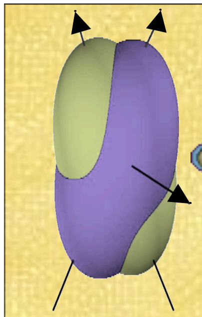


Fermion Systems with Fuzzy Symmetries

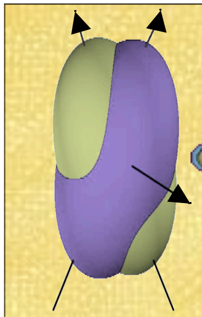
Leveraging the Known to Understand the Unknown

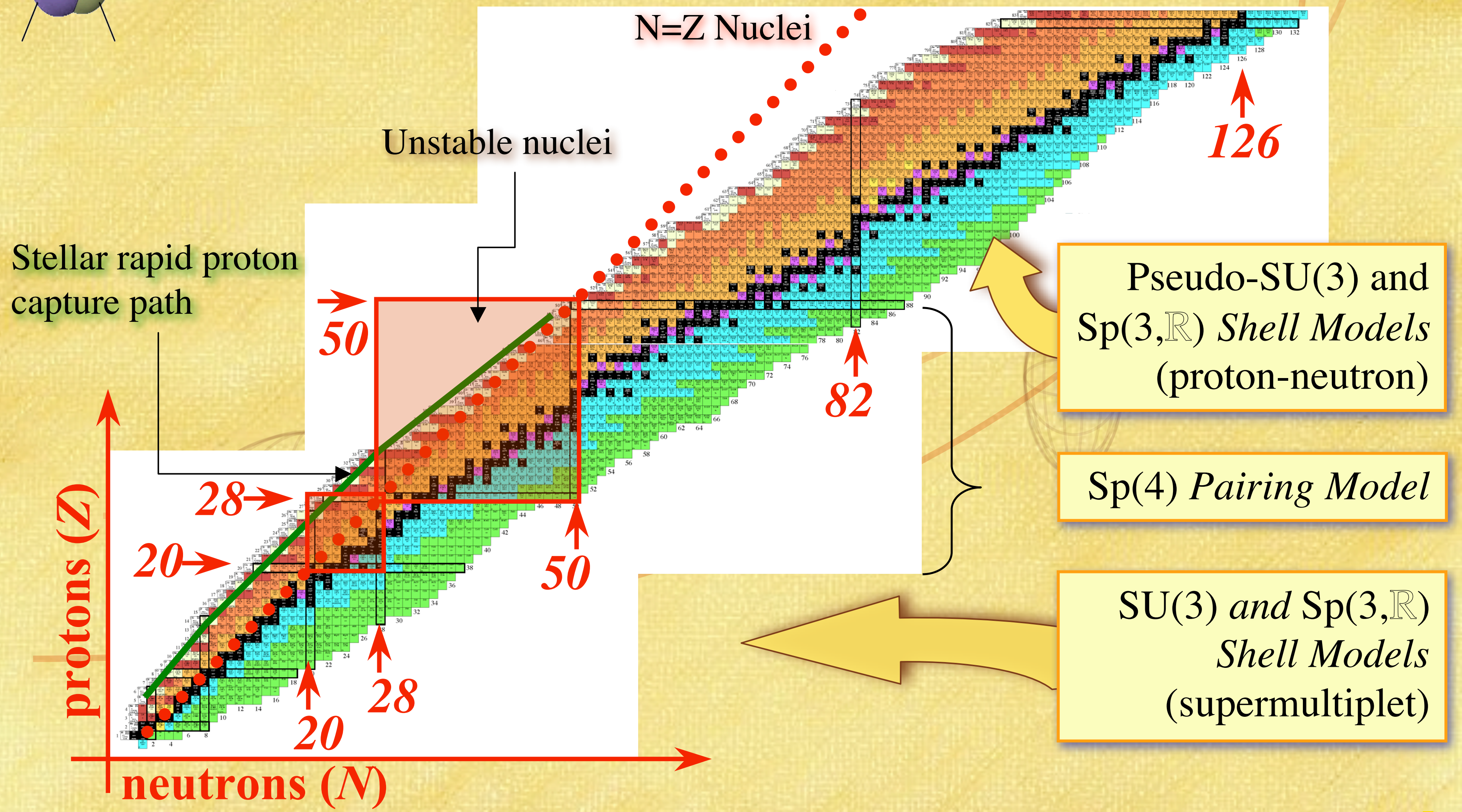
J. P. Draayer

3rd ANL/MSU/INT/JINA
RIA Theory Meeting, April 2006

Louisiana State University

Light to Heavy Nuclei





N=Z Nuclei

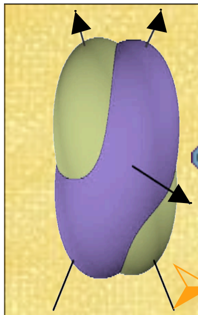
Unstable nuclei

Stellar rapid proton capture path

Pseudo-SU(3) and Sp(3,$\mathbb{R}$) *Shell Models* (proton-neutron)

Sp(4) *Pairing Model*

SU(3) *and* Sp(3,$\mathbb{R}$) *Shell Models* (supermultiplet)



The Models

- SU(3) and Sp(3,ℝ) models for a microscopic description of *collective phenomena* (plus pseudo-symmetry counterparts)

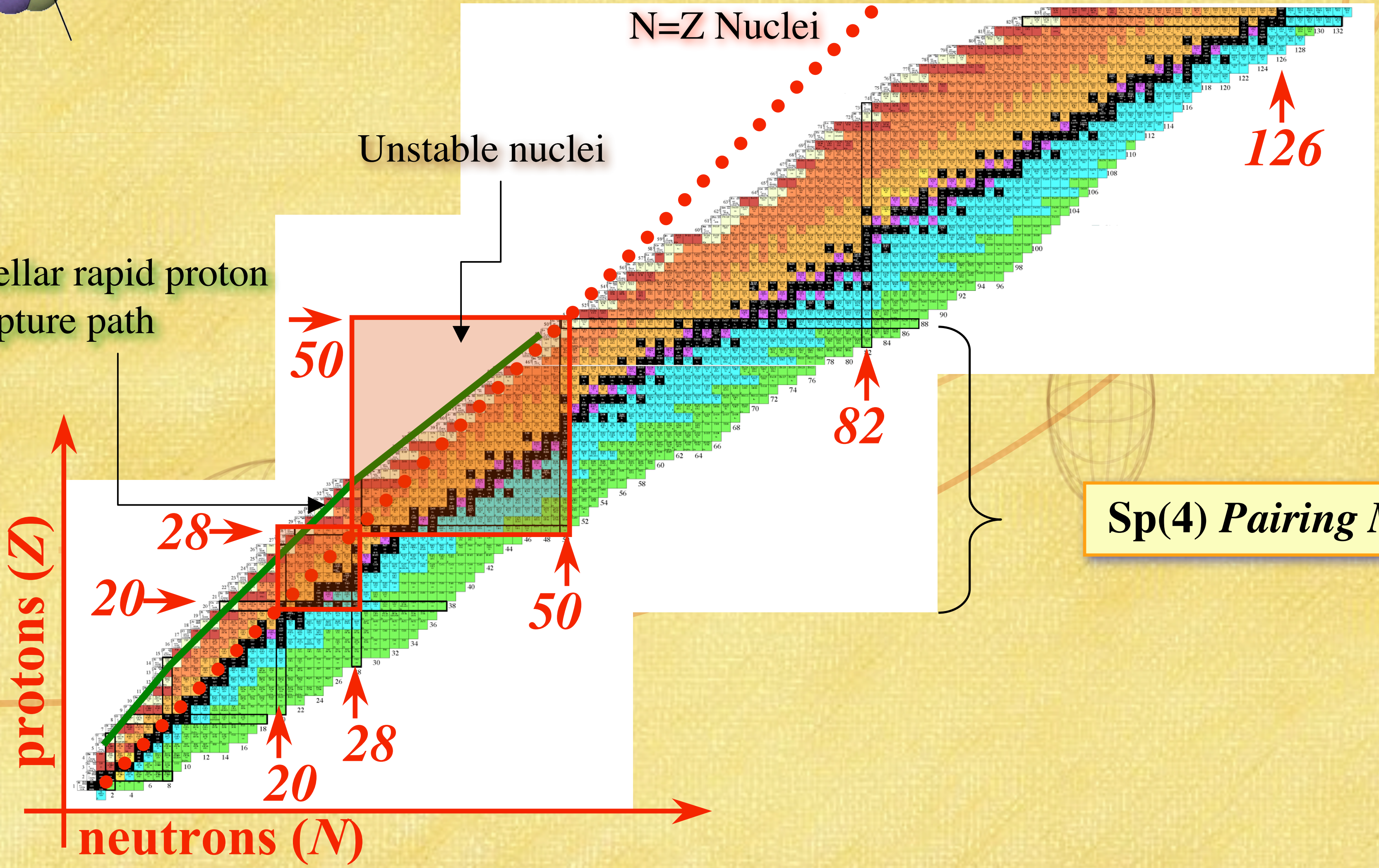
$$\mathrm{U}(4\Omega)\supset \begin{array}{l} \mathrm{U}(\Omega)\supset \boxed{\mathrm{SU}(3)}\supset \mathrm{SO}(3) \\ \otimes \\ \mathrm{U}(4)\supset \mathrm{SU}(4)\supset \mathrm{SU}_S(2)\otimes \mathrm{SU}_T(2) \end{array}$$

$$\begin{array}{l} \boxed{\mathrm{Sp}(3,\mathbf{R})}\supset \mathrm{U}(1)\otimes \mathrm{SU}(3)\supset \mathrm{SO}(3) \\ \otimes \\ \mathrm{SU}(4)\ \supset \mathrm{SU}_S(2)\otimes \mathrm{SU}_T(2) \end{array}$$

- Sp(4) model ($\mathfrak{sp}(4)\sim\mathfrak{so}(5)$) for a microscopic description of *pp+nn+pn* isovector (isospin T=1) *pairing correlations* in pairing-governed isobaric analog 0^+ states:

$$\begin{array}{lcc} \mathrm{U}(4\Omega)\supset & \mathrm{Sp}(4\Omega)\otimes & \boxed{\mathrm{SO}(5)} \\ & \cup & \\ & \mathrm{SO}(3) & \end{array}$$

Medium Mass Nuclei



N=Z Nuclei

Unstable nuclei

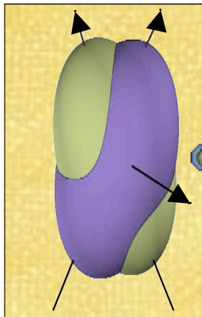
Stellar rapid proton capture path

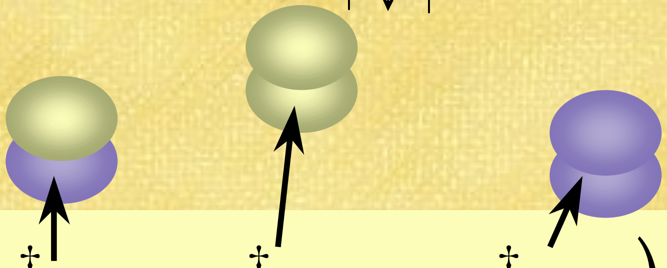
Sp(4) *Pairing Model*

Sp(4) Dynamical Symmetry

$$(A_1^\dagger)^{n_1}(A_0^\dagger)^{n_0}(A_{-1}^\dagger)^{n_{-1}}|0\rangle$$

basis states

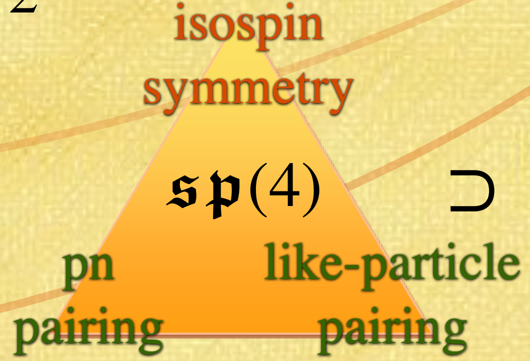




$$H = -\varepsilon\hat{N} - G\left(A_0^\dagger A_0 + A_1^\dagger A_{-1} + A_{-1}^\dagger A_1\right) - \frac{E}{2\Omega}\left(T^2 - \frac{3\hat{N}}{4}\right) - C\,\frac{\hat{N}(\hat{N}-1)}{2}$$

Isovector (isospin 1)
***J*=0 pairing** interaction

diagonal isoscalar
(isospin 0) ***pn*** force
[symmetry term]



isospin symmetry

$\mathfrak{sp}(4)$

pn pairing

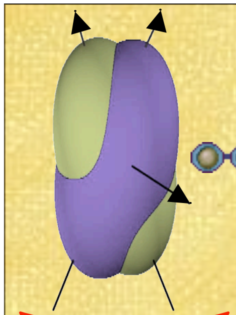
like-particle pairing

$$\mathfrak{sp}(4) \supset \begin{cases} \mathfrak{su}^T(2) \oplus \mathfrak{u}^N(1) \\ \mathfrak{su}^{pn}(2) \oplus \mathfrak{u}^{T_0}(1) \\ \mathfrak{su}^{pp}(2) \oplus \mathfrak{su}^{nn}(2) \end{cases}$$

Algebraic: in terms of second-order *su*(2) Casimir invariants

Microscopic: fermion realization, protons $(c^\dagger_{jm,\frac{1}{2}}, c_{jm,\frac{1}{2}})$ & neutrons $(c^\dagger_{jm,-\frac{1}{2}}, c_{jm,-\frac{1}{2}})$

