

# QCD Bound-State Problem:

Off-shell Persistence of Composite Pions and Kaons

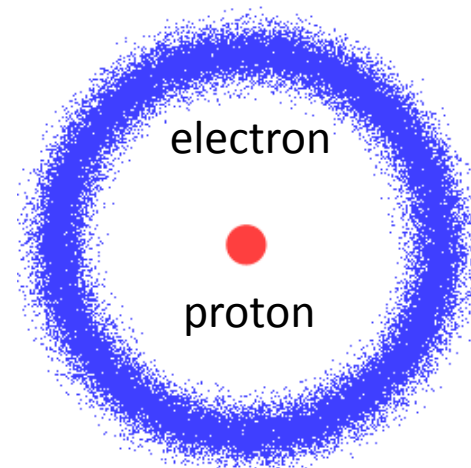
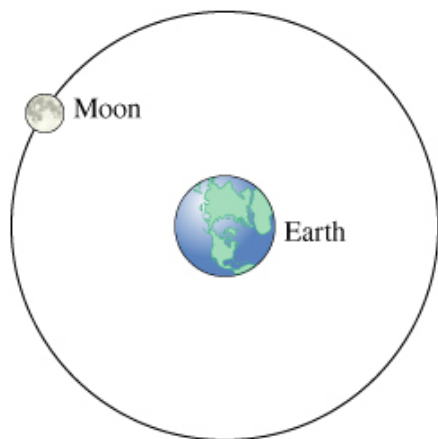
**Sixue Qin**

Argonne National Laboratory

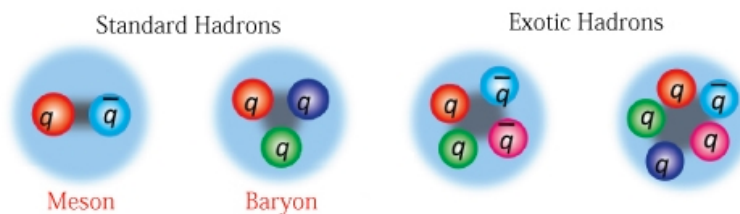
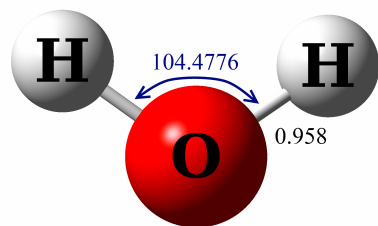
# 1 Background: What are bound-states?

$$E_{\text{sys}} < \sum E_i$$

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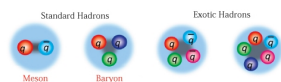
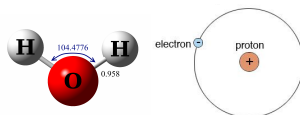
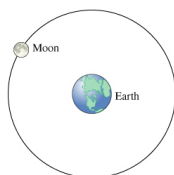
$$E_{\text{sys}} < \sum E_i$$



# 1 Background: Why do we study bound-states?

## Experiment

“Easy” objects  
involving  
interactions

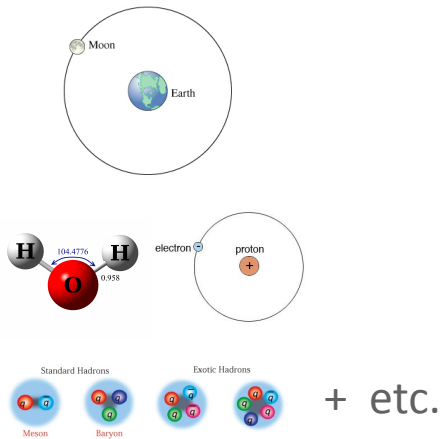


+ etc.

# 1 Background: Why do we study bound-states?

Experiment

“Easy” objects  
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interactions



Theory

“Simple” objects  
involving  
dynamics

Newtonian Mechanics

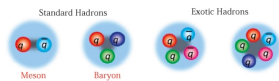
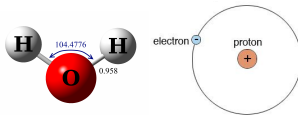
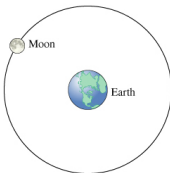
Quantum Mechanics

Quantum Field Theory + etc.

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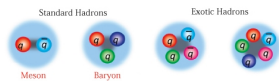
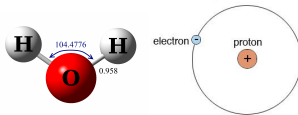
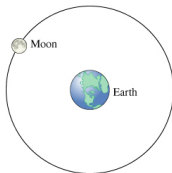
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# 1 Background: Why do we study bound-states?

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“Easy” objects  
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+ etc.

What matter is possible  
&  
how is it constituted?

Theory

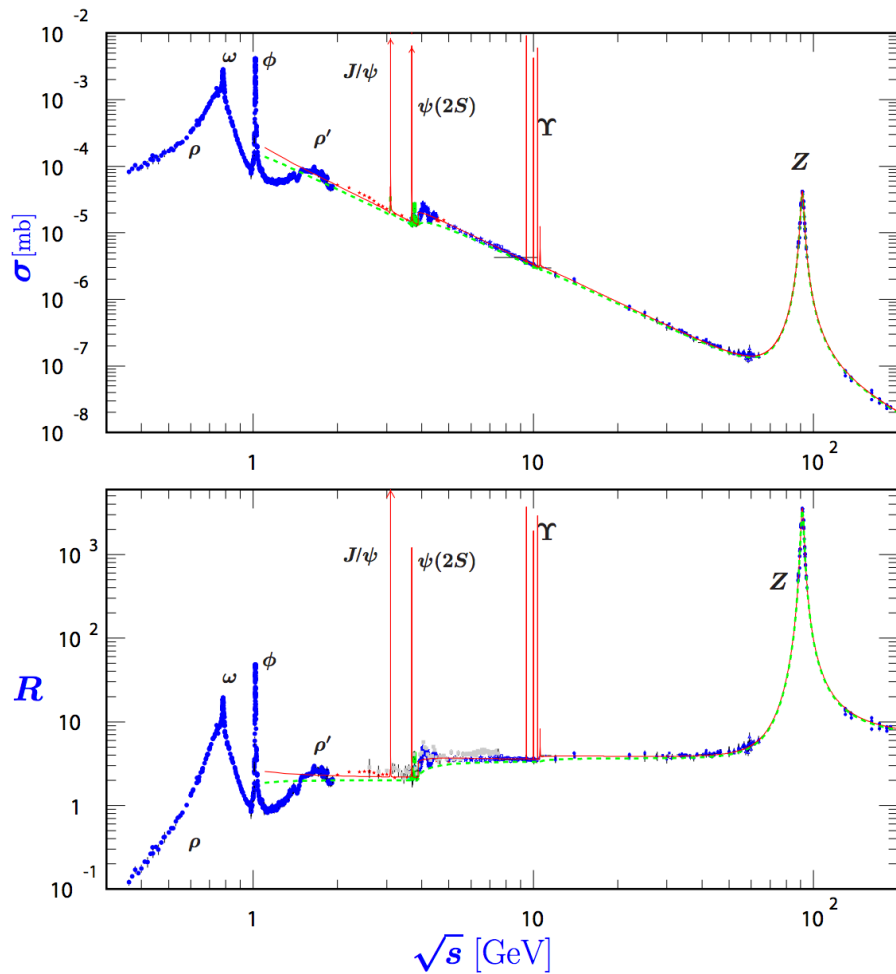
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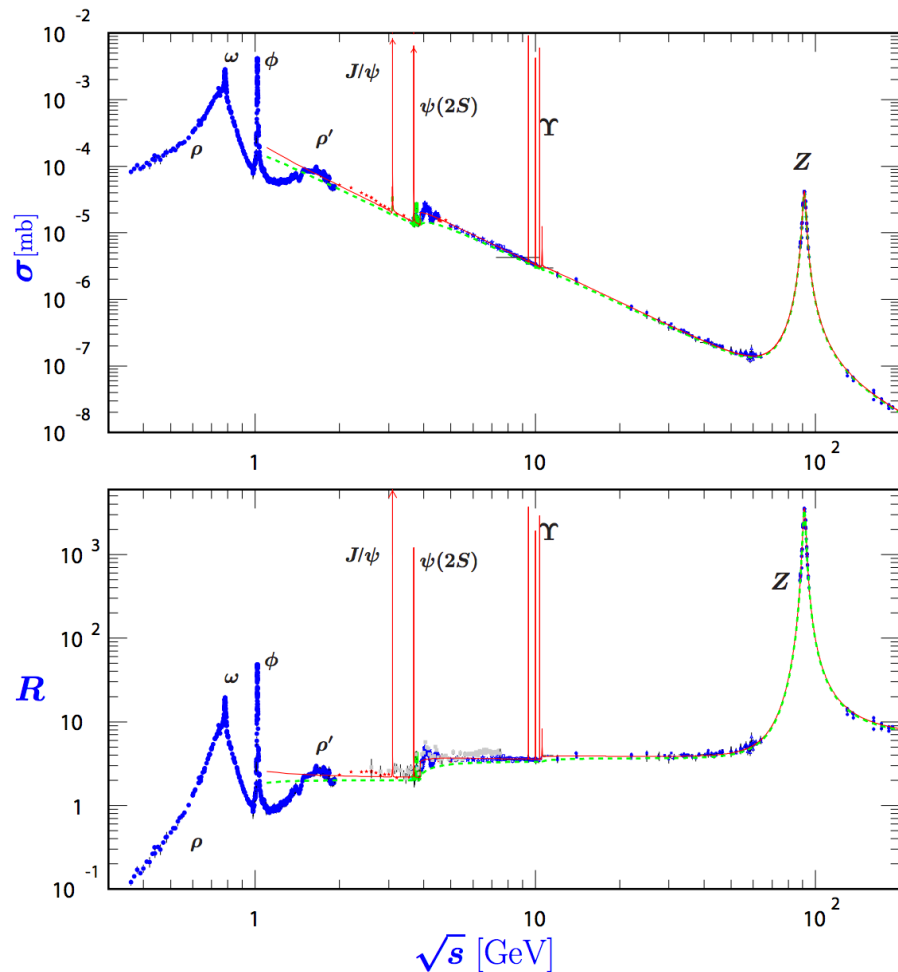
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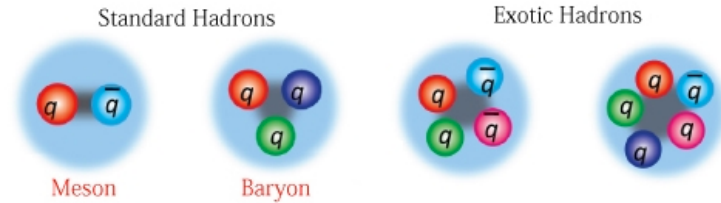
$e^+ e^-$  hadronic annihilation



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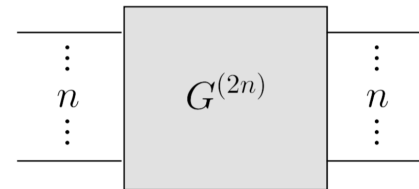


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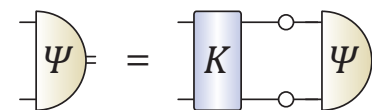


Quantum Field Theory

- Green functions



- Bethe-Salpeter equation



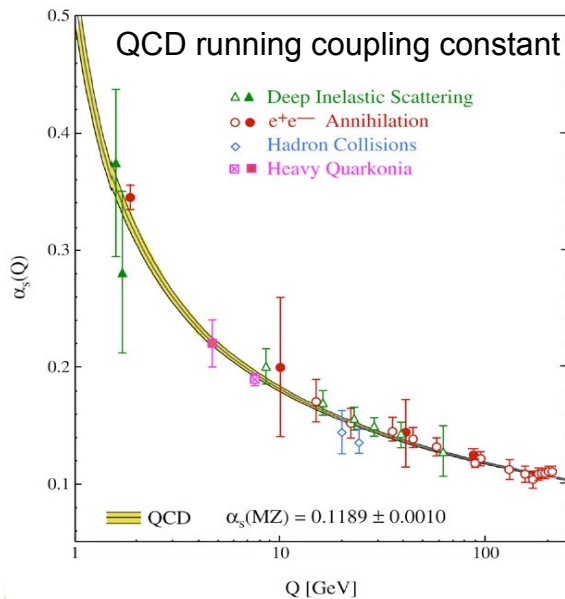
# 1 Background: Why is QCD bound-state problem difficult?

