Pion-Nucleon Sigma Term in Global Color Symmetry Model

Yu-xin Liu

Department of Physics, Peking University,

Beijing 100871, China

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Outline

I. Introduction

II. Brief Description of the GCM

III. The $\sigma_{\pi N}$ in Free Space and in Nuclear Matter

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V. Summary and Remarks

I. Introduction

Pion-nucleon sigma term is significant in

Nucleon mass decomposition $M_N = M_{N,b} + \sigma_{\pi N}$

s-quark content in nucleon $\frac{2\langle \bar{s}s \rangle_\rho}{\langle \bar{u}u + \bar{d}d \rangle_0} = 1 - \frac{\sigma_0}{\sigma_{\pi N}}$

Chiral symmetry breaking effect in nucleon

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_{\pi N}}{2m_q \left| \langle \bar{q}q \rangle_0 \right|} \rho$$

Chiral symmetry restoration in nuclear matter

may probably be helpful in searching

Higgs boson and supersymmetric particles, Dark matter

Can not be measured directly !! $\sigma_{\pi N} = \sigma(0) \neq \sigma(2m_\pi^{\ 2})$

Pion-nucleon sigma term has been studied in

Chiral perturbation theory

Chiral quark models,

Lattice QCD,

……

Different approach gives quite different result

as small as	(18 ± 5) MeV	PRD59, 054504 (1999)
as large as	(80 ~ 90) MeV	EPJ A22, 89 (2004);
generally	(45 ± 7) MeV	PLB253, 252, 260; PRD63, 054026
recently	(50 ~ 80) MeV	PRD69, 034003; PRC71,065201

Two Puzzles in Strong Interaction at low energy:

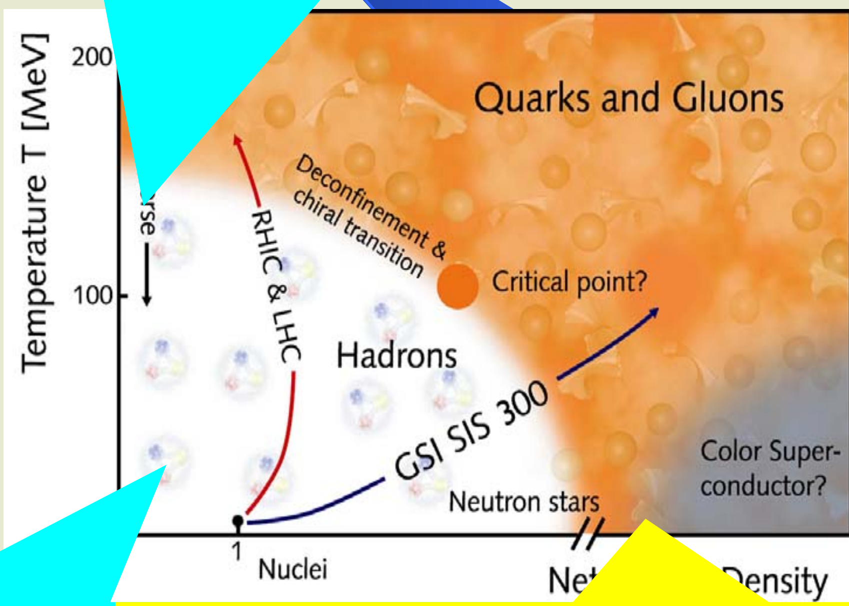
- **Chiral Symmetry and its Spontaneous Breaking**
- **Color Confinement**

Lattice QCD, pQCD (Factorization
Re-summation), •••

Characteristics Identifying Quark Deconfinement and Chiral Symmetry Restoration:

Hadron Properties

Vacuum Structure



Confinement
Flux Tube, C
Intuitive view

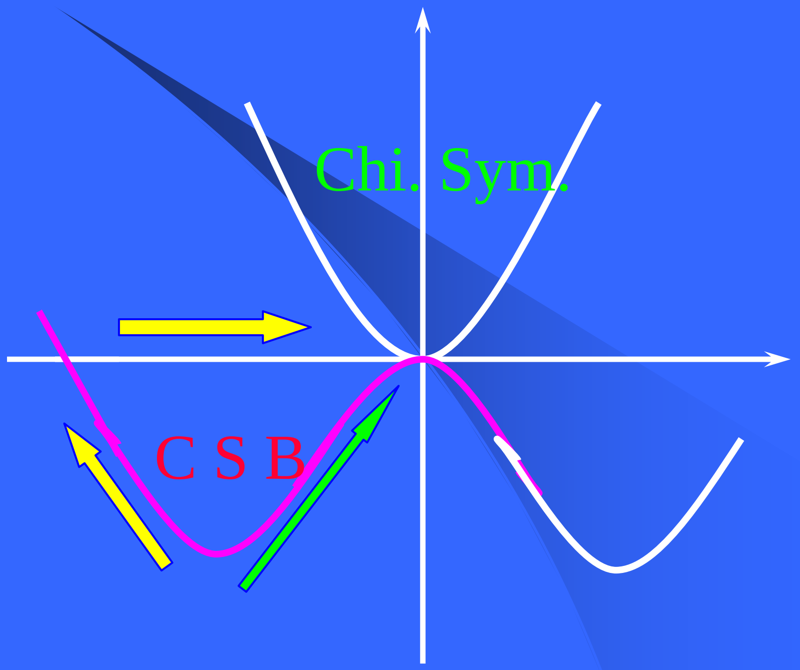
NJL model, QMC, QMF,
Truncated DSE, Instanton,
GCM, …

How the Chiral Symmetry is Restored ?

Quark condensates are usually taken as characteristics of

Vacuum Structure

Order Parameters.