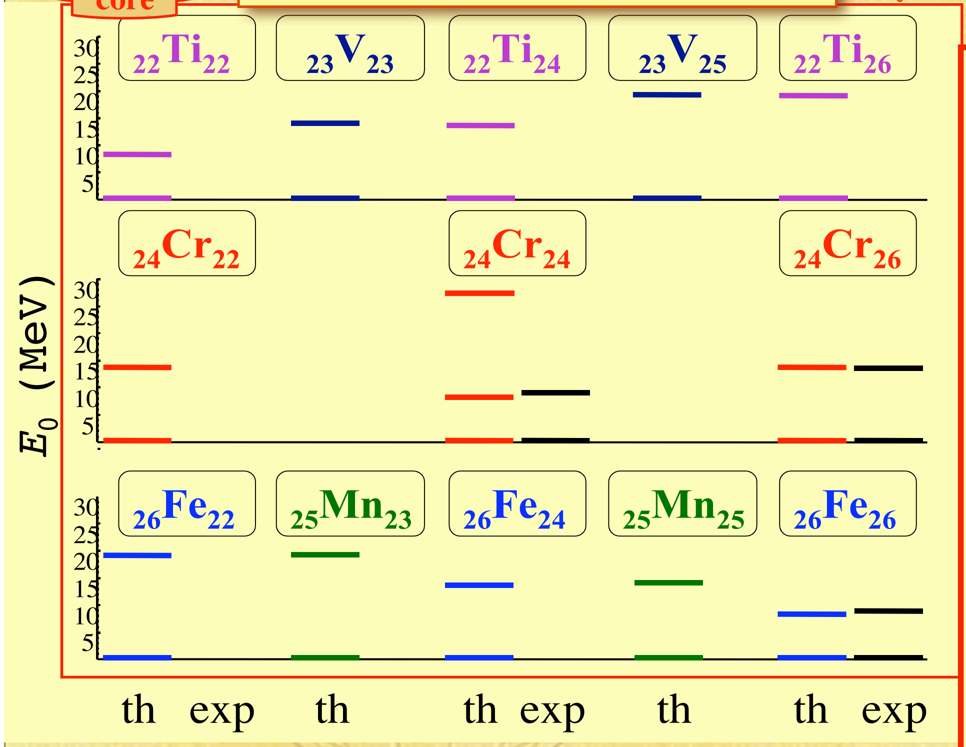
Reproduction of Isobaric Analog 0^+ State Energies

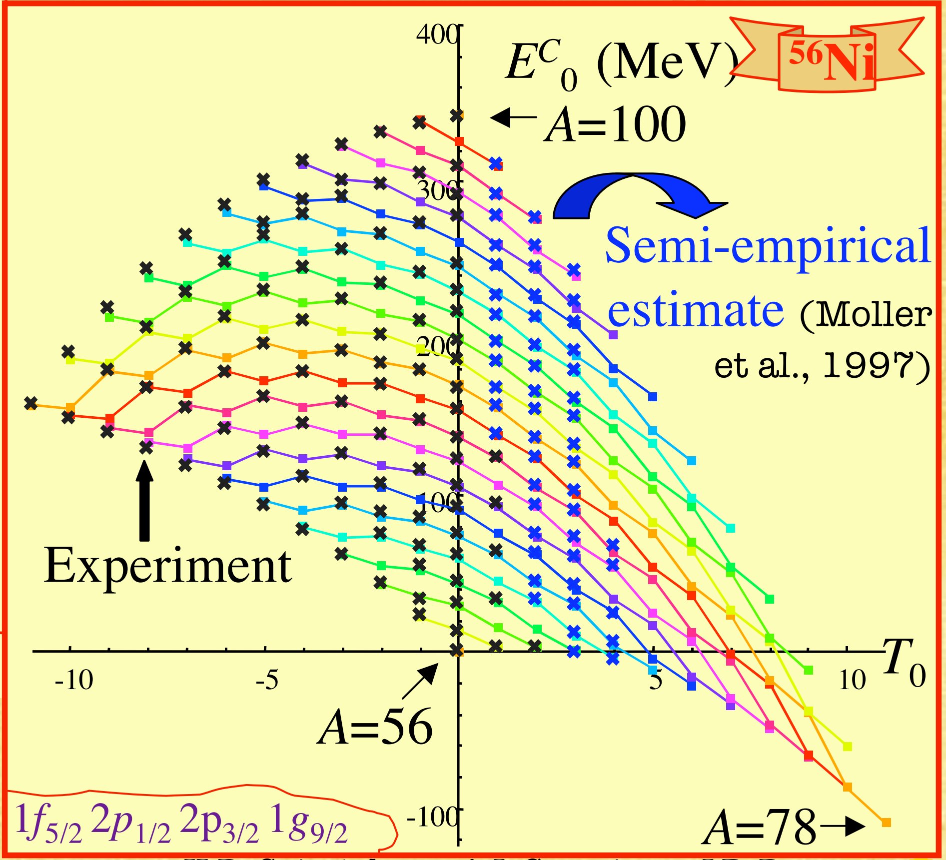


Energy spectra of pairing-governed 0^+ IAS of **319** nuclei and only **6** parameters

^{40}Ca core

Without parameter variation

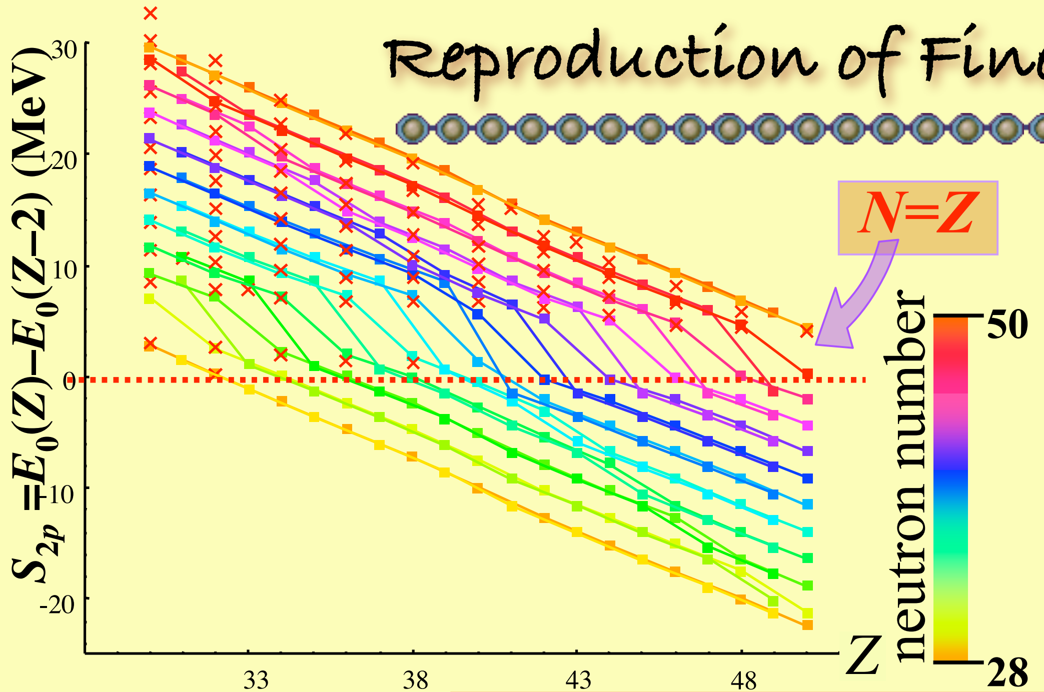




Semi-empirical estimate (Moller et al., 1997)

K.D. Sviratcheva, A.I. Georgieva, J.P. Draayer

Reproduction of Fine Structure



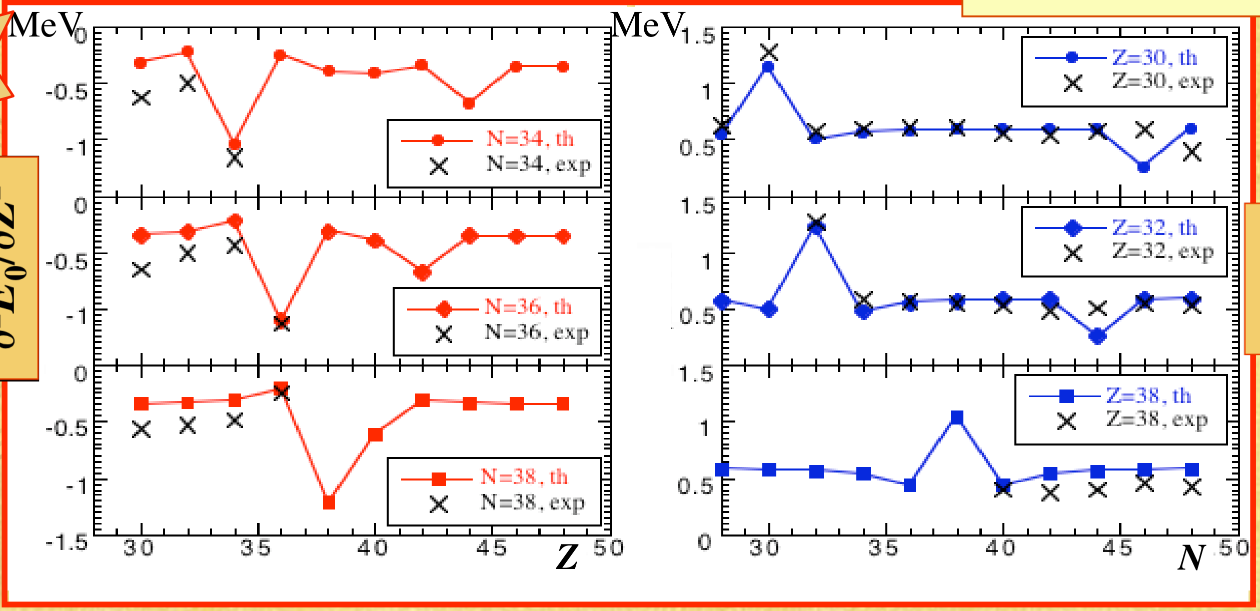
- Two-Proton Drip Line
- $N=Z$ Irregularities

Interaction between the last proton and the last neutron

$\frac{\partial^2 E_0}{\partial Z \partial N}$

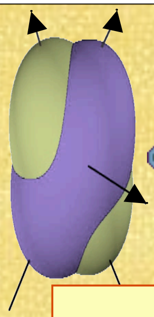
^{56}Ni core

$\partial^2 E_0/\partial Z^2$

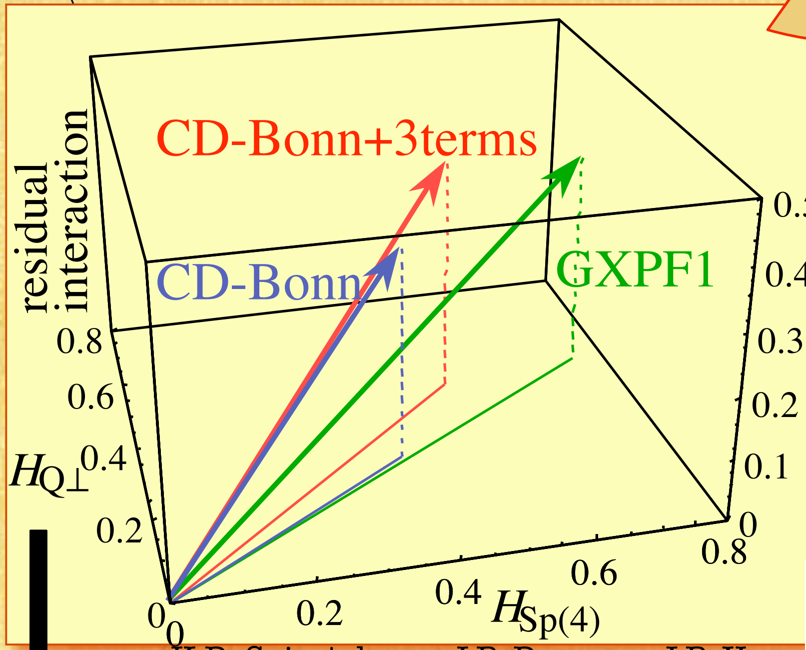


Non-pairing like-particle interaction

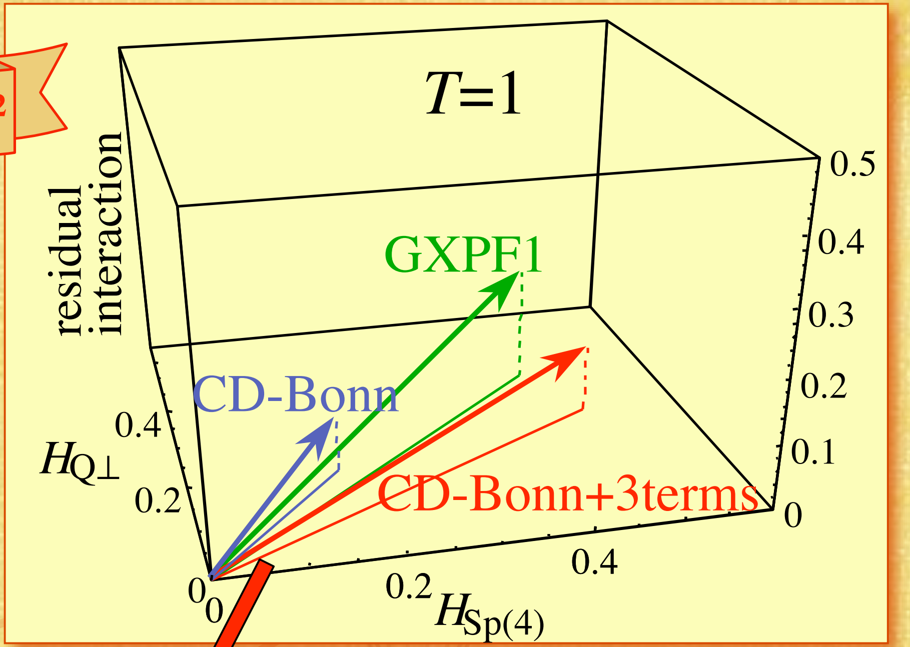
Strong Correlation of Sp(4) Hamiltonian with Realistic Interactions



