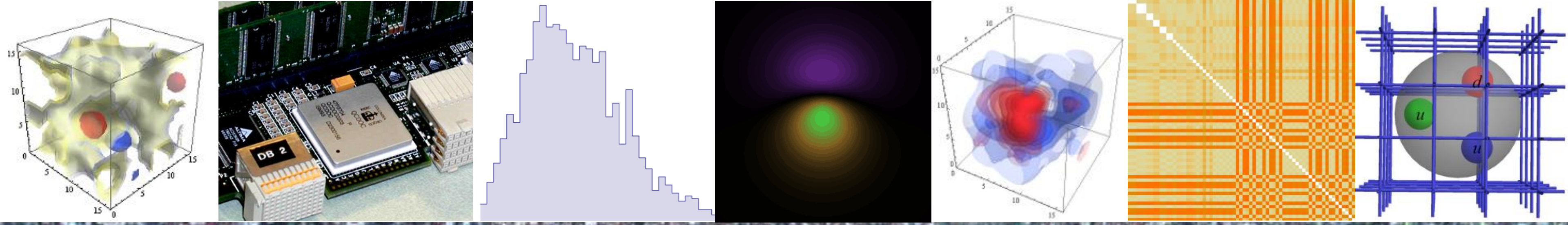


Pion & Kaon Structure

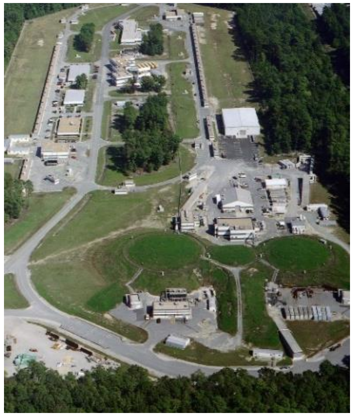
Bjorken-x Dependence of the DAs

Huey-Wen Lin

Meson Distribution Amplitudes

§ DAs are universal

- Involved in many processes



§ Pseudoscalar mesons

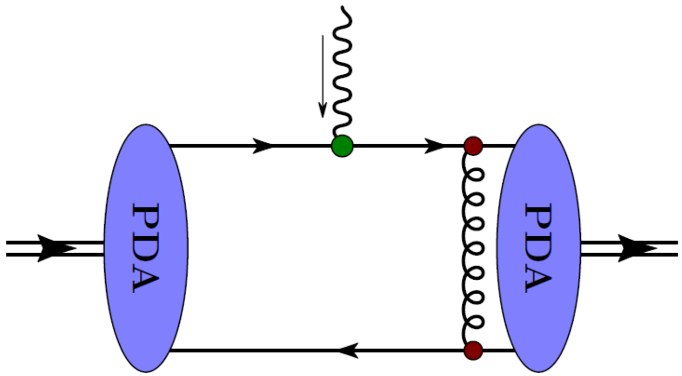
- Pion form factor to 6 GeV^2 (JLab Hall C)
- Weak exclusive B and Λ_b decays (LHCb)

§ Vector mesons

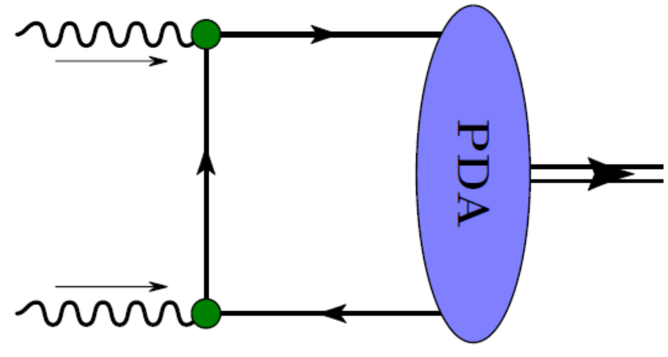
- Deeply virtual exclusive ρ production: $eN \to eN\rho$ (JLab, EIC)
- Weak exclusive $B \to V\mu^+\mu^-, V\ell\nu_\ell, V\pi$ decays (LHCb)
  e.g. $B \to K^*\mu^+\mu^-$ or $B_s \to \phi\mu^+\mu^-$

Meson Distribution Amplitudes

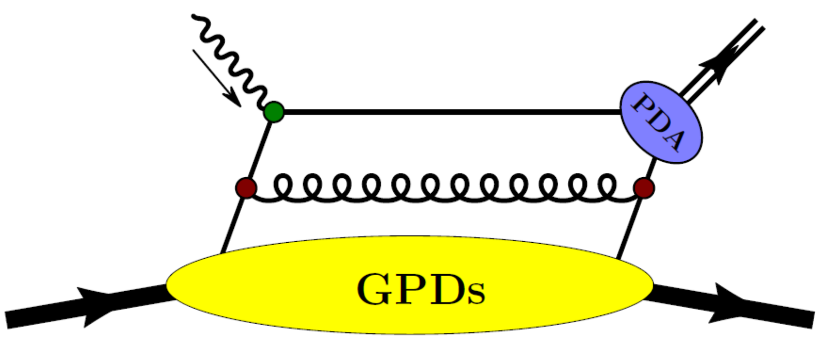
§ PDAs enter numerous hard exclusive scattering processes

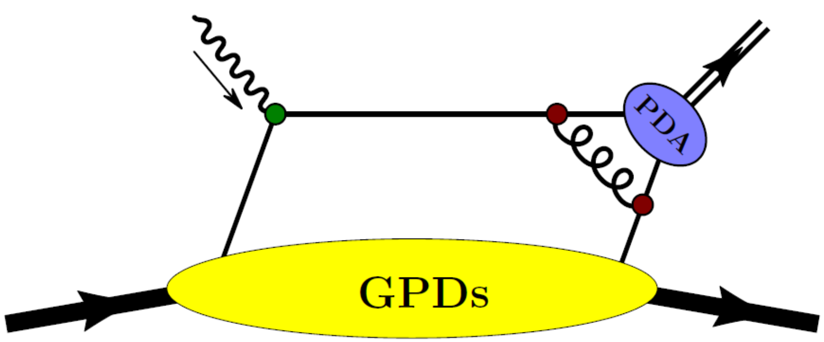


$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$



$$Q^2 F_{\gamma^*\gamma\pi}(Q^2) \to 2 f_\pi$$





Diagrams from Ian Cloet

Meson Distribution Amplitudes

- Distrubution amplitudes (DAs) are nonperturbative inputs for the theoretical description of hard exclusive processes
- Intuitive picture in the infinite momentum frame:

$$|M\rangle = |\bar{q}q\rangle + |\bar{q}qg\rangle + |\bar{q}q\bar{q}q\rangle + \cdots$$

