Glimpses of practical metrology and imperfect boson sampling: Two studies in interferometry

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Quantum Computing: Beginnings to Current Frontiers

A celebration of the work of Paul Benioff

Argonne National Laboratory, 2016 May 26

Collaborators: M. D. Lang, Z. Jiang; S. Rahimi-Keshari, T. C. Ralph

High-impact-factor syndrome (HIFS)

High-impact-factor syndrome (HIFS) is a disease of scientists and administrators. The most virulent manifestation of the disease lies in judging the accomplishments of individual scientists, especially junior scientists, in terms of the number of publications in high-impact-factor (HIF) journals.

C. M. Caves, "High-impact-factor syndrome," *APSNews* **23**(10), 8,6 (2014 November). Back-page opinion piece on HIFS.

If you think your institution would benefit from straight talk about HIFS, contact C. M. Caves at ccaves@unm.edu .

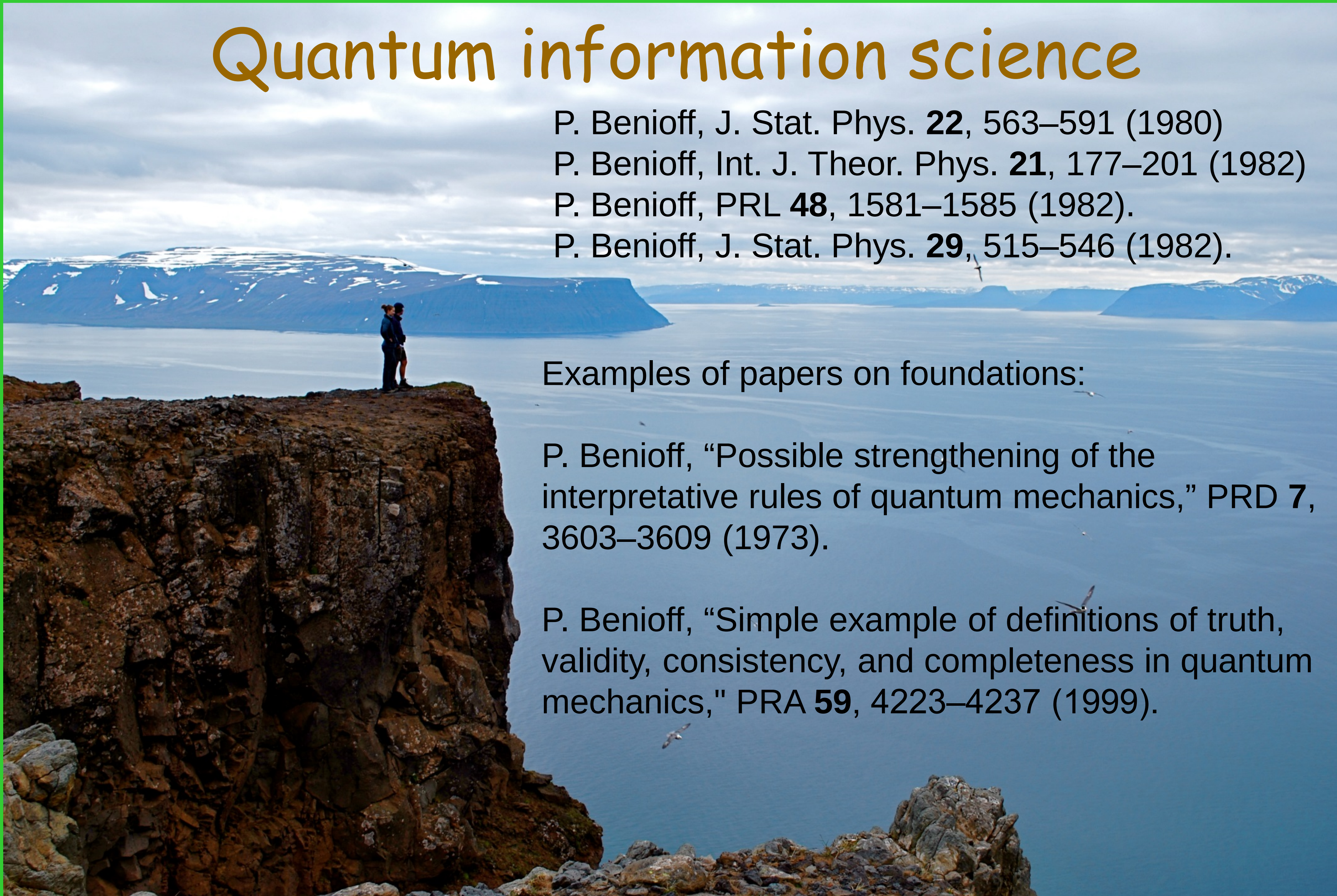
Quantum information science

P. Benioff, J. Stat. Phys. **22**, 563–591 (1980)
P. Benioff, Int. J. Theor. Phys. **21**, 177–201 (1982)
P. Benioff, PRL **48**, 1581–1585 (1982).
P. Benioff, J. Stat. Phys. **29**, 515–546 (1982).