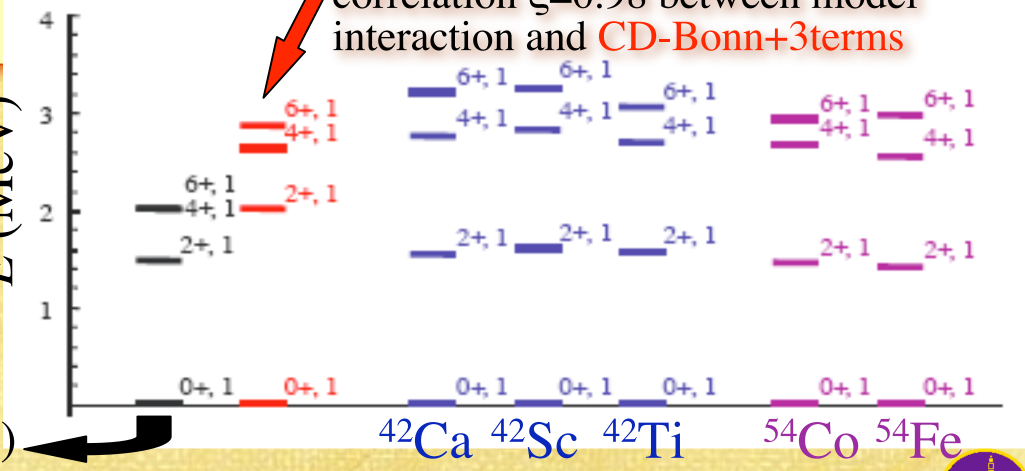
$1f_{7/2}$



K.D. Sviratcheva, J.P. Draayer, J.P. Vary

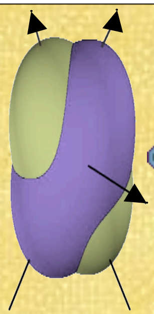


correlation $\zeta=0.98$ between model interaction and CD-Bonn+3terms

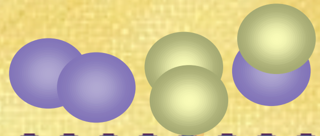


Part of the quadrupole-quadrupole two-body interaction that is not contained in Sp(4) model interaction

model interaction: $H_{\mathrm{Sp}(4)}+H_{\mathrm{Q}\perp}(\chi=0.07)$



Sp(4) Model



Range of applicability

- $1f_{7/2}$ and upper fp+$1g_{9/2}$ shells
- even-*A* nuclei, $40 \leq A \leq 100$

Main positive results

- Microscopic description of **like-particle and proton-neutron pairing correlations** in isobaric analog 0^+ states (*IAS*) [ground states in even-even nuclei and some odd-odd nuclei]
- Reproduction of available experimental energies of the 0^+ *IAS* with only 6 parameters
- Reproduction of the observed detailed structure [beyond mean-field effects] such as the *N*=*Z* anomalies, isovector pairing gaps and staggering

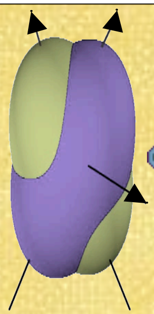
Principal Limitations

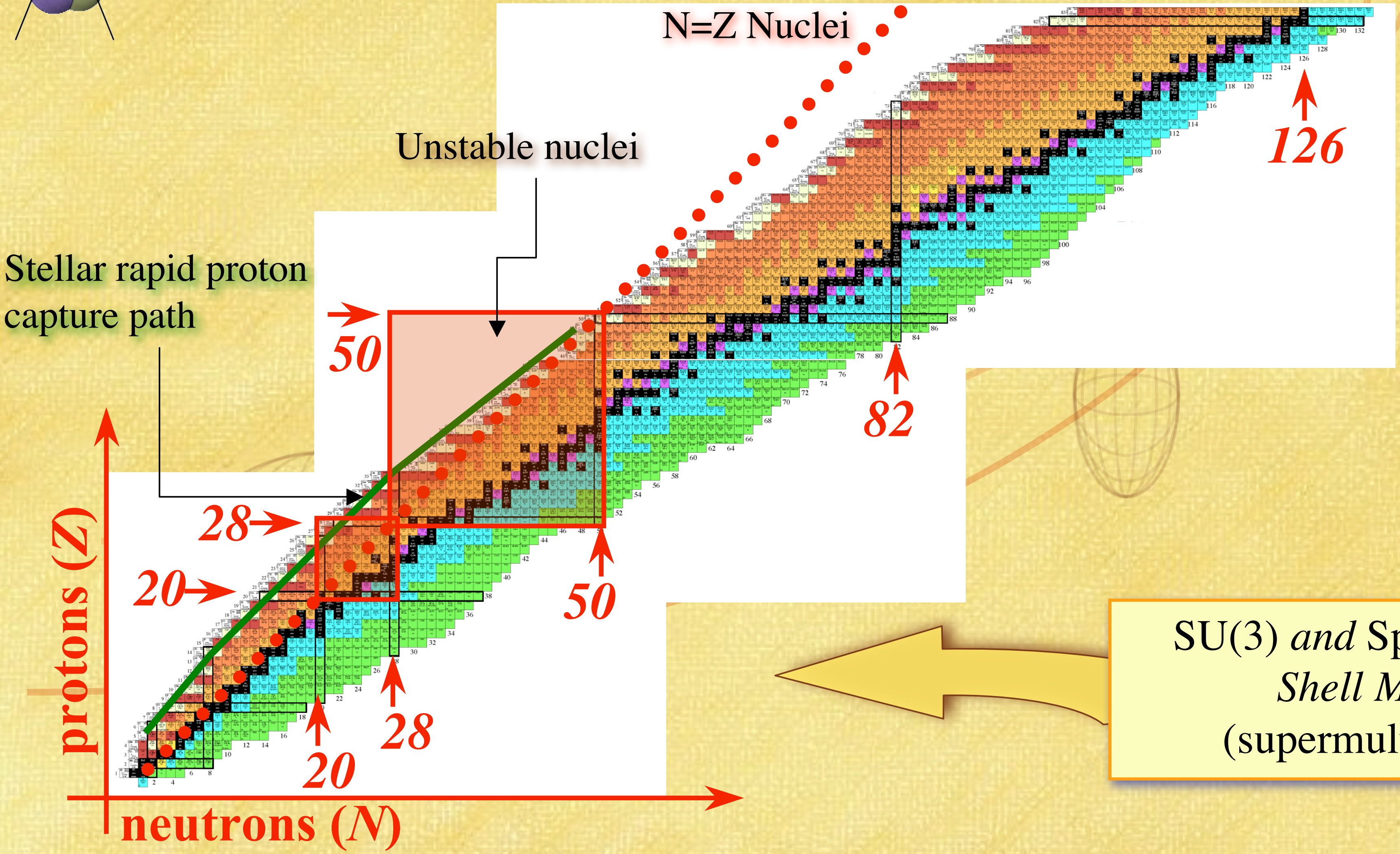
- Isoscalar *pn* force is *J*-independent
- Important part of *Q.Q* is missing
- *Total* seniority zero configurations and therefore only $J^{\pi} = 0^+$ states

Possible extensions

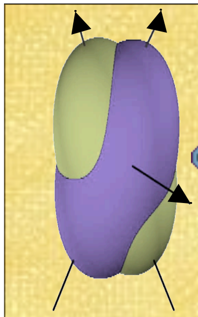
- Non-zero seniority irreps
- Introduce mixing due, for example, to *Q.Q* interaction, or even full Sp(4) breaking

Light Nuclei





SU(3) *and* Sp(3,$\mathbb{R}$)
Shell Models
(supermultiplet)



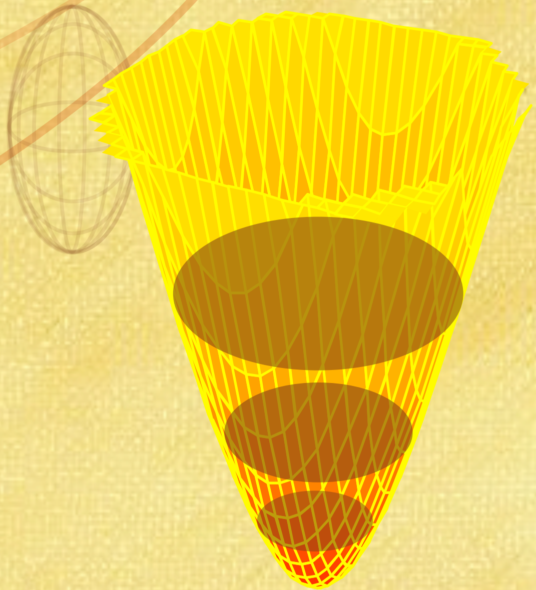
Elliott's SU(3) Model

microscopic **collective**

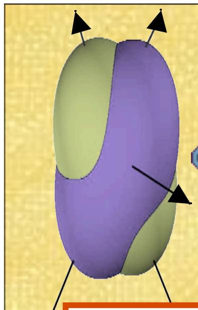
$x_i, p_i \Longleftrightarrow b_i^\dagger \otimes b_i \Longleftrightarrow L_{1,m}$ Angular Momentum

$Q_{2,m}$ Quadrupole Moment

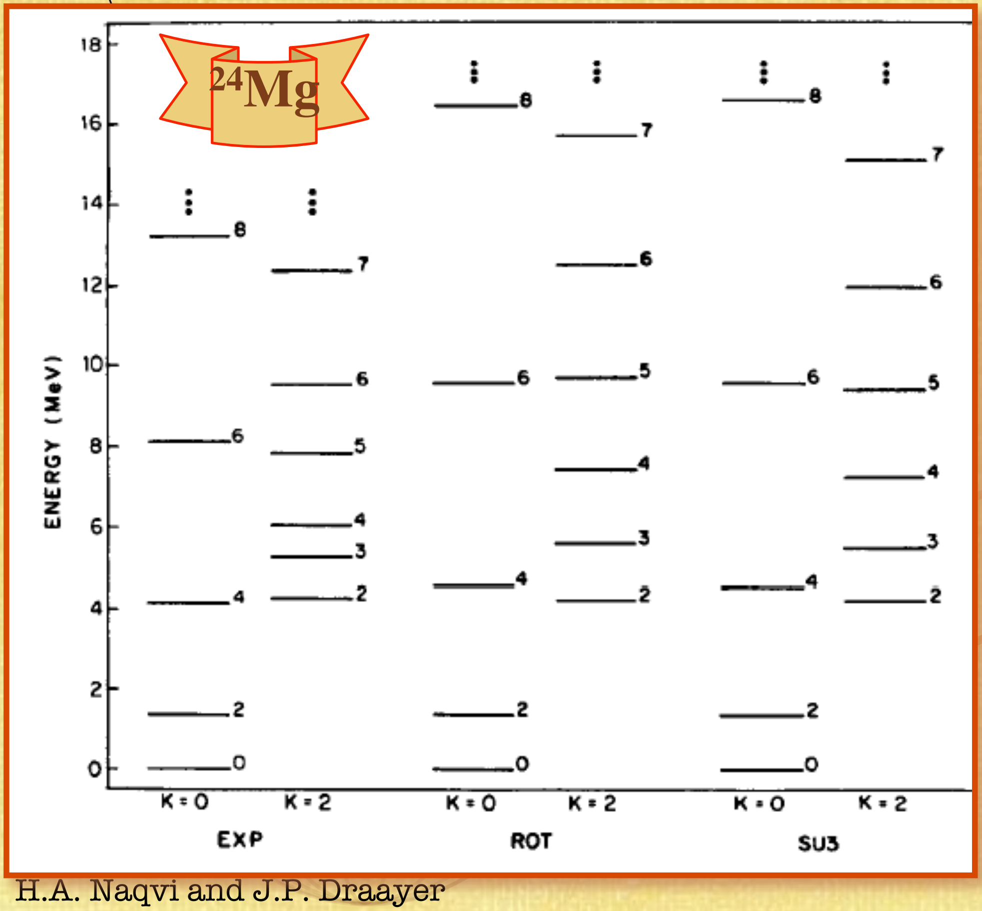
- SU(3) is the exact symmetry group of the spherical oscillator [which is a reasonable approximation for the average potential experienced by nucleons in nuclei]
- SU(3) is the dynamical symmetry group of the deformed oscillator [when, as is usually the case, the deformation is generated by quadrupole interactions]
- In many cases, a single/few-irrep(s) calculation suffices to achieve good agreement with experimental data



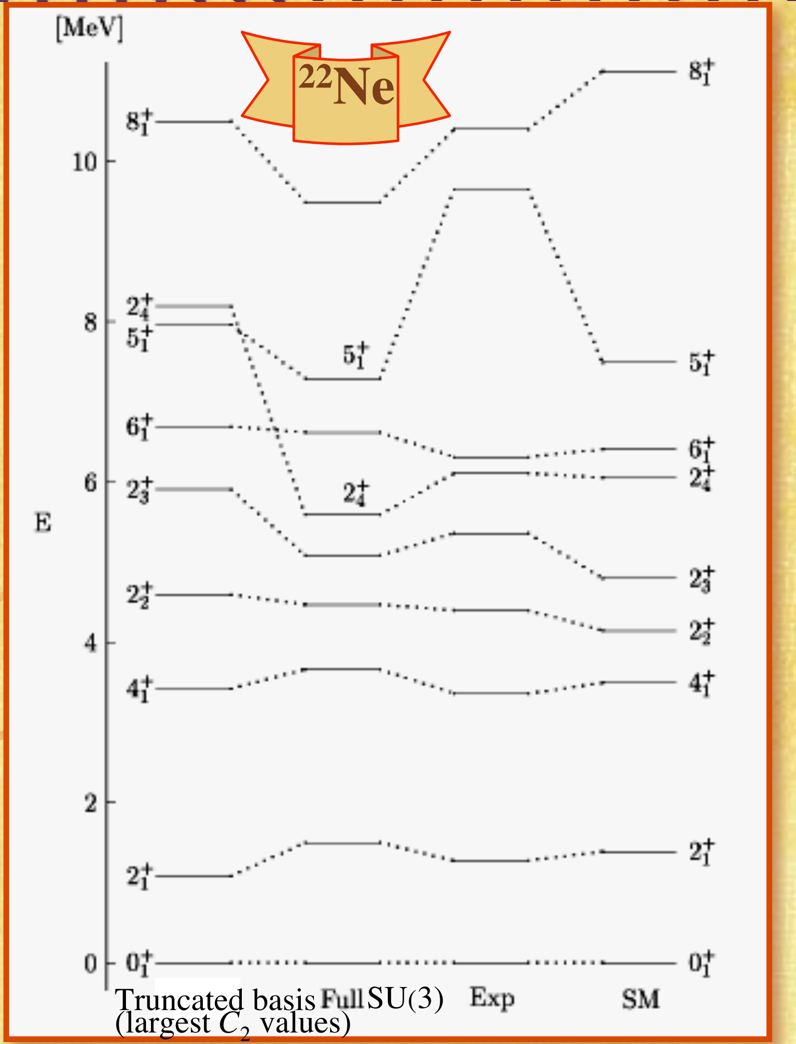
Elliott's SU(3) Model: Light Deformed Nuclei



$$H_{SU(3)} = H_0 + aL^2 + bX_3^a + cX_4^a$$

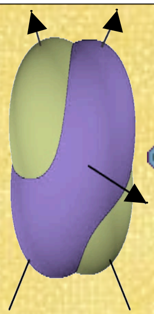


H.A. Naqvi and J.P. Draayer



C.E. Vargas, J.G.Hirsch, J.P.Draayer

$$H = H_{s.p.,\pi} + H_{s.p.,\nu} - \frac{1}{2}\chi Q\cdot Q - G_\pi H_{pair,\pi} - G_\nu H_{pair,\nu} + aK_J^2 + bJ^2 + A_{sym}C_2$$



