QCD Bound-State Problem:

Off-shell Persistence of Composite Pions and Kaons

Sixue Qin

Argonne National Laboratory

2017-06-02 @ PIEIC2017, ANL

1 Background: What are bound-states?

$$E_{\mathrm{sys}} < \sum E_i$$

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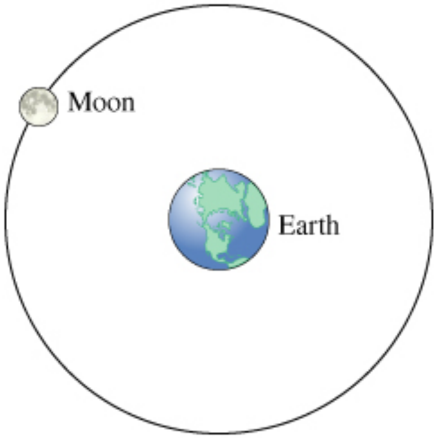

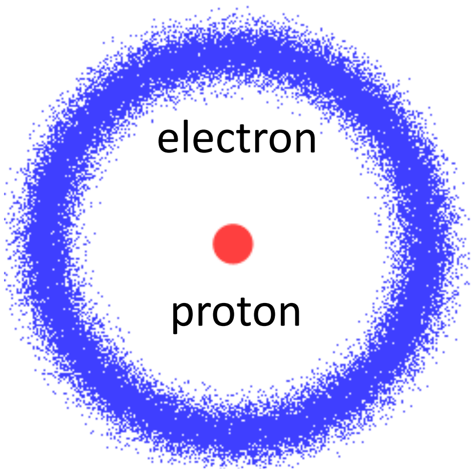

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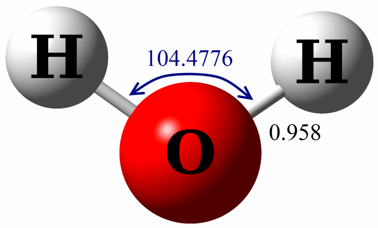
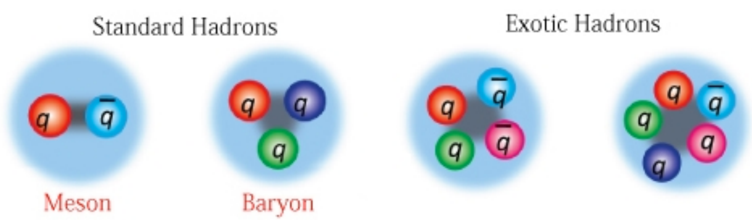
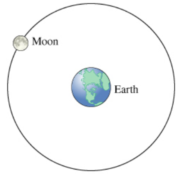


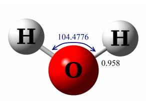


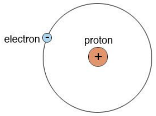
1 **Background:** Why do we study bound-states?

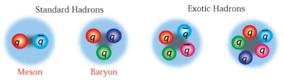
Experiment

"Easy" objects involving interactions









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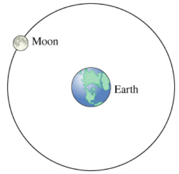

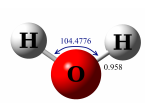

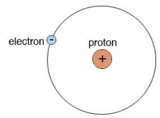

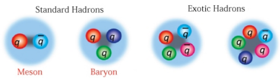

+ etc.

Theory

"Simple" objects
involving
dynamics

Newtonian Mechanics

Quantum Mechanics

Quantum Field Theory

+ etc.

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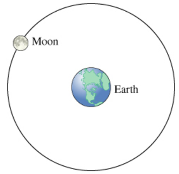

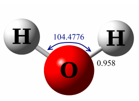

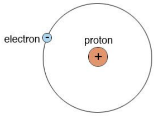

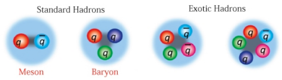

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Experiment

"Easy" objects involving interactions

What matter is possible & how is it constituted?

Theory

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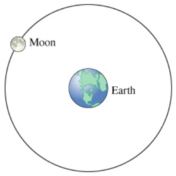

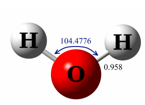

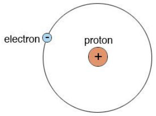

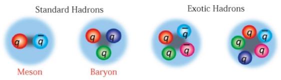

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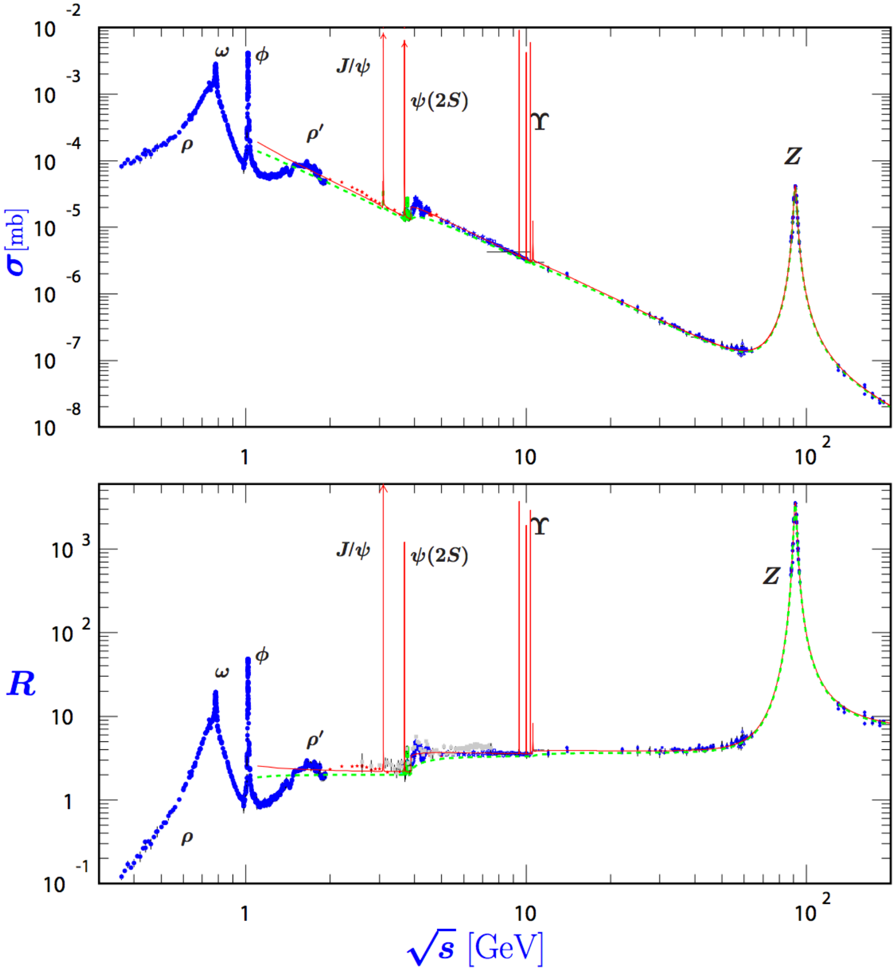

e⁺ e⁻ hadronic annihilation

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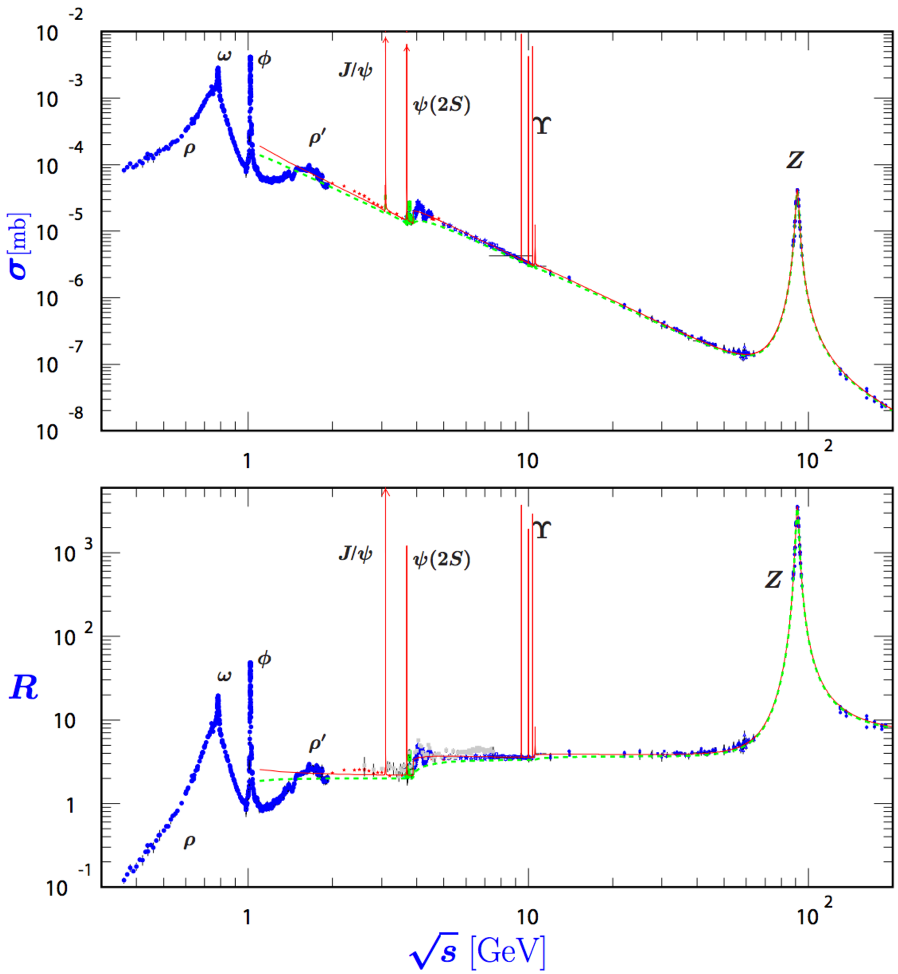
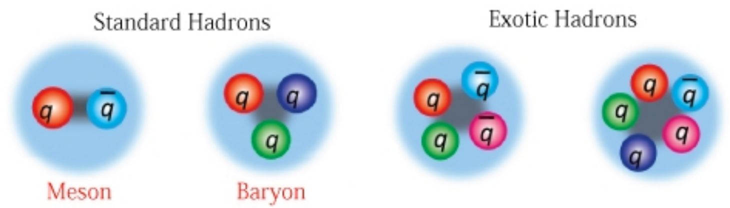
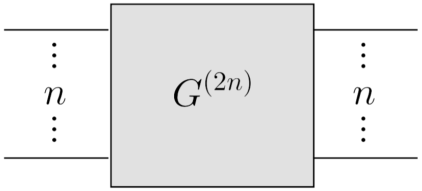


$e^+ e^-$ hadronic annihilation

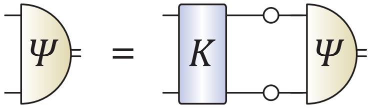


Quantum Field Theory

- Green functions



- Bethe-Salpeter equation



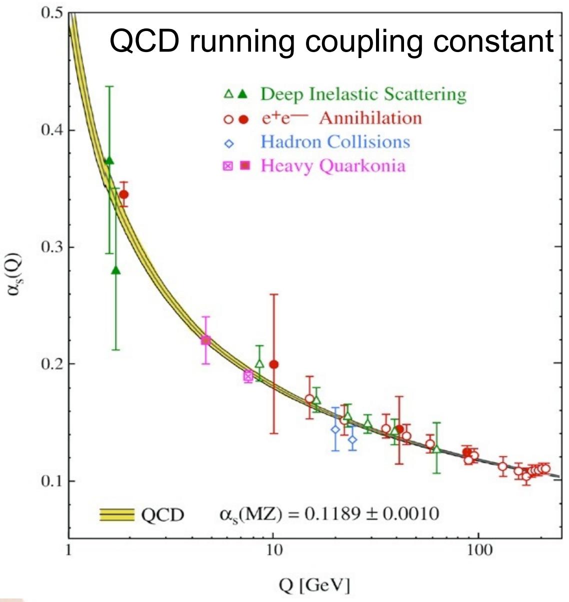
1 Background: Why is QCD bound-state problem difficult?

