Structure functions and form factors of a nucleon in the medium

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There are many indications for medium modifications of nucleon properties. For example,

1. **Structure functions**:
   - **EMC effect**: How to explain it in an effective quark theory?
   - **Polarized EMC effect**: Can we make some predictions? Possible to measure?
2. **Form factors**:
   Medium modifications are best probed in quasielastic processes, i.e.,
   - **Nuclear response functions** for inclusive quasielastic e-scattering: How to explain the quenching of the longitudinal response function in an effective quark theory?
   - $(\vec{e}, e'\vec{p})$ reactions. (Not considered here.)

Our aim is to discuss these medium modifications in an effective quark theory (NJL model), which also reproduces

- Properties of single nucleon
- Nuclear matter equation of state (saturation, etc)

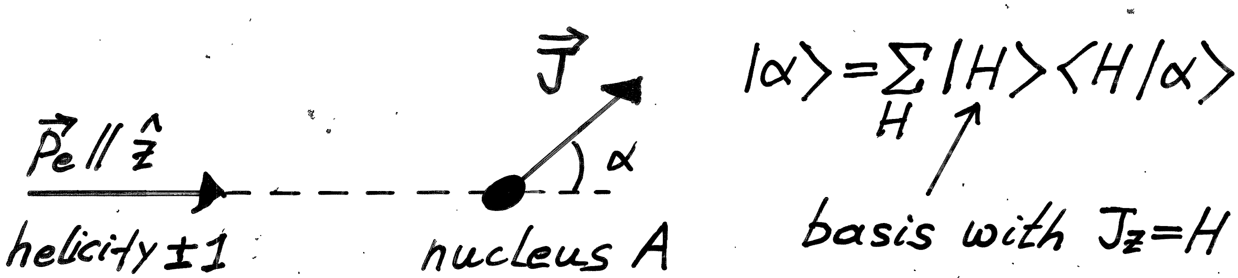
Structure functions

Most prominent effect of medium modifications:

EMC effect:

$$\boxed{R(x) = \frac{F_{2A}(x_A)}{F_{2N,\text{free}}(x)}} \quad \left(x_A = A \cdot \frac{Q^2}{2M_A\nu} = x \cdot \frac{M_N}{(M_A/A)}\right)$$

When we study effects of *nuclear polarization* in $e - A$ DIS, some interesting problems arise.



1. Only few (valence) nucleons contribute to nuclear polarization $\Rightarrow$ $1/A$ effect (relative to F_{2A}.)
2. For $J > 1/2$, new kinds of structure functions appear, e.g.,

$$\boxed{g_{1A}^{(H)}(x_A) = \frac{1}{2}\sum_q e_q^2 \Delta q_A^{(H)}(x_A) = \frac{1}{2}\sum_q e_q^2 \left(q_{A\uparrow}^{(H)} - q_{A\downarrow}^{(H)}\right)}$$

where $H = -J, \ldots + J$. [$(2J+1)/2$ F_{1A}'s and g_{1A}'s for half-integer J (or $J+1$ for integer J). See: Jaffe, Manohar, NPB **321** (1989) 343.]

Interpretation: $q_{A\uparrow}^{(H)}(x_A) \ldots$ Probability to find quark (flavor q) with light cone (LC) momentum fraction x_A/A and $s_z = +1/2$ in the nucleus with $J_z = H$.

3. Is there a '**polarized EMC effect**'?
Best candidates: Single proton particle or hole states, A not too large (^{11}B, ^{15}N, etc).
In the convolution model,

$$
\begin{aligned}
g_{1A}^{(H)}(x_A) &= \frac{1}{2}\left(\frac{4}{9}\Delta u_A^{(H)}(x_A) + \frac{1}{9}\Delta d_A^{(H)}(x_A)\right) \\
&= \int_{x_A}^{A} \frac{\mathrm{d}y_A}{y_A}\, \Delta f_{N/A}^{(H)}(y_A) \cdot g_{1p}\left(\frac{x_A}{y_A}\right)
\end{aligned}
$$

⇒ "Spin dependent EMC ratio"

$$
\boxed{R_s^{(H)}(x) = \frac{g_{1A}^{(H)}(x_A) \,/\, \langle\Sigma_N\rangle_A^{\text{(Schmidt)}}}{g_{1p,\text{free}}(x)}}
$$

expresses the medium effects.