Examples of papers on foundations:

P. Benioff, “Possible strengthening of the interpretative rules of quantum mechanics,” PRD **7**, 3603–3609 (1973).

P. Benioff, “Simple example of definitions of truth, validity, consistency, and completeness in quantum mechanics," PRA **59**, 4223–4237 (1999).



Holstrandir Peninsula overlooking Ísafjarðardjúp
Westfjords, Iceland

Foundations
Quantum optics
Quantum measurement theory
Quantum communications
Physical theory of computation

AMO physics
Condensed-matter physics
Quantum optics
Quantum gravity

Quantum control
Quantum algorithms
Quantum computation
Quantum technologies
Open quantum systems
Quantum measurements
Quantum Shannon theory
Quantum error correction
Quantum communications
Quantum phase transitions
Quantum metrology and sensing
Quantum computational complexity
Foundations of quantum mechanics
Black-hole information paradox

Quantum information science

Quantum information science

A new way of thinking

Computer science

Computational complexity depends on physical law.

New physics

Quantum mechanics as liberator.

What can be accomplished with quantum coherence that can't be done in a classical world?

Explore what quantum systems can do, instead of being satisfied with what Nature hands us.

Quantum engineering

~~**Old physics**~~

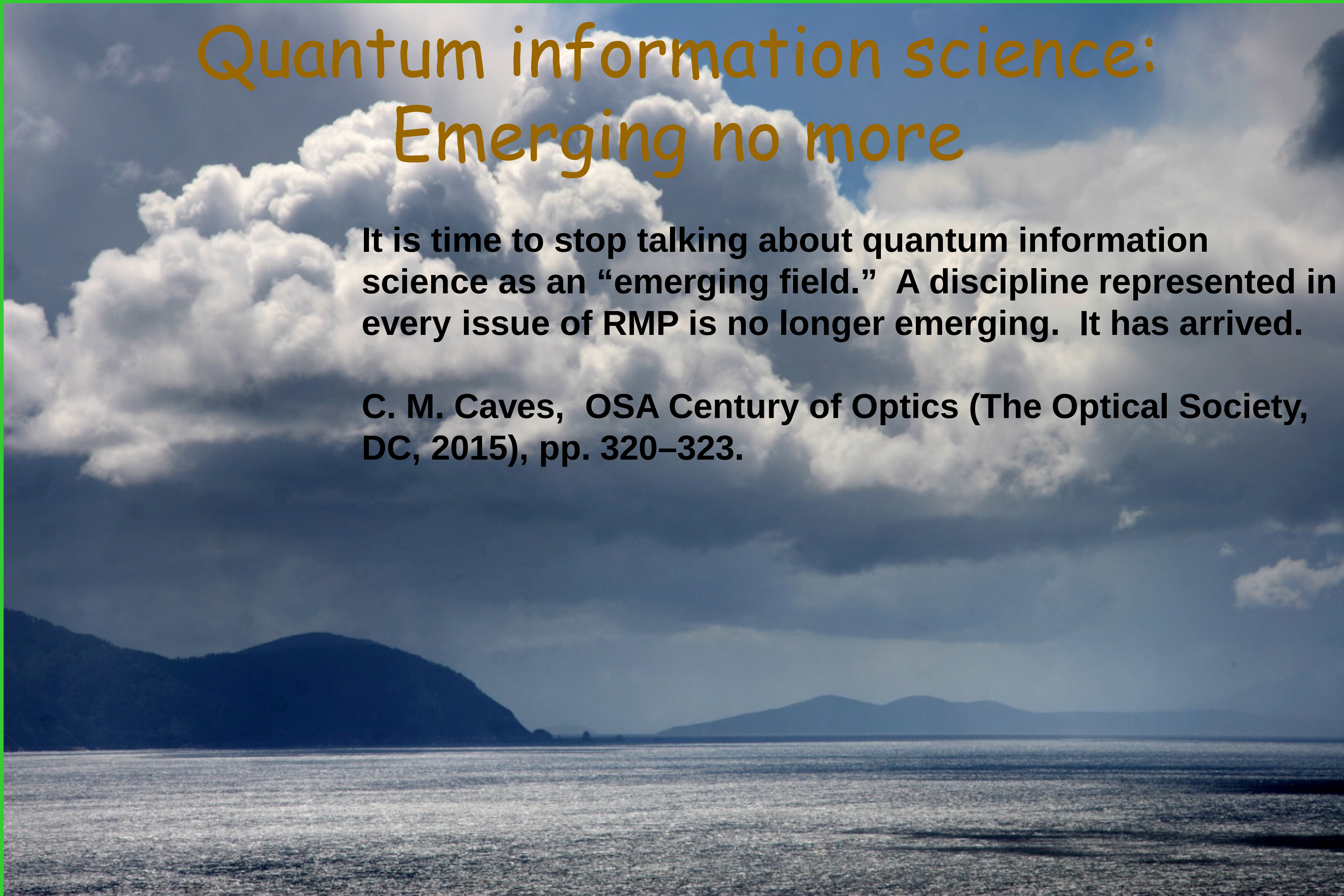
~~*Quantum mechanics as nag.*~~

~~***The uncertainty principle restricts what can be done.***~~

Quantum information science: Emerging no more

It is time to stop talking about quantum information science as an "emerging field." A discipline represented in every issue of RMP is no longer emerging. It has arrived.

C. M. Caves, OSA Century of Optics (The Optical Society, DC, 2015), pp. 320–323.

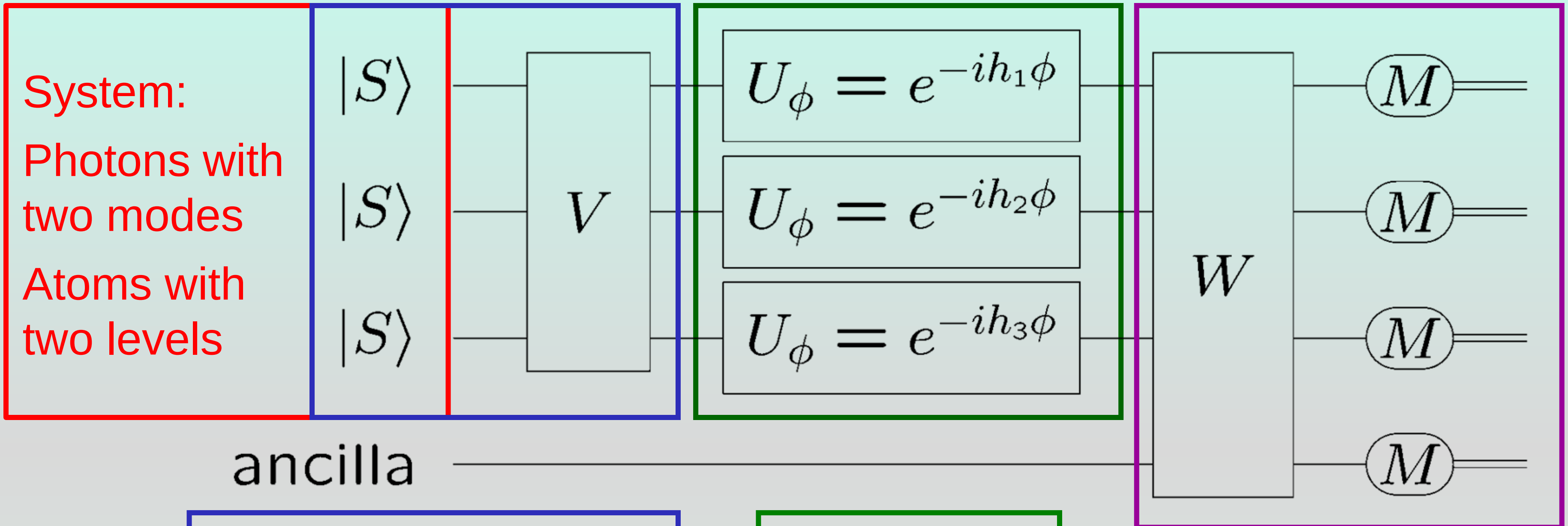


View from Cape Hauy
Tasman Peninsula, Tasmania


Cable Beach
Western Australia
Practical quantum metrology

Parameter estimation

S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. **247**, 135 (1996).
V. Giovannetti, S. Lloyd, and L. Maccone, PRL **96**, 041401 (2006).