• Relativistic bound states

"These problems are those involving bound states […] such problems necessarily involve a breakdown of ordinary perturbation theory. […] The pole therefore can only arise from a divergence of the sum of all diagrams […]"

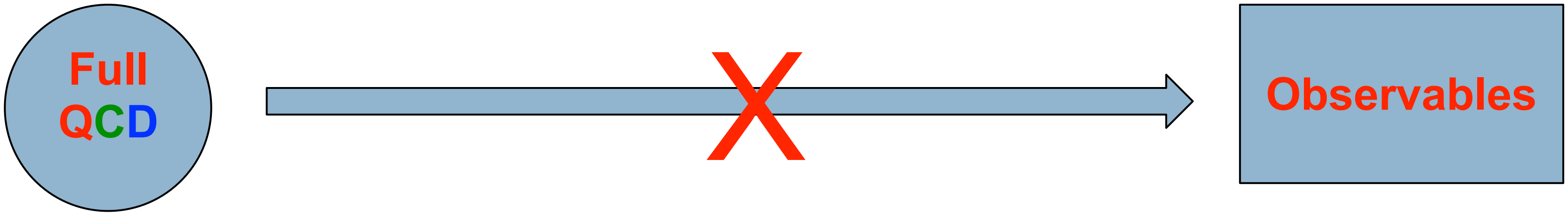
The QFT book vol1 p564 Weinberg

• Strongly coupled systems



- Asymptotic freedom: Bonds between particles become asymptotically weaker as energy increases and distance decreases (Nobel Prize).
- Quark and Gluon Confinement: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
- Dynamical Chiral Symmetry Breaking: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners.*

1 Background: Non-perturbative approaches of QCD



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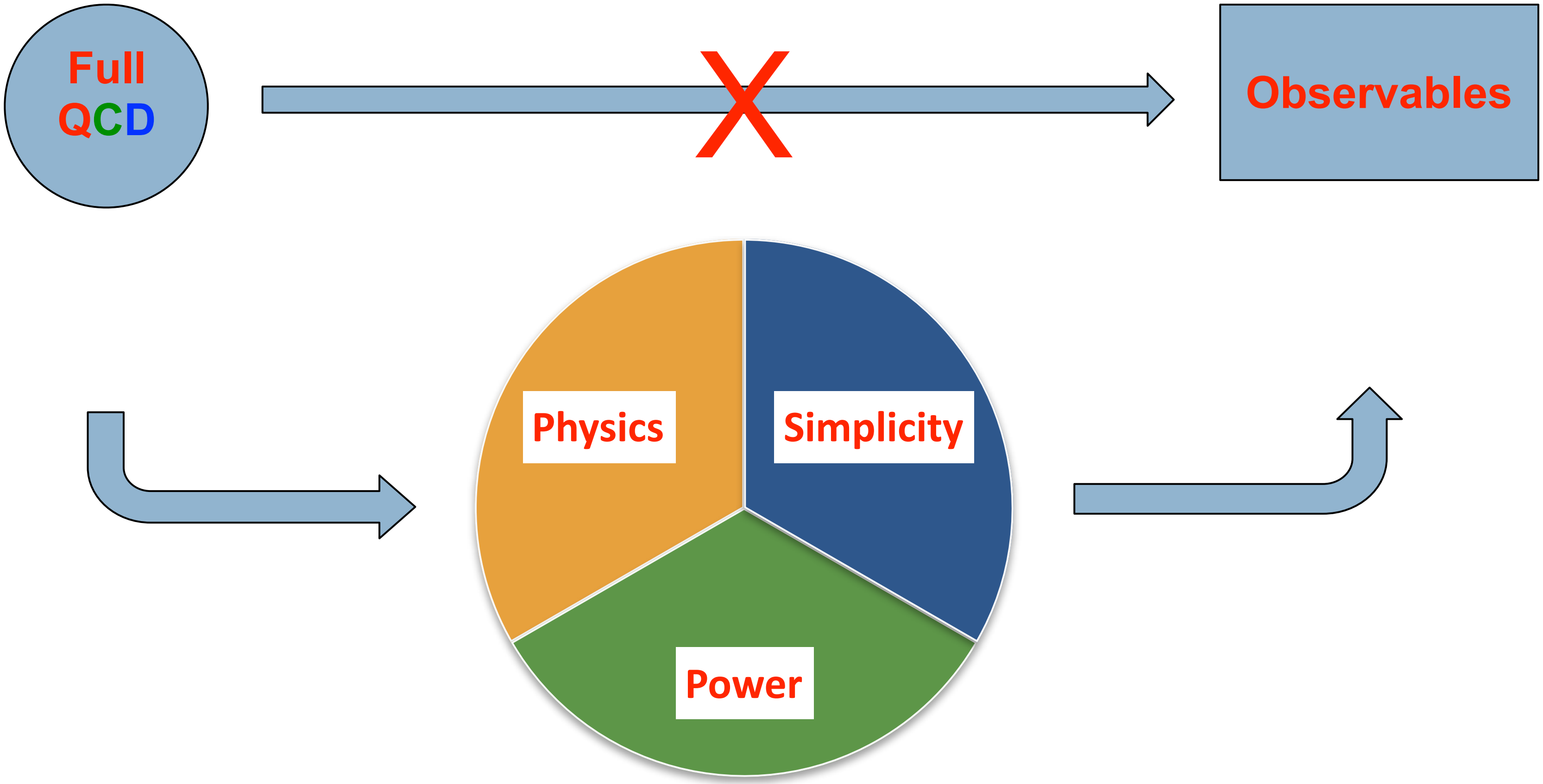

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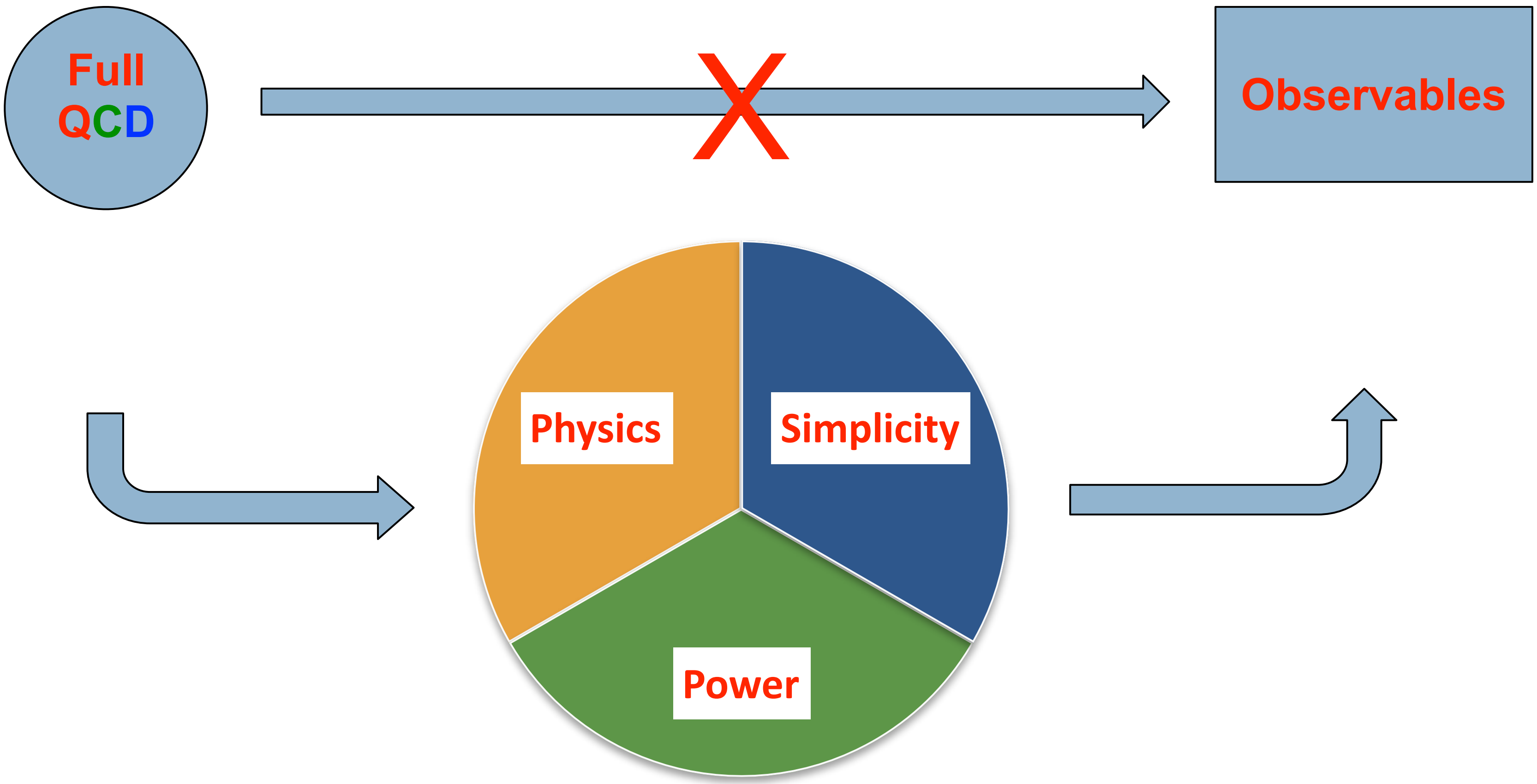

Lattice QCD, Dyson-Schwinger equations, chiral perturbation, AdS/QCD, NJL model, …

2 DSE: EoM of QCD's Green functions

Classical Mechanics	Quantum Field Theory
Degrees of freedom	Degrees of freedom
Generalized coord.	Fields on spacetime
Principle of Least Action	Principle of Least Action
$\dfrac{\delta S[q]}{\delta q} = 0$	$\left\langle \dfrac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$
Equations of Motion (EoM)	Equations of Motion (EoM)
Euler-Lagrange Equation	Dyson-Schwinger Equations