Elliott's SU(3) Model

Range of applicability

Light nuclei: $A \leq 28$

Main positive results

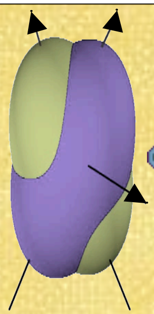
- Microscopic description of **collective modes** in deformed nuclei
- Reproduction of experimental rotational energy spectra and electromagnetic transitions
- Tremendous reduction in model space

Principal Limitations

- Model space within a shell
- Effective charge required
- Pairing, *l.l (orbit-orbit), and l.s (spin-orbit)* break SU(3), but full technology in place

Possible extensions

- Symplectic extension: Sp(3,$\mathbb{R}$)
  [D.J. Rowe and G. Rosensteel]
- No-Core Shell Model + Sp(3,$\mathbb{R}$)
  [T. Dytrych, C. Bahri, K.D. Sviratcheva, J.P. Draayer, J.P. Vary]
- Pseudo-SU(3) for heavy nuclei



SU(3) and the Symplectic Extension ...

Configuration Shell-Model
(multi-$\hbar\omega$)

$4\hbar\omega \left\{ 2\hbar\omega \left\{ 0\hbar\omega \right.\right. \rightarrow$

horizontal
slices

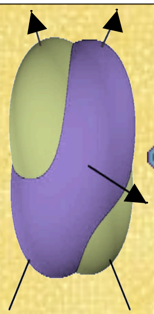
Symplectic Shell-Model
(multi-$\hbar\omega$)

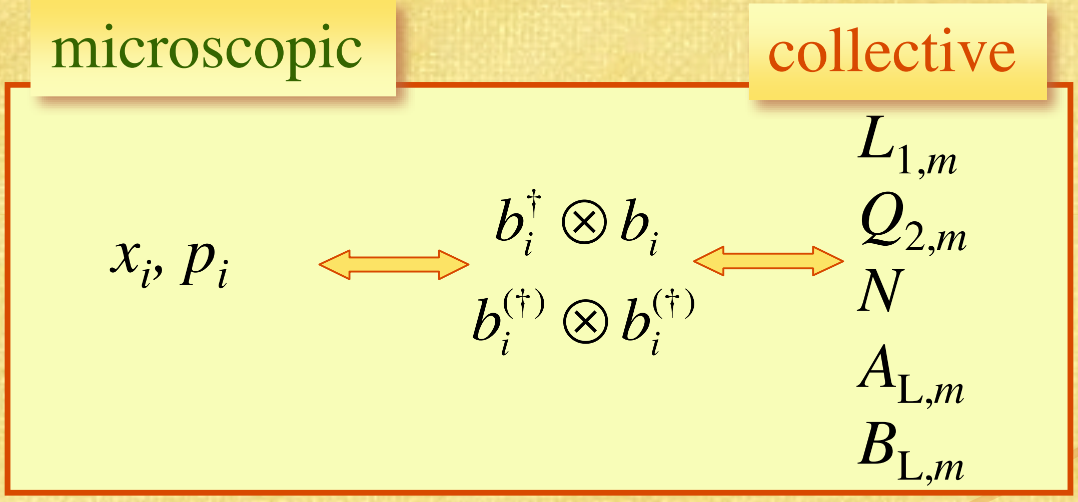
SU(3) limit ($0\hbar\omega$)

vertical
slices

translation invariance okay for both

Sp(3,R)⊃SU(3) Model





microscopic

collective

$x_i, p_i \Longleftrightarrow b_i^\dagger \otimes b_i,\ b_i^{(\dagger)} \otimes b_i^{(\dagger)} \Longleftrightarrow L_{1,m},\ Q_{2,m},\ N,\ A_{L,m},\ B_{L,m}$

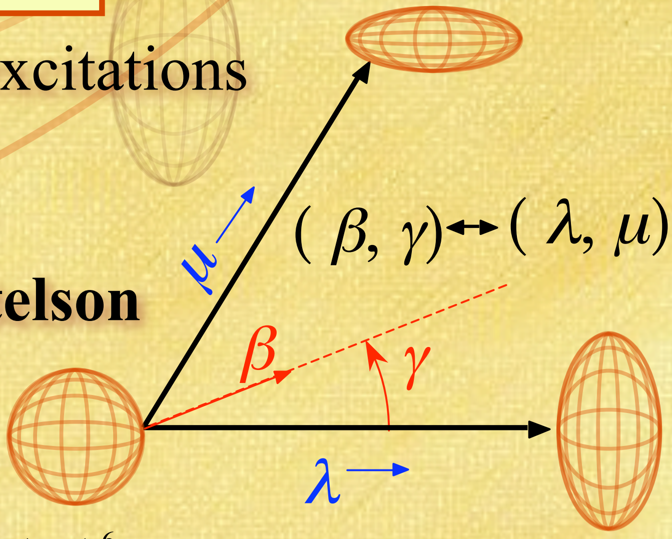
Angular Momentum

Quadrupole Moment

Boson number

Inter-shell excitations

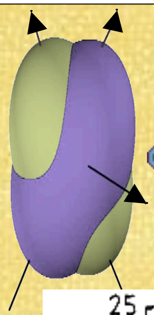
- Multiple-shell correlations including core excitations
- No effective charge
- Microscopic formulation of the **Bohr-Mottelson** collective model



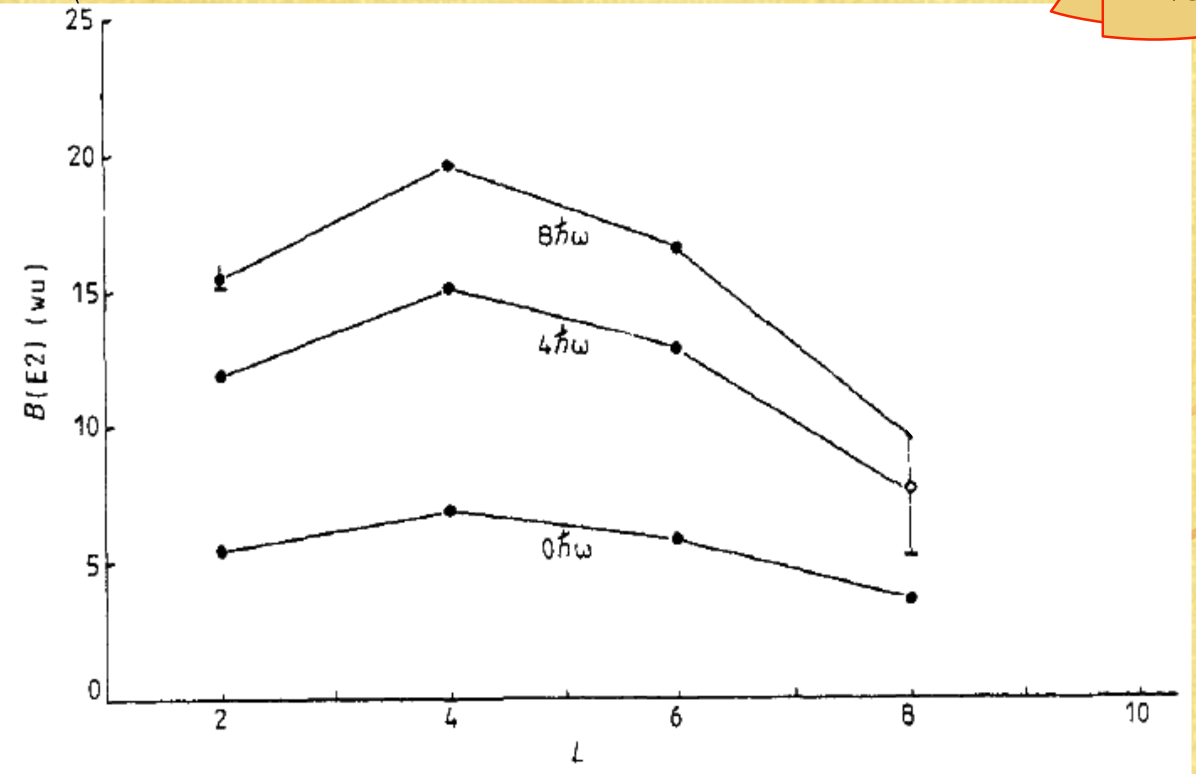
$$\frac{1}{12}[Q\otimes Q]^{L=0} = \frac{3}{20\pi}A^2\left(\frac{R_0}{b}\right)^4\beta^2$$

$$-\frac{1}{108}\sqrt{\frac{7}{2}}[Q\otimes Q\otimes Q]^{L=0} = \frac{1}{20\pi\sqrt{5\pi}}A^3\left(\frac{R_0}{b}\right)^6\beta^3\cos 3\gamma$$

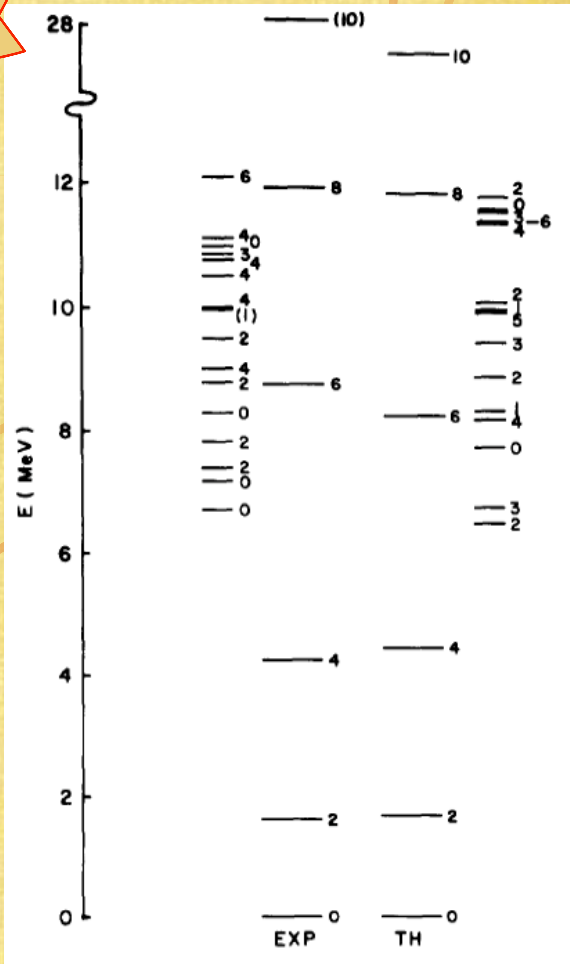
Sp(3,R)⊃SU(3) Model



^{20}Ne

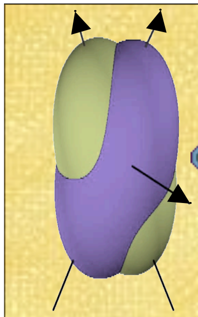


G. Rosensteel and D.J. Rowe



J.P. Draayer, K.J. Weeks, G. Rosensteel

$$H = \hbar\omega H_0 + b_2 Q\cdot Q + b_3 (Q\times Q)\cdot Q + b_4 (Q\cdot Q)^2 + \sum_j \varepsilon_j n_j + G_0 P$$



$Sp(3,\mathbb{R})\supset SU(3)$ Model

Range of applicability

Light nuclei: $A \leq 28$

Main positive results

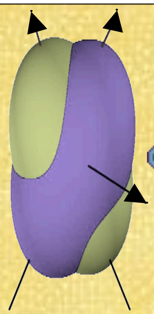
- Microscopic description of monopole and quadrupole **collective modes** in deformed nuclei
- Reproduction of experimental rotational energy spectra and electromagnetic transitions **without effective charges**
- Account of important multi-shell correlations

Principal Limitations

- Light nuclei
- Important short- and intermediate range correlations may not be fully accounted for, but required technologies exist

Possible extensions

- Pseudo-$Sp(3, \mathbb{R})$ for heavy nuclei
- $Sp(3, \mathbb{R})$+No-Core Shell Model
  [T. Dytrych, C. Bahri, K.D. Sviratcheva, J.P. Draayer, J.P. Vary]