## • Relativistic bound states

*“These problems are those involving bound states [...] such problems necessarily involve a breakdown of ordinary perturbation theory. [...] The pole therefore can only arise from a divergence of the sum of all diagrams [...]”*

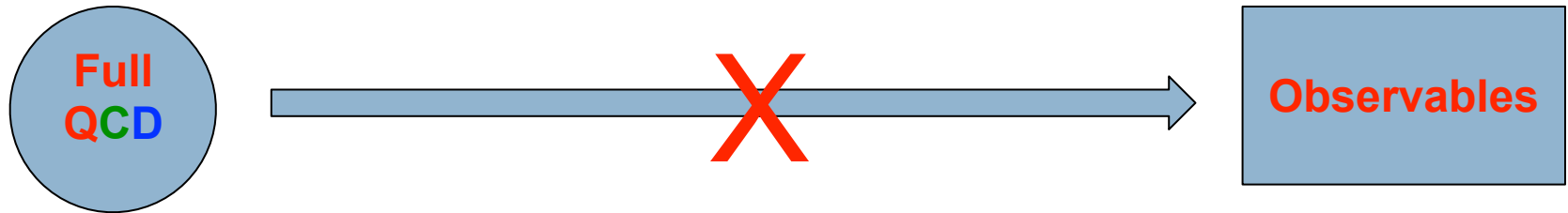
The QFT book vol1 p564 Weinberg

## • Strongly coupled systems

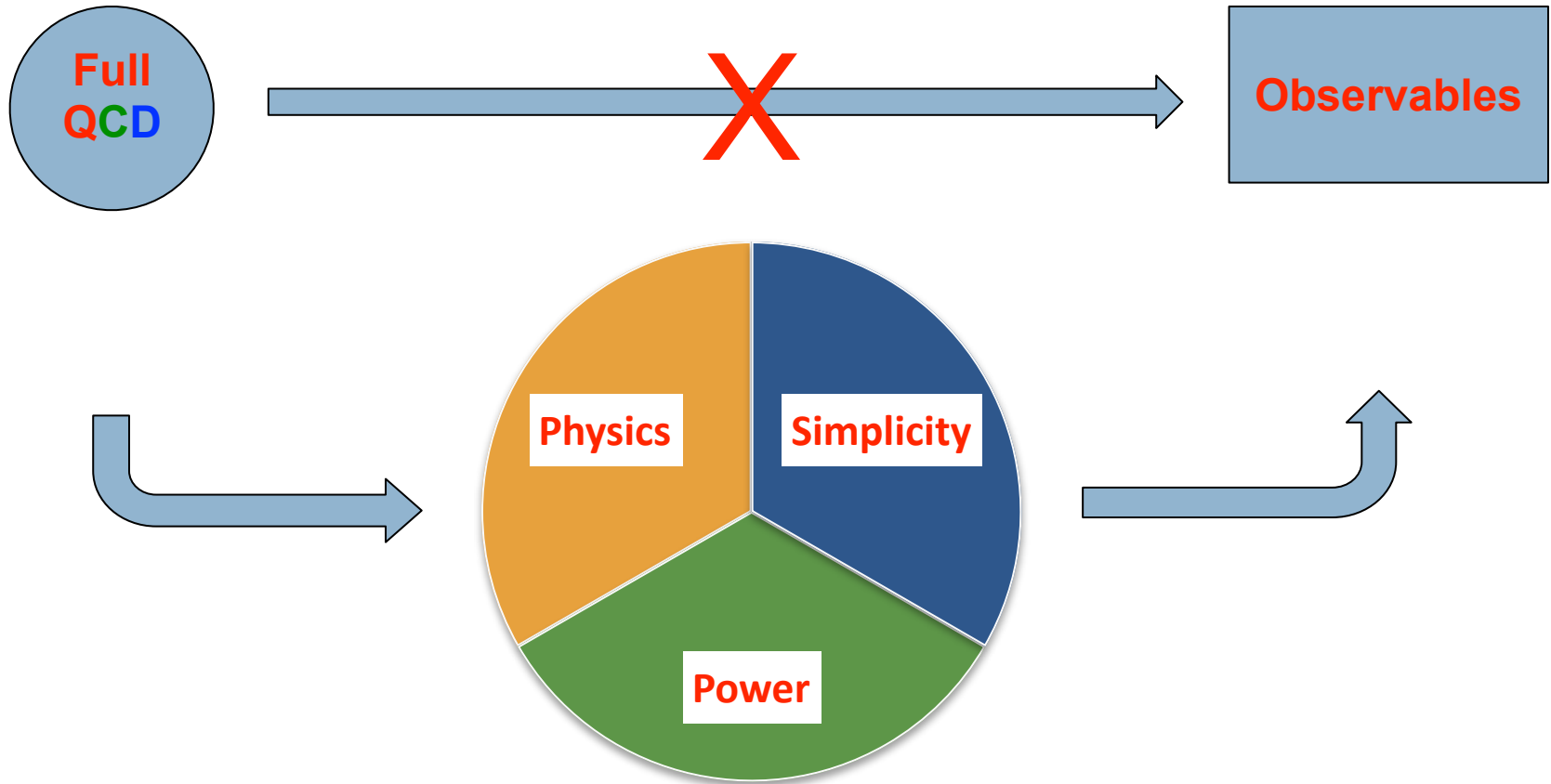


- **Asymptotic freedom:** Bonds between particles become asymptotically weaker as energy increases and distance decreases (Nobel Prize).
- **Quark and Gluon Confinement:** No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
- **Dynamical Chiral Symmetry Breaking:** Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners*.

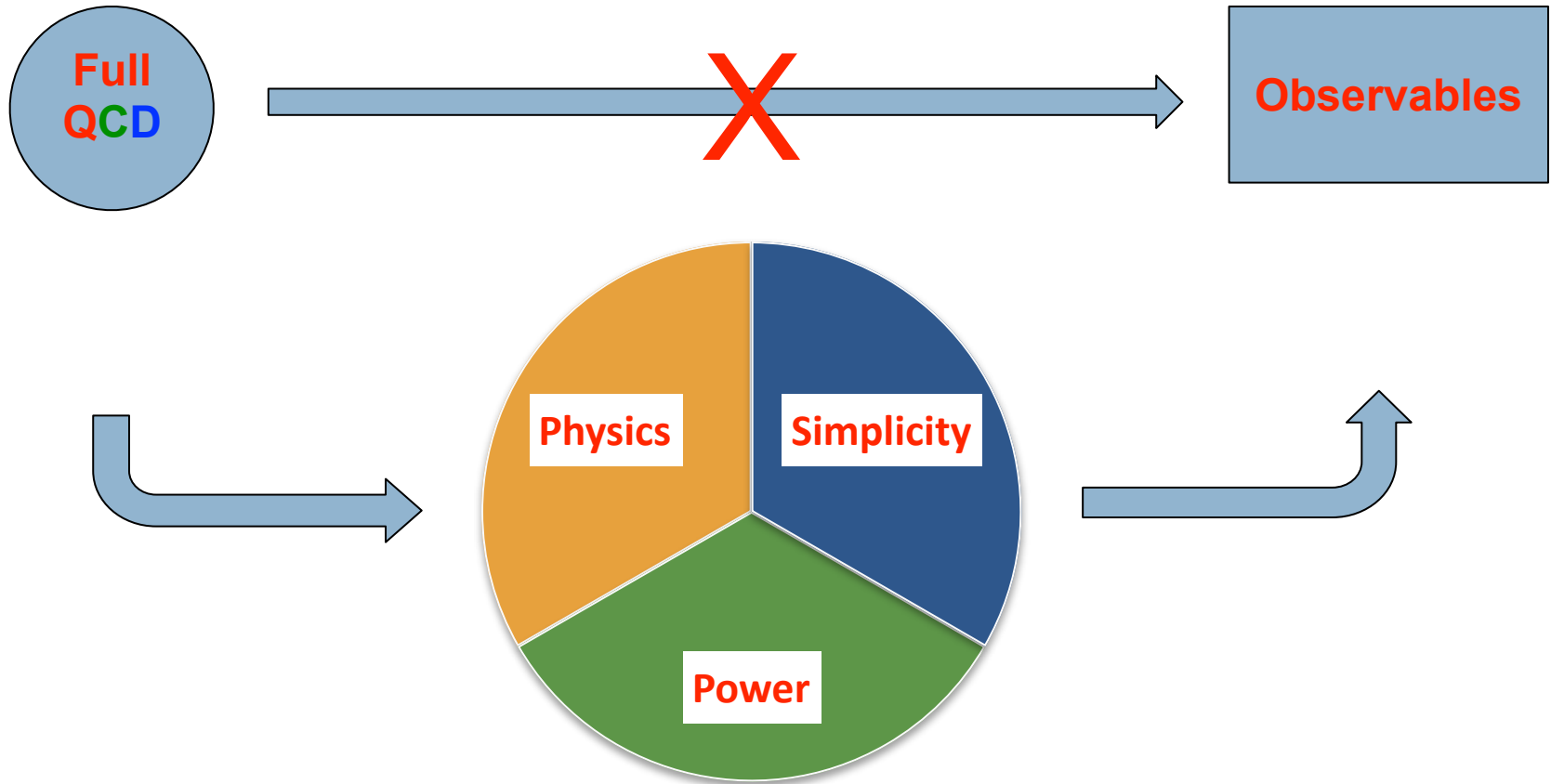
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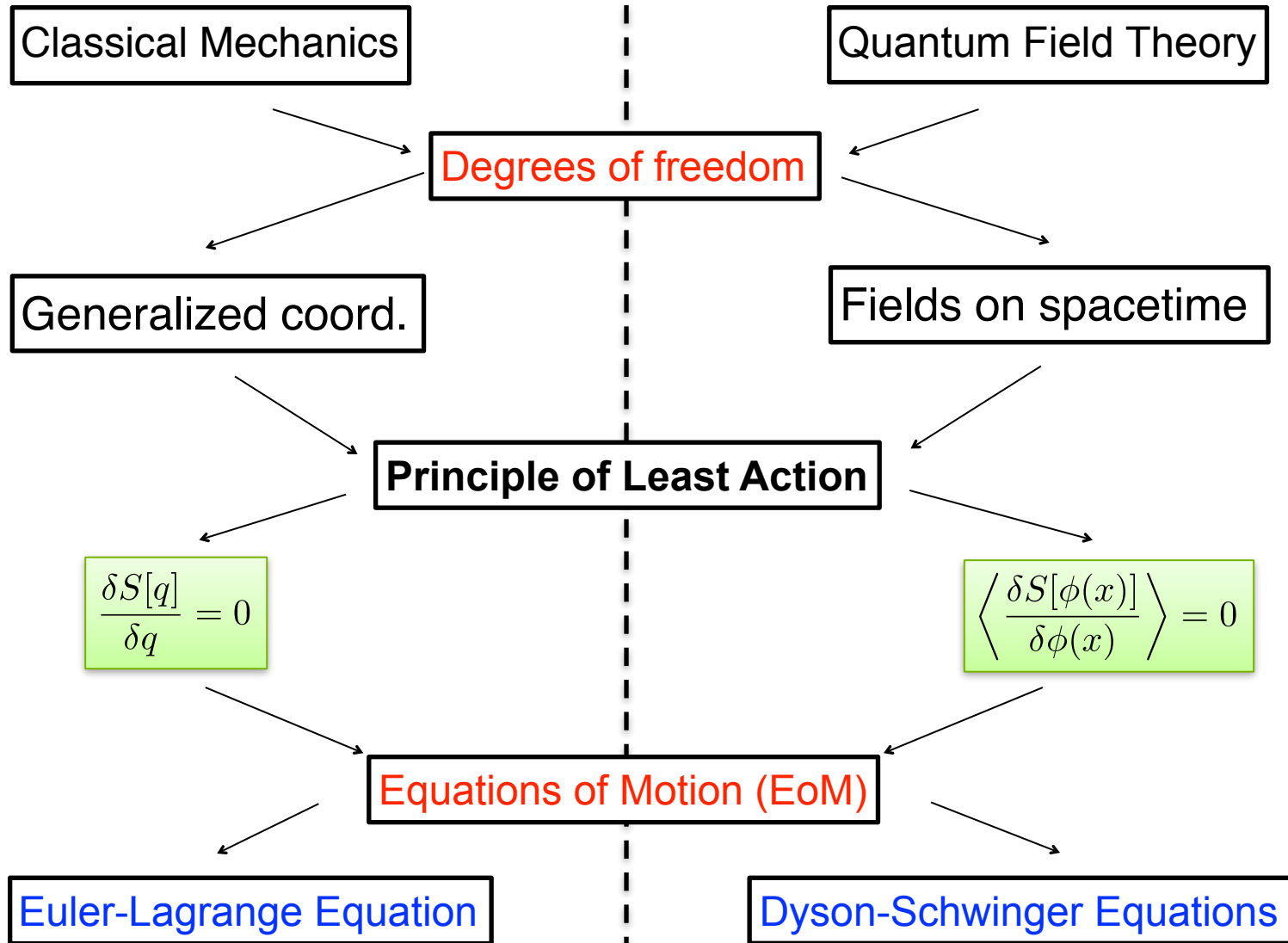