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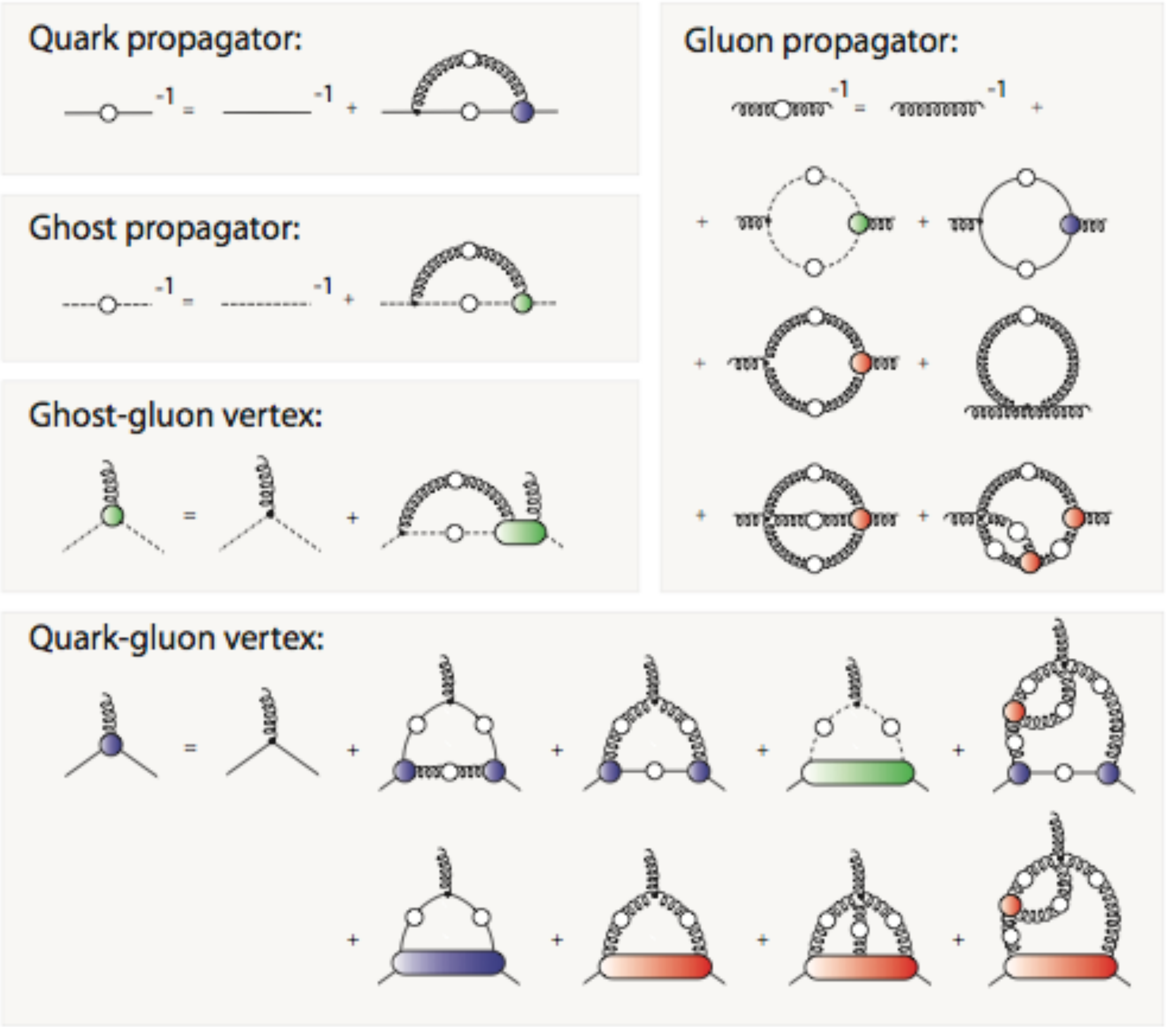
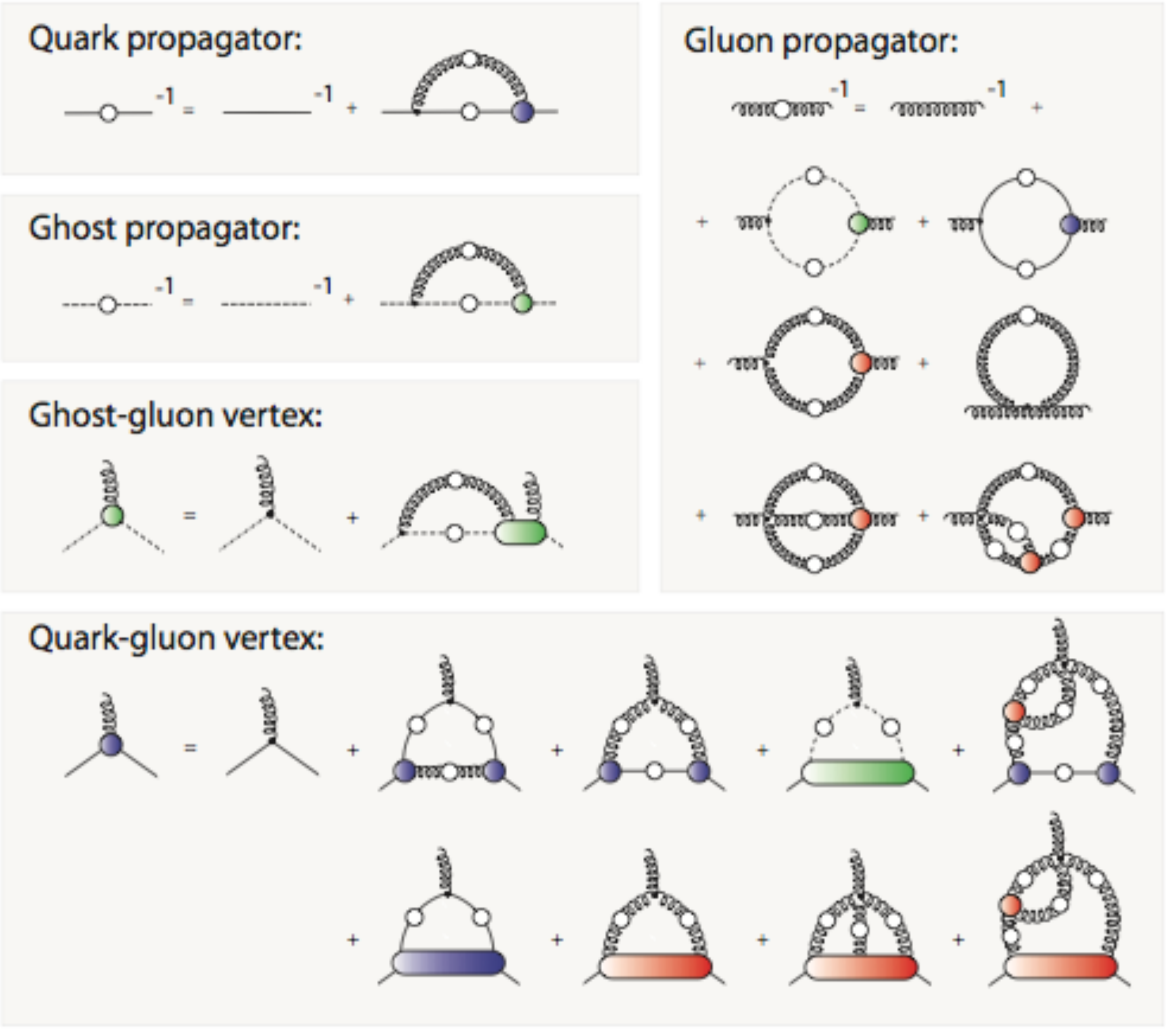


G. Eichmann, arXiv:0909.0703

2 DSE: EoM of QCD's Green functions

- Most equations are very complicated.

- Green functions of different orders couple together.



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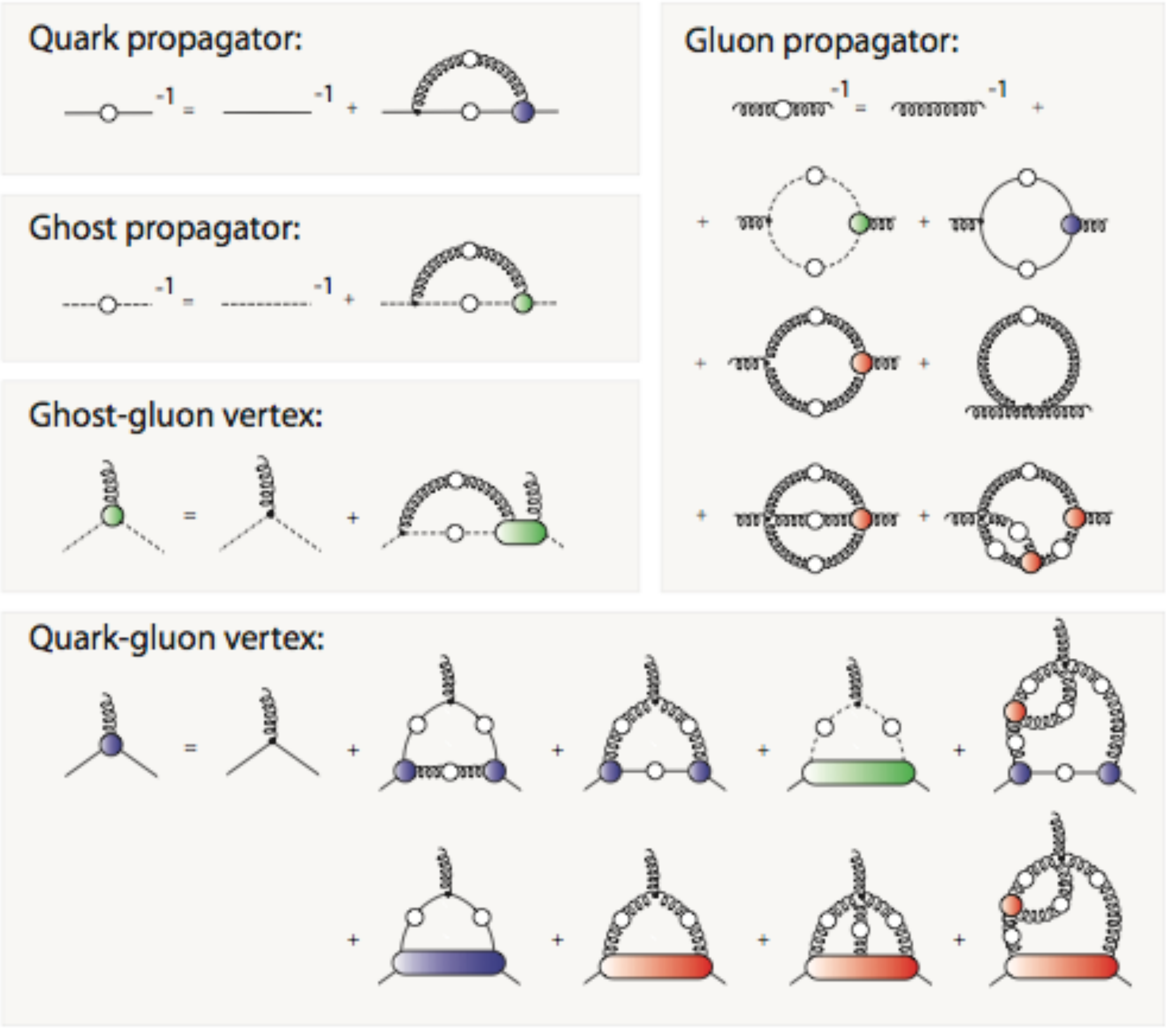
2 DSE: EoM of QCD's Green functions

- Most equations are very complicated.
  - ❑ *Modeling*
- Green functions of different orders couple together.
  - ❑ Truncation



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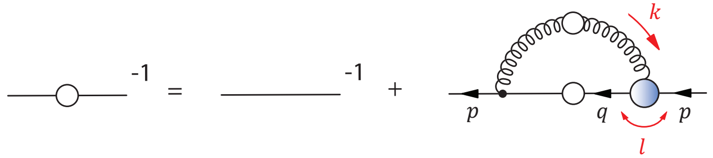
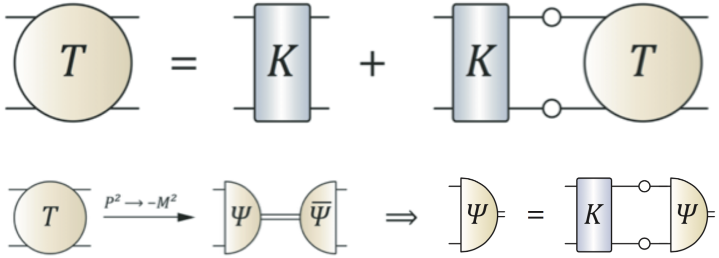
2 DSE: Most frequently used equations

- **One-body gap equation**

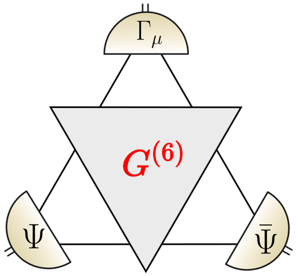


- **Two-body Bethe-Salpeter equation**



- **Three-body form factor equation**

$$2P_\mu F(Q^2) =$$



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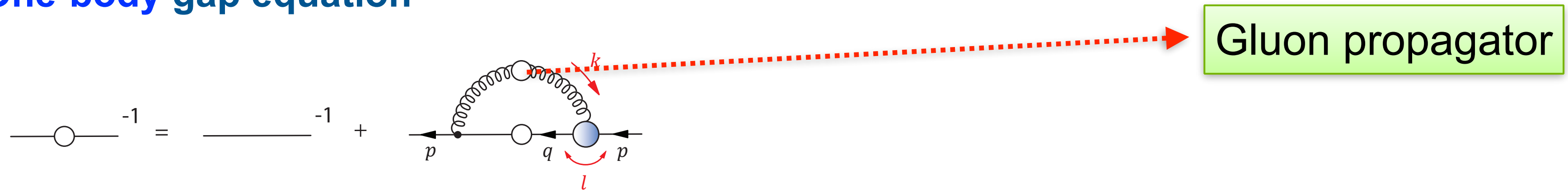