State preparation

Dynamics

$$h = \sum_{j=1}^{N} h_j$$

Measurement

Generalized uncertainty principle
Quantum Cramér-Rao bound

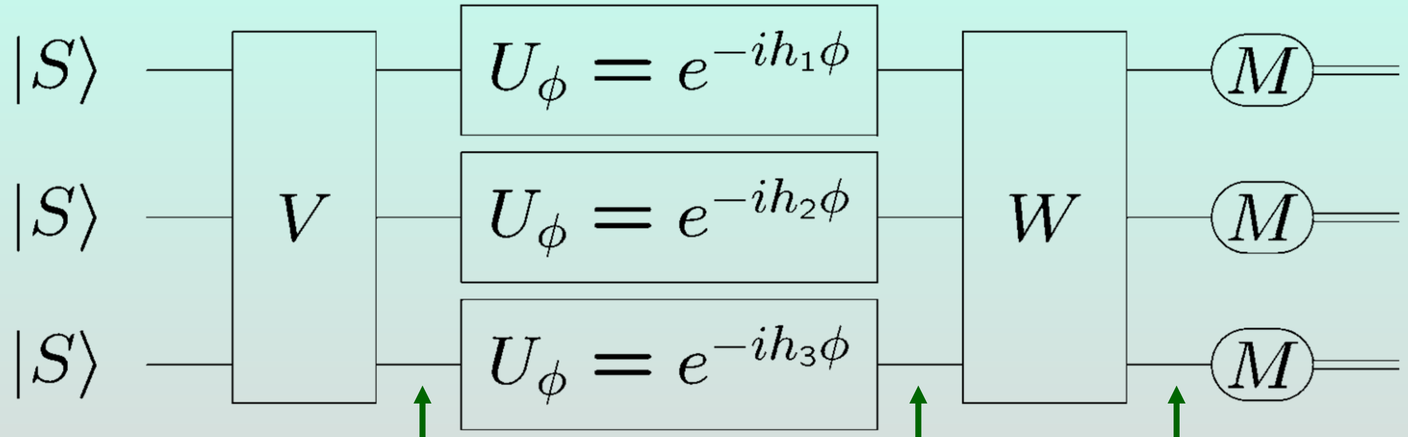
Product inputs

$$\Delta h \le \frac{1}{2}\sqrt{N}(\Lambda - \lambda)$$

$$\Delta\phi \ge \frac{1}{\sqrt{N}(\Lambda - \lambda)}$$

$$\Delta\phi_{\text{est}} \ge \frac{1}{2\Delta h} \ge \frac{1}{N(\Lambda - \lambda)} \qquad \Delta h \le \frac{1}{2}N(\Lambda - \lambda)$$

Achieving the 1/N "Heisenberg limit"



cat state

$$\frac{1}{\sqrt{2}}(|\Lambda,\ldots,\Lambda\rangle + |\lambda,\ldots,\lambda\rangle)$$

$$\frac{1}{\sqrt{2}}\left(e^{-iN\Lambda\phi}|\Lambda,\ldots,\Lambda\rangle + e^{-iN\lambda\phi}|\lambda,\ldots,\lambda\rangle\right)$$

$$e^{-iN(\Lambda+\lambda)\phi/2}\Big(\cos[N(\Lambda-\lambda)\phi/2]|\Lambda,\ldots,\Lambda\rangle - i\sin[N(\Lambda-\lambda)\phi/2]|\lambda,\ldots,\lambda\rangle\Big)$$

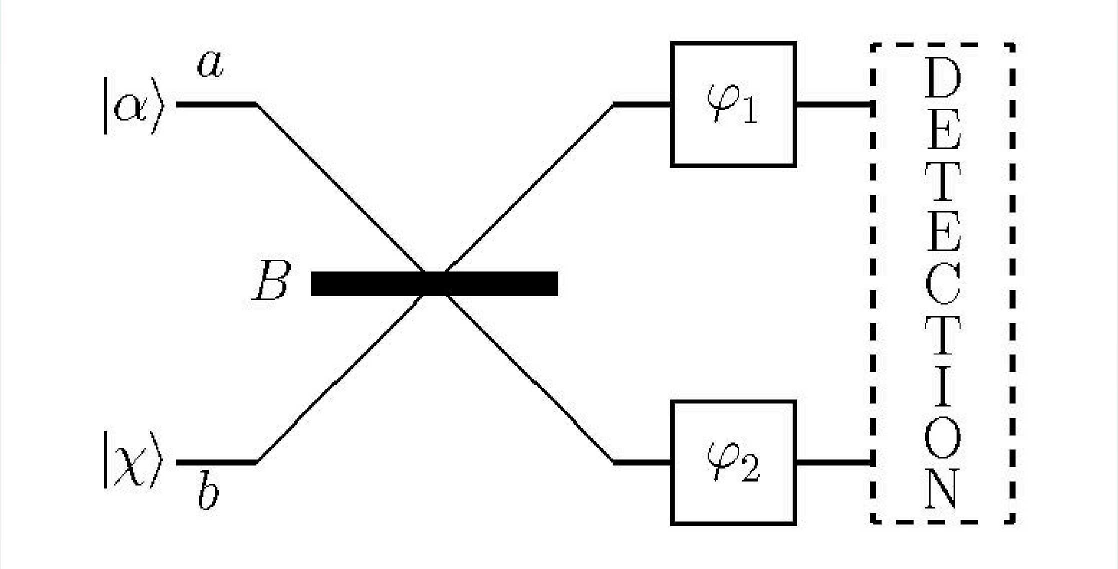
Fringe pattern with period $2\pi/N(\Lambda-\lambda)$

$$\Delta\phi = \frac{1}{N(\Lambda-\lambda)}$$

Quantum limit on practical optical interferometry

1. Cheap photons from a laser (coherent state)
2. Low losses on the detection timescale
3. Beamsplitter to make differential phase detection insensitive to laser fluctuations