- Theoretical approaches:

  Composite-operator,

  Sum rules, QMC, NJL,

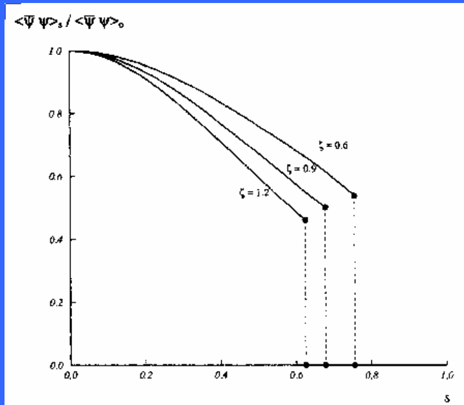
  Walecka model,

  Dirac-Brueckner,

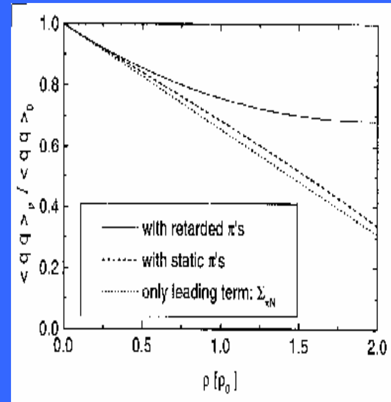
  S-D Equation,

  Instanton dilute liquid model, …

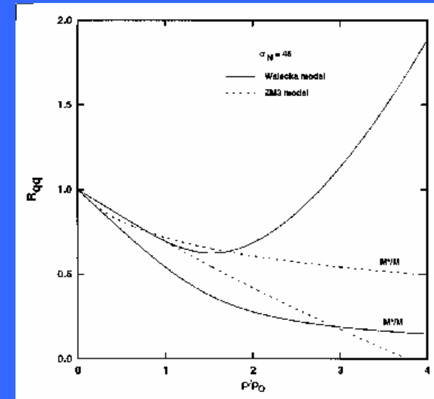
Different results have been obtained!!



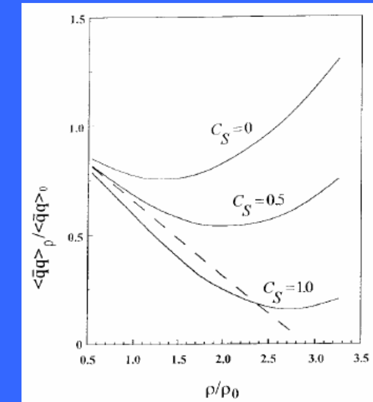
(Comp.-Op., PRD41,1610('90))



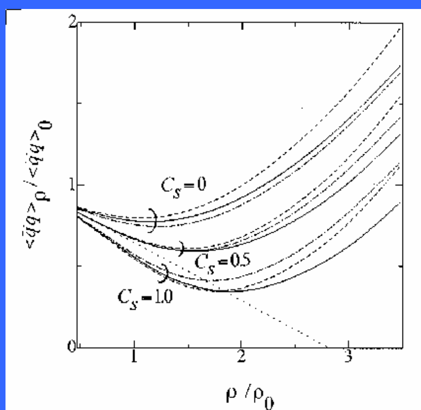
(QSR, NP A642, 171 ('98))



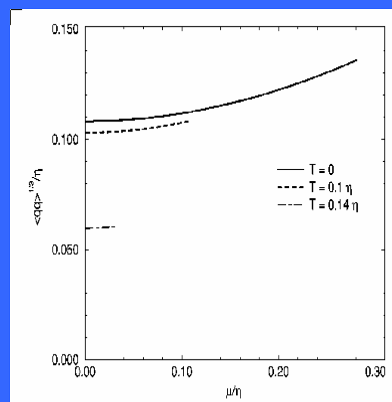
(Walecka, PR C55, 521 ('97))



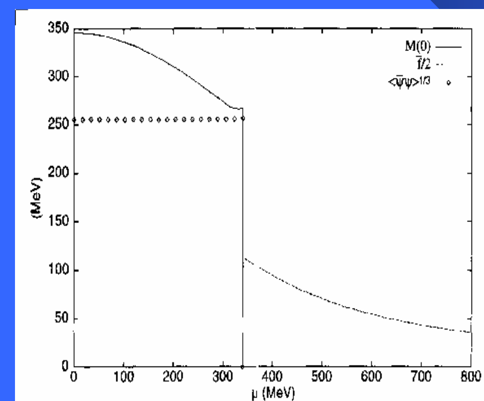
(RBHF PL B367, 40 ('96))



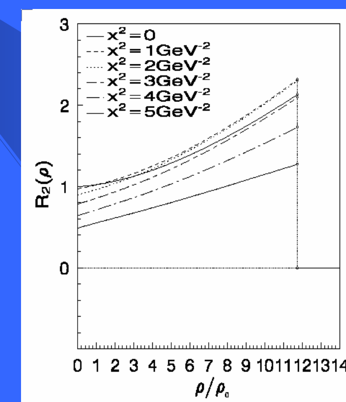
(D-S E. PR C55, 1577('97))



(DS E PR C57, 2821('98))



(IDLM, NP A642, 83('98))



(GCM, PRC68, 035204('03))

How to understand the difference is a puzzle !!

