Quantum Error Correction: Dream or Nightmare

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Back in 1982

Quantum Mechanical Hamiltonian Models of Turing Machines

Paul Benioff[1]

Received October 5, 1981; revised June 9, 1982

Quantum mechanical Hamiltonian models, which represent an aribtrary but finite number of steps of any Turing machine computation, are constructed here on a finite lattice of spin-1/2 systems. Different regions of the lattice correspond to different components of the Turing machine (plus recording system). Successive states of any machine computation are represented in the model by spin configuration states. Both time-independent and time-dependent Hamiltonian models are constructed here. The time-independent models do not dissipate

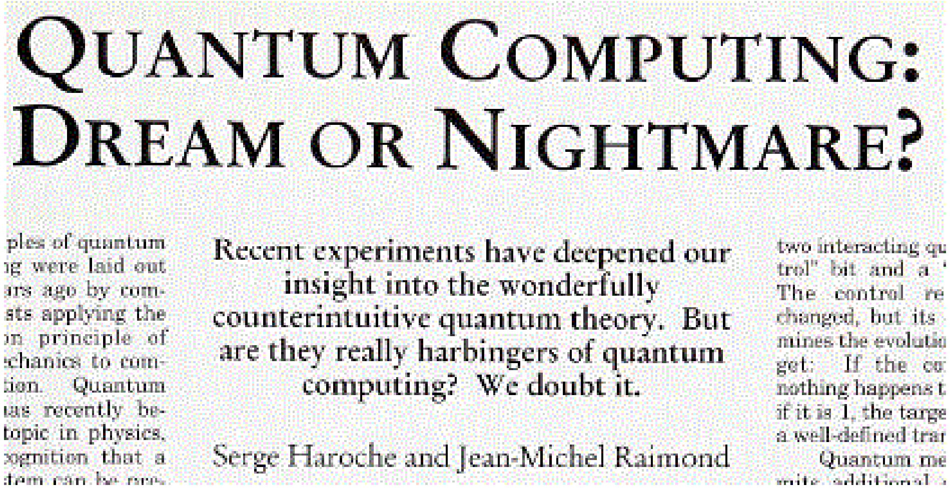
Time-independent Hamiltonian on a 2D lattice to execute a 1D quantum circuit (Lloyd & Terhal: *Adiabatic and time-independent universal computing on a 2D lattice with simple 2-qubit interactions,* New Journal of Physics 2016).

Quantum Error Correction?

Back in 1996…

After Peter Shor’s factoring algorithm came out…

Serge Haroche & Jean-Michel Raimond wrote in Physics Today



QUANTUM COMPUTING: DREAM OR NIGHTMARE?

Recent experiments have deepened our insight into the wonderfully counterintuitive quantum theory. But are they really harbingers of quantum computing? We doubt it.

Serge Haroche and Jean-Michel Raimond

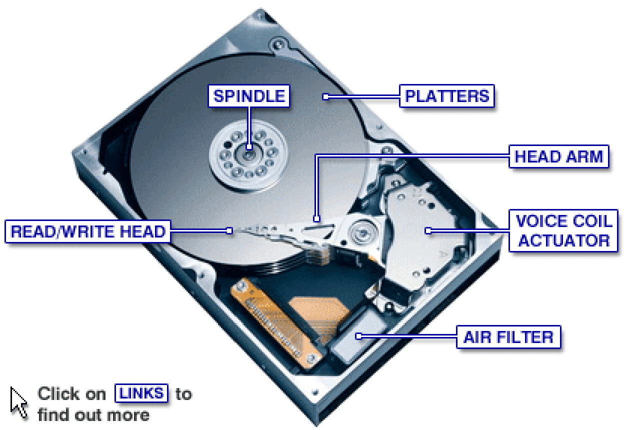
Their main points of criticism

- To do a computation with N operations and get a sensible answer, the error rate in each step should scale as 1/N or less. Such low error rates (10^{-10} or less) are unphysical.

- Watchdog strategies or quantum error-correction is an experimenter's nightmare due to its complexity.

- Computing is different from creating coherent macroscopic quantum states, i.e. Bose-Einstein condensate (or superconducting state) as it involves information and manipulation.

What is error correction?



PROBABILISTIC LOGICS AND THE SYNTHESIS OF RELIABLE ORGANISMS FROM UNRELIABLE COMPONENTS

J. von Neumann

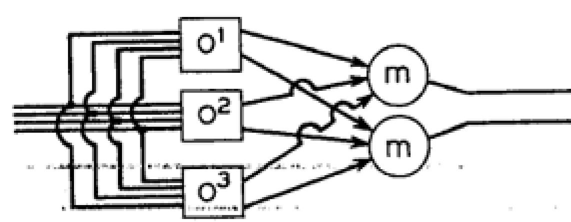
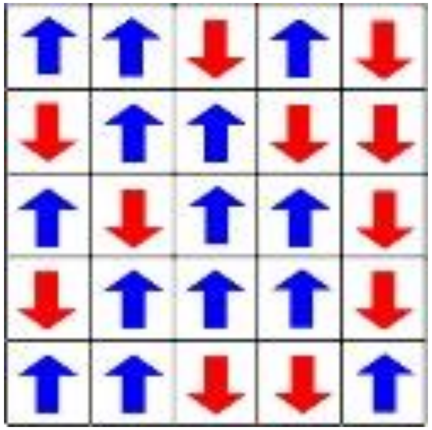


FIGURE 26

From 1956

What is error correction?



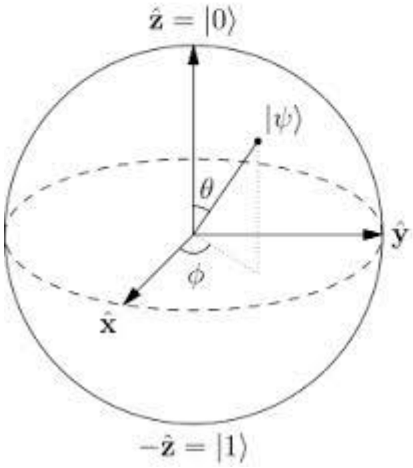
2D Ferromagnetic Ising model.
Below critical temperature T_c: symmetry-breaking and stable magnetization.