  - Superposition of states with different numbers of partons
  - Hadron wavefuctions at small transverse distance of the constituents (distribution of longitudinal momentum)
- In hard exclusive process, valence contribution dominates
  - Higher Fock states are suppressed at large momentum transfer
  - Appear always in convolutions in expressions for hard exclusive processes
  - Difficult to extract from experiment without contamination from other hadronic uncertainties

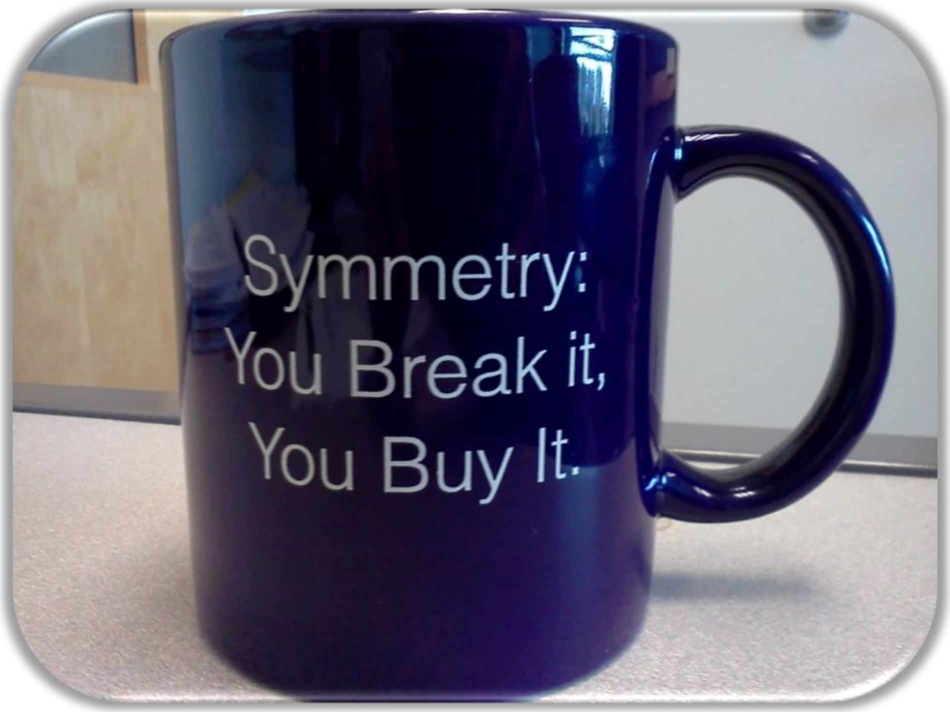
DAs on the Lattice

- Lightcone definition
  - $\langle 0 | \bar{d}(-z)\gamma_\mu\gamma_5[-z,z]u(z) | \pi^+ \rangle = i f_\pi p_\mu \int_0^1 d\xi\, e^{-i\xi p\cdot z} \phi_\pi(\xi,\mu)$
    - Nonlocal matrix elements with Wilson line $[-z,z]$ connecting u and $\bar{d}$
    - $\xi = x-(1-x) = 2x-1$
- Lattice calculations rely on operator product expansion, only provide moments
  - $\langle \xi^n \rangle = \int d\xi\, \xi^n \phi(\xi,\mu)$
  - $\langle \xi^0 \rangle = 1, \langle \xi^2 \rangle \to \left\langle 0 \left| \bar{d}\overleftrightarrow{D}_{(\mu}\overleftrightarrow{D}_\nu\gamma_{\rho)}\gamma_5 u \right| \pi(p) \right\rangle \ldots$
- In principle, use inverse transformation to retrieve true DA

$$\phi(\xi,\mu) = 6x(1-x)\left(1 + \sum_{n=2,4,\ldots} a_n^\pi(\mu) C_n^{3/2}(2x-1)\right)$$

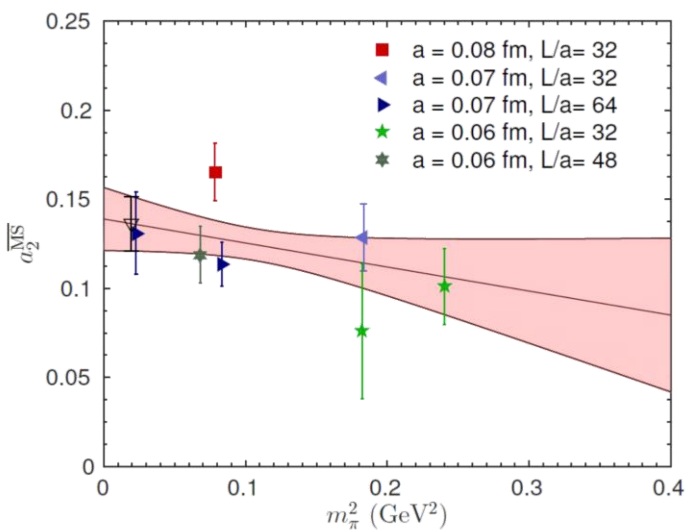
Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative error grows in higher moments
  - Calculation would be costly and difficult

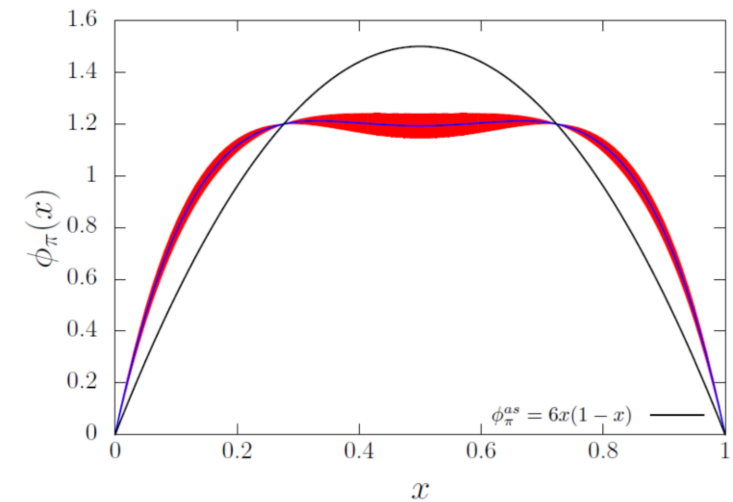


§ Pion DAs

- $a_2^\pi, \langle \xi^2 \rangle$ from $\left\langle 0 \left| \bar{d} \overleftrightarrow{D}_{(\mu} \overleftrightarrow{D}_\nu \gamma_{\rho)} \gamma_5 u \right| \pi(p) \right\rangle$
- RQCD: $N_f = 2, m_\pi \in [150, 490]$ MeV, $m_\pi L \in [3.4, 6.7]$