Freedom: state input to the second input port; optimize with a mean number constraint.
Entanglement: mixing this state with coherent state at the beamsplitter.



Generalized uncertainty principle QCRB

$$\Delta\phi_{d,\mathrm{est}}^2 \geq \frac{1}{\Delta N_d^2} \equiv \frac{1}{\mathcal{F}}$$

$$\mathcal{F} = 2|\alpha|^2\langle(\Delta p)\rangle^2 + \bar{N}_b \leq |\alpha|^2\left(2\bar{N}_b + \sqrt{\bar{N}_b(\bar{N}_b+1)} + 1\right) + \bar{N}_b$$

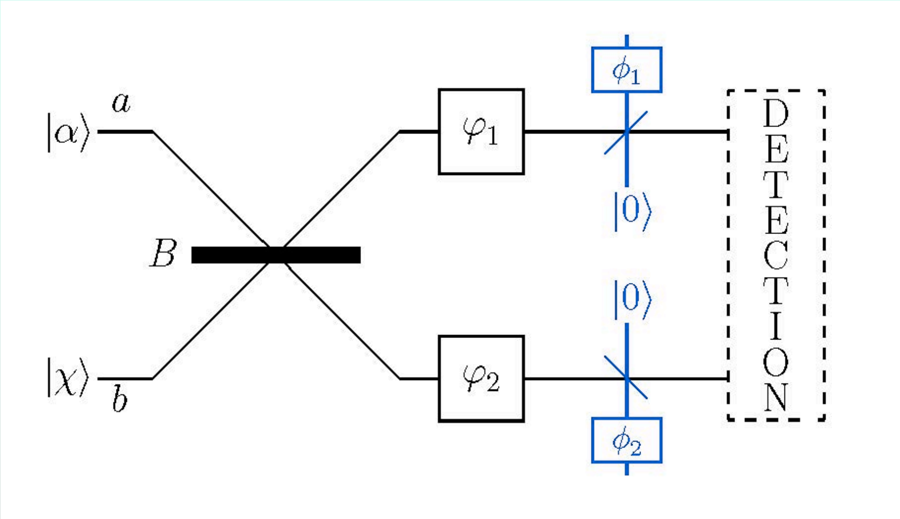
$$= |\alpha|^2 e^{2r} + \sinh^2 r$$

M. D. Lang and C. M. Caves, PRL **111**, 17360 (2013).

Optimum achieved by differenced photodetection in a Mach-Zehnder configuration.

Achieved by squeezed vacuum into the second input port

Practical optical interferometry: Photon losses



M. D. Lang , UNM PhD dissertation, 2015.

$$1-\eta = \left(\begin{array}{c}\text{fractional loss}\\ \text{in each arm}\end{array}\right)$$

$$\Delta\phi^2_{d,\text{est}} \ge \boxed{\frac{1}{\mathcal{F}_Q} \ge \frac{1}{\mathcal{C}_Q} \ge \frac{1}{I_Q}}$$

B. M. Escher, R. L. de Matos Filho, and L. Davidovich, Nat. Phys. **7**, 406–411 (2011).

$$\mathcal{C}_Q = \left(\begin{array}{c}\text{Upper bound on quantum Fisher information}\\ \text{maximized over fake phase shifts } \phi_1 \text{ and } \phi_2\\ \text{and over all states input to second input port}\end{array}\right)$$

Z. Jiang, PRA **89**, 032128 (2014).

$$\mathcal{F}_Q = \left(\begin{array}{c}\text{Quantum Fisher information}\\ \text{for squeezed vacuum}\\ \text{input to second input port}\end{array}\right)$$

$$I_Q = \boxed{\frac{|\alpha|^2+\bar{N}_b}{\dfrac{1-\eta}{\eta}+\dfrac{1}{2\langle(\Delta p)^2\rangle}} \simeq \frac{\eta}{1-\eta}|\alpha|^2}$$

When $|\alpha|^2 \gg \bar{N}_b$, all agree to within corrections of order $\bar{N}_b/|\alpha|^2$.