Errors= spin flips

Ferromagnetic 2-spin interactions = 'parity checks'

Encode a qubit into a 2D Ising model?

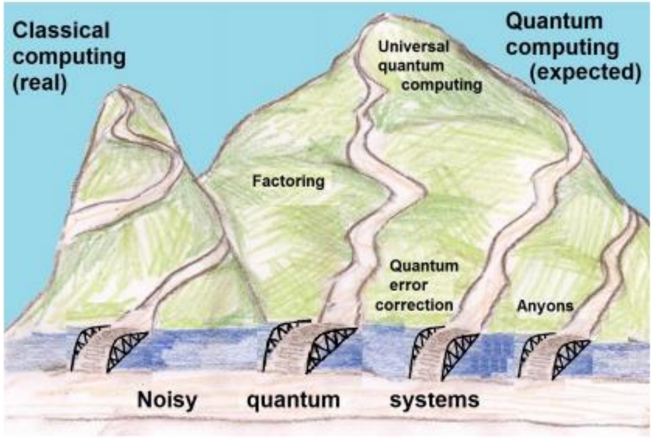
$$|0\rangle = |\uparrow\uparrow \cdots \uparrow\rangle, |1\rangle = |\downarrow\downarrow \cdots \downarrow\rangle.$$



But a rotation $e^{-iS_z\pi/2}$ on a single spin can map $|\uparrow\uparrow \cdots \uparrow\rangle + |\downarrow\downarrow \cdots \downarrow\rangle$ onto $|\uparrow\uparrow \cdots \uparrow\rangle - |\downarrow\downarrow \cdots \downarrow\rangle$ $(S_Z|\uparrow\rangle = |\uparrow\rangle, S_Z|\downarrow\rangle = -|\downarrow\rangle)$.

QUANTUM INFORMATION

Models of Computation



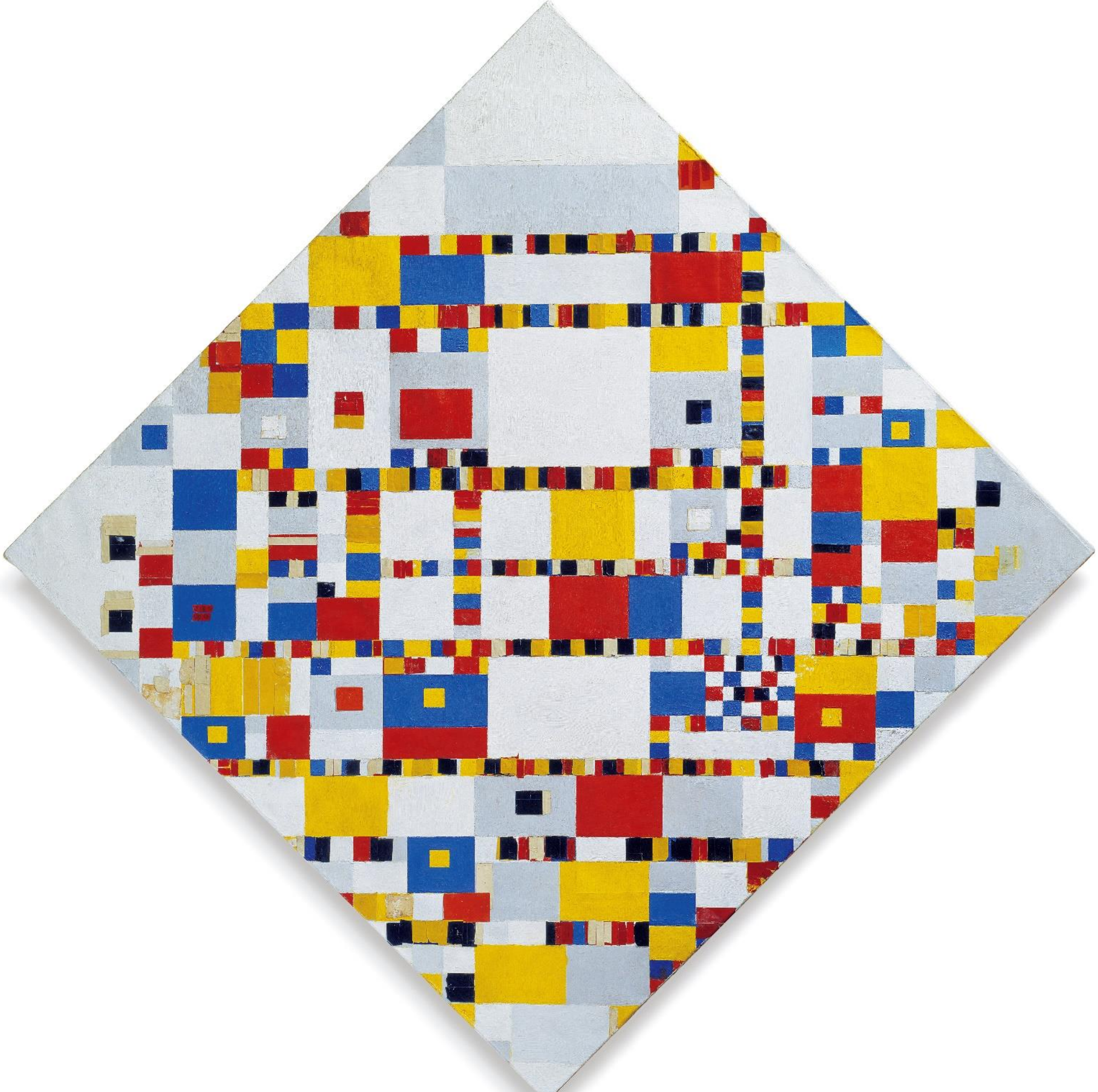
From *The quantum computer puzzle*, (Gil Kalai, 2016)

Domain of universal fault-tolerant quantum computation

Intermediate models:
probably not efficiently simulatable classically, but do not give full quantum computation either
Cannot simulate each other