Sp(3,R)+NCSM Model

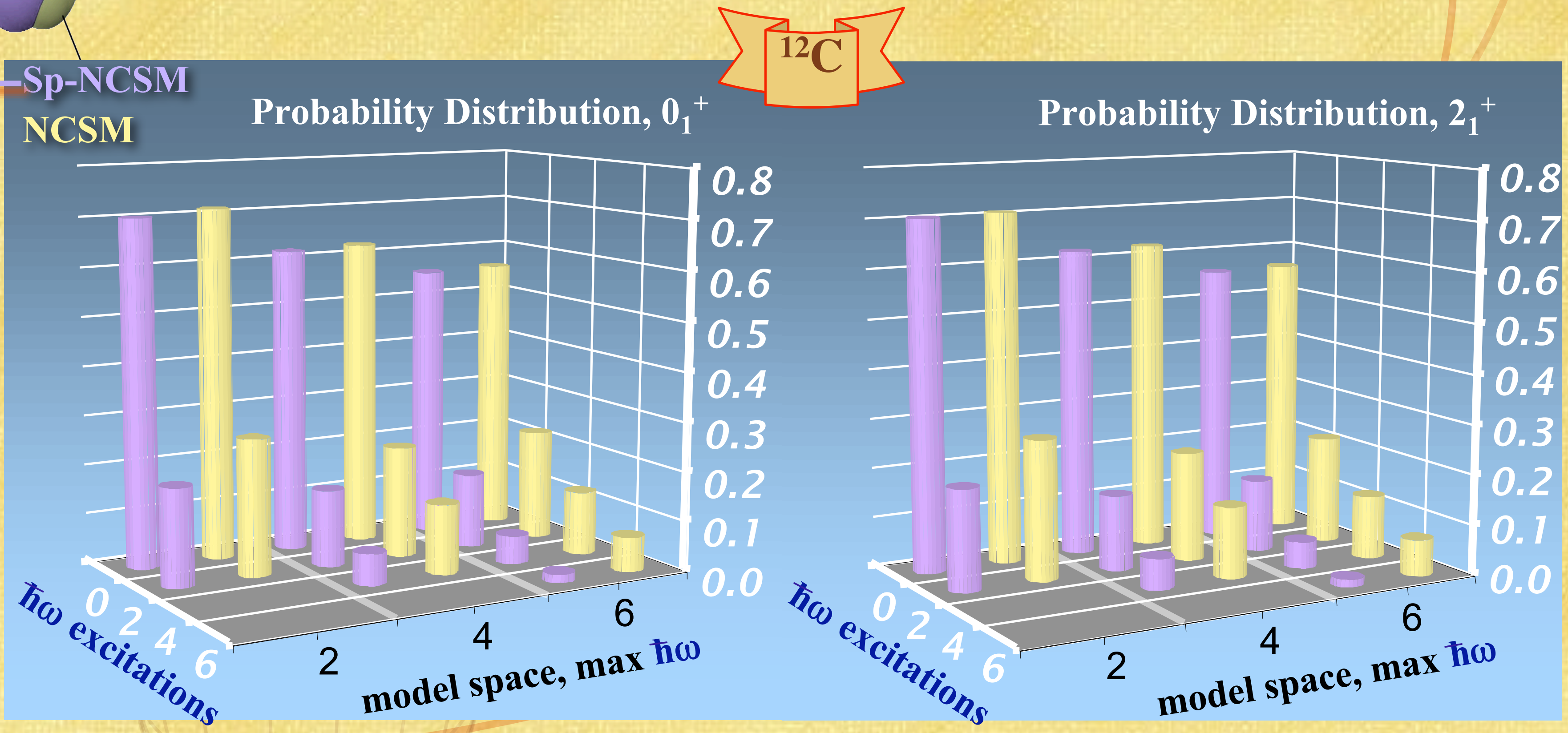
microscopic **collective**

$x_i, p_i \Leftrightarrow b_i^{\dagger} \otimes b_i \quad b_i^{(\dagger)} \otimes b_i^{(\dagger)} \Leftrightarrow$

$L_{1,m}$	Angular Momentum
$Q_{2,m}$	Quadrupole Moment
N	Boson number
$A_{L,m}$ $B_{L,m}$	Inter-shell excitations

- ➢Realistic interaction
- ➢Short- and intermediate-range correlations
- ➢High-ħω collective excitations

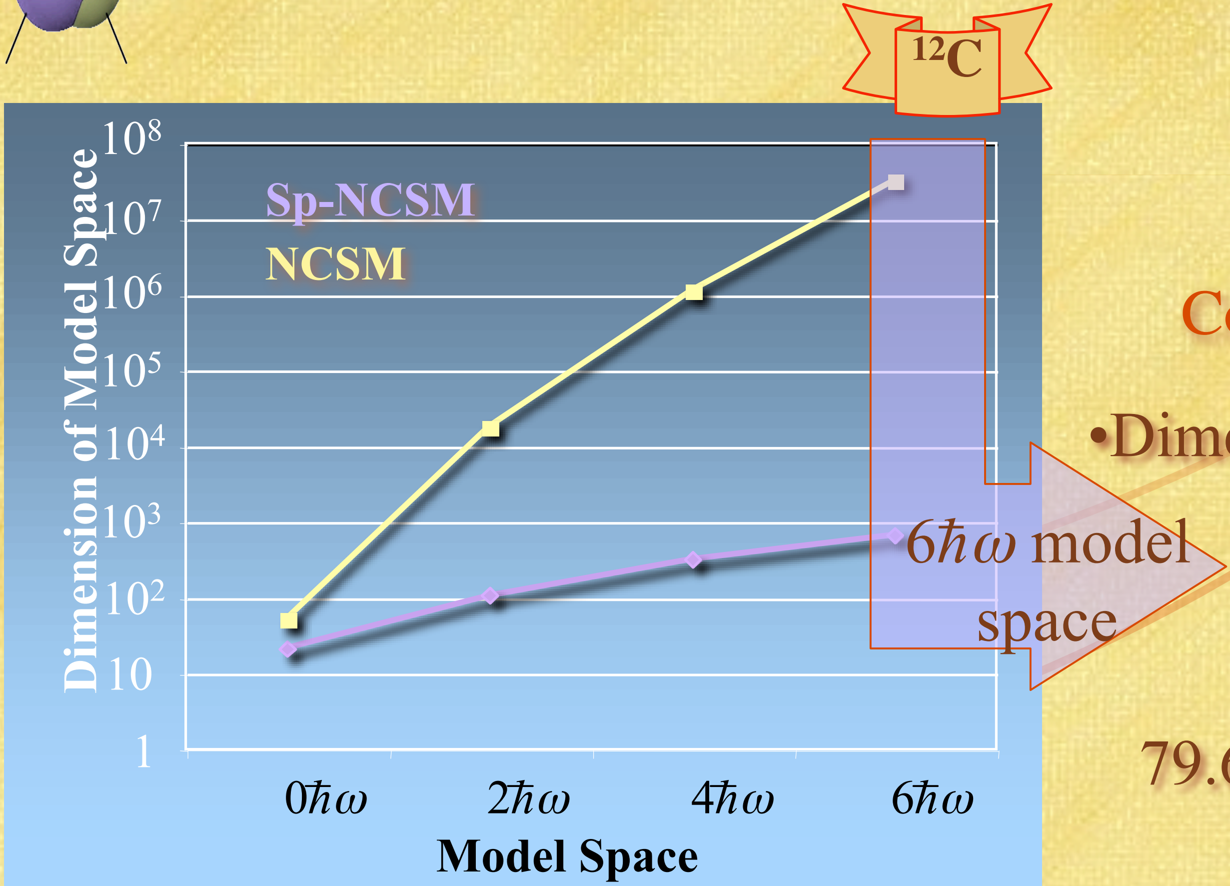
Sp(3,R)+NCSM Model



T. Dytrych, C. Bahri, K.D. Sviratcheva, J.P. Draayer, J.P. Vary

Only **3** symplectic irreps

Sp(3,R)+NCSM Model



^{12}C

$6\hbar\omega$ model space

Compared to NCSM

- Dimension of model space

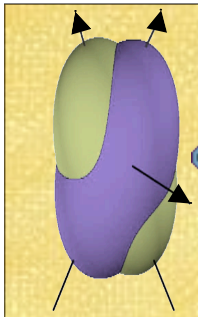
0.003%

- Overlap

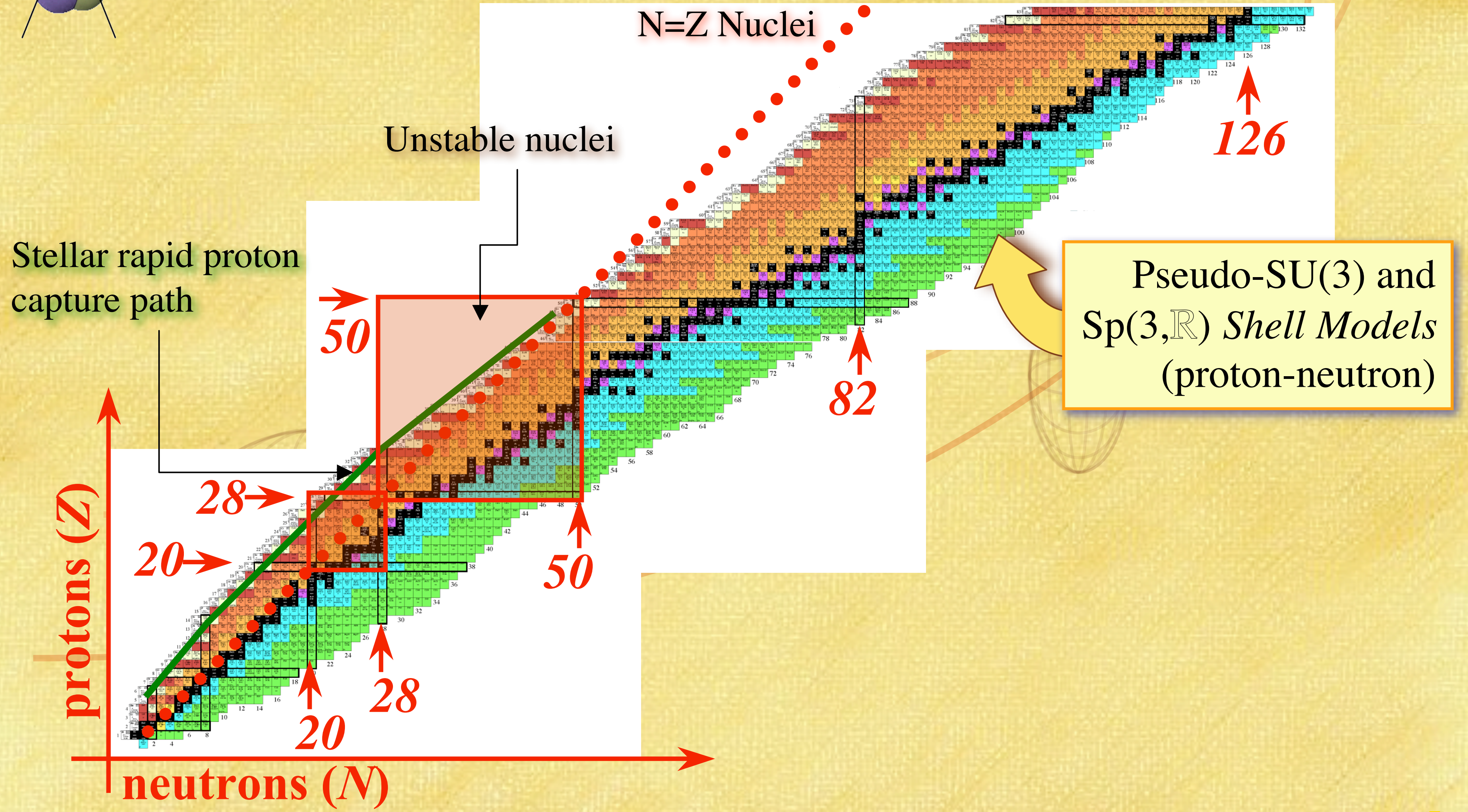
79.6%(0_1^+), 79.1%(2_1^+)

- B(E2: $2_1^+ \rightarrow 0_1^+$)

81%



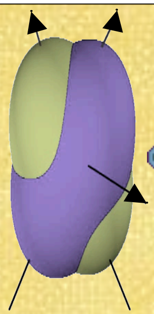
Heavy Nuclei



Pseudo-SU(3) and Sp(3,$\mathbb{R}$) *Shell Models* (proton-neutron)

Pseudo-spin

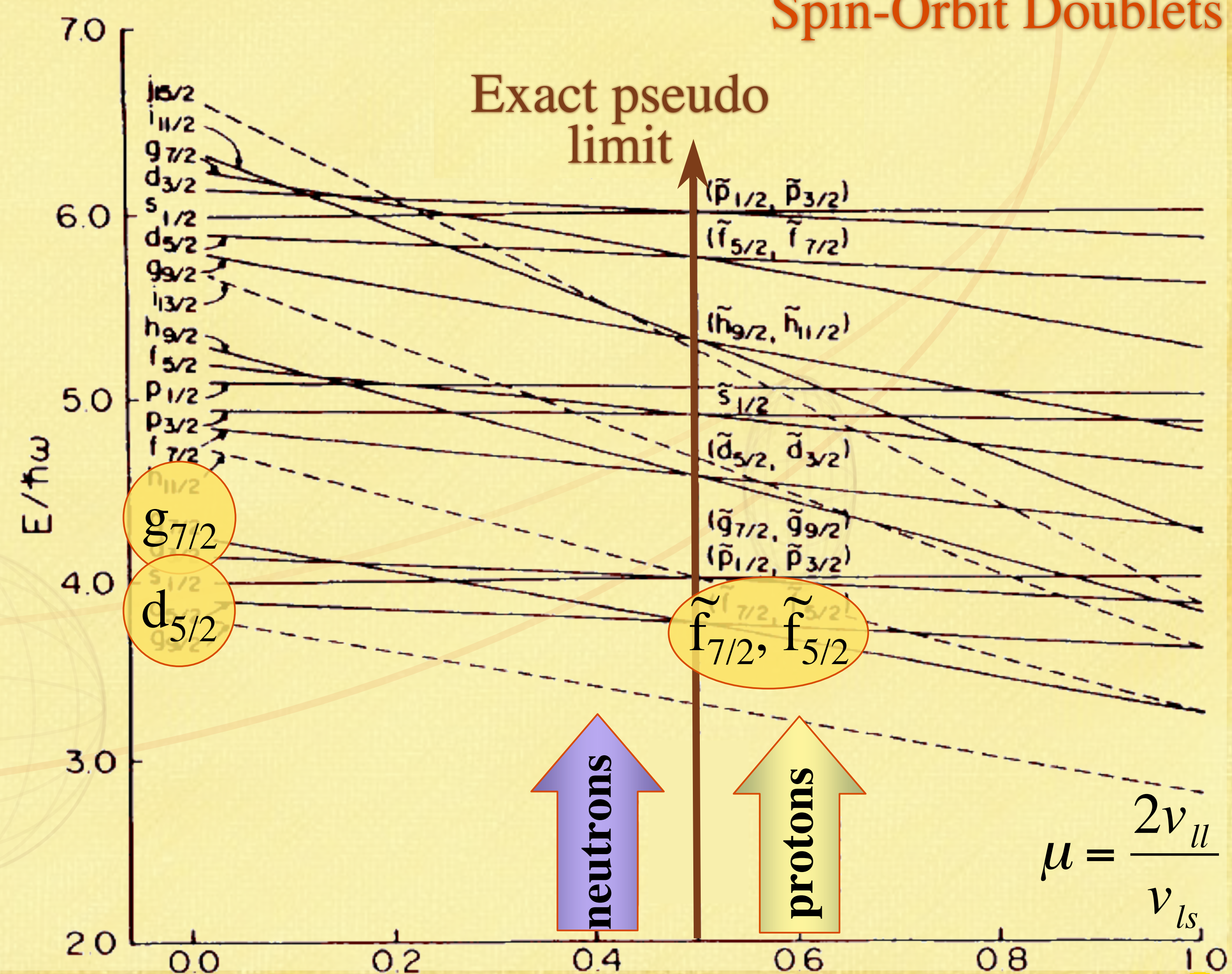
1969: K. T. Hecht and A. Adler;
A. Arima A, M. Harvey M K. Shimizu