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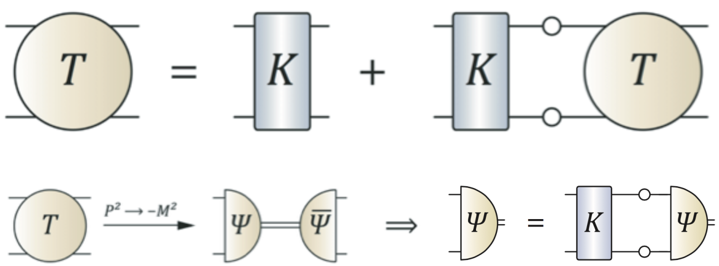

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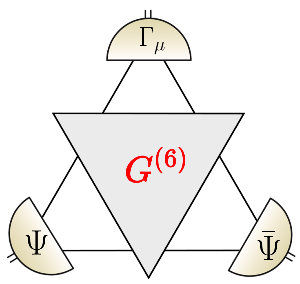

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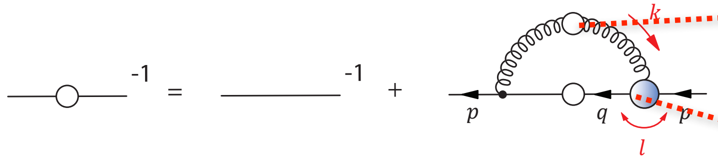

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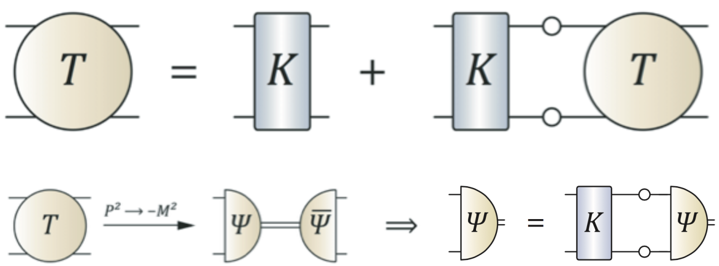

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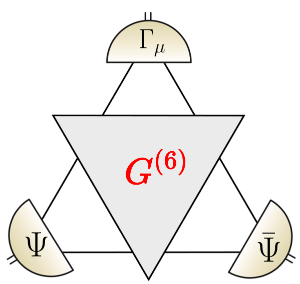

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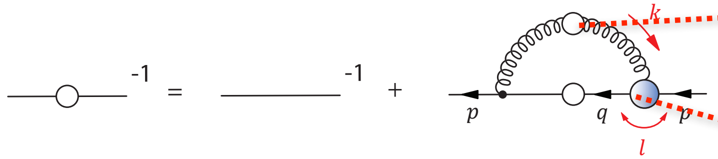

Gluon propagator

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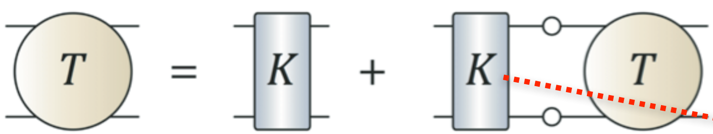

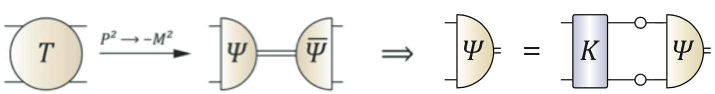

Quark-gluon vertex

Scattering Kernel

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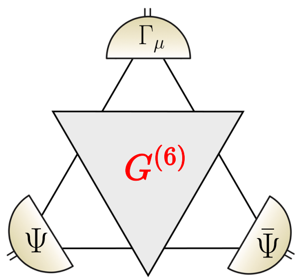

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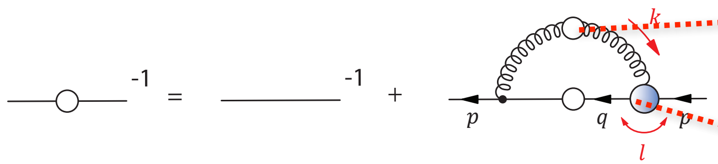

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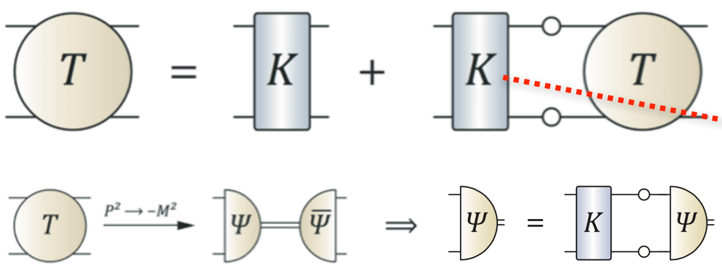

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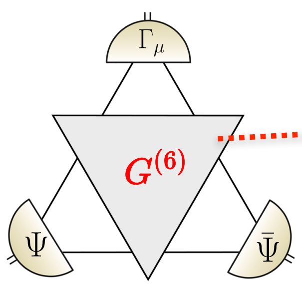

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6-point Green function

2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

II. Quark-gluon vertex

III. Scattering kernel

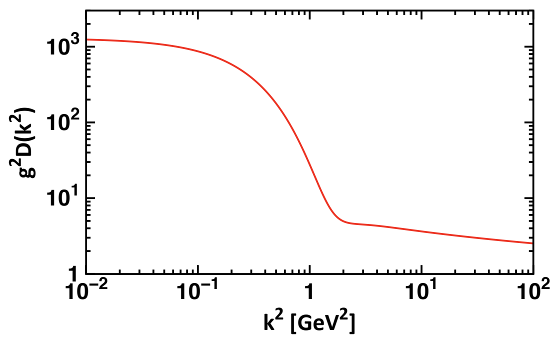
IV. 6-point Green function

2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

massive gluon model

$$g^2 D_{\mu\nu}^{ab}(k) = \delta_{ab} D_{\mu\nu}^{\text{free}}(k) \mathcal{G}(k^2)$$



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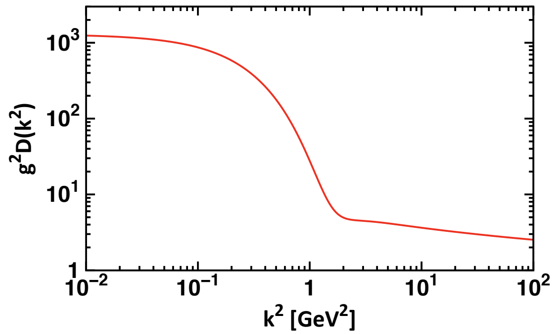
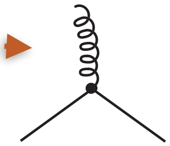


II. Quark-gluon vertex

rainbow approximation



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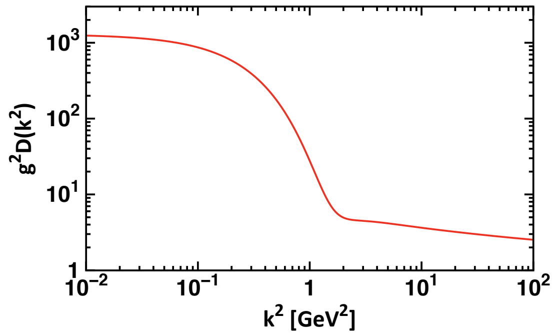

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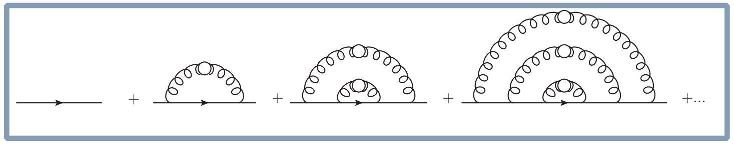

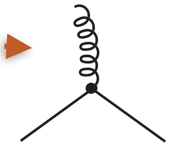

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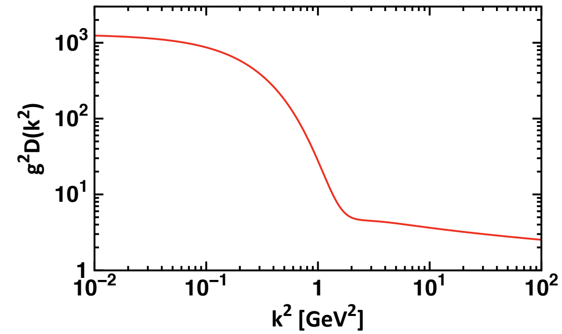

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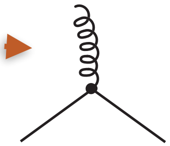
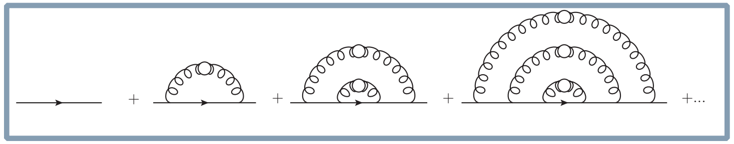
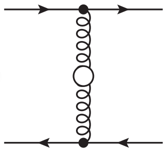




III. Scattering kernel

ladder approximation



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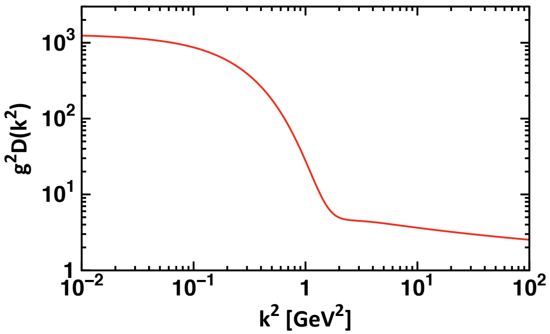

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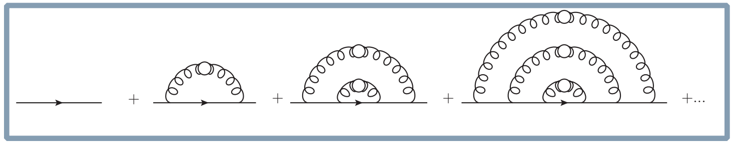
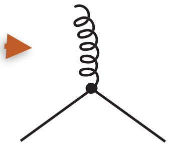
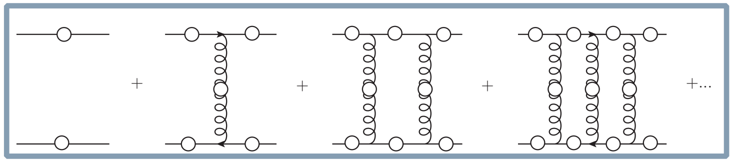
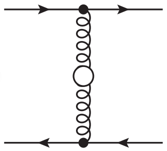




III. Scattering kernel

ladder approximation





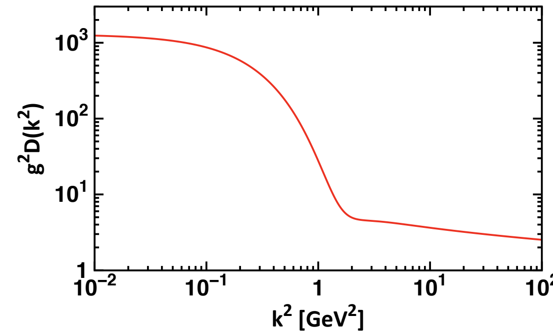
IV. 6-point Green function

2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

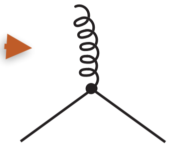
massive gluon model

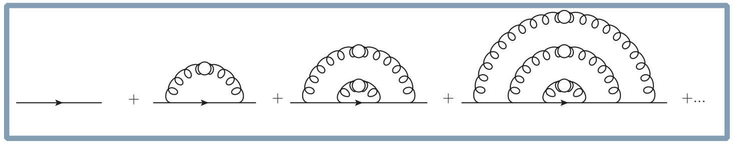
$$g^2 D^{ab}_{\mu\nu}(k) = \delta_{ab} D^{\text{free}}_{\mu\nu}(k) \mathcal{G}(k^2)$$



II. Quark-gluon vertex

rainbow approximation





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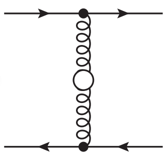
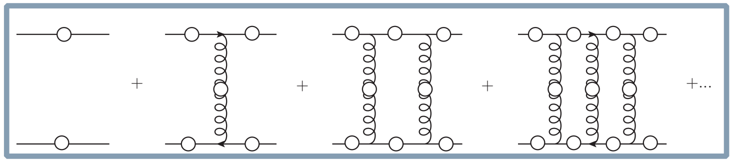
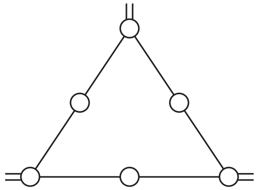




IV. 6-point Green function

triangle diagram

$$\Lambda_\mu(P,Q) = 2P_\mu F(Q^2)$$



2 DSE: Simplest approximation of QCD DSEs

✦ In the chiral limit, the color-singlet av-WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

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$$\lim_{P^2\to M_{\pi_n}^2} \Gamma_{5\mu}(k,P) \sim \frac{2i\gamma_5 f_{\pi_n} E_{\pi_n}(k,P) P_\mu}{P^2 + M_{\pi_n}^2} < \infty \qquad f_{\pi_n} = 0$$

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DCSB means **much more** than massless pseudo-scalar meson.

