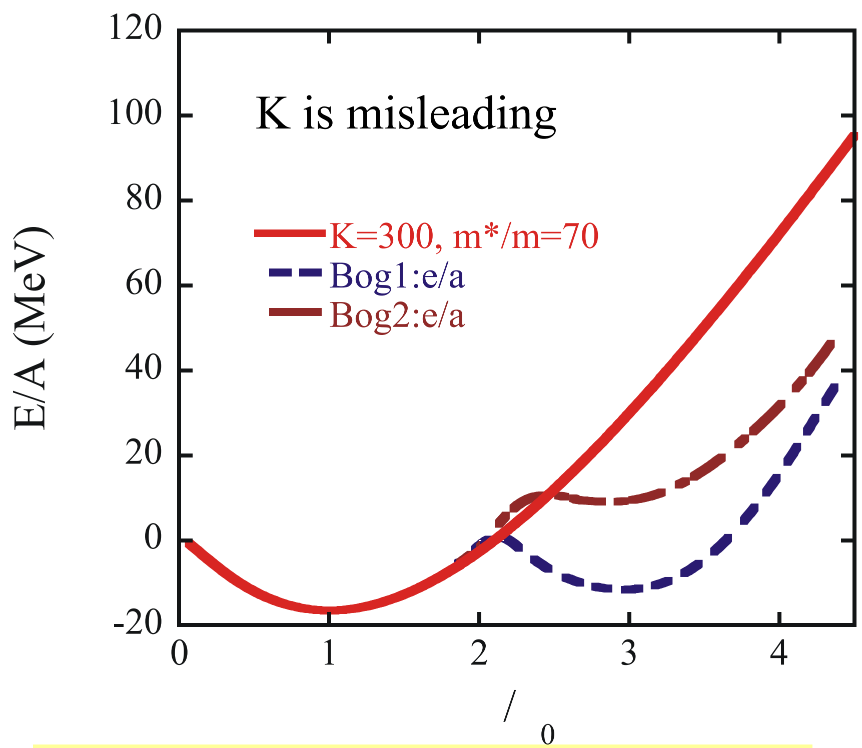
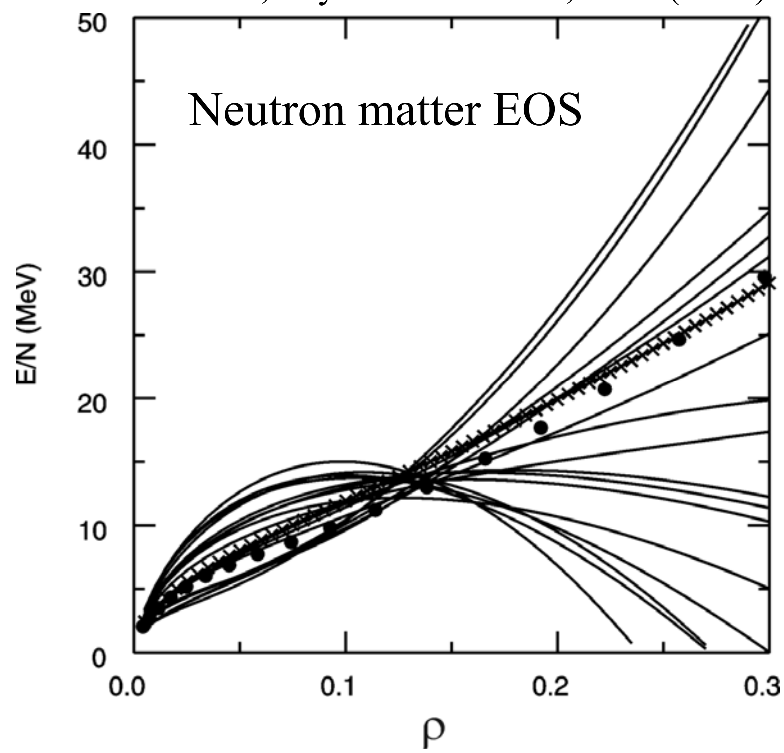
Measurements on the Asymmetric EoS

William Lynch, Yingxun Zhang, Dan Coupland, Pawel Danielewicz, Micheal Famiano, Zhuxia Li, Betty Tsang

- Motivations
- Sources of constraints on the EOS.
- Laboratory constraints from nuclear collisions
  - symmetric matter
  - symmetry energy
- Summary and outlook

Symmetric matter and neutron matter





$E/A\,(\rho,\delta) = E/A\,(\rho,0) + \delta^2 \cdot S(\rho)$

$\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$

$$P = \rho^2 \left. \frac{\partial (E/A)}{\partial \rho} \right|_{s/a}$$

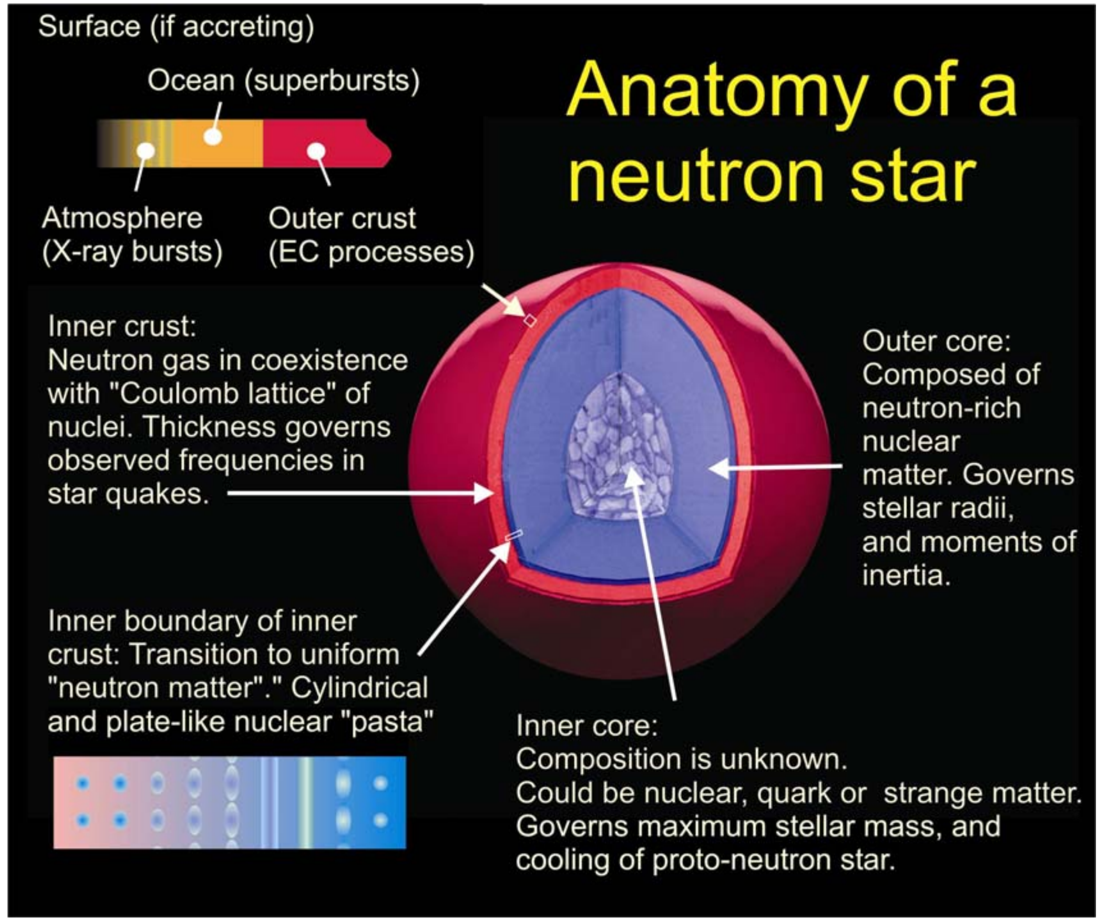
- The density dependence of symmetry energy is largely unconstrained.
- What is “stiff” or “soft” is density dependent

Constraints and how to obtain them

- Both astrophysical and laboratory observables can constrain the EOS, $\varepsilon(\rho,T,\delta)$ or $P(\rho,T,\delta)$ indirectly.
  - Usually P nor ε are not directly measured.
- Questions:
  - For what densities or asymmetries is the constraint relevant?
  - What is the precision of the constraint?
  - What is the accuracy or model dependence of the constraint?

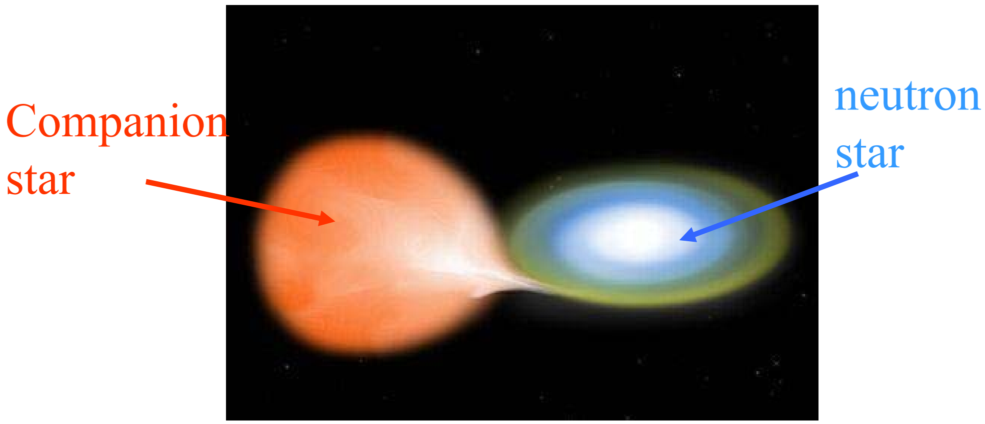
EOS, Symmetry Energy and Neutron Stars

- Neutron Star stability against gravitational collapse
- Stellar density profile
- Internal structure: occurrence of various phases.
- Observational consequences:
  - Cooling rates of proto-neutron stars
  - Cooling rates for X-ray bursters.
  - Stellar masses, radii and moments of inertia.
    - Possible study of low mass X-ray binaries

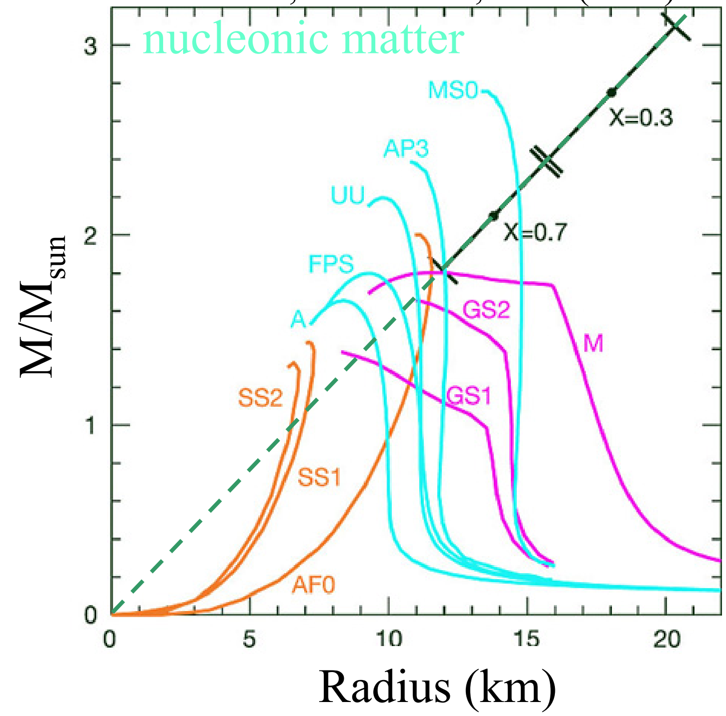


- Beyond capabilities of Chandra or XMM.
- Requires "International X-ray Observatory"
  - Cost ~ \$2B RY: Possible launch date 2020.

