooling of isolated neutron stars.
- Thermal profiles of accreting neutron stars.
- Long term cooling of magnetars.
- Thermal relaxation in quiescence.



Neutron Star Thermal Evolution

- Long term cooling of isolated neutron stars.
- Thermal profiles of accreting neutron stars.
- Long term cooling of magnetars.
- Thermal relaxation in quiescence.



Temperature gradient in the crust plays a role

Conduction in Terrestrial Solids

$$
\begin{aligned}
Q &= \kappa \frac{\partial T}{\partial x} \\
\kappa &\simeq \frac{1}{3} C_V \, v \, \lambda
\end{aligned}
$$

$$
\begin{aligned}
C_e &\propto \mathcal{N}(0) \frac{T}{\epsilon_F} \\
C_{\mathrm{lPh}} &\propto \frac{T^3}{v_{\mathrm{lPh}}^3}
\end{aligned}
$$

Electrons dominate transport in conductors: $\frac{C_e \, v_e}{C_{\mathrm{lPh}} \, v_{\mathrm{lPh}}} \simeq 3$

In insulators lattice phonons dominate. Their conduction can be large.

Phonon Conduction in Solids

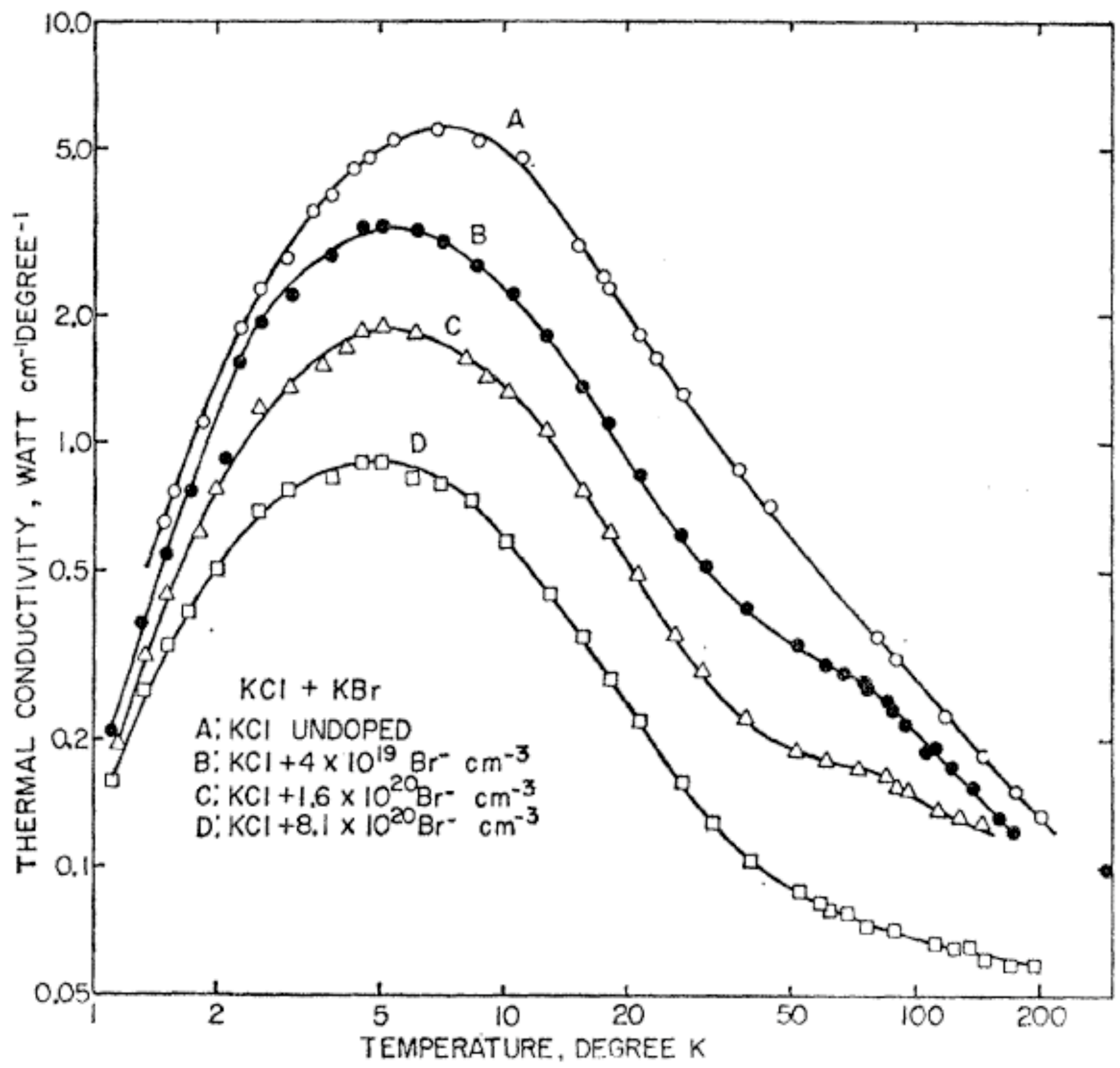
At low temperature phonons have very large mean free path.

Rayleigh scattering off impurities dominates at long-wavelength

$$\sigma_R = \pi r_0^6 \, q^4 \simeq \frac{A}{v^4} T^4$$

$$\lambda = \frac{1}{n_I \, \sigma_R}$$

$$\kappa \simeq \frac{1}{3} C_V \, v \, \lambda \simeq B \, \frac{v^2}{n_I \, T}$$



Baumann & Pohl (1967)
Ziman (1960)

Phonon Conduction in Solids

At low temperature phonons have very large mean free path.

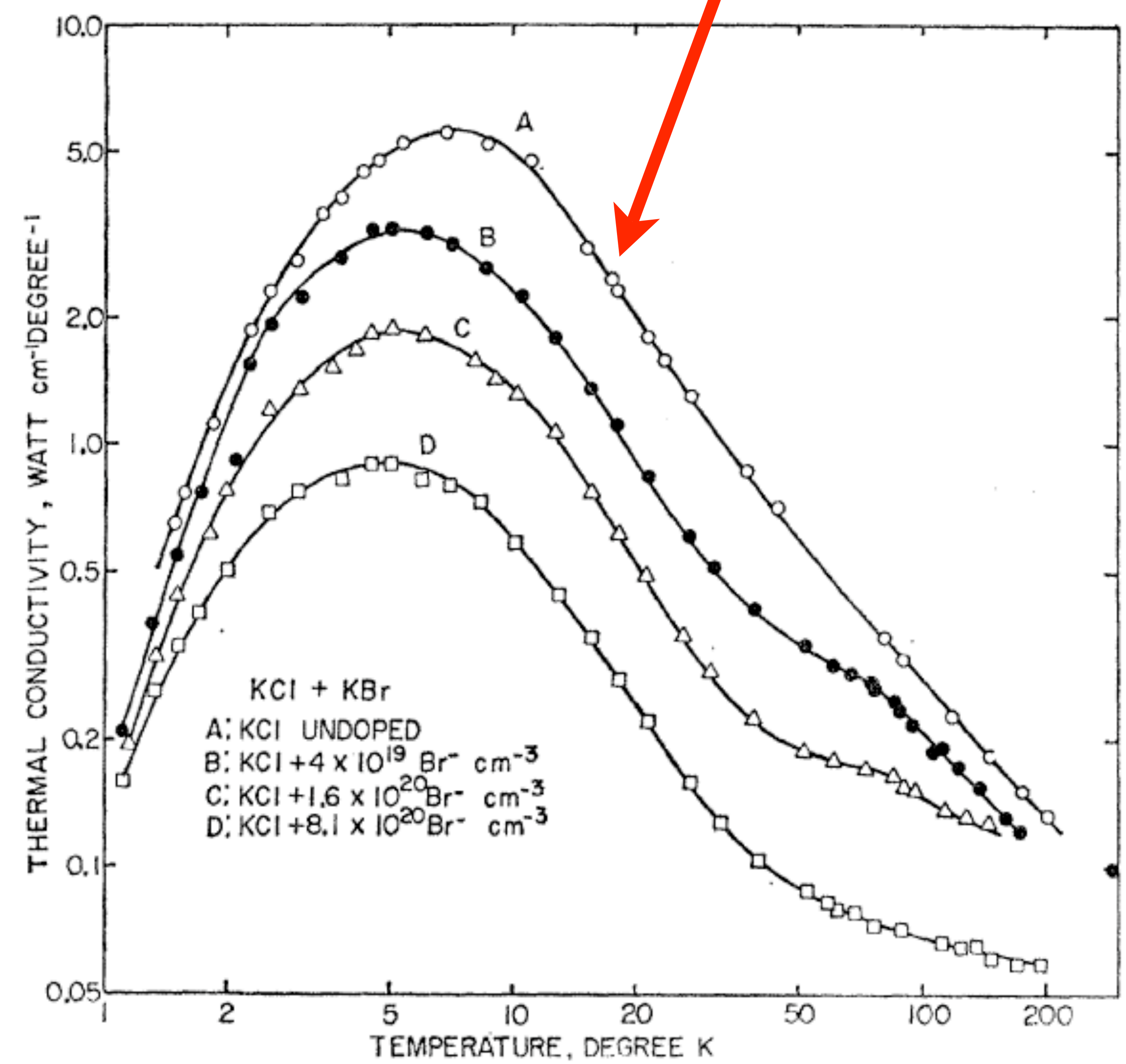
Rayleigh scattering off impurities dominates at long-wavelength

$$\sigma_R = \pi r_0^6 \, q^4 \simeq \frac{A}{v^4} T^4$$

$$\lambda = \frac{1}{n_I \, \sigma_R}$$

$$\kappa \simeq \frac{1}{3} C_V \, v \, \lambda \simeq B \, \frac{v^2}{n_I \, T}$$

Rayleigh Scattering



Baumann & Pohl (1967)
Ziman (1960)

Phonon Conduction in Solids

At low temperature phonons have very large mean free path.

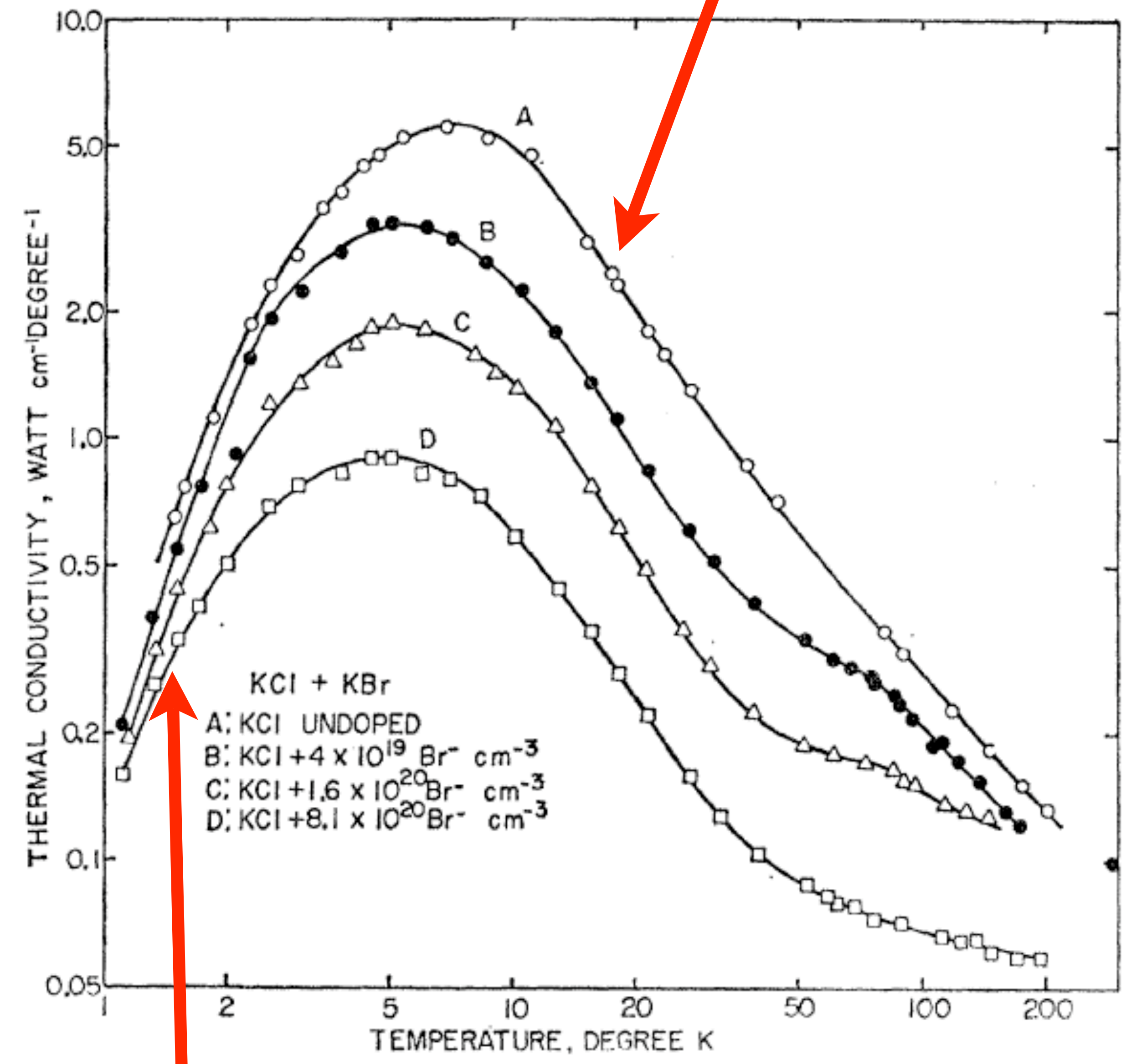
Rayleigh scattering off impurities dominates at long-wavelength

$$\sigma_R = \pi r_0^6\, q^4 \simeq \frac{A}{v^4}\, T^4$$

$$\lambda = \frac{1}{n_I\, \sigma_R}$$

$$\kappa \simeq \frac{1}{3} C_V\, v\, \lambda \simeq B\, \frac{v^2}{n_I\, T}$$

Rayleigh Scattering



Boundary Scattering

Baumann & Pohl (1967)
Ziman (1960)

Thermal Conduction in the Crust

Outer Crust:

- Liquid Phase: Electrons & Ions
- Solid Phase: Electrons & Phonons

- Electrons carry heat
- The electron mean free path is limited due to interactions with ions/phonons
- Fermi momentum of electrons $k_F > 1/a$

Electron Thermal Conduction

$$\kappa = \frac{\pi^2}{3}\, n_e \, \left(\frac{T}{\mu_e}\right) \, \lambda_e$$

Electrons are degenerate & relativistic

Electron mean free path set by collisions with ions.

- Energy transfer ~ T
- Momentum transfer ~ k_{Fe}

$$\lambda_e = \tau_e = \frac{1}{\nu_e}$$

Electrons or Phonons ?

$$\frac{\kappa_{\mathrm{el}}}{\kappa_{\mathrm{lPh}}} = \frac{C_{\mathrm{el}}}{C_{\mathrm{lPh}}} \frac{1}{c} \frac{\lambda_{\mathrm{e}}}{\lambda_{\mathrm{lPh}}}$$

Typically electrons dominate
- unless there is a large magnetic field.