4. Informations on both nuclear effects and quark effects.
For example, the spin sum in the nucleus $(N_{q\uparrow/A} - N_{q\downarrow/A})/2$ is:

$$
\begin{aligned}
&\int \mathrm{d}x_A\, \Delta q_A^{(H)}(x_A) \\
&= \left(\int \mathrm{d}z\, \Delta q_N(z)\right) \times \left(\int \mathrm{d}y_A\, \Delta f_{N/A}^{(H)}(y_A)\right) \\
&= \langle\Sigma_q\rangle_N \times \langle\Sigma_N\rangle_A \\
&= \left(N_{q\uparrow/N\uparrow} - N_{q\downarrow/N\uparrow}\right) \times \left(N_{N\uparrow/A} - N_{N\downarrow/A}\right)
\end{aligned}
$$

↗ spin sum for nucleon

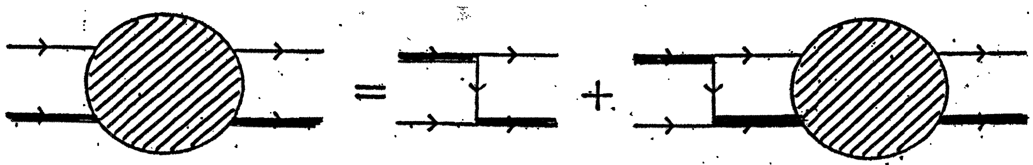
↑ nuclear polarization factor. (N.R. $\langle\Sigma_N\rangle_A^{\text{Schmidt}}$)

[There may be interesting connections to other spin phenomena in nuclei: magnetic moments, Gamow-Teller transitions, etc.]

Model calculations

Input:

- **Effective quark theory for single nucleon:**
  Nambu-Jona-Lasinio (NJL) model, Faddeev method



see: N. Ishii et al, Nucl. Phys. A **587** (1995) 617.

For finite density calculations, we use a simple "static approximation" for the quark exchange kernel $\stackrel{\text{simplify}}{\Longrightarrow}$ Quark-Diquark Model: We include both scalar (0^+) and axial vector (1^+) diquarks. (Only small amount of a.v. diquark admixture needed to have $g_A = 1.26$.)

- **Nuclear matter equation of state (EOS)**
  calculated in mean field approximation $\Longrightarrow$
  Mean scalar and vector fields couple to the quarks in the nucleons (cf. Guichon's model).

For example, the energy density is simply expressed as

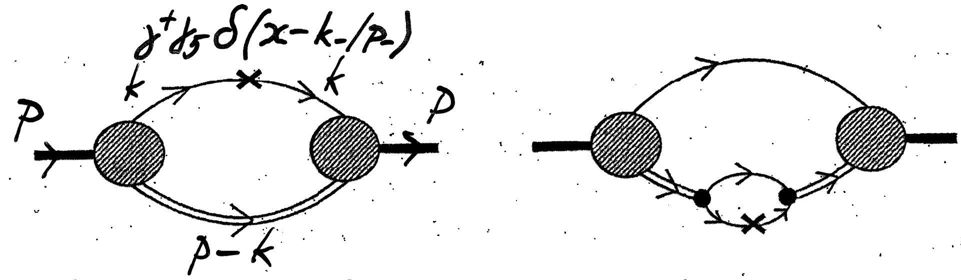
$$\mathcal{E}_N(M) = \mathcal{E}_{\text{vac}}(M) + \gamma_N \int^{p_F} \frac{d^3k}{(2\pi)^3} \sqrt{M_N(M)^2 + k^2} + \mathcal{E}_\omega$$

- Constituent quark mass $M \simeq$ scalar field.
- $\mathcal{E}_\omega \ldots$ Arises from mean vector field.
- $\mathcal{E}_{\text{vac}}(M) =$ quark loop $\simeq$ Mexican hat potential.
- Effects of nucleon structure are summarized in the function $M_N(M)$ (from quark-diquark equation).

Unphysical quark decay thresholds are avoided by using an infrared cut-off in the proper-time regularization scheme ($\Leftrightarrow$ aspect of confinement).