2 DSE: Summary

✦ **Gluon propagator**: Solve the gluon DSE or extract information from lattice QCD. The dressing function of gluon has a mass scale as that of quark.

✦ **Quark-gluon vertex + Scattering kernel**: Analyze continuous (WGTIs or STIs) & discrete symmetries. The kernel (RL) preserves the chiral symmetry which makes pion to play a twofold role: Bound-state and Goldstone boson.

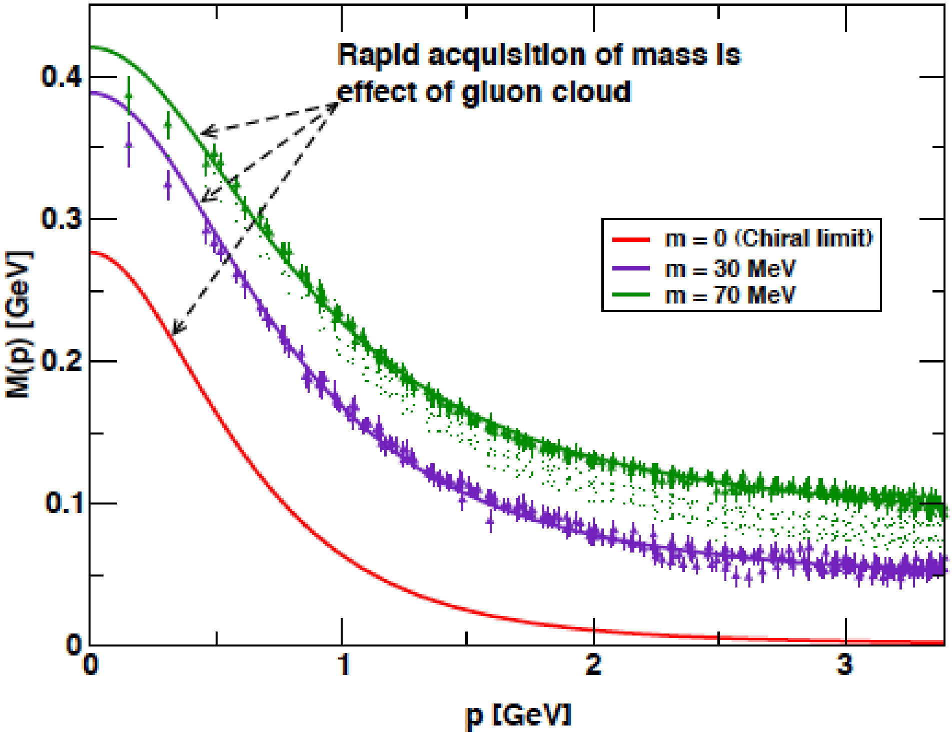
✦ **Form factor**: Generalize the wave function normalization condition. The form factor (the triangle diagram) preserves the current conservation.

3 Application: Realization of DCSB & Confinement

✦ **DCSB**:

1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

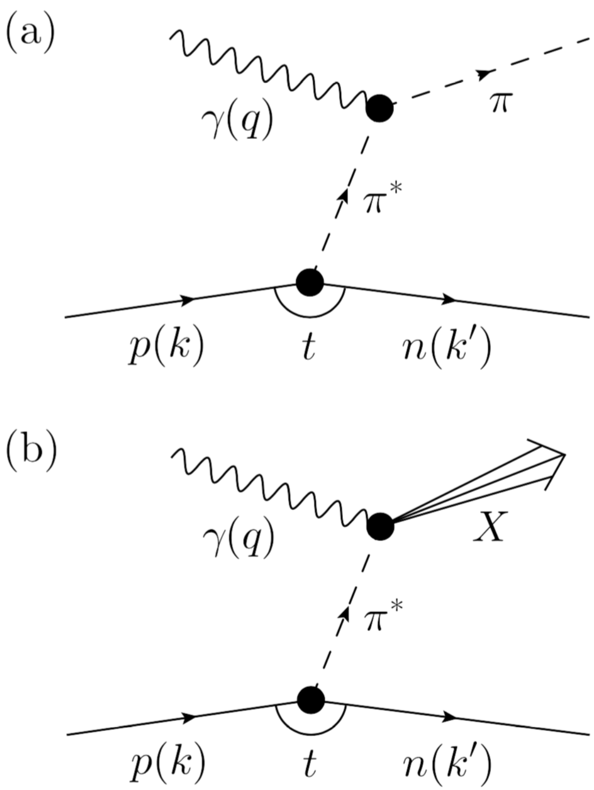
$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



✦ **Confinement**:

Although we exactly know few knowledge about confinement, the positivity violation of quark spectral density supports a fact that a asymptotically free quark is unphysical. In this sense, we say that quarks are confined.

3 Application: Off-Shell pions and kaons



Sullivan processes, in which a nucleon's pion cloud is used to provide access to the pion's (a) elastic form factor and (b) parton distribution functions.

- Experiments use a nucleon's virtual pion cloud as a pion target, e.g., the processes are usually involved:

$$\pi^* + \gamma \to \pi$$

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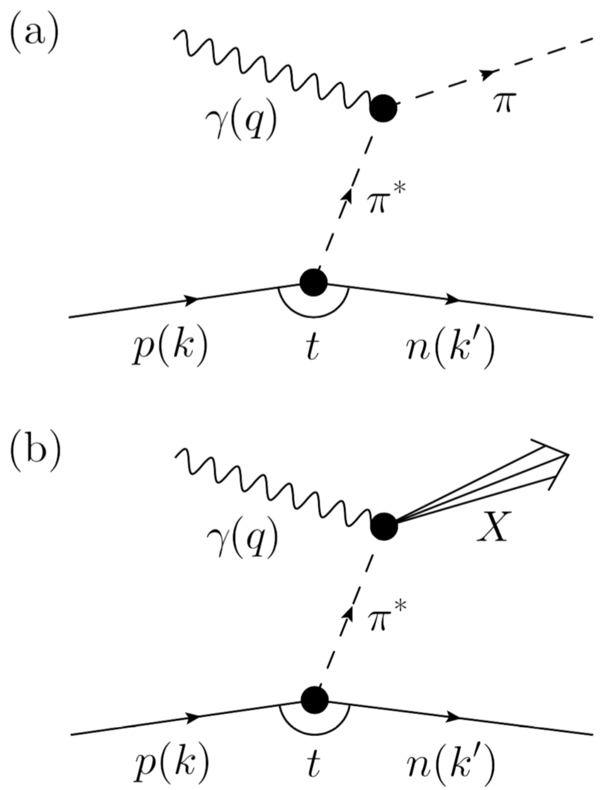

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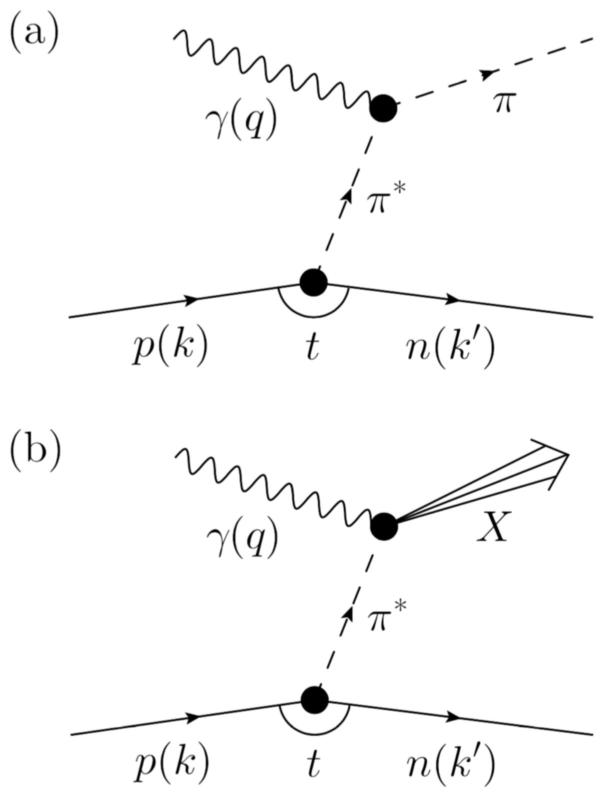

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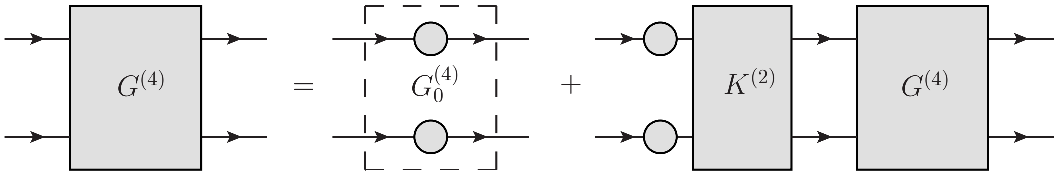
- How does the pion's virtuality affect its properties and further affect the related processes?

- Is there a critical virtuality above which a Sullivan-like process cannot provide reliable access to a meson target?

3 Application: Off-Shell pions and kaons

- ✦ In QFT, bound-states are encoded in Green functions.



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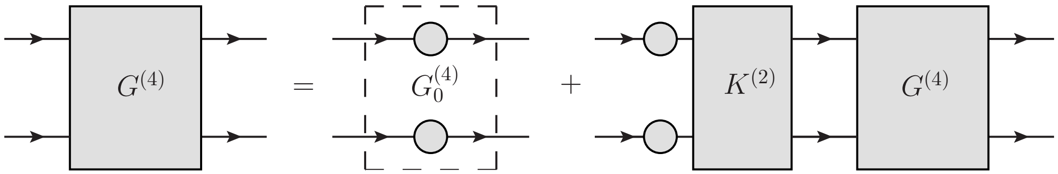

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✦ The kernel can be decomposed by its orthogonal eigenbasis, which are classified by J^P quantum number and radial quantum number n_r,

$$K^{(2)} = \sum_i \lambda_i^{-1} \, |\Gamma_i\rangle\langle\Gamma_i| \qquad |\Gamma_i\rangle = \lambda_i \, K^{(2)} \cdot G_0^{(4)} \cdot |\Gamma_i\rangle \qquad \langle\Gamma_i|G_0^{(4)}|\Gamma_j\rangle = \delta_{ij}$$

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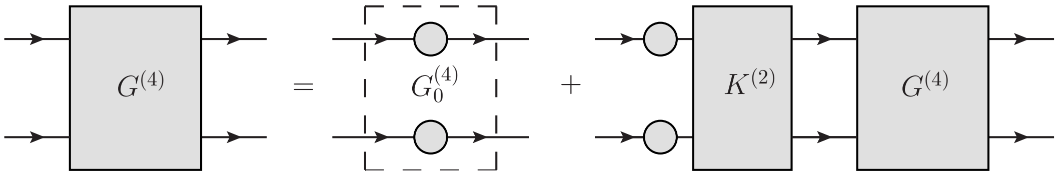