Magnetic field suppresses transverse conduction

$$\kappa_{\perp} = \frac{\kappa_{\|}}{1 + (\omega_g \; \tau_e)^2}$$

$$\kappa_{\|} = \kappa_{\mathrm{el}}(B=0)$$

$\omega_g = \frac{eB}{\mu_e}$ =Gyrofrequency

τ_{e} = Collision time

Canuto and Ventura (1977)
Uripin & Yakovlev (1980)

Electrons or Phonons ?

$$\frac{\kappa_{\rm el}}{\kappa_{\rm lPh}} = \frac{C_{\rm el}}{C_{\rm lPh}} \; \frac{1}{c} \; \frac{\lambda_{\rm e}}{\lambda_{\rm lPh}} \simeq \frac{\mu_e^2}{T^2} \; \frac{1}{c} \; \frac{\lambda_{\rm e}}{\lambda_{\rm lPh}} \gg 1$$

Typically electrons dominate
- unless there is a large magnetic field.

Magnetic field suppresses transverse conduction

$$\kappa_\perp = \frac{\kappa_\|}{1 + (\omega_g \; \tau_e)^2}$$

$$\kappa_\| = \kappa_{\rm el}(B = 0)$$

$\omega_g = \dfrac{eB}{\mu_e}$ =Gyrofrequency

$\tau_{\rm e}$ = Collision time

Canuto and Ventura (1977)
Uripin & Yakovlev (1980)

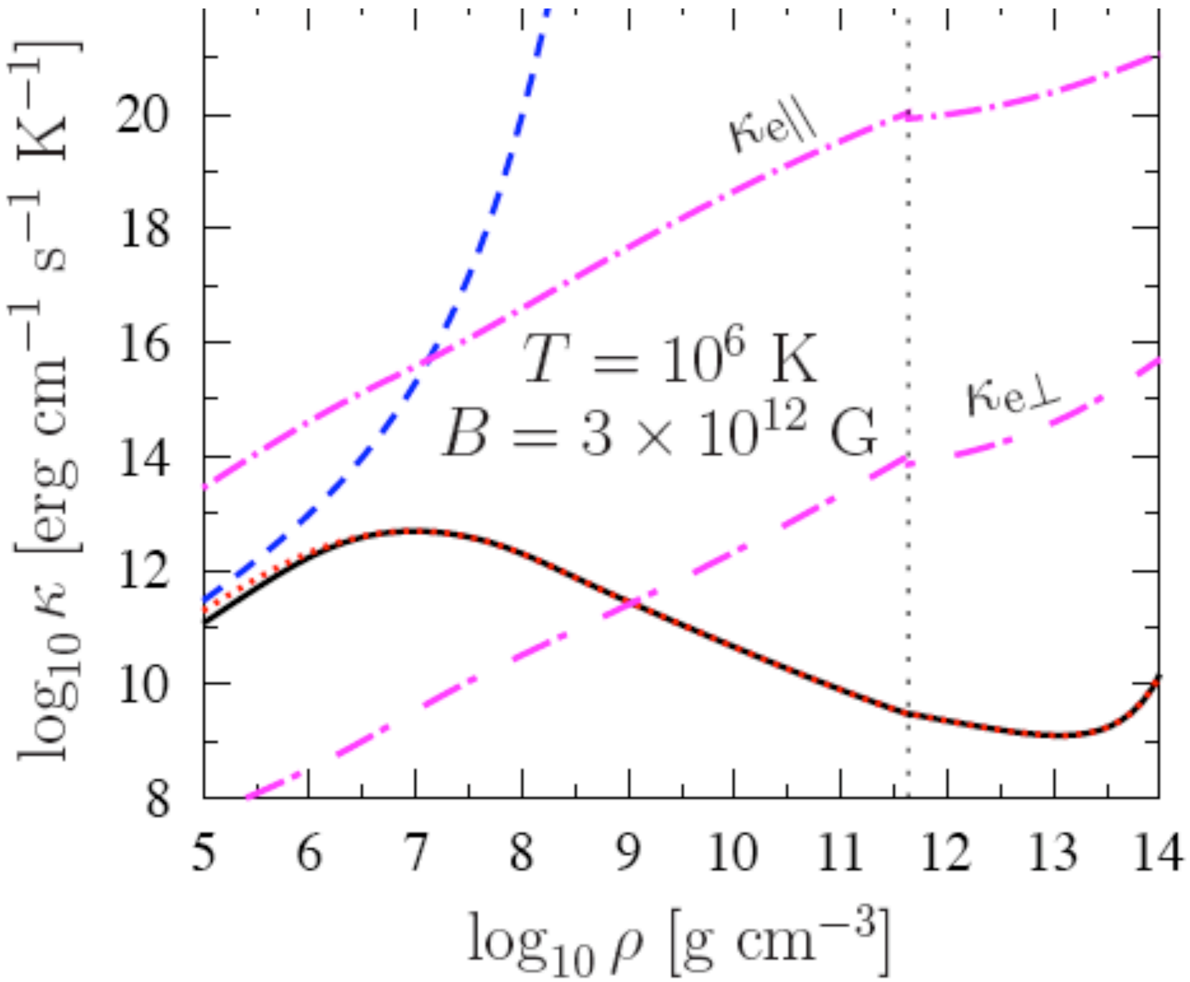
Phonon Conduction in the Outer Crust

Lattice Phonons have large mean free paths.

$$\lambda_{\mathrm{lPh}} \gg \lambda_{\mathrm{e}}$$

Mean free path set by:

1. Impurity scattering
2. Absorption by Electrons



Perez-Azorin (2006) Chugunov and Haensel (2007)

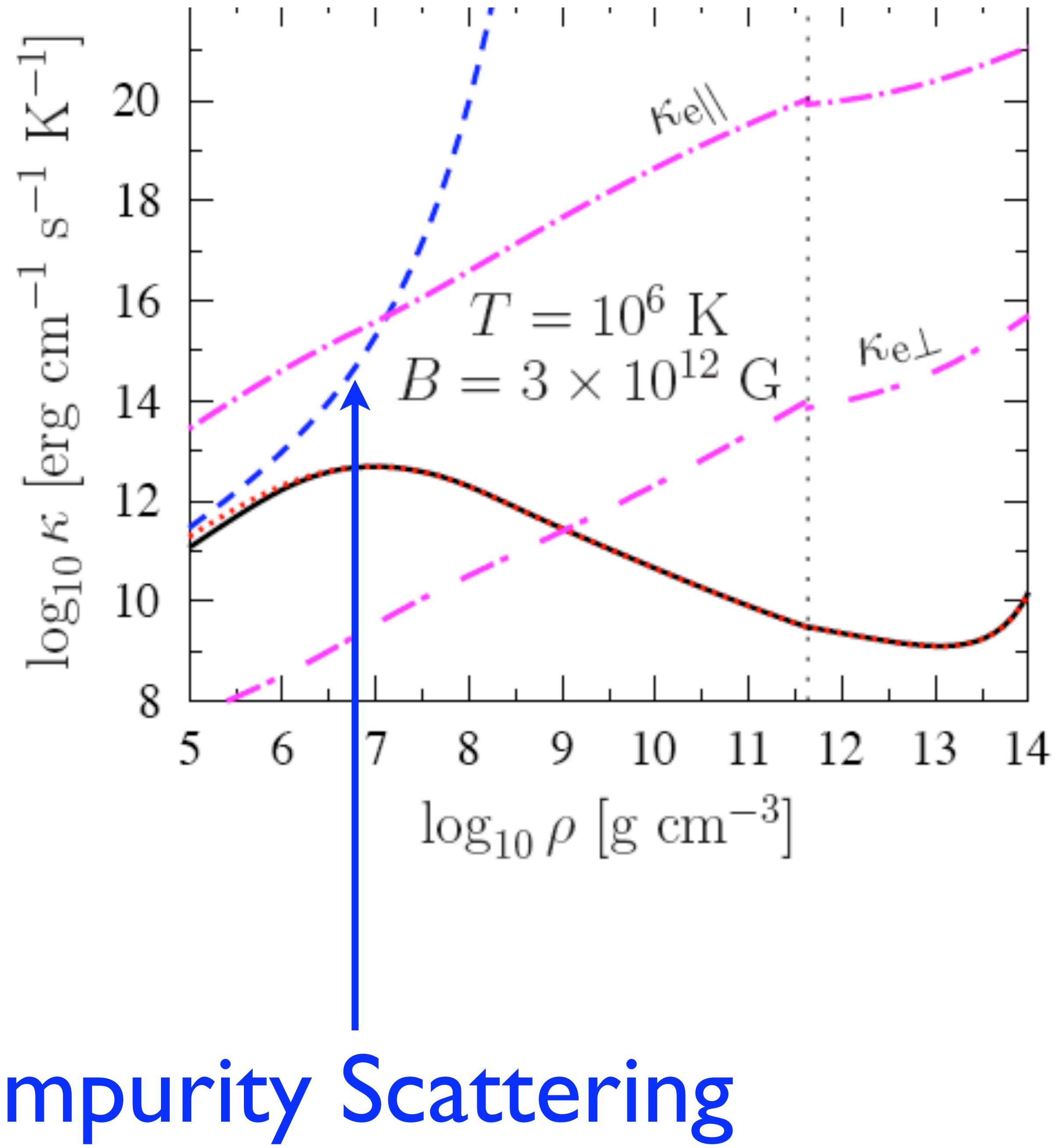
Phonon Conduction in the Outer Crust

Lattice Phonons have large mean free paths.

$$\lambda_{\mathrm{lPh}} \gg \lambda_{\mathrm{e}}$$

Mean free path set by:

1. Impurity scattering
2. Absorption by Electrons



Perez-Azorin (2006) Chugunov and Haensel (2007)

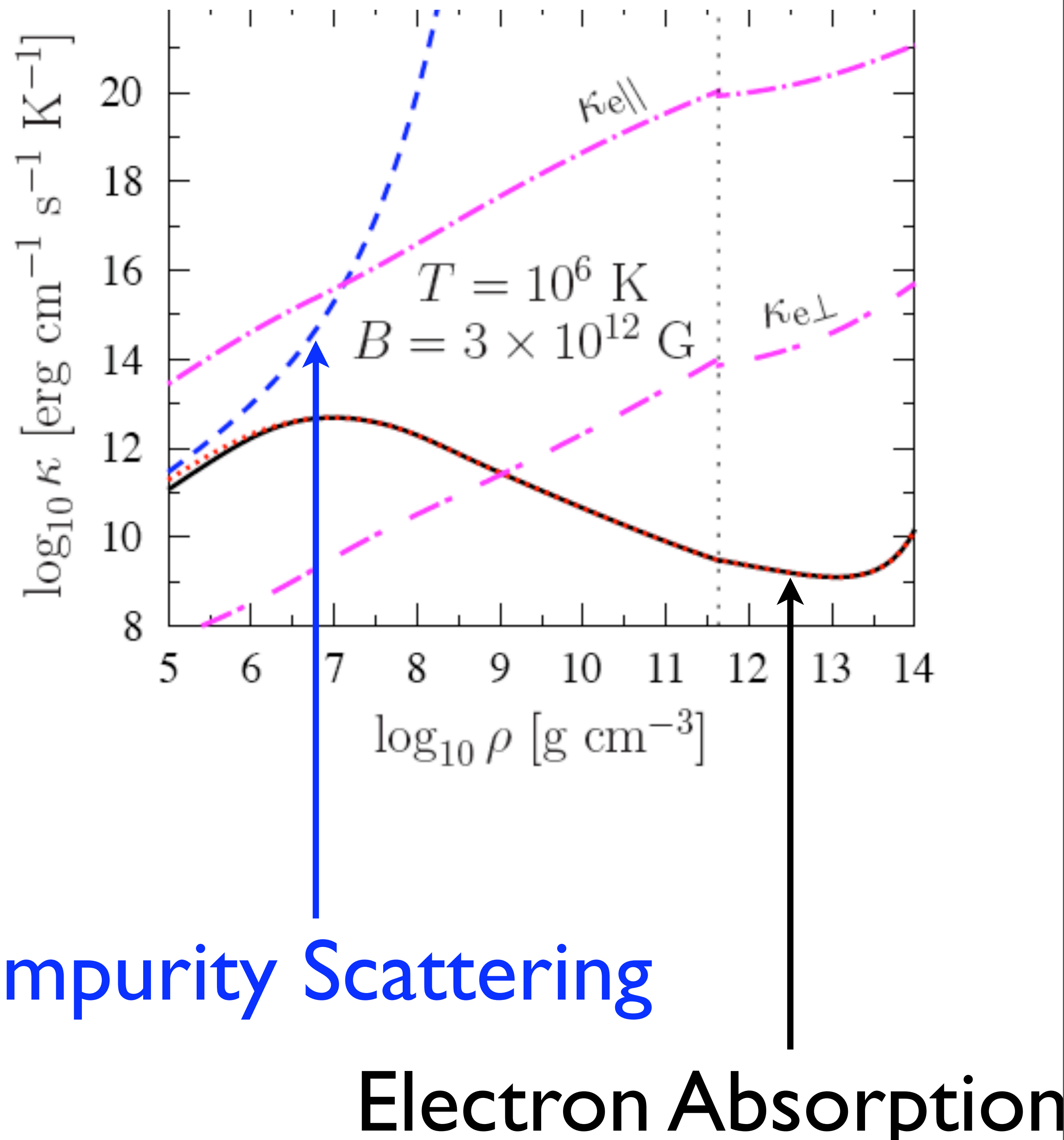
Phonon Conduction in the Outer Crust

Lattice Phonons have large mean free paths.

$$\lambda_{\mathrm{lPh}} \gg \lambda_{\mathrm{e}}$$

Mean free path set by:

1. Impurity scattering
2. Absorption by Electrons



Perez-Azorin (2006) Chugunov and Haensel (2007)

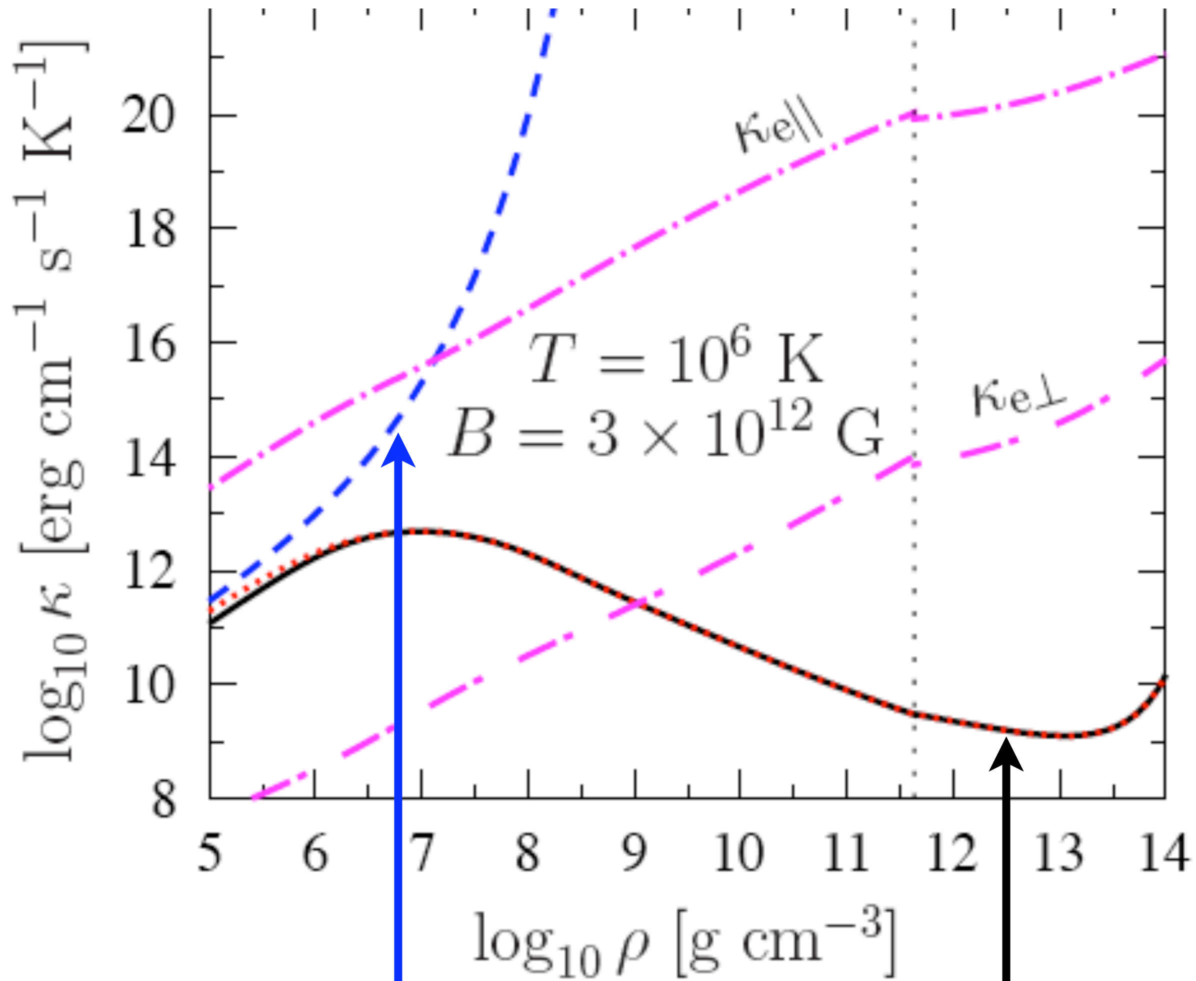
Phonon Conduction in the Outer Crust

Lattice Phonons have large mean free paths.