II. Brief Description of the GCM

1. The Main Point of Global Color Symmetry Model

- The global color symmetry model (GCM) is based upon an effective quark-quark interaction defined through a truncation of QCD

$$S=\int d^4x\left\{\bar{q}\left[\gamma_\mu\cdot\partial_\mu+m\right]\delta(x-y)q(y)-\frac{g^2}{2}j_\mu^a(x)D_{\mu\nu}(x-y)j_\mu^a(y)\right\},$$

where $j_\mu^a=\bar{q}(x)\frac{\lambda}{2}\gamma_\mu q(x)$ is the quark current,

$D_{\mu\nu}(x-y)$ is the effective two-point gluon propagator.

R.T. Cahill, C.D. Roberts, Phys. Rev. D 32 (1985) 2419

- Taking $D_{\mu\nu}(x-y) = \delta_{\mu\nu} D(x-y)$

  and applying the Fierz reordering

$$\Lambda^{\theta} = \frac{1}{2} K^a \otimes C^b \otimes F^c$$

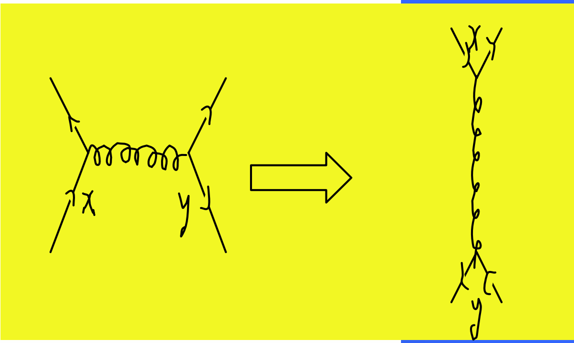
  with

$$\{K^a\} = \{I, i\gamma_5, \frac{i}{\sqrt{2}}\gamma_\mu, \frac{i}{\sqrt{2}}\gamma_\mu\gamma_5\}$$

$$\{C^b\} = \{\frac{4}{3}I, \frac{i}{\sqrt{3}}\lambda\}$$

$$\{F^c\} = \{\frac{1}{\sqrt{2}}I, \frac{\vec{\tau}}{\sqrt{2}}\}$$

  to the quark field, one has the current term



$$\frac{g^2}{2}\int d^4x d^4y j^a_\mu(x) D_{\mu\nu}(x-y) j^a_\mu(y)$$

$$= -\frac{g^2}{2}\int d^4x d^4y j^{\theta}(x,y) D(x-y) j^{\theta}(y,x)$$

- The $j^{\theta}(x,y) = \bar{q}(x)\Lambda q(y)$ is a bilocal current, and can be transferred into an auxiliary Bose-field $B^{\theta}(x,y)$.

- The GCM action can be rewritten as

$$S[\bar{q},q,B^{\theta}(x,y)] = \int\int d^4x d^4y \left[\bar{q}(x)G^{-1}(x,y;B^{\theta})q(y) + \frac{B^{\theta}(x,y)B^{\theta}(y,x)}{2g^2D(x-y)}\right], \quad (1)$$

with

$$G^{-1}(x,y;B^{\theta}) = \gamma \cdot \partial\delta(x-y) + \Lambda^{\theta}B^{\theta}(x,y). \quad (2)$$

- The bilocal field $B^{\theta}(x,y)$ can be generally given as

$$B^{\theta}(x,y) = B^{\theta}_{v}(x-y) + \sum_{i\theta}\phi^{\theta}_{i}\left(\frac{x+y}{2}\right)\Gamma^{\theta}_{i}(x-y), \quad (3)$$

- The vacuum configuration $B^{\theta}_{v}(x,y)$ can be obtained by solving the equation

$$\frac{\partial S[B^{\theta}(x,y)]}{\partial B^{\theta}(x,y)}\Big|_{B^{\theta}(x,y)=B^{\theta}_{v}(x-y)} = 0.$$

- In practical calculation, one can fix the B_{v}^{θ} in two ways.

  One is by solving the rainbow Dyson-Schwinger equation

$$G^{-1}(x,y) = i\gamma \cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{t^a}{2}\gamma_\mu G(x,y)\gamma_\mu \frac{t^a}{2} \qquad (4)$$

  with $G^{-1} = iA(p^2)\gamma \cdot p + \Lambda^{\theta} B_{v}^{\theta}(p^2)$.

  Another is by modeling dressed quark propagator directly

  For example

$$A = 1, \qquad B_{v}^{\theta}(q) = m(q^2)\,. \qquad (5)$$

  where $m(q)$ is the dynamical quark mass.

Lue, Liu, Zhao, Zong,
Phys. Rev. C 58 (1998) 1195

- The relative excitation can be fixed as

$$\Gamma_{\sigma} = B_{v}^{\theta}, \qquad \Gamma_{\pi} = i\gamma_5 \vec{\tau} B_{v}^{\theta}\,.$$

Effective degrees of freedom becomes quark and chiral mesons

- It has been successful in describing hadron properties, chiral parameters, and so on.

Prog. Part. Nucl. Phys. 39 (1997) 117;
Phys. Rev. D49 (1994) 125;
Phys. Rev. C53 (1996) 2410; …… .