- Certain constant depth quantum computations
- Analog and digital (noisy) quantum simulation
- Quantum Annealing/ Stoquastic adiabatic QC
- Linear Optics on Gaussian states and p,q measurements
- **Nonadaptive** Linear Optics with single photons (now called Boson Sampling)
- Clifford computation and fault-tolerant Clifford memory
- Fermionic Linear Optics

Domain of classical computation



Surface Code in Progress

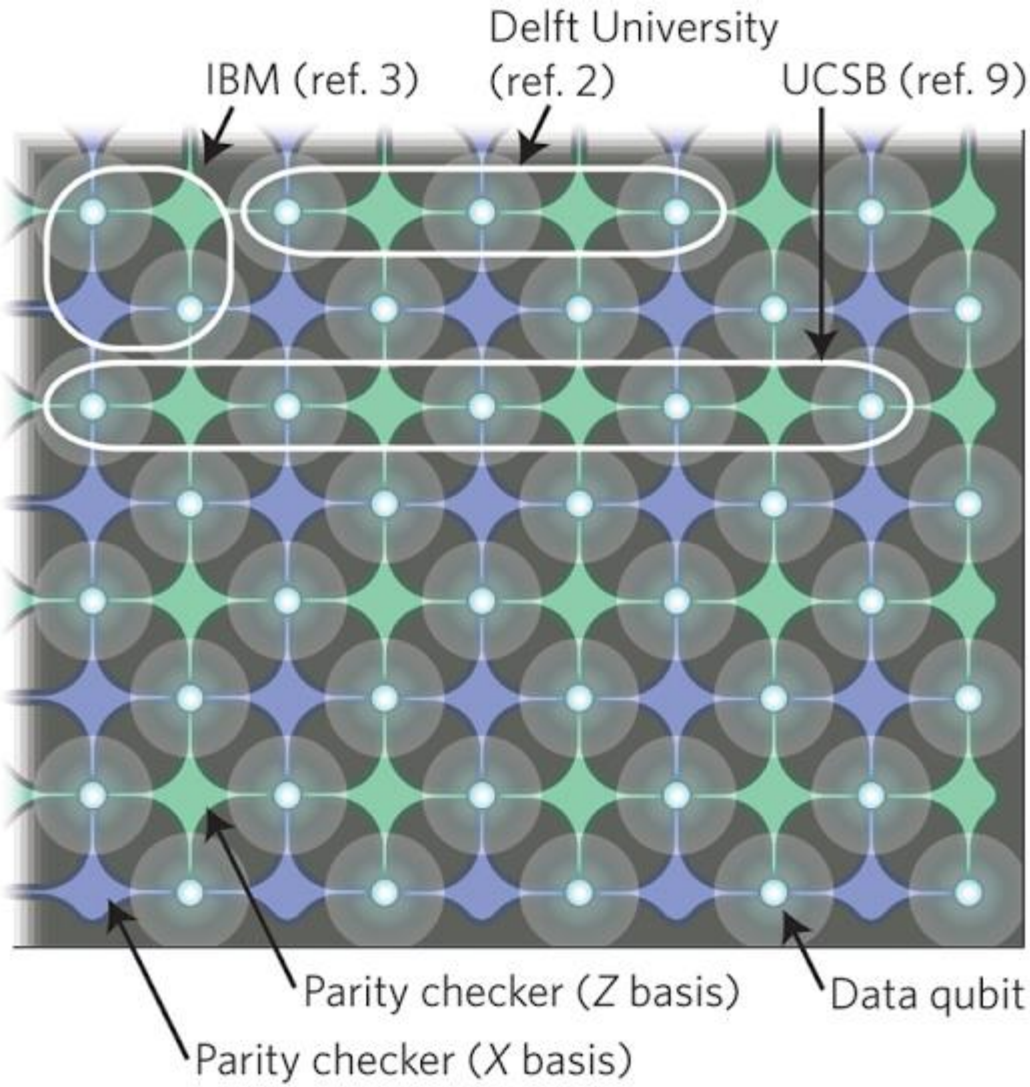
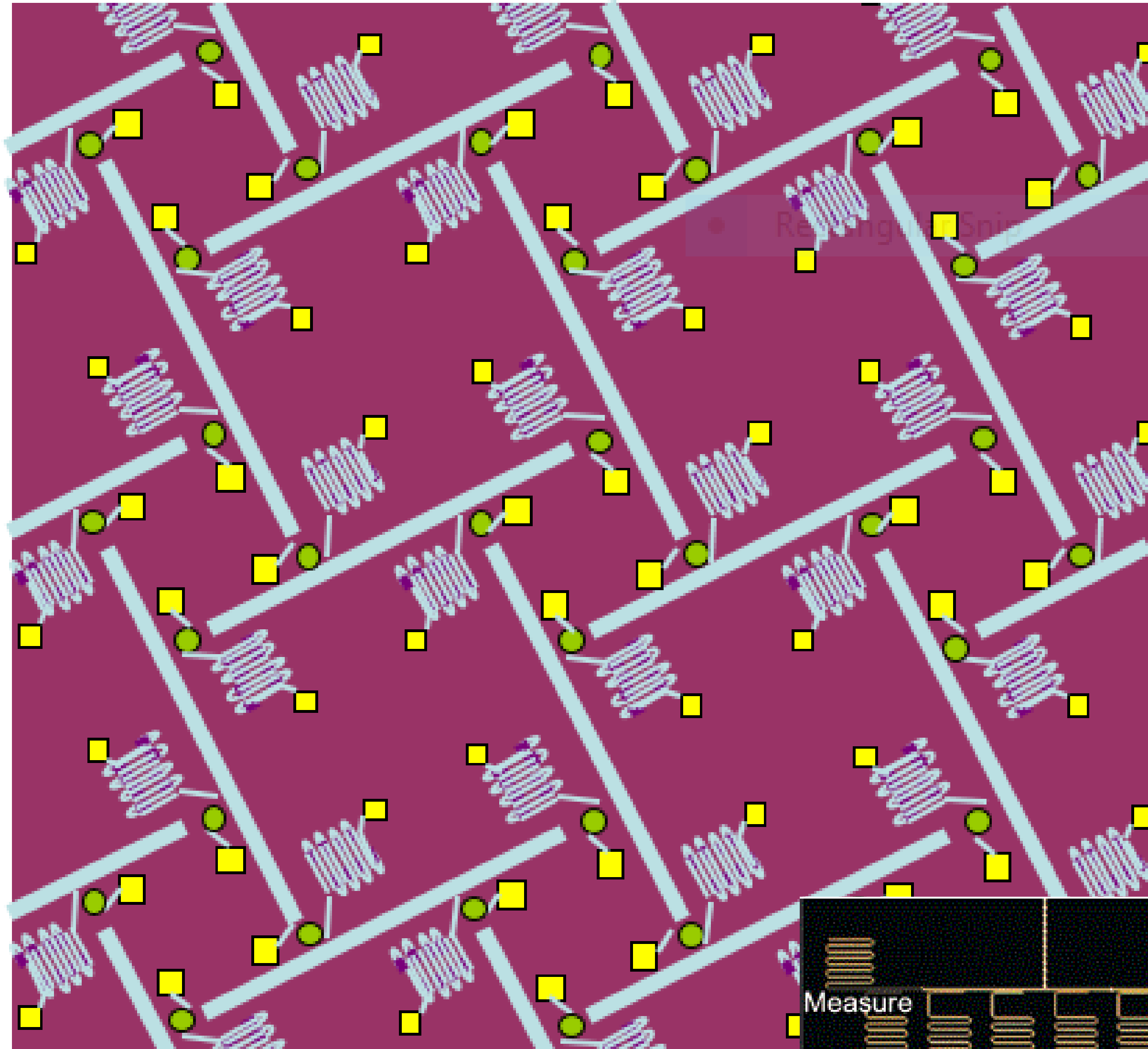
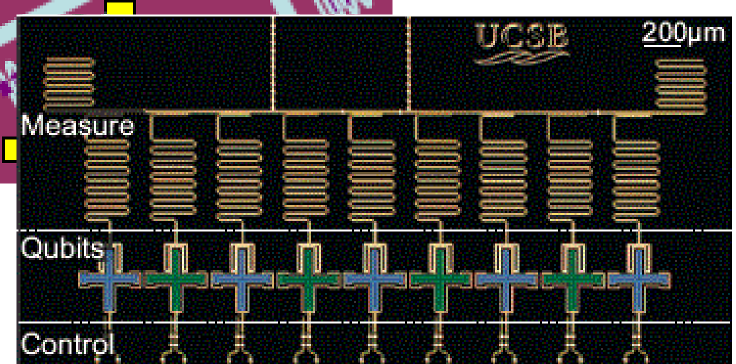