$$\lambda_{\mathrm{lPh}} \gg \lambda_{\mathrm{e}}$$

Mean free path set by:

1. Impurity scattering
2. Absorption by Electrons



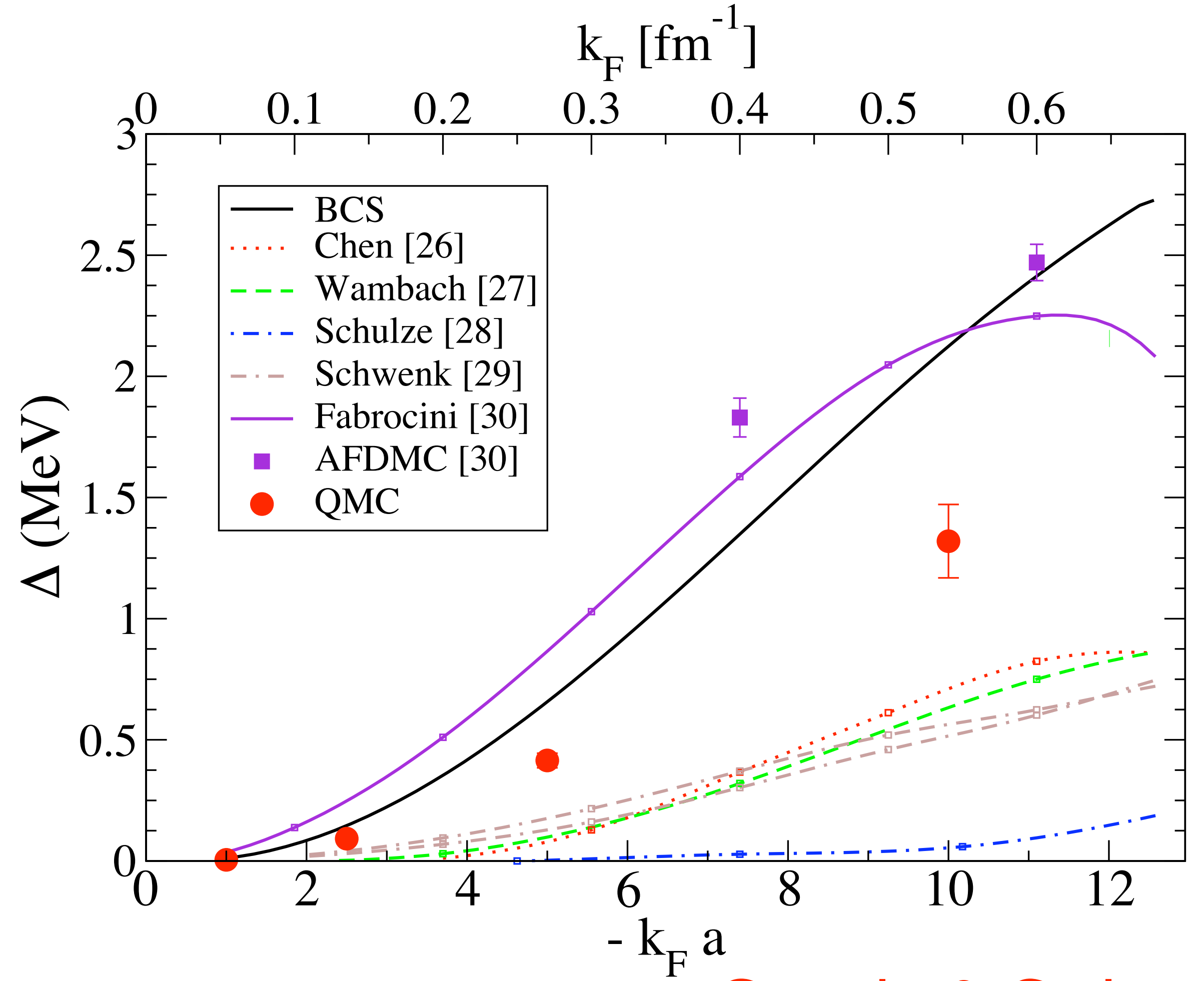
Impurity Scattering

Electron Absorption

Perez-Azorin (2006) Chugunov and Haensel (2007)

Heat Transport in the Inner Crust

- Neutron matter in the crust is superfluid.
- Neutron particle-hole excitations are gapped



Gezerlis & Carlson (2008)

Low energy degrees of freedom:

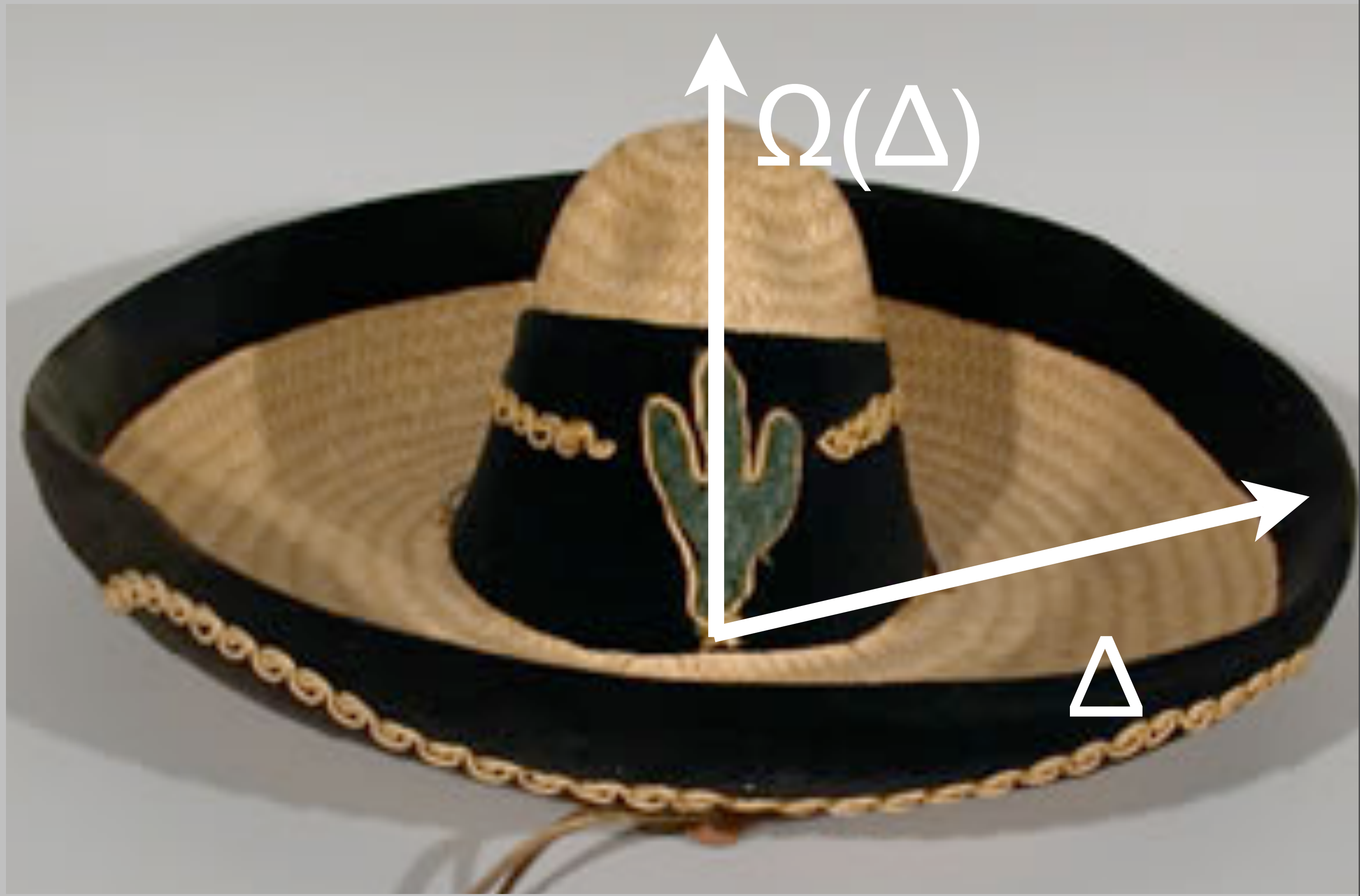
1. Electrons
2. Lattice Phonons (1 long. + 2 Trans.)
3. Superfluid Phonons

Pairing in neutron matter

Attractive interactions destabilize the Fermi surface:

$$
\begin{aligned}
H &= \sum_{k,s=\uparrow,\downarrow} \left(\frac{k^2}{2m} - \mu\right) a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s} \\
\Delta &= g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle
\end{aligned}
$$

Cooper pairs leads to superfluidity



Energy gap for fermions:

$$E(p) = \sqrt{\left(\frac{p^2}{2M} - \mu\right)^2 + \Delta^2}$$

New collective mode:
Superfluid Phonon

$$\omega(k) = v_s\ k$$

sPh mean free path

Rayleigh Scattering $\sigma_R = \pi\ r_0^2 \left(\frac{q^4 r_0^4}{1 + q^4 r_0^4} \right)$

r_0 =Typical nuclear radii
q = sPh momentum

$q\ r_0 \simeq 10^{-3} \left(\frac{T}{10^7 \mathrm{K}} \right) \ll 1$

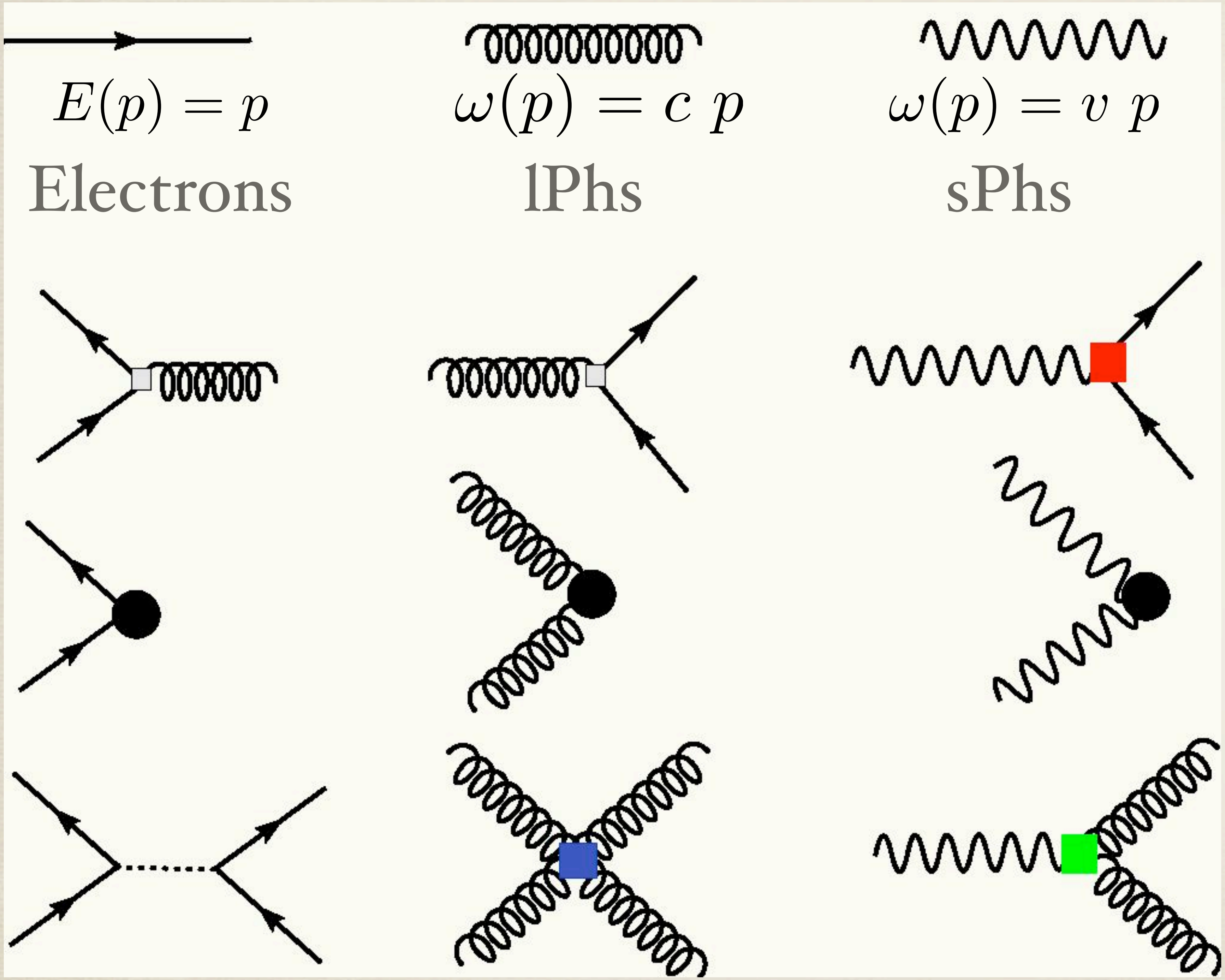
Scattering dominated by impurities: $\lambda_{\mathrm{R}} = \frac{1}{n_I\ \sigma_R} = \frac{v_s^4}{81\ \pi\ n_I\ r_0^6\ T^4}$

Very large mean free path!

$$\lambda_{\mathrm{Ray}} = 450 \left(\frac{v_s}{0.1} \right)^4 \left(\frac{x}{10} \right)^3 \left(\frac{10\ \mathrm{fm}}{r_0} \right)^3 T_7^{-4}\ \mathrm{cm}$$

If only impurity scattering is relevant: $\kappa_{\mathrm{sPh}}(T = 10^8 \mathrm{K}) \simeq 10^{21} \frac{\mathrm{ergs}}{\mathrm{cm\ s\ K}}$