2. GCM in Strongly Interacting Matter

(1) General Method

- The quark field and the bilocal field in nuclear matter with chemical potential μ

$$q(x) \Longrightarrow q' = e^{\mu x_4} q(x) \,, \tag{6}$$

$$B^{\theta}(x,y) \Longrightarrow B^{\theta}(x,y;\mu) = e^{\mu x_4} B^{\theta}(x,y) e^{-\mu y_4} \,. \tag{7}$$

- The action of the GCM at finite chemical potential is

$$S = \int d^4x d^4y \bar{q}'(x) \left[\gamma \cdot \partial \delta(x-y) - \mu \gamma_4 + \Lambda^{\theta} B^{\theta}(x,y;\mu) \right] q'(y) + \int d^4x d^4y \frac{B^{\theta}(x,y) B^{\theta}(y,x)}{2g^2 D(x-y)} \,, \tag{8a}$$

i.e.,

$$S = -\mathrm{Trln} \left[\gamma \cdot \partial \delta(x-y) - \mu \gamma_4 + \Lambda^{\theta} B^{\theta} \right] + \int d^4x d^4y \frac{B^{\theta}(x,y) B^{\theta}(y,x)}{2g^2 D(x-y)} \,. \tag{8b}$$

Generally, the bilocal field $B^\theta(x,y;\mu)$ can be written as

$$B^\theta(x,y;\mu) = B_0^\theta(x,y;\mu) + \sum_i \Gamma_i^\theta \phi_i^\theta\left(\frac{x+y}{2}\right), \tag{9}$$

- The fluctuation amplitude

$$\Gamma_\sigma^\theta = B_0^\theta(x,y;\mu), \tag{10a}$$

$$\Gamma_\pi^\theta = i\gamma_5\vec{\tau} B_0^\theta(x,y;\mu). \tag{10b}$$

- The vacuum configuration can be determined by the saddle-point condition $\frac{\partial S}{\partial B_0^\theta} = 0$. Consequently, one gets the quark self-energy

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D(x-y) \frac{t^a}{2}\gamma_\nu \frac{1}{i\gamma\cdot\tilde{q}+\Sigma(q,\mu)}\gamma_\nu \frac{t^a}{2}. \tag{11}$$

where $\tilde{q} = \{\vec{q}, q_4 + i\mu\}$. $q_4 = i\omega_n = i(2n+1)\pi T$

- Taking the conventional decomposition for the quark self-energy

$$G^{-1}(p,\mu) = iA(p,\mu)\vec{\gamma}\cdot\vec{p} + iC(p,\mu)\gamma_4(p_4+i\mu) + VB(\tilde{p})\,, \quad (12)$$

$$\Sigma(p,\mu) = i[A(p,\mu)-1]\vec{\gamma}\cdot\vec{p} + i[C(p,\mu)-1]\gamma_4(p_4+i\mu) + VB(\tilde{p})\,, \quad (13)$$

where $V = \sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5$ with restriction $\sigma^2 + \vec{\pi}^2 = 1$.

The A, B and C can be fixed by solving the rainbow D-S Eqs

$$[A(\tilde{p})-1]\vec{p}^2 = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{A(\tilde{q})\vec{q}\cdot\vec{p}}{A^2(\tilde{q})\vec{q}^2+C^2(\tilde{q})\tilde{q}_4^2+B^2(\tilde{q})}\,, \quad (14a)$$

$$[C(\tilde{p})-1]\tilde{p}_4^2 = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{C(\tilde{q})\tilde{q}_4\tilde{p}_4}{A^2(\tilde{q})\vec{q}^2+C^2(\tilde{q})\tilde{q}_4^2+B^2(\tilde{q})}\,, \quad (14b)$$

$$B(\tilde{p}) = \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{B(\tilde{q})}{A^2(\tilde{q})\vec{q}^2+C^2(\tilde{q})\tilde{q}_4^2+B^2(\tilde{q})}\,. \quad (14c)$$

(2) Simple Approximation

Expanding the Eq.(11) in power of μ, we have

$$\mathcal{G}[\mu] = G - \mu G\Gamma^{(1)}G - \frac{\mu^2}{2}[G\Gamma^{(2)}G - 2G\Gamma^{(1)}G\Gamma^{(1)}G] + \cdots\,, \qquad (15)$$

with

$$\Gamma^{(1)}(p) = \left.\frac{\partial \mathcal{G}^{-1}[\mu](p)}{\partial \mu}\right|_{\mu=0}, \quad \Gamma^{(2)}(p) = \left.\frac{\partial^2 \mathcal{G}^{-1}[\mu](p)}{\partial \mu^2}\right|_{\mu=0}.$$

To fix the expansion coefficients $\Gamma^{(1)}$ and $\Gamma^{(2)}$, we define other two functions as

$$\Lambda^{(1)}(p) = \Gamma^{(1)}(p) - \frac{\partial G^{-1}(p)}{\partial(-ip_4)}; \quad \Lambda^{(2)}(p) = \Gamma^{(2)}(p) - \frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2}.$$

Zong, Chang, Hou, Sun, Liu, Phys. Rev. C 71, 015205 (2005)

Applying operations $\partial/\partial\mu$ and $\partial/\partial(-ip_4)$ on both sides of Eqs.(4) and (11), respectively, we obtain

$$\Lambda^{(1)}(p) = -\frac{4}{3}\int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu G(q)\Lambda^{(1)}(q)G(q)\gamma_\nu \,, \qquad (16)$$

and

$$\begin{aligned}\Lambda^{(2)}(p) = -\frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D_{\mu\nu}(p-q)\gamma_\mu\{G(q)\Lambda^{(2)}(q)G(q) \\ -2[G(q)\Gamma^{(1)}(q)G(q)\Gamma^{(1)}(q)G(q) \\ -G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)\frac{\partial G^{-1}(q)}{\partial(-iq_4)}G(q)]\}\gamma_\nu \,. \end{aligned} \qquad (17)$$

Eq.(16) is just the meson Bethe-Salpeter Equation with ladder approximation

Solutions of Eq.(16)

<i> $\Lambda^{(1)}(p) = 0$, <ii> $\Lambda^{(1)}(p) = i\gamma_5 \frac{B(p^2)}{f_\pi}$.