Fig. from S. Benjamin & J. Kelly, *Superconducting Qubits: Solving a wonderful problem*.
News & Views, Nature Materials 14, 561–563 (2015)

Noise Threshold:
0.6%- 1% error rate
for each component

e.g. DiVincenzo architecture for surface code
using microwave resonators and transmon qubits



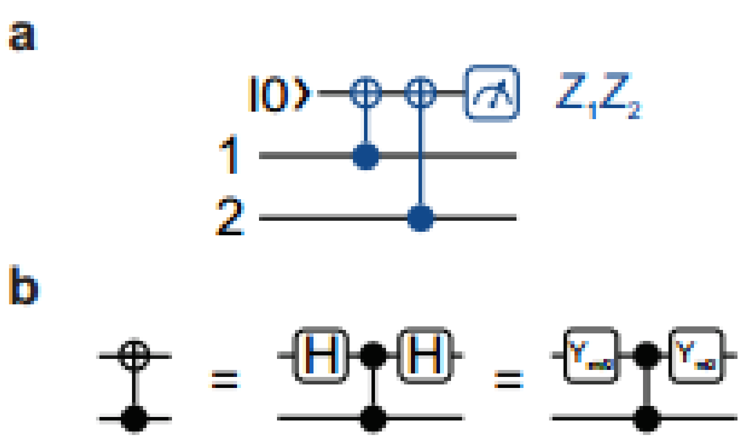


The conundrum of small codes

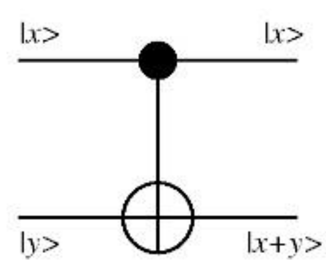
Three-bit repetition code

$$|\bar{0}\rangle = |000\rangle, |\bar{1}\rangle = |111\rangle.$$

Parity checks are $Z_1 Z_2$ and $Z_2 Z_3$ measured non-destructively, e.g.



Single X errors detected
and corrected.
In quantum code we also
measure also parity X-checks!