Dissipative Processes



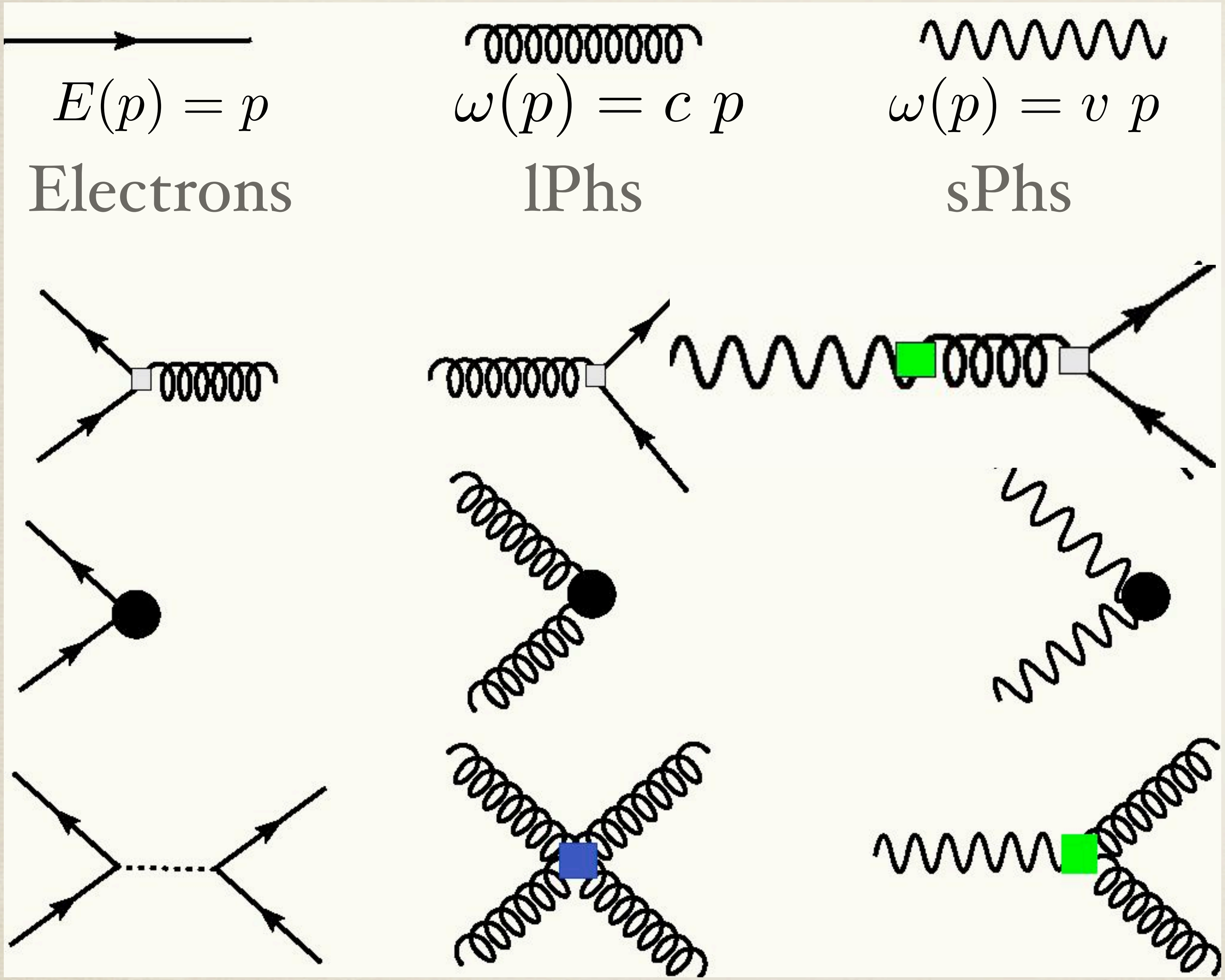
$$\begin{aligned} c &\simeq 0.01 - 0.1 \\ v &\simeq 0.01 - 0.1 \end{aligned}$$

Electron-phonon processes

Impurity (Rayleigh) scattering

Multi electron and phonon processes

Dissipative Processes



$$
\begin{aligned}
c &\simeq 0.01 - 0.1 \\
v &\simeq 0.01 - 0.1
\end{aligned}
$$

Electron-phonon processes

Impurity (Rayleigh) scattering

Multi electron and phonon processes

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EFT}}^{\mathrm{sPh}} &= \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\,\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots \\
\mathcal{L}_{\mathrm{EFT}}^{\mathrm{lPh}} &= \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\,\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots
\end{aligned}
$$

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$\mathcal{L}_{\mathrm{EFT}}^{\mathrm{sPh}} = \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\;\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots$$

$$\mathcal{L}_{\mathrm{EFT}}^{\mathrm{lPh}} = \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\;\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots$$

kinetic terms

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$\mathcal{L}_{\mathrm{EFT}}^{\mathrm{sPh}} = \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\,\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots$$

$$\mathcal{L}_{\mathrm{EFT}}^{\mathrm{lPh}} = \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\,\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots$$

kinetic terms

coupling to Fermions

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$
\begin{aligned}
\mathcal{L}_{\rm EFT}^{\rm sPh} &= \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\;\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots \\
\mathcal{L}_{\rm EFT}^{\rm lPh} &= \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\;\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots
\end{aligned}
$$

kinetic terms

coupling to Fermions

self-coupling

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$\mathcal{L}_{\rm EFT}^{\rm sPh} = \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\,\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots$$

$$\mathcal{L}_{\rm EFT}^{\rm lPh} = \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\,\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots$$

kinetic terms

coupling to Fermions

self-coupling

$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\,\partial_o\phi\,\partial_i\xi_i + \frac{1}{\Lambda^2}\,\partial_o\phi\,\partial_i\xi^i\partial_i\xi^i + \cdots$$

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$\mathcal{L}^{\rm sPh}_{\rm EFT} = \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\;\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots$$

$$\mathcal{L}^{\rm lPh}_{\rm EFT} = \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\;\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots$$

kinetic terms

coupling to Fermions

self-coupling

$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\,\partial_o\phi\;\partial_i\xi_i + \frac{1}{\Lambda^2}\,\partial_o\phi\;\partial_i\xi^i\partial_i\xi^i + \cdots$$

lPh-sPh mixing

Low Energy Effective Theory

Phonon coupling is derivative - Low momentum phonons interact weakly !

$$\mathcal{L}_{\rm EFT}^{\rm sPh} = \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\,\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots$$

$$\mathcal{L}_{\rm EFT}^{\rm lPh} = \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\,\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots$$

kinetic terms

coupling to Fermions

self-coupling

$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\,\partial_o\phi\,\partial_i\xi_i + \frac{1}{\Lambda^2}\,\partial_o\phi\,\partial_i\xi^i\partial_i\xi^i + \cdots$$

lPh-sPh mixing

sPh→2 lPh

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\mathrm{el-Ion}} = \int d^3x \int d^3y \; V(x-y) \; \psi^\dagger(x)\psi(x) \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\mathrm{Ion}} + \delta\rho(y)$$

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\rm el-Ion} = \int d^3x \int d^3y \; V(x-y)\; \psi^\dagger(x)\psi(x) \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\rm Ion} + \delta\rho(y)$$

Fluctuation in density due to displacement field : $\vec{d}(y)$

$$\delta\rho(y) = -n_{\rm Ion}\; \nabla\cdot\vec{d}(y) + \cdots$$

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\text{el}-\text{Ion}} = \int d^3x \int d^3y \, V(x-y) \, \psi^\dagger(x)\psi(x) \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\text{Ion}} + \delta\rho(y)$$

Fluctuation in density due to displacement field : $\vec{d}(y)$

$$\delta\rho(y) = -n_{\text{Ion}} \nabla \cdot \vec{d}(y) + \cdots$$

$$\vec{d}(y) = \frac{1}{\sqrt{n_{\text{Ion}} \, AM}} \boxed{\vec{\xi}(x)}$$

Canonically normalized lattice phonon field

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\mathrm{el-Ion}} = \int d^3x \int d^3y \, V(x-y) \, \psi^\dagger(x)\psi(x) \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\mathrm{Ion}} + \delta\rho(y)$$

Fluctuation in density due to displacement field : $\vec{d}(y)$

$$\delta\rho(y) = -n_{\mathrm{Ion}} \, \nabla \cdot \vec{d}(y) + \cdots$$

$$\vec{d}(y) = \frac{1}{\sqrt{n_{\mathrm{Ion}} \, AM}} \boxed{\vec{\xi}(x)}$$

Canonically normalized lattice phonon field

$$\mathcal{H}_{\mathrm{el-lPh}} = \boxed{\frac{4\pi Z e^2}{q_{\mathrm{TF}}^2} \, \frac{n_{\mathrm{Ion}}}{\sqrt{n_{\mathrm{Ion}} AM}}} \int d^3x \, \psi^\dagger(x)\psi(x) \, \nabla \cdot \vec{\xi}(x)$$

Electron-Phonon Coupling

Fetter & Walecka

$$\mathcal{H}_{\mathrm{el-Ion}} = \int d^3x \int d^3y \; V(x-y) \; \psi^\dagger(x)\psi(x) \boxed{\Psi^\dagger(y)\Psi(y)}$$

$$\Psi^\dagger(y)\Psi(y) = n_{\mathrm{Ion}} + \delta\rho(y)$$

Fluctuation in density due to displacement field : $\vec{d}(y)$

$$\delta\rho(y) = -n_{\mathrm{Ion}} \; \nabla \cdot \vec{d}(y) + \cdots$$

$$\vec{d}(y) = \frac{1}{\sqrt{n_{\mathrm{Ion}} \; AM}} \boxed{\vec{\xi}(x)}$$

Canonically normalized lattice phonon field

$$\mathcal{H}_{\mathrm{el-lPh}} = \boxed{\frac{4\pi Z e^2}{q_{\mathrm{TF}}^2} \; \frac{n_{\mathrm{Ion}}}{\sqrt{n_{\mathrm{Ion}} AM}}} \int d^3x \; \psi^\dagger(x)\psi(x) \; \nabla \cdot \vec{\xi}(x)$$

$$\mathcal{H}_{\mathrm{el-lPh}} = \boxed{\frac{1}{f_{\mathrm{el-ph}}}} \int d^3x \; \psi^\dagger(x)\psi(x) \; \nabla \cdot \vec{\xi}(x)$$

Neutron-IPh Interaction

$$\mathcal{H}_{\rm n-Ion} = \int \; d^3x \int d^3y \; V_{\rm n-A}(x-y) \; \psi_n^\dagger(x)\psi_n(x) \; \Psi^\dagger(y)\Psi(y)$$

$$V_{\rm n-A} = \frac{2\pi \; a_{\rm n-A}}{A \; M} \; \delta^3(x)$$

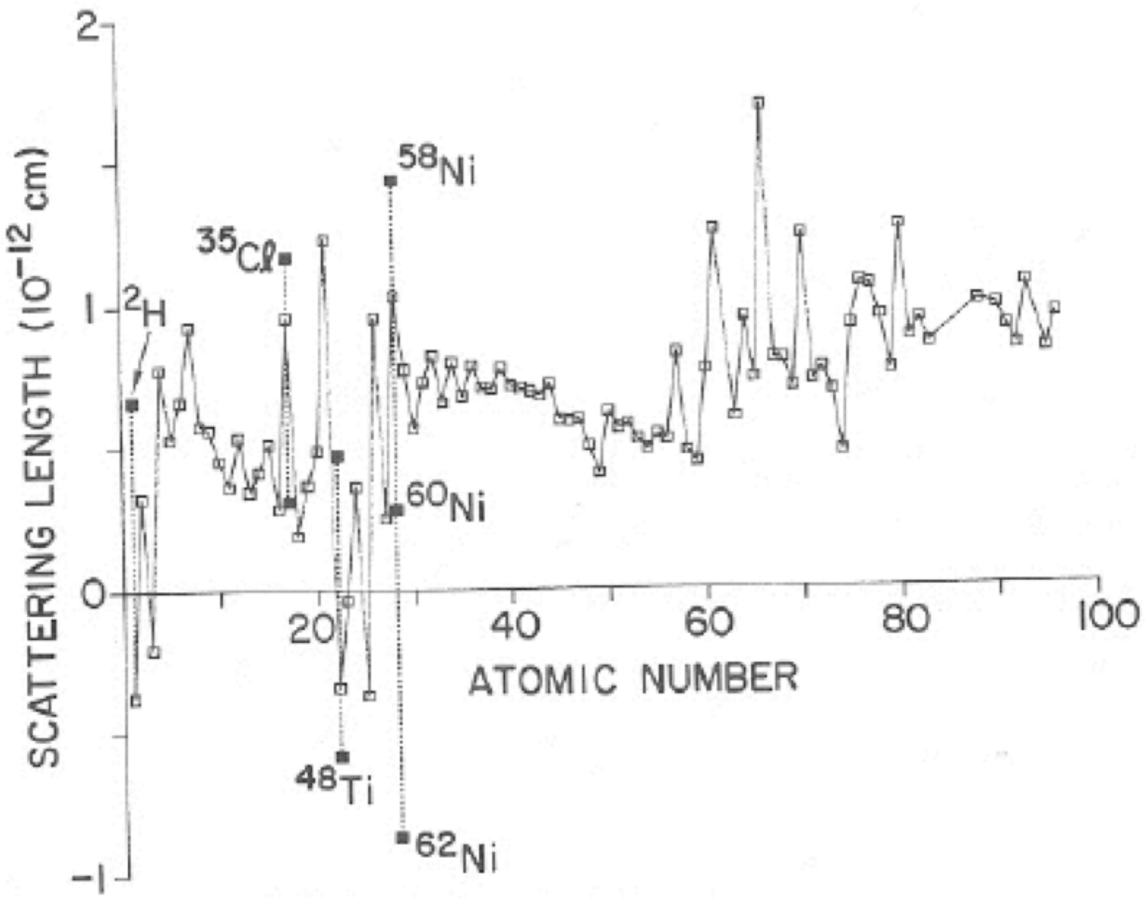
Low-energy neutron-nucleus potential (Fermi Potential)

Neutron-IPh Interaction

$$\mathcal{H}_{\mathrm{n-Ion}} = \int \ d^3x \int d^3y \ V_{\mathrm{n-A}}(x-y) \ \psi_n^\dagger(x)\psi_n(x) \ \Psi^\dagger(y)\Psi(y)$$

$$V_{\mathrm{n-A}} = \frac{2\pi \ a_{\mathrm{n-A}}}{A \ M} \ \delta^3(x)$$

Low-energy neutron-nucleus potential (Fermi Potential)