# 1 Background: Non-perturbative approaches of QCD



Lattice QCD, **Dyson-Schwinger equations**, chiral perturbation, AdS/QCD, NJL model, ...

## 2 DSE: EoM of QCD's Green functions



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Quark propagator:

$$\text{---}\text{---}\text{---}^{-1} = \text{---}\text{---}\text{---}^{-1} + \text{---}\text{---}\text{---}$$

Ghost propagator:

$$\text{---}\text{---}\text{---}^{-1} = \text{---}\text{---}\text{---}^{-1} + \text{---}\text{---}\text{---}$$

Ghost-gluon vertex:

$$\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

Quark-gluon vertex:

$$\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

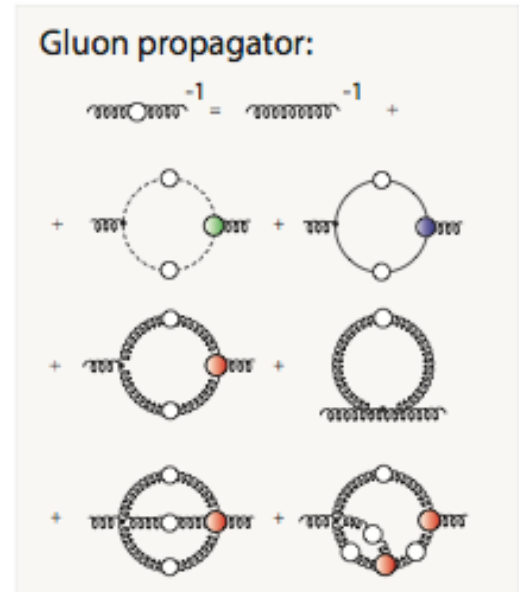
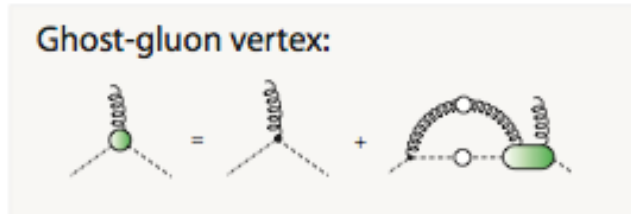
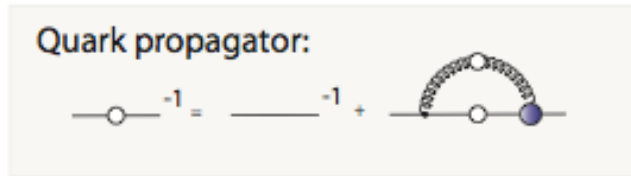
Gluon propagator:

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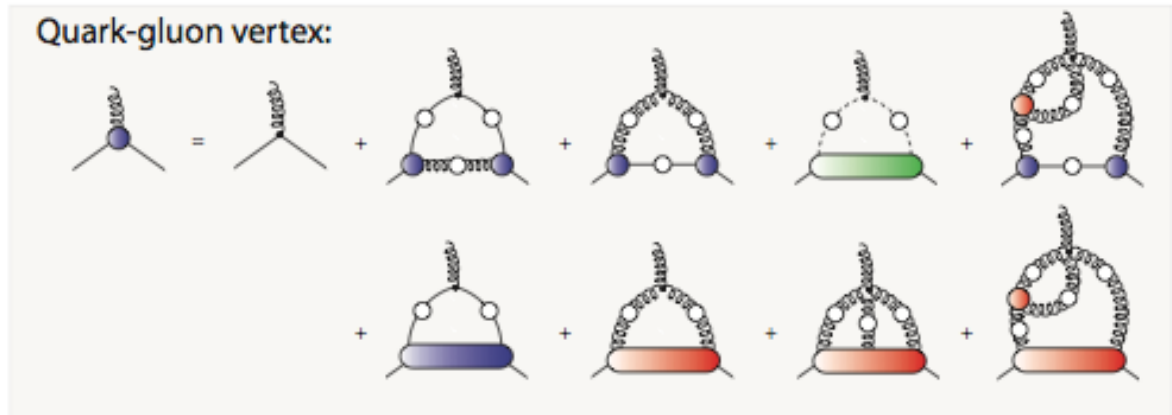
$$+ \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}$$

## 2 DSE: EoM of QCD's Green functions

- ◆ Most equations are very **complicated**.



- ◆ Green functions of different orders **couple together**.





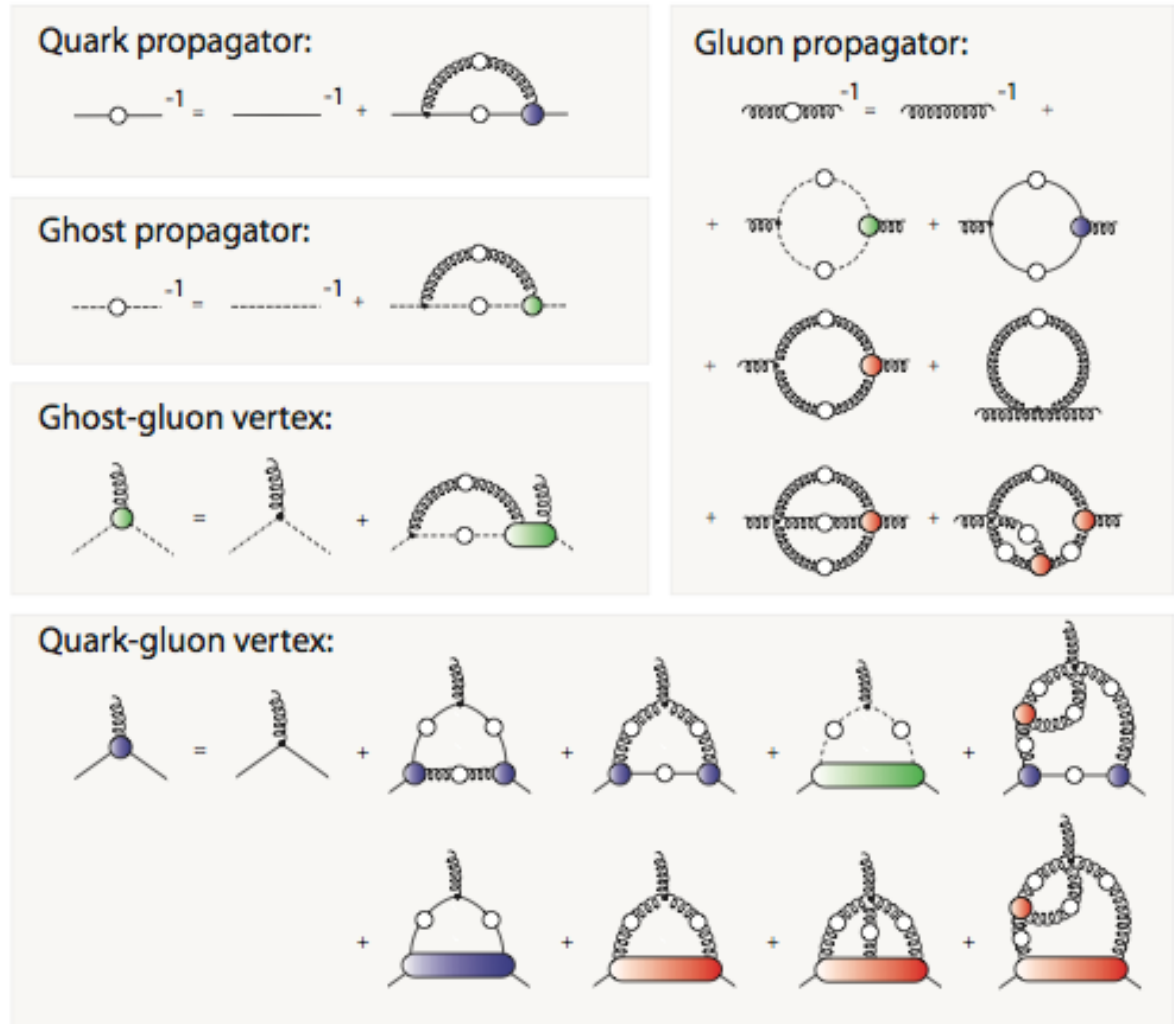
## 2 DSE: EoM of QCD's Green functions

- ◆ Most equations are very **complicated**.

### □ Modeling

- ◆ Green functions of different orders **couple together**.