Output:

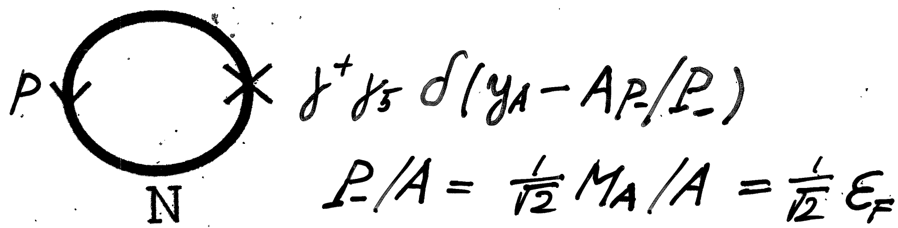
- **Quark distributions** $q_N(x)$, $\Delta q_N(x)$ in the nucleon calculated from Feynman diagrams:



in the presence of mean scalar and vector fields.

[This covariant approach satisfies all sum rules!]

- **Nucleon distributions**:



Convolution used to get the nuclear structure functions.

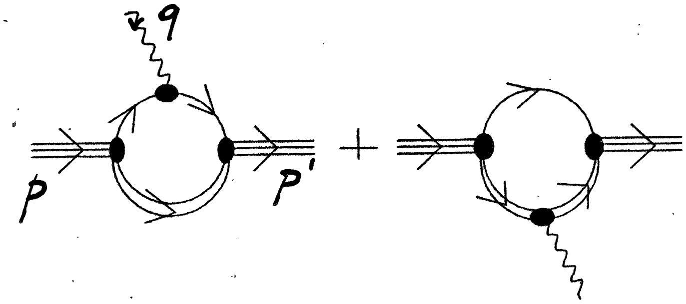
Here:

– Spin-independent case: Use nuclear matter result for nucleon distribution $f_{N/A}(y_A)$.

– Spin-dependent case: One should calculate $\Delta\, f^{(H)}_{N/A}(y_A)$ for finite nuclei. (In progress ...)

To get first estimates of polarized EMC effect: Use the same $f_{N/A}(y_A)$ as for spin-independent case.

- **Nucleon form factors**

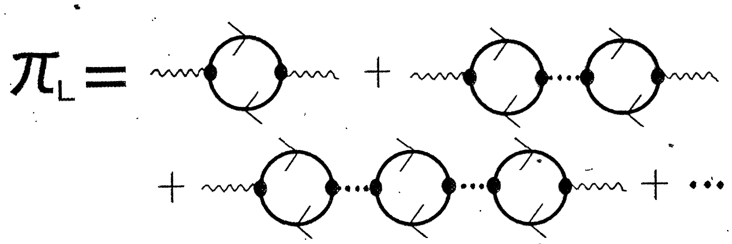


$$j^\mu(p',p) = \overline{u}_N(p')\left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2(Q^2)\right]u_N(p)$$

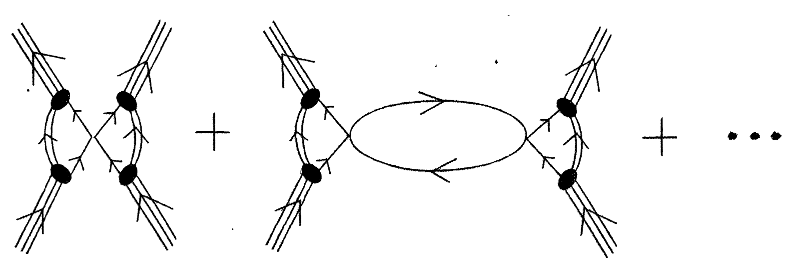
($Q^2 > 0$; u_N and M_N are in-medium quantities.)
These form factors should replace the free nucleon form factors in the current operator of a conventional nuclear calculation. For example ...