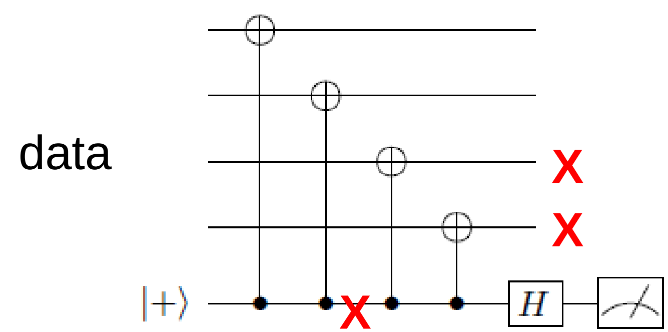


Using notation $S_Z = Z, S_X = X$

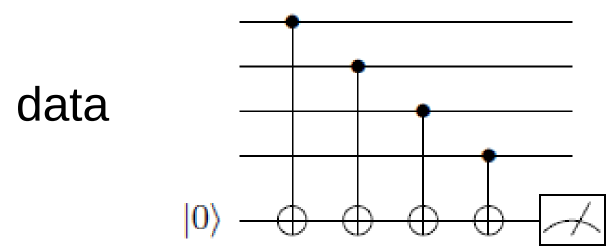
The conundrum of small codes

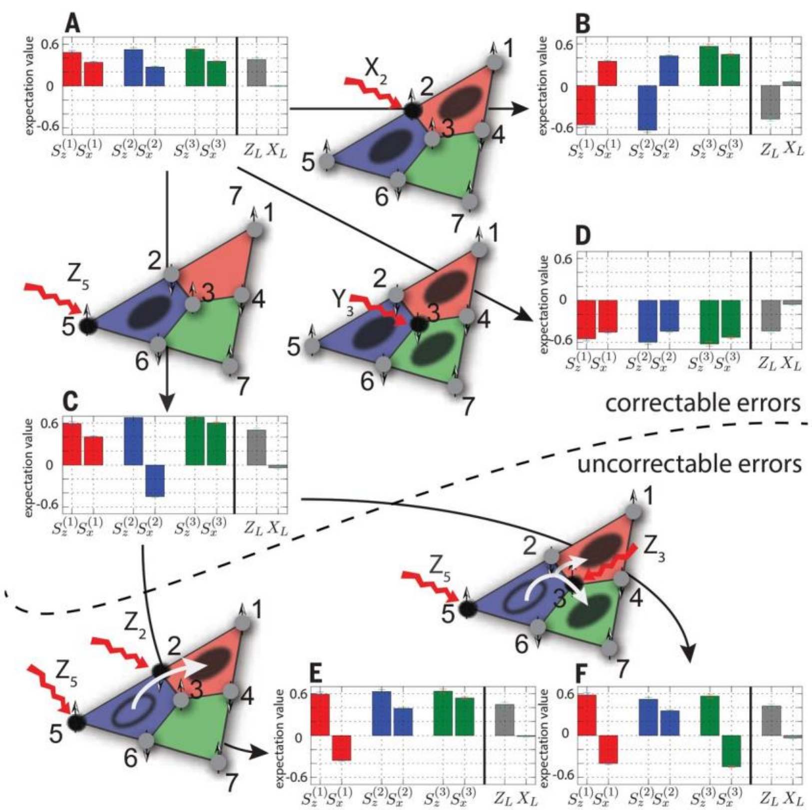
Seven qubit code (Steane) encoding 1 qubit, able to correct a single error.

Parity check circuits



Bit flip error!





Nigg *et al*, Science (2014)

The conundrum of small codes

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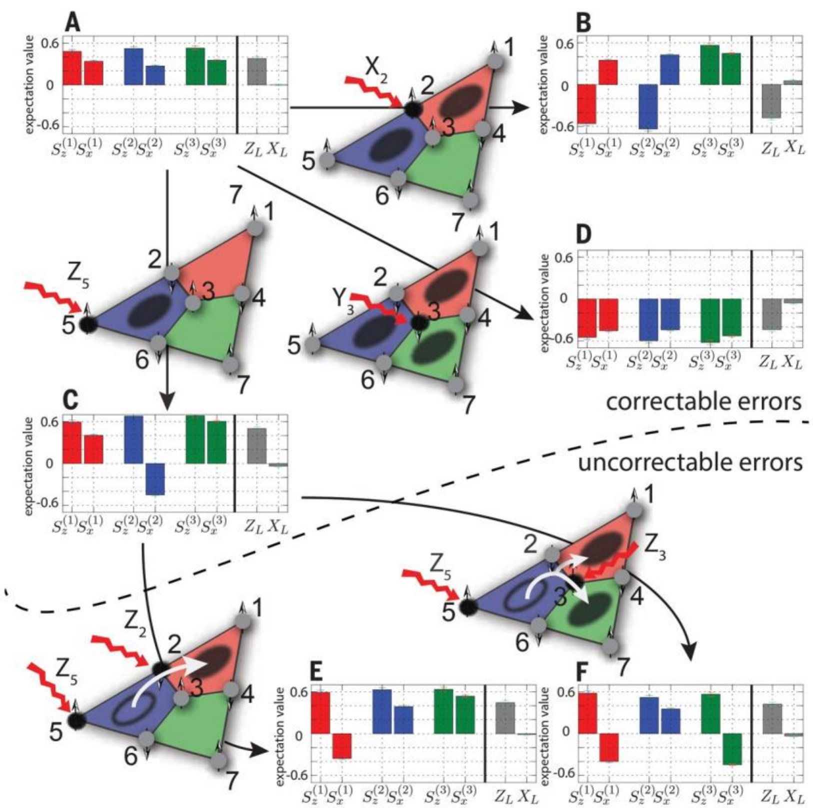
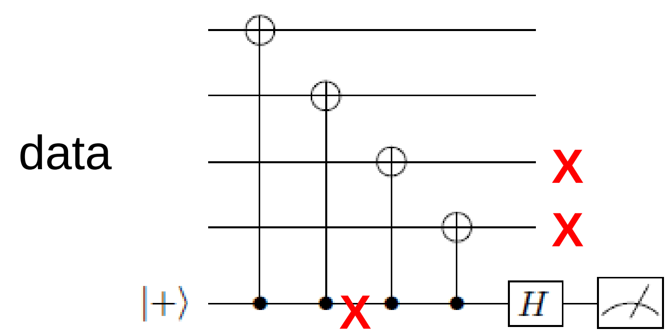
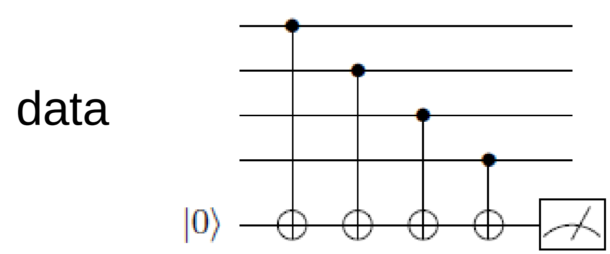


