

Transverse Meson Structure from Exclusive Measurements



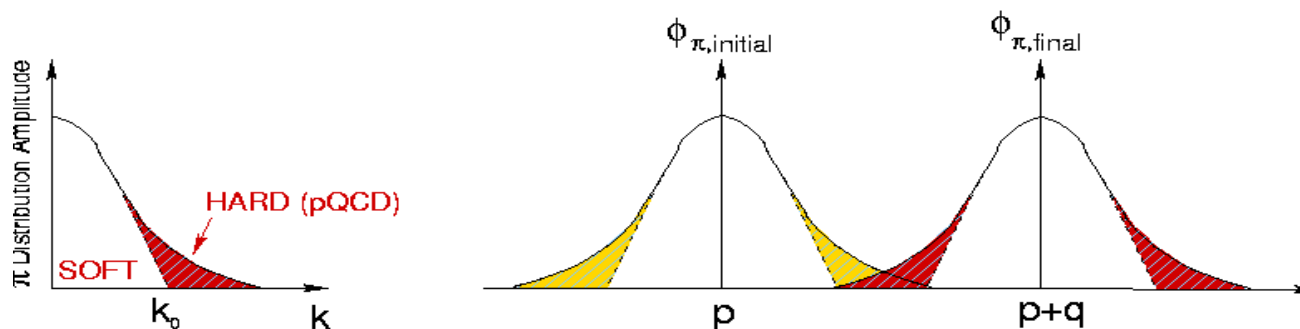
Meson Form Factors



Simple $q\bar{q}$ valence structure of mesons presents the ideal testing ground for our understanding of bound quark systems.

In quantum field theory, the form factor is the overlap integral:

$$F_{\pi}(Q^2) = \int \phi_{\pi}^*(p) \phi_{\pi}(p+q) dp$$



The meson wave function can be separated into φ_{π}^{soft} with only low momentum contributions ($k < k_0$) and a hard tail φ_{π}^{hard} .

While φ_{π}^{hard} can be treated in pQCD, φ_{π}^{soft} cannot.

From a theoretical standpoint, the study of the Q^2 -dependence of the form factor focuses on finding a description for the hard and soft contributions of the meson wave-function.

pQCD and the Charged Pion Form Factor



At large Q^2 , perturbative QCD (pQCD) can be used

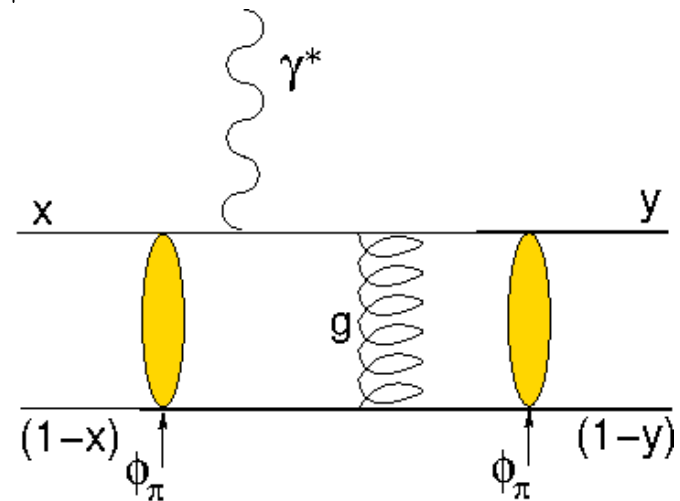
$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log \left(\frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \right|^2 \left[1 + O \left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

at asymptotically high Q^2 , only the hardest portion of the wave function remains

$$\phi_\pi(x) \xrightarrow{Q^2 \rightarrow \infty} \frac{3 f_\pi}{\sqrt{n_c}} x(1-x)$$

and F_π takes the very simple form

$$F_\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{16\pi \alpha_s(Q^2) f_\pi^2}{Q^2}$$



where $f_\pi = 92.4$ MeV is the $\pi^+ \rightarrow \mu^+ \nu$ decay constant.

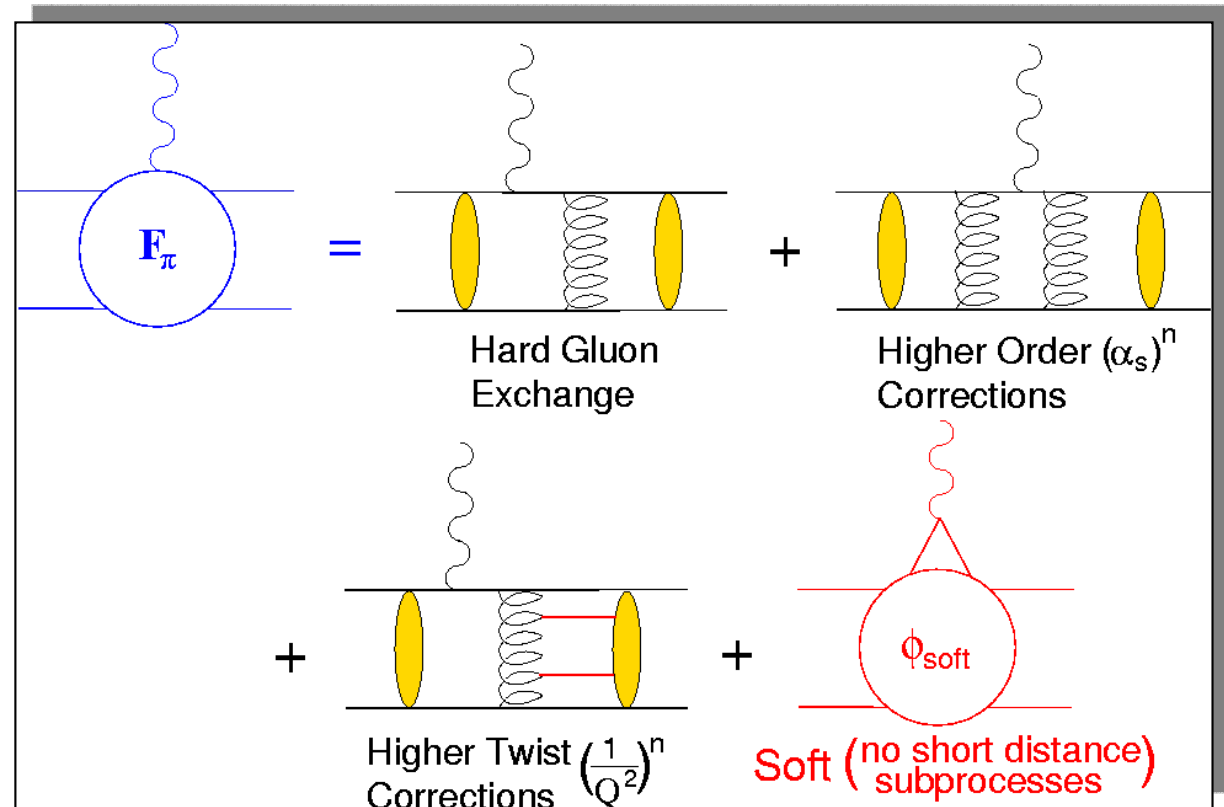
G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

Pion Form Factor at Finite Q^2



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- At finite momentum transfer, higher order terms contribute.
- Calculation of higher order, “hard” (short distance) processes difficult, but tractable.



$Q^2 F_\pi$ should behave like $\alpha_s(Q^2)$ even for moderately large Q^2 .

→ Pion form factor seems to be best tool for experimental study of nature of the quark-gluon coupling constant renormalization.

[A.V. Radyushkin, JINR 1977, arXiv:hep-ph/0410276]

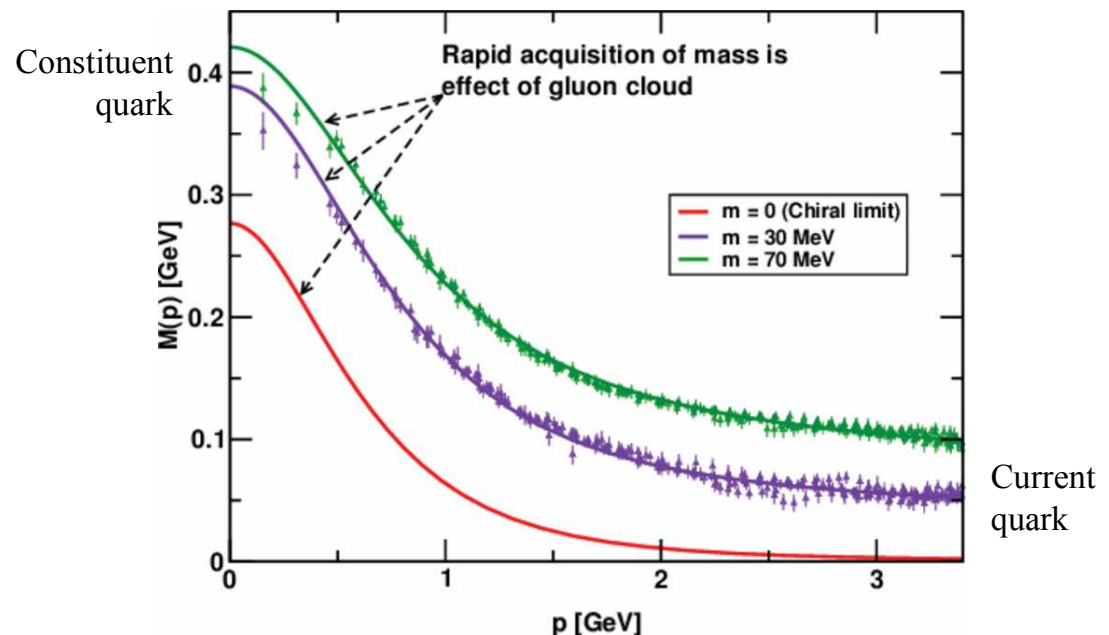
Recent Theoretical Advances



Amazing progress in the last few years.