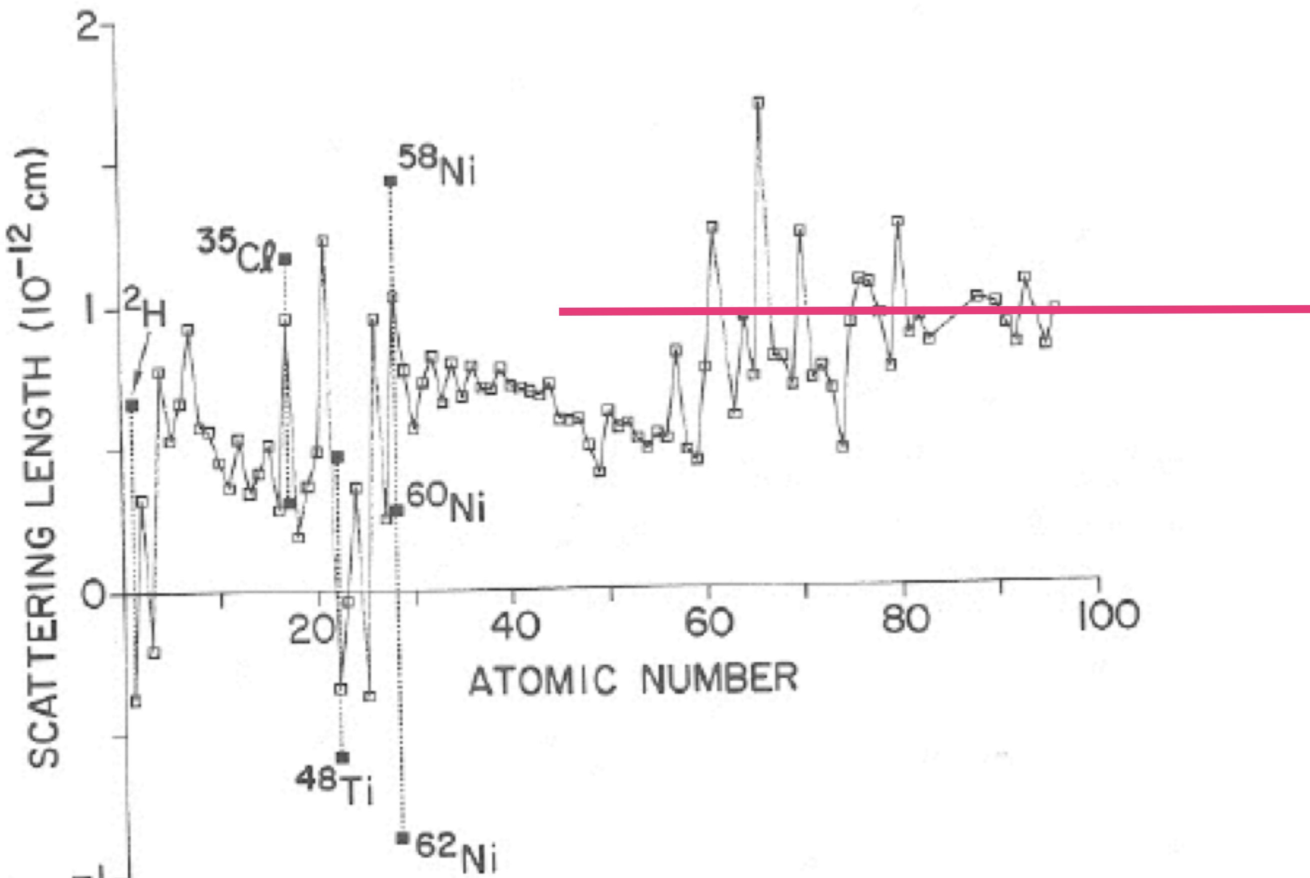


Neutron-IPh Interaction

$$\mathcal{H}_{\mathrm{n-Ion}} = \int\ d^3x \int d^3y\ V_{\mathrm{n-A}}(x-y)\ \psi_n^\dagger(x)\psi_n(x)\ \Psi^\dagger(y)\Psi(y)$$

$$V_{\mathrm{n-A}} = \frac{2\pi\ a_{\mathrm{n-A}}}{A\ M}\ \delta^3(x)$$

Low-energy neutron-nucleus potential (Fermi Potential)

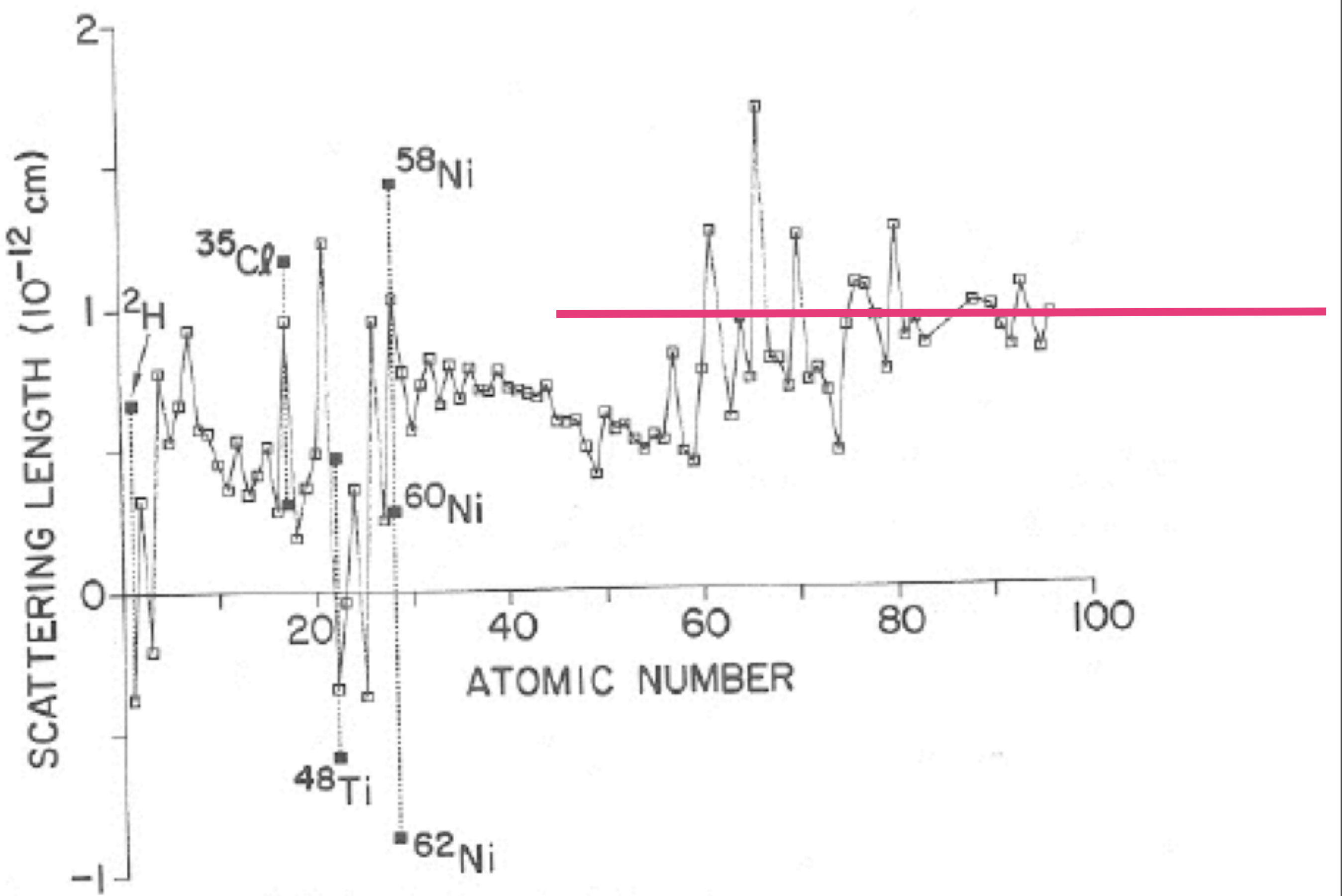


Neutron-lPh Interaction

$$\mathcal{H}_{\rm n-Ion} = \int d^3x \int d^3y \; V_{\rm n-A}(x-y) \; \psi_n^\dagger(x)\psi_n(x) \; \Psi^\dagger(y)\Psi(y)$$

$$V_{\rm n-A} = \frac{2\pi \; a_{\rm n-A}}{A \; M} \; \delta^3(x)$$

Low-energy neutron-nucleus potential (Fermi Potential)



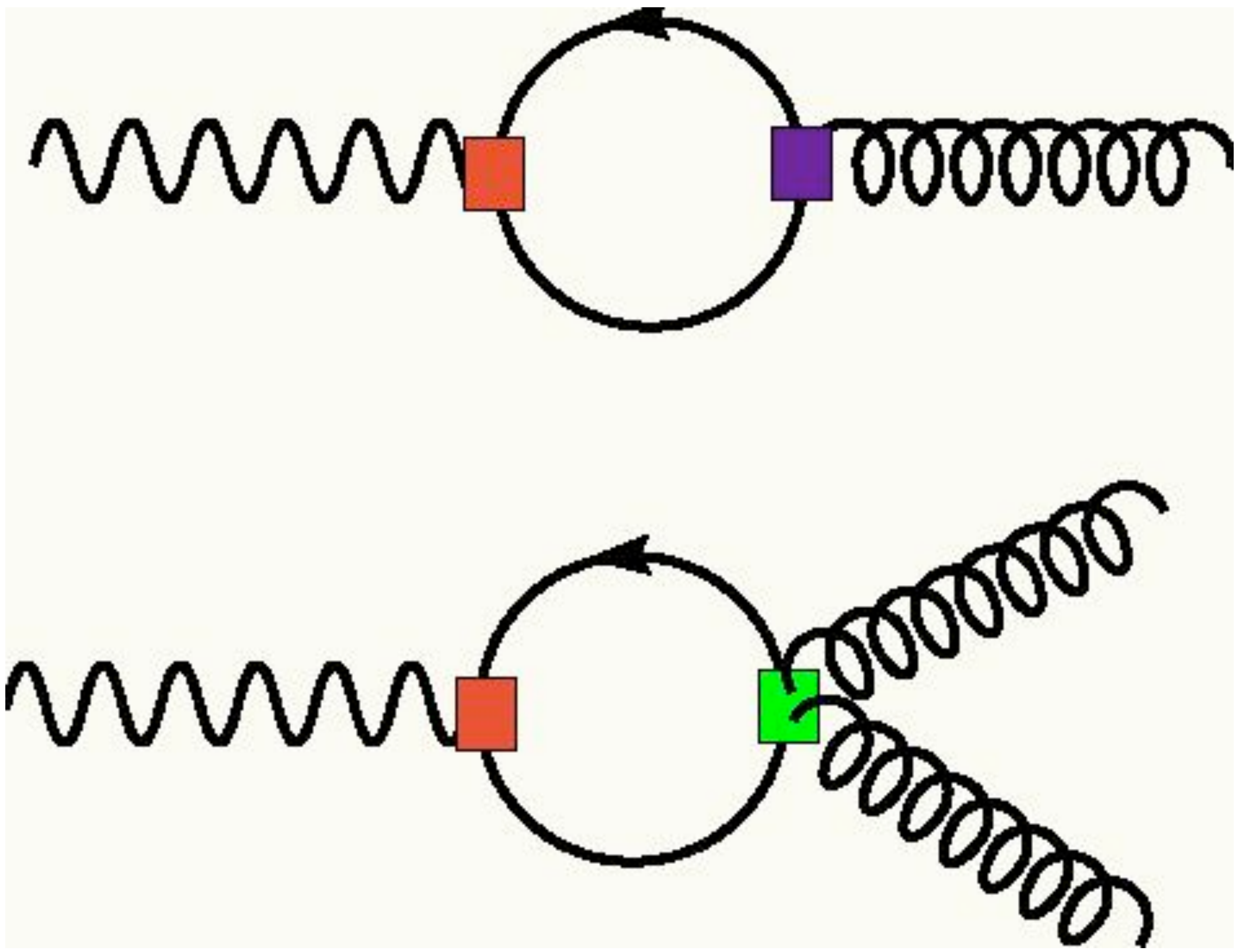
$$\mathcal{H}_{\rm n-lPh} = \frac{1}{f_{\rm n-lPh}} \int d^3x \; \psi_n^\dagger(x)\psi_n(x) \; \partial_i \xi_i(x)$$

$$\frac{1}{f_{\rm n-lPh}} = -2\pi a_{\rm n-A} \sqrt{\frac{n_{\rm Ion}}{A \; M^3}}$$

sPh-lPh Interactions

$$\mathcal{L}_{\mathrm{sPh-lPh}} = g_{\mathrm{mix}}\ \partial_o\phi\ \partial_i\xi_i + \frac{1}{\Lambda^2}\ \partial_o\phi\ \partial_i\xi^i\partial_i\xi^i + \cdots$$

Integrate-out neutron degree of freedom

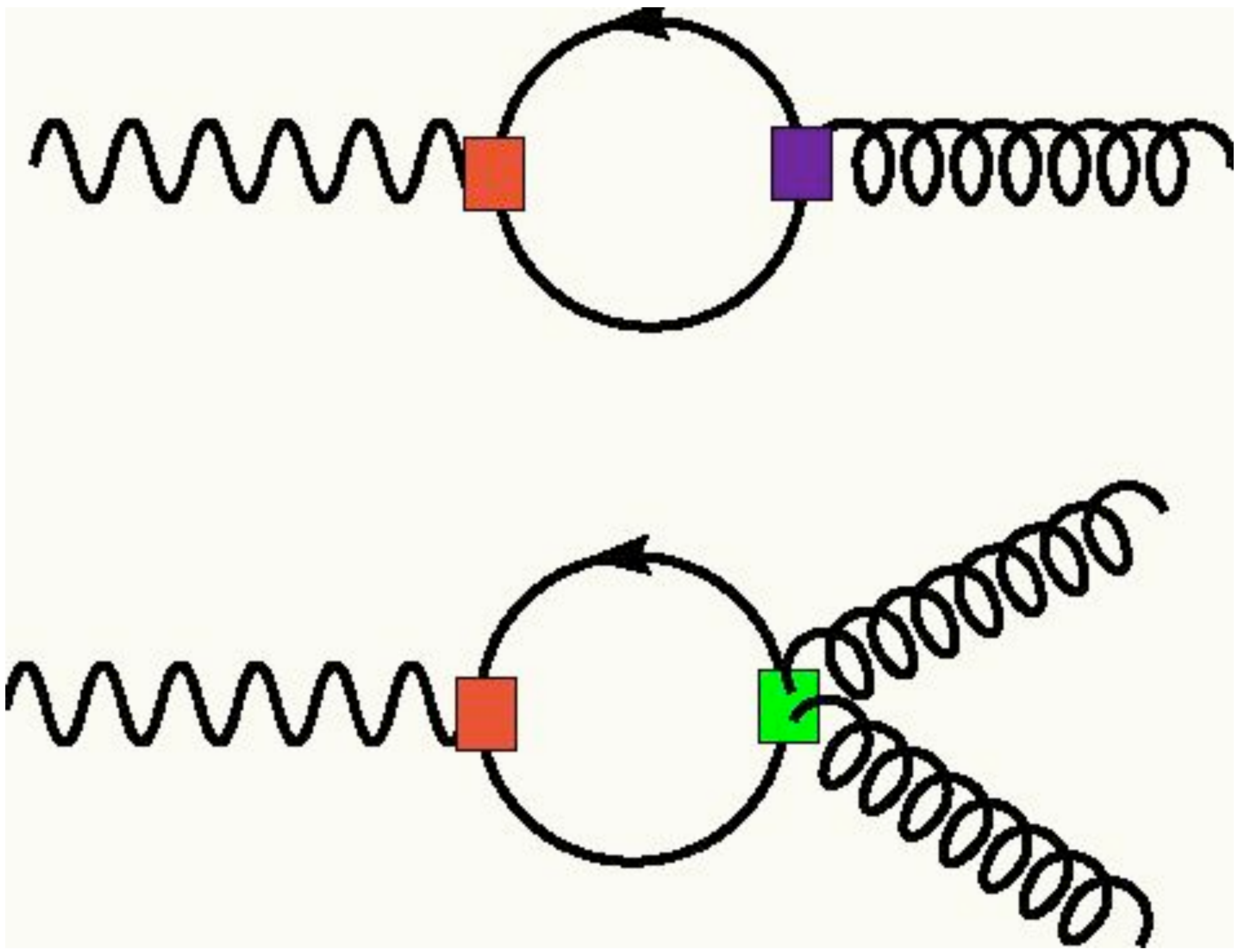


$$\begin{aligned} g_{\mathrm{mix}} &= 2a_{\mathrm{n-Ion}}\sqrt{\frac{n_{\mathrm{Ion}}\ k_{\mathrm{Fn}}}{A\ M^2}} \\ \Lambda^2 &= \sqrt{n_{\mathrm{Ion}}\ A\ M} \end{aligned}$$

sPh-lPh Interactions

$$\mathcal{L}_{\rm sPh-lPh} = g_{\rm mix}\ \partial_o\phi\ \partial_i\xi_i + \frac{1}{\Lambda^2}\ \partial_o\phi\ \partial_i\xi^i\partial_i\xi^i + \cdots$$