Our models

model-1:

$$\Gamma^{(1)}(p) = \frac{\partial G^{-1}(p)}{\partial(-ip_4)}; \qquad \Gamma^{(2)}(p) = \frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2}.$$

model 2:

$$\Gamma^{(1)}(p) = \frac{\partial G^{-1}(p)}{\partial(-ip_4)} + \frac{i\gamma_5 B}{f_\pi}; \quad \Gamma^{(2)}(p) = \frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2} + i\gamma_5 E$$

3. Relation Between the Chemical Potential and the Nuclear Matter Density

The pressure density

$$P[\mu] = Tr\ln[G^{-1}] - \tfrac{1}{2}Tr[\Sigma G]$$

The nuclear matter density

$$\rho = \tfrac{\partial P}{\partial \mu} = \tfrac{2}{3\pi^2}\mu^3 + 4\tfrac{\partial}{\partial \mu}\int \tfrac{d^4q}{(2\pi)^4}[A(\tilde{q}^2) - 1]$$

GCM, Global Color Symmetry Model:

an effective field theory model of QCD

With the GCM, one can explore

- the QCD foundation of bag models,
- the chiral symmetry breaking and restoration,
- the quark confinement and deconfinment, ••••••••••.

III. The $\sigma_{\pi N}$ in Free Space and in Nuclear Matter

1. In Free Space

According to Hellmann-Feynman theorem

$$\sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q}$$

Assuming $M_N = 3M_q$

we have

$$\sigma_{\pi N} = 3m_q \frac{\partial M_q}{\partial m_q}$$

The M_q is the constituent quark mass and can be determined by the solution of the D-S equation

$$M_q = \frac{B}{A}$$

Chang, Liu, Guo, hep-ph/0508192

Taking

$$\Gamma(p^2) = \frac{\partial G^{-1}(p)}{\partial m_q}$$

we have

$$\Gamma(p,\mu) = 1 - \frac{4}{3}\int \frac{d^4q}{(2\pi)^2} D_{\mu\nu}\gamma_\mu G(q,\mu)\Gamma(q,\mu)G(q,\mu)\gamma_\nu$$

It can be decomposed as

$$\Gamma(p,\mu) = i\gamma\cdot pC(p^2) + D(p^2)$$

Then the chiral susceptibility can be given as

$$\chi(p^2) = \frac{\partial M_q(p^2)}{\partial m_q} = \frac{A(p^2)D(p^2) - B(p^2)C(p^2)}{A^2(p^2)}$$

Taking $p^2 = M_q{}^2$, we have

$$\sigma_{\pi N} = 3m_q[\chi(p^2 = M_q{}^2)_{m_q\to 0}]$$

Taking the Munczek model of the effective gluon propagator

$$D_{\mu\nu}(k) = 4\pi^2\eta^2\delta(k)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

we have the solution of D-S equation at chiral limit

$$\left.\begin{aligned} &A = 2, && B = \sqrt{\eta^2 - 4p^2}, \\ &C = -\frac{2}{\sqrt{\eta^2 - 4p^2}}, && D = \frac{1}{2}\frac{\eta^2 + 8p^2}{\eta^2 - 8p^2}, \end{aligned}\right\} \quad \text{for } p^2 < \frac{\eta^2}{4}$$

$$\left.\begin{aligned} &A = \frac{1}{2}(1 + \sqrt{1 + \frac{2\eta^2}{p^2}}, && B = 0, \\ &C = 0, && D = \frac{p^2 + \eta^2 + p^2\sqrt{1 + \frac{2\eta^2}{p^2}}}{p^2 - \eta^2 + p^2\sqrt{1 + \frac{2\eta^2}{p^2}}}, \end{aligned}\right\} \quad \text{for } p^2 > \frac{\eta^2}{4}$$