RQCD (V.M. Braun et al), 1503.03656



- RQCD: $a_2^{\overline{\rm MS}}(2\text{ GeV}) = 0.136(15)(15)$ Braun 1503.03656
- RBC/UKQCD: $a_2^{\overline{\rm MS}}(2\text{ GeV}) = 0.233(30)(60)$ Arthur 1011.5906
- QCDSF/UKQCD: $a_2^{\overline{\rm MS}}(2\text{ GeV}) = 0.211(114)$ Braun hep-lat/0606012

Problem with Reconstruction

§ Possible reconstructions

- Using Gegenbauer polynomial expansion of the pion DA

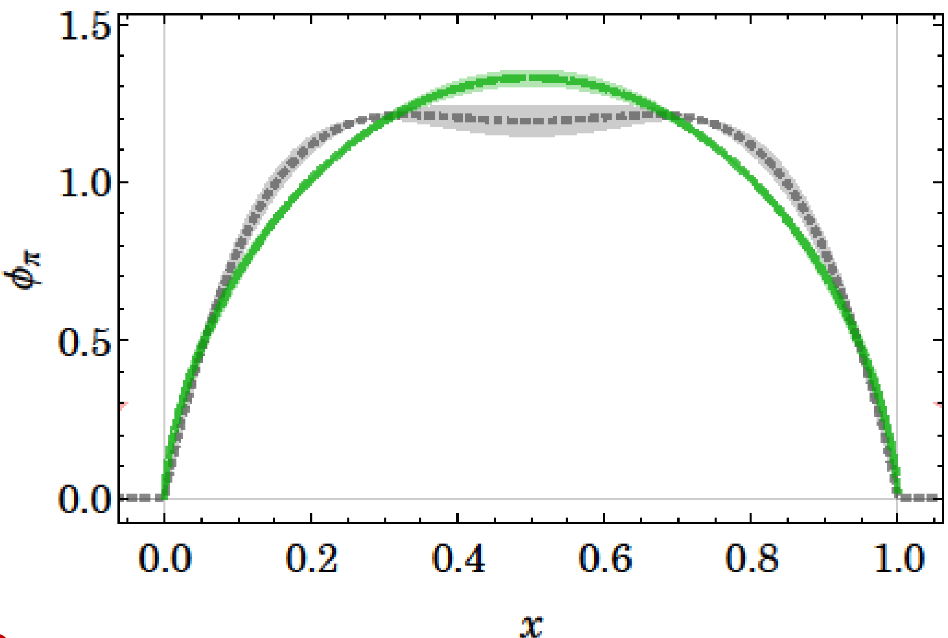
$$\boxed{6x(1-x)\left(1+a_2 C_2^{3/2}(2x-1)\right)}$$

  - and $a_2 = 0.136(15)(15)$

- Another through

$$\boxed{A\left(x(1-x)\right)^B}$$

  - with $\langle \xi^2 \rangle = 0.2361(41)(39)$



LP3 Collaboration, 1702.00008

§ A new approach is needed to extract DAs on the lattice

A Promising New Direction
with Examples of PDFs

New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

§ Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)

§ Weinberg introduced a more convenient description using correlation functions along the lightcone

e.g. nucleon quark distribution

$$q(x,\mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left\langle P \left| \overline{\psi}(\xi_-)\gamma_+ \exp\left(-ig\int_0^{\xi_-} d\eta_-\, A_+(\eta_-)\right)\psi(0)\right| P\right\rangle$$

Renormalization scale μ

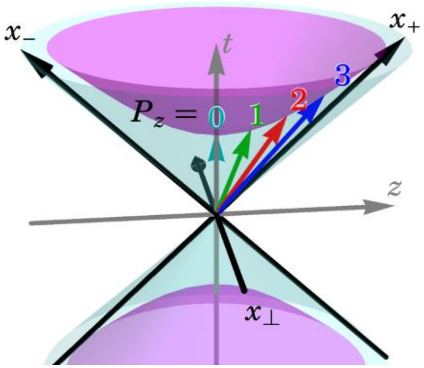
Gluon potential A_+

Lightcone coordinate $\xi_\pm = (t+z)/\sqrt{2}$

New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)



- § Going back to the IMF concept
- § Finite-momentum quark distribution (quasi-distribution)
  - Suggested operator:

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$x = k_z/P_z$

Lattice z coordinate

Product of lattice gauge links

Nucleon momentum $P_\mu = \{P_0, 0, 0, P_z\}$

- § Take the infinite-P_z limit to recover lightcone functions
  - Just another limit to take, like taking $a \to 0$ or $V \to \infty$

New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

Finite-P_z corrections needed

- Neglect typical lattice corrections for now:

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Finite $P_z \leftrightarrow \infty$ perturbative matching

$$Z(x,\mu/P_z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;

Ma and Qiu, 1404.6860

Dominant correction (for nucleon); known scaling form

HWL et al. 1402.1462

J.-W. Chen et al, 1603.06664

§ Benefit from our pQCD colleagues

New Direction

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

Finite-P_z corrections needed

- Neglect typical lattice corrections for now:

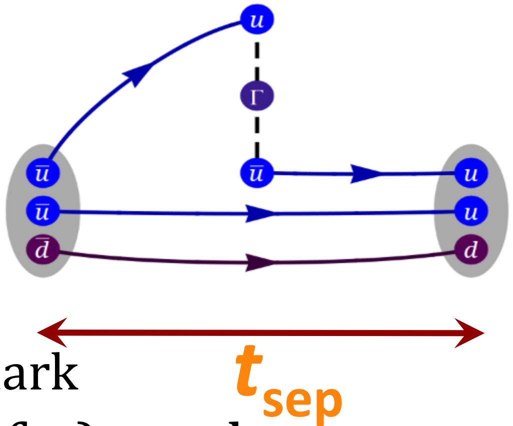
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

complicated higher-twist operator;
smaller P_z correction for nucleon
J.-W. Chen et al, 1603.06664 and reference within
(extrapolate it away)

§ Some similarity in more broadly-studied HQET…

$$O\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda}, \frac{\Lambda}{\mu}\right) o(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right) + \cdots$$

Some Lattice Details



§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm** ($L \approx 2.88$ fm)
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separations ($t_{sep} \approx 0.96$ and 1.2 fm) used

§ Properties known on these lattices

- Lattice Z_Γ for bilinear operator ~ 1
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