Previous attempt: X-ray bursters



F. Ozel, Nature **441**, 1115 (2006).

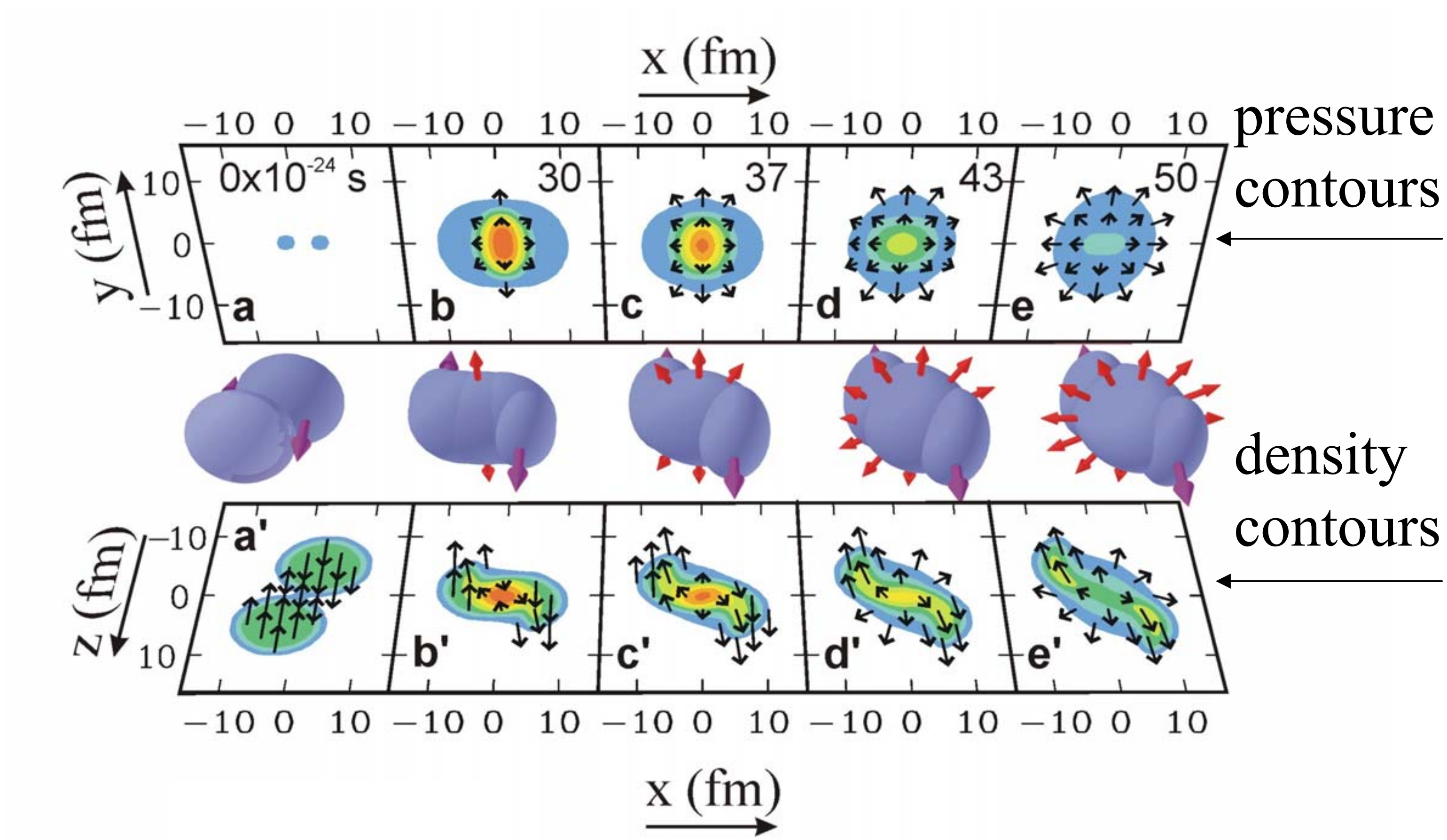


- EXO 0748 – 676 is a neutron star in a Binary system, which emits bursts of X-rays.
- Recent X-ray observations with XMM-Newton have identified red-shifted lines of O and Fe.
  - red shift ⇒M/R
- Other measurements:
  - F_{cool}/T_c^4
  - Eddington flux

- Rules out most EOS's
  - "...If this object is typical, then condensates[2] and unconfined quarks[1] do not exist in the centres of neutron stars." F. Ozel, Nature **441**, 1115 (2006).

- Independent confirmation from Laboratory measurements may be relevant

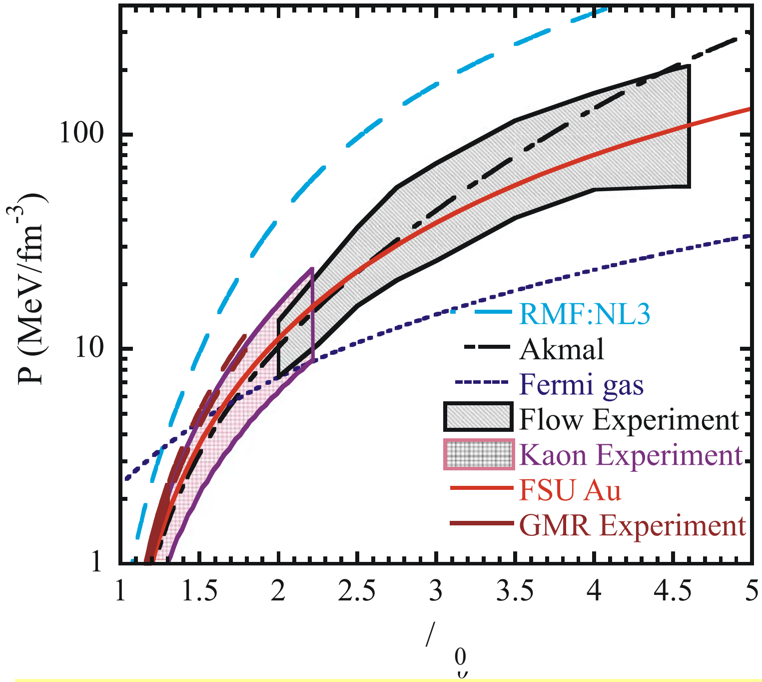
Constraining the symmetric matter EOS at high densities by laboratory collisions

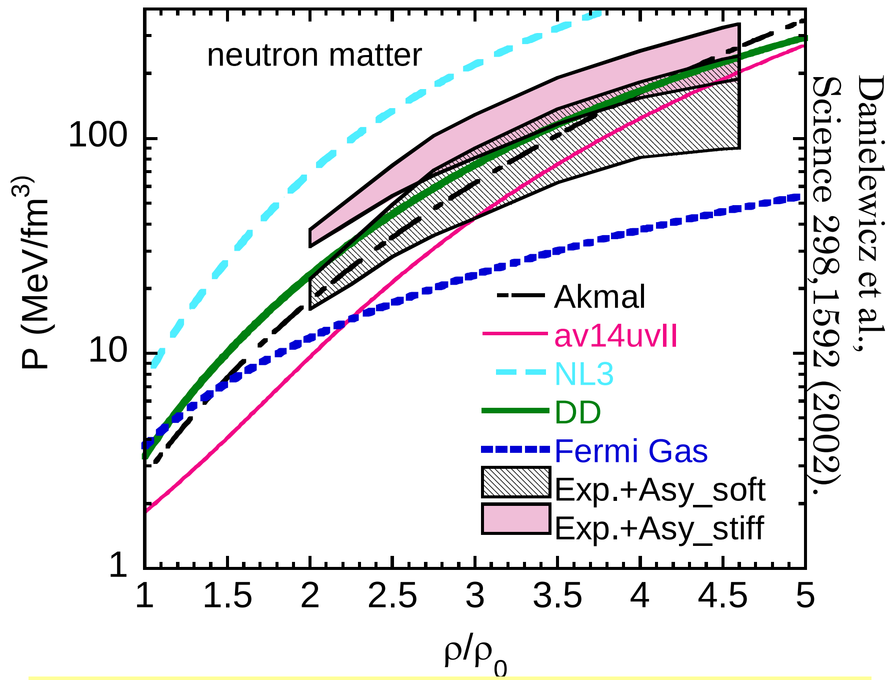


- The blocking by the spectator matter provides a clock with which to measure the expansion rate.

Constraints from collective flow on EOS at $\rho > 2\rho_0$.

$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$ $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A \approx 1$





- Flow confirms the softening of the EOS at high density.
- Constraints from kaon production are consistent with the flow constraints and bridge gap to GMR constraints.
- Note: analysis requires additional constraints on m* and σ_{NN}.

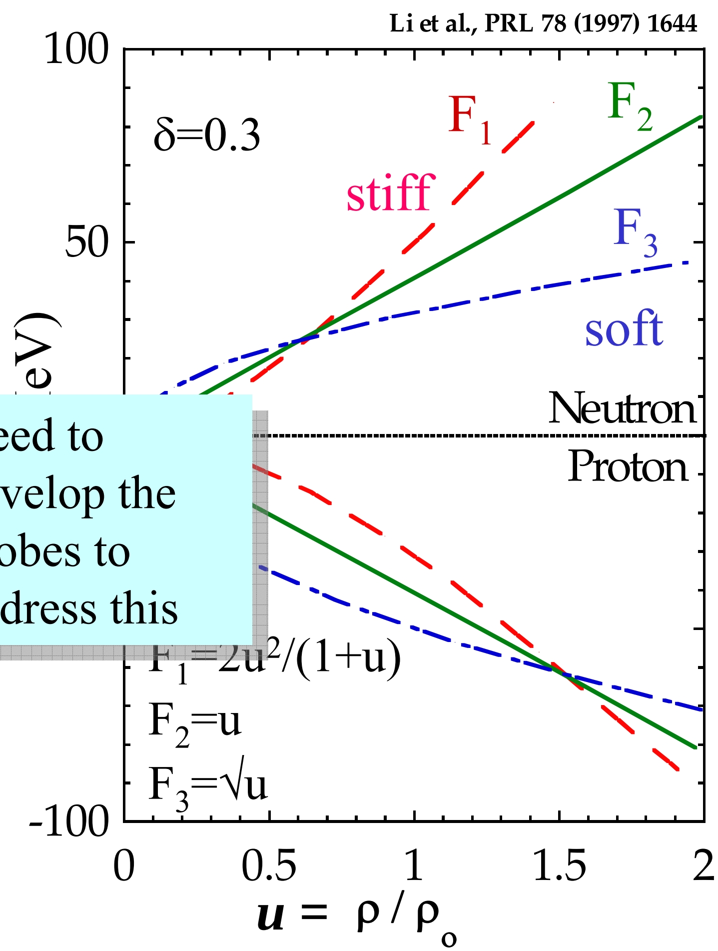
- The symmetry energy dominates the uncertainty in the n-matter EOS.
- Both laboratory and astronomical constraints on the density dependence of the symmetry energy are urgently needed.

Probes of the symmetry energy

$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)$; $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$

- Low densities ($\rho<\rho_0$):
  - Fragment isotopic distributions:
  - Neutron/proton spectra and flows
  - Isospin diffusion
  - Neutron, proton radii, E0 and E1 collective modes.
- High densities ($\rho\approx 2\rho_0$) :
  - Neutron/proton spectra and flows
  - π^+ vs. π^- production, k, hyperon production.
- Common features of some of these studies
  - Vary isospin of detected particle
  - Vary isospin asymmetry $\delta=(N-Z)/A$ of reaction.

- Need to develop the probes to address this



Theory

- Theoretical description must follow the reaction dynamics self-consistently from contact to detection.
- Theoretical tool: transport theory:
  - Example BUU eq. (Bertsch Phys. Rep. 160, 189 (1988).):

$$\frac{\partial f_1}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla}_{\mathbf{r}} f_1 - \vec{\nabla}_{\mathbf{r}} U \cdot \vec{\nabla}_{\mathbf{p}} f_1$$

$$= \frac{4}{(2\pi)^3} \int d^3k_2 d\Omega \frac{d\sigma_{nn}}{d\Omega} v_{12} \left[f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4) \right]$$

    - f is the Wigner transform of the one-body density matrix
  - The experimental domain penetrates to the domain of liquid-gas phase transition. To model fluctuations, one can try moleclular dynamics QMD or Anti-Symmetrized Molecular Dynamics (AMD). BUU and QMD have been cross-calibrated at E/A=2 GeV, but not at these lower energies,
  - The most accurately predicted observables are those that can be calculated from $f(\vec{r}, \vec{p}, t)$ i.e. flows and other average properties of the events that are not sensitive to fluctuations.

Why Isospin Diffusion and n/p ratios?

- Depends on quantities that can be more accurately calculated in BUU transport theory.
- May be less sensitive to uncertainties in the production mechanism for complex fragments.
- May be less sensitive to secondary decay.

Isospin Diffusion and the EOS

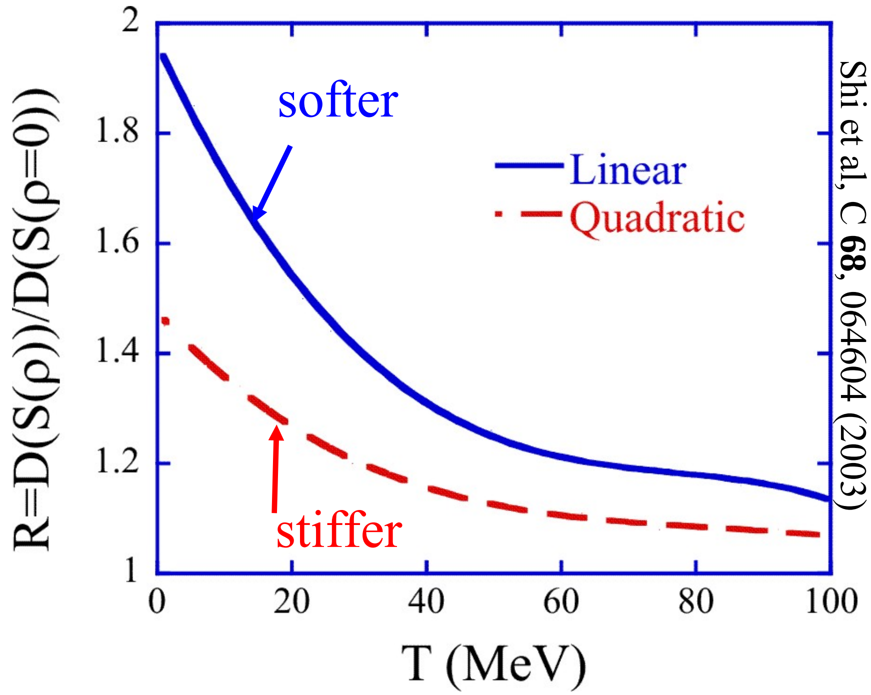
- In a reference frame where the matter is stationary:

  $$\vec{j}_n - \vec{j}_p = -\rho D_\delta \vec{\nabla}\delta - \delta D_\rho \vec{\nabla}\rho$$

  $$\delta = (\rho_n - \rho_p)/\rho$$

  - D_δ the isospin diffusion coef.
  - D_ρ is a drift coefficient.
- Two effects contribute to diffusion
  - Random walk
  - Potential (EOS) driven flows
- D_δ governs the relative flow of neutrons and protons
  - D_δ decreases with σ_{np}
  - D_δ increases with $S_{int}(\rho)$