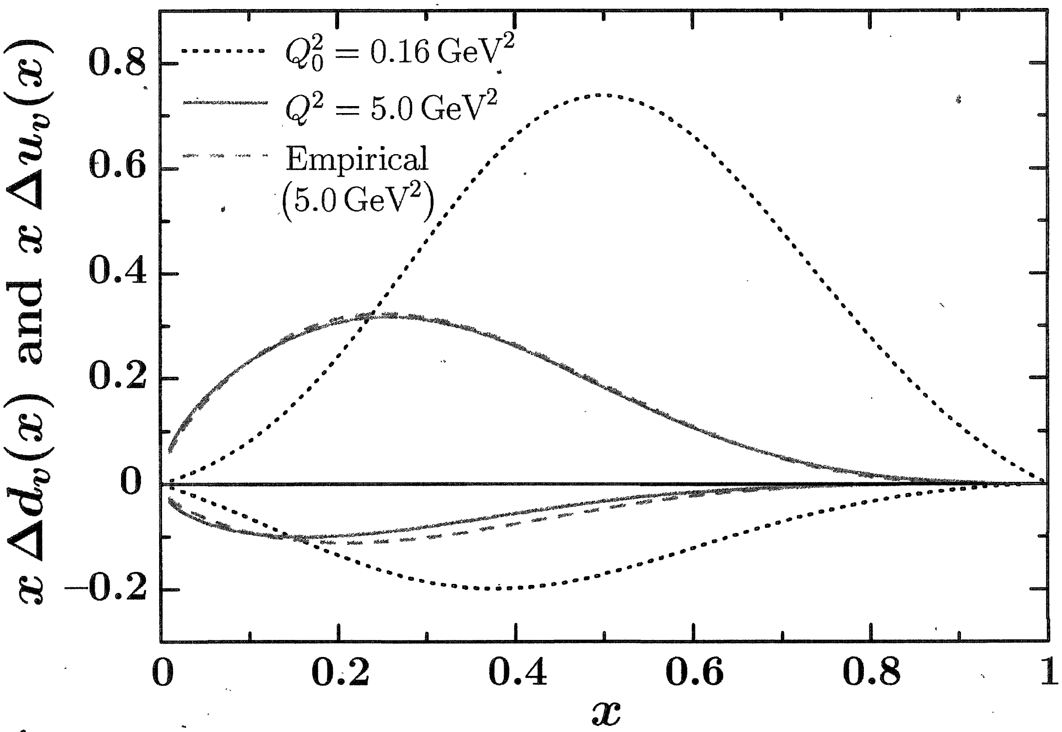
- **Longitudinal response function** in nuclear matter:

$$S_L(\omega, \mathbf{q}) = \frac{2Z}{\pi\rho}\mathrm{Im}\,\Pi_L(\omega, \mathbf{q})$$