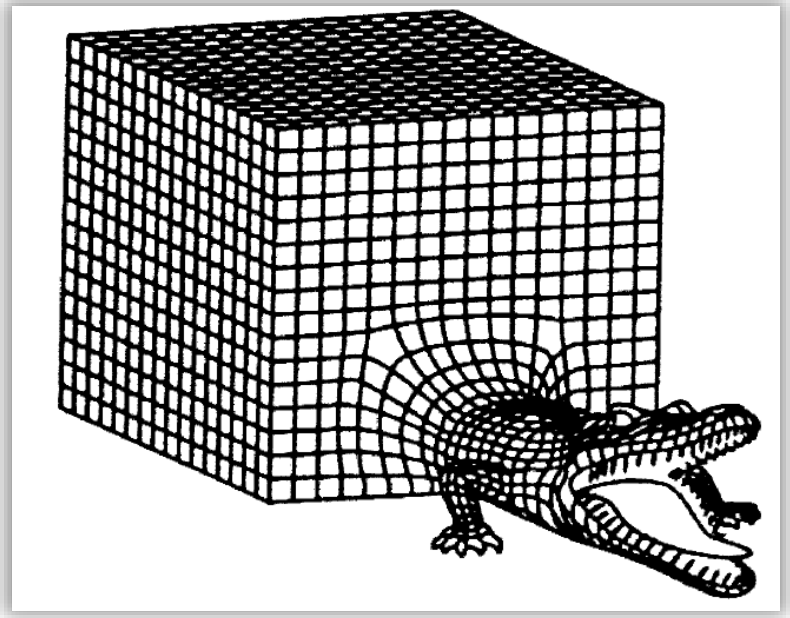
§ Feasible with today's resources!

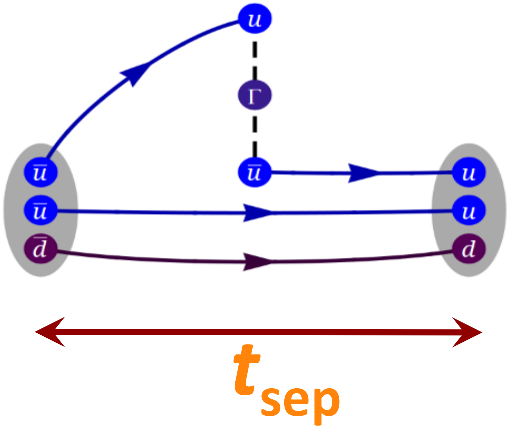
1402.1462 [hep-ph]; 1603.06664 [hep-ph]

Warning!

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)





NO SYSTEMATICS YET!

§ Demonstration that the method works

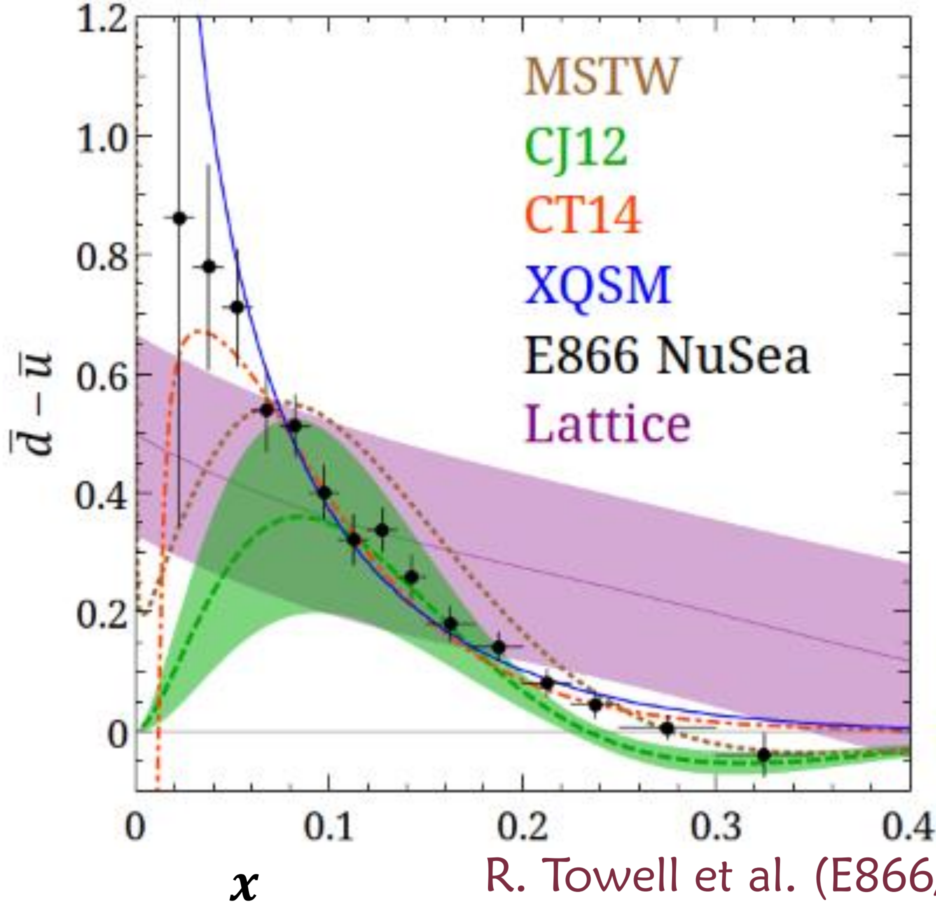
- Intend to motivate future LQCD work on many quantities

Sea Flavor Asymmetry

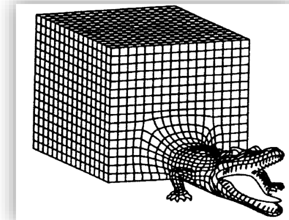
§ First time in LQCD history to study antiquark distribution!