Diffusion coefficient for $\rho=0.5\rho_0$



Probe: Isospin diffusion in peripheral collisions

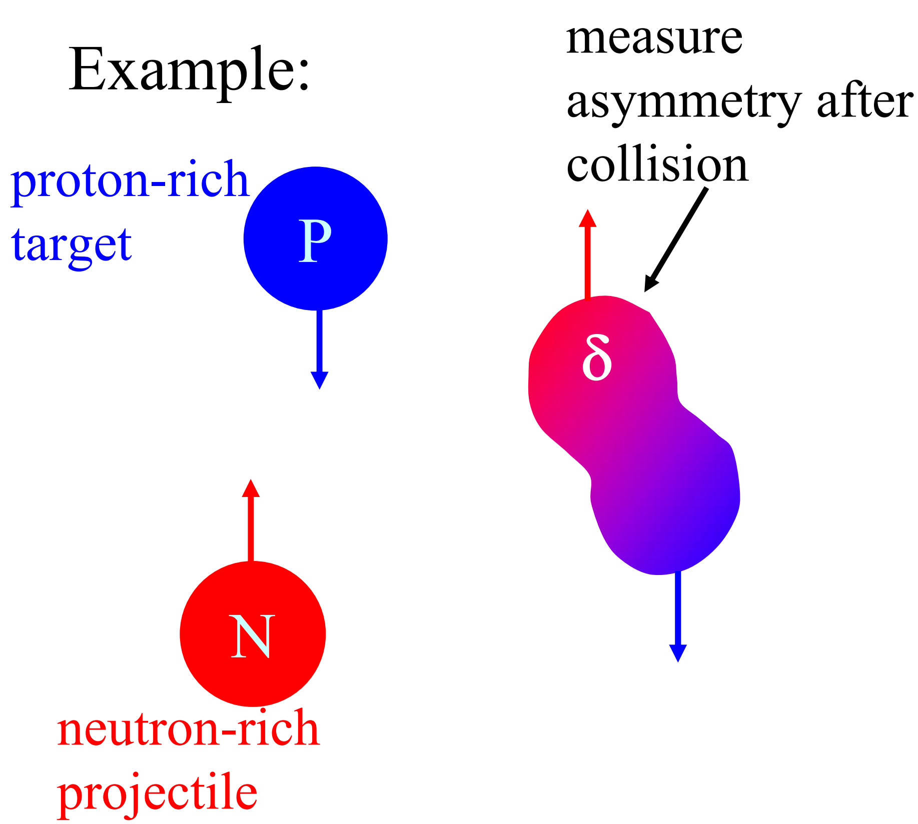
- Collide projectiles and targets of differing isospin asymmetry
- Probe the asymmetry $\delta=(N-Z)/(N+Z)$ of the projectile spectator during the collision.
- The use of the isospin transport ratio $R_i(\delta)$ isolates the diffusion effects:

$$R_i(\delta) = 2 \cdot \frac{\delta - (\delta_{both\_neut.-rich} + \delta_{both\_prot.-rich})/2}{\delta_{both\_neut.-rich} - \delta_{both\_prot.-rich}}$$

- Useful limits for R_i for $^{124}Sn+^{112}Sn$ collisions:
  - $R_i = \pm 1$: no diffusion
  - $R_i \approx 0$: Isospin equilibrium

Systems
- mixed $^{124}Sn+^{112}Sn$
- n-rich $^{124}Sn+^{124}Sn$
- p-rich $^{112}Sn+^{112}Sn$

Example:



What influences isospin diffusion?

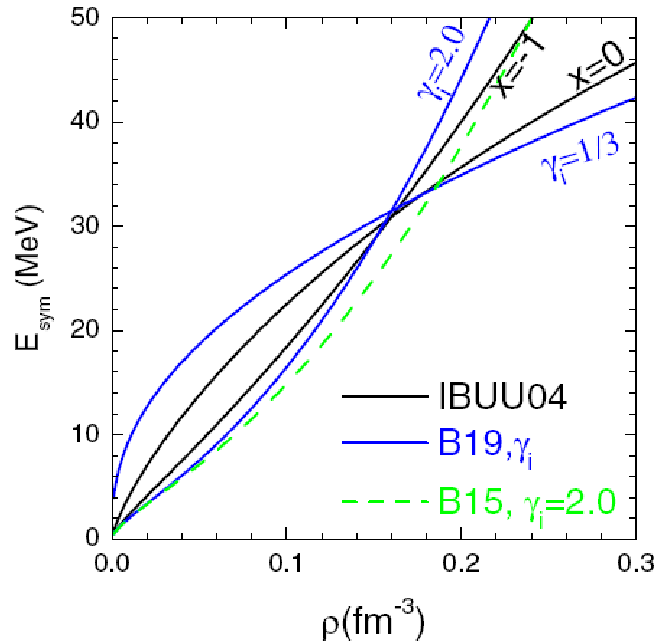
Simple expectation:

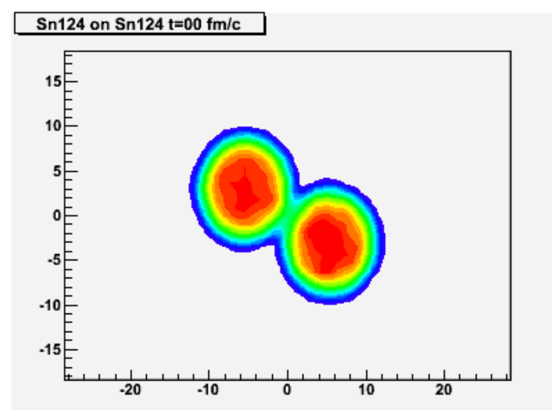
Where *D*

- Increases with larger symmetry energy
- Decreases with with larger n-p cross section

We tested this be investigating the dependence on:

- S_{int} and γ_i in the symmetry energy
  - $S(\rho) = 12.5\cdot(\rho/\rho_0)^{2/3} + S_{int}\cdot(\rho/\rho_0)^{\gamma_i}$
- Cross sections
- Momentum dependence in the mean field
- Impact parameter
- Compressibility of the mean field

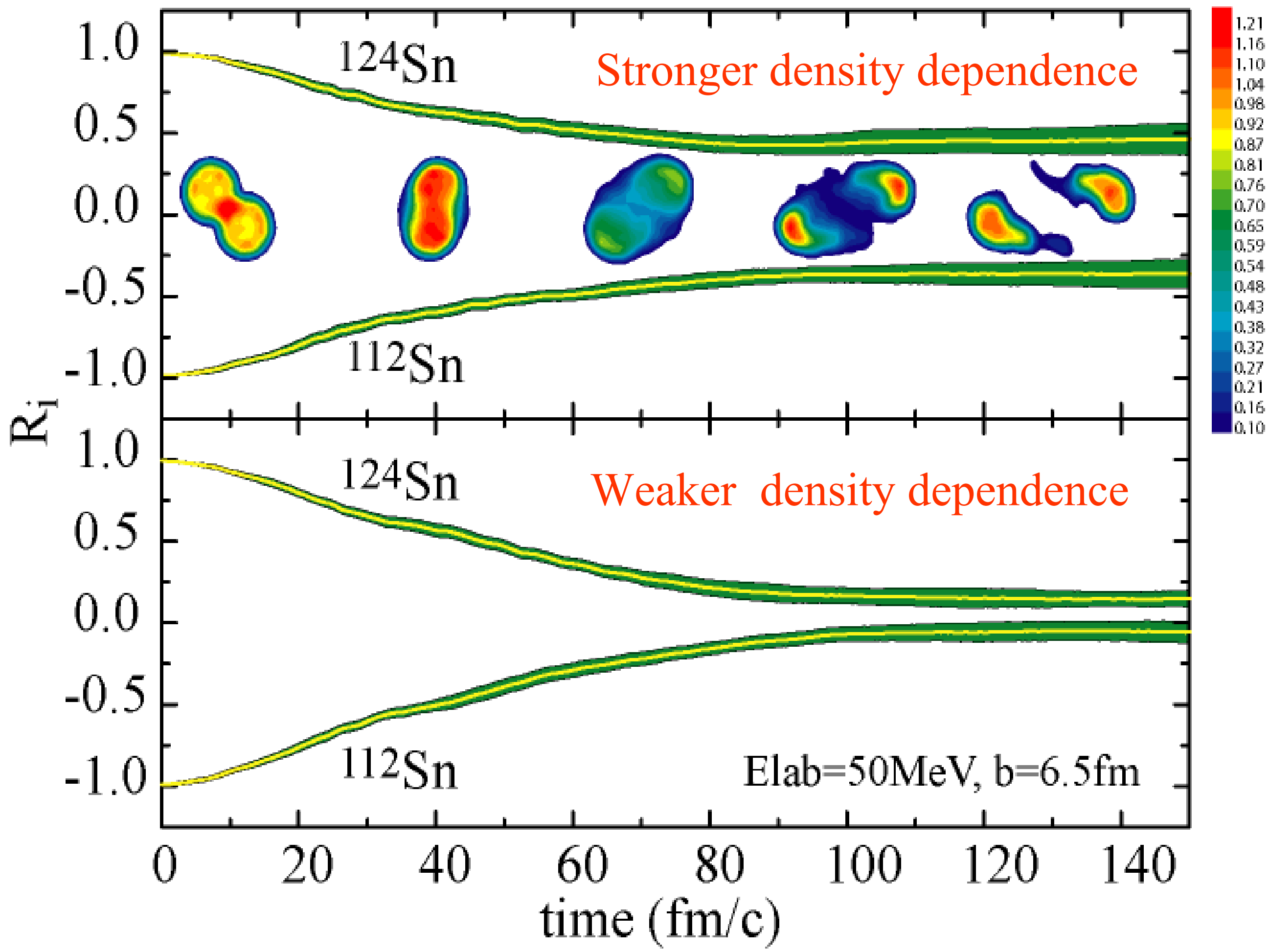




Sensitivity to symmetry energy

$$R_i(\delta) = 2 \cdot \frac{\delta - (\delta_{Neutron-rich} + \delta_{Proton-rich})/2}{\delta_{Neutron-rich} - \delta_{Proton-rich}}$$

- The asymmetry of the spectators can change due to diffusion, but it also can changed due to pre-equilibrium emission.
- The use of the isospin transport ratio $R_i(\delta)$ isolates the diffusion effects:



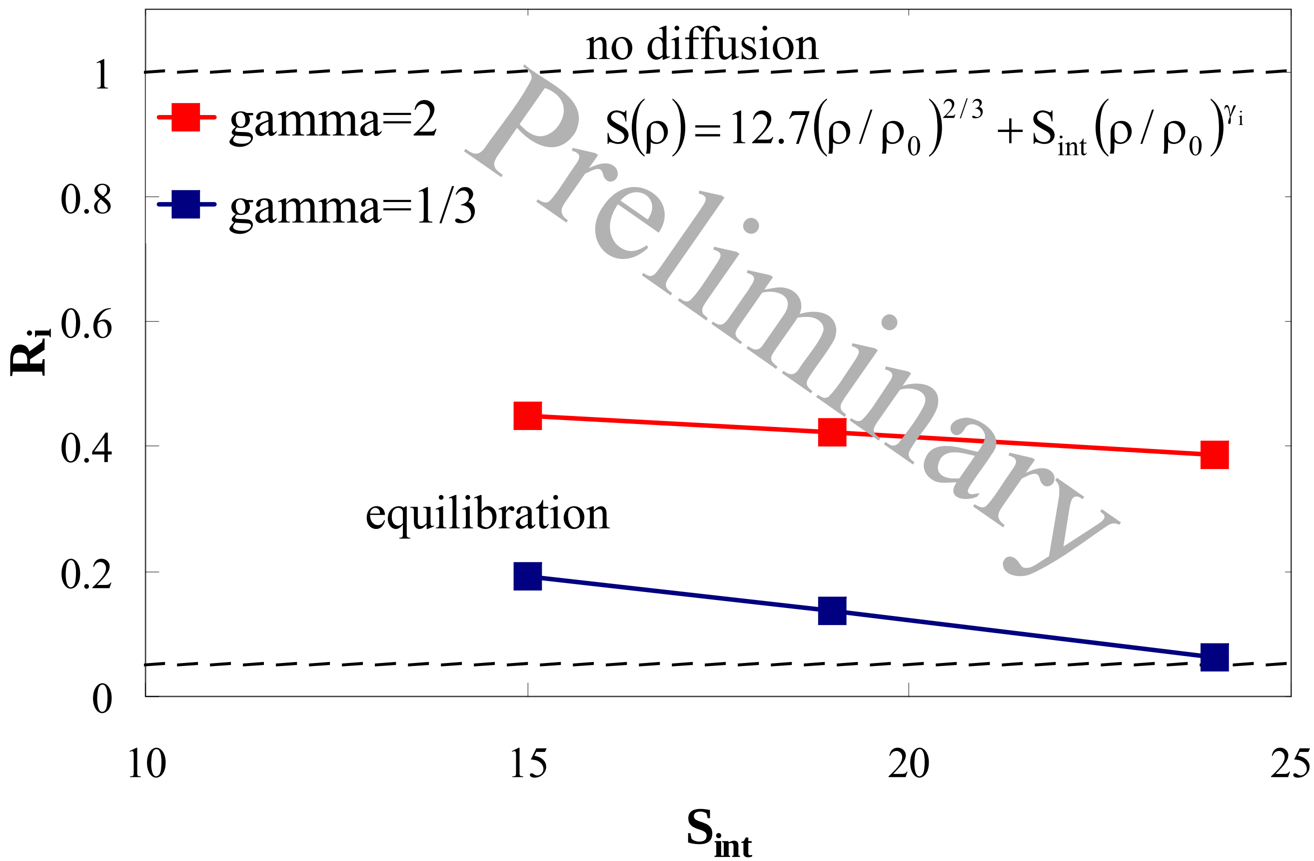
Lijun Shi, thesis

Tsang et al., PRL92(2004)