- We now have a much better understanding how **Dynamical Chiral Symmetry Breaking (DCSB)** generates hadron mass.
- Quenched lattice QCD data on the dressed quark wave function were analyzed in a Bethe-Salpeter Equation framework by Bhagwat, et al.
- For the first time, the evolution of the current quark of pQCD into constituent quark was observed as its momentum becomes smaller.

- The constituent-quark mass arises from a cloud of low-momentum gluons attaching themselves to the current quark.
- **This is DCSB:** an essentially non-perturbative effect that generates a quark *mass from nothing*: namely, it occurs even in the chiral limit.



M.S. Bhagwat, et al., PRC **68** (2003) 015203.
L. Chang, et al., Chin.J.Phys. **49** (2011) 955.

Implications for Pion Structure



Craig Roberts (2016): *“No understanding of confinement within the Standard Model is practically relevant unless it also explains the connection between confinement and DCSB, and therefore the existence and role of pions.”*

- For the pQCD derivation on slide #3, the normalization for F_π has been based on the conformal limit of the pion’s twist-2 PDA.

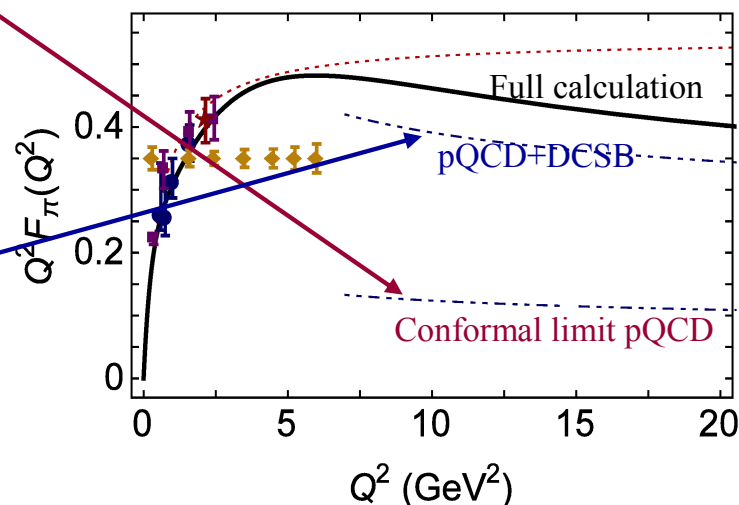
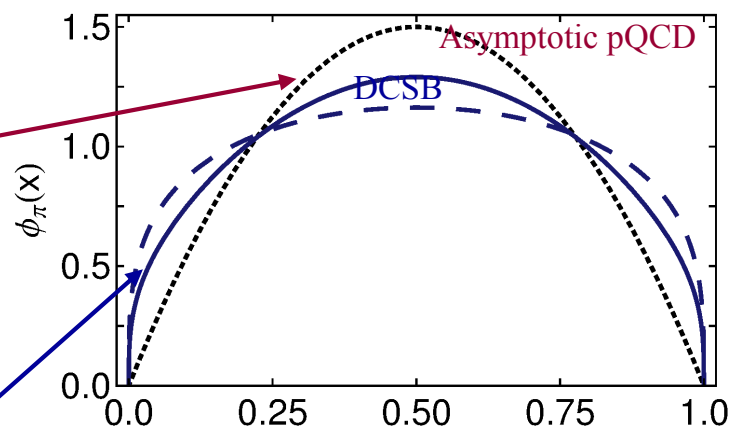
$$\phi_\pi^{cl}(x) = 6x(1-x)$$

- This leads to “too small” F_π values in comparison with present & projected JLab data.

- Recent works incorporating DCSB effects indicate that at experimentally accessible energy scales the actual pion PDA is broader, concave function, close to

$$\phi_\pi(x) = (8/\pi)\sqrt{x(1-x)}$$

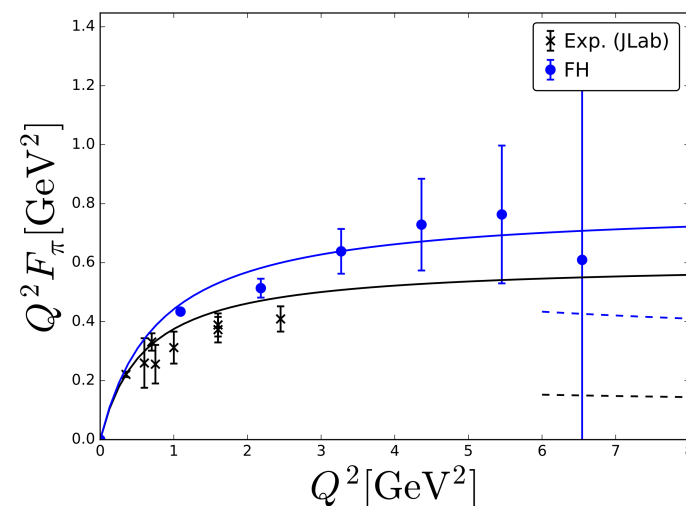
- Simply inputting this $\phi_\pi(x)$ into the pQCD expression for F_π brings the calculation much closer to the data.
- Underestimates full computation by ~15% for $Q^2 \geq 8 \text{ GeV}^2$. Addresses issue raised in 1977.



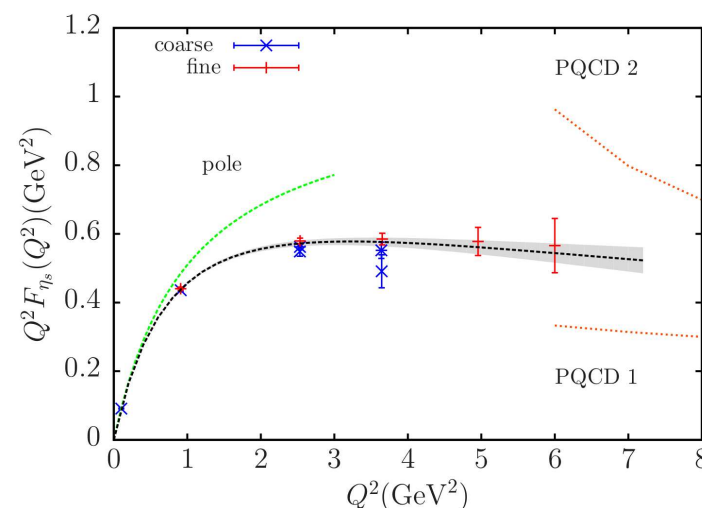
New Lattice QCD at Higher Q^2



- Lattice QCD calculations traditionally have difficulty predicting hadron structure at high-momentum transfer.
- Form factors drop rapidly with Q^2 , so one is attempting to extract a much weaker signal from datasets with finite statistics.
- QCDSF/UKQCD/CSSM Collab. address with new technique relating matrix elements to energy shifts.
- Simulate single set of u, d, s gauge configurations corresponding to $m_\pi \approx 470$ MeV.
- Confident future LQCD will provide insight into transition of perturbative to non-perturbative QCD.
- HPQCD Collab. study pseudoscalar η_s meson made of valence s quarks accurately tuned on full QCD ensembles of gluon field configurations.
- Qualitatively similar to pion since $m_s < \Lambda_{QCD}$, but numerically much faster.
- F_π result flat for $2 < Q^2 < 6$ GeV², far above asymptotic QCD value.
- Confident of future LQCD calcs. at higher Q^2 .

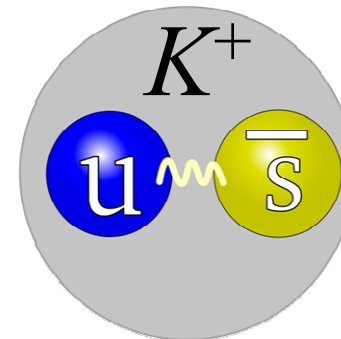
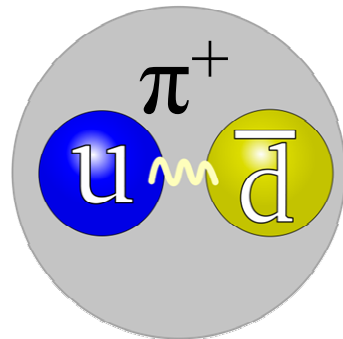


Chambers et al., arXiv: 1702.01513



Koponen et al., arXiv: 1701.04250

The Charged Kaon – a second QCD test case



- The properties of the K^+ are also strongly influenced by DCSB.
 - K^+ PDA also is broad, concave and asymmetric.
 - While the heavier s quark carries more bound state momentum than the u quark, the shift is markedly less than one might naively expect based on the difference of u, s current quark masses.
[C. Shi, et al., PRD 92 (2015) 014035].

- In the hard scattering limit, pQCD predicts that the π^+ and K^+ form factors will behave similarly:

$$\frac{F_K(Q^2)}{F_\pi(Q^2)} \xrightarrow{Q^2 \rightarrow \infty} \frac{f_K^2}{f_\pi^2}$$

- It is important to compare the magnitudes and Q^2 -dependences of both form factors.