Parity check circuits

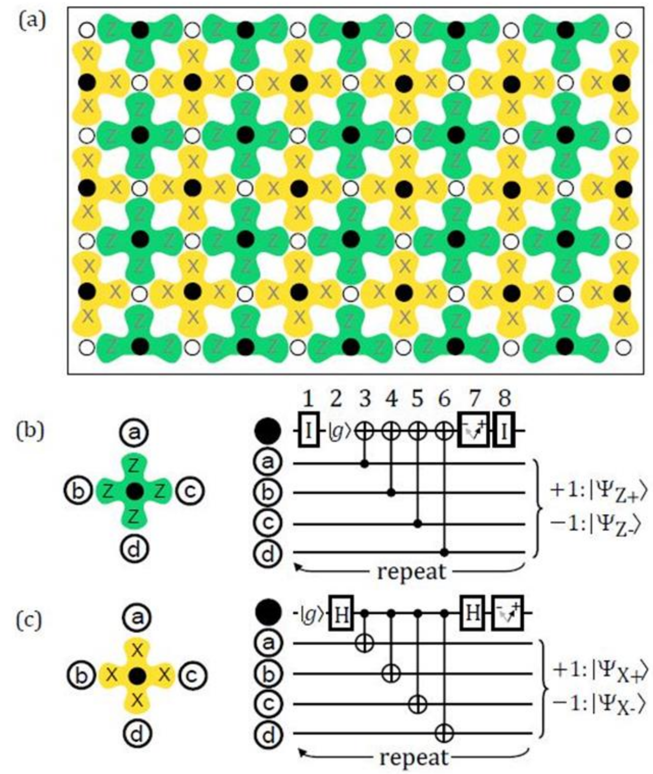


Bit flip error!



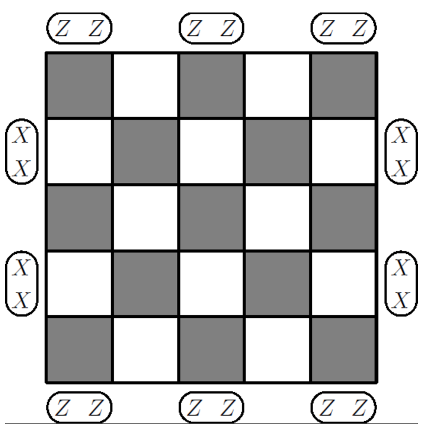
The means through which you get parity info. can also be the means through which you mess up your qubit!

Why The Surface Code



Surface code for storing 1 logical qubit using d^2 physical qubits. Can correct d/2 errors (and more)

Smallest one: d=3 Surface-17
Below d=6



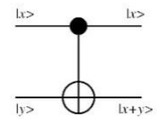
Measure of encoding success?
Get a encoded qubit with a longer lifetime τ ($F(t) \approx e^{-t/\tau}$).
How fast are the encoded gates, t_{gate} (QEC slows things down!)? Improve $\frac{t_{gate}}{\tau}$!

Qubits on vertices.
Black squares=XXXX checks
White squares=ZZZZ checks

Logic

Hadamard , Phase, CNOT are Clifford gates

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$



All quantum power comes from the T gate

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

When implementing universal QC with
T gates one needs to process error information
online, without running a backlog.

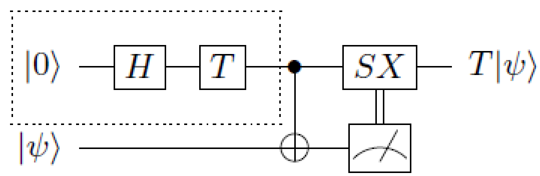


FIG. 6 Using the ancilla $T|+\rangle$ in the dashed box, one can realize the T gate by doing a corrective operation SX.

For 2D stabilizer codes you cannot do the T gate via a constant-depth fault-tolerant circuit (Bravyi, Koenig 2013). Thus lots of overhead via ‘magic-state-distillation technique’. In a 3D color code you can do a T gate without extra qubit overhead (smallest example [[15,1,3]]) but threshold is likely much worse than 1%.

Some Numbers/Estimates

Is space-time volume to factor N=2000 bit number realistic?

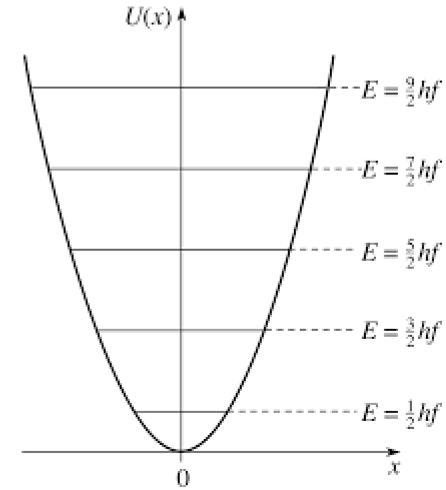
- Factoring a number with N=2000 bits needs 40 N^3 $=O(10^{11})$ Toffolis (modular exponentiation) and about 2N=4000 logical qubits
- Each logical qubit (surface code) uses p=14,500 physical qubits (assume physical error rate=threshold/10), so 58 Mqbits
- Ancilla Factory. Each Toffoli needs 7 encoded T ancillas, so $O(10^{12})$ encoded ancillas. Generating and purifying one ancilla takes 800,000 physical qubits (and 500 surface code cycles).