Relative importance of S_{int} and γ_i

$^{124}Sn+^{112}Sn$, E_{lab}/A =50MeV, b=6 fm

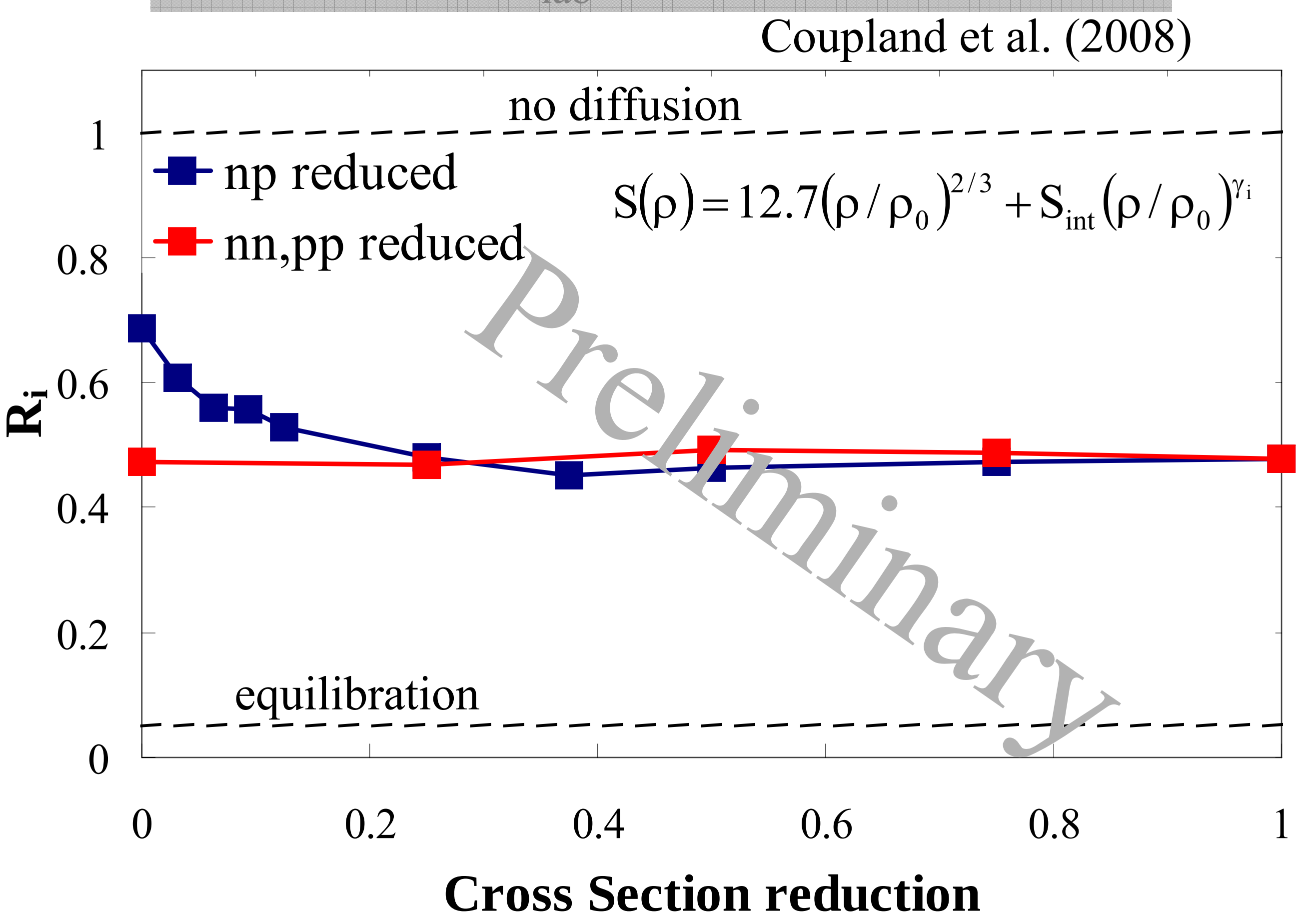
Coupland et al. (2008)



The diffusion is more sensitive to γ_i than to S_{int}

Variation with in-medium cross section

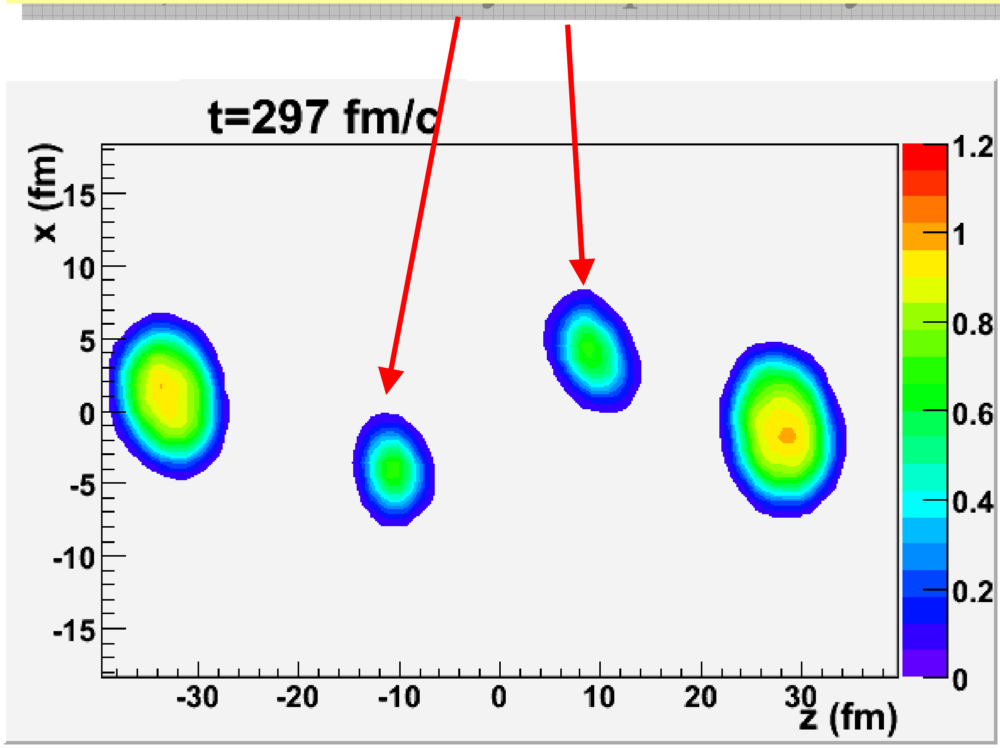
$^{124}Sn+^{112}Sn$, E_{lab}/A =50MeV, b=6 fm



- Reduced np cross section → reduced equilibration!
  - nn, pp cross sections have little effect
- Trend is opposite to simple expectations
  - actually probing diffusion in phase space.

Momentum Dependence

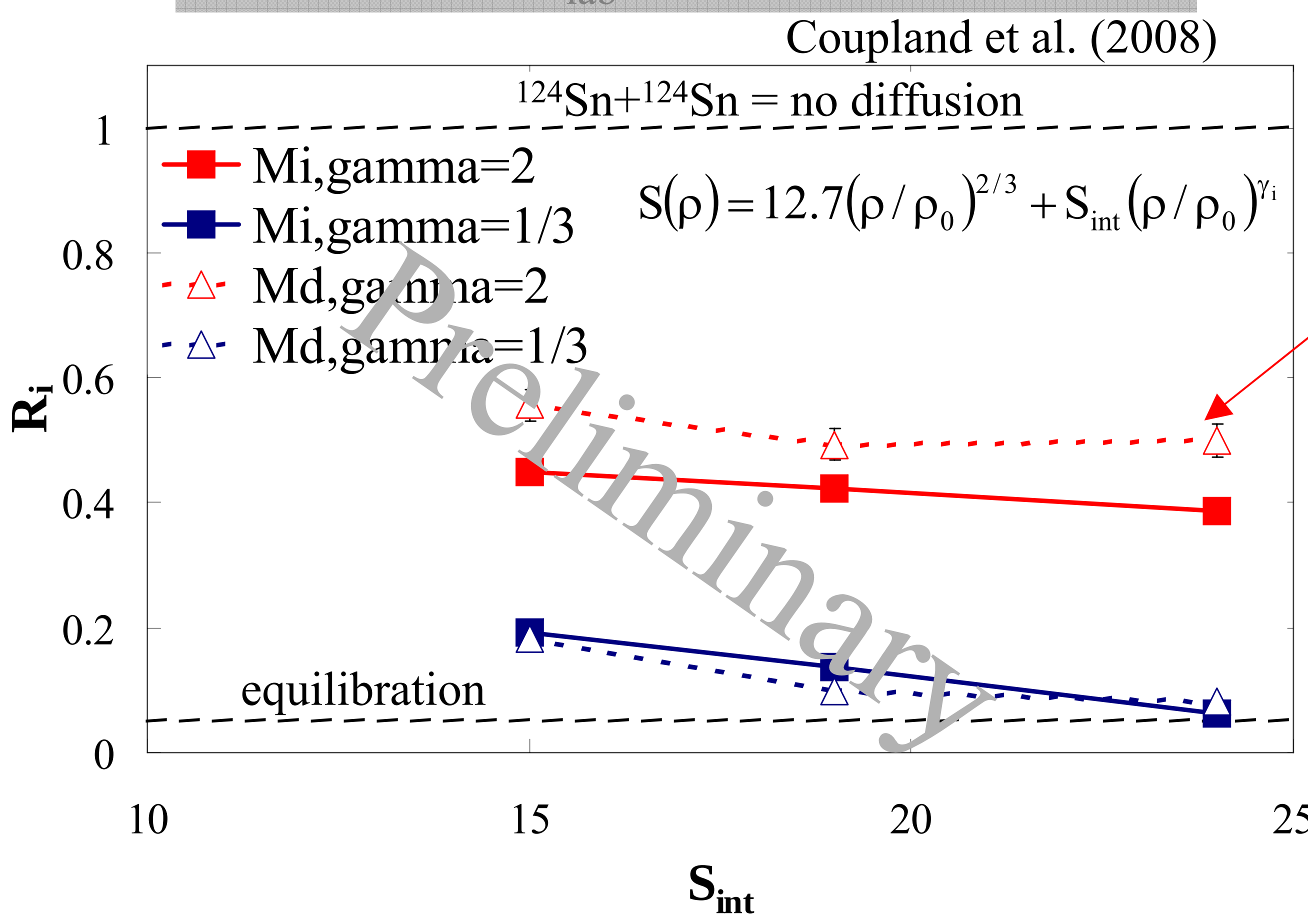
- Characterized by effective mass $m^* = 0.7\ m_N$
- Larger neck fragments formed – the dynamics of the system may be changed by fluctuations of the mean field, enhanced by the proximity to the LGPT



- Suggests that comparisons to both BUU and molecular dynamics calculations should be performed.

Effect of Momentum Dependence

$^{124}Sn+^{112}Sn$, E_{lab}/A =50MeV, b=6 fm



Decreases diffusion for larger γ_i, weaker symmetry energy

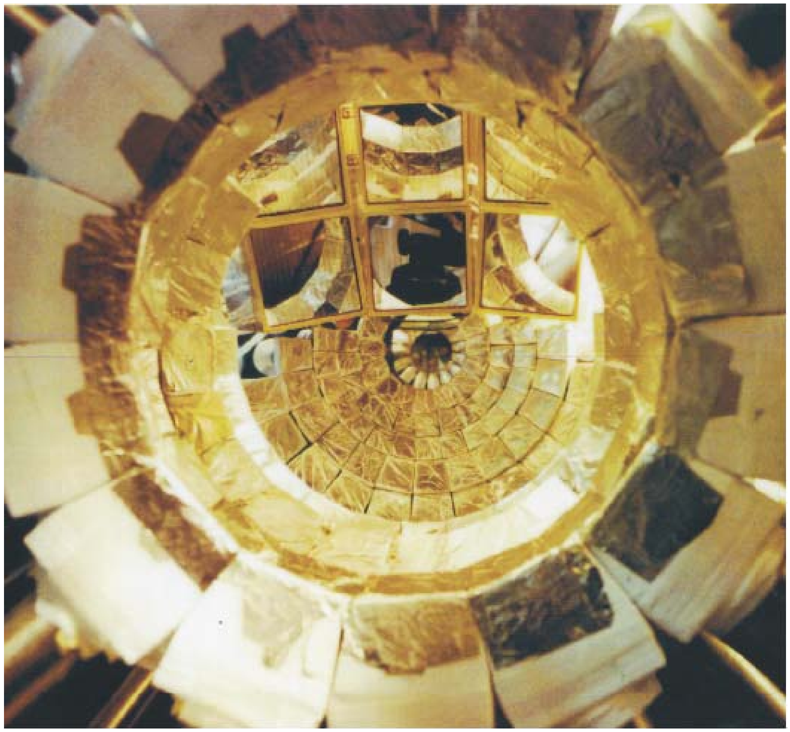
Experimental measurements of isospin diffusion

- Experimental device:

  Miniball with LASSA array

  Experiment:

  $^{112,124}Sn + ^{112,124}Sn$, E/A =50 MeV



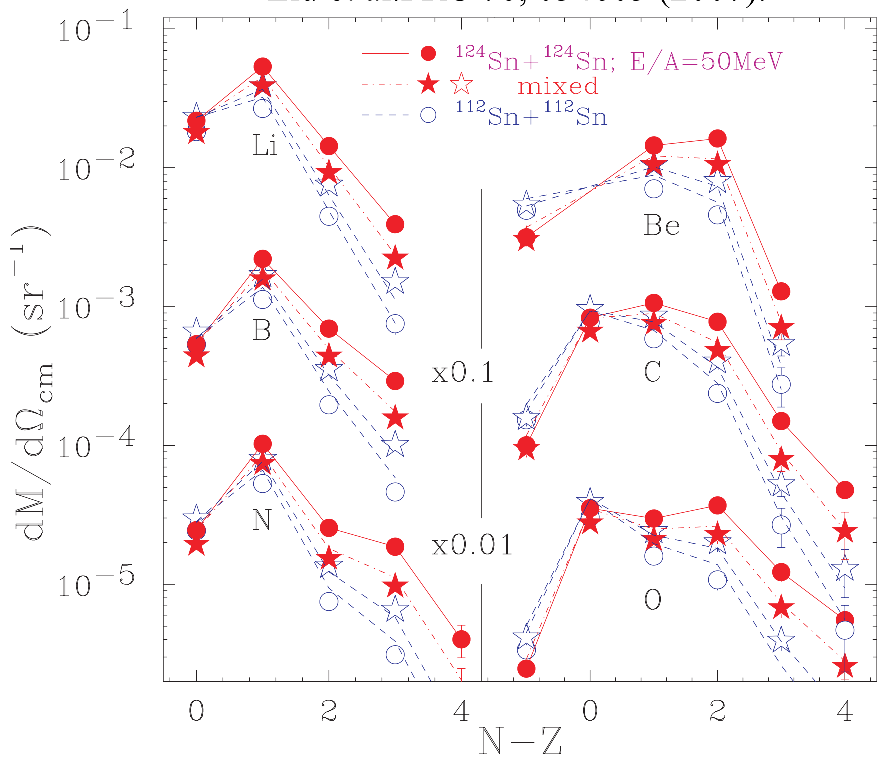
Probing the asymmetry of the Spectators

- The main effect of changing the asymmetry of the projectile spectator remnant is to shift the isotopic distributions of the products of its decay

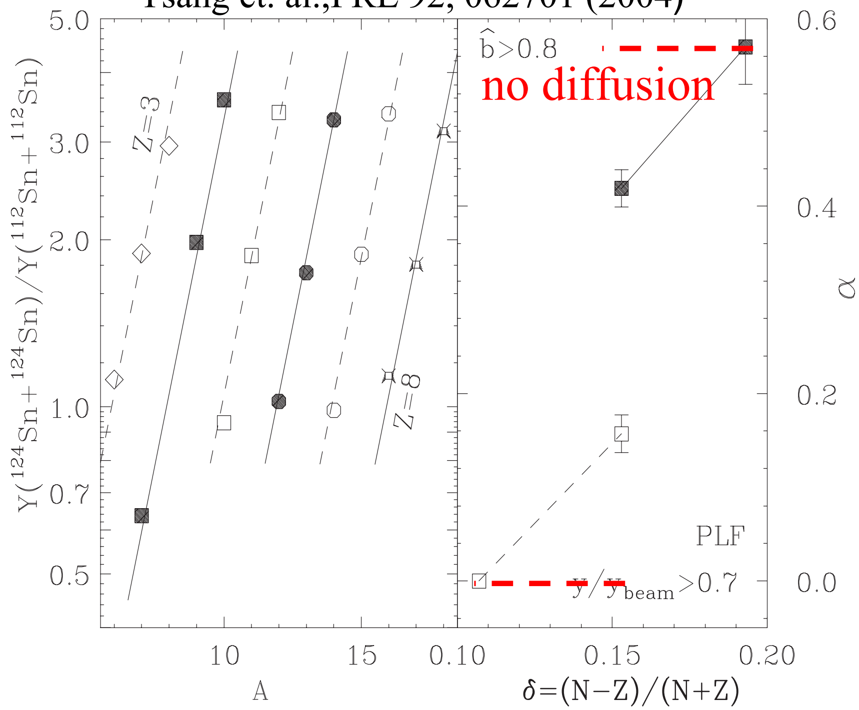
- This can be described by the isoscaling parameters α and β:

$$\frac{Y_2(N,Z)}{Y_1(N,Z)} = C\exp(\alpha N + \beta Z)$$

Liu et al.PRC 76, 034603 (2007).



Tsang et. al.,PRL 92, 062701 (2004).



Determining $R_i(\delta)$

$$R_i(\delta) = 2 \cdot \frac{\delta - (\delta_{Neutron-rich} + \delta_{Proton-rich})/2}{\delta_{Neutron-rich} - \delta_{Proton-rich}}$$

$$\frac{Y_2(N,Z)}{Y_1(N,Z)} = C\exp(\alpha N + \beta Z)$$

- In statistical theory, certain observables depend linearly on δ:

$$\alpha = a\delta_2 + b,$$

$$X_7 = \ln\left[Y\left({}^7\mathrm{Li}\right)/Y\left({}^7\mathrm{Be}\right)\right] \propto c \cdot \delta_2 + d$$

- Calculations confirm this
- We have experimentally confirmed this

- Consider the ratio $R_i(X)$, where $X = \alpha$, X_7 or some other observable:

$$R_i(X) = 2 \cdot \frac{X - (X_{Neutron-rich} + X_{Proton-rich})/2}{X_{Neutron-rich} - X_{Proton-rich}}$$

- If X depends linearly on δ_2:

$$X = a \cdot \delta_2 + b$$

- Then by direct substitution:

$$R_i(X) = R_i(\delta_2)$$

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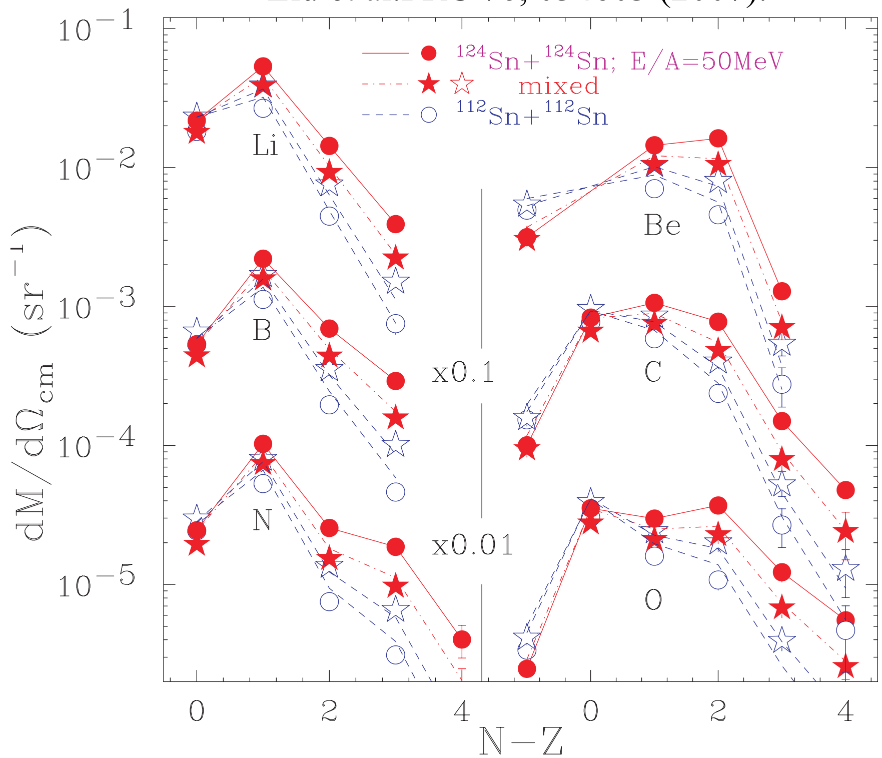
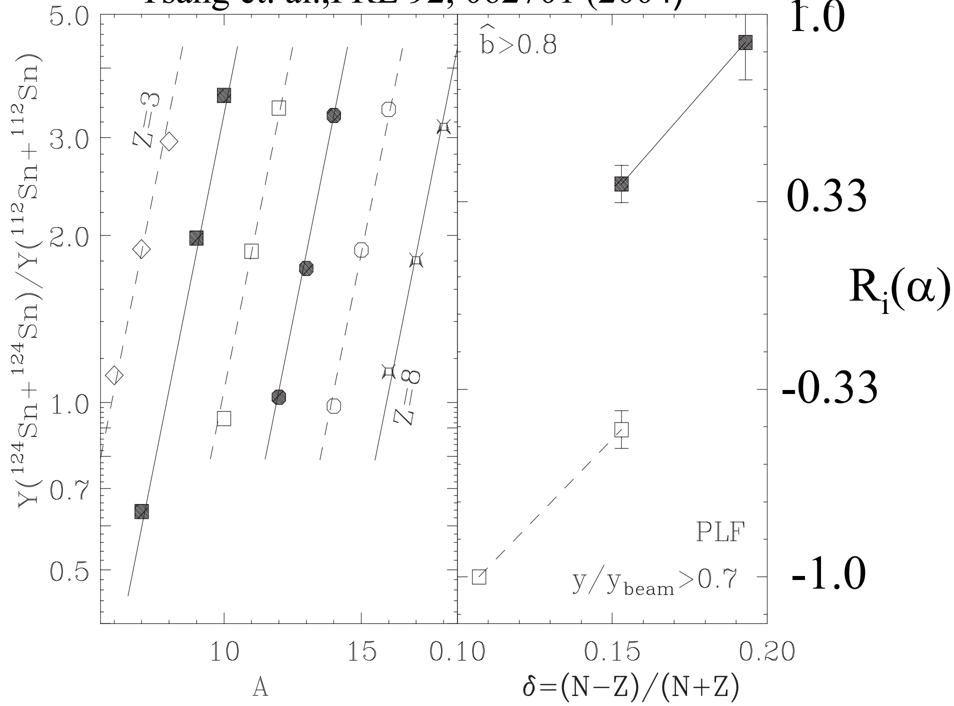
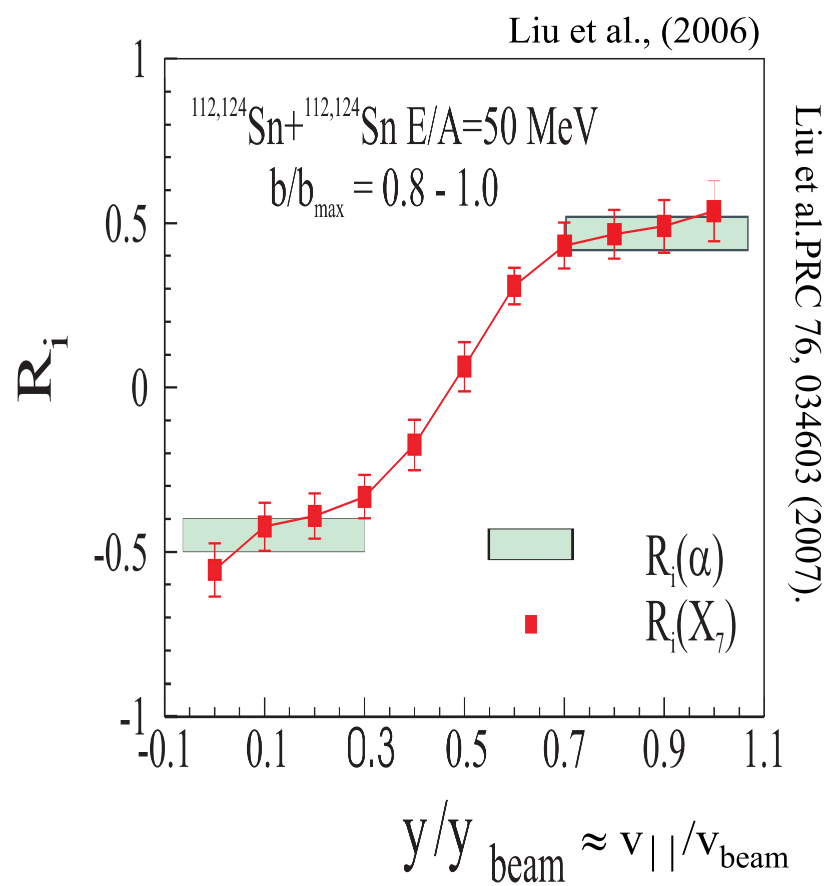


Tsang et. al.,PRL 92, 062701 (2004)