Measurement of π^+ Form Factor – Low Q^2



At low Q^2 , F_π can be measured model-independently via high energy elastic π^- scattering from atomic electrons in Hydrogen

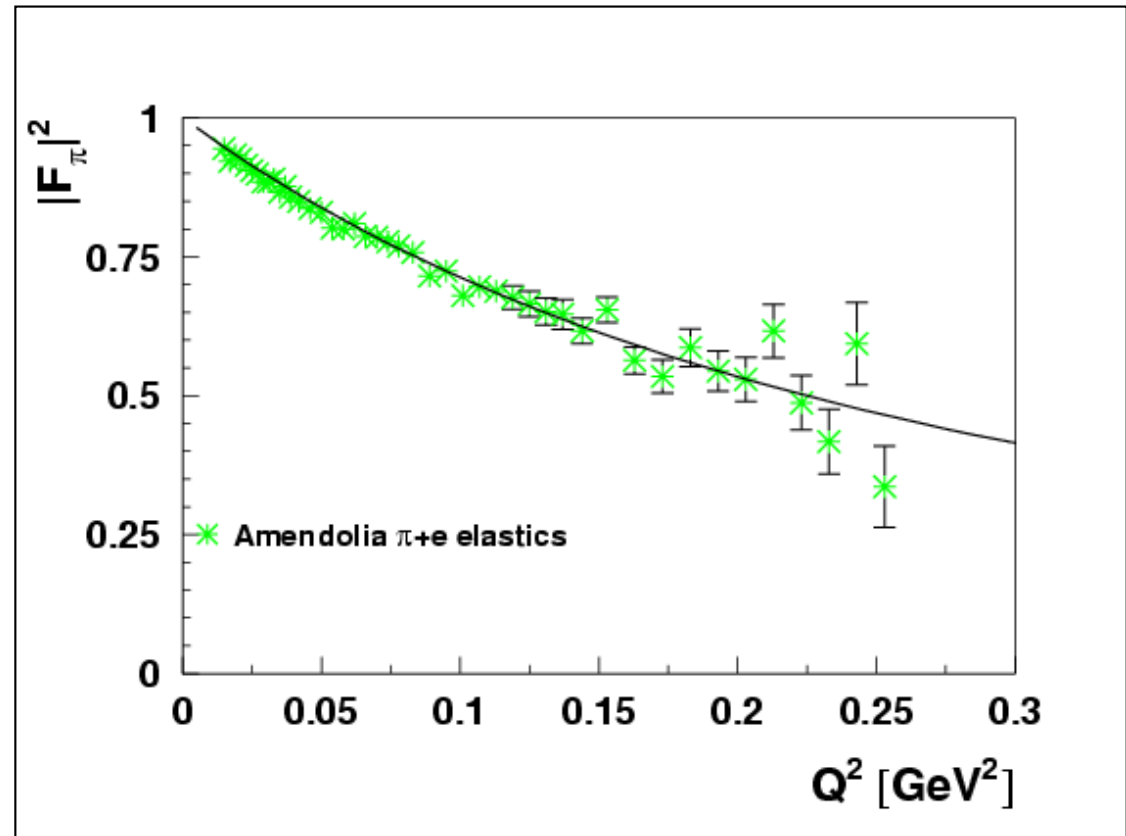
- CERN SPS used 300 GeV pions to measure form factor up to $Q^2 = 0.25 \text{ GeV}^2$ [*Amendolia, et al, NP B277(1986)168*]

- Data used to extract pion charge radius

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

Maximum accessible Q^2 roughly proportional to pion beam energy

$Q^2=1 \text{ GeV}^2$ requires 1 TeV pion beam



Measurement of π^+ Form Factor – Larger Q^2



At larger Q^2 , F_π must be measured indirectly using the “pion cloud” of the proton via pion electroproduction $p(e, e'\pi^+)n$

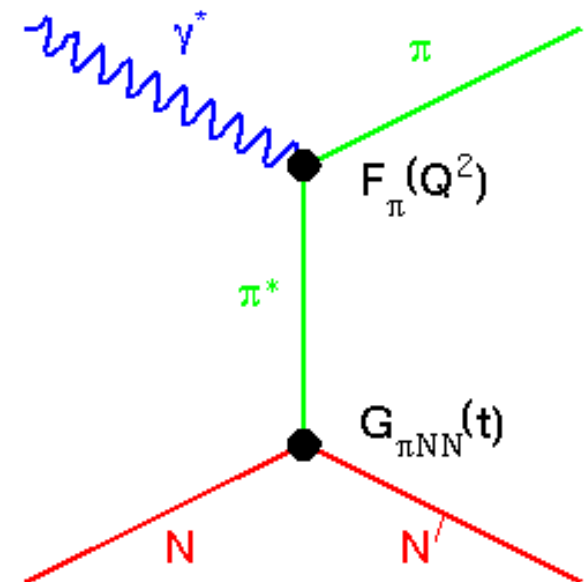
$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + \dots$$

- At small $-t$, the pion pole process dominates the longitudinal cross section, σ_L
- In Born term model, F_π^2 appears as,

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

Drawbacks of this technique

1. Isolating σ_L experimentally challenging
2. Theoretical uncertainty in form factor extraction.



Measurement of K^+ Form Factor



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- Similar to π^+ form factor, elastic K^+ scattering from electrons used to measure charged kaon form factor at low Q^2

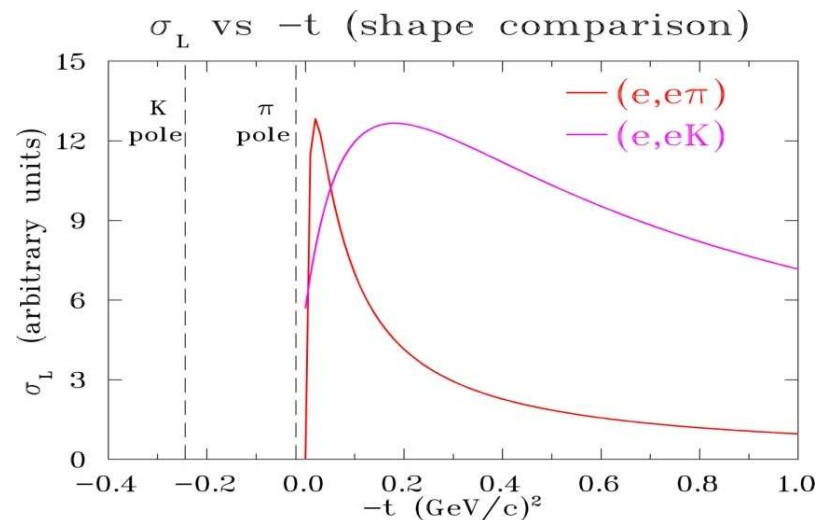
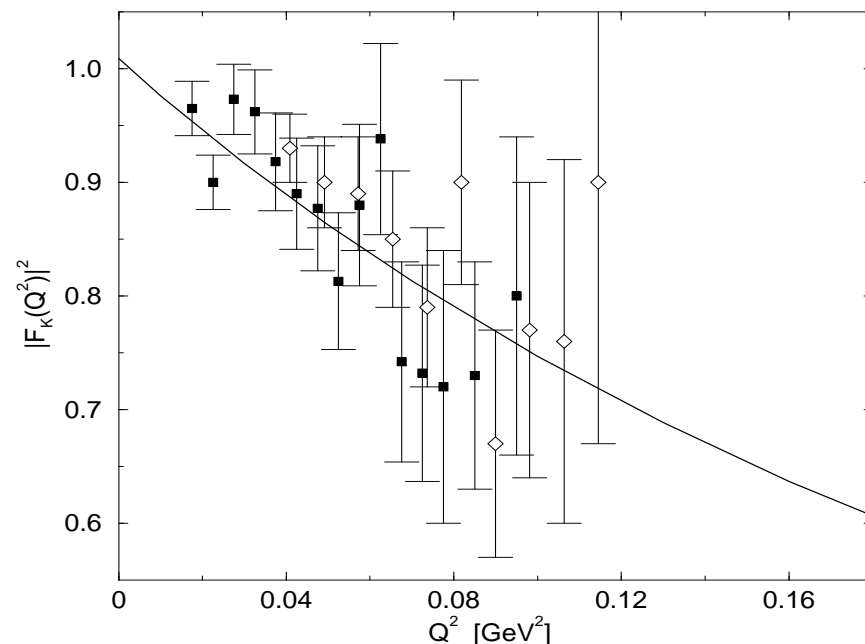
[Amendolia, et al, PL **B178**(1986)435]

- Can “kaon cloud” of the proton be used in the same way as the pion to extract kaon form factor via $p(e, e'K^+)\Lambda$?

- Kaon pole further from kinematically allowed region.

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_K^2)} g_{K\Lambda N}^2(t) F_K^2(Q^2, t)$$

- Many of these issues will be explored in JLab E12-09-11.

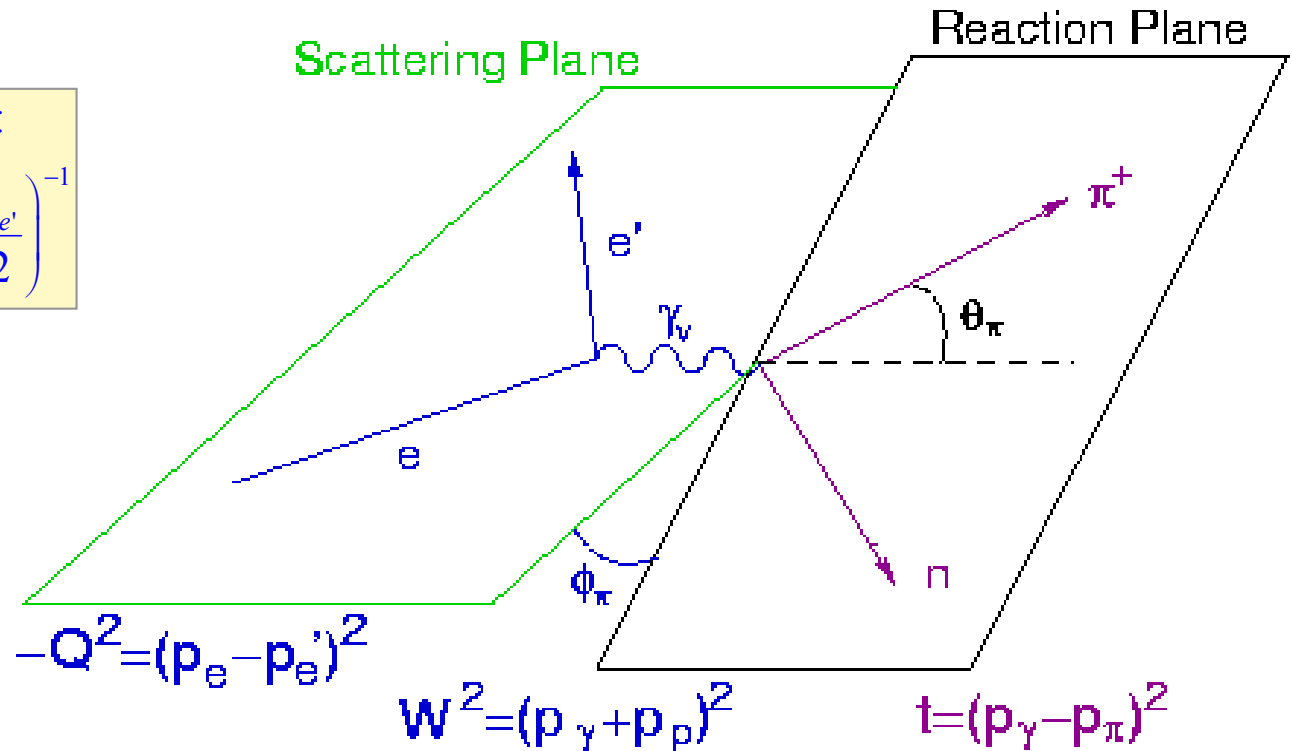




$$2\pi \frac{d^2\sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

Virtual-photon polarization:

$$\varepsilon = \left(1 + 2 \frac{(E_e - E_{e'})^2 + Q^2 \tan^2 \frac{\theta_{e'}}{2}}{Q^2} \right)^{-1}$$