Eigenvalues of the spherical oscillator + spin-orbit and orbit-orbit interactions

$$H_0 + v_{ls} l \cdot s + v_{ll} l^2$$

Spin-Orbit Doublets



$$\mu = \frac{2v_{ll}}{v_{ls}}$$

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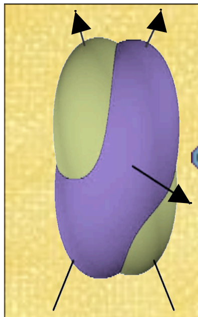
RIA Theory Meeting, April 2006

Fermion Systems with
Fuzzy Symmetries



Pseudo-spin

1969: K. T. Hecht and A. Adler;
A. Arima A, M. Harvey M K. Shimizu

Spin-Orbit Doublets

Eigenvalues of the spherical oscillator + spin-orbit and orbit-orbit interactions

$$H_0 + v_{ls} l \cdot s + v_{ll} l^2$$

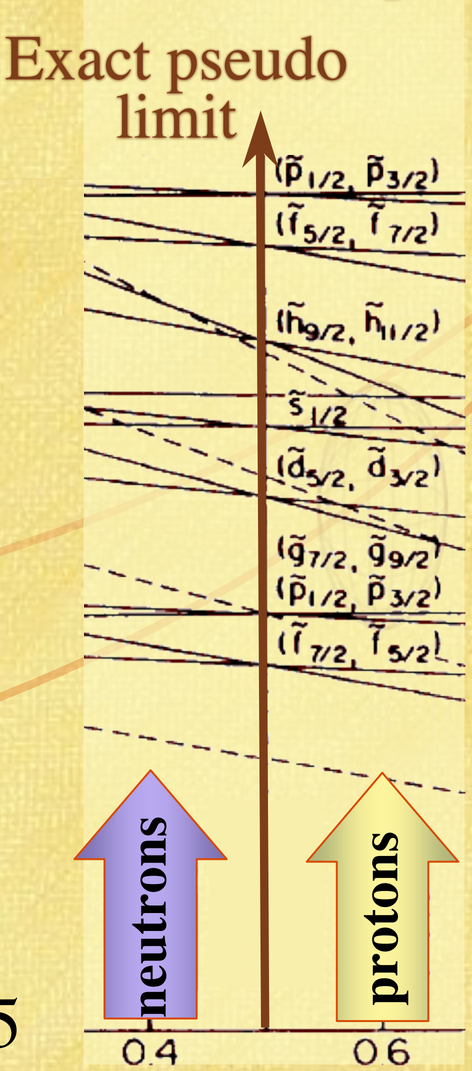
pseudo

$$\begin{array}{cccc} n-1 & j & m & l \pm \frac{1}{2} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \tilde{n} & \tilde{j} & \tilde{m} & \tilde{l} \mp \frac{1}{2} \end{array}$$

$$\tilde{H}_0 + (4v_{ll} - v_{ls})\tilde{l} \cdot \tilde{s} + v_{ll}\tilde{l}^2 + const$$

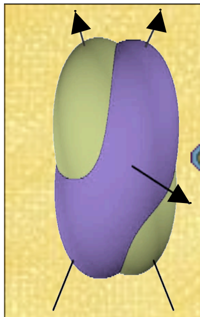
For heavy nuclei ($A \geq 100$)

$$4v_{ll} \approx v_{ls} \Leftrightarrow \mu = 0.5$$



Pseudo spin-orbit doublets can be treated as (almost) degenerate levels with the same pseudo-angular momentum and (almost) decoupled pseudo-spin.

$$\mu = \frac{2v_{ll}}{v_{ls}}$$



Pseudo-SU(3) Model

R.D. Ratna Raju, J.R Draayer
and K.T. Hecht

$\tilde{L}_{1,m}$ Pseudo-Angular Momentum

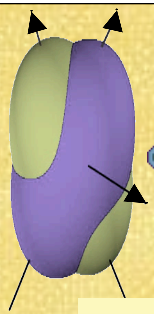
$\tilde{Q}_{2,m}$ Pseudo-Quadrupole Moment

- Pseudo-SU(3) is an (almost) good symmetry in heavy nuclei ($A \geq 100$) when deformation dominates
- Pseudo-spin scheme is an excellent starting point for a many-particle description of heavy nuclei, whether or not they are deformed
- As for the SU(3) model, in many cases a leading-irrep calculation achieves good agreement with experimental data
- Normal⇔Pseudo Mapping, e.g.:

$$Q.Q = \kappa \tilde{Q}.\tilde{Q} + \underbrace{\cdots}$$

<1% contribution to excitation energies and electromagnetic transition strengths

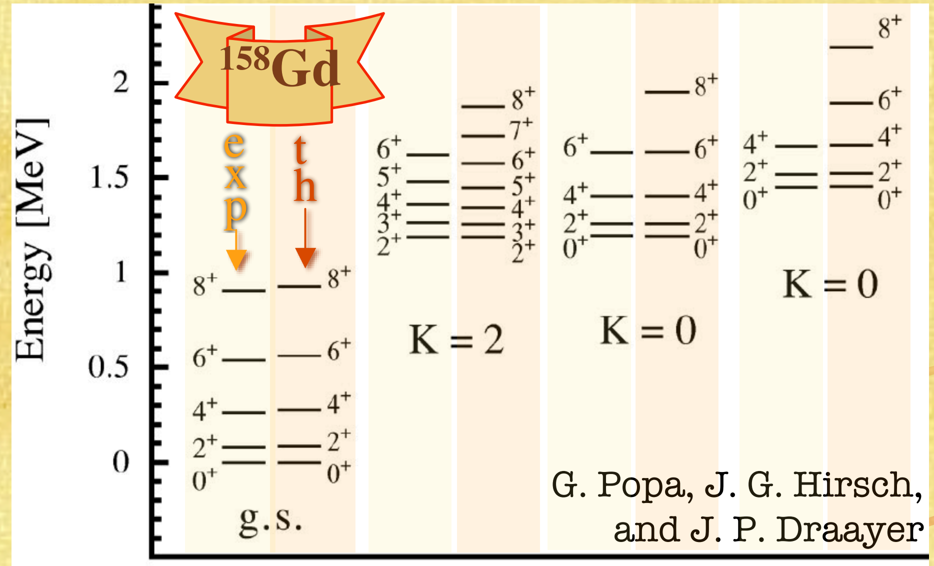
Pseudo-SU(3) Model



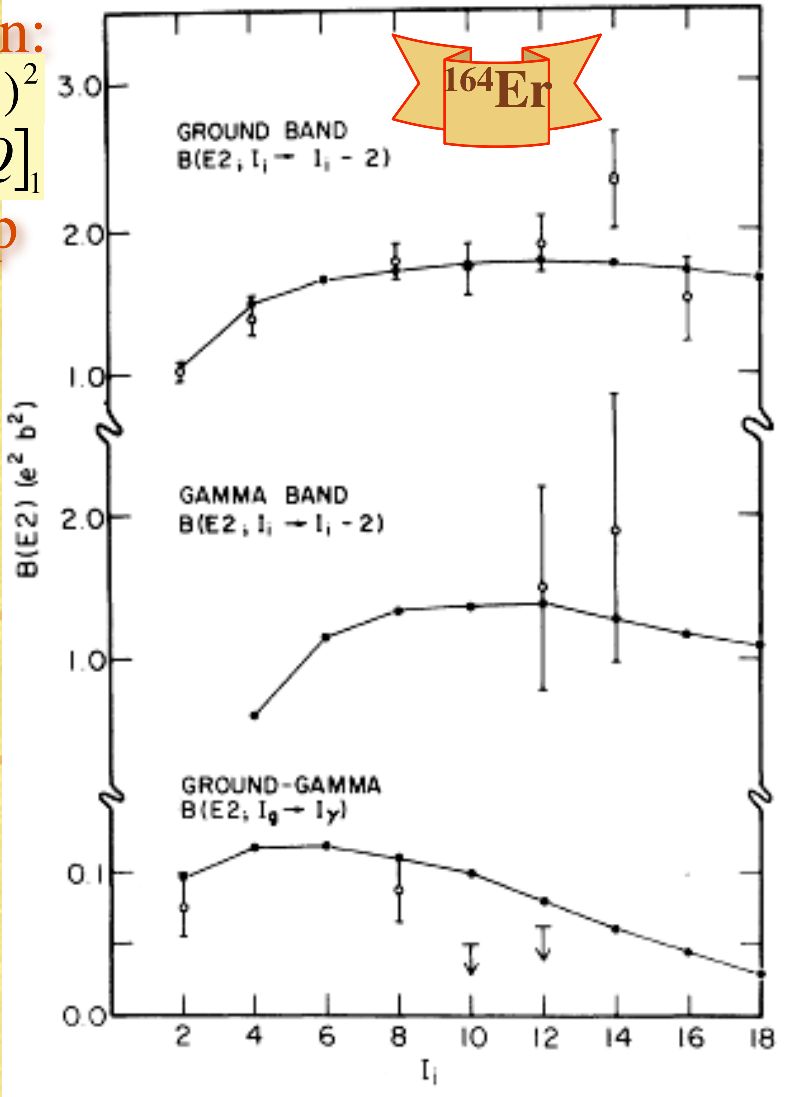
- Residual interaction:

$$aX_2 + bX_3 + cX_4 + d(X_2)^2 + \kappa_2 Q\cdot Q + \kappa_3\left[Q\times Q\right]\cdot Q + \kappa_4\left[L\times Q\right]_1\cdot\left[L\times Q\right]_1$$

- Single leading irrep



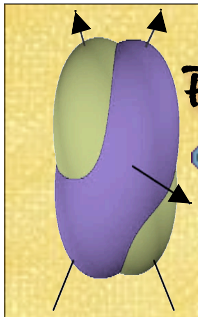
G. Popa, J. G. Hirsch, and J. P. Draayer



J.R Draayer and K.J. Weeks

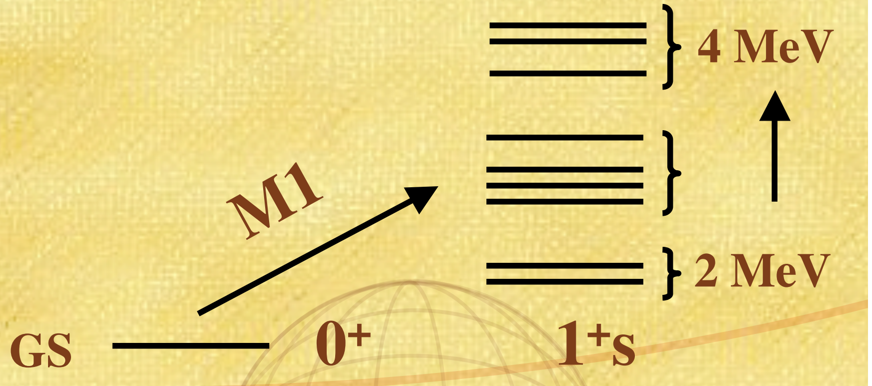
- $$H_0 + D_\pi \sum_i l_{i,\pi}^2 + D_\nu \sum_i l_{i,\nu}^2 - \frac{1}{2}\chi Q\cdot Q - G_\pi H_{P,\pi} - G_\nu H_{P,\nu} + aK_J^2 + bJ^2 + A_{sym}C_2$$