Integrate-out neutron degree of freedom



$$g_{\rm mix} = 2a_{\rm n-Ion}\sqrt{\frac{n_{\rm Ion}\ k_{\rm Fn}}{A\ M^2}}$$

$$\Lambda^2 = \sqrt{n_{\rm Ion}\ A\ M}$$

In the neutron star crust:

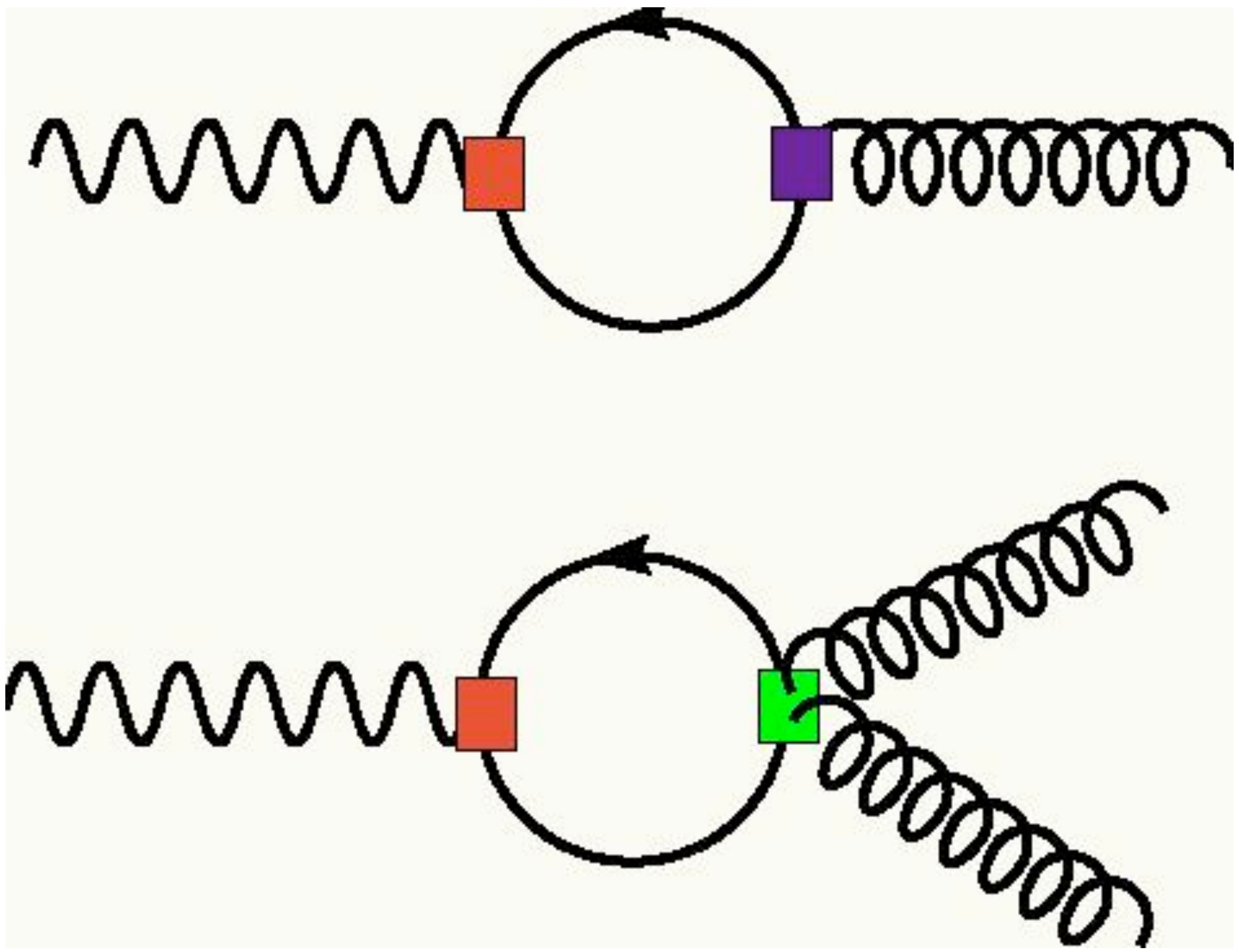
$$g_{\rm mix} \simeq 10^{-3}$$

$$\Lambda \simeq 50\ {\rm MeV}$$

sPh-lPh Interactions

$$\mathcal{L}_{\mathrm{sPh-lPh}} = g_{\mathrm{mix}}\ \partial_o\phi\ \partial_i\xi_i + \frac{1}{\Lambda^2}\ \partial_o\phi\ \partial_i\xi^i\partial_i\xi^i + \cdots$$

Integrate-out neutron degree of freedom



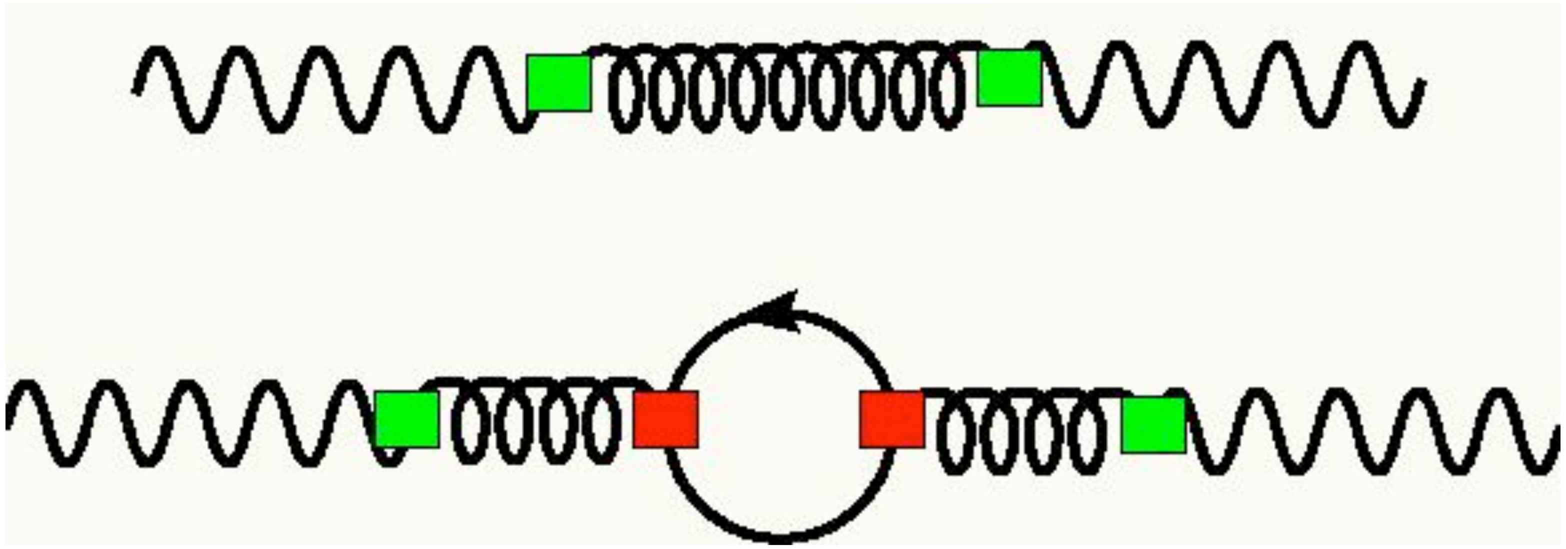
$$\begin{aligned} g_{\mathrm{mix}} &= 2a_{\mathrm{n-Ion}}\sqrt{\frac{n_{\mathrm{Ion}}\ k_{\mathrm{Fn}}}{A\ M^2}} \\ \Lambda^2 &= \sqrt{n_{\mathrm{Ion}}\ A\ M} \end{aligned}$$

In the neutron star crust:

$$\begin{aligned} g_{\mathrm{mix}} &\simeq 10^{-3} \\ \Lambda &\simeq 50\ \mathrm{MeV} \end{aligned}$$

Now we are ready to calculate the sPh mean free path

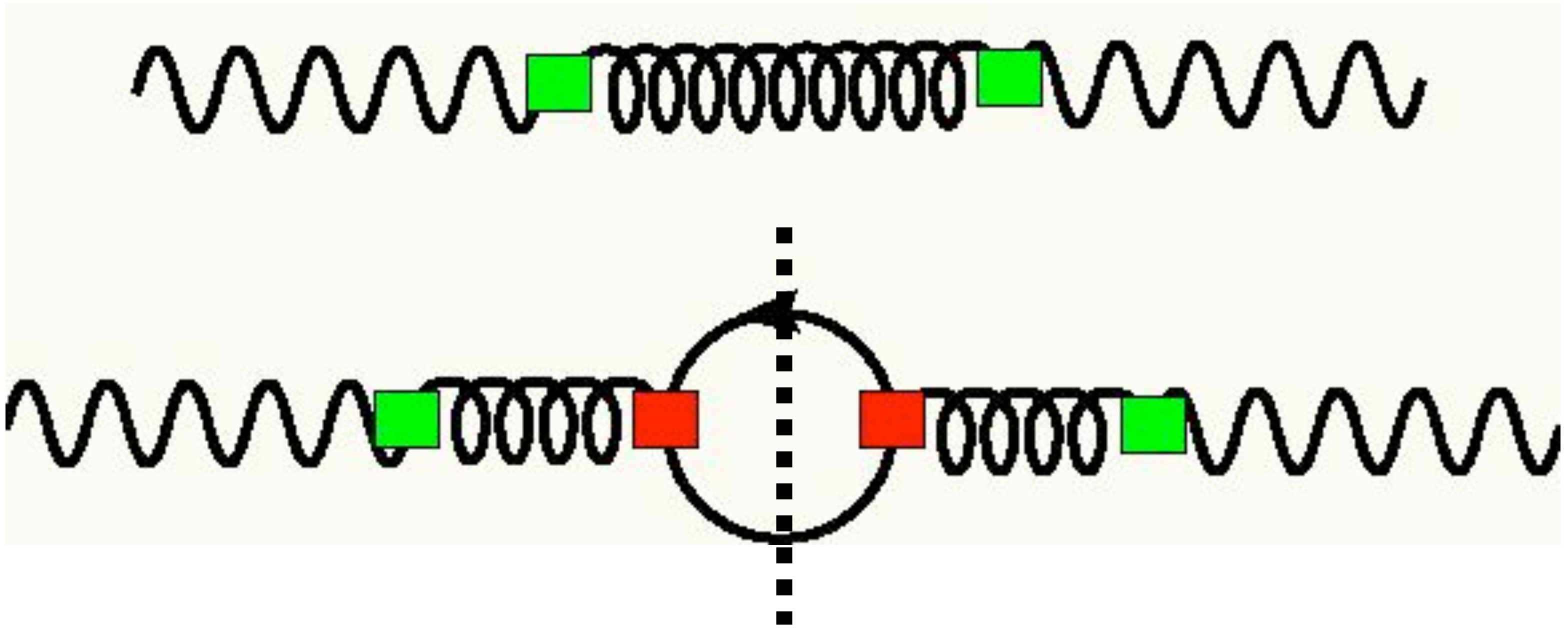
Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

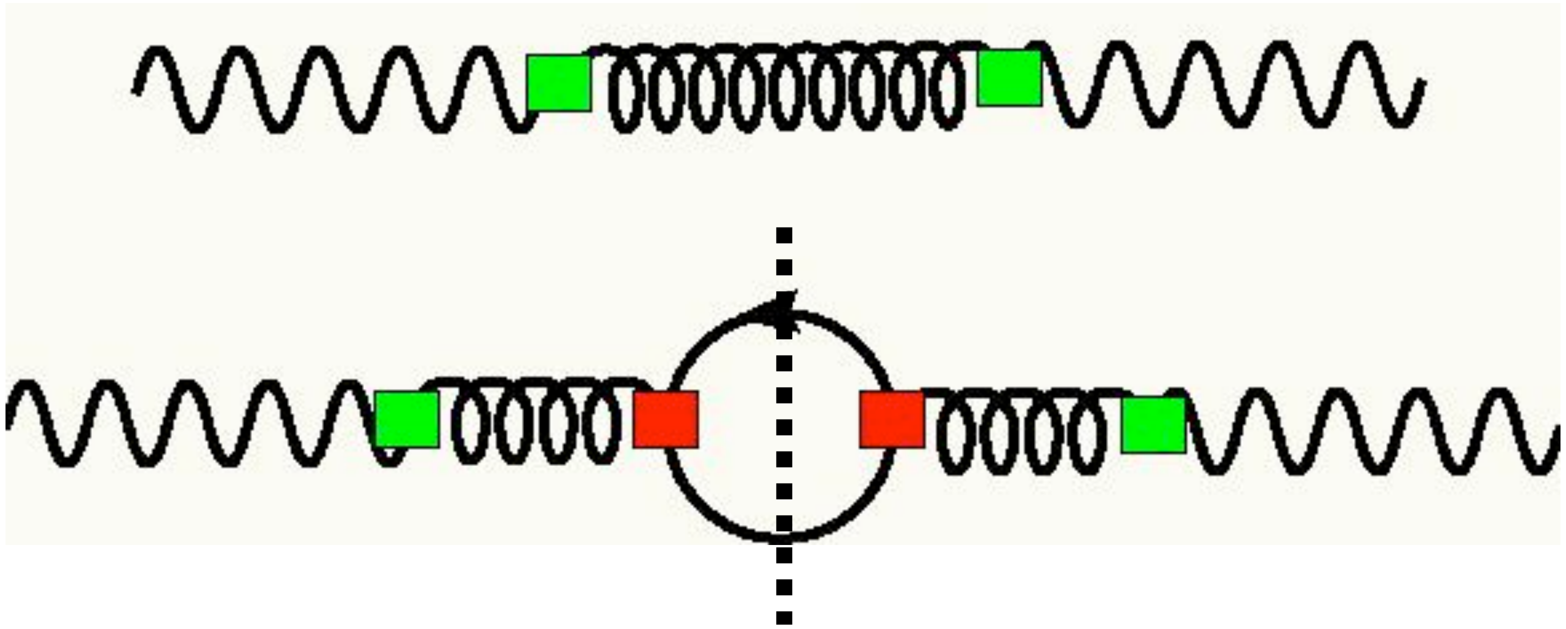
Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