- $M_\pi \approx 310$ MeV

HWL et al. 1402.1462



$$\bar{q}(x) = -q(-x)$$



Lost resolution in small-x region

Future improvement:
 larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

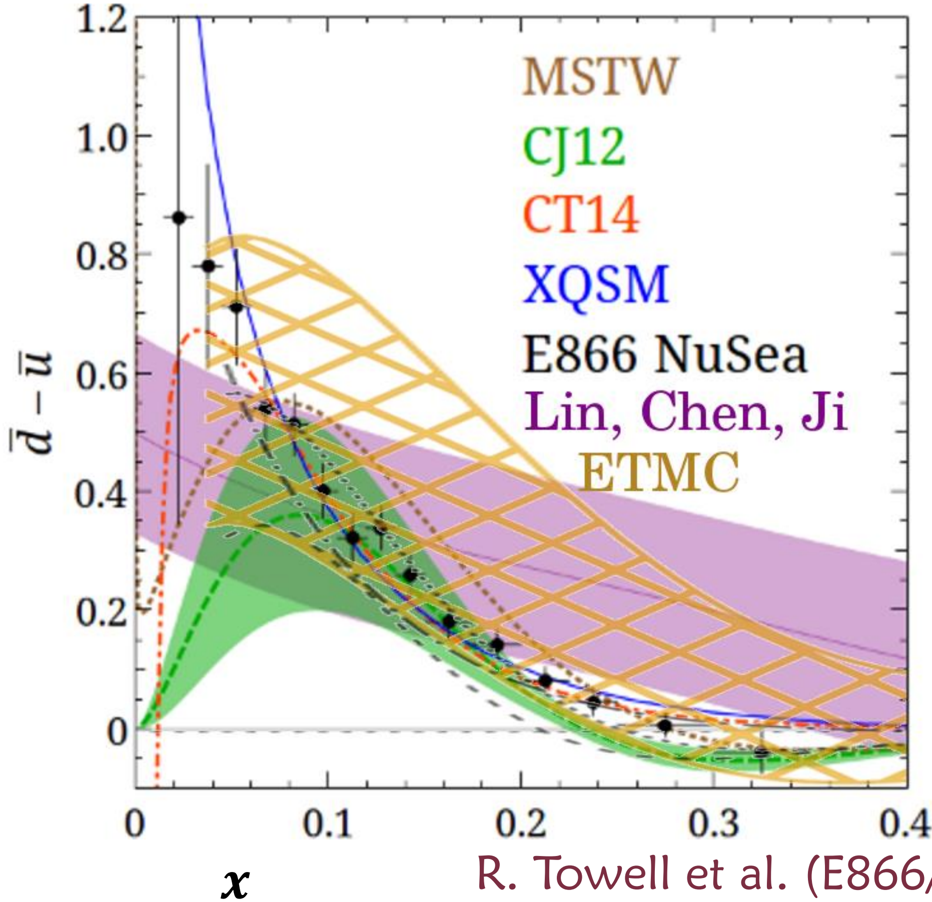
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al. 1402.1462



Compared with E866

Too good to be true?

Lost resolution in small-x region

Similar results repeated by ETMC, at $M_\pi \approx 373$ MeV

ETMC, 1504.07455

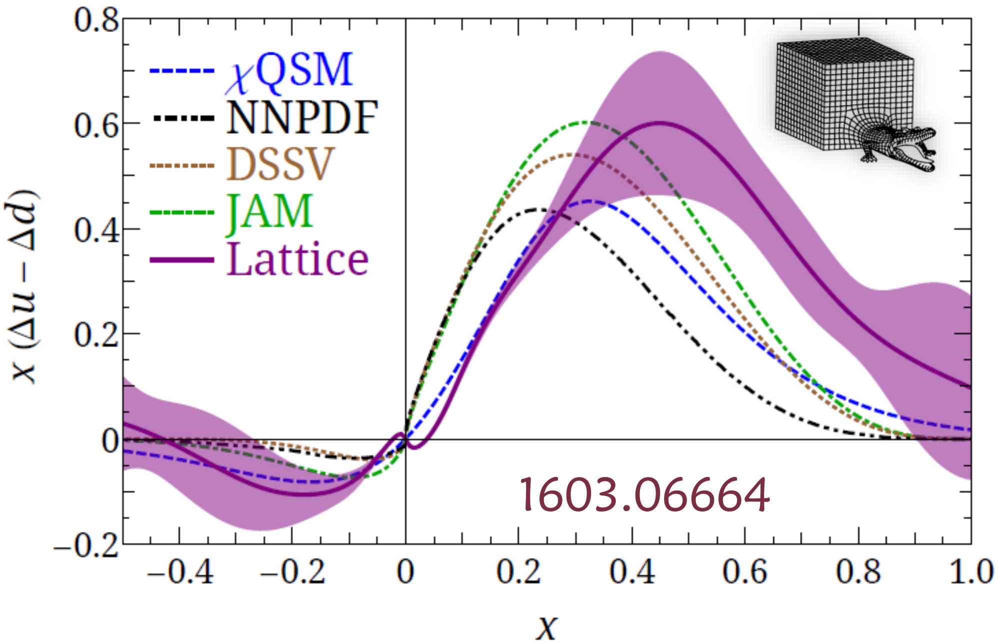
(7)

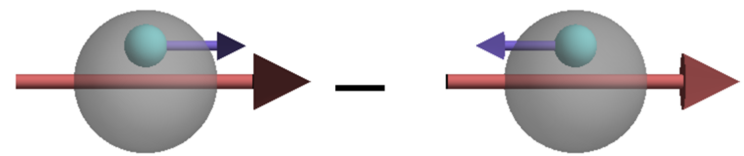
Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Helicity Distribution

§ Exploratory study   ☙ $M_\pi \approx 310$ MeV





Removing
$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$
+ $O(\Lambda_{\mathrm{QCD}}^2/P_z^2)$

☙ We see polarized sea asymmetry $\int dx\,\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.14(9)$

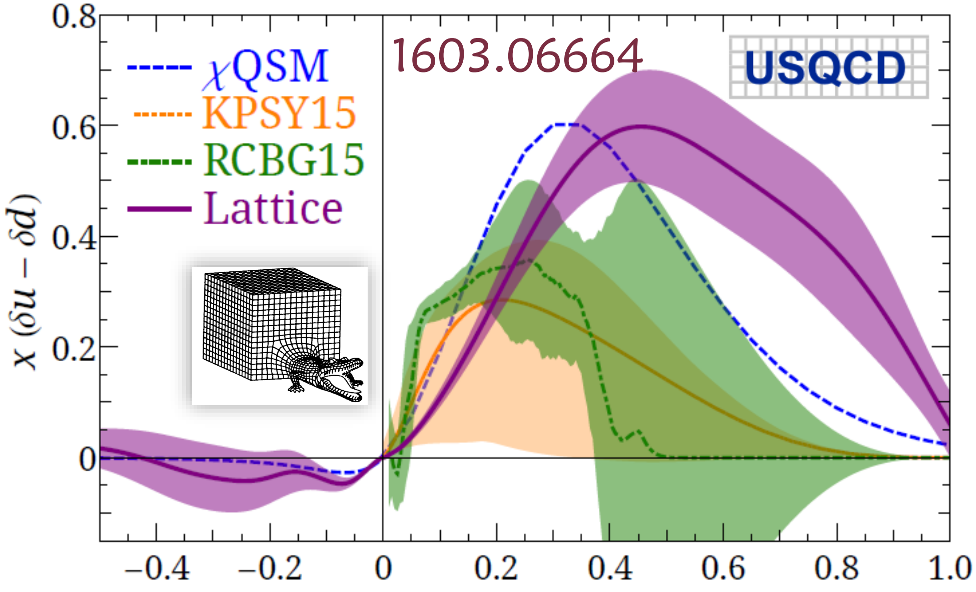
☙ Both STAR and PHENIX at RHIC see $\Delta\bar{u} > \Delta\bar{d}$

1404.6880 and 1504.07451

☙ Other experiments, Fermilab DY exp'ts (E1027/E1039), future EIC

Transversity Distribution

§ Exploratory study ⁂ $M_\pi \approx 310$ MeV



$$\int dx\,(\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.10(8)$$

Removing

$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

+ $O(\Lambda_{\mathrm{QCD}}^2/P_z^2)$

$$\delta\bar{q}(x) = -\delta q(-x)$$

1505.05589; 1503.03495

- We found sea asymmetry of $\int dx\,(\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.10(8)$
- Chiral quark-soliton model $\int dx\,(\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.082$

P. Schweitzer et al., PRD 64, 034013 (2001)

- SoLID at JLab, Drell-Yan exp't at FNAL (E1027+E1039), EIC, ...