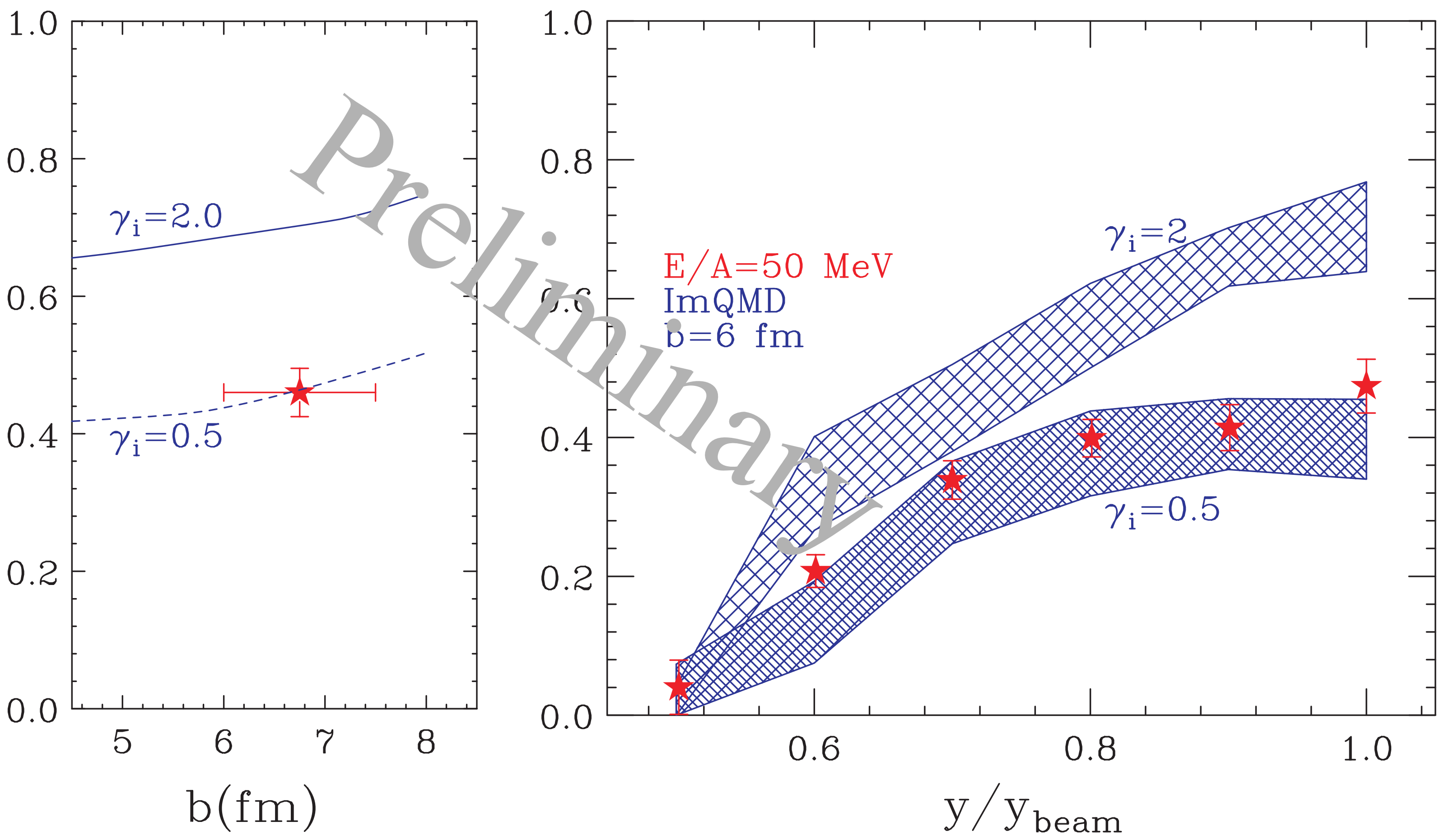
Quantitative values

- Reactions:
  - $^{124}Sn+^{112}Sn$: diffusion
  - $^{124}Sn+^{124}Sn$: neutron-rich limit
  - $^{112}Sn+^{112}Sn$: proton-rich limit
- Exchanging the target and projectile allowed the full rapidity dependence to be measured.
- Gates were set on the values for $R_i(\alpha)$ near beam rapidity.
  - $R_i(\alpha) \approx 0.47 \pm 0.05$ for $^{124}Sn+^{112}Sn$
  - $R_i(\alpha) \approx -0.44 \pm 0.05$ for $^{112}Sn+^{124}Sn$
- Obtained similar values for $R_i(\ln(Y(^7Li)/Y(^7Be))$
  - Allows exploration of dependence on rapidity and transverse momentum.



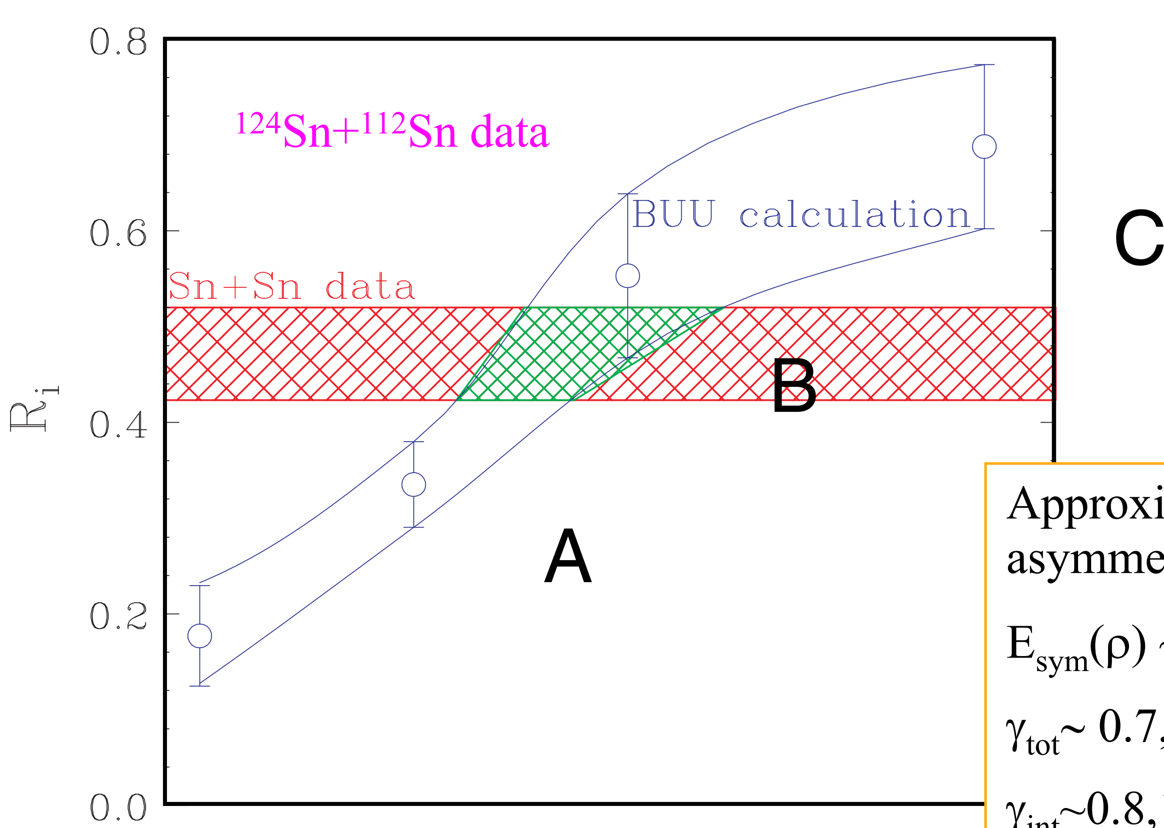
Comparison to QMD calculations

- IQMD calculations were performed for γ_i=0.5 and 2.0, S_{int}=19 MeV.
- Momentum dependent mean fields with $m_n^*/m_n = m_p^*/m_p = 0.7$ were used.



Constraints from BUU calculations

- BUU calculations were performed for S_{int}=19 and various values of γ_i MeV.
- Momentum dependent mean fields with $m_n^*/m_n \approx 0.75$, $m_p^*/m_p \approx 0.65$ at $\rho \approx \rho_0$ and δ=0.2 were used.



M.B. Tsang et. al., PRL 92, 062701 (2004)

L.W. Chen, C.M. Ko, and B.A. Li, PRL 94, 032701 (2005)

C.J. Horowitz and J. Piekarewicz, PRL 86, 5647 (2001)

B.A. Li and A.W. Steiner, nucl-th/0511064

Approximate representation of the various asymmetry terms used in BUU calcuations:

$E_{sym}(\rho) \sim 32(\rho/\rho_0)^\gamma\ [(\rho_n - \rho_p)/(\rho_n + \rho_p)]^2$

$\gamma_{tot} \sim$ 0.7, 1.0, 1.6 (for cases A, B, C)

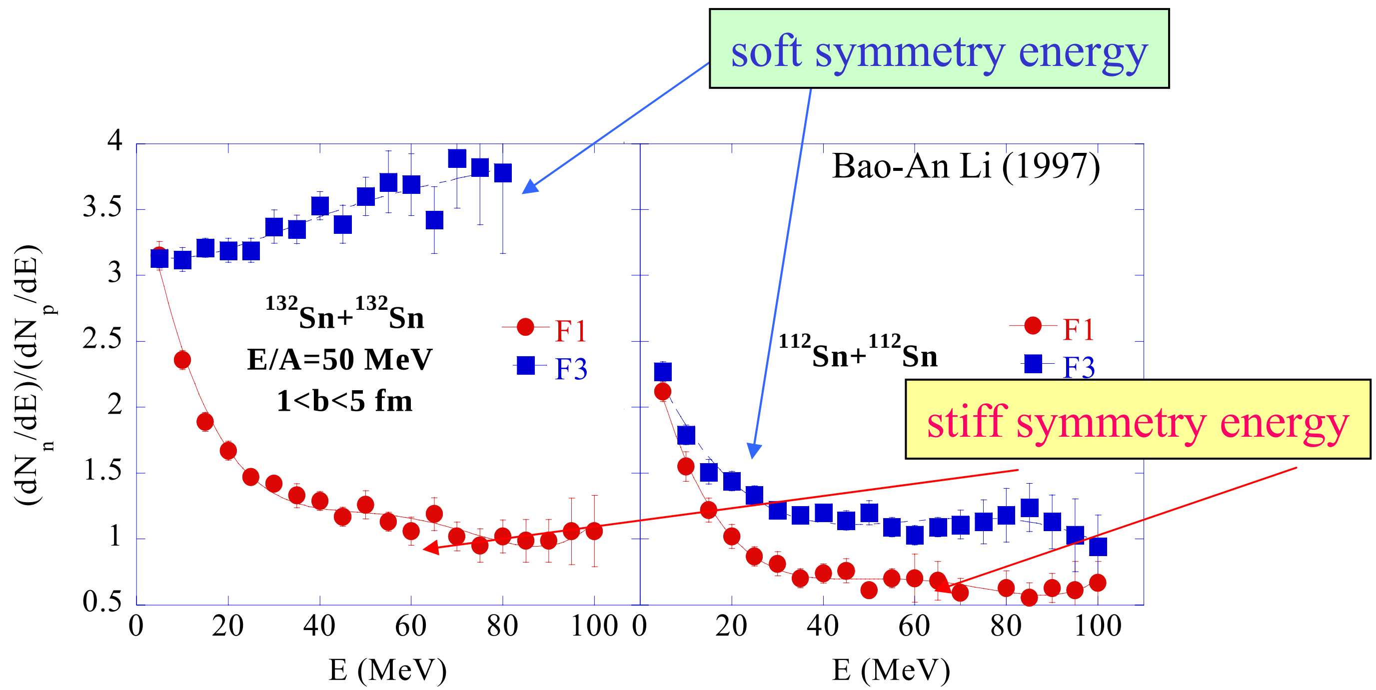
$\gamma_{int} \sim$0.8,1.4, ?

Tightening the constraints at sub-saturation density

- Better constraining the sources of emission and the emission timescale ▶
- Improving and understanding better the impact parameter determination. ▶
- Lowering the incident energy or using rare isotope beams ▶
- Improving the constraints on $S(\rho_0)$, m* and σ_{NN} ? ▶
- Investigating with other observables. ▶

Measurement of n/p spectral ratios: also probes the asymmetry term at $\rho \leq \rho_0$.

- Isospin fractionation: Expulsion of neutrons from neutron-rich fragments by symmetry energy.Larger for soft symmetry energy
- Has been probed by direct measurements of n vs. proton emission rates (more direct) or by fragment isotopic distributions.



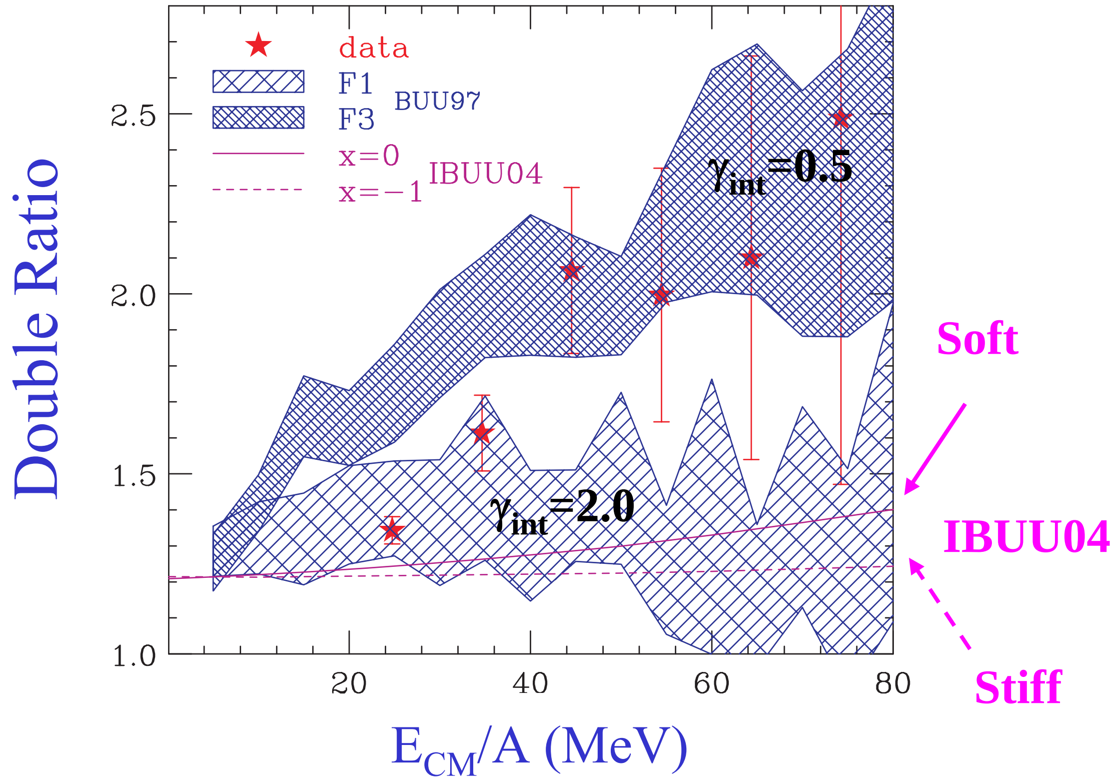
•Above were results from IBUU97 transport theory of Bao-An Li.