Mixing and Dissipation



Mixing leads to oscillations

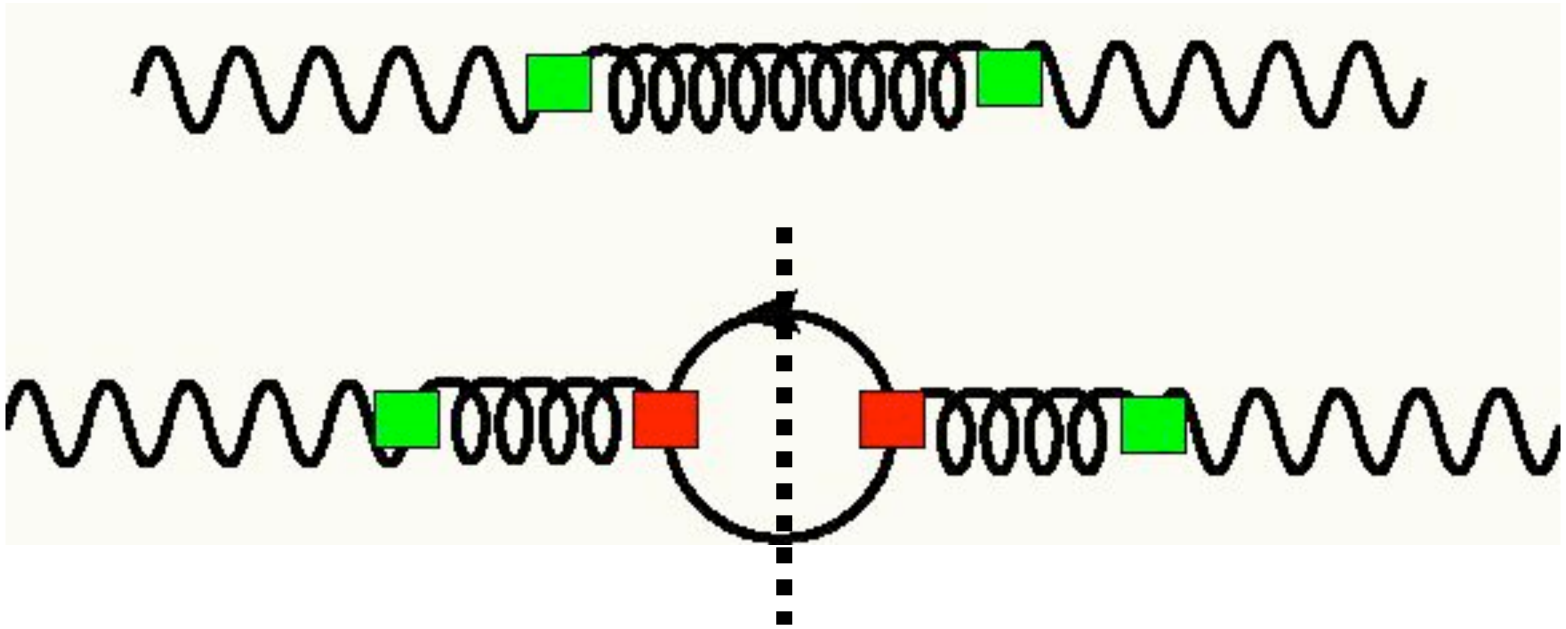
Dissipation of lPh leads to dissipation of sPh

$$\lambda_{\rm abs}(\omega) = \frac{v_s^2}{g_{\rm mix}^2}\,\frac{1+(1-\alpha^2)^2\,(\omega\,\tau_{\rm lPh})^2}{\alpha\,(\omega\,\tau_{\rm lPh})^2}\,\lambda_{\rm lPh}(\omega)$$

sPh mean free path

lPh mean free path

Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

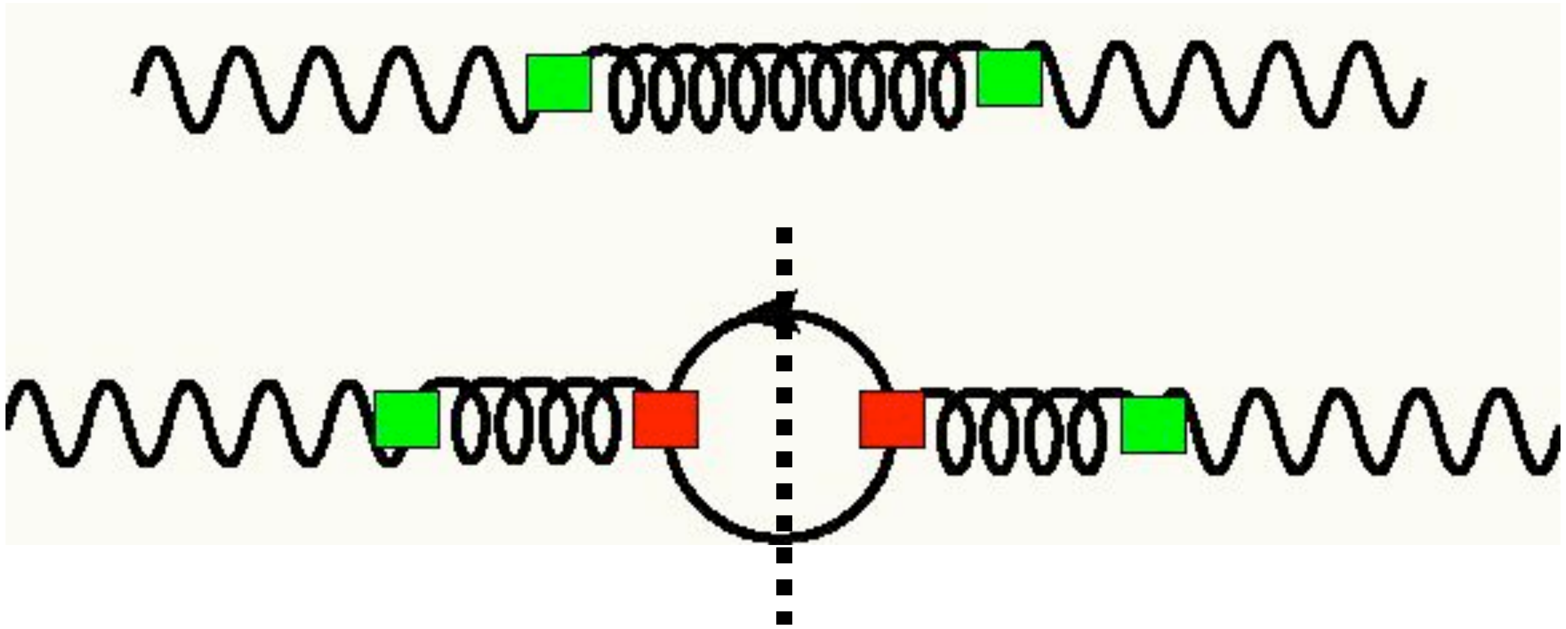
$$\lambda_{\text{abs}}(\omega) = \frac{v_s^2}{g_{\text{mix}}^2} \frac{1 + (1-\alpha^2)^2 \ (\omega \ \tau_{\text{lPh}})^2}{\alpha \ (\omega \ \tau_{\text{lPh}})^2} \ \lambda_{\text{lPh}}(\omega)$$

sPh mean free path

lPh mean free path

$$\begin{aligned} \alpha &= \frac{c_s}{v_s} \\ \lambda_{\text{lPh}} &= c_s \ \tau_{\text{lPh}} \end{aligned}$$

Mixing and Dissipation



Mixing leads to oscillations

Dissipation of lPh leads to dissipation of sPh

$$\lambda_{\rm abs}(\omega) = \frac{v_s^2}{g_{\rm mix}^2}\, \frac{1+(1-\alpha^2)^2\ (\omega\ \tau_{\rm lPh})^2}{\alpha\ (\omega\ \tau_{\rm lPh})^2}\ \lambda_{\rm lPh}(\omega)$$

sPh mean free path

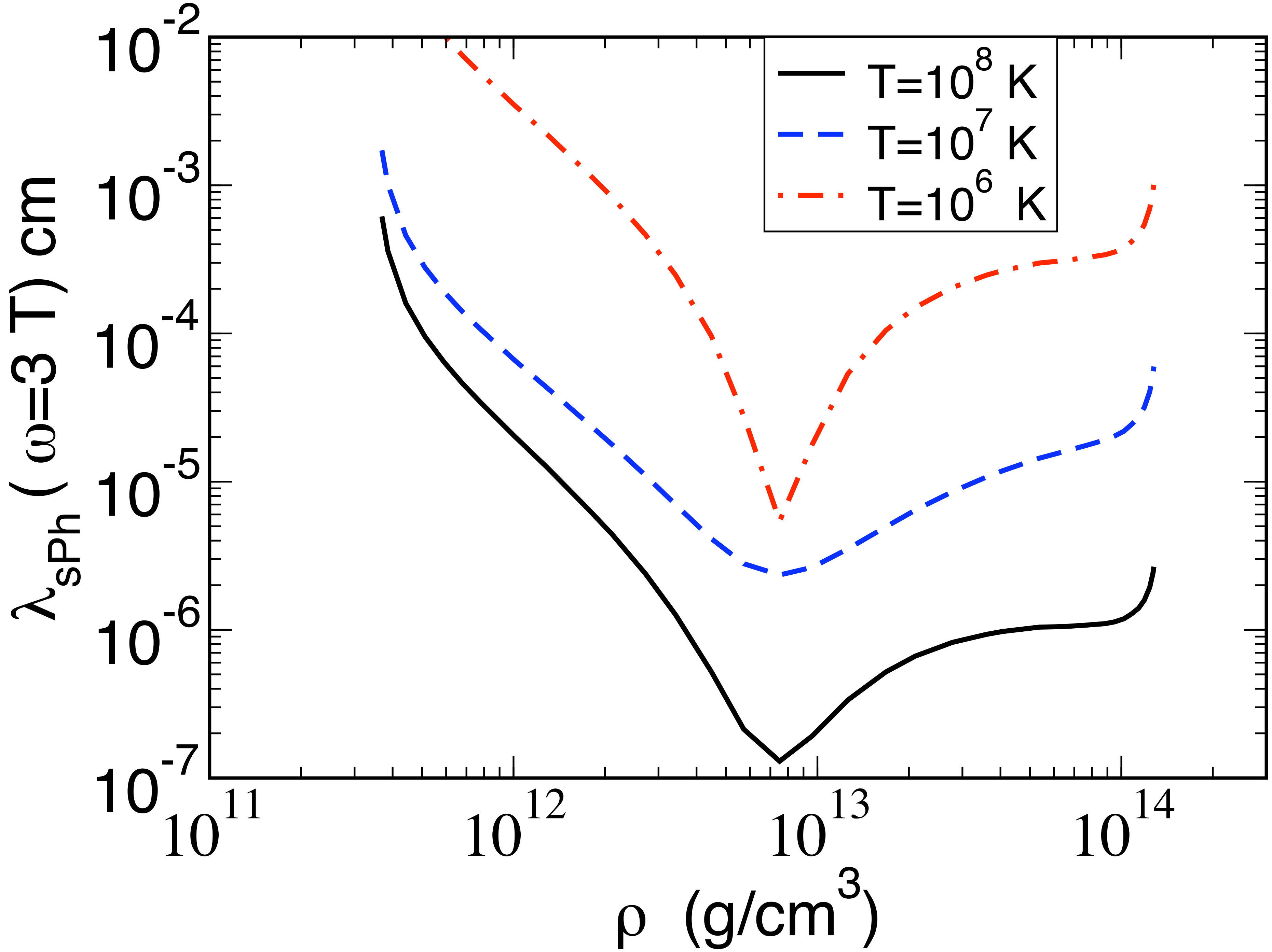
lPh mean free path

$$\alpha = \frac{c_s}{v_s}$$

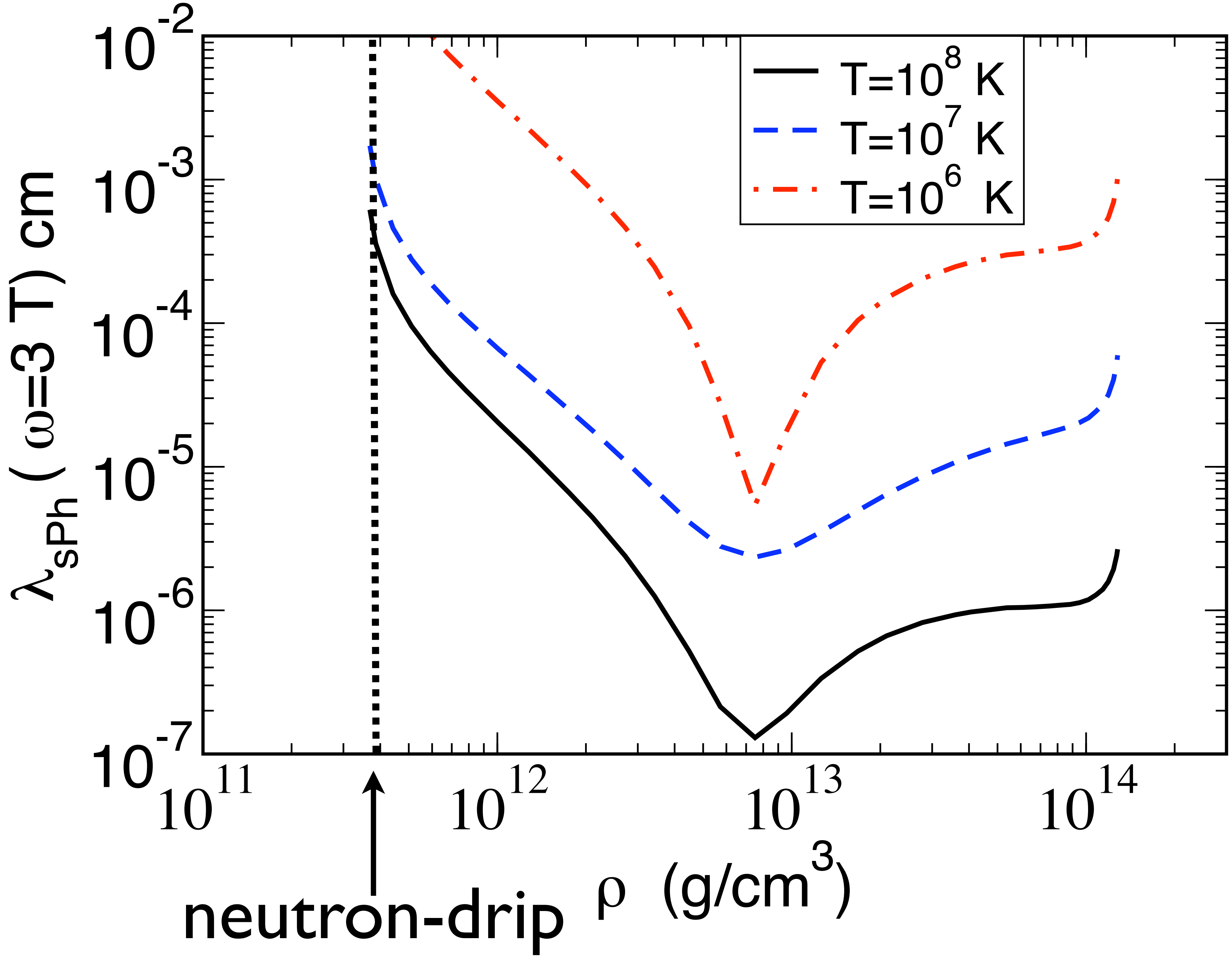
$$\lambda_{\rm lPh} = c_s\ \tau_{\rm lPh}$$