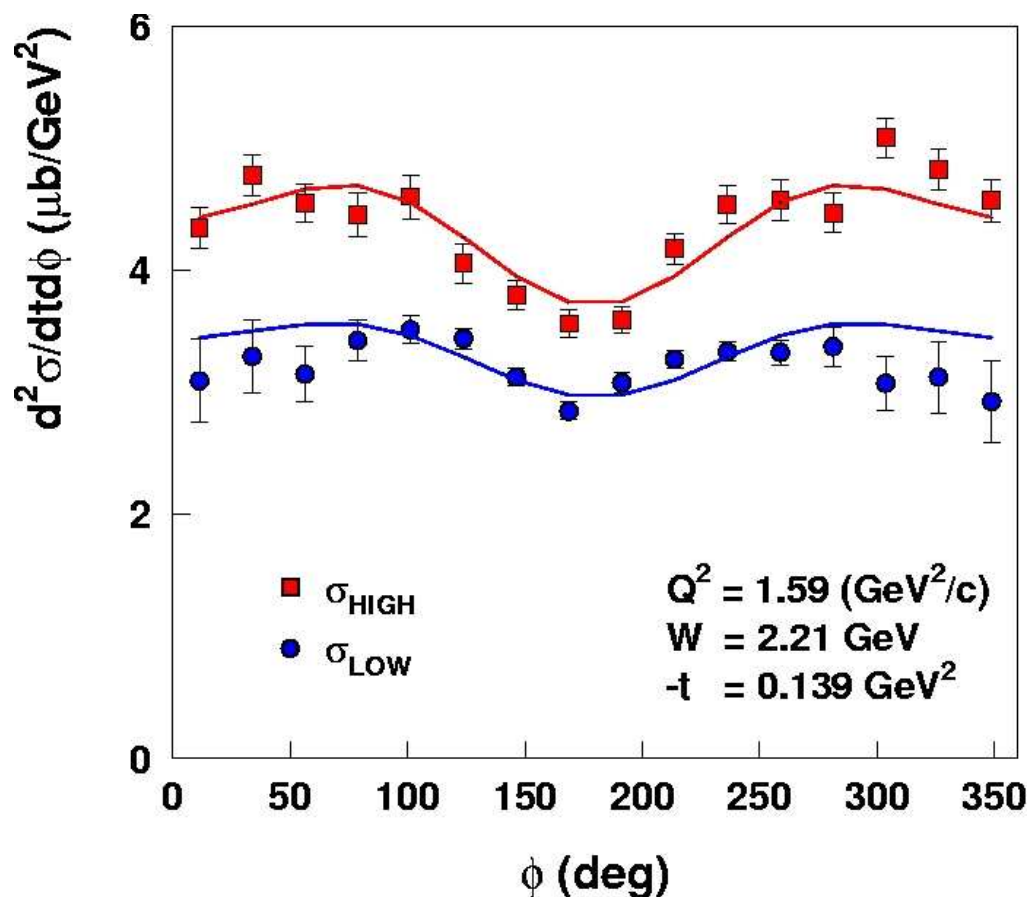
- L-T separation required to separate σ_L from σ_T .
- Need to take data at smallest available $-t$, so σ_L has maximum contribution from the π^+ pole.

Measuring $d\sigma_L/dt$ at JLab



$$2\pi \frac{d^2\sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

- Rosenbluth separation required to isolate σ_L
 - Measure cross section at fixed (W, Q^2, t) at 2 beam energies
 - Simultaneous fit at 2 ε values to determine σ_L , σ_T , and interference terms
- Control of point to point systematic uncertainties crucial due to $1/\Delta\varepsilon$ error amplification in σ_L
- Careful attention must be paid to spectrometer acceptance, kinematics, efficiencies, ...



Horn, et al, PRL 97(2006)192001

Chew–Low Method to determine Pion Form Factor

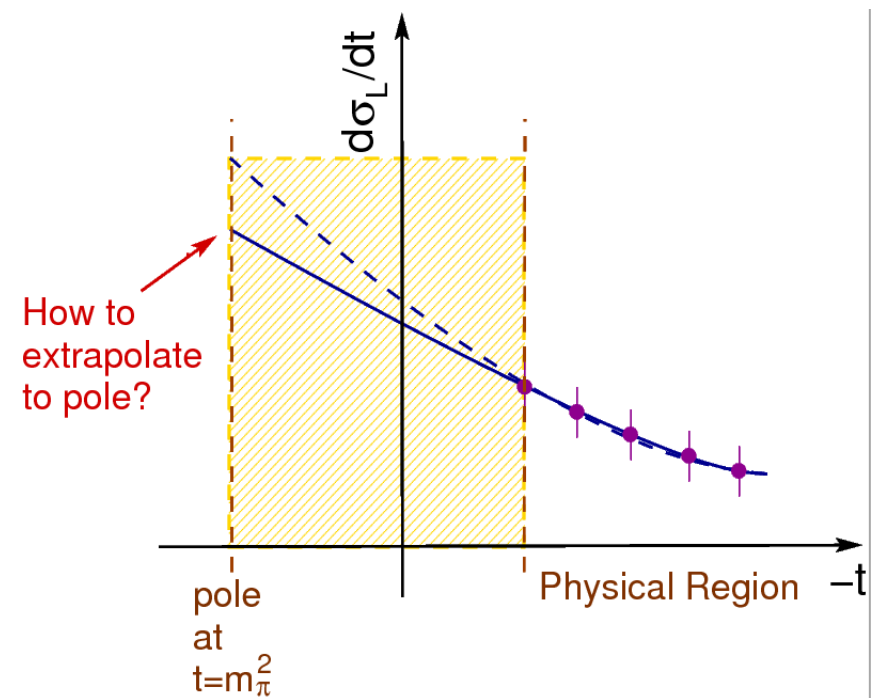


$p(e, e'\pi^+)n$ data are obtained some distance from the $t=m_\pi^2$ pole.

→ “Chew Low” extrapolation method requires knowing the analytic dependence of $d\sigma_L/dt$ through the unphysical region.

Extrapolation method last used in 1972 by Devenish & Lyth

- Very large systematic uncertainties.
 - Failed to produce reliable result.
- Different polynomial fits equally likely in physical region gave divergent form factor values when extrapolated to $t=m_\pi^2$.



The Chew-Low Method was subsequently abandoned.

Chew–Low Method Check with PseudoData



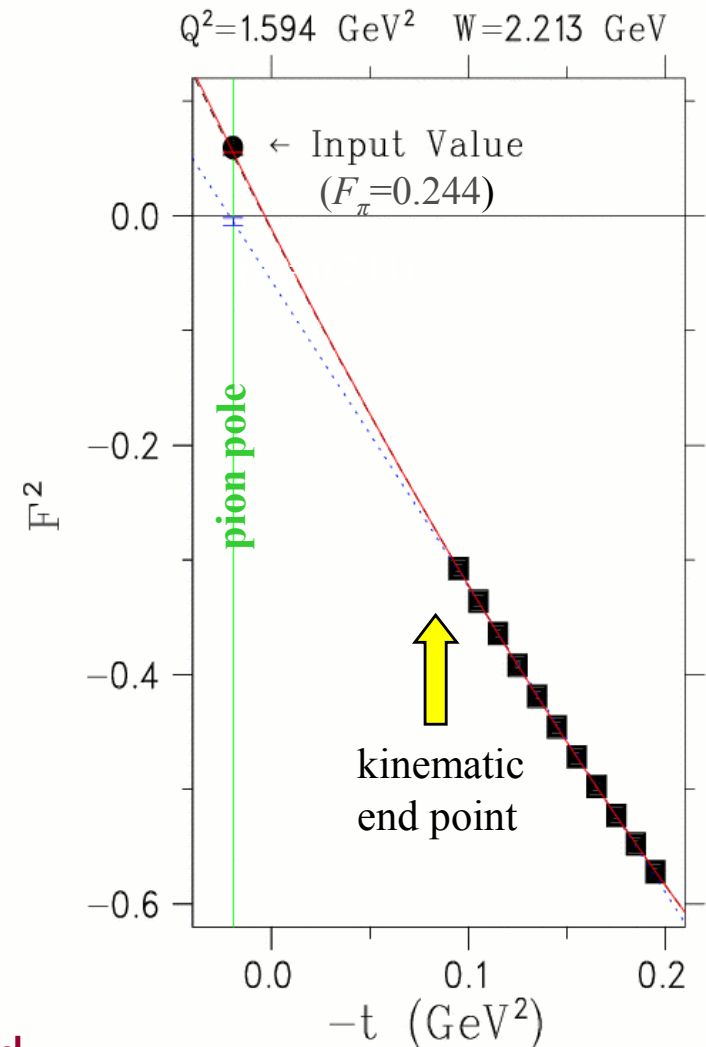
Plot $F^2 = \frac{N}{4\hbar c(eg_{\pi NN})^2} \frac{(t - m_\pi^2)^2}{-Q^2 m_\pi^2} \frac{d\sigma_L}{dt}$ vs. $-t$

- Pure pole cross section gives straight line through origin, with value $F_\pi^2(Q^2)$ at pole.
- Other contributions introduce non-linearities since don't contain $(t - m_\pi)^2$ factor, but don't influence F^2 value at pole.
 - Do not know if behavior of F^2 with $-t$ is linear, quadratic, or higher order.