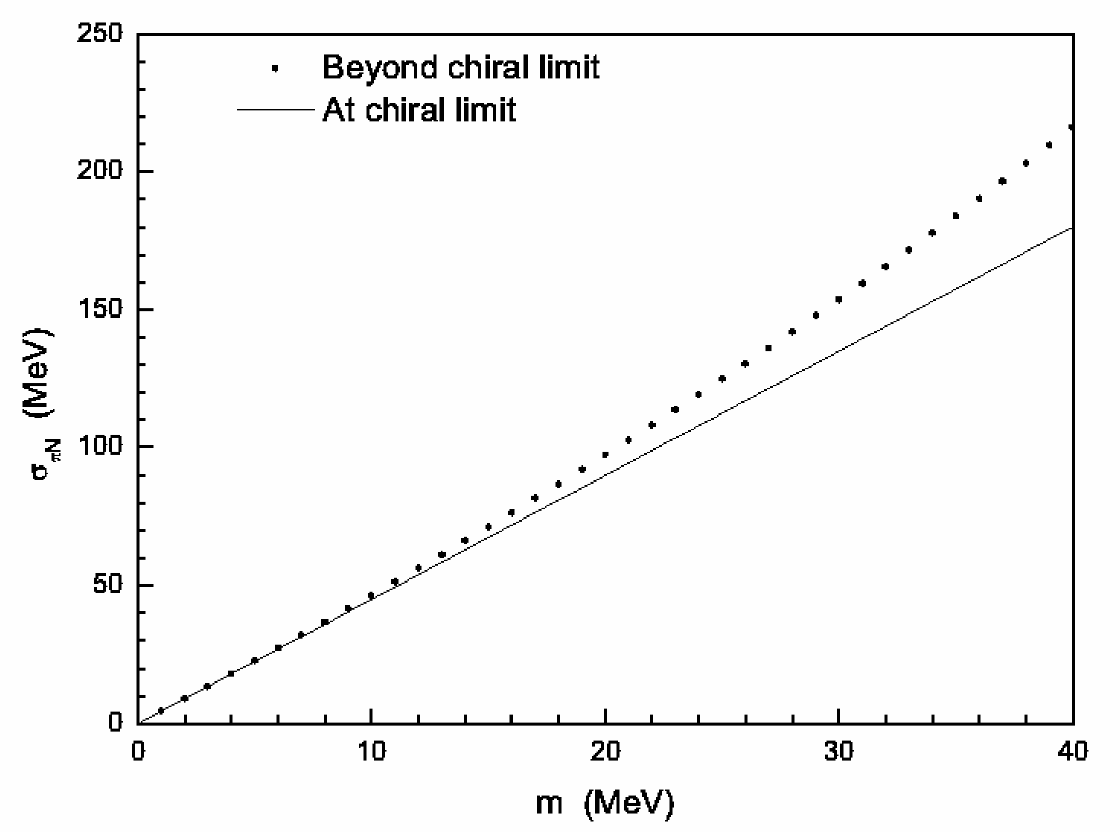
Then

$$\chi(p^2) = \frac{3}{4}\frac{\eta^2}{\eta^2 - p^2}$$

$$\sigma_{\pi N} = \frac{9}{2} m_q$$

Beyond the chiral limit, the D-S equation should be solved numerically.

The deduced pion-nucleon sigma term reads



2. In Nuclear Matter

In the Munczek model, the solution of the D-S equation with finite chemical potential μ can be expressed in the same form as that in free space with replacement $p_4 \to p_4 + i\mu$

However, the boundary condition can not be fixed directly, we take

$$\mathrm{Re}[\tilde{p}^2 - \frac{\eta^2}{4}] < a\mu^2$$

Then

$$\sigma_{\pi N}(\mu) = \frac{9}{2} m_q \frac{\eta^2}{\eta^2 + 4\mu^2}$$

IV. Application to Quark Condensate

1. Direct Application

Model independent relation

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{2\left|\langle \bar{q}q \rangle_0\right|} \frac{d\varepsilon}{dm_q}$$

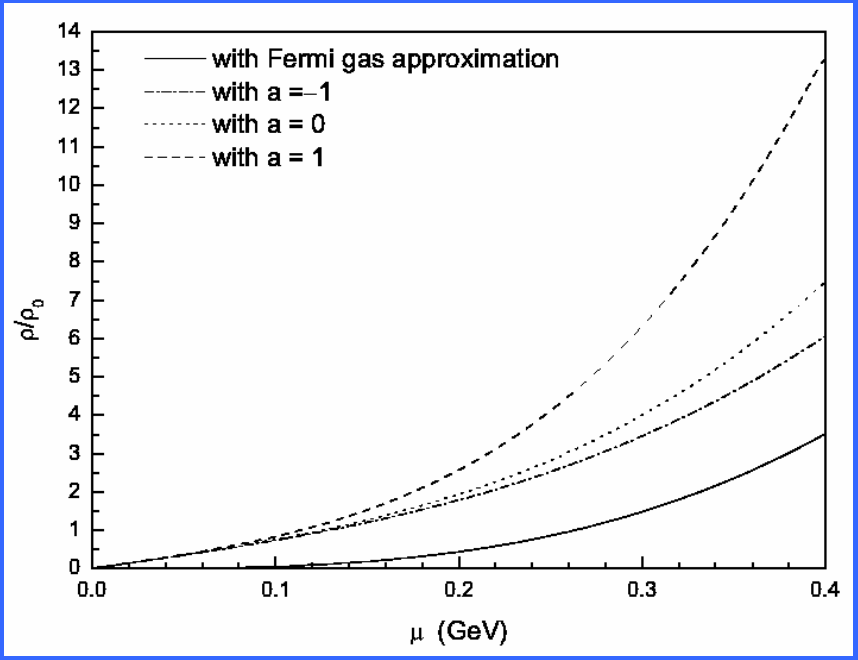
with approximation $\varepsilon = \rho M_N + \delta\varepsilon$

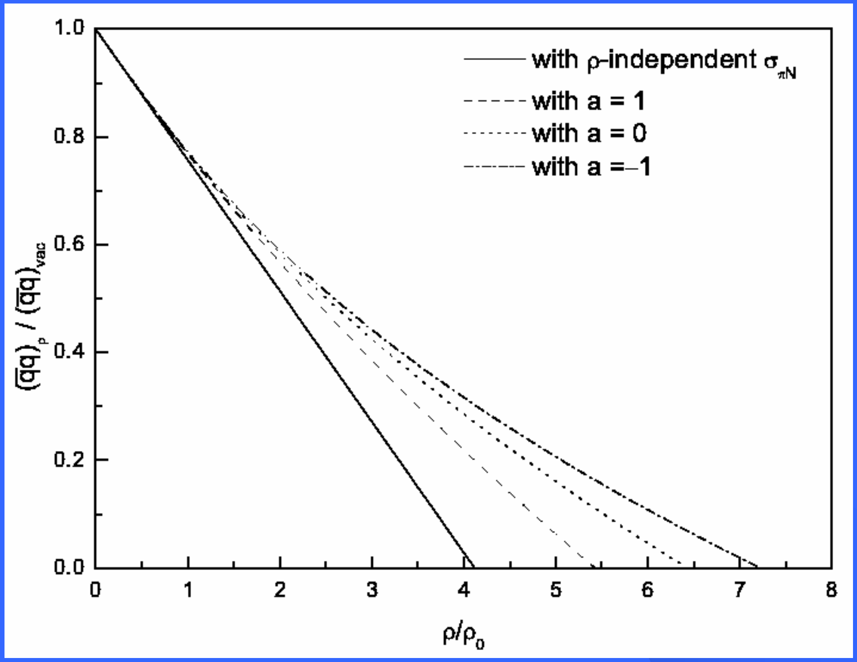
we have

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_{\pi N}}{2m_q\left|\langle \bar{q}q \rangle_0\right|} \rho$$