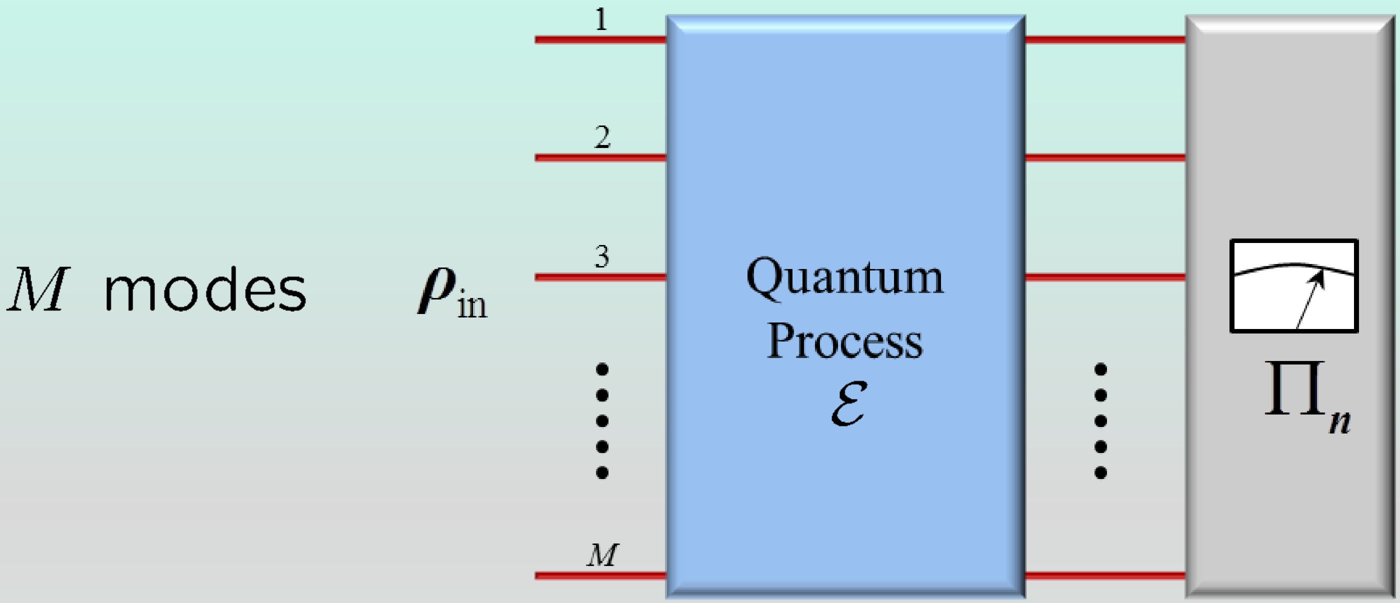
Optimum achieved by differenced photodetection in a Mach-Zehnder configuration.

Imperfect boson sampling

Pinnacles National Park
Central California

Quantum-optical experiments

S. Rahimi-Keshari, T. C. Ralph, and C. M. Caves, PRX, to be published; arXiv:1511.06526.



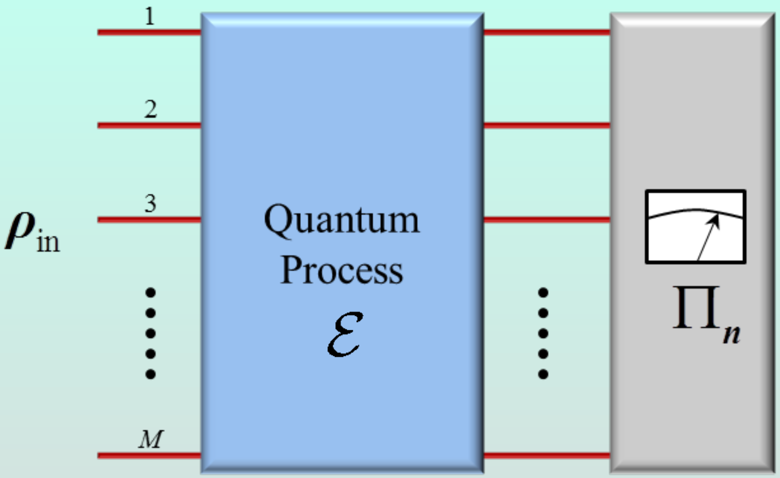
Output distribution

$$p(\boldsymbol{n}) = \mathrm{Tr}[\Pi_n \mathcal{E}(\rho_{\text{in}})]$$

Question: Can one sample efficiently from the output distribution classically?