All fits missed the input F_π .

- no consistent trend on order of polynomial best able to reproduce input value (6-15% deviation, $Q^2=0.6-2.45 \text{ GeV}^2$).

- Experimental σ_L data have only 4-6 t -bins and statistical and systematic uncertainties of 5-10%.
 - Extrapolation with real data will be even more uncertain.



For details see: G.M. Huber et al., PRC 78(2008)045203.



Only reliable approach is to use a model incorporating the π^+ production mechanism and the 'spectator' nucleon to extract F_π from σ_L .

- JLab F_π experiments use the Vanderhaeghen-Guidal-Laget (VGL) Regge model as it has proven to give a reliable description of σ_L across a wide kinematic domain.
[Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]
- More models would allow a better understanding of the model dependence of the F_π result. There has been considerable recent interest:
 - *T.K. Choi, K.J. Kong, B.G. Yu, arXiv: 1508.00969.*
 - *T. Vrancx, J. Ryckebusch, PRC 89(2014)025203.*
 - *M.M. Kaskulov, U. Mosel, PRD 81(2010)045202.*
 - *S.V. Goloskokov, P. Kroll, Eur.Phys.J. C65(2010)137.*

Our philosophy remains to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ can be extracted as better models become available.

Extract $F_\pi(Q^2)$ from JLab σ_L data



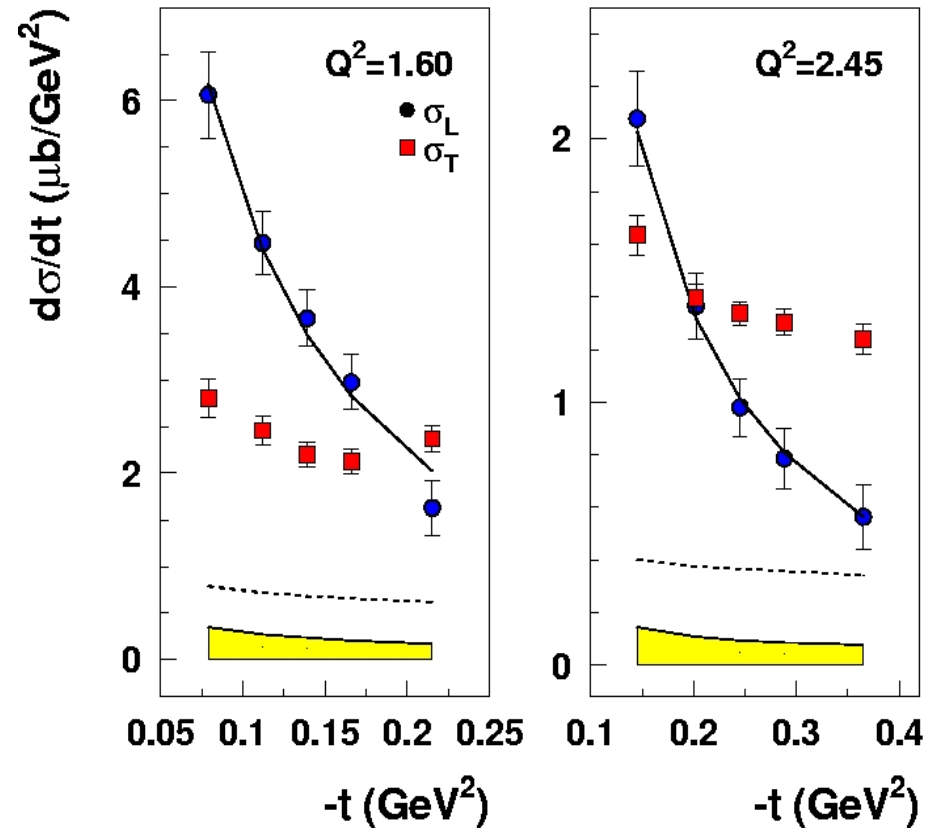
Model incorporates π^+ production mechanism and spectator neutron effects:

VGL Regge Model:

- Feynman propagator $\left(\frac{1}{t - m_\pi^2} \right)$
replaced by π and ρ Regge propagators.
 - Represents the exchange of a series of particles, compared to a single particle.
- Free parameters: $\Lambda_\pi, \Lambda_\rho$ (trajectory cutoff)
[Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]
- At small $-t$, σ_L only sensitive to F_π

$$F_\pi = \frac{1}{1 + Q^2 / \Lambda_\pi^2}$$

Fit to σ_L to model
gives F_π at each Q^2



Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.

Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

$$\Lambda_\pi^2 = 0.513, 0.491 \text{ GeV}^2, \Lambda_\rho^2 = 1.7 \text{ GeV}^2.$$

F π -2 data: T. Horn et al., PRL 97(2006)192001.

JLab Current and Projected Data



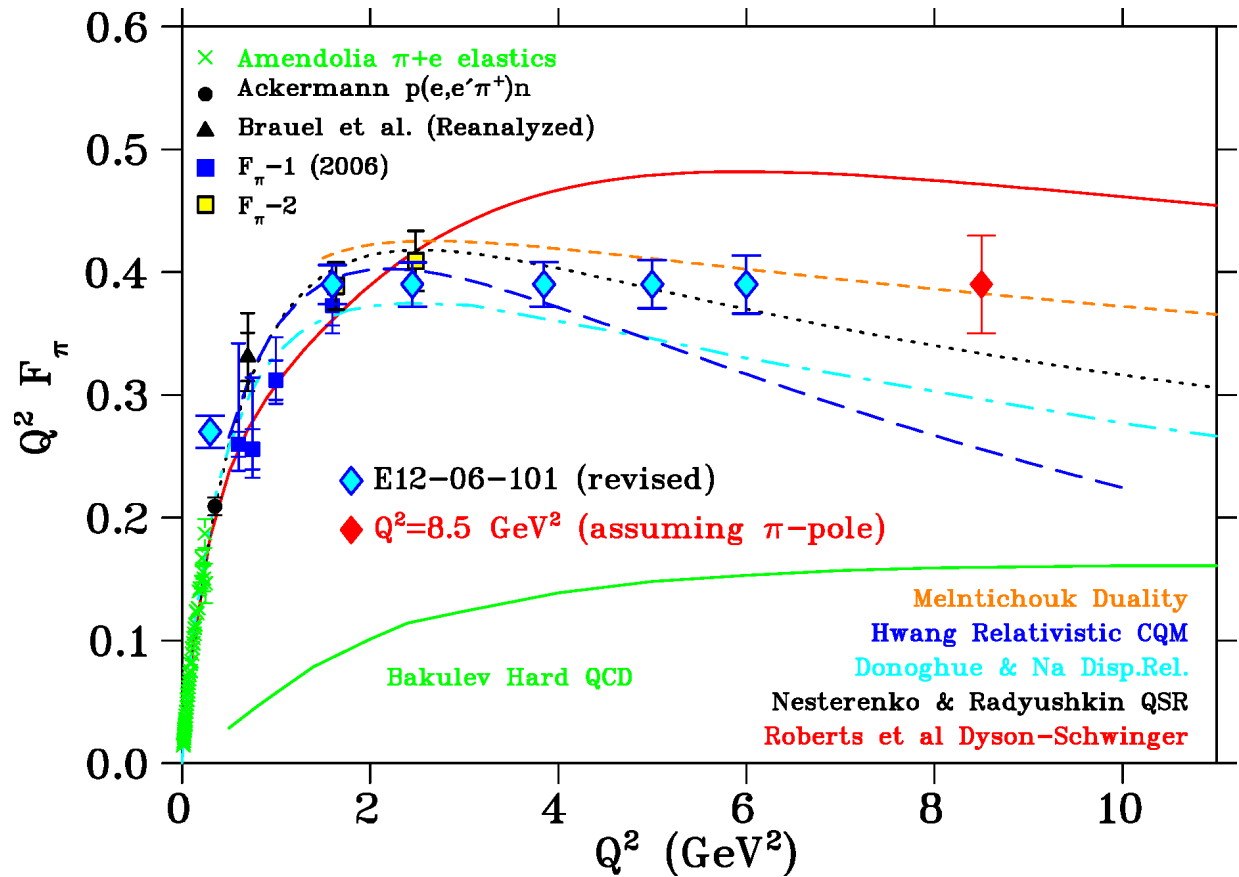
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JLab 12 GeV upgrade will allow measurement of F_π to much higher Q^2 .

No other facility worldwide can perform this measurement.

New overlap points at $Q^2=1.6, 2.45$ will be closer to pole to constrain- t_{min} dependence.

New low Q^2 point will provide best comparison of the electroproduction extraction of F_π vs. elastic $\pi+e$ data.