n/p emission and flow ($1/3 \leq \rho/\rho_0 \leq 1.5$)

(Famiano et al, PRL 97, 052701(2006))

Double Ratio:

$$\frac{^{124}Sn + ^{124}Sn; Y(n)/Y(p)}{^{112}Sn + ^{112}Sn; Y(n)/Y(p)}$$

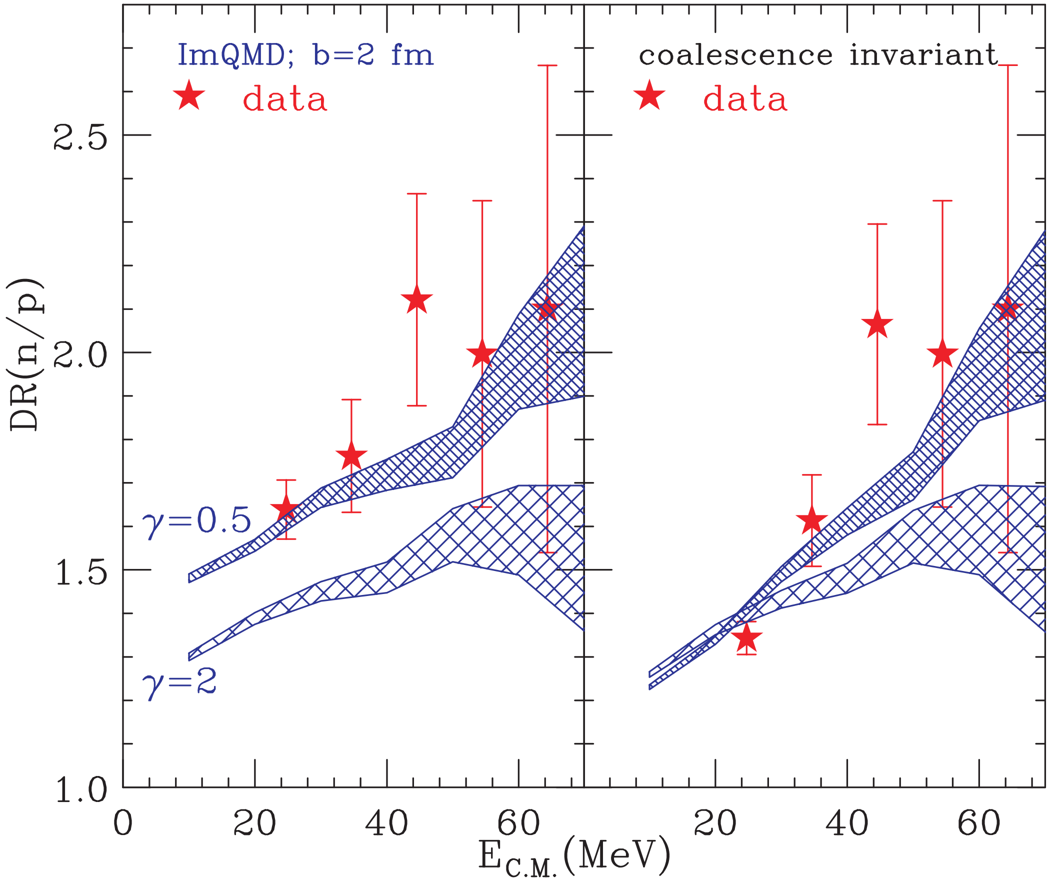


- Double ratio cancels out uncertainties in the neutron efficiencies.
- BUU does not calculate cluster emission. Comparisons can be made to coalescence invariant yields which include both free nucleons and those bound in clusters.
  - Original comparison was made to BUU97, which does not include momentum dependence of the nuclear mean field and the isospin dep. cross sections.
  - Comparison between IBUU04, BUU97 and data is problematic.

Possible issues and strategies:

- Comparison between BUU97 and IBUU03
  - Different codes: BUU97 cannot be located
  - IBUU04 has momentum dependent mean fields with different neutron and proton effective masses. BUU97 does not.
  - IBUU04 has different NN cross sections.
- Comparison with data:
  - Cluster production changes the n/p spectral ratios. Production of isoscaler alpha particles enhances the asymmetry of the free nucleons: (Dempsey, Shi)
  - Some transport properties in either or both calculations may be incorrect (i.e. symmetry energies, σ_{NN}, m^*_n , m^*_p .)
- Strategy:
  - Examine the role of cluster production with ImQMD and possibly AMD.
  - Investigate the influence of each BUU transport property.

Cluster production predictions using ImQMD



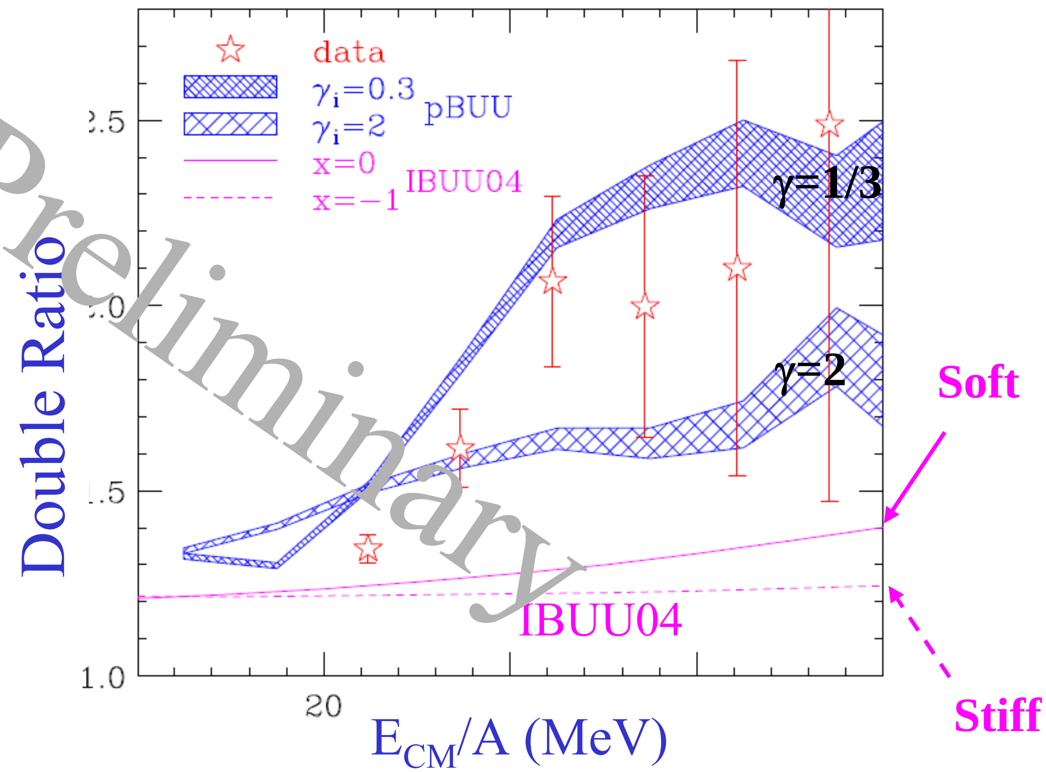
Yingxun Zhang PLB (2008).

- Calculation includes same symmetry energy as BUU97, as well as momentum dependent mean fields, free cross sections.
- Calculations reproduce free and coalescence invariant ratios.
- Shows that cluster production is important to final values. Role of cluster production in the dynamics needs more investigation.

BUU investigations: Comparison of Danielewicz code to IBUU04

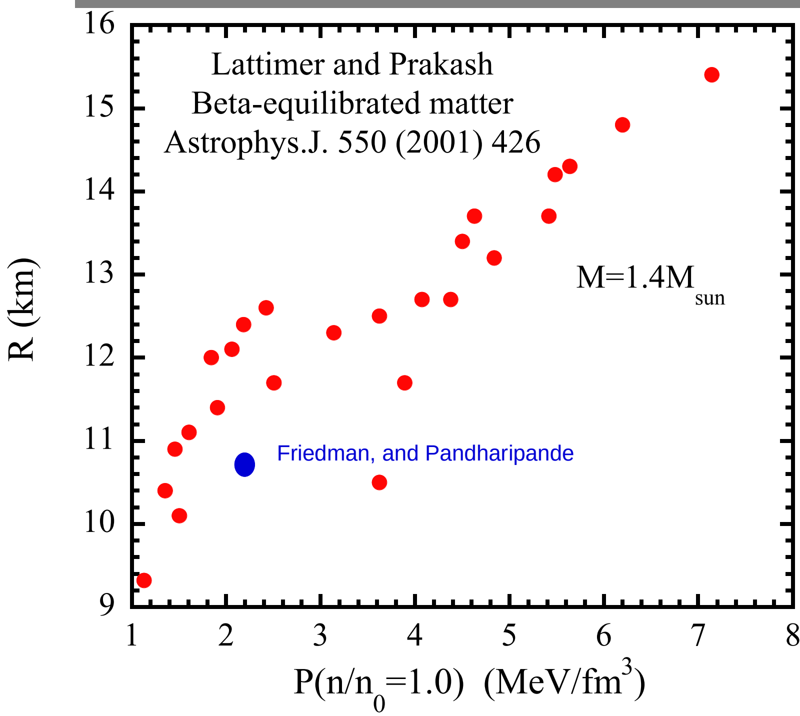
Double Ratio:

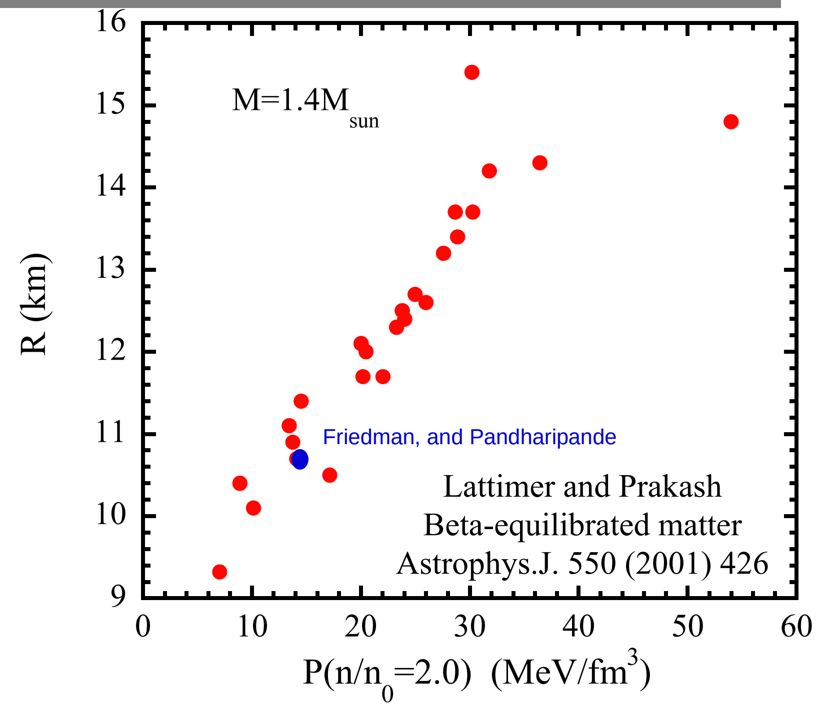
$$\frac{^{124}Sn+^{124}Sn;Y(n)/Y(p)}{^{112}Sn+^{112}Sn;Y(n)/Y(p)}$$



- Danielewicz code (PBUU) double ratios appear similar to the BUU97 results of Bao-an Li, but different from IBUU04 results.
  - Allows thorough explorations of the influence of nucleon-nucleon cross sections, nucleon effective masses, and cluster production effects.
- PBUU similar predictions regarding the isospin diffusion that are similar to IBUU04 (Coupland R14 5)
  - May enable consistent constraints on the symmetry energy.

Extrapolation to neutron star radius





- The neutron star radius is not strongly correlated with the symmetry pressure at saturation density.
  - This portends difficulties in uniquely constraining neutron star radii for constraints at subsaturation density.

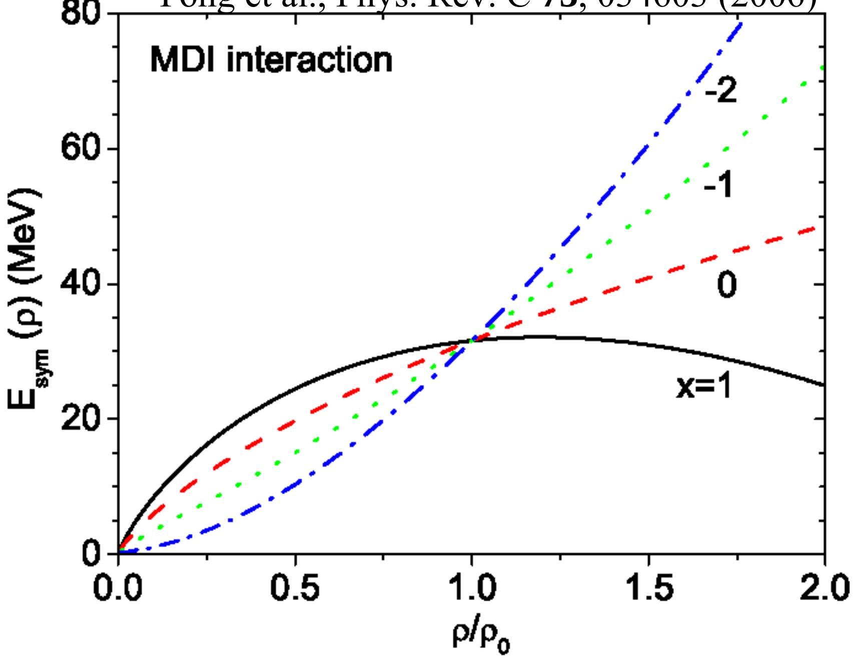
- The correlation between the pressure at twice saturation density and the neutron star radius is much stronger.
  - additional measurements at supra-saturation density will lead to stronger constraints.