Boson sampling

S. Aaronson and A. Arkhipov, Theory of Computing **9**(4), 143 –252 (2013).



ρ_{in}: $N \simeq \sqrt{M}$ single photons into the first N modes

Quantum process $\mathcal{E}$: passive linear-optical network (LON)

$$\mathcal{E}(\rho_{\text{in}}) = \mathcal{U}\rho_{\text{in}}\mathcal{U}^\dagger \qquad \boldsymbol{a} = \begin{pmatrix} a_1 & \dots & a_M \end{pmatrix}$$

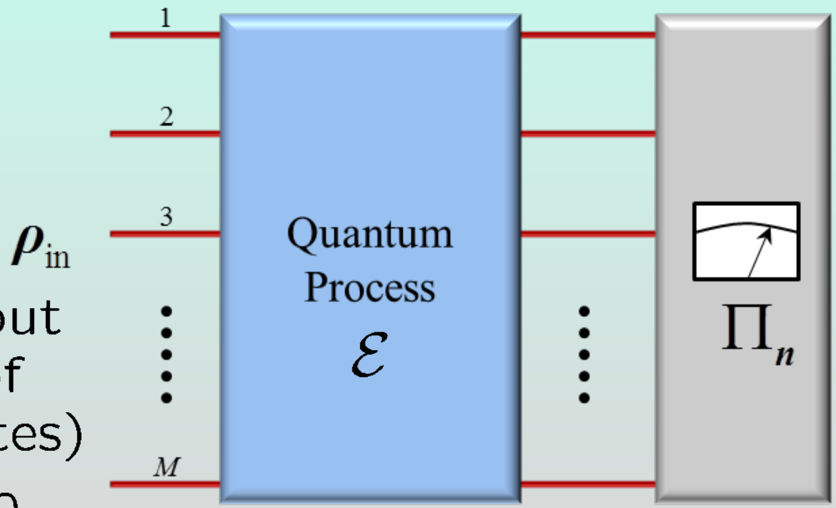
$$\mathcal{U}^\dagger \boldsymbol{a} \mathcal{U} = \boldsymbol{a}\boldsymbol{U} \qquad \boldsymbol{U} = M \times M \text{ matrix}$$

$\Pi_{\boldsymbol{n}}$: on-off photodetection

$p(\boldsymbol{n})$ is the *permanent* of a (sub)matrix of $\boldsymbol{U}$.

It is very likely that one cannot sample efficiently from the output distribution.

Classical inputs and classical processes



Classical input (mixture of coherent states)

$P(\boldsymbol{\alpha}|\rho_{\text{in}}) \geq 0$

P function of input

PQD $W^{(t)}(\boldsymbol{\alpha}|\rho_{\text{in}})$
$t = 1$

Classical process

$P_{\mathcal{E}}(\boldsymbol{\beta}|\boldsymbol{\alpha}) \geq 0$

P function of $\mathcal{E}(|\alpha\rangle\langle\alpha|)$

Lossless LON:

$P_{\mathcal{E}}(\boldsymbol{\beta}|\boldsymbol{\alpha}) = \delta^{2M}(\boldsymbol{\beta} - \boldsymbol{\alpha}\boldsymbol{U})$

Any measurement

$\Pi^M Q_\Pi(\boldsymbol{n}|\boldsymbol{\beta}) = \langle\boldsymbol{\beta}|\Pi_{\boldsymbol{n}}|\boldsymbol{\beta}\rangle \geq 0$

Q function of $\Pi_{\boldsymbol{n}}$

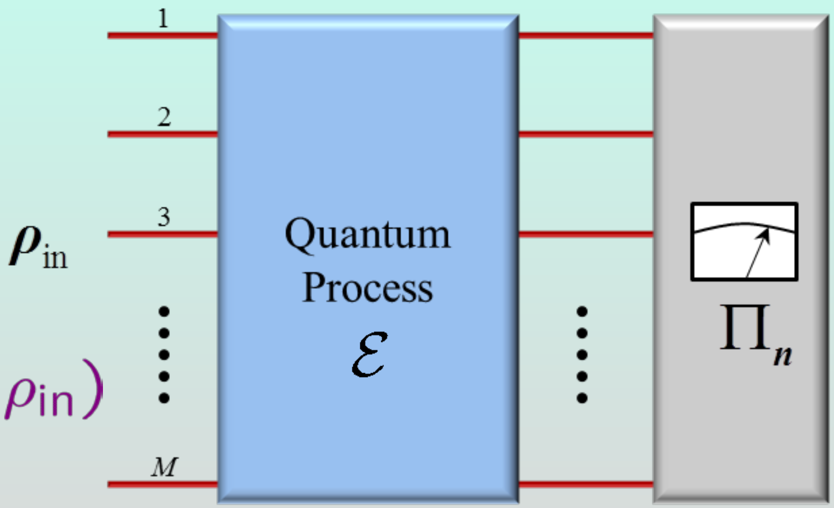
PQD $W_\Pi^{(-s)}(\boldsymbol{n}|\boldsymbol{\beta})$
$s = 1$