### □ Truncation



## 2 DSE: Most frequently used equations

- One-body gap equation

$$\text{---} \circ \text{---}^{-1} = \text{---}^{-1} + \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---}^{-1} \circ \text{---} \circ \text{---}^{-1} + \dots$$

- Two-body Bethe-Salpeter equation

$$T = K + K \circ T$$

$$T \xrightarrow{p^2 \rightarrow -M^2} \Psi \bar{\Psi} \Rightarrow \Psi = K \circ \Psi$$

- Three-body form factor equation

$$2P_\mu F(Q^2) = G^{(6)}$$



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Gluon propagator

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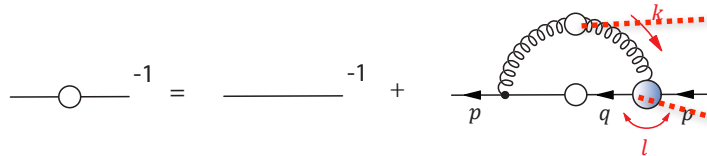
Quark-gluon vertex

- Three-body form factor equation

$$2P_\mu F(Q^2) = \text{---} \Gamma_\mu \text{---} \text{---} \Psi \text{---} \bar{\Psi} \text{---}$$

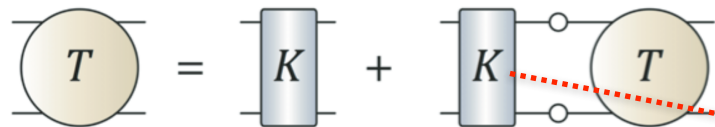
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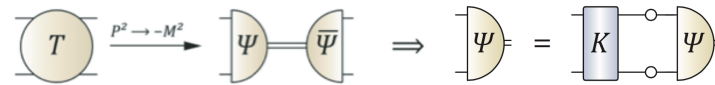


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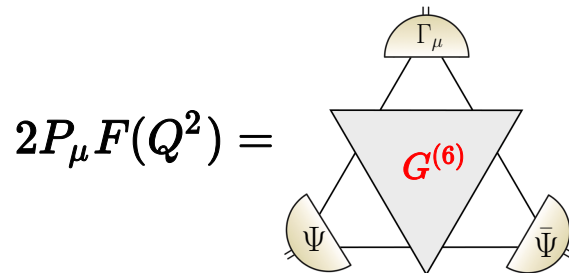


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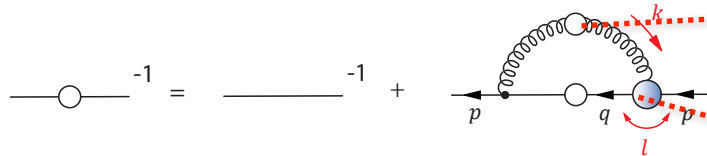
Scattering Kernel

- Three-body form factor equation



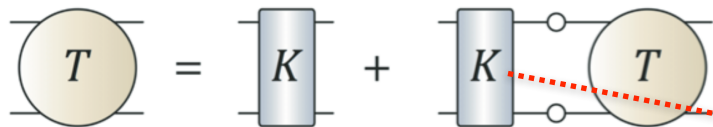
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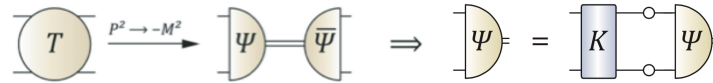


Gluon propagator

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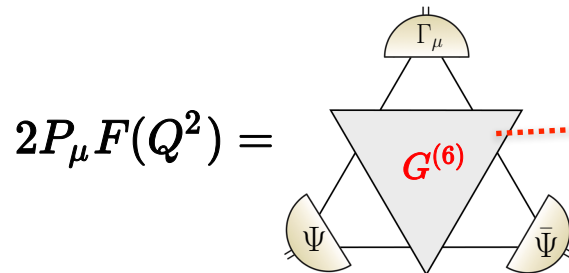


Quark-gluon vertex



Scattering Kernel

- Three-body form factor equation



6-point Green function

## 2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

II. Quark-gluon vertex

III. Scattering kernel

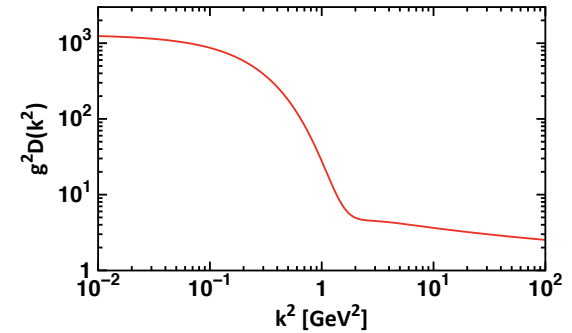
IV. 6-point Green function

## 2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

massive gluon model  $\dashrightarrow$

$$g^2 D_{\mu\nu}^{ab}(k) = \delta_{ab} D_{\mu\nu}^{\text{free}}(k) \mathcal{G}(k^2)$$



II. Quark-gluon vertex

III. Scattering kernel

IV. 6-point Green function

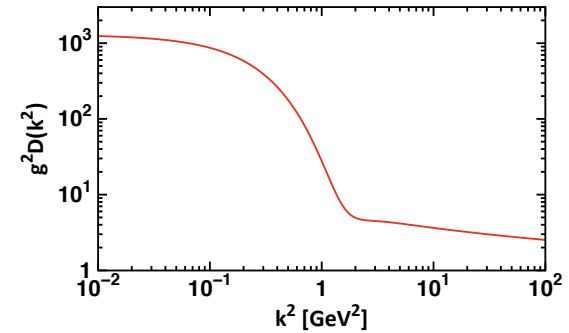


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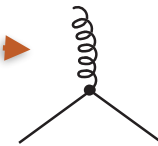
massive gluon model

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II. Quark-gluon vertex

rainbow approximation



III. Scattering kernel

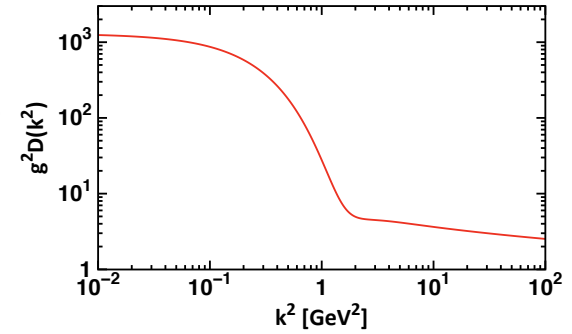
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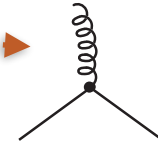
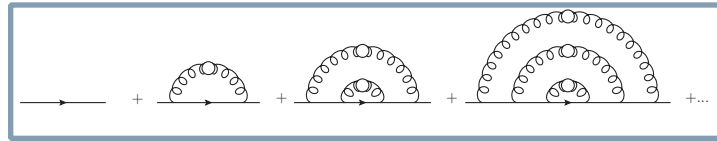
massive gluon model  $\rightarrow$

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### II. Quark-gluon vertex

rainbow approximation  $\rightarrow$



### III. Scattering kernel

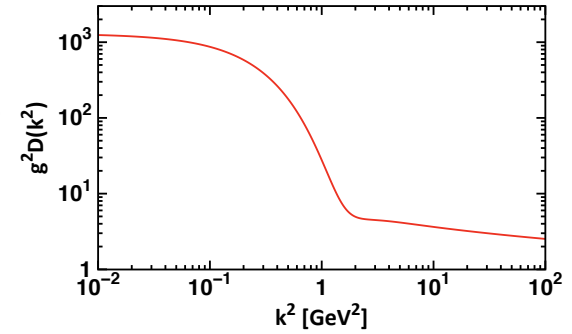
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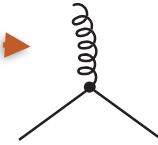
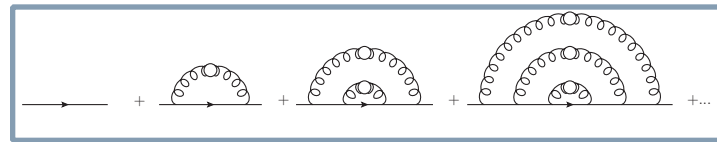
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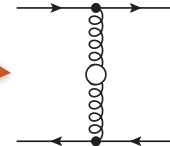
### II. Quark-gluon vertex

rainbow approximation



### III. Scattering kernel

ladder approximation



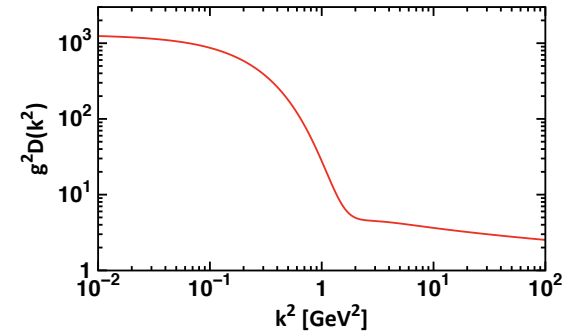
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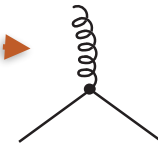
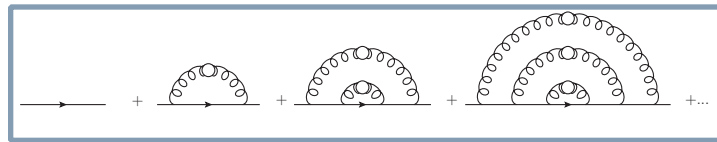
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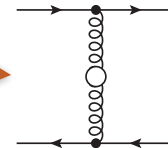
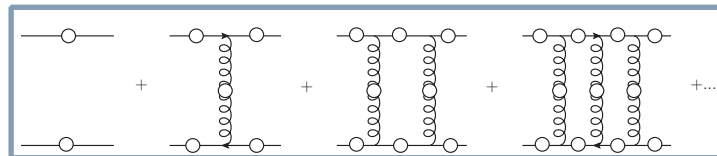
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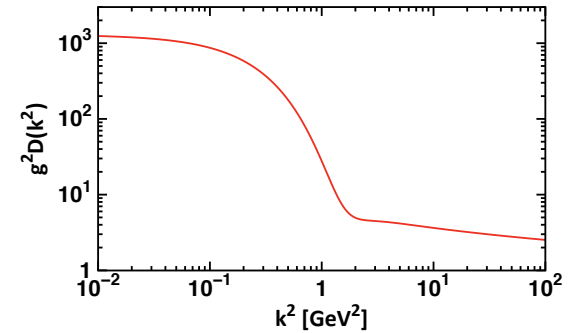
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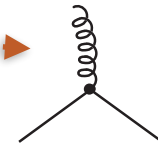
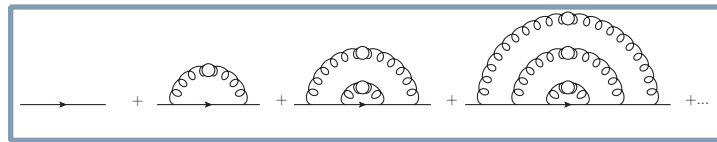
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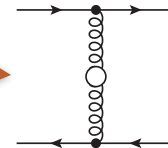
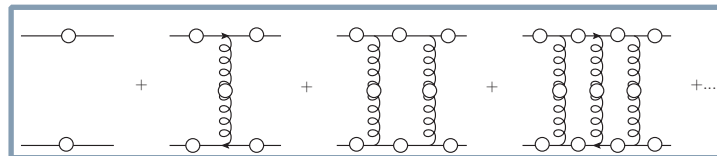
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rainbow approximation



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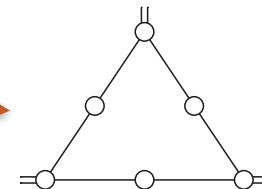
ladder approximation



### IV. 6-point Green function

triangle diagram

$$\Lambda_\mu(P, Q) = 2P_\mu F(Q^2)$$



## 2 DSE: Simplest approximation of QCD DSEs

◆ In the chiral limit, the color-singlet av-WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k, P) = S^{-1} \left( k + \frac{P}{2} \right) i\gamma_5 + i\gamma_5 S^{-1} \left( k - \frac{P}{2} \right)$$

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- ◆ Assuming **DCSB**, i.e., the mass function is nonzero, we have the following identity

$$\lim_{P \rightarrow 0} P_\mu \Gamma_{5\mu}(k, P) = 2i\gamma_5 B(k^2) \neq 0$$

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- ◆ The axial-vector vertex must involve a **pseudo scalar pole (Goldstone theorem)**

$$\Gamma_{5\mu}(k, 0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \quad f_\pi E_\pi(k^2) = B(k^2)$$



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$$\Gamma_{5\mu}(k, 0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \quad f_\pi E_\pi(k^2) = B(k^2)$$

- ◆ Assuming there is a radially excited pion, its decay constant vanishes

$$\lim_{P^2 \rightarrow M_{\pi_n}^2} \Gamma_{5\mu}(k, P) \sim \frac{2i\gamma_5 f_{\pi_n} E_{\pi_n}(k, P) P_\mu}{P^2 + M_{\pi_n}^2} < \infty \quad f_{\pi_n} = 0$$



## 2 DSE: Simplest approximation of QCD DSEs

- ◆ In the chiral limit, the color-singlet av-WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k, P) = S^{-1} \left( k + \frac{P}{2} \right) i\gamma_5 + i\gamma_5 S^{-1} \left( k - \frac{P}{2} \right)$$

- ◆ Assuming **DCSB**, i.e., the mass function is nonzero, we have the following identity

$$\lim_{P \rightarrow 0} P_\mu \Gamma_{5\mu}(k, P) = 2i\gamma_5 B(k^2) \neq 0$$

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**DCSB** means **much more** than **massless** pseudo-scalar meson.