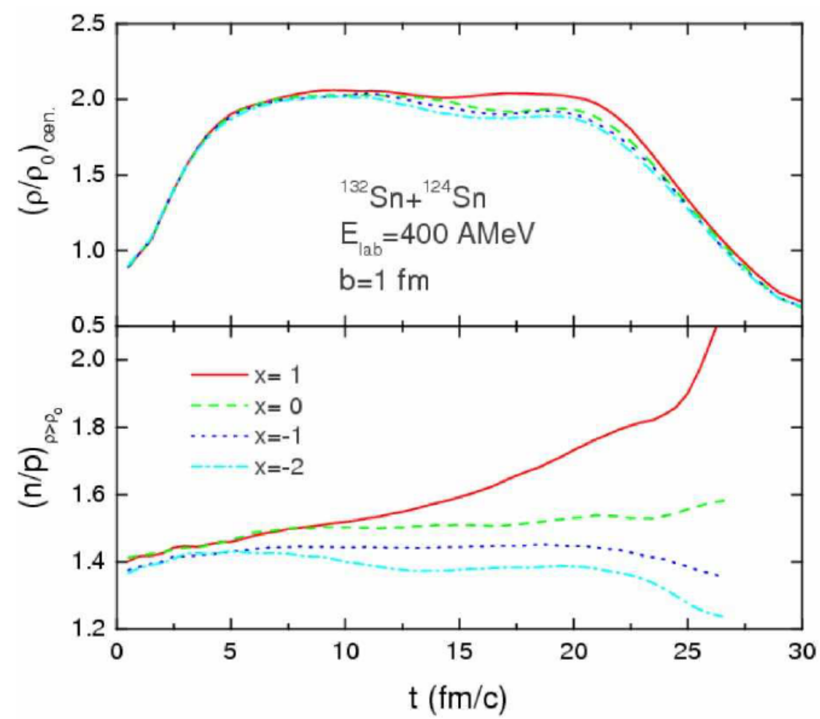
➢ Would be advisable to have multiple probes that can sample different densities

Asymmetry term studies at $\rho \approx 2\rho_0$

- Densities of $1.7\rho_0$ at E/A ≈200 MeV, $2\rho_0$ at E/A ≈400 MeV can be achieved.
  - Provides information about direct Urca cooling in proto-neutron stars, stability and phase transitions of dense neutron star interior.
- $S(\rho)$ influences diffusion of neutrons from dense overlap region at b=0.
  - Diffusion is reduced, neutron-rich dense region is formed for soft $S(\rho)$.

Yong et al., Phys. Rev. C **73**, 034603 (2006)

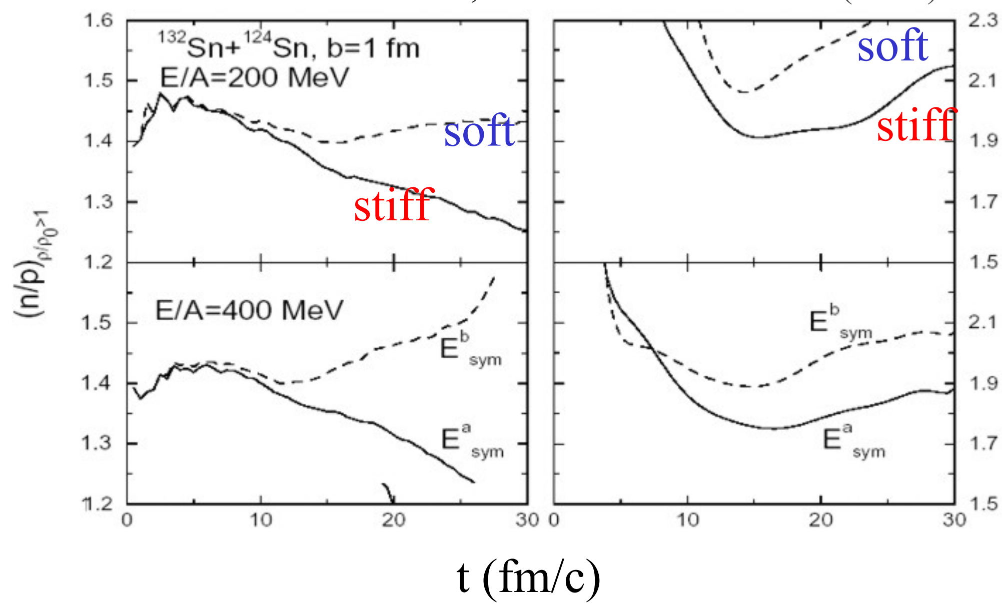




High density probe: pion production

- Larger values for ρ_n / ρ_p at high density for the soft asymmetry term (x=0) causes stronger emission of negative pions for the soft asymmetry term (x=0) than for the stiff one (x=-1).
- π^- / π^+ means $Y(\pi^-)/Y(\pi^+)$
  - In delta resonance model, $Y(\pi^-)/Y(\pi^+) \approx (\rho_n/\rho_p)^2$
  - In equilibrium, $\mu(\pi^+)-\mu(\pi^-)=2(\mu_p-\mu_n)$
- The density dependence of the asymmetry term changes ratio by about 10% for neutron rich system.

Li et al., arXiv:nucl-th/0312026 (2003).



- This can be explored with stable or rare isotope beams at the MSU CCF, ISF, RIKEN, or GSI.
  - Sensitivity to $S(\rho)$ occurs primarily near threshold in A+A

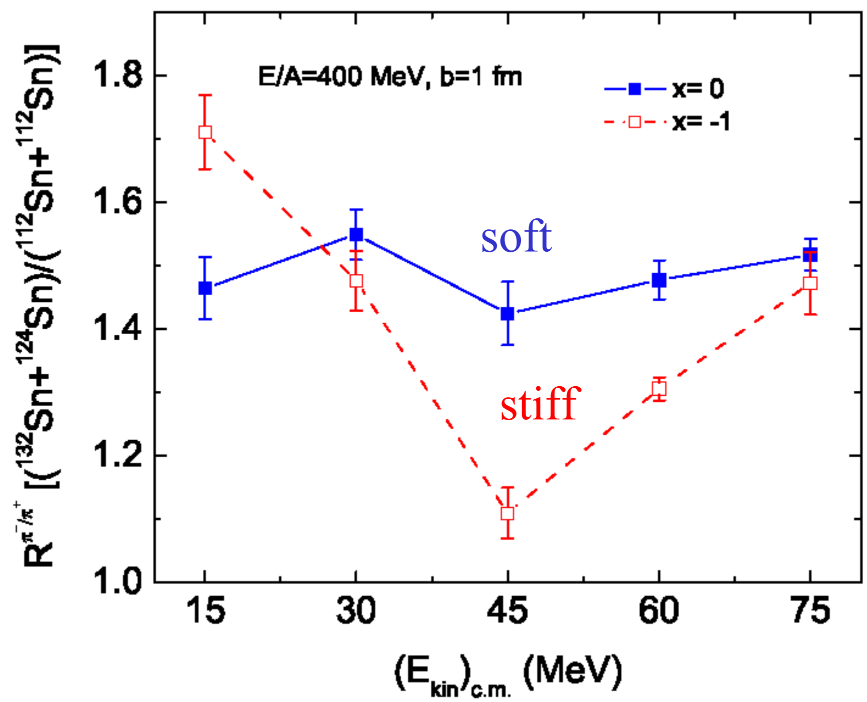
Double ratio: pion production

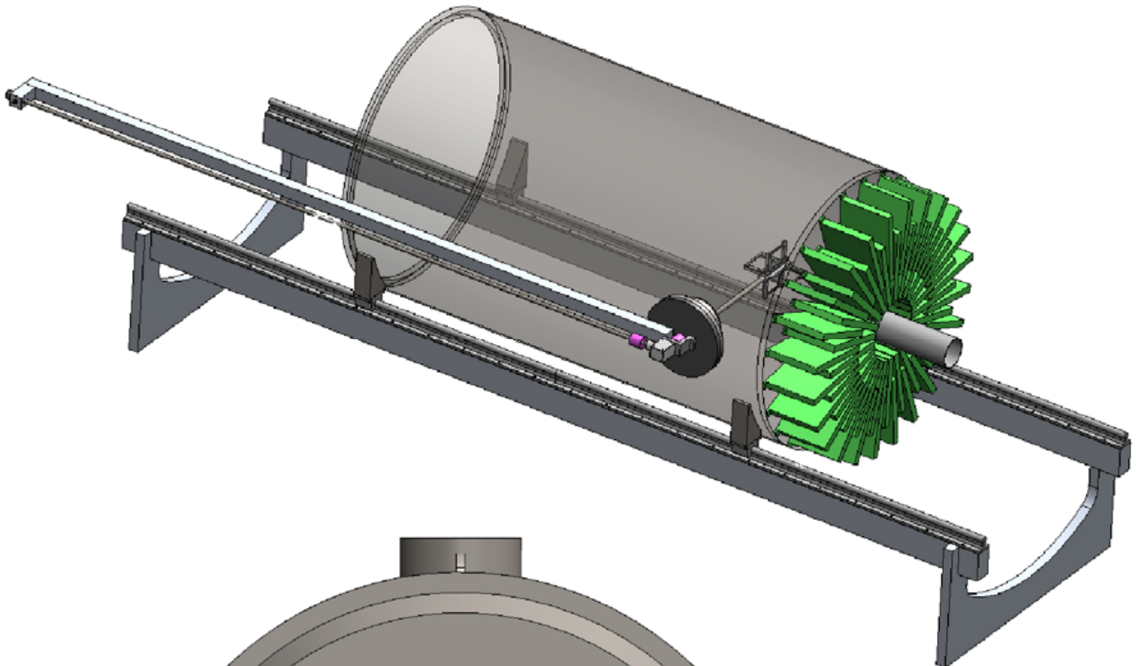
- Double ratio involves comparison between neutron rich $^{132}Sn+^{124}Sn$ and neutron deficient $^{112}Sn+^{112}Sn$ reactions.

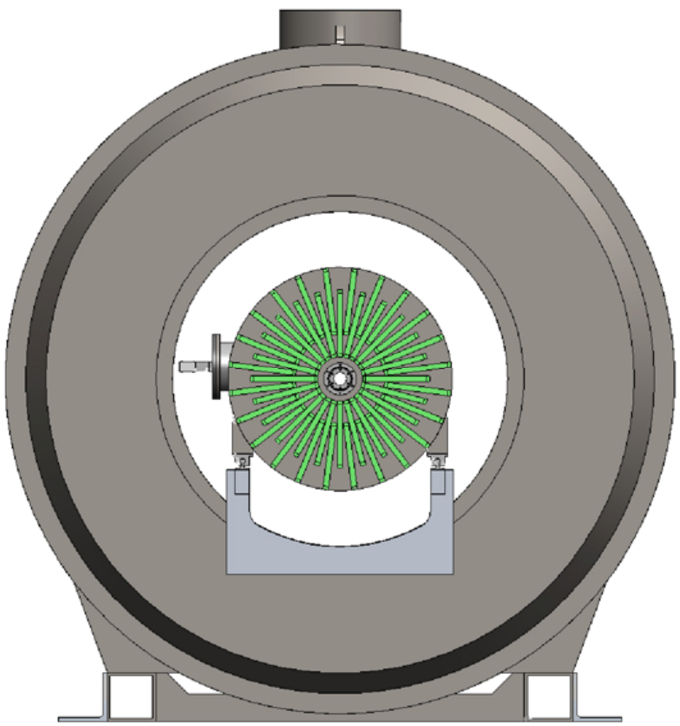
$$R^{\pi^-/\pi^+}\left(^{132}Sn+^{124}Sn\right)/\left(^{112}Sn+^{112}Sn\right) = \frac{\left[Y\left(\pi^-\right)_{132+124} / Y\left(\pi^+\right)_{132+124}\right]}{\left[Y\left(\pi^-\right)_{112+112} / Y\left(\pi^+\right)_{112+112}\right]}$$

- Double ratio maximizes sensitivity to asymmetry term.
  - Largely removes sensitivity to difference between π^- and π^+ acceptances.

Yong et al., Phys. Rev. C **73**, 034603 (2006)







Can be probed at FRIB with proposed Active Target –TPC: AT-TPC
Cost: $1.3 MD
~1/1000 of IXO

Summary and Outlook

- Heavy Ion collisions with rare isotope beams can address some compelling issues.
- The EOS of dense asymmetric matter is of fundamental importance to nuclei and neutron stairs.
- Calculations suggest a number of promising observables that can probe the density dependence of the symmetry energy.
  - Isospin diffusion, isotope ratios, and n/p spectral ratios may provide some constraints at $\rho \leq \rho_{0,}$.
  - π^+ vs. π^- production, Kaon and Σ hyperon production, neutron/proton spectra and flows may provide constraints at $\rho \approx 2\rho_0$ and above.
- The availability of fast stable and rare isotope beams at a variety of energies at MSU, GANIL, RIKEN and GSI allows the exploration of the symmetry energy at a range of densities.