PQD transition function
$T_{\mathcal{E}}^{(s,t)}(\boldsymbol{\beta}|\boldsymbol{\alpha})$

Classical simulation using classical waves

$$p(\boldsymbol{n}) = \int d^{2M}\boldsymbol{\beta}\; \pi^M Q_\Pi(\boldsymbol{n}|\boldsymbol{\beta}) \int d^{2M}\boldsymbol{\alpha}\, P_{\mathcal{E}}(\boldsymbol{\beta}|\boldsymbol{\alpha})\, P(\boldsymbol{\alpha}|\boldsymbol{\rho}_{\text{in}})$$

Simulation using arbitrary operator orderings



PQD $W^{(t)}(\boldsymbol{\alpha}|\rho_{\text{in}})$

PQD $W_{\Pi}^{(-s)}(\boldsymbol{n}|\boldsymbol{\beta})$

PQD transition function

$T_{\mathcal{E}}^{(s,t)}(\boldsymbol{\beta}|\boldsymbol{\alpha})$

Classical simulation using classical waves if all the PQDs are efficiently computable and nonnegative (and no more singular than a δ function).

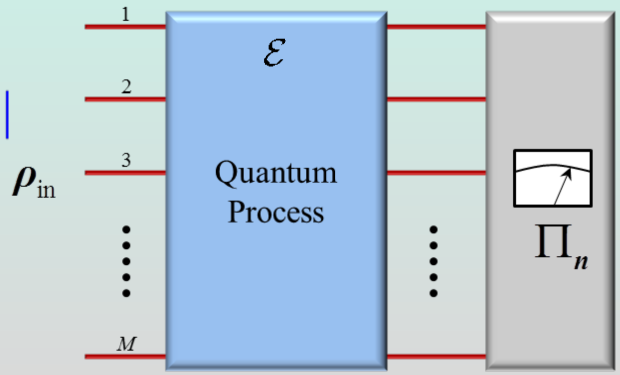
$$p(\boldsymbol{n}) = \int d^{2M}\boldsymbol{\beta}\; \pi^M W_{\Pi}^{(-s)}(\boldsymbol{n}|\boldsymbol{\beta}) \int d^{2M}\boldsymbol{\alpha}\, T_{\mathcal{E}}^{(s,t)}(\boldsymbol{\beta}|\boldsymbol{\alpha})\, W^{(t)}(\boldsymbol{\alpha}|\rho_{\text{in}})$$

We actually only need the output and measurement PQDs to be efficiently computable and nonnegative.

Classical simulation of imperfect boson sampling

LON with depth d
and uniform loss
$\eta_L = \eta_0^d = \eta_0^{\log_\ell M}$

N single photons
$\rho = (1-\mu)|0\rangle\langle 0| + \mu|1\rangle\langle 1|$
Purity μ
Mode mismatching η_B



On-off photodetectors
Efficiency η_D
Random-count probability p_D

Classical simulation by this method if and only if
$$p_D \geq \eta = (\text{overall loss}) = \mu\eta_B\eta_L\eta_D = \mu\eta_B\eta_D\eta_0^{\log_\ell M}.$$

$N = \min(M, \sqrt{M}/\eta)$
Input purity $\mu = 0.5$
Input mode mismatching $\eta_B = 0.1$
Ports per optical element $\ell = 2$
Loss per optical element $\eta_0 = 0.98$
Photodetector efficiency $\eta_D = 0.95$

Classical simulation of imperfect boson sampling

Classical simulation by
this method if and only if

$$p_D \geq \eta = \mu\eta_B\eta_L\eta_D.$$

$N = \min(M, \sqrt{M}/\eta)$
Input purity $\mu = 0.5$
Input mode mismatching $\eta_B = 0.1$
Ports per optical element $\ell = 2$
Loss per optical element $\eta_0 = 0.98$
Photodetector efficiency $\eta_D = 0.95$

M	η_L	$\boxed{\eta}$	$\sqrt{M}/\eta$	N	$N\eta$	η	$\boxed{p_D^{(\mathrm{eff})}}$
10	0.94	0.044	71	10	0.44	0.044	0.046
100	0.87	0.042	241	100	4.2	0.042	0.049
1 600	0.81	0.038	1 044	1 044	40	0.038	0.034

Dark-count probability $< \eta$ relatively easy to achieve.

Mode-mismatched photons

Mode-mismatched photons that reach and are counted at the photodetectors are the chief challenge for boson sampling.
Simulation criterion is roughly that the number of random counts exceed the number of mode-matched counts.

Thanks for your attention