Back to Meson PDA

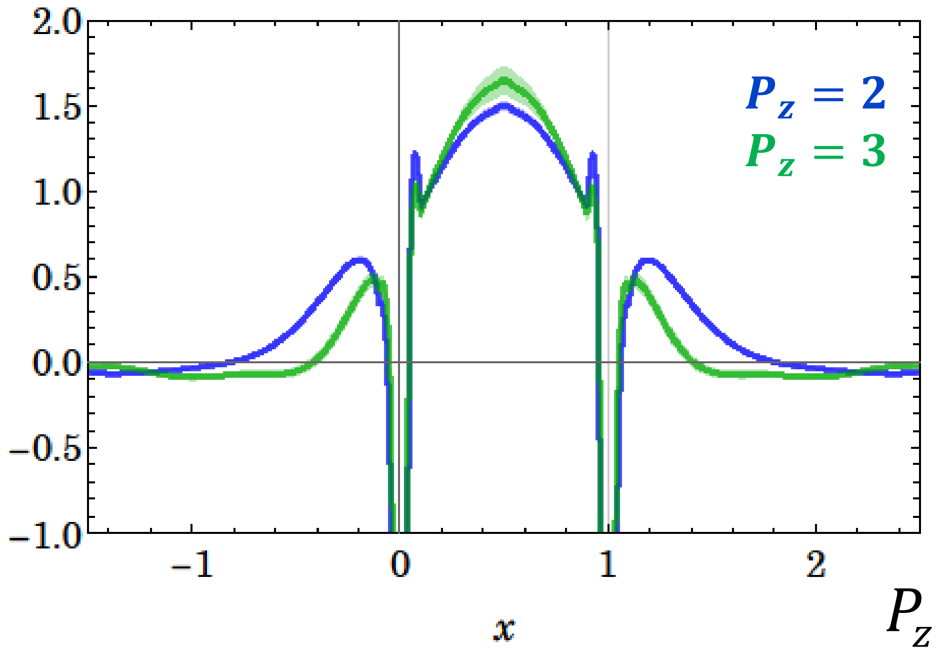
Pion Distribution Amplitude

§ First calculate the quasi-distribution

- Momentum-boosted dependence

$$\tilde{\phi}(x,P_z)=\frac{i}{f_\pi}\int\frac{dz}{2\pi}e^{-i(x-1)P_z z}\langle\pi(P)|\bar{\psi}(0)\gamma_z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

$$\tilde{\phi}(x,\Lambda,P_z)=\int_0^1 dy\, Z_\phi(x,y,\Lambda,\mu,P_z)\phi(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2},\frac{m_\pi^2}{P_z^2}\right)$$