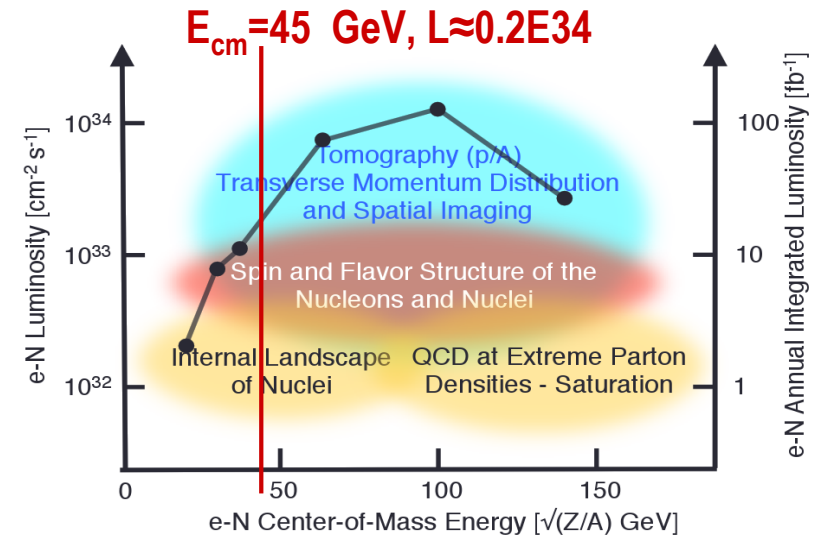


The $\sim 10\%$ measurement of F_π at $Q^2=8.5 \text{ GeV}^2$ is at higher $-t_{min}=0.45 \text{ GeV}^2$. Requires additional measurements (not yet approved) to verify π -pole dominance in σ_L .

EIC Exclusive $p(e, e'\pi^+n)$ Kinematics



- $\epsilon > 0.995$ fairly straightforward.
 - 5 GeV(e^-) x 100 GeV(p), allows access to a wide kinematic range.
- Lab cross sections in $\mu\text{b}/\text{sr}^2/\text{GeV}$.
 - C. Weiss, V. Guzey (2008) extrapolation of soft model cross section to high Q^2 , assuming QCD scaling behavior and $W^2 \gg Q^2$.



Q^2	W	$P_{e'}$	$\theta_{e'}$	P_π	θ_π	P_n	θ_n	$-t$	$d^3\sigma$
10.0	7.0	5.4	35.6	16.9	-10.6	83.7	-0.01	0.032	1.1
15.0	7.0	5.6	43.0	23.2	-9.4	77.2	-0.02	0.066	0.34
20.0	7.5	5.8	49.0	25.7	-9.8	74.4	-0.02	0.085	0.12
25.0	8.5	6.0	54.2	25.0	-11.2	74.9	-0.02	0.081	0.039
30.0	9.0	6.2	58.8	26.1	-11.7	73.6	-0.02	0.090	0.019
35.0	9.5	6.4	62.8	26.8	-12.3	72.7	-0.02	0.098	0.010

High $\epsilon > 0.995$ Detector Requirements



- **Only way to assure exclusivity of the $p(e, e' \pi^+ n)$ reaction is by detecting the recoil neutron.**
 - Neutrons are emitted at small angle ($\theta < 0.05^\circ$), momentum 73-84% of the proton beam. Resolution?
- **Scattered electron (5 GeV e^- x 100 GeV p):**
 - Scattered electron angles of 35° - 63° (wrt incident electron beam).
 - Resolution requirements modest ($\delta P/P \approx 5 \times 10^{-3}$, $\delta \theta \approx 1 \text{ mr}$)
 - Kinematics were chosen to avoid regions where cross sections drop rapidly, needing high resolution for small systematic errors.
- **17-26 GeV/c π^+ detected at forward angle (9.5° - 12.5°)**
 - Will need reliable PID. e.g. ePHENIX concept in White Paper has Aerogel & RICH up to $\sim 40^\circ$.
- **Requirements appear to be compatible with both eRHIC and JLEIC detector conceptual designs.**
 - **The critical issue is identification of the exclusive events.**

5x100 Exclusive $p(e, e'\pi^+n)$ Kinematics

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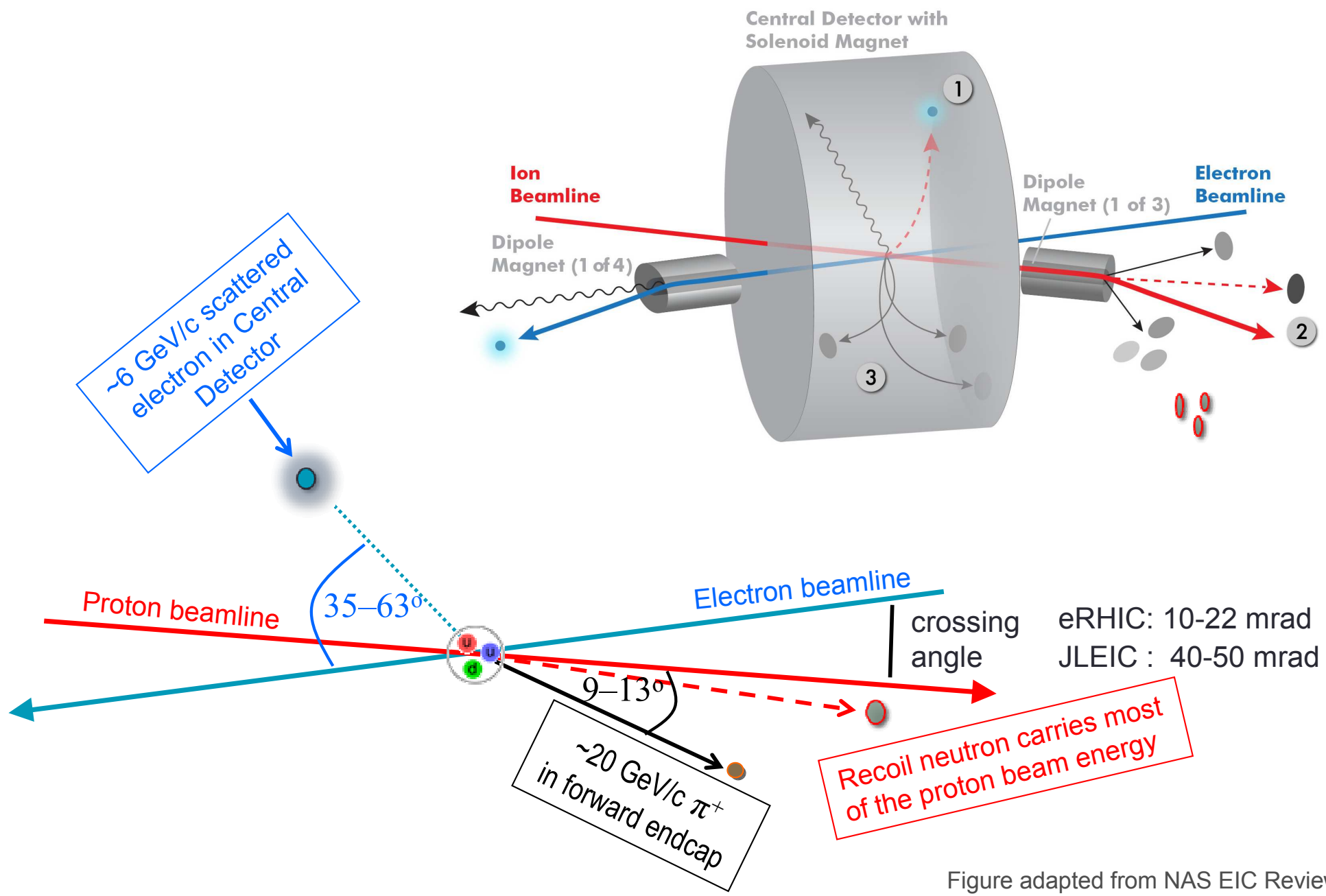


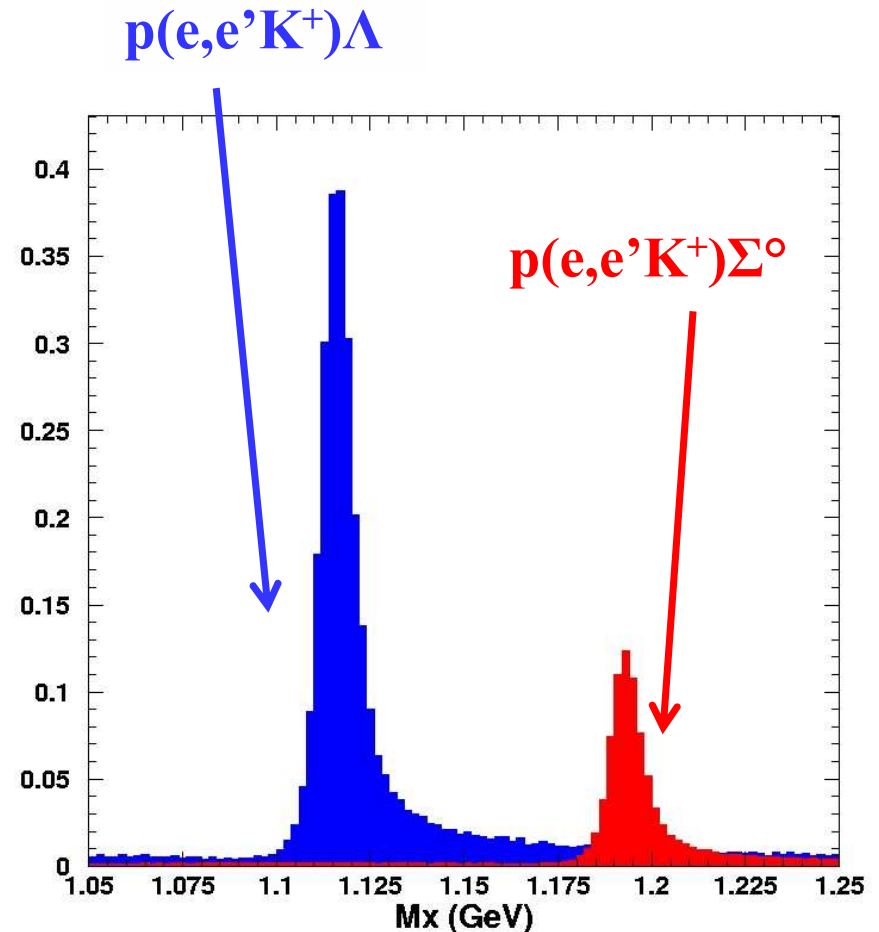
Figure adapted from NAS EIC Review draft by A. Deshpande, March 2017

$p(e, e' K^+ \Lambda)$ Requirements



$$M_X = \sqrt{(E_{\text{det}} - E_{\text{init}})^2 - (p_{\text{det}} - p_{\text{init}})^2}$$

- At EIC CM energies, exclusive π , K cross sections are likely more comparable, statistics likely to be less of an issue than at JLab.
- **Assuring exclusivity poses many challenges.**
 - $\Lambda\Sigma$ final states are closer together in missing mass than n , $n+\pi$.
 - Λ $c\tau=7.89$ cm.
 - Planned vertex detectors cover central rapidity range, while in these kinematics Λ is at very small angle to proton beamline.
- Would need $\Lambda \rightarrow \pi p$ simulation. Requirements similar to EIC K^+ structure function measurement.