## 2 DSE: Summary

- ◆ **Gluon propagator**: Solve the gluon DSE or extract information from lattice QCD. The dressing function of gluon has a **mass scale** as that of quark.
- ◆ **Quark-gluon vertex + Scattering kernel**: Analyze continuous (WGTIs or STIs) & discrete symmetries. The kernel (RL) preserves the chiral symmetry which makes pion to play a **twofold role**: Bound-state and Goldstone boson.
- ◆ **Form factor**: Generalize the wave function normalization condition. The form factor (the triangle diagram) preserves the **current conservation**.

### 3 Application: Realization of DCSB & Confinement

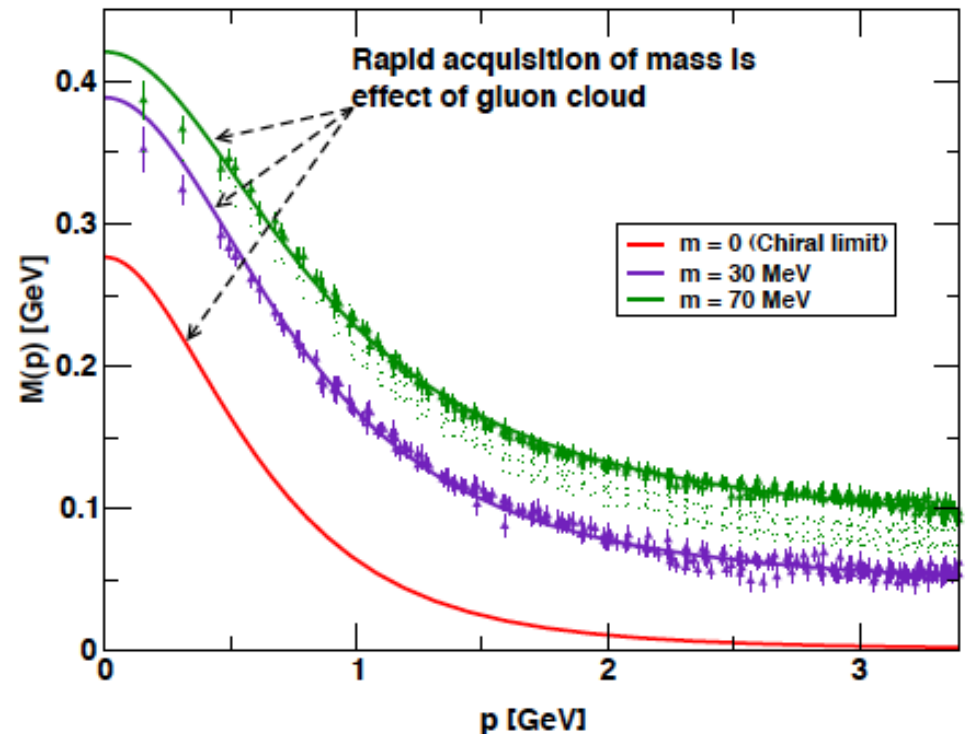
#### ◆ DCSB:

1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

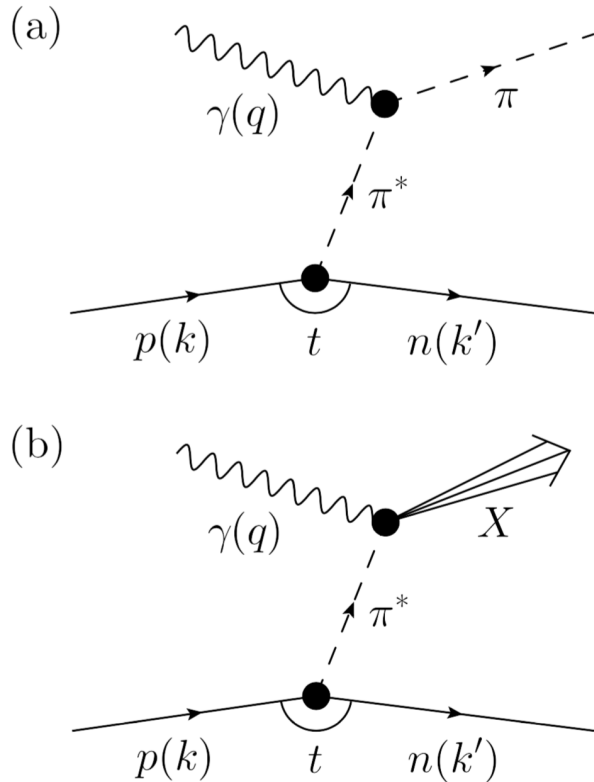
#### ◆ Confinement:

Although we exactly know few knowledge about confinement, the positivity violation of quark spectral density supports a fact that a asymptotically free quark is unphysical. In this sense, we say that quarks are **confined**.

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



### 3 Application: Off-Shell pions and kaons



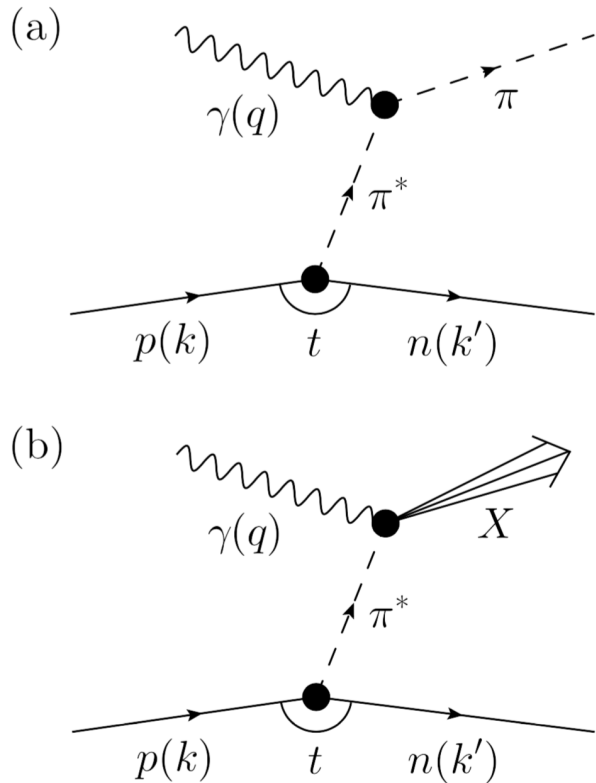
- ◆ Experiments use a nucleon's **virtual pion cloud** as a pion target, e.g., the processes are usually involved:

$$\pi^* + \gamma \rightarrow \pi$$

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Sullivan processes, in which a nucleon's pion cloud is used to provide access to the pion's (a) elastic form factor and (b) parton distribution functions.

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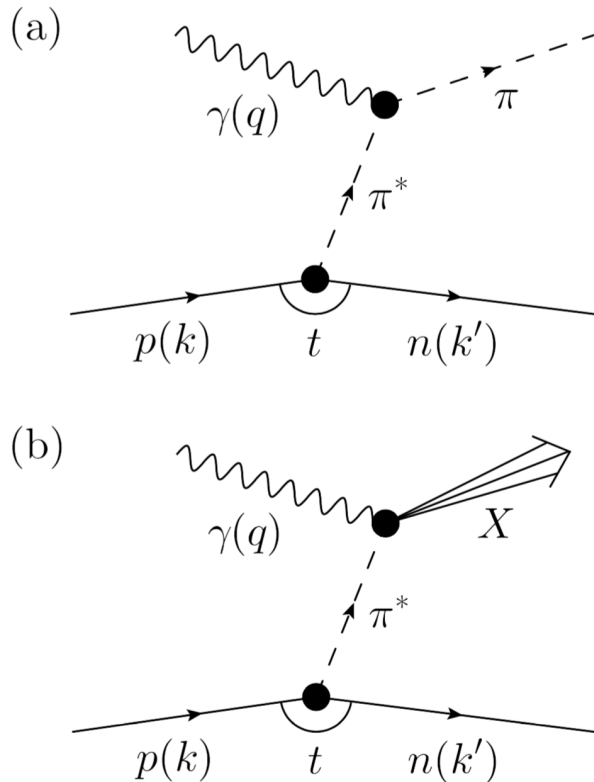
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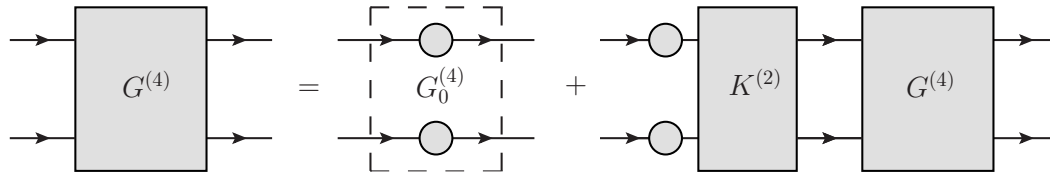
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- ◆ How does the pion's virtuality affect its **properties** and further affect the related processes?
- ◆ Is there a **critical virtuality** above which a Sullivan-like process **cannot** provide reliable access to a meson target?

### 3 Application: Off-Shell pions and kaons

◆ In QFT, bound-states are encoded in Green functions.

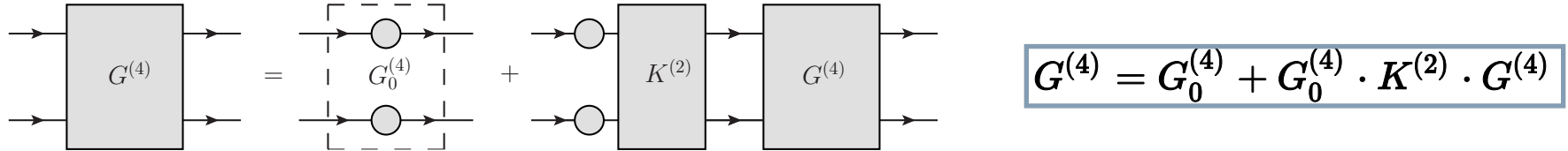


$$G^{(4)} = G_0^{(4)} + G_0^{(4)} \cdot K^{(2)} \cdot G^{(4)}$$



### 3 Application: Off-Shell pions and kaons

- ◆ In QFT, bound-states are encoded in Green functions.