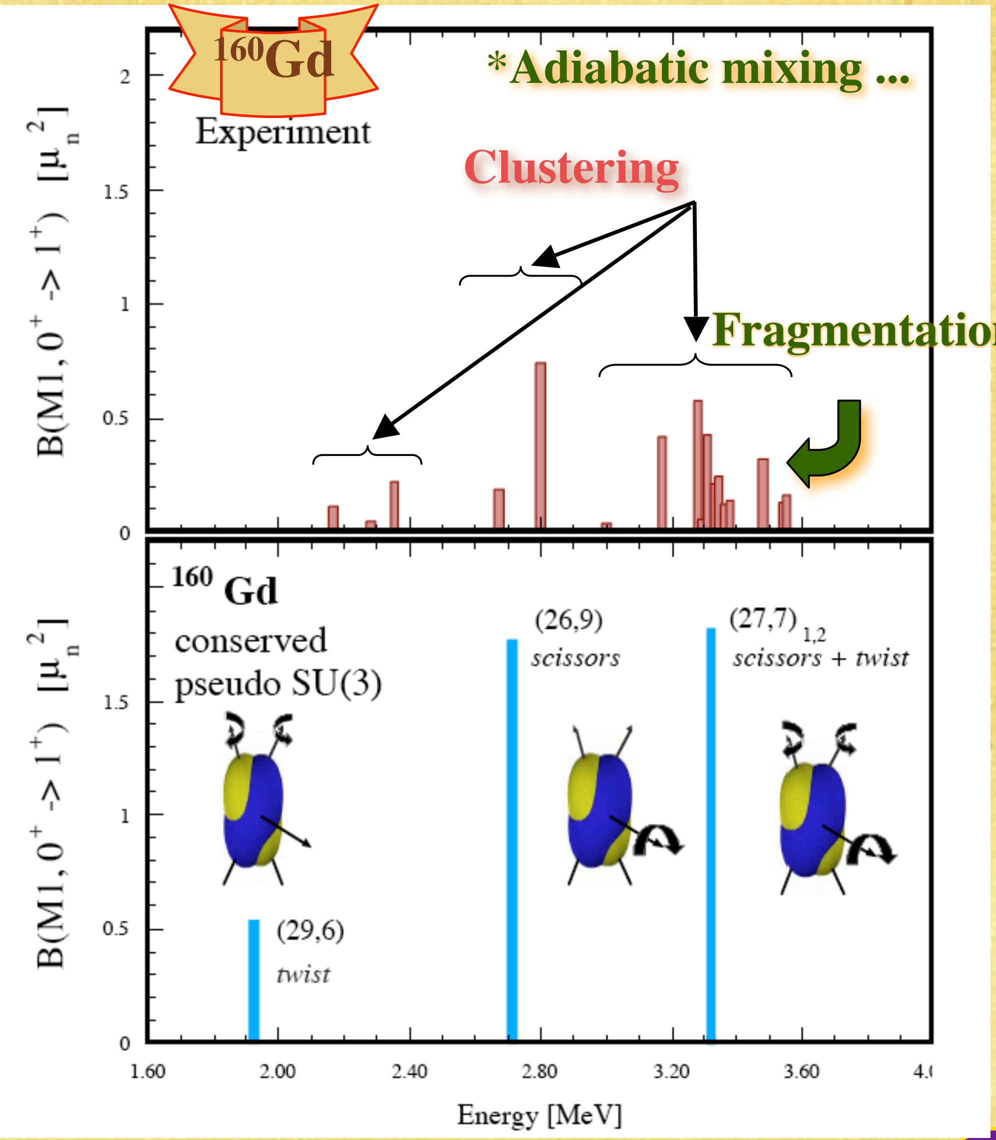
- mixed irrep shell-model basis



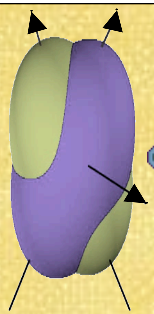
Pseudo-SU(3) Model: Scissors and Twist Modes

Typical M1 Spectrum



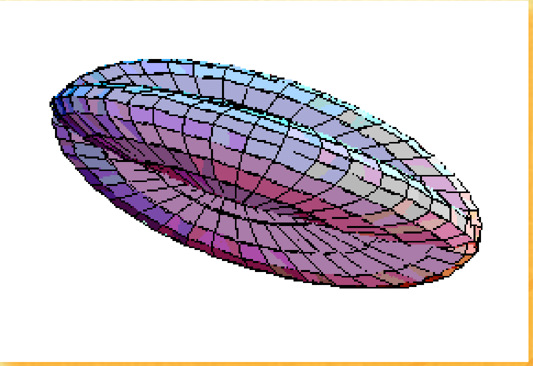


T. Beuschel, J. P. Draayer,
D. Rompf, J. G. Hirsch



Direct Product Coupling ★

Coupling proton and neutron irreps to total (coupled) SU(3):



$$(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu)$$

$$\rightarrow \quad (\lambda_\pi + \lambda_\nu, \mu_\pi + \mu_\nu)$$ **GROUND STATE**

$$+ \quad (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1)$$ **SCISSORS**

$$+ \quad (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2)$$ **TWIST**

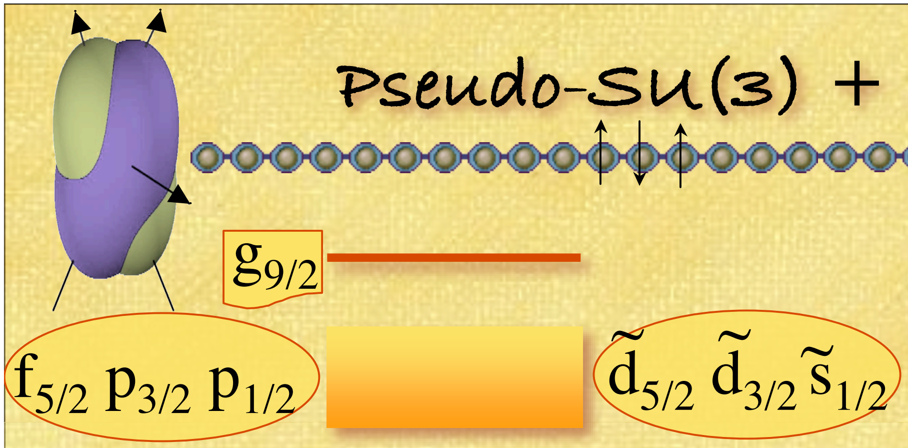
$$+ \quad (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1)^2$$ **SCISSORS + TWIST**

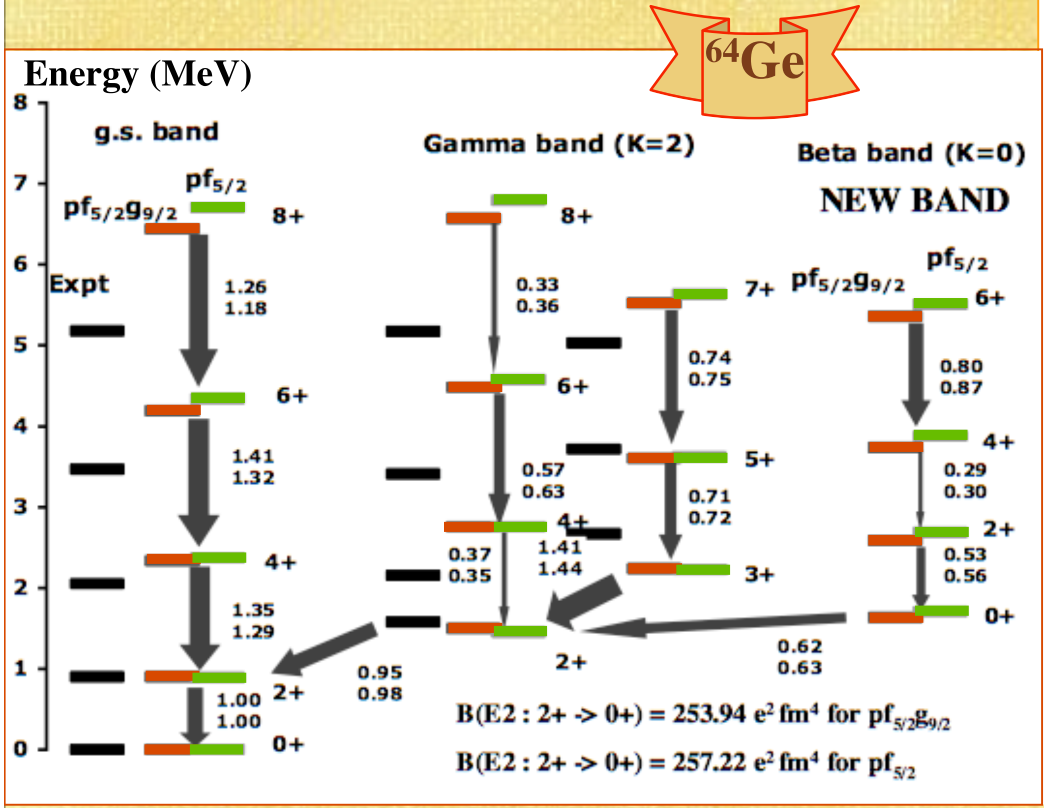
$$+ \quad \ldots$$

$$\rightarrow \sum_{m,l} \oplus (\lambda_\pi + \lambda_\nu - 2m + l, \mu_\pi + \mu_\nu + m - 2l)^k \longrightarrow$$ **… multiplicity**

★**… Orientation of the π-ν system is quantized with the <u>multiplicity</u> denoted by $k = k(m,l)$**

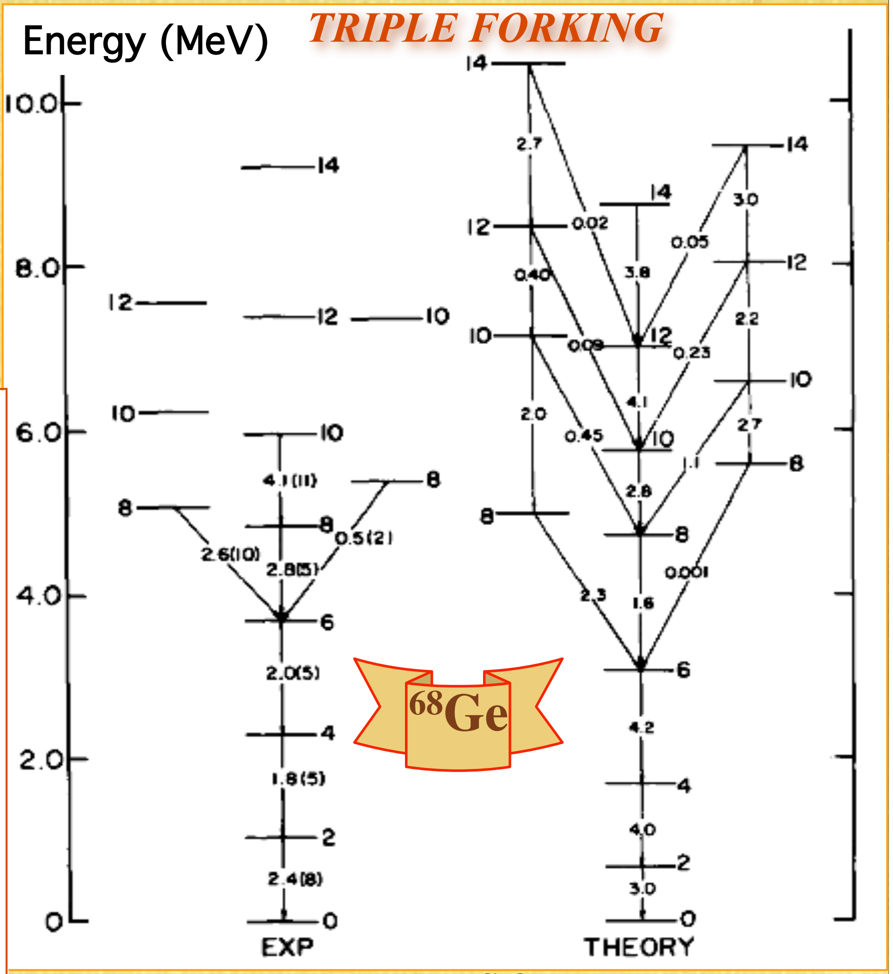
Pseudo-SU(3) + intruder level Picture





B(E2 : 2+ -> 0+) = 253.94 e^2 fm^4 for $pf_{5/2}g_{9/2}$

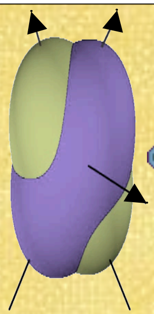
B(E2 : 2+ -> 0+) = 257.22 e^2 fm^4 for $pf_{5/2}$

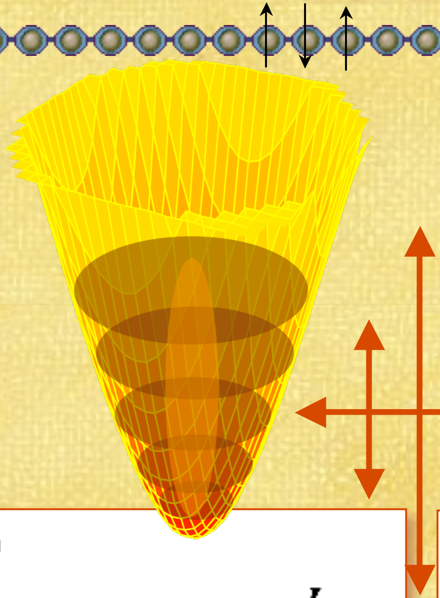


K.J. Weeks, C.S. Han, J.P. Draayer

K. P. Drumev, C. Bahri, V. G. Gueorguiev and J. P. Draayer

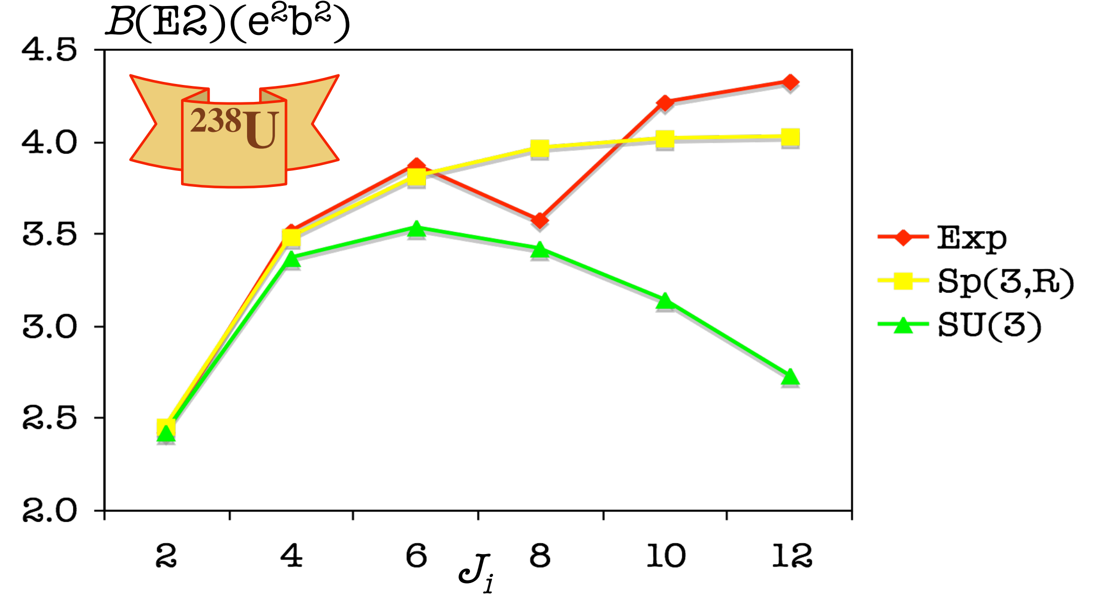
Pseudo-Sp(3,R) Model



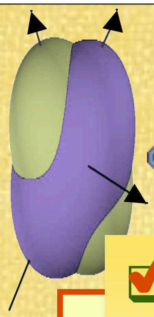


$\widetilde{SU}(3)$ limit $(0\hbar\omega)$

E_i(MeV)		J_i
Exp	Sp(3,R)	
0.0449	0.0435	2
0.1487	0.1451	4
0.3072	0.3048	6
0.5178	0.5225	8
0.7757	0.7982	10
1.0765	1.1320	12