JLab Hall C simulation at $Q^2=2.0$ GeV²,
 $W=3.0$ GeV and high ϵ

How to separate σ_L from σ_T in e-p Collider



$$\varepsilon = \frac{2(1-y)}{1+(1-y)^2} \quad \text{where the fractional energy loss } y = \frac{Q^2}{x(s_{tot} - M_N^2)}$$

- Systematic uncertainties in σ_L are magnified by $1/\Delta\varepsilon$.
 - Desire $\Delta\varepsilon > 0.2$.
- **To access $\varepsilon < 0.8$, one needs $y > 0.5$.**
 - This can only be accessed with small s_{tot} ,
i.e. low proton collider energies (5–15 GeV),
where luminosities are too small for a practical measurement.
- **A conventional L-T separation is impractical, need some other way to identify σ_L .**

σ_L via Beam and Target Polarization



Although the technique has not been tested for this reaction, it is in principle possible to extract $R = \sigma_L / \sigma_T$ using polarization degrees of freedom

For parallel kinematics
(outgoing meson along \vec{q})
in proton rest frame

$$R = \frac{\sigma_L}{\sigma_T} = \frac{1}{\varepsilon_L} \left(\frac{1}{\chi_z} - 1 \right)$$

Longitudinal polarization
of virtual photon

$$\varepsilon_L = \left(Q^2 / \omega_{cm}^2 \right) \varepsilon$$

z-component of proton
“reduced” polarization in
exclusive pseudoscalar
meson production

$$\chi_z = \frac{1}{2P_e P_p \sqrt{1 - \varepsilon^2}} A_z$$

A_z = double-spin asymmetry

Schmieden, Tiator Eur.Phys.J. A **8**(2000)15 7.

Polarization Technique Considerations



- A point in favor of this technique is that P_p (component of proton polarization parallel to \vec{q}) should be readily optimizable at EIC.
- Need to keep in mind that the $R=\sigma_L/\sigma_T$ polarization relation only strictly applies in parallel kinematics.
 - The detector geometry enforces very tight constraints, as recoil neutron angle is very sensitive to θ_{CM} .

$$\sigma_L \propto P_e P_p \sqrt{1 - \varepsilon^2} A_z$$

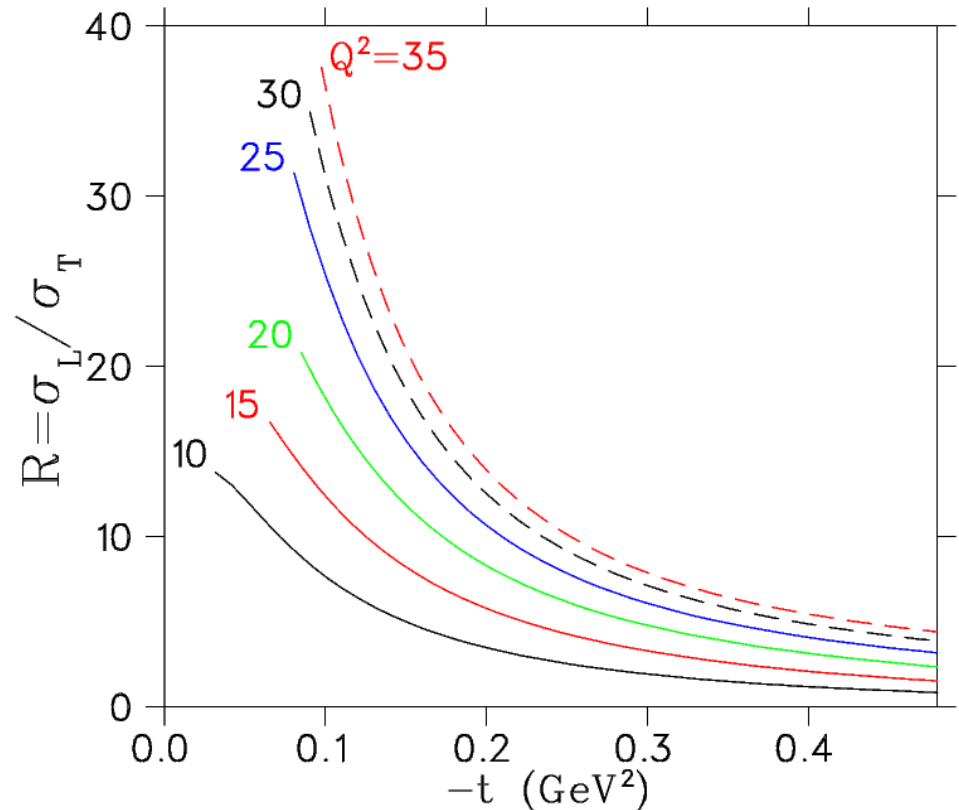
- Figure of merit for this technique vanishes for $\varepsilon \approx 1.0$.
- $\varepsilon \approx 0.95$ gives $\sqrt{1 - \varepsilon^2} \approx 0.31$
- Requires $E_{CM} < 20$ GeV, e.g. 3x25. Luminosity low.
- At best, this could be used as a spot-check only in specific kinematics. Generally not feasible.

Isolate σ_L using a Model



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- In the hard scattering regime, QCD scaling predicts $\sigma_L \propto Q^{-6}$ and $\sigma_T \propto Q^{-8}$.
- At high Q^2 , W accessible at EIC, phenomenological models predict $\sigma_L \gg \sigma_T$ at small $-t$.
- The most practical choice might be to use a model to isolate dominant $d\sigma_L/dt$ from measured $d\sigma_{UNS}/dt$.
- **In this case, it is very important to confirm the validity of the model used.**



- T. Vrancx, J. Ryckebusch, PRC **89**(2014)025203.
- Predictions are for $\epsilon > 0.995$, Q^2, W kinematics shown earlier.

π^-/π^+ data to check t -channel dominance



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- π t -channel diagram is purely isovector (G-parity conservation).

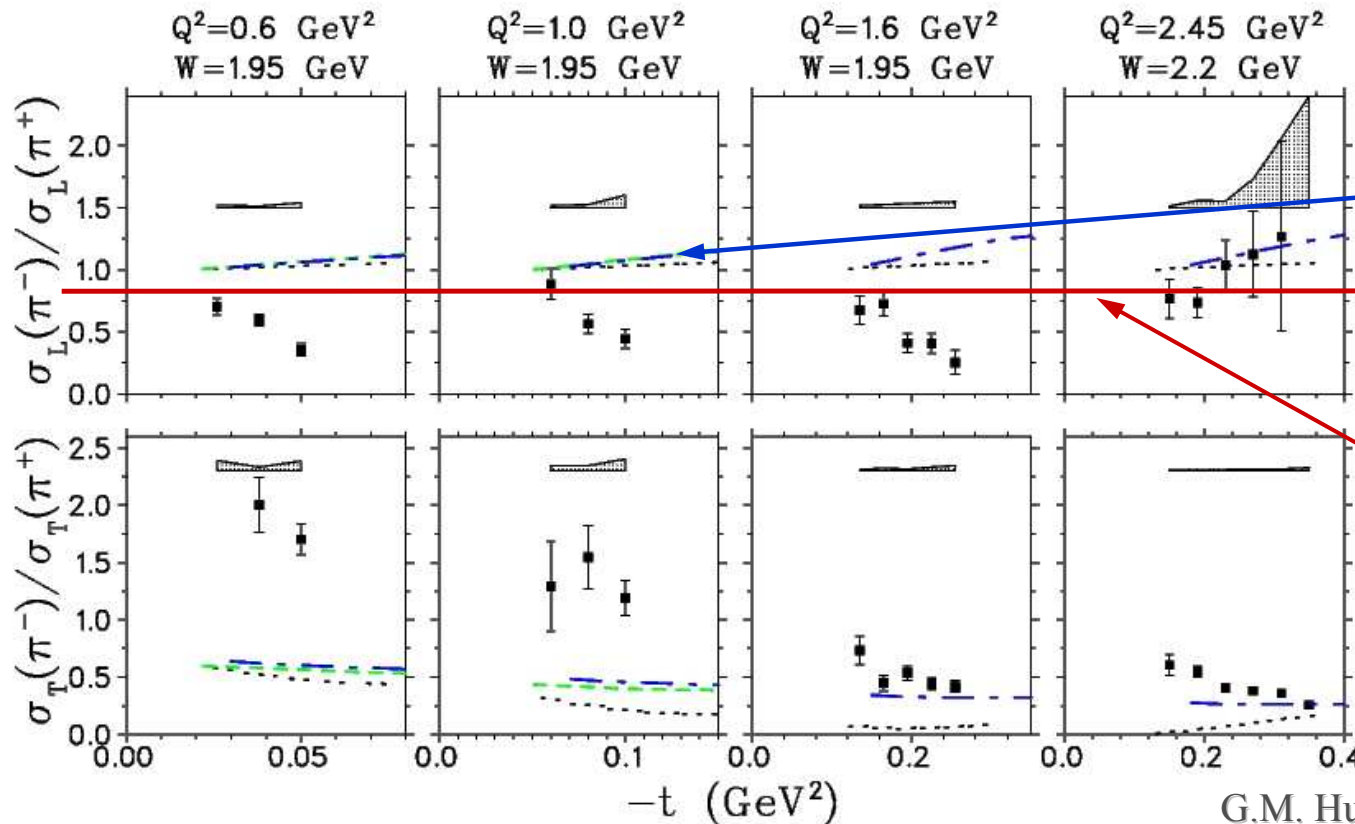
$$R_L = \frac{\sigma_L[n(e, e' \pi^-) p]}{\sigma_L[p(e, e' \pi^+) n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

- Isoscalar backgrounds (such as $b_1(1235)$ contributions to t -channel) will dilute ratio.