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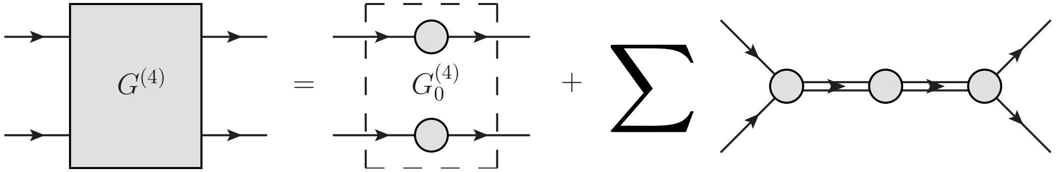
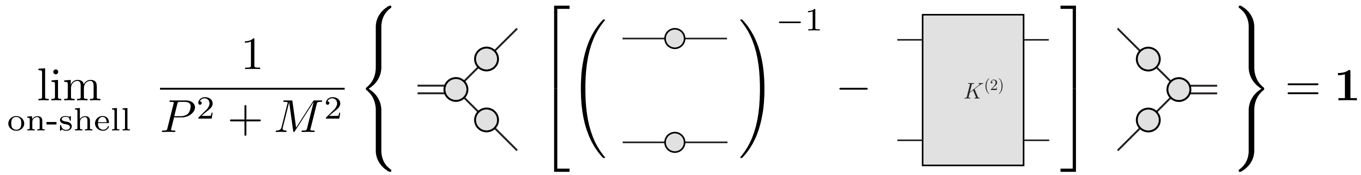
- Accordingly, the four-point Green function can be decomposed:



$$\boxed{G^{(4)} = G_0^{(4)} + \sum_i |\chi_i\rangle \frac{1}{\lambda_i(P^2) - 1} \langle\chi_i|}$$

3 Application: Off-Shell pions and kaons

- The **wave function** of the bound state has to satisfy the following condition



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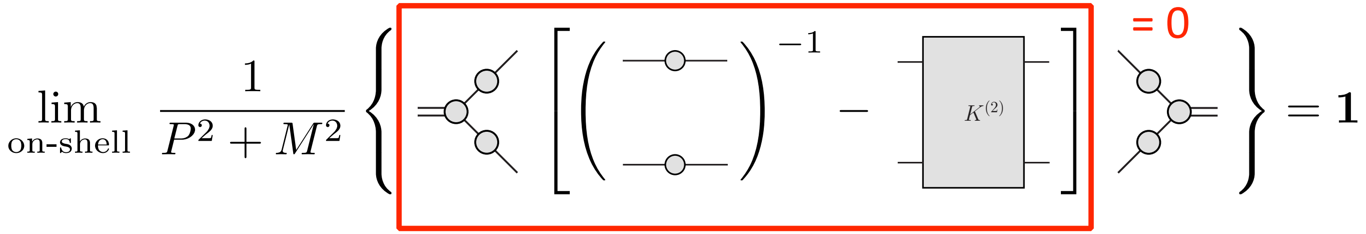

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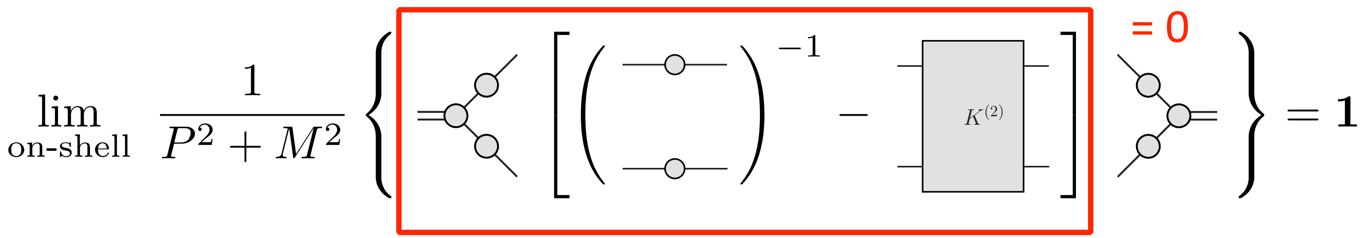
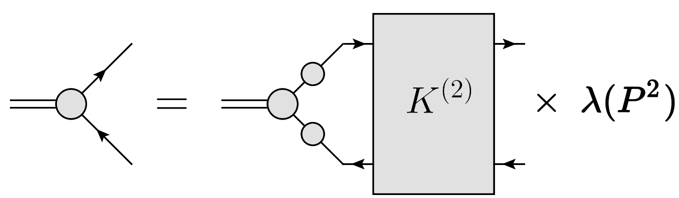


- The generalized homogeneous **Bethe-Salpeter** equation can be obtained as



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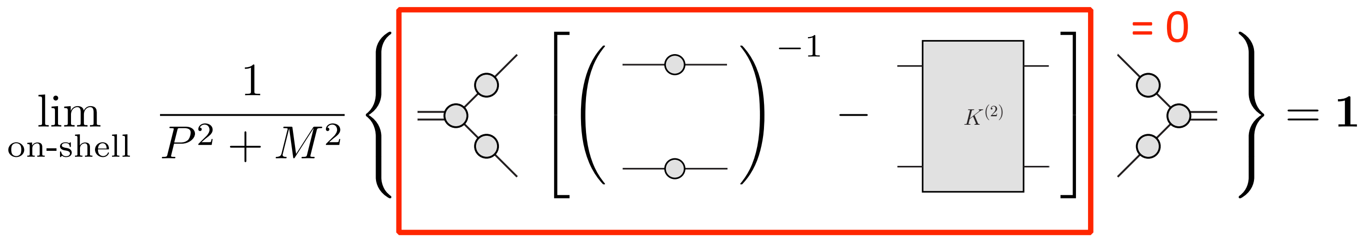

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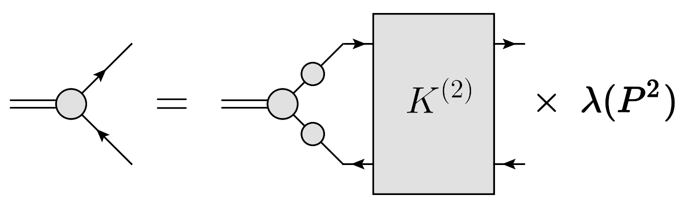
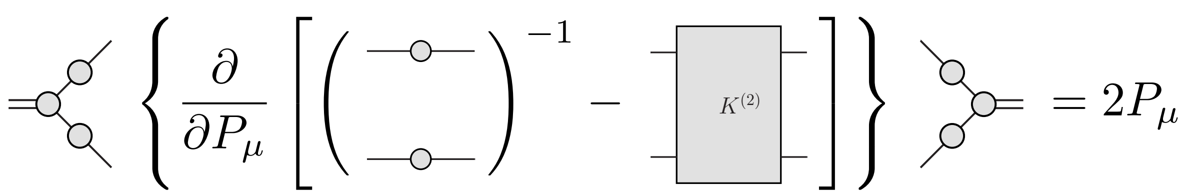


✦ The solved wave function must be **normalized** as following



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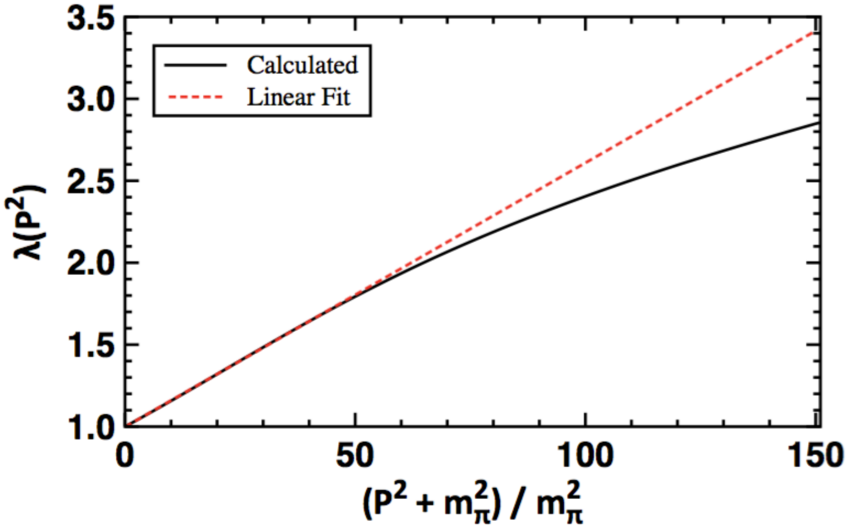
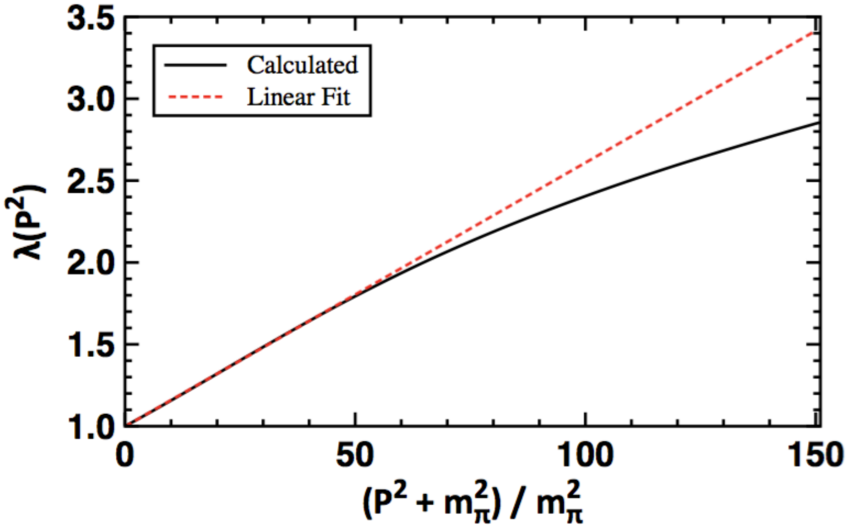


- The eigenvalue is linear to the virtuality less than **45** ($P^2 = (v-1)m_\pi^2$)

$$\lambda(v) = 1 + 0.016\, v$$

3 Application: Off-Shell pions and kaons



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$$\frac{1}{\lambda(P^2)-1} \sim \frac{1}{P^2 + m_\pi^2}$$

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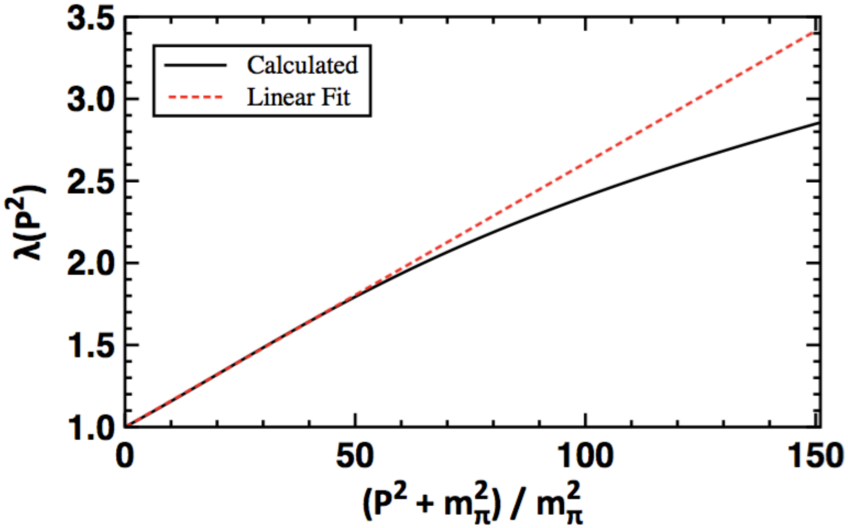