- ◆ The kernel can be decomposed by its **orthogonal** eigenbasis, which are classified by  **$J^P$**  quantum number and radial quantum number  **$n_r$** ,

$$K^{(2)} = \sum_i \lambda_i^{-1} |\Gamma_i\rangle \langle \Gamma_i| \quad |\Gamma_i\rangle = \lambda_i K^{(2)} \cdot G_0^{(4)} \cdot |\Gamma_i\rangle \quad \langle \Gamma_i | G_0^{(4)} | \Gamma_j \rangle = \delta_{ij}$$

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- ◆ Accordingly, the four-point Green function can be decomposed:

$$G^{(4)} = G_0^{(4)} + \sum_i |\chi_i\rangle \frac{1}{\lambda_i(P^2) - 1} \langle \chi_i|$$



### 3 Application: Off-Shell pions and kaons

- ◆ The **wave function** of the bound state has to satisfy the following condition

$$\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \begin{array}{c} \text{Diagram 1} \\ \left[ \left( \text{Diagram 2} \right)^{-1} - \text{Diagram 3} \right] \\ \text{Diagram 4} \end{array} \right\} = \mathbf{1}$$

The equation is a mathematical condition involving Feynman diagrams. The left side is a limit as the system goes on-shell, multiplied by the inverse of the propagator  $P^2 + M^2$ . The right side is a bracketed expression of four diagrams: a vertex diagram, the inverse of a two-line diagram, a box labeled  $K^{(2)}$ , and another vertex diagram. The result is the identity  $\mathbf{1}$ .

### 3 Application: Off-Shell pions and kaons

- ◆ The **wave function** of the bound state has to satisfy the following condition

$$\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \left[ \text{diagram} \left( \left( \text{diagram} \right)^{-1} - \text{diagram } K^{(2)} \right) \text{diagram} \right] \right\} = 0 = 1$$

The equation shows a limit as the system goes on-shell. The denominator is  $P^2 + M^2$ . The numerator is a large curly bracket containing a diagrammatic expression. The expression is a tree-level vertex diagram (a circle with two external lines) multiplied by a square bracket containing the inverse of a two-particle propagator diagram (two parallel lines with a circle in between) minus a shaded box labeled  $K^{(2)}$ . This entire bracketed expression is then multiplied by another tree-level vertex diagram. A red "= 0" is written above the second vertex diagram, and the final result of the limit is "= 1".

### 3 Application: Off-Shell pions and kaons

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The diagram in the equation is enclosed in a red box. The diagram consists of a vertex with two incoming lines and two outgoing lines. The two outgoing lines are connected to a box labeled  $K^{(2)}$ . The two incoming lines are connected to a box labeled  $K^{(2)}$ . The entire diagram is enclosed in a red box. The diagram is followed by a red "= 0" and a closing brace. The entire expression is followed by "= 1".

- ◆ The generalized homogeneous **Bethe-Salpeter** equation can be obtained as

$$\text{diagram} = \text{diagram} \times \lambda(P^2)$$

The diagram on the left is a vertex with two incoming lines and two outgoing lines. The diagram on the right is a vertex with two incoming lines and two outgoing lines, connected to a box labeled  $K^{(2)}$ . The entire diagram is followed by  $\times \lambda(P^2)$ .

### 3 Application: Off-Shell pions and kaons

- ◆ The **wave function** of the bound state has to satisfy the following condition

$$\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \left[ \text{diagram} \left( \left( \text{diagram} \right)^{-1} - \text{diagram } K^{(2)} \right) \text{diagram} \right] \right\} = 0 = 1$$

The equation shows a limit as the system goes on-shell. The expression is a product of a propagator  $\frac{1}{P^2 + M^2}$  and a bracketed term. The bracketed term consists of a diagram on the left, a large square bracket containing the inverse of a two-particle diagram minus a diagram with a box labeled  $K^{(2)}$ , and a diagram on the right. A red "= 0" is written above the right diagram, and a red "= 1" is written to the right of the large bracket.

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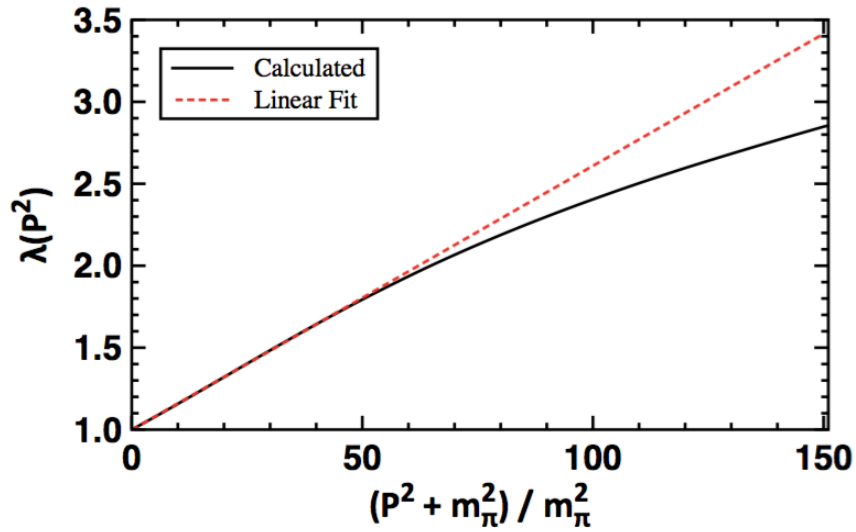
The diagram shows an equality between a two-particle vertex diagram and a more complex diagram involving a two-particle diagram, a box labeled  $K^{(2)}$ , and a vertex diagram, all multiplied by  $\lambda(P^2)$ .

- ◆ The solved wave function must be **normalized** as following

$$\text{diagram} \left\{ \frac{\partial}{\partial P_\mu} \left[ \left( \text{diagram} \right)^{-1} - \text{diagram } K^{(2)} \right] \right\} \text{diagram} = 2P_\mu$$

The diagram shows a vertex diagram multiplied by a derivative with respect to  $P_\mu$  of a bracketed term. The bracketed term is the inverse of a two-particle diagram minus a diagram with a box labeled  $K^{(2)}$ . This is then multiplied by another vertex diagram, and the result is set equal to  $2P_\mu$ .

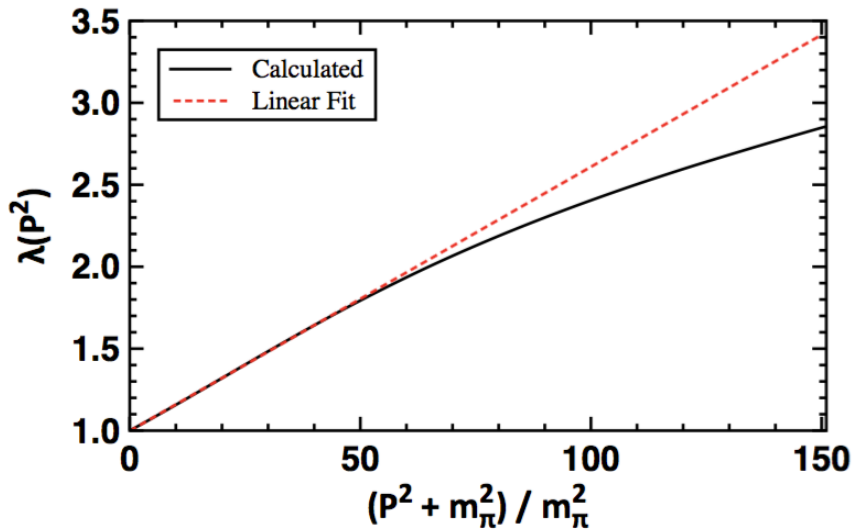
### 3 Application: Off-Shell pions and kaons



◆ The eigenvalue is **linear** to the virtuality less than **45** ( $P^2 = (v - 1)m_\pi^2$ )

$$\lambda(v) = 1 + 0.016 v$$

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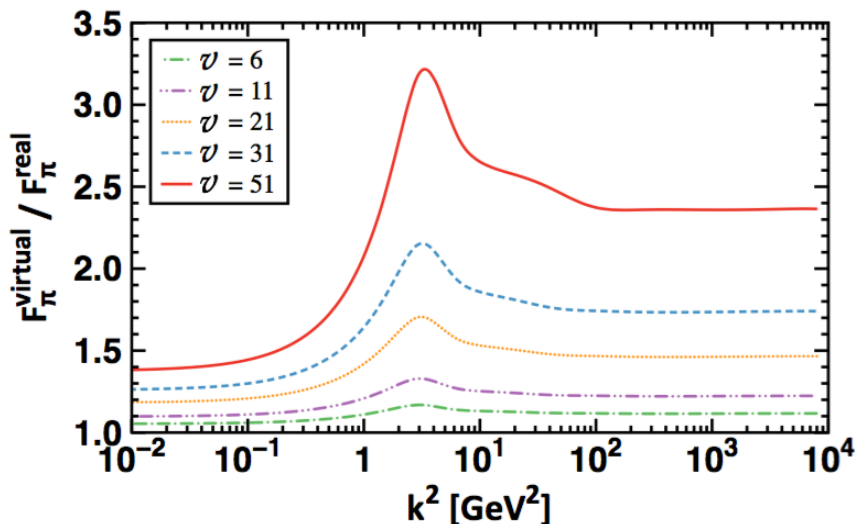
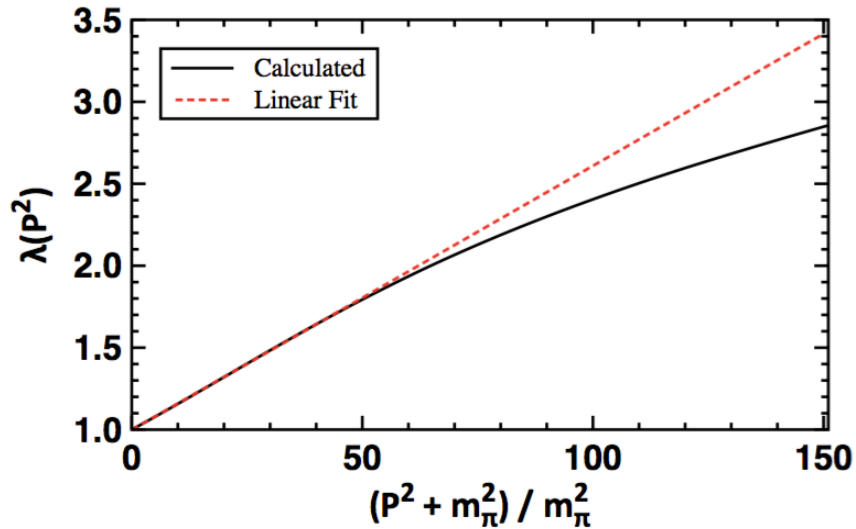
$$\frac{1}{\lambda(P^2) - 1} \sim \frac{1}{P^2 + m_\pi^2}$$

the change in  $\lambda$  is purely **kinematic** and, hence, the pion pole **dominates** the quark-antiquark scattering matrix.