O. Castaños, P.O.Hess, J.P. Draayer and P. Rochford (1991)



Origin of Pseudo-spin

☑ Phenomenology
☑ Relativistic mean-field results
☐ σ-model with vector potential

⟺ Pseudo-spin ⟺ $\widetilde{\mathrm{SU}}(3)$

collective

$$v_{ll} = -\frac{\hbar^2}{2MR^2}$$

nucleon mass
radius of the well

Using the Klein-Gordon equation (for small kinetic energy)

Intuitively: from $l(l+1)$ splitting of s.p. energies for a spherical well in a large mass limit

$$v_{ls} = -\frac{\hbar^2}{2MR^2}\frac{6B}{1-B}$$

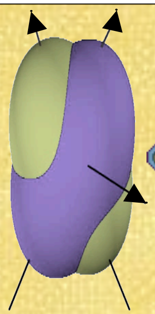
B is related to the strength of the scalar and vector coupling constants

- Starting with the Dirac equation (with only the time component of the scalar and vector potentials)
- Using the non-relativistic limit of the relativistic mean-field theory
- Averaging of the spin-orbit interaction over the region inside radius *R* (surface region of importance)

$$\mu = \frac{2v_{ll}}{v_{ls}} = \frac{1-B}{3B}$$

quantum field theory models → $\mu = 0.45 - 0.69$

C. Bahri, J. P. Draayer, S.A. Moszkowski



Microscopic Pseudo-spin Transformation

A.L. Blokhin, C. Bahri, J. P. Draayer

$$\tilde{\mathbf{l}}^2 = U\mathbf{l}^2U^{-1} = \mathbf{l}^2 + 2\mathbf{l}\cdot\sigma + 2 = 2\mathbf{j}^2 - \mathbf{l}^2 + \frac{1}{2}$$

$[U,\mathbf{j}] = 0$ ⇐ Rotational invariance

$[U,\mathcal{P}] = 0 \quad [U,\mathcal{T}] = 0$ ⇐ Parity and time-reversal symmetry

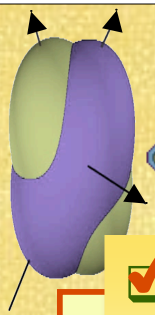
$UU^\dagger = U^\dagger U = 1$ ⇐ Unitary and conservation of symmetry

$$\boxed{U = \left(d\cdot d^\dagger\right)^{-1/2} d}$$

$$d = (\cos\theta\, r_0\mathbf{p} + i\sin\theta\, \mathbf{r}/r_0)\cdot\boldsymbol{\sigma}$$

$[U,\mathbf{p}] = 0$ ⇐ Translational invariance

The *p*-helicity (θ=0°) $\boxed{U_p = \boldsymbol{\sigma}\cdot\mathbf{p}/p}$



Origin of Pseudo-spin

- ☑ Phenomenology
- ☑ Relativistic mean-field results
- ☑ σ-model with vector potential

⟺ Pseudo-spin ⟺ $\widetilde{\mathrm{SU}}(3)$

collective

$$[\mathcal{H}, \hat{h}] = -\hat{h}\,(g_s \beta S + g_v V)$$

≈0

$$\hat{h}S = \hat{h}V \approx 0$$

Helicity/pseudo-spin (approximate) conservation

Hamiltonian for a single nucleon in the presence of two fields

$$\mathcal{H} = \vec{\alpha}\vec{p} - g_s \beta S + g_v V$$

$$\hat{h}S = a\gamma_5 S$$

Vector meson field with **chiral symmetry**

- ➢ For heavy nuclei [large R]
- ➢ Fast moving nucleons [$v >> p/M$]
- ➢ M=0 (trivial)
- ➢ Helicity conserves exactly in the plane ⊥ to nucleon spin

$$g_s|\hat{h}S| \sim \frac{\pi n M}{pR}\left|\frac{(\vec{\sigma}\cdot\vec{r})(\vec{\tau}\cdot\vec{e})}{r}\right|$$

≈0 ≈0

Pseudo-spin and Pseudo-SU(3)/Sp(3,R) Models

Range of applicability { Heavy nuclei: $A \geq 100$

Main positive results

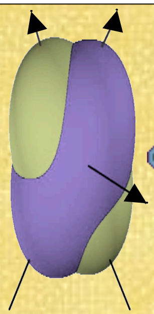
- **Fundamental nature of pseudo-spin symmetry in nuclei**
- Reproduction of experimental rotational energy spectra and electromagnetic transitions in heavy nuclei
- Understanding the M1 transitions in heavy nuclei

Principal Limitations

☂ Challenge for odd-A nuclei (?)

Possible extensions

❀ Pseudo-Sp(3, $\mathbb{R}$): still unexplored…

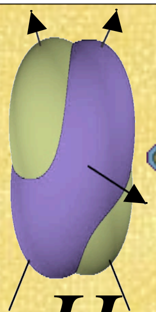


Fuzzy Symmetry Fysics

The End

(Thanks for the Invite!)

Sp(4) Model: Isospin Breaking



$$(A_1^\dagger)^{n_1}(A_0^\dagger)^{n_0}(A_{-1}^\dagger)^{n_{-1}}\left|0\right\rangle$$

basis states ❖

$$H = -\varepsilon \hat{N}$$

$$-G\left(A_0^\dagger A_0 + A_1^\dagger A_{-1} + A_{-1}^\dagger A_1\right)$$

Isovector (isospin 1) *J*=0 pairing interaction

$$-\frac{E}{2\Omega}\left(\enclose{circle}{T^2} - \frac{\hat{N}}{2} - \frac{\hat{N}^2}{4}\right)$$

Diagonal isoscalar (isospin 0) *pn* force

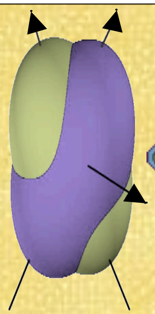
Symmetry term

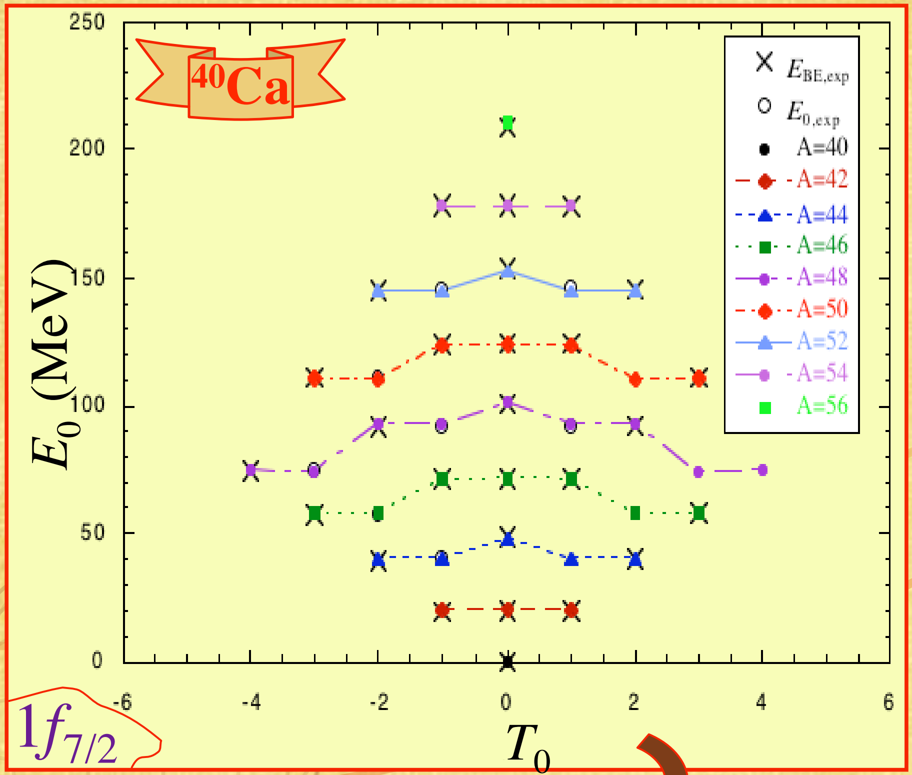
$$-FA_0^\dagger A_0 - \left(C + \frac{E}{4\Omega}\right)\frac{\hat{N}(\hat{N}-1)}{2} - D\left(T_0^{\ 2} - \frac{\hat{N}}{4}\right)$$

Symmetry **breaking**

- ❖ Describe pairing-governed isobaric analog 0^+ states (IAS)
- ❖ Include ground states for all even-even and only some (N~Z) odd-odd nuclei

Lowest Isobaric Analog 0^+ State (Binding) Energy

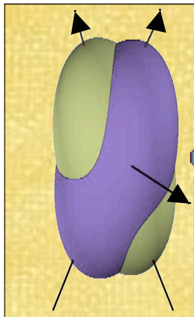




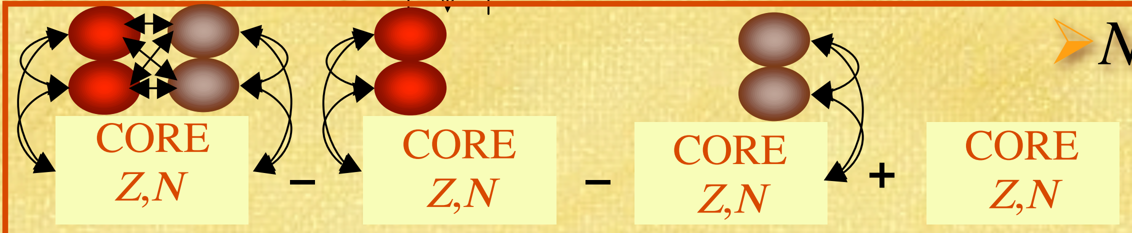
Agree well with experiment: binding energies (×) and energies of the lowest 0^+ IAS (o).

Coulomb correction

J. Retamosa, E. Caurier, F. Nowacki and A. Poves, Phys. Rev. C 55, 1266 (1997).



Reproduction of Fine Structure II



CORE *Z,N* − CORE *Z,N* − CORE *Z,N* + CORE *Z,N*

- *N=Z* Irregularities

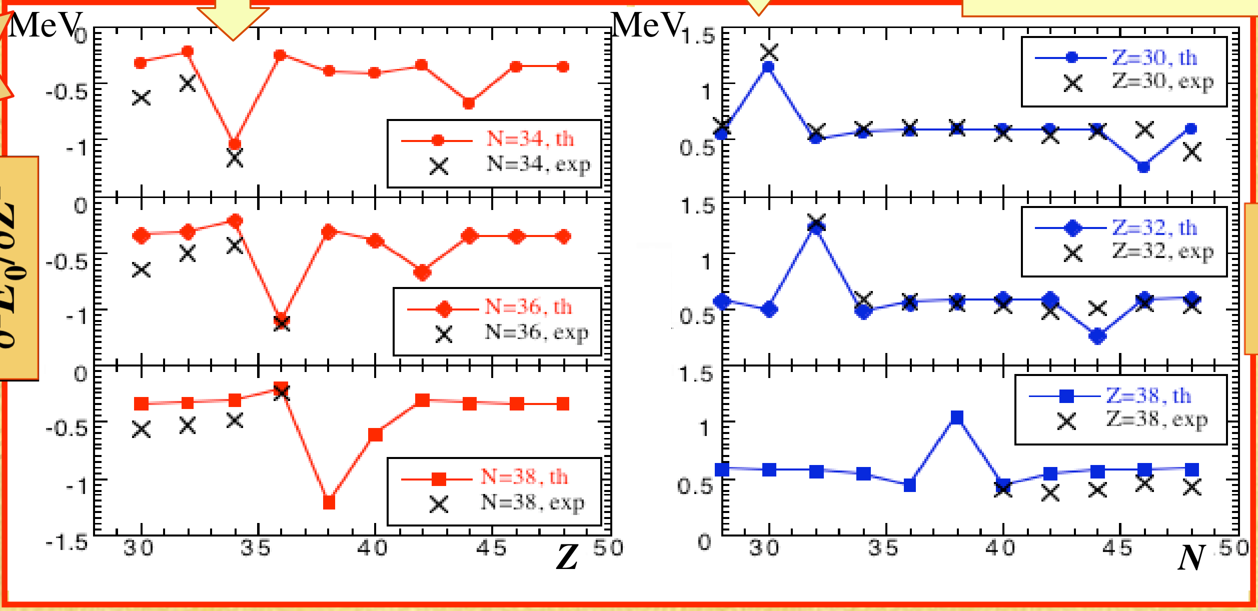
$$\frac{E_0(N_+ + 2, N_- + 2) - E_0(N_+ + 2, N_-) - E_0(N_+, N_- + 2) + E_0(N_+, N_-)}{4}$$

$$E_0(N_\pm + 2) - 2E_0(N_\pm) + E_0(N_\pm - 2)$$

Interaction between the last proton and the last neutron

^{56}Ni core

$\partial^2 E_0 / \partial Z^2$



$$\frac{\partial^2 E_0}{\partial Z \partial N}$$

Non-pairing like-particle interaction