Phys. Rev. C45, 1881 (1992) ; Nucl. Phys. A 642, 171 (1998); ……

Numerical result:





Chang, Liu, Guo, hep-ph/0508192

2. Understanding the variation behavior of $\langle\bar{q}q\rangle_\mu$

$$|\langle\bar{q}q\rangle_\mu| = Tr_{D,C}\int\frac{d^4q}{(2\pi)^4}\mathcal{G}(q)\,.$$

With the $\mathcal{G}[\mu]$ determined above, we have

$$\left|\langle\bar{q}q\rangle\right| = a + c\mu + b\mu^2$$

Chang, Liu, et al., hep-ph/0508094

with

$$a = Tr_{D,C}\int\frac{d^4q}{(2\pi)^4}G(q)\,,$$

$$c = -Tr_{D,C}\int\frac{d^4q}{(2\pi)^4}G(q)\Gamma^{(1)}(q)G(q)\,,\equiv 0$$

$$b = -Tr_{D,C}\int\frac{d^4q}{(2\pi)^4}\Big\{\frac{1}{2}G(q)\Gamma^{(2)}(q)G(q) - G(q)\Gamma^{(1)}(q)G(q)\Gamma^{(1)}(q)G(q)\Big\}\,,$$

With effective gluon propagator

$$D_{\mu\nu}(k) = t_{\mu\nu} 4\pi^2 d \frac{\chi^2}{k^4 + \Delta},$$

where $d = \frac{12}{27}$ and $t_{\mu\nu} = \delta_{\mu\nu}$

- ♠ Calculated chiral quark condensate in vacuum and the response to the chemical potential

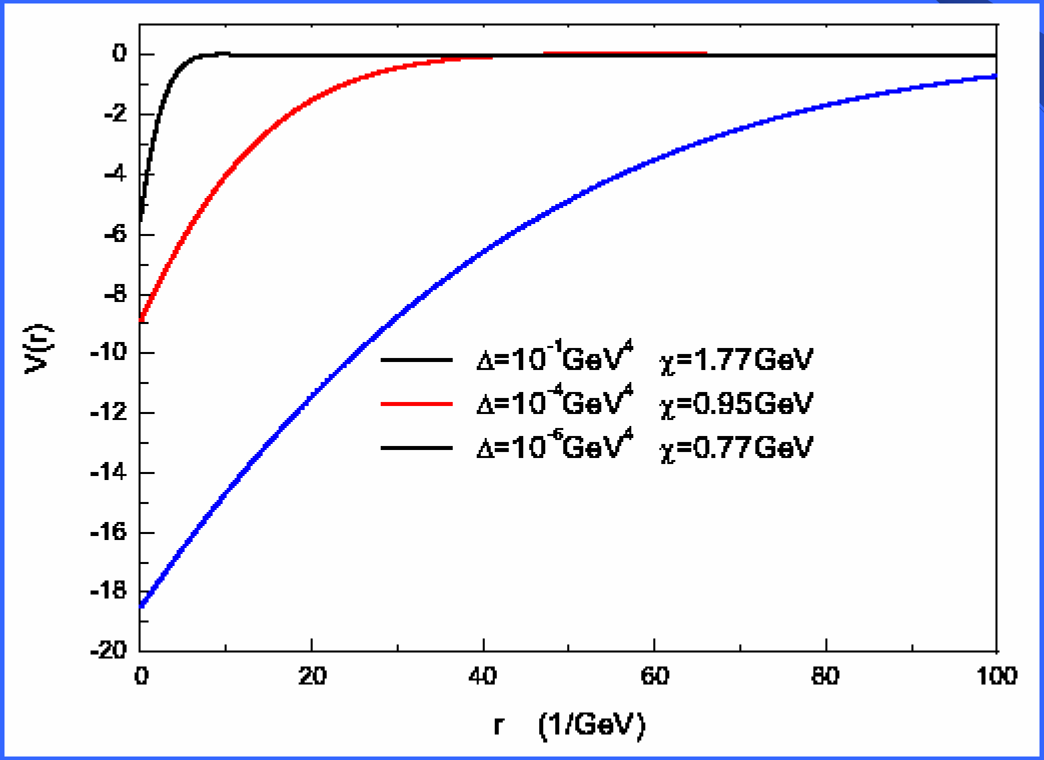
Δ [GeV^4]	χ [GeV]	$a^{1/3}$	b/a in model-1	b/a in model-2
10^{-1}	1.77	290	0.177	-4.073
10^{-2}	1.33	250	0.325	-6.309
10^{-4}	0.95	217	0.594	-10.913
10^{-6}	0.77	205	0.782	-15.450

parameters are taken from Phys. Lett. B 405, 8 (1997), with f_π being fixed as

$f_\pi = 87 MeV$

The classical potential corresponding to the effective gluon propagator

$$V(r) = -\frac{d\pi\chi^2}{r\sqrt{\Delta}} e^{-r\sqrt[4]{\frac{\Delta}{4}}} \sin[r\sqrt[4]{\frac{\Delta}{4}}] \,.$$



Variation of the condensate in vacuum:

increase with the increasing of confinement strength

Physics meaning of the chiral quark condensate:

Mean spectral density at zero energy of quark over the gluon configurations and the four-volume,

Quantitative expression: Banks-Casher relation

$$-\langle \bar{q} q \rangle = \pi \nu_q(0)$$

confinement ⟷ attractive potential

increase of confinement strength

→ attractive potential stronger → $\nu(0)$ increases

Response to the chemical potential: positive in model-1; negative in model-2.