### 3 Application: Off-Shell pions and kaons



- ◆ The eigenvalue is **linear** to the virtuality less than **45** ( $P^2 = (\nu - 1)m_\pi^2$ )

$$\lambda(\nu) = 1 + 0.016 \nu$$

- ◆ Recalling the Green function's structure

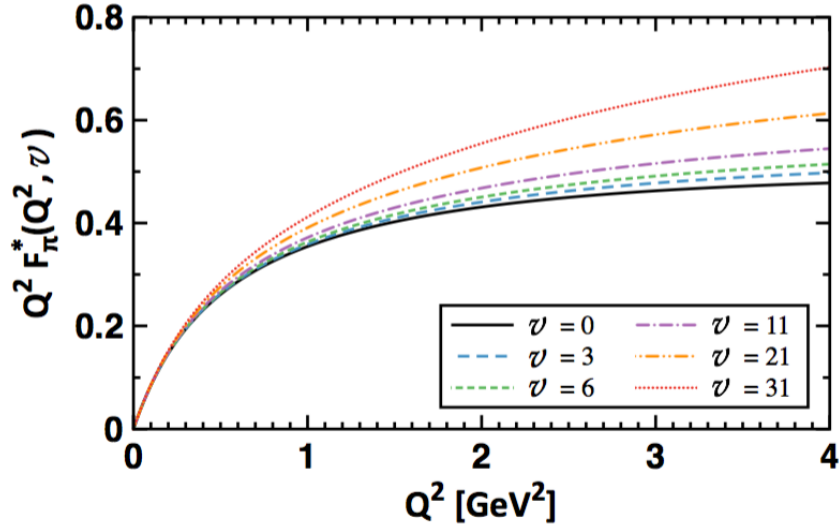
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the change in  $\lambda$  is purely **kinematic** and, hence, the pion pole **dominates** the quark-antiquark scattering matrix.

- ◆ The **UV shifts** of the BS amplitudes grow with the virtuality less than **31** and that growths are almost linear. This leads to a **linear** growth of the **in-pion condensate**:

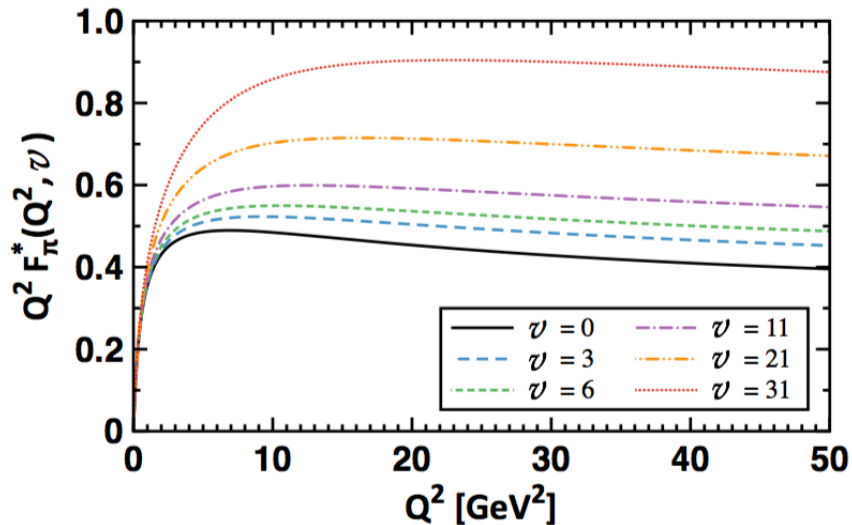
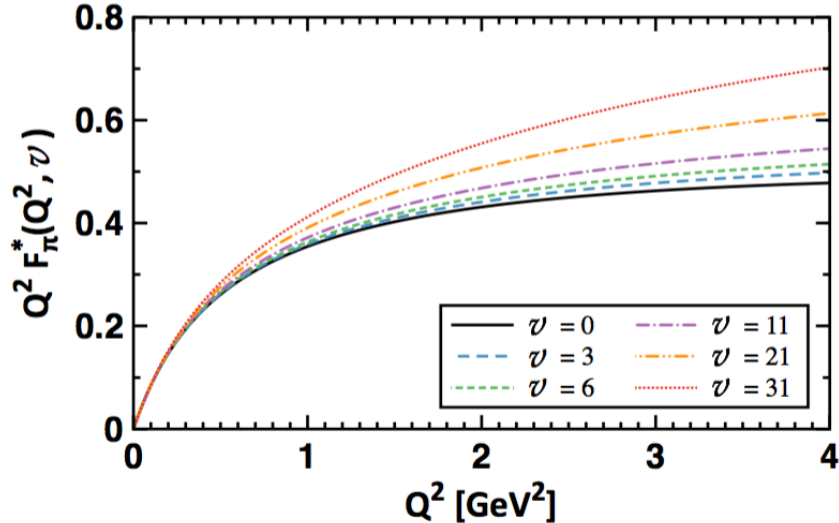
$$\kappa_\pi^\zeta(\nu) \approx \kappa_\pi^\zeta(0)[1 + 0.032\nu]$$

### 3 Application: Off-Shell pions and kaons



- ◆ With the virtuality increasing, the pion has a smaller radius and becomes more point-like.

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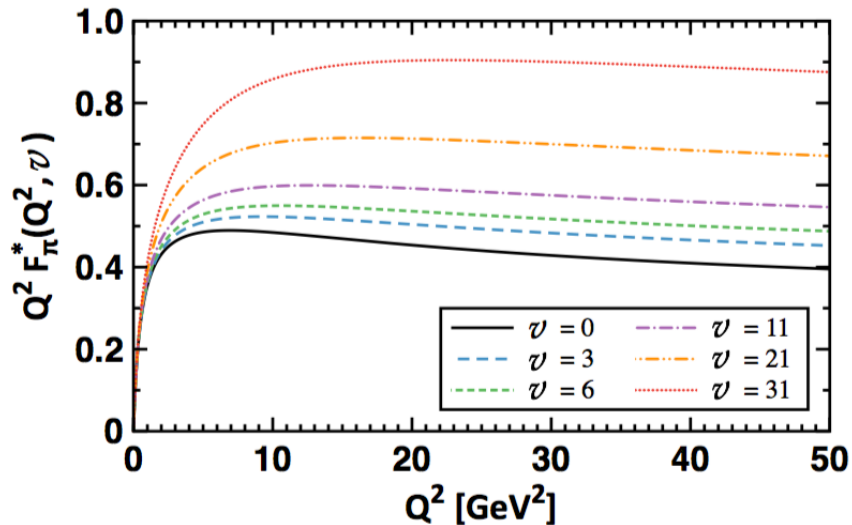
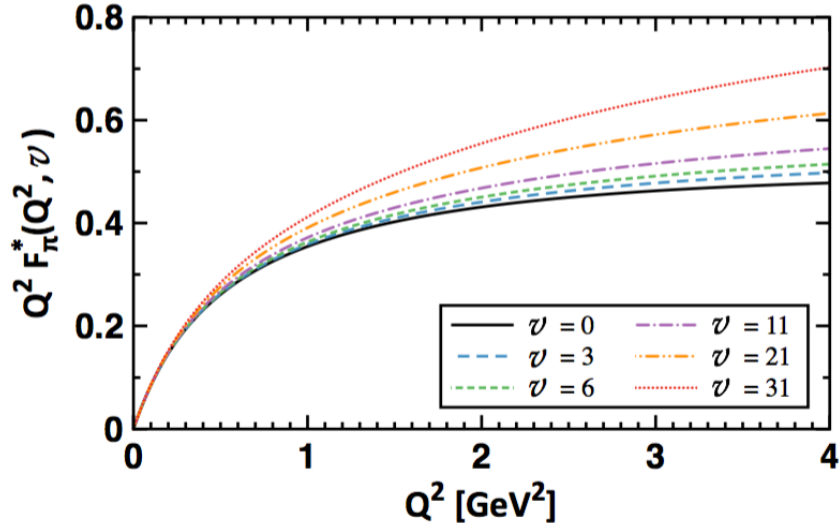


- ◆ With the virtuality increasing, the pion has a **smaller radius** and becomes more **point-like**.
- ◆ The computed form factor can be interpolated by a **monopole** multiplied by a simple factor that restores the correct **QCD anomalous dimension**.

$$F_\pi^*(Q^2, \nu) = \frac{1}{1 + Q^2/m_0^2} \mathcal{A}(Q^2, \nu)$$

$$\mathcal{A}(Q^2, \nu) = \frac{1 + Q^2 a_0^2(\nu)}{1 + Q^2 [a_0^2(\nu)/b_u^2(\nu)] \ln(1 + Q^2/\Lambda_{\text{QCD}}^2)}$$

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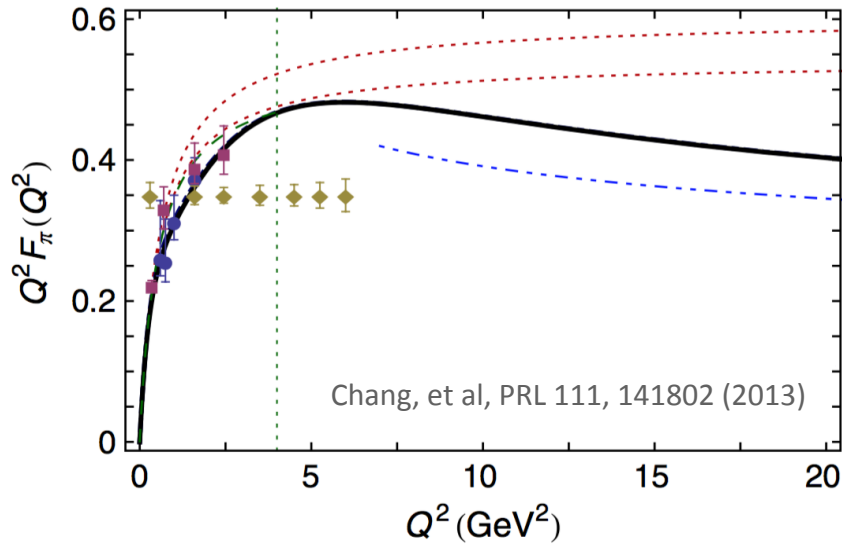
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- ◆ For  $\nu \lesssim \nu_S$ , the  $Q^2 \gtrsim 10$  GeV<sup>2</sup> form factor responds **linearly** to changes in the BS amplitudes and such modifications should become **evident** on this domain.