- **Qualitatively in agreement with our F_{π^-} analysis:**

- We found evidence for small additional contribution to σ_L at $W=1.95$ GeV not taken into account by the VGL model.

- We found no evidence for this contribution at $W=2.2$ GeV.



Vrancx-Ryckebusch Model:

- VR extend VGL with hard DIS process of virtual photons off nucleons. [PRC 89(2014)025203]

$R_L=0.8$ consistent with $|A_S/A_V| < 6\%$.

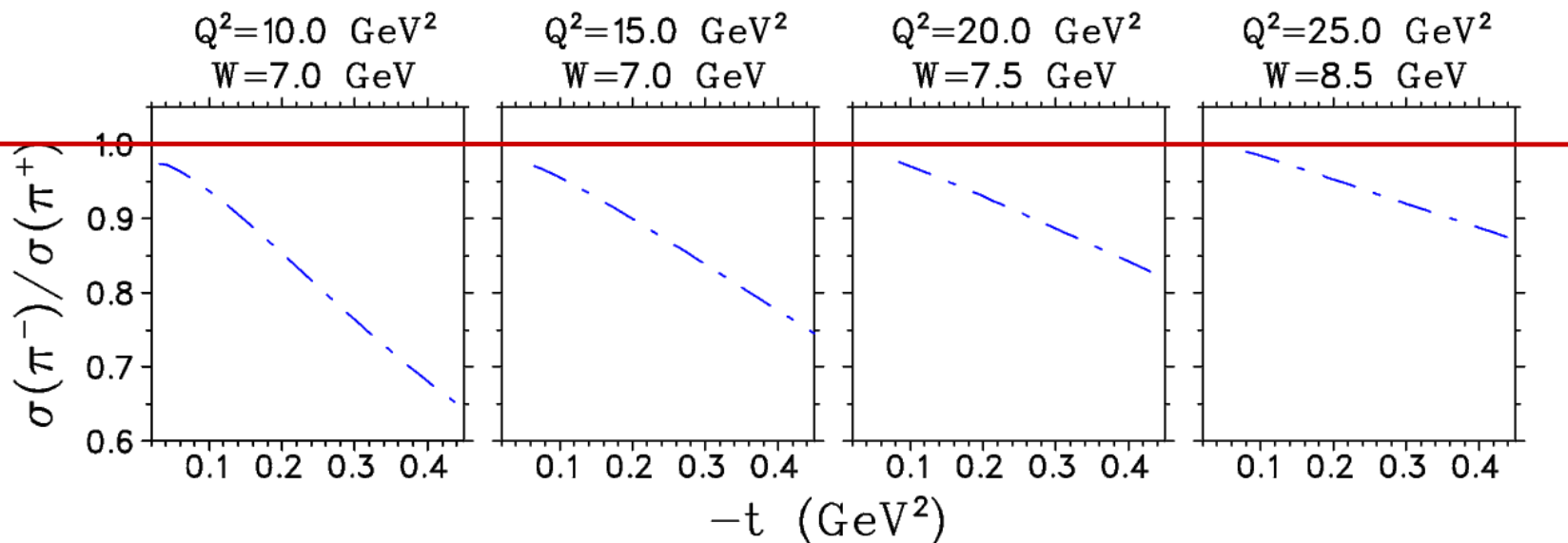
Similar approach to confirm $\sigma_L \gg \sigma_T$ at EIC



- Exclusive ${}^2\text{H}(e, e' \pi^+ n)n$ and ${}^2\text{H}(e, e' \pi^- p)p$ in same kinematics as $p(e, e' \pi^+ n)$
- π t -channel diagram is purely isovector (G parity conservation).

$$R = \frac{\sigma[n(e, e' \pi^- p)]}{\sigma[p(e, e' \pi^+ n)]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

- The π^-/π^+ ratio will be diluted if σ_T is not small, or if there are significant non-pole contributions to σ_L .
- Compare measured π^-/π^+ ratio to model expectations.

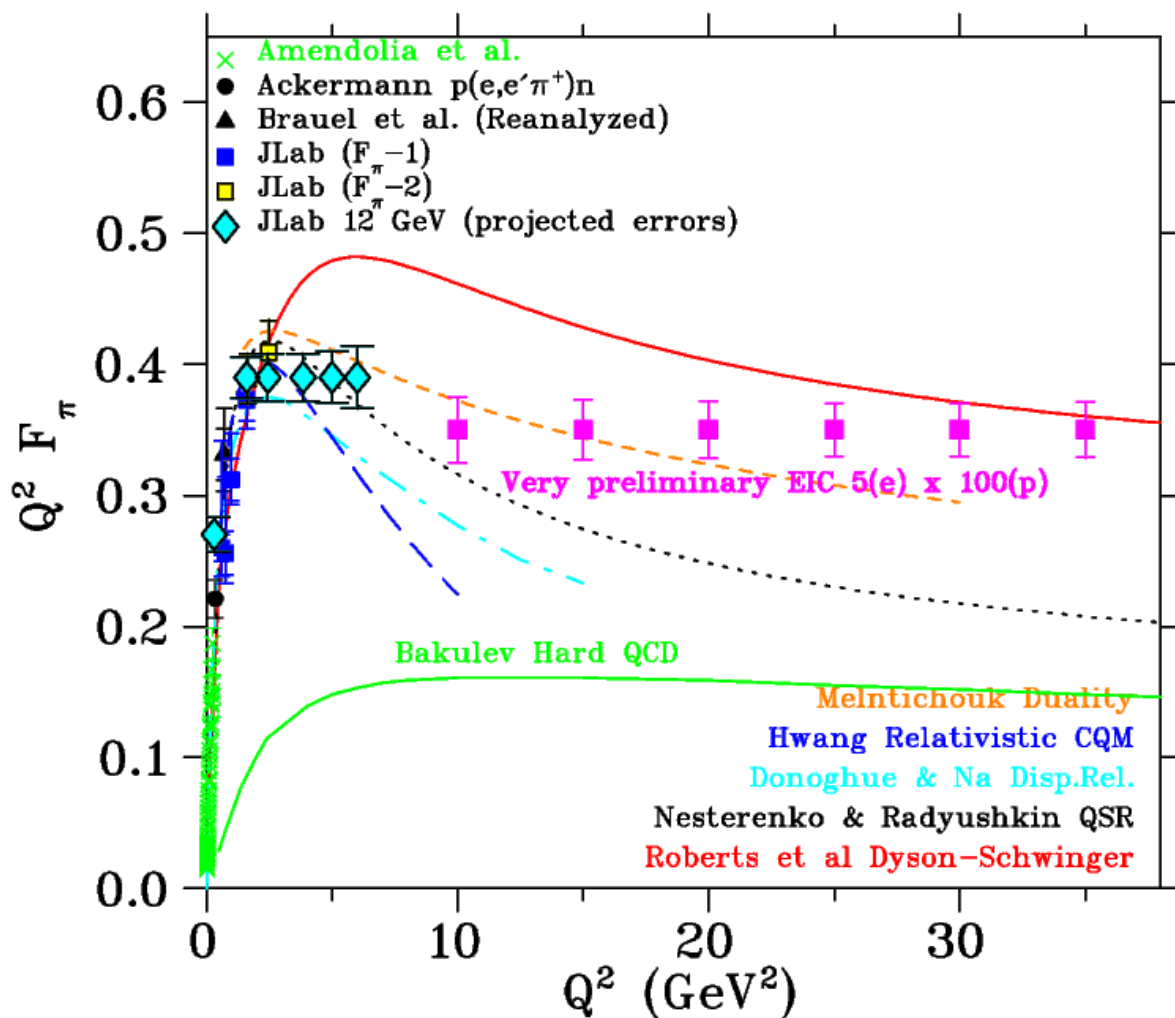


$R = 1.0$

EIC Kinematic Reach (Very Tentative)



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Assumptions:

- $5(e^-) \times 100(p)$.
- Integrated $L=20 \text{ fb}^{-1}/\text{yr}$.
- Identification of exclusive $p(e, e' \pi^+ n)$ events.
- 10% exp. syst. unc.
- $R=\sigma_L/\sigma_T$ from VR model, and π pole dominance at small $-t$ confirmed in ${}^2\text{H } \pi^-/\pi^+$ ratios.
- 100% syst. unc. in model subtraction to isolate σ_L .

Much more study needed to confirm assumptions.



- Higher Q^2 data on the pion form factor are vital to our better understanding of hadronic physics
 - Pion properties are intimately connected with dynamical chiral symmetry breaking (DCSB), which explains the origin of more than 98% of the mass of visible matter in the universe.
 - F_π is our best hope to directly observe QCD's transition from confinement-dominated physics at large length-scales to perturbative QCD at short length-scales.
- **Measurement of F_π at EIC involves significant challenges.**
 - Need good identification of $p(e, e' \pi^+ n)$ triple coincidences.
 - Conventional L-T separation not possible due to low proton ring energies required to access $\epsilon < 0.8$.
 - Use of polarization degrees of freedom with $\epsilon \approx 0.95$ seems very difficult due to low E_{CM} required.
 - As $\sigma_L \gg \sigma_T$ expected, most likely possibility is to use model to extract σ_L from $d\sigma_{UNS}/dt \rightarrow$ Used also for $Q^2 = 10 \text{ GeV}^2$ Cornell expt (1978).
 - Best to use exclusive π^-/π^+ ratio in e+d collisions to validate model.
 - **Looks promising, but more studies are needed.**