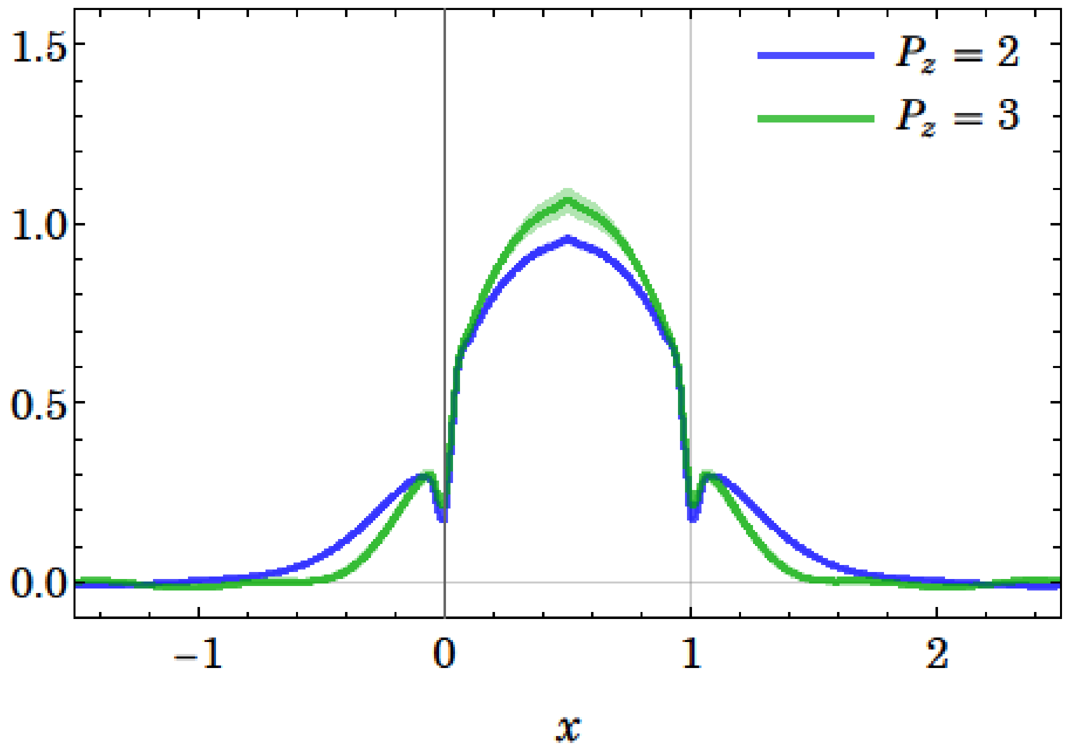
$P_z \in \{0.86, 1.29\}$ GeV

Pion Distribution Amplitude

§ Renormalize the Wilson line

- δm taken from static-quark potential on the same lattice

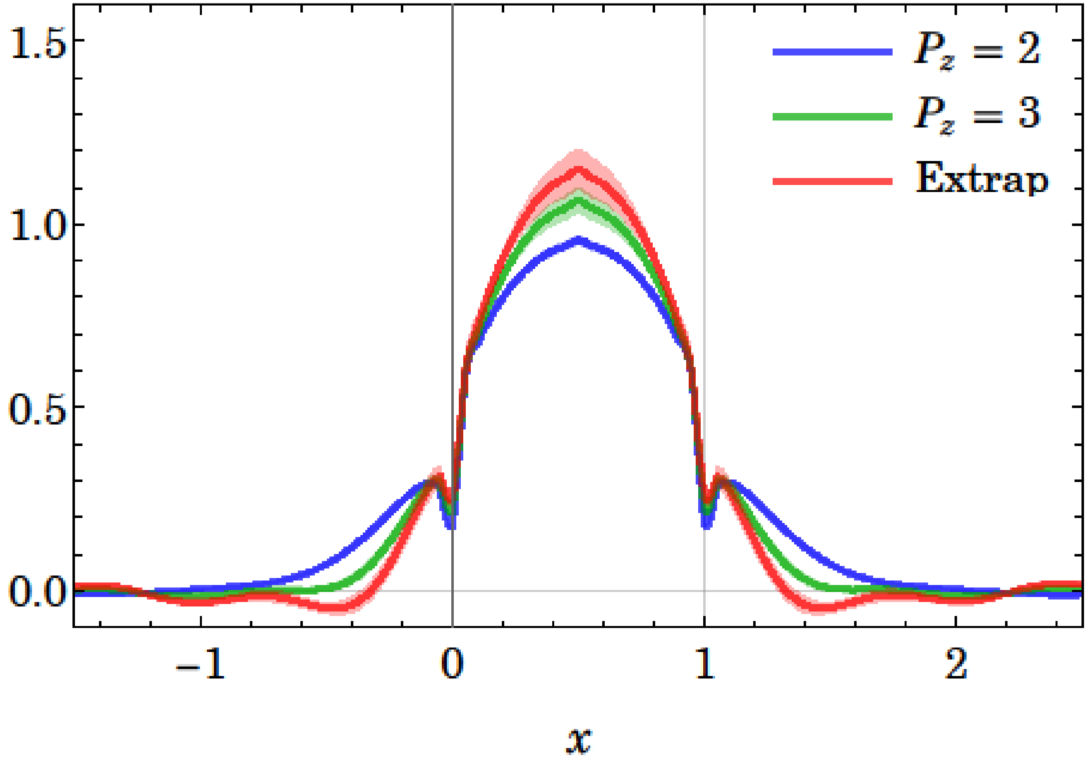
$$\tilde{\phi}_{\text{imp}}(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|} \langle \pi(P) | \bar{\psi}(0) \gamma_z \gamma_5 \Gamma(0, z) \psi(z) | 0 \rangle$$



Pion Distribution Amplitude

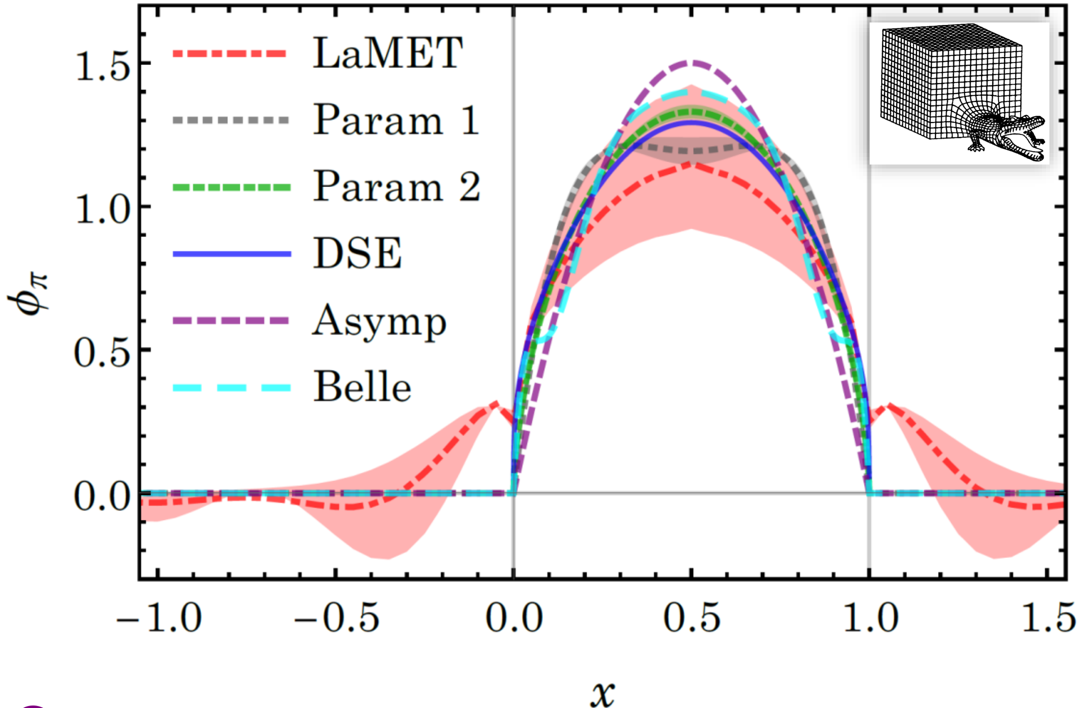
§ Removing the linear divergence in Wilsonline

- δm taken from static-quark potential on the same lattice
- Remove higher-twist contribution at $O(\Lambda_{\mathrm{QCD}}^2/P_z)$ using $\alpha(x)+\beta(x)/P_z^2$