### 3 Application: Off-Shell pions and kaons

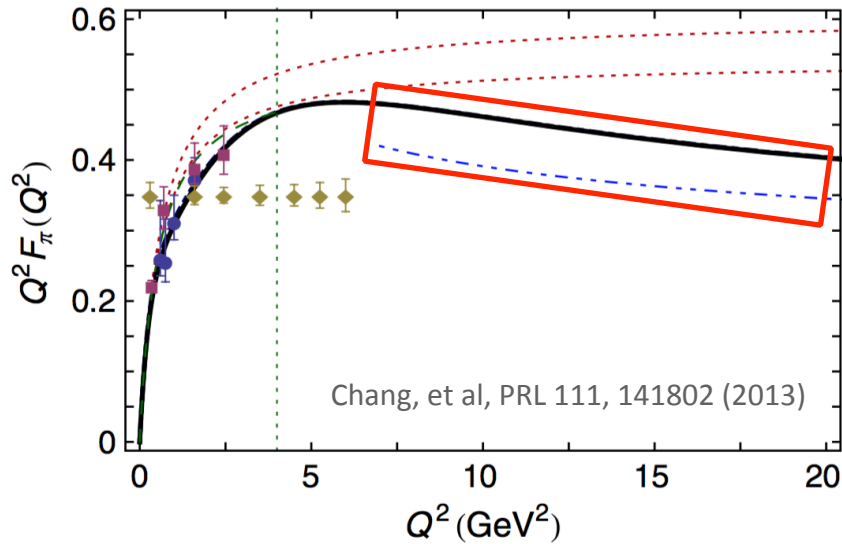


- ◆ The pion's twist-two valence-quark PDA is connected with the large- $Q^2$  form factor:

$$Q^2 F_\pi(Q^2) \stackrel{Q^2 \gg \Lambda_{\text{QCD}}^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,$$

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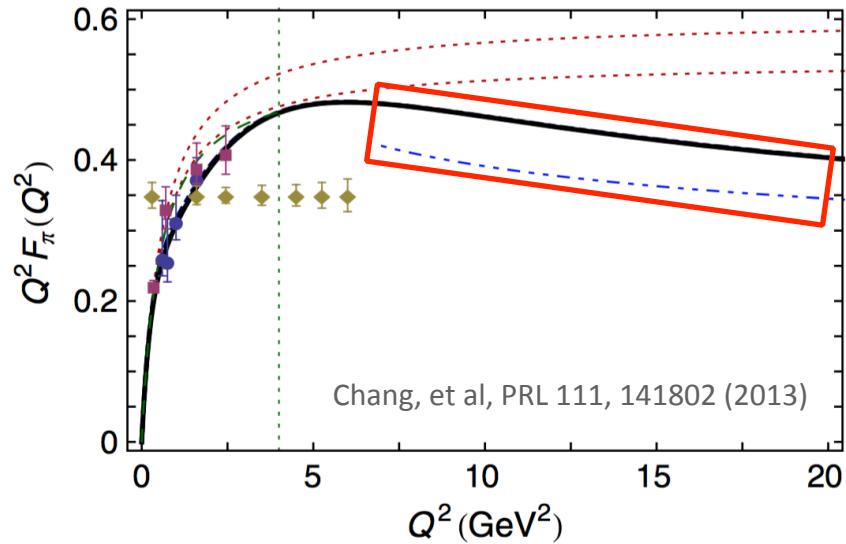


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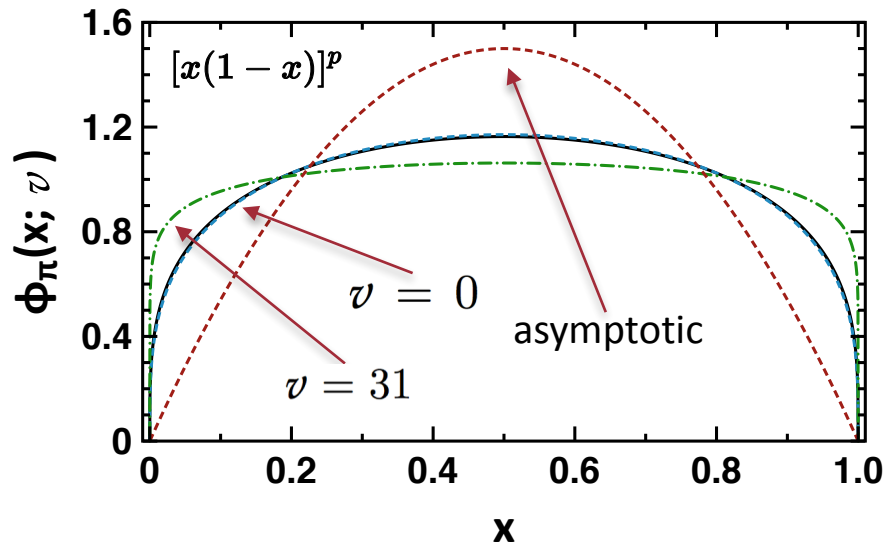
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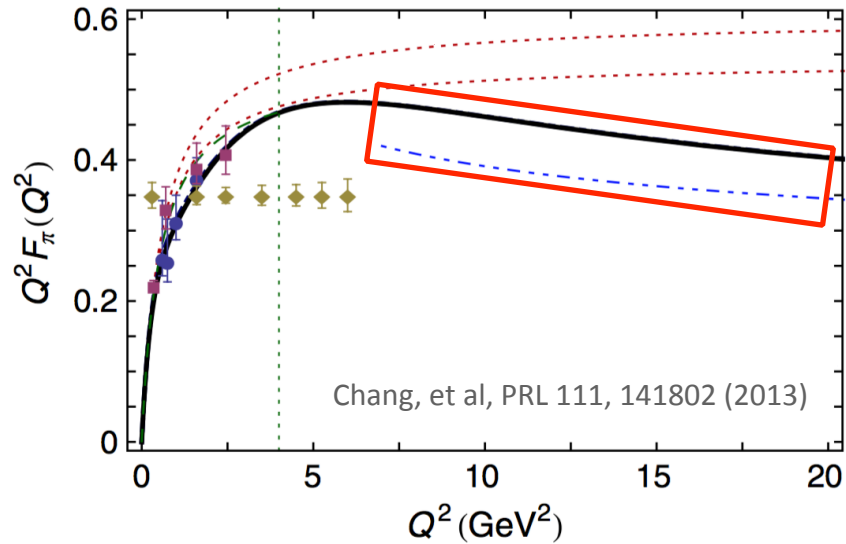
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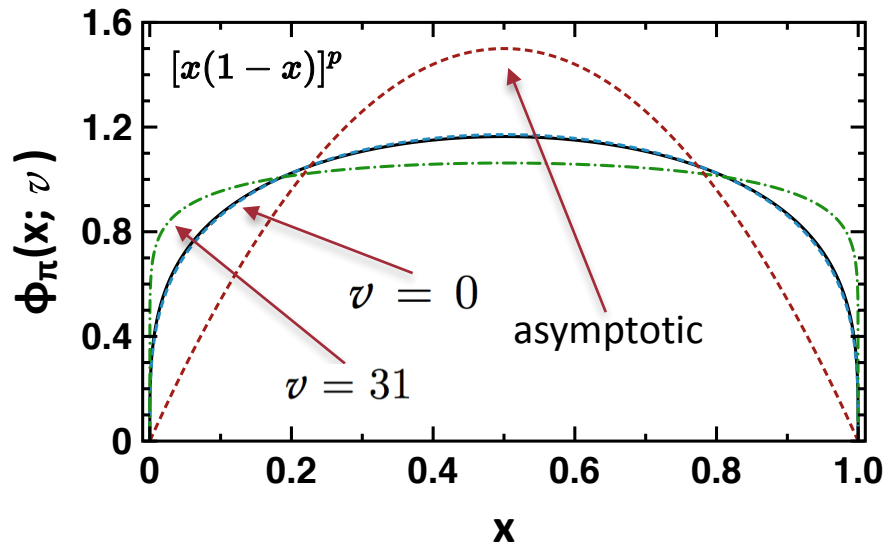
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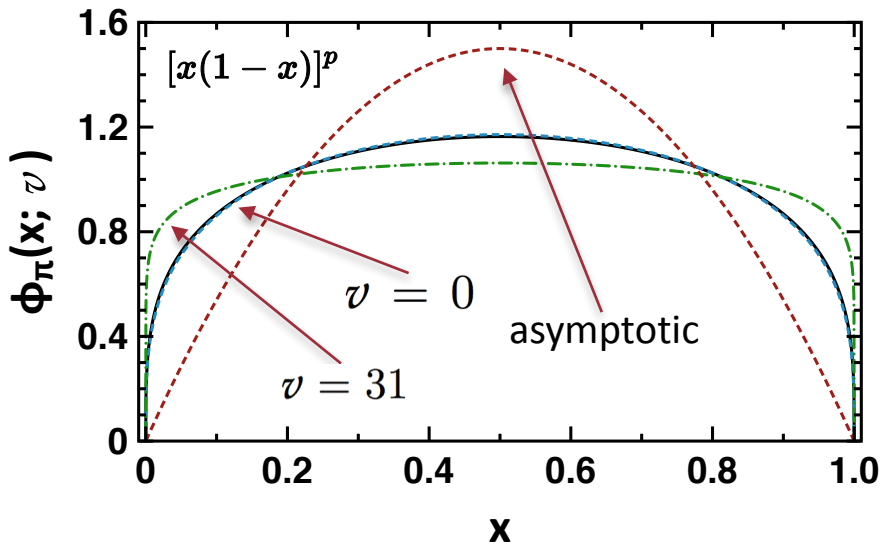
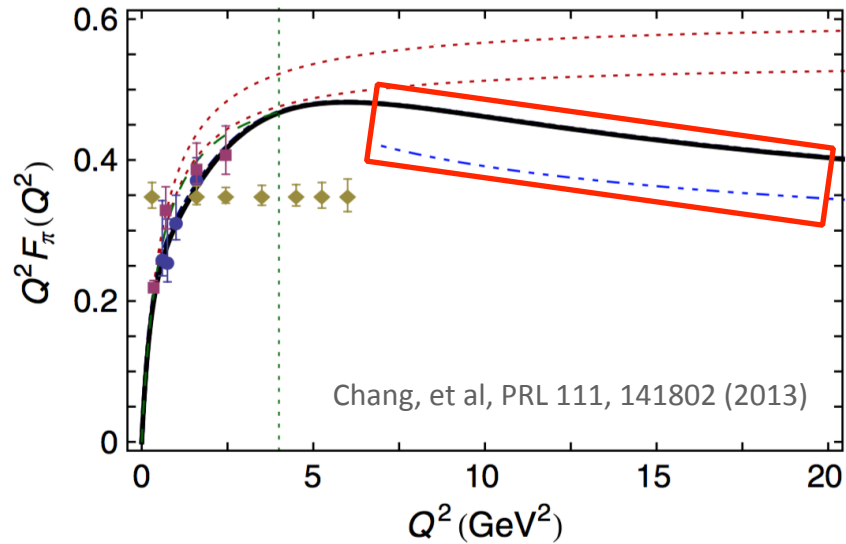
$$F_\pi(Q^2) = \int_{-1}^1 dx H_{\pi^+}^u(x, 0, Q^2),$$

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### 3 Application: Off-Shell pions and kaons



- ◆ The pion's twist-two valence-quark **PDA** is connected with the **large- $Q^2$**  form factor:

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$$F_\pi(Q^2) = \int_{-1}^1 dx H_{\pi^+}^u(x, 0, Q^2),$$

$$u^\pi(x) = H_{\pi^+}^u(x > 0, 0, 0),$$

- ◆ The **critical virtuality**, below which the virtual particles serve as a **valid target**, is

$$\begin{array}{ccc} u\bar{u} & u\bar{s} & s\bar{s} \\ \hline \xrightarrow{\hspace{10em}} & & \end{array}$$

$-t \lesssim 0.6 \text{ GeV}^2$        $-t \lesssim 0.9 \text{ GeV}^2$        $-t \lesssim 1.3 \text{ GeV}^2$

# Summary

- ◆ **Bound-states** are **ideal** objects connecting experiments and theories. **QCD bound-state** problems are difficult because of its relativistic and strongly-couple properties.
- ◆ Based on LQCD and QCD's symmetries, the **simplest method** to construct the **gluon propagator, quark-gluon vertex, scattering kernel**, and **form factor**, is demonstrated.
- ◆ A **model-independent** scheme to study the **off-shell** bound state is proposed. Off-shell pions and kaons are studied to suggest critical **virtualities** for experiments.

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# Outlook

◆ With the **sophisticated method** to solve the DSEs, we can push the approach to a wide range of applications in **QCD bound-state** problems.

◆ Hopefully, after more and more **successful applications** are presented, the DSEs may provide a **faithful path** to understand **QCD** and a powerful tool for general physics.