Away from resonance $\lambda_{\rm sPh} \simeq 10^5\ \lambda_{\rm lPh}$

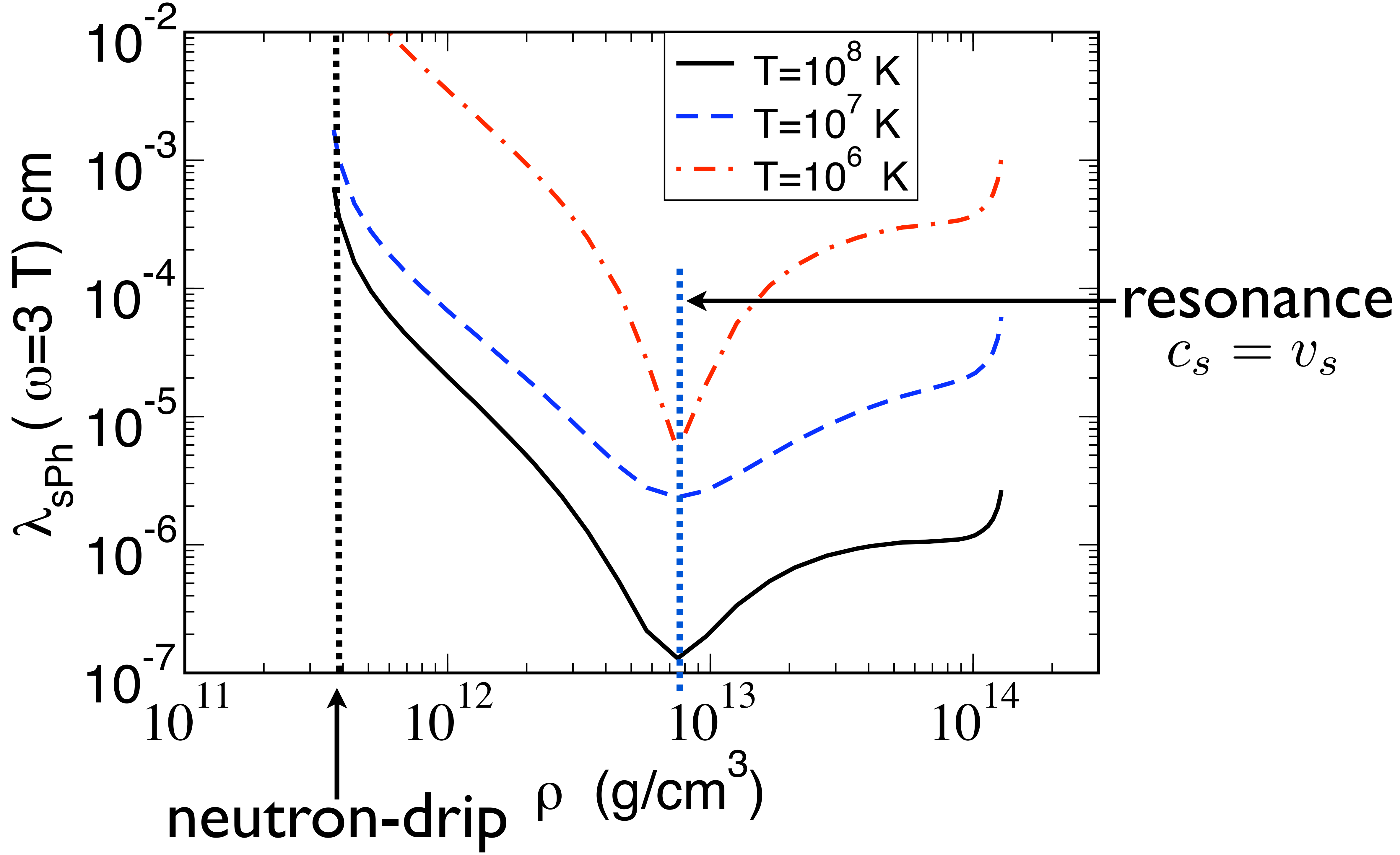
Superfluid Phonon Mean Free Path



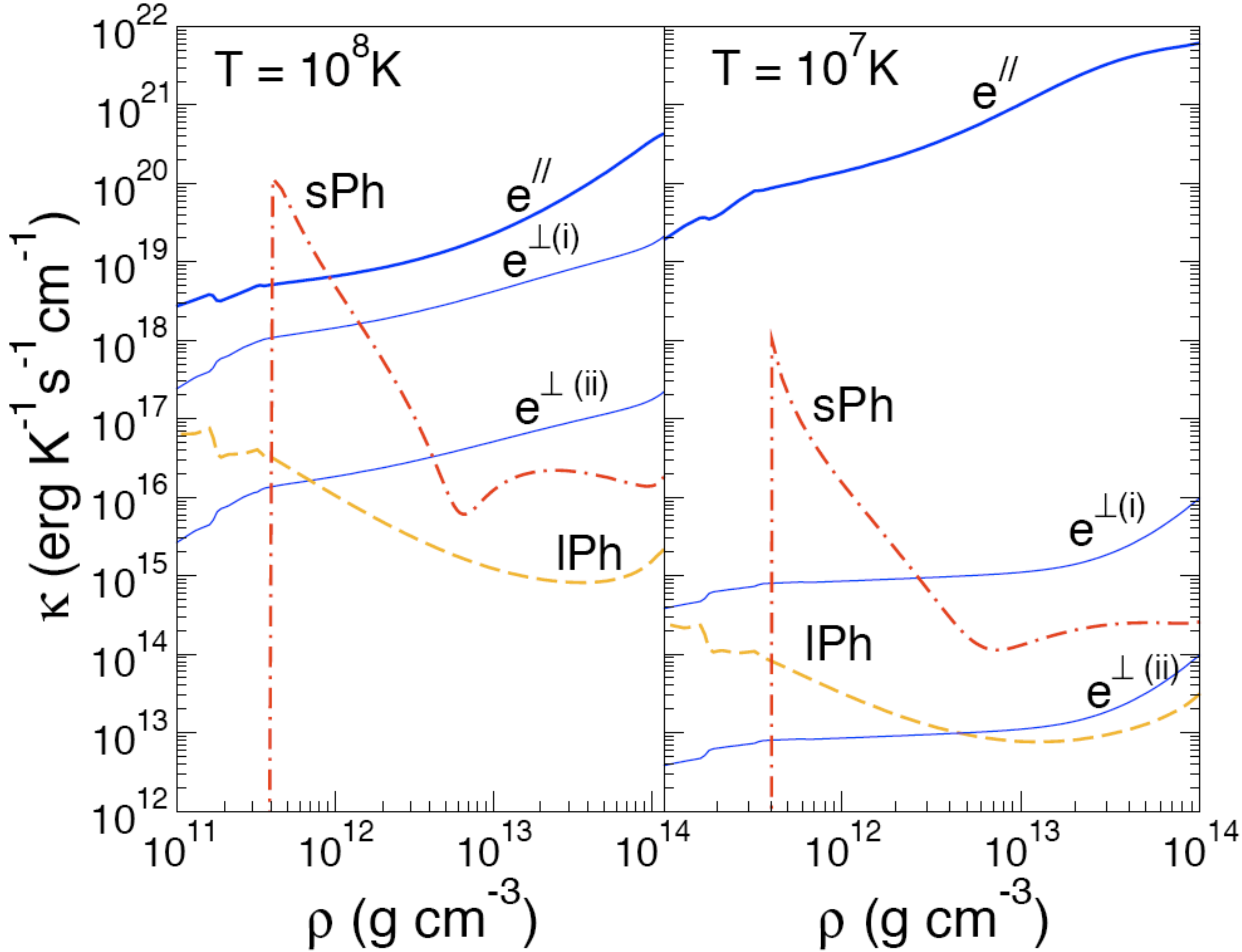
Superfluid Phonon Mean Free Path



Superfluid Phonon Mean Free Path

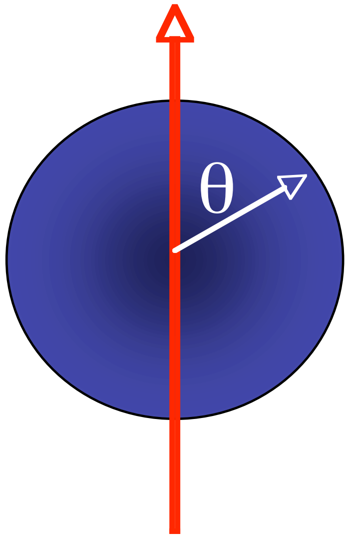


Thermal Conductivity



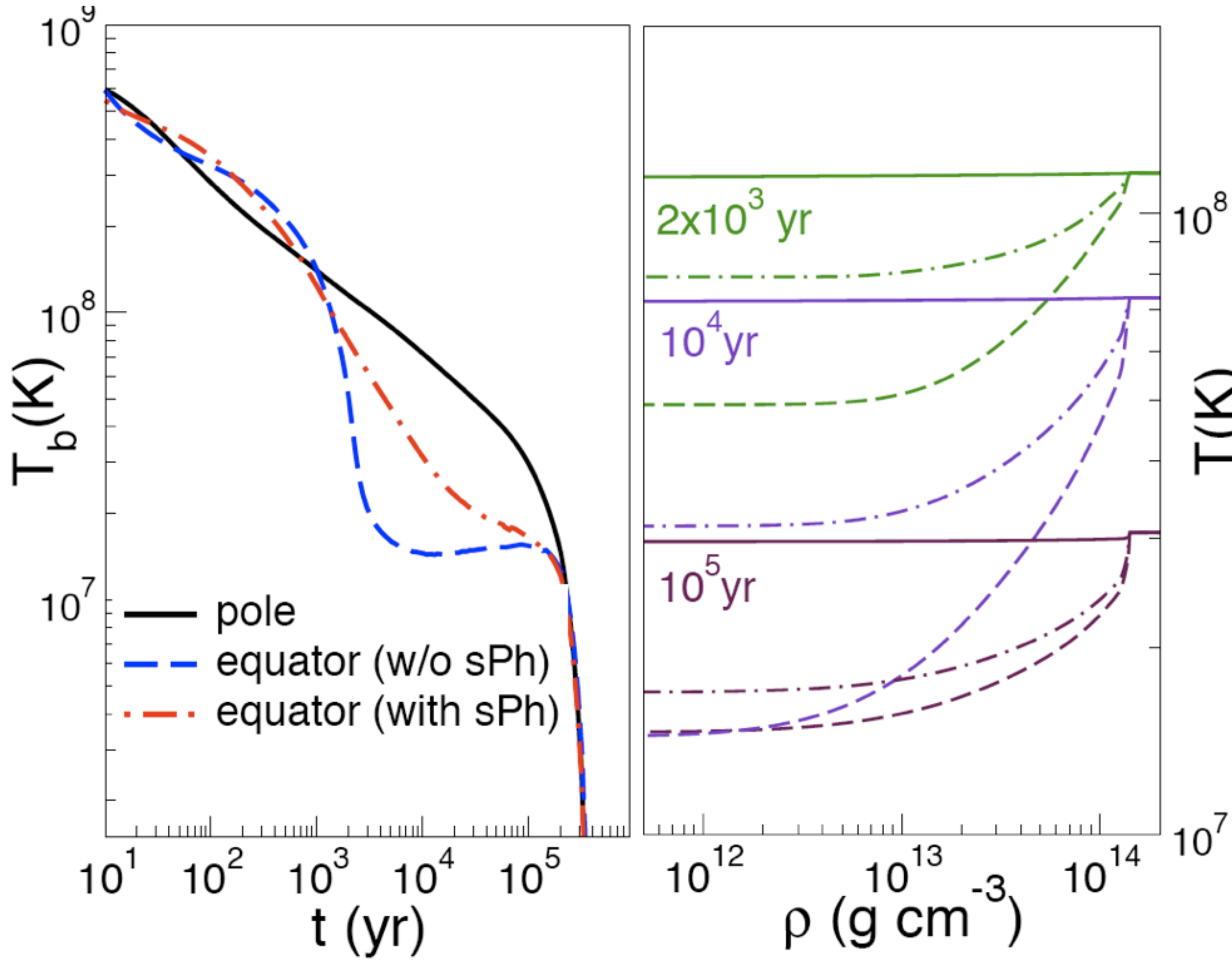
Consequences for Magnetar Cooling

Surface temperature anisotropy due anisotropic conduction.



sPh can limit the anisotropy

$$T^4(\theta) = T_{\rm eff}^4 \left(\cos^2\theta + \frac{\kappa^\perp}{\kappa^\parallel}\sin^2\theta\right)$$



Conclusions

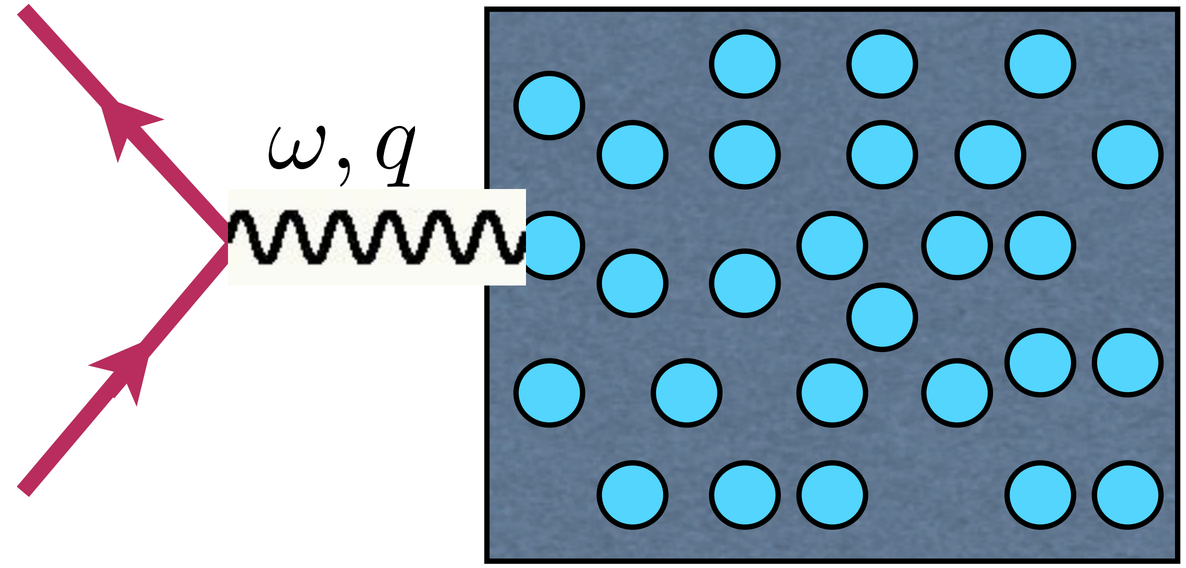
- New mode for heat conduction in the inner crust.
- Low energy EFT for sPhs, lPhs and electrons.
- sPh conduction is likely to be important for thermal evolution of magnetars.

Conclusions

- New mode for heat conduction in the inner crust.
- Low energy EFT for sPhs, lPhs and electrons.
- sPh conduction is likely to be important for thermal evolution of magnetars.

Large temperature anisotropy on the surface can probe efficiency of phonon transport in the crust.

Electron Scattering and the Dynamic Structure Factor



$$\frac{1}{\lambda_e} = \nu_e = \frac{4\pi Z^2\ e^4\ n_{\text{ions}}}{k_{Fe}^2\ v_{Fe}}\ \Lambda_\kappa$$

$$\Lambda_\kappa = \int_0^{2k_{\text{Fe}}} dq\ q^3\ V_{\text{eff}}(q)^2\ S_\kappa(q)\ \left[1 - \frac{v_{\text{Fe}}^2 q^2}{4k_{\text{Fe}}^2}\right]$$

Flowers & Itoh (1976)
Yakovlev & Urpin (1980)
Potekhin et al. (1999)

Coulomb Logarithm

$$\Lambda_\kappa = \int d\omega\ S(q,\omega)\ \left[\frac{z + f(q)\ z^3}{1 - e^{-z}}\right] \qquad z = \frac{\omega}{T}$$

Dynamic Structure Factor

$$S(q,\omega) = \int dt\ e^{i\omega t}\ \langle \rho(q,t)\rho(-q,0)\rangle$$

Potekhin (1999)

Plasma physics of the outer crust:

$$\Gamma = \frac{Z^2\ e^2}{a\ kT} \quad \Gamma_c \simeq 175$$

$$a \simeq 125 \left(\frac{A}{50}\ \frac{1}{\rho_{10}}\right)^{1/3} \text{fm}$$

$$kT = 4.4\ 10^{-5} \frac{T}{10^8\ K}\ \text{fm}^{-1}$$

$$\omega_{\text{plasmon}} = \sqrt{\frac{4\pi e^2 Z^2 n_I}{AM}}$$

$$q_{\text{TFe}} = \sqrt{\frac{4e^2}{\pi}}\ k_{\text{Fe}}$$