Pion Distribution Amplitude

§ Comparison with other PDA results



$$\delta m = (0.38 \pm 0.28)\delta m_{1-\mathrm{loop}}$$

LP3 Collaboration, 1702.00008

L. Chang et al, Phys. Rev. Lett. 110, 132001 (2013)

S. S. Agaev et al, Phys. Rev. D86, 077504 (2012)

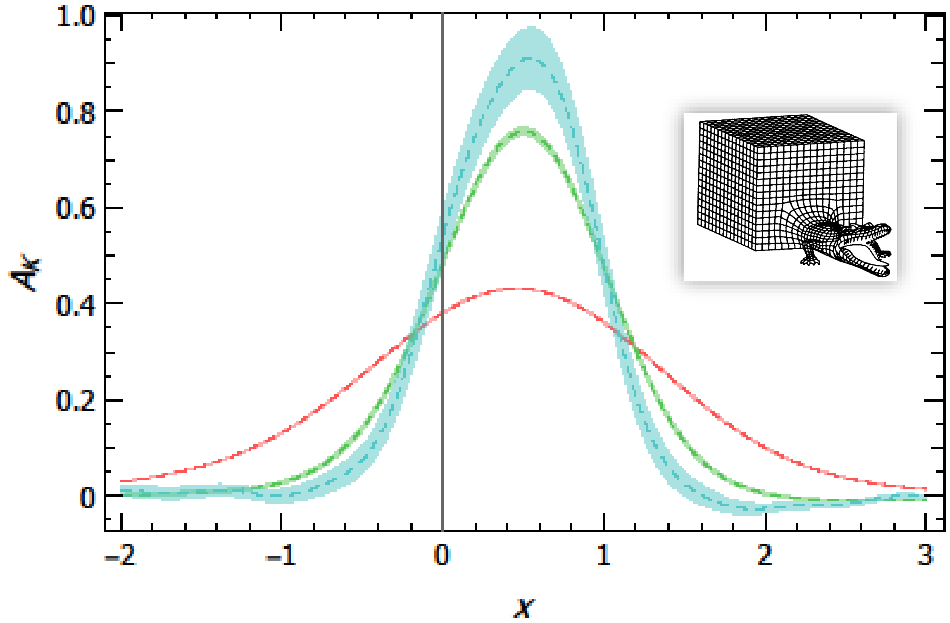
§ Caveats

- Not physical pion mass ($M_\pi \approx 310$ MeV)
- Nonperturbative renormalization needed
- Indicates need to check for higher-momentum convergence

Kaon Distribution Amplitude

§ Very preliminary

- Only preliminary quasi-distribution
- Not physical pion mass ($M_\pi \approx 310$ MeV, $M_K \approx 520$ MeV)



- Lattice momenta discretized by finite size of volume

$P_z \in \{0.43, 0.86, 1.29\}$ GeV

Theoretical Challenges

There are 2 key issues that need to be addressed

§ Large-momentum issues

- Progress is being made:
  - RQCD, *Novel quark smearing for hadrons with high momenta in lattice QCD*, Phys. Rev. D 93, 094515 (2016)
- Systematics due to $(pa)^n$
  - Excited states get worse with larger $p \Rightarrow$ more t_{sep} + finer a

§ Renormalization Issues

- Currently assume the renormalization is multiplicative

$$q_{\text{norm}}(x,\mu,P_z) = \frac{q(x,\mu,P_z)}{\int dx\, q(x,\mu,P_z)} \times g_V^{\overline{\text{MS}}}(2\ \text{GeV})$$

- Progress is on its way:

  LP3 and ETMC

Challenge = Opportunity

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

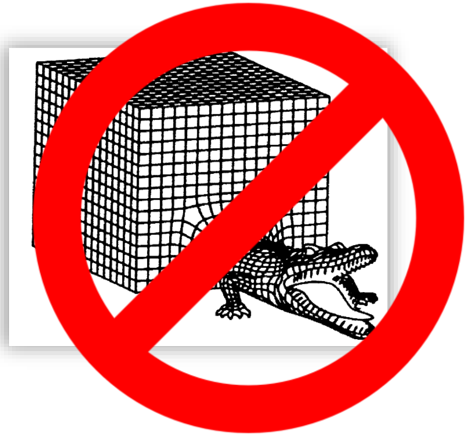
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary & Outlook

Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full x-distribution
- Most importantly, this can be done with today's computer
- First lattice approach to study sea asymmetry
- First look into the meson PDA



§ Systematic control
- Working on renormalization,
  statistics (all-mode averaging?),
  larger momentum boost,
  finer lattice-spacing ensembles, …

§ Closer collaboration with our heavy-quark colleagues
- Certain similar issues: large-q form factors, …

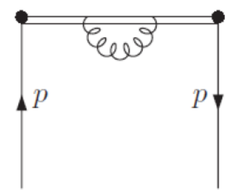
§ Opens doors to much future lattice-QCD structure work
- Many first calculations waiting to be done!

Backup Slides

Power Divergence

Renormalization of power divergence

- Power divergence comes from Wilson line self energy [Ishikawa et al. 16′, Chen et al. 16′]
  - At one-loop, a linear div. is associated with



  - It is well-known that linear divergence associated with Wilson line can be removed by a mass renormalization (e.g. in auxiliary z-field formalism)
  - In a sense, the auxiliary field can be understood as a Wilson line extending between $[z, \infty]$

$$Z(z) = L(z, \infty) \text{ satisfies } \left[\partial_z - igA_z(z)\right] Z(z) = 0$$

  - Analogous to a heavy quark field
  - Non-local Wilson line can be interpreted as a two-point function of z-field

$$L(z,0) = Z(z)Z^\dagger(0)$$

  - Renormalizes analogously to a heavy quark two-point function [Dotsenko and Vergeles 80′, Dorn 86′]

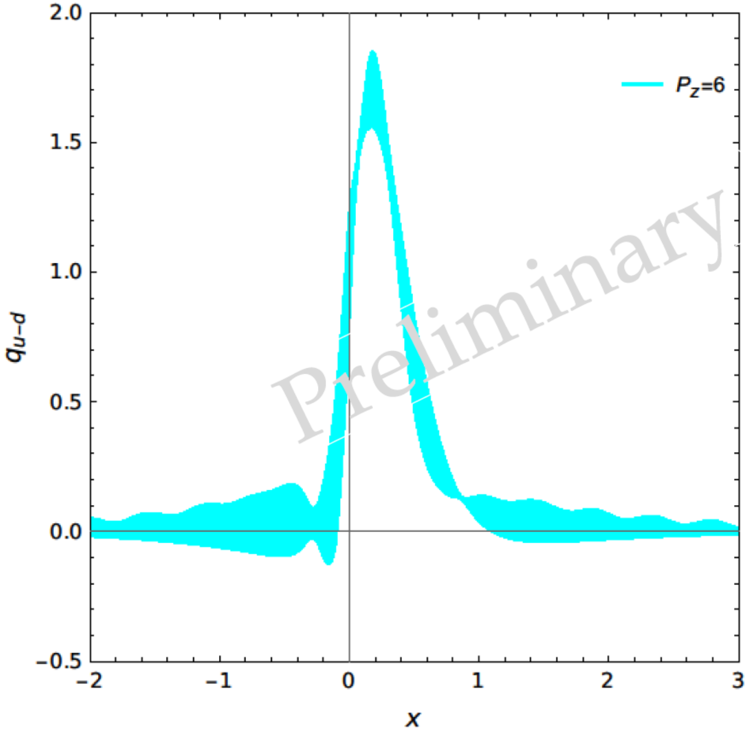
$$L^{\text{ren}}(z,0) = \mathcal{Z}_Z^{-1} e^{-\delta m|z|} L(z,0)$$

Power Divergence

Improved quasi quark distribution

$$\tilde{q}_{\rm imp}(x,\Lambda,p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0,0_\perp,z)\gamma^z L(z,0)\psi(0)|p\rangle$$

- Wilson line renormalization removes power div.



near physical pion mass, $L \approx 6fm, a = 0.09fm$