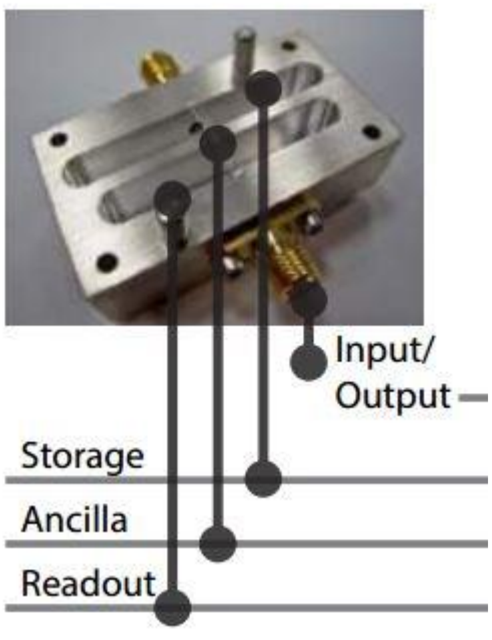
Qubit into a microwave mode

- Lots of space in a harmonic oscillator…

$$H = \hbar\omega(a^{\dagger}a + 1/2)$$

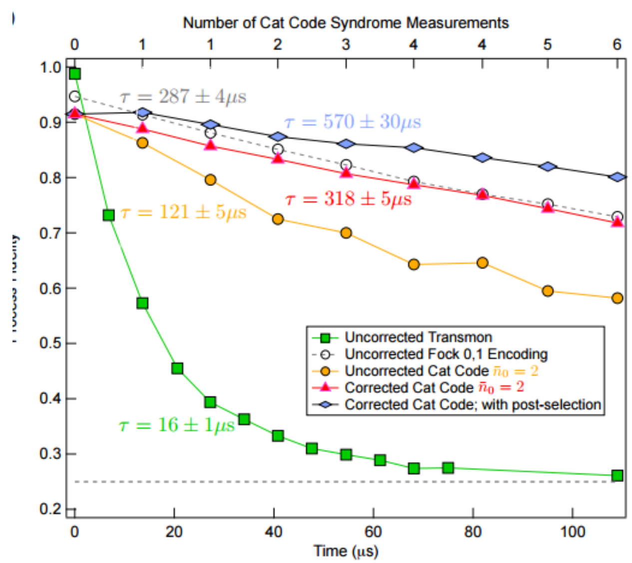
- What states offer ‘protection’, form a code?





Yale group superconducting experiments extending lifetime of qubit using a cat code, Ofek *et al.* arXiv.org: 1602.04768

Less space, fewer sources of noise?
Small bosonic codes?

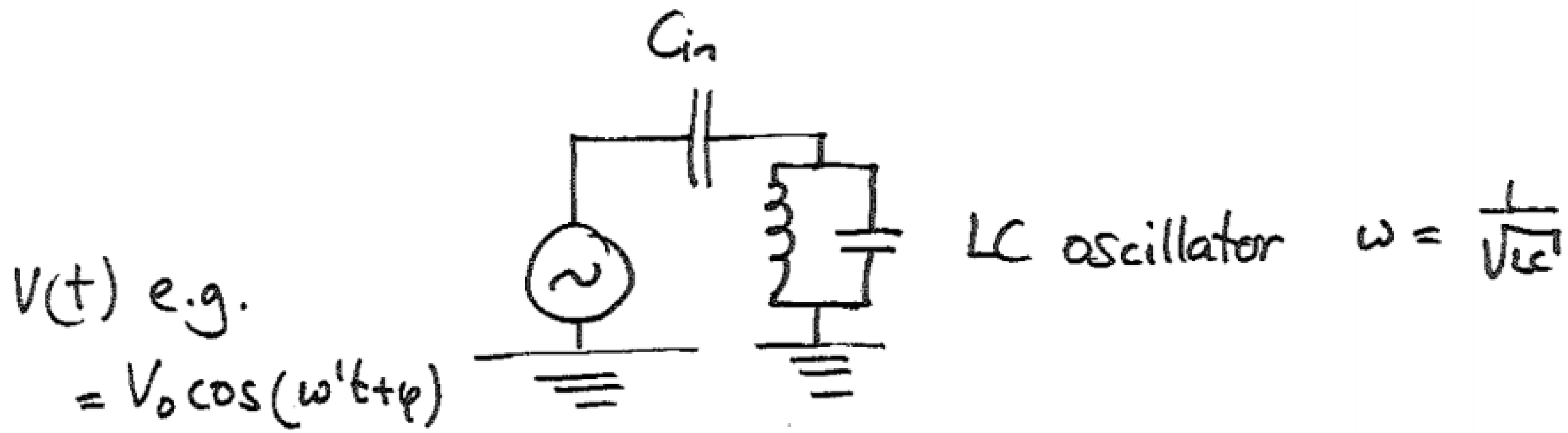


Displacement Sensor

Assume weak time-dependent unknown force $F(t)$ on oscillator so Hamiltonian $H(t) = \hbar\omega(a^\dagger a + 1/2) - \hat{q}F(t)$.

For example LC oscillator

$$H(t) = \hbar\omega(a^\dagger a + 1/2) + g\,V(t)(a + a^\dagger),\ \hat{q} = q = \frac{1}{\sqrt{2}}(a + a^\dagger)$$



What are the limits in determining the displacement caused by V(t)?

Fundamental Limit?

From Rev. Mod. Phys. (1980)

On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*

Carlton M. Caves, Kip S. Thorne, Ronald W. P. Drever,[†] Vernon D. Sandberg,[‡] and Mark Zimmermann[§]

B. Uncertainty principle and ways to measure the oscillator

In classical theory it is possible to measure the oscillator's complex amplitude $X \equiv X_1 + iX_2$ with complete precision. Not so in quantum theory. Equations (2.1) and (2.5) imply that $\hat{X}_1$ and $\hat{X}_2$ do not commute:

$$[\hat{X}_1, \hat{X}_2] = i\hbar/m\omega \,. \tag{2.8}$$

Therefore the variances of $\hat{X}_1$ and $\hat{X}_2$ in any oscillator state must satisfy

$$\Delta X_1 \Delta X_2 \geqslant \tfrac{1}{2}|\langle[\hat{X}_1, \hat{X}_2]\rangle| = \hbar/2m\omega \,, \tag{2.9a}$$

which is the complex-amplitude analog of the Heisenberg uncertainty principle for position and momentum:

$$\Delta x \Delta p \geqslant \tfrac{1}{2}\hbar \,. \tag{2.9b}$$

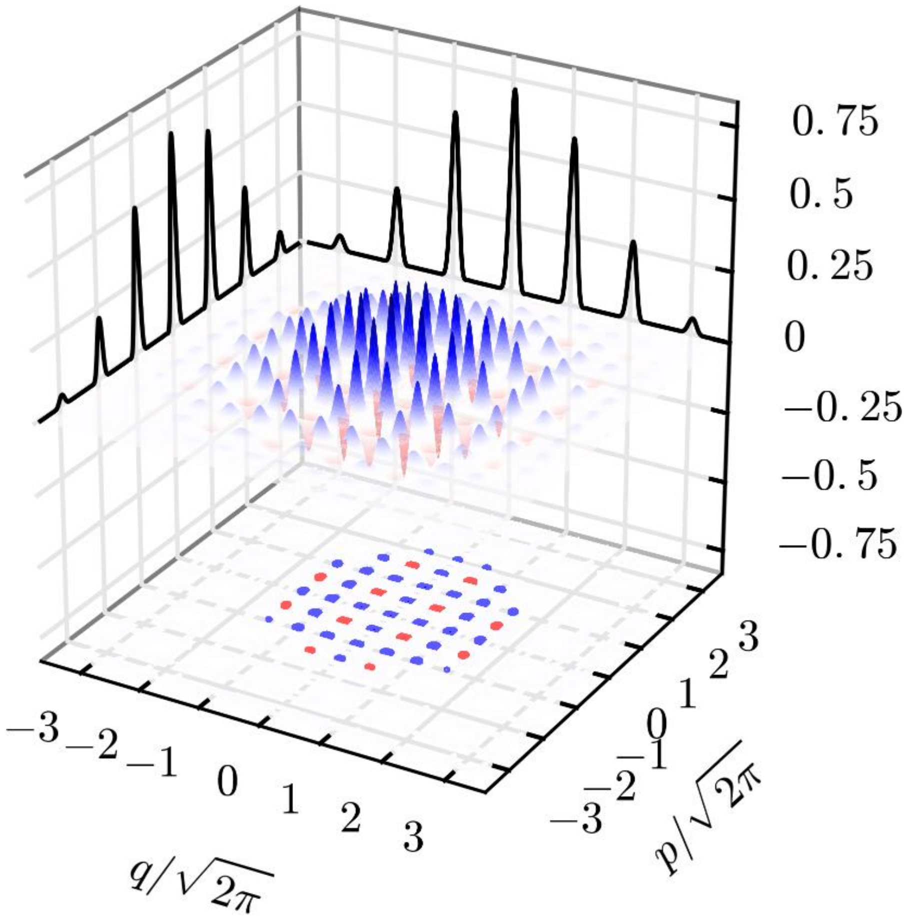
But why measure p and q? We want to measure 2 parameters. Fundamental quantum limit is subtle.

Terhal, Duivenvoorden, Single-mode Displacement Sensor, arXiv:org:1603.02242, submitted to PRL

Displacement Sensor

Grid state $|\psi_{grid}\rangle$ is a highly sensitive displacement sensor

$$p \approx 0 \, mod \sqrt{2\pi}, \; q \approx 0 \, mod \sqrt{2\pi}.$$



$$\bar{n} \approx \frac{1}{2\Delta^2}$$

σ of Gaussian envelope $\backsim \frac{1}{\Delta}$
and σ of individual peaks $\sim\Delta$

Maximum strength of displacement on vacuum input $\bar{n} \leq \pi/2$

Grid states introduced by Gottesman, Preskill, Kitaev in 2001 for quantum error correction.

How well can one do?

Using Quantum Cramer-Rao Bound one can find for estimates $\tilde{u}$ and $\tilde{v}$ (of the parameters u and v in displacement $e^{-i\,u\hat{p}+i\,v\,\hat{q}}$)

$Var(\tilde{u}) + Var(\tilde{v}) \geq 2$ (for coherent/thermal/squeezed states)

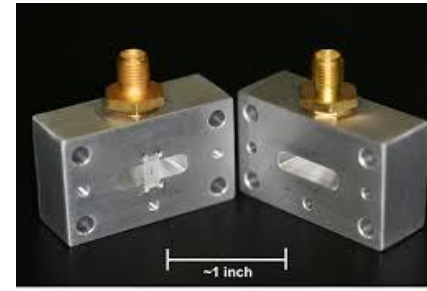
$Var(\tilde{u}) + Var(\tilde{v}) \rightarrow {}^{1}/_{(2\bar{n}+1)}$ for 2-mode squeezed (EPR) state, one mode undergoing displacement

Our Result

$Var(\tilde{u}) + Var(\tilde{v}) = O({}^{1}/_{\sqrt{\bar{n}}})$ for grid state with phase estimation 'parity' measurement, for small u, v.

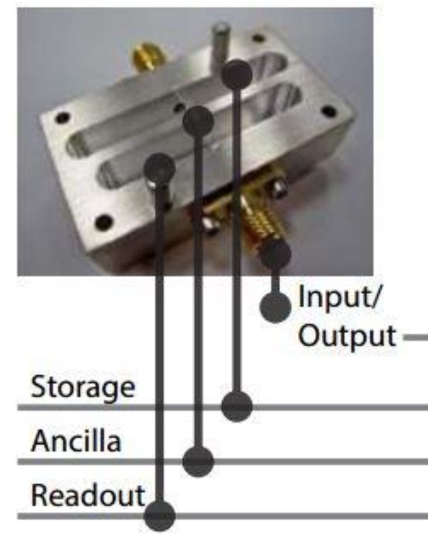
Cats in cavities, e.g. Vlastakis *et al.*, Science 2013, Ofek *et al.:* arXiv.org:1602.04768

Sensor state in Circuit-QED Hardware

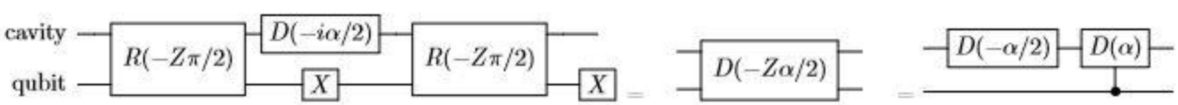


- High-Q micro-cavity, say, $1\ msec$ or more.
- High quality qubit, say, $T_1, T_2 \approx O(10-100)\ \mu sec$
- Strong dispersive qubit-cavity coupling $\chi Z a^\dagger a$

(e.g. $\frac{\chi}{2\pi} = 2.5 MHz$, cavity/qubit detuning *1 GHz*, non-linearities *O(1) kHz*)



- Dispersive coupling allows for qubit-controlled cavity rotation $(R(\theta Z) = \exp(-i\theta\ a^\dagger a\ Z))$ which can be directly used for qubit-controlled displacement.



- Controlled-rotations take $T = {}^{\pi}/_{\chi} = 200\ nanosec$.
- Use no more than 50 photons. Squeezing required.
- Initial schemes worked out in Terhal/Weigand, PRA 2016.

Conclusion

Creation of Grid or GKP code states may be experimentally feasible. They can be useful for encoding a qubit into an oscillator as well as for displacement sensing.