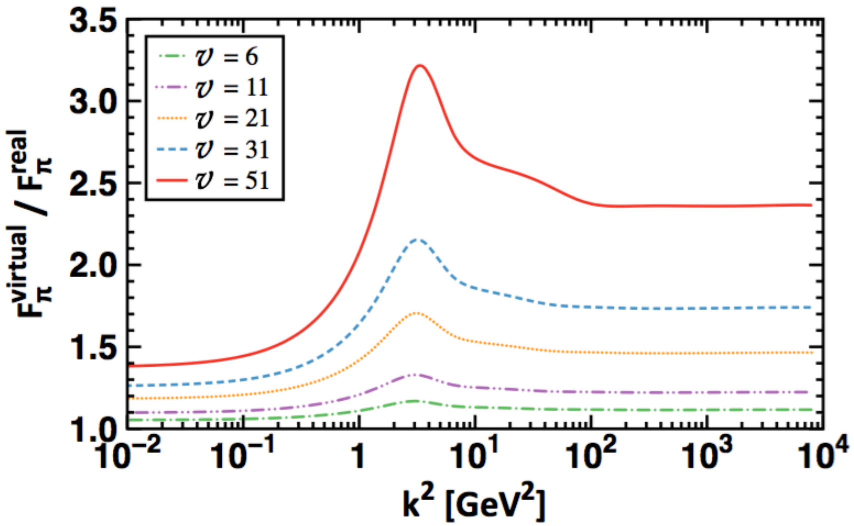

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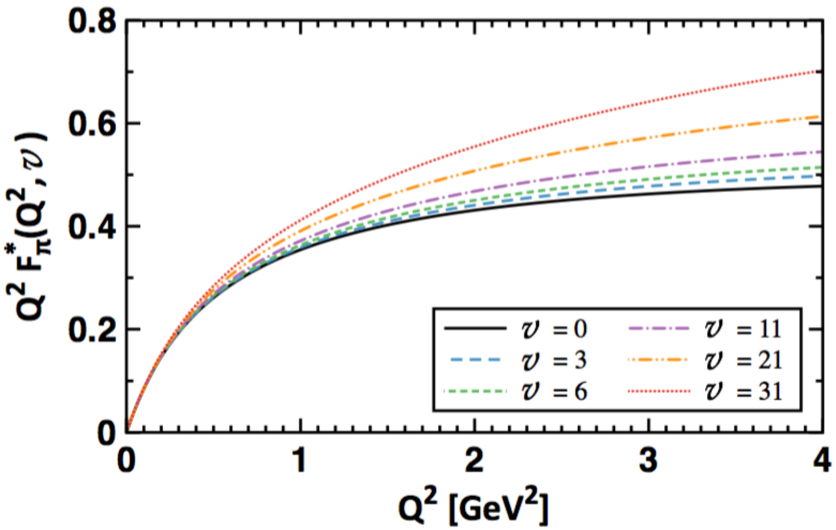
✦ The UV shifts of the BS amplitudes grow with the virtuality less than **31** and that growths are almost linear. This leads to a linear growth of the in-pion condensate:

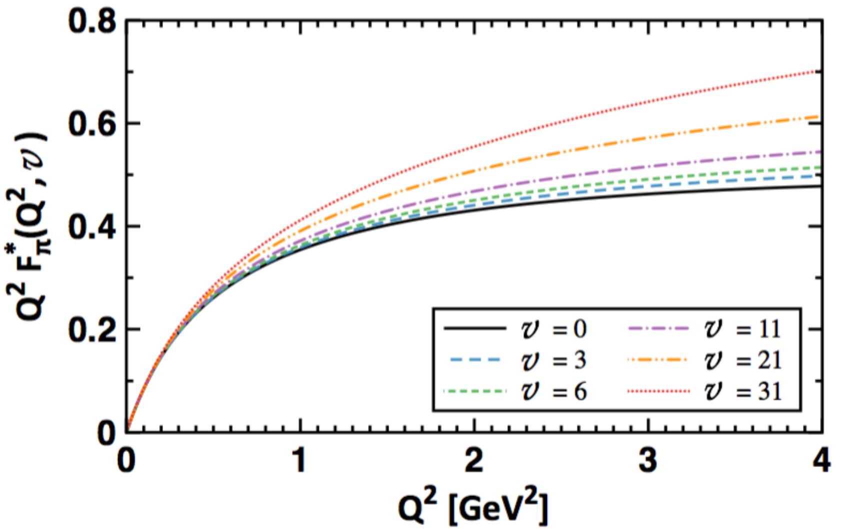
$$\kappa_\pi^\zeta(v) \approx \kappa_\pi^\zeta(0)[1 + 0.032 v]$$

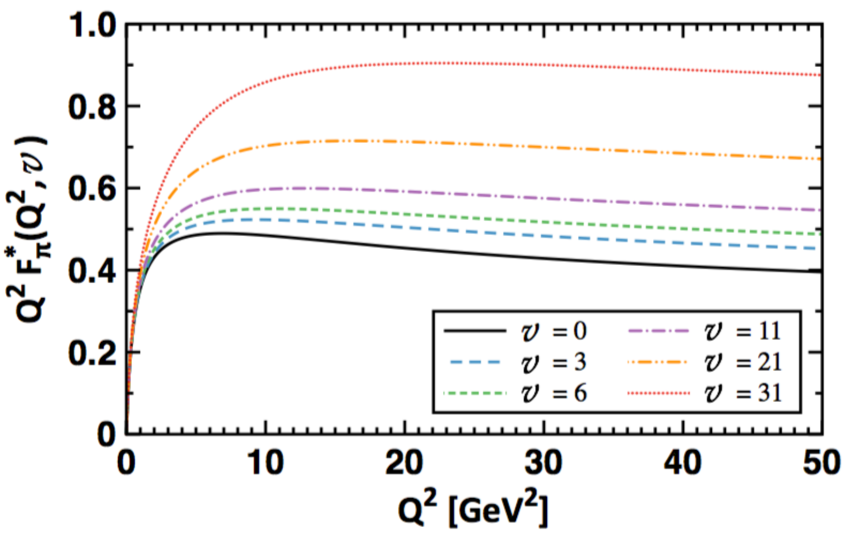
3 Application: Off-Shell pions and kaons



✦ With the virtuality increasing, the pion has a smaller radius and becomes more point-like.

3 Application: Off-Shell pions and kaons





- With the virtuality increasing, the pion has a smaller radius and becomes more point-like.

- The computed form factor can be interpolated by a monopole multiplied by a simple factor that restores the correct QCD anomalous dimension.

$$F_\pi^*(Q^2, v) = \frac{1}{1 + Q^2/m_0^2} \mathcal{A}(Q^2, v)$$

$$\mathcal{A}(Q^2, v) = \frac{1 + Q^2 a_0^2(v)}{1 + Q^2 [a_0^2(v)/b_u^2(v)] \ln(1 + Q^2/\Lambda_{\rm QCD}^2)}$$

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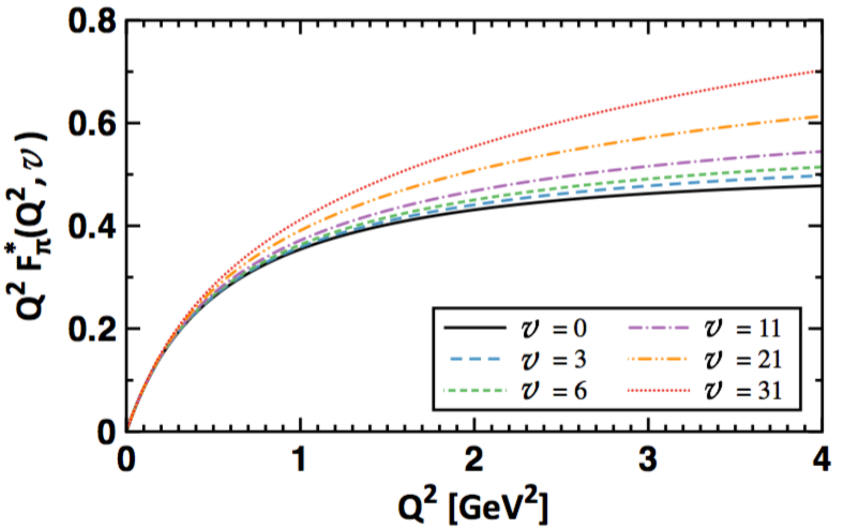

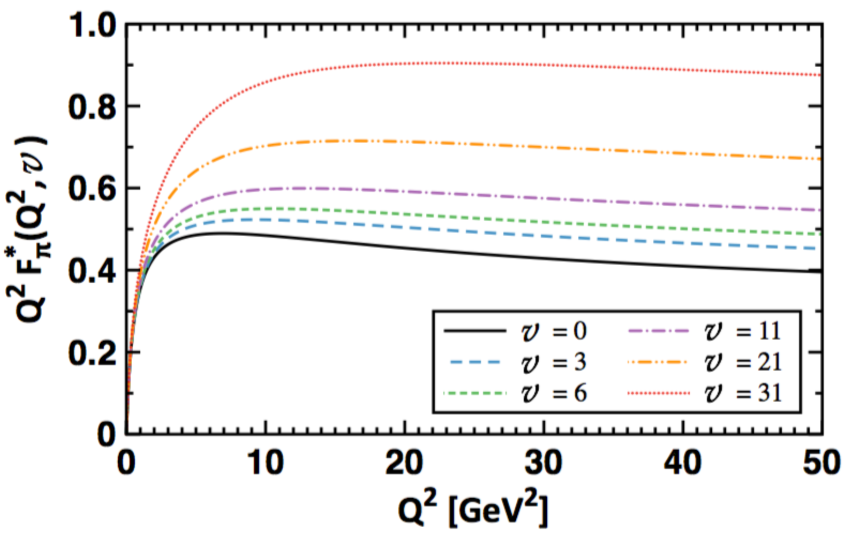

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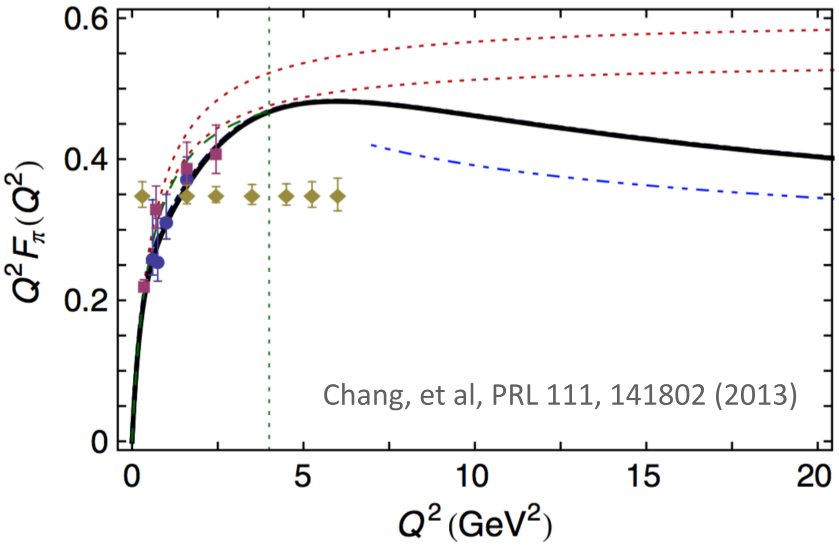
- For $v \lesssim v_S$, the $Q^2 \gtrsim 10\,\mathrm{GeV}^2$ form factor responds linearly to changes in the BS amplitudes and such modifications should become evident on this domain.