Increase of chemical potential

→ confinement more strong → $\Delta\nu(0) > 0$

Model-1: $\Delta\nu_q(0) = \Delta\nu(0) > 0$

→ The response is positive !

Model-2: $\Delta\nu_q(0) + \Delta\nu_\pi(0) = \Delta\nu(0)$

$$\left.\begin{array}{r}\text{the pion decreases } \Delta\nu(0) \\ \Delta\nu_\pi(0) > 0\end{array}\right\} \rightarrow \Delta\nu_q(0) < 0$$

→ The response is negative !

With effective gluon propagator

$$D_{\mu\nu}(k) = t_{\mu\nu} 4\pi^2 \xi \frac{k^2}{\omega^2} e^{-k^2/\omega^2},$$

where $t_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu / k^2$

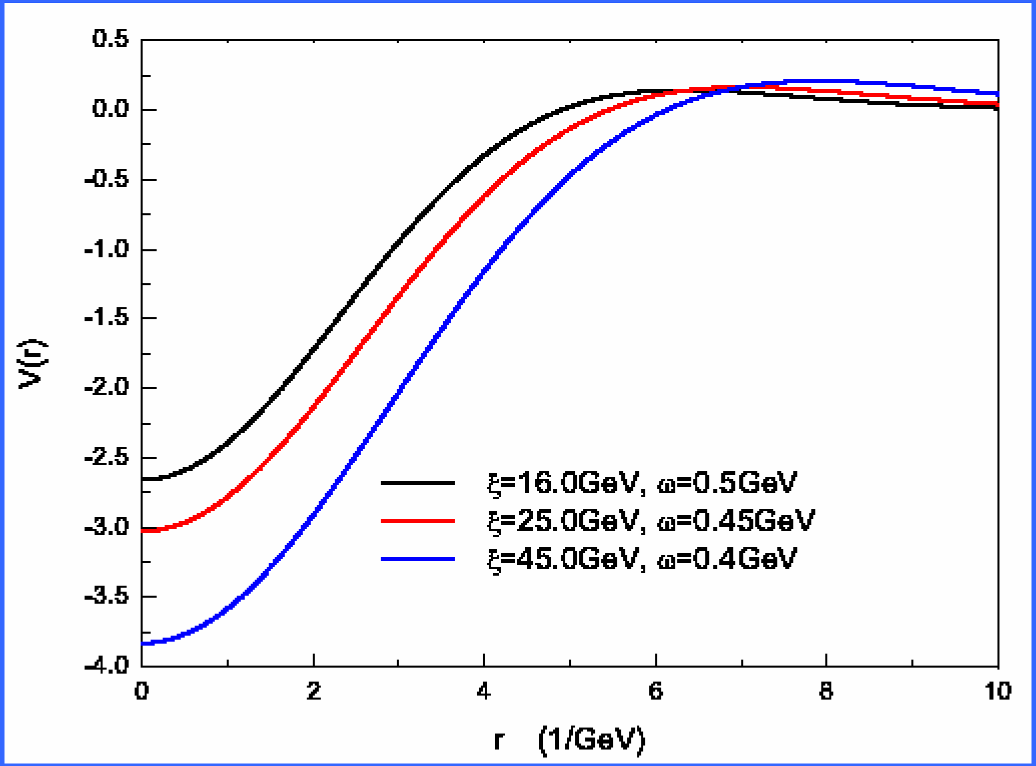
- Calculated chiral quark condensate in vacuum and the response to the chemical potential

ξ [GeV]	ω [GeV]	$a^{1/3}$	b/a in model-1	b/a in model-2
16.0	0.5	251	-1.441×10^{-5}	-5.954
25.0	0.45	246	-1.396×10^{-5}	-6.571
45.0	0.4	246	-1.281×10^{-5}	-7.237

parameters are taken From Phys. Rev. D 65, 094026 (1997), with f_π being fitted as $f_\pi = 93 MeV$

The classical potential corresponding to the effective gluon propagator

$$V(r) = \frac{\sqrt{\pi}\xi\omega^3}{8}(\omega^2 r^2 - 6)e^{-\frac{\omega^2 r^2}{4}}.$$



An interaction with confinement and screening at middle distance and "asymptotic free" property at small distance.

Variation characteristics of the quark condensate:

- ♠ The condensate in the vacuum increases with the ascend of the confinement strength.
- ♠ The response in model-2 is negative.
- ♠ The response in model-1 is negative, due to the "asymptotic free" property.

V. Summary and Remarks

- The pion-nucleon sigma term in free space and in nuclear matter are studied in the GCM (truncated DSE)
- It is found that $\sigma_{\pi N}(\mu) = \frac{9}{2} m_q \frac{\eta^2}{\eta^2 + 4\mu^2}$ in the Munczek model of effective gluon propagator.
- The chiral quark condensate decreases slowly than the linear behavior as the density dependence of the $\sigma_{\pi N}$ is taken into account.
- The decreasing characteristic of the chiral quark condensate is attributed to the pion configuration.

Remark (2)

- The quark confinement strengthens the chiral quark condensates.
- The difference of the variation behavior of the condensate arises from the difference of the configuration of the vacuum.
- The vacuum may consist both quark condensate and Goldstone bosons.

Remark (3)

- The dependence of the effective gluon propagator on the chemical potential has not yet been taken into account.
- Quantitative description of the variation rate of the response against the confinement strength is expected.
- The chiral quark condensate may not be a good characteristic to identify the restoration process.

Thanks !!!