3 Application: Off-Shell pions and kaons



- The pion's twist-two valence-quark PDA is connected with the large-Q^2 form factor:

$$Q^2F_\pi(Q^2) \overset{Q^2\gg\Lambda^2_{\rm QCD}}{\approx} 16\pi\alpha_s(Q^2)f_\pi^2 w_\varphi^2,$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\,\frac{1}{x}\varphi_\pi(x)\,,$$

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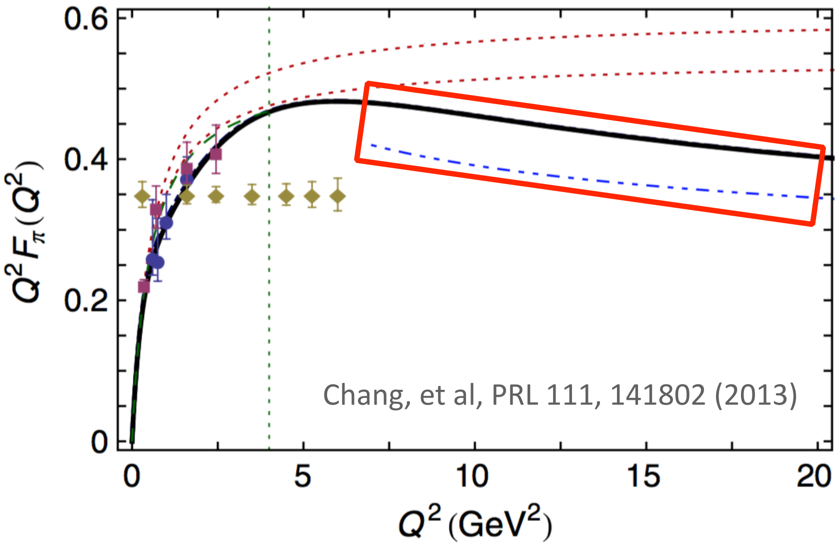

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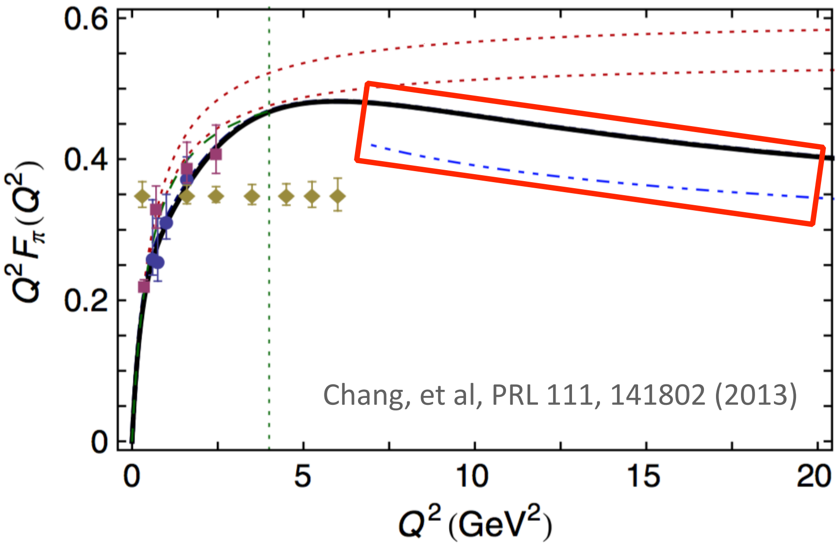

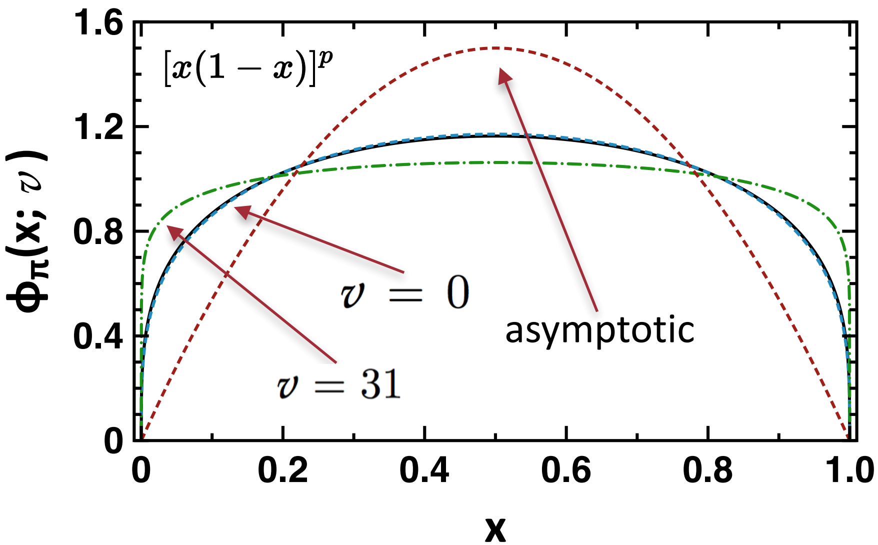

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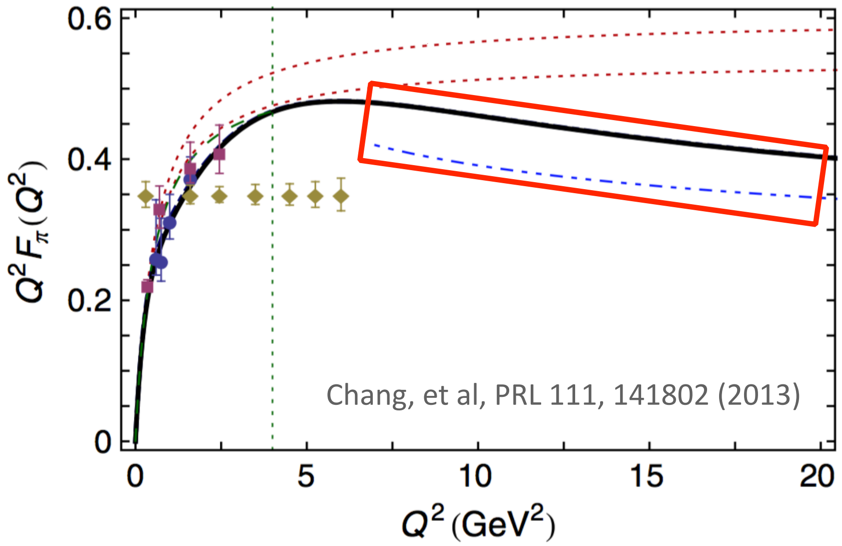
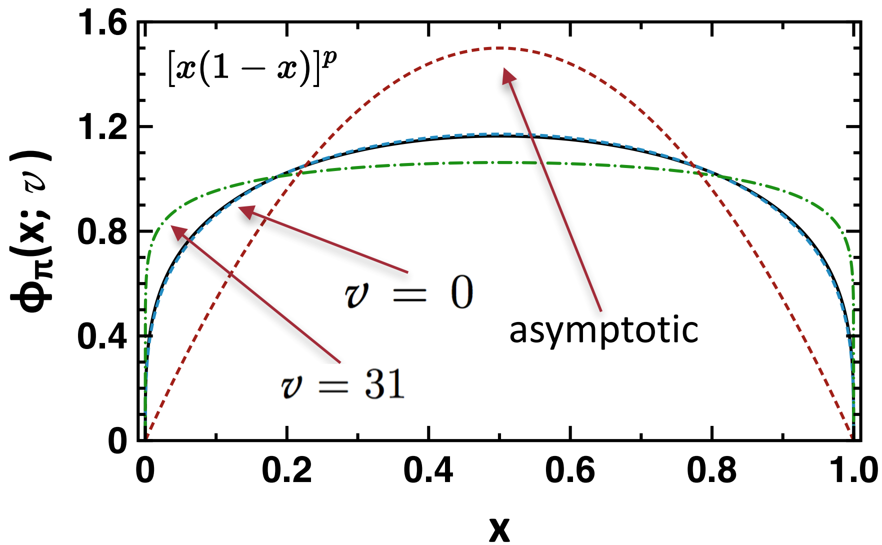
3 Application: Off-Shell pions and kaons





- ✦ The pion's twist-two valence-quark PDA is connected with the large-Q^2 form factor:

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)\,,$$

- ✦ we can use GPDs to translate the behavior of $F_\pi^*(Q^2, v)$ into insights regarding the impact of virtuality on extractions of the pion's valence-quark PDF:

$$F_\pi(Q^2) = \int_{-1}^1 dx\, H^u_{\pi^+}(x, 0, Q^2)\,,$$

$$u^\pi(x) = H^u_{\pi^+}(x > 0, 0, 0)\,,$$

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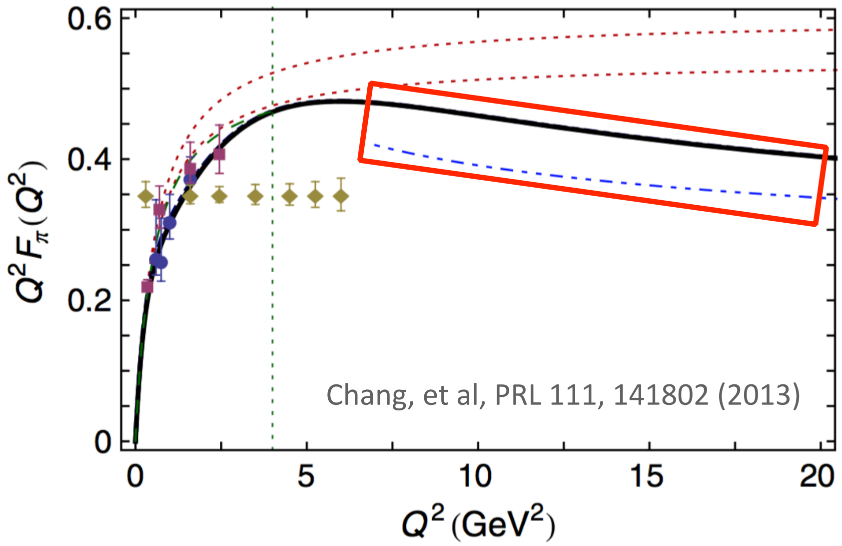

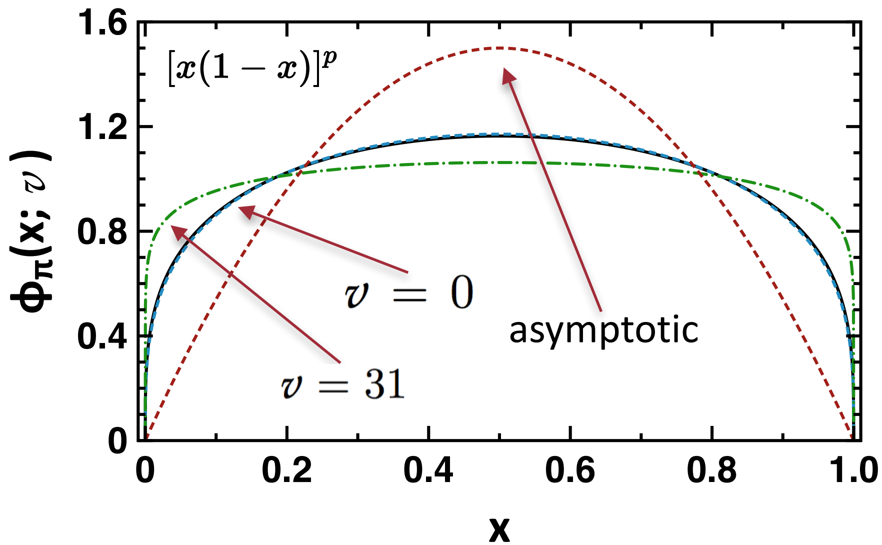

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- we can use GPDs to translate the behavior of $F^*_\pi(Q^2, \mathcal{v})$ into insights regarding the impact of virtuality on extractions of the pion's valence-quark PDF:

$$F_\pi(Q^2) = \int_{-1}^1 dx\, H^u_{\pi^+}(x,0,Q^2)\,,$$

$$u^\pi(x) = H^u_{\pi^+}(x>0,0,0)\,,$$

- The critical virtuality, below which the virtual particles serve as a valid target, is

$u\bar{u}$	$u\bar{s}$	$s\bar{s}$
$-t \lesssim 0.6$ GeV^2	$-t \lesssim 0.9$ GeV^2	$-t \lesssim 1.3$ GeV^2

Summary

✦ **Bound-states** are **ideal** objects connecting experiments and theories. **QCD bound-state** problems are difficult because of its relativistic and strongly-couple properties.

✦ Based on LQCD and QCD's symmetries, the **simplest method** to construct the **gluon propagator, quark-gluon vertex, scattering kernel,** and **form factor**, is demonstrated.

✦ A **model-independent** scheme to study the **off-shell** bound state is proposed. Off-shell pions and kaons are studied to suggest critical **virtualities** for experiments.

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Outlook

◆ With the **sophisticated method** to solve the DSEs, we can push the approach to a wide range of applications in QCD **bound-state** problems.

◆ Hopefully, after more and more **successful applications** are presented, the DSEs may provide a **faithful path** to understand **QCD** and a powerful tool for general physics.