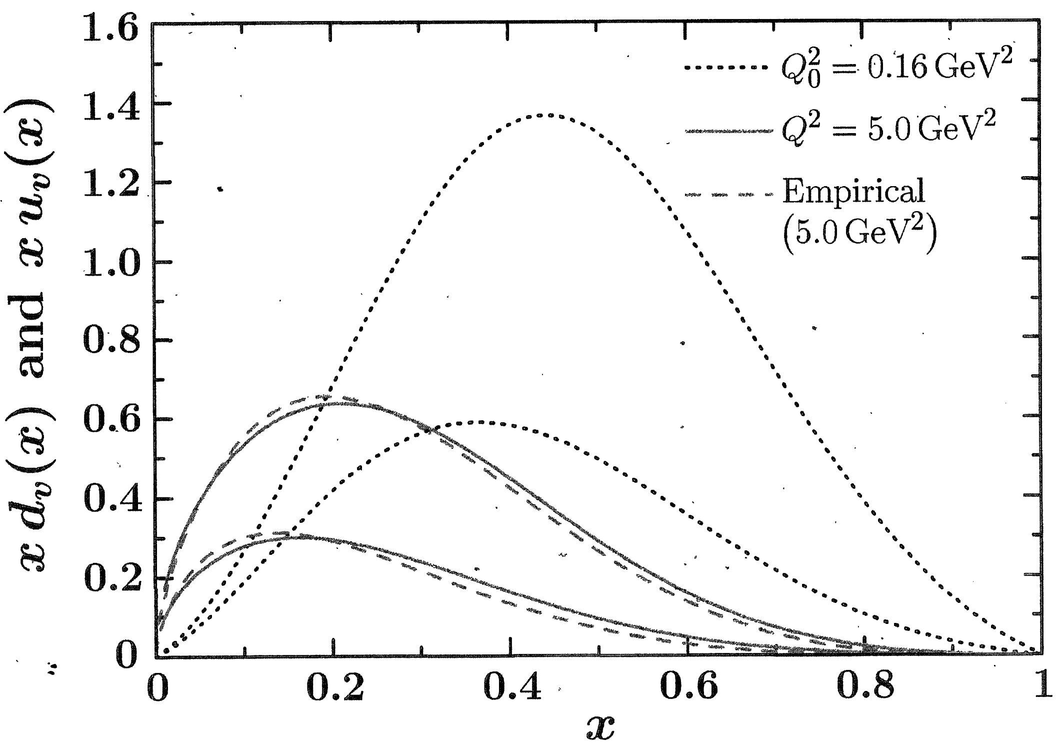


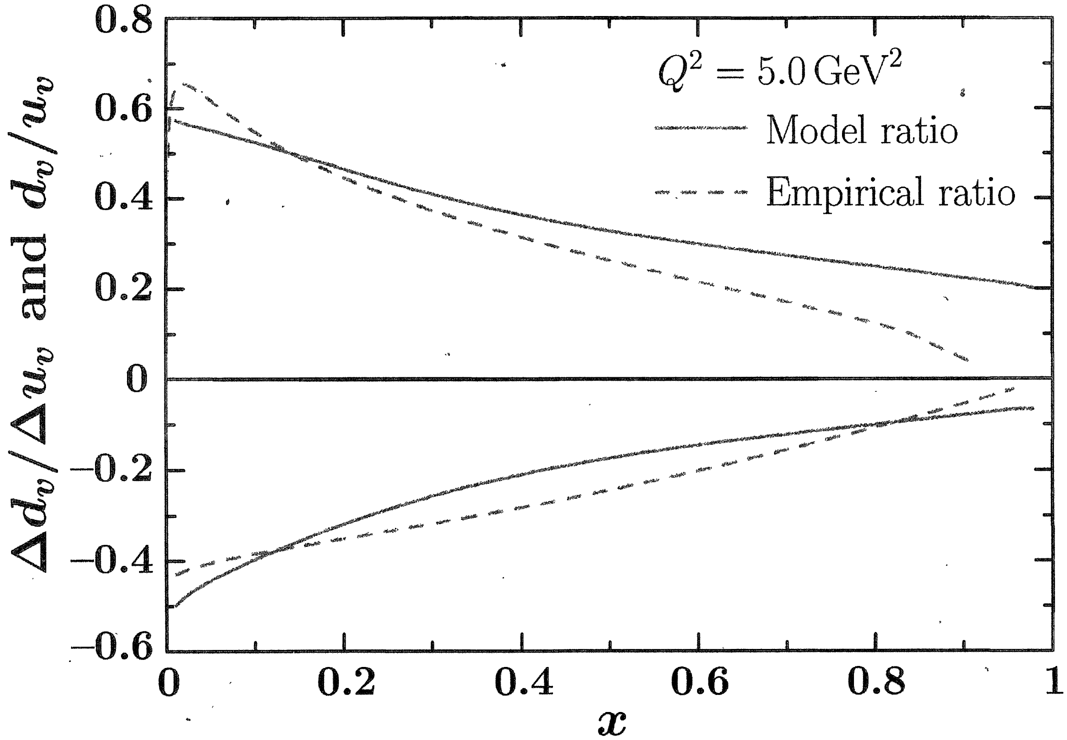
To evaluate this in the RPA, we also need the nuclear force in our quark-diquark model:

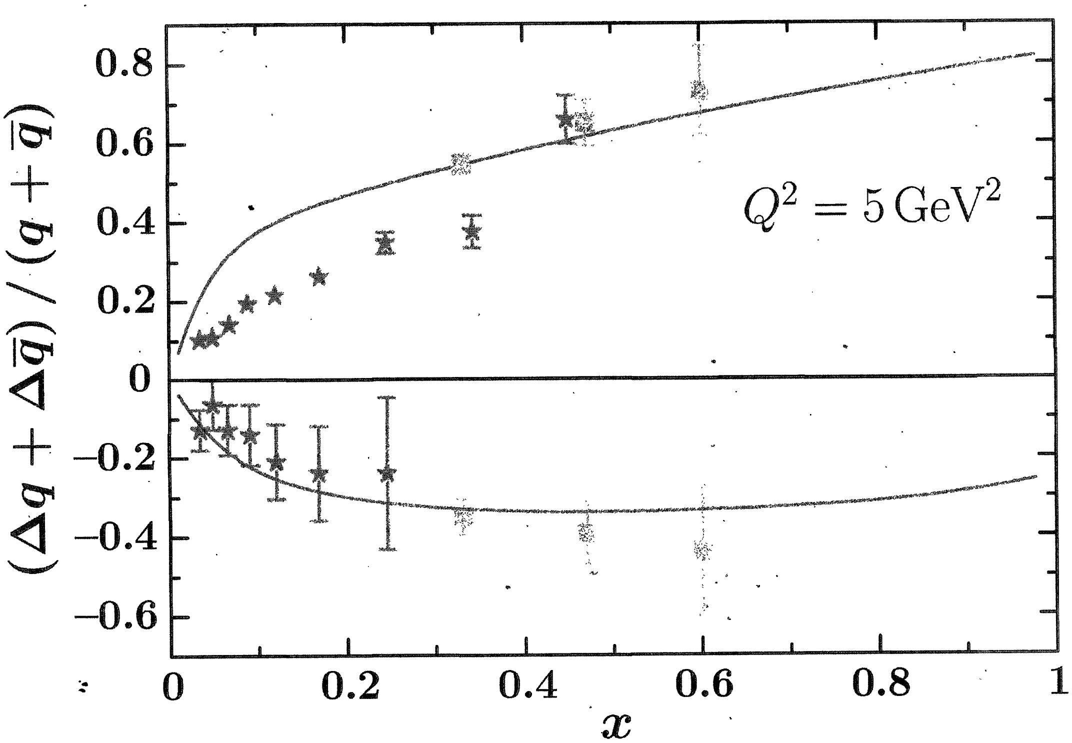




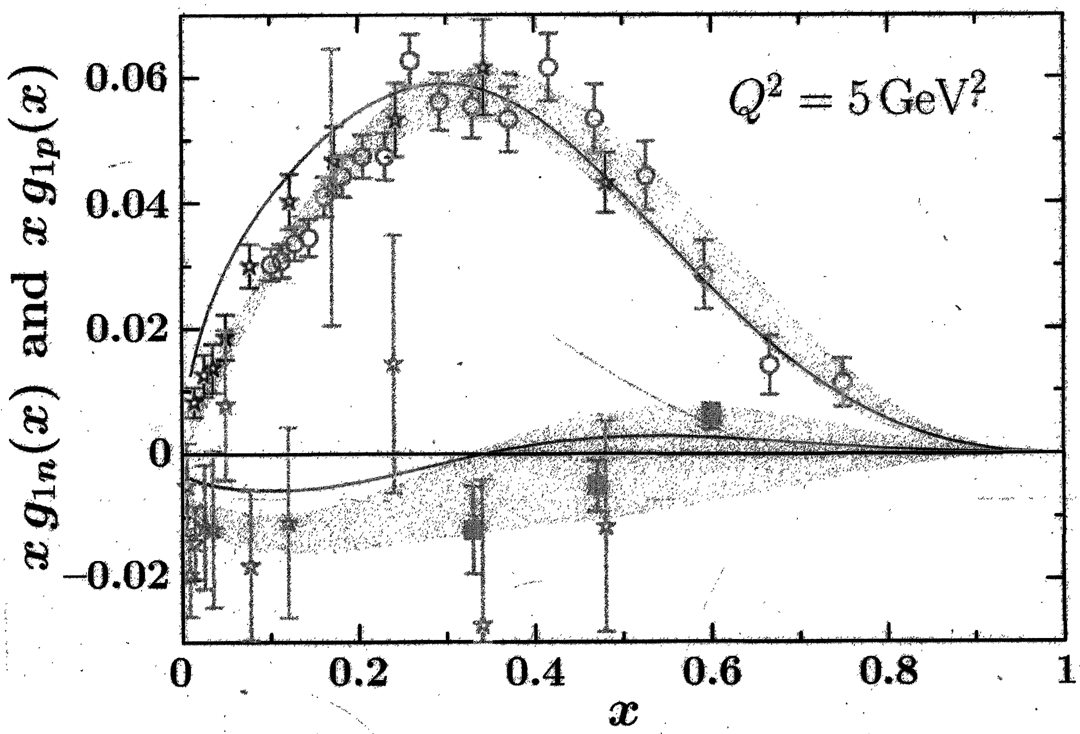
$$\begin{cases} \Delta u_v - \Delta d_v = 1.26 \\ \Delta u_v + \Delta d_v = 0.581 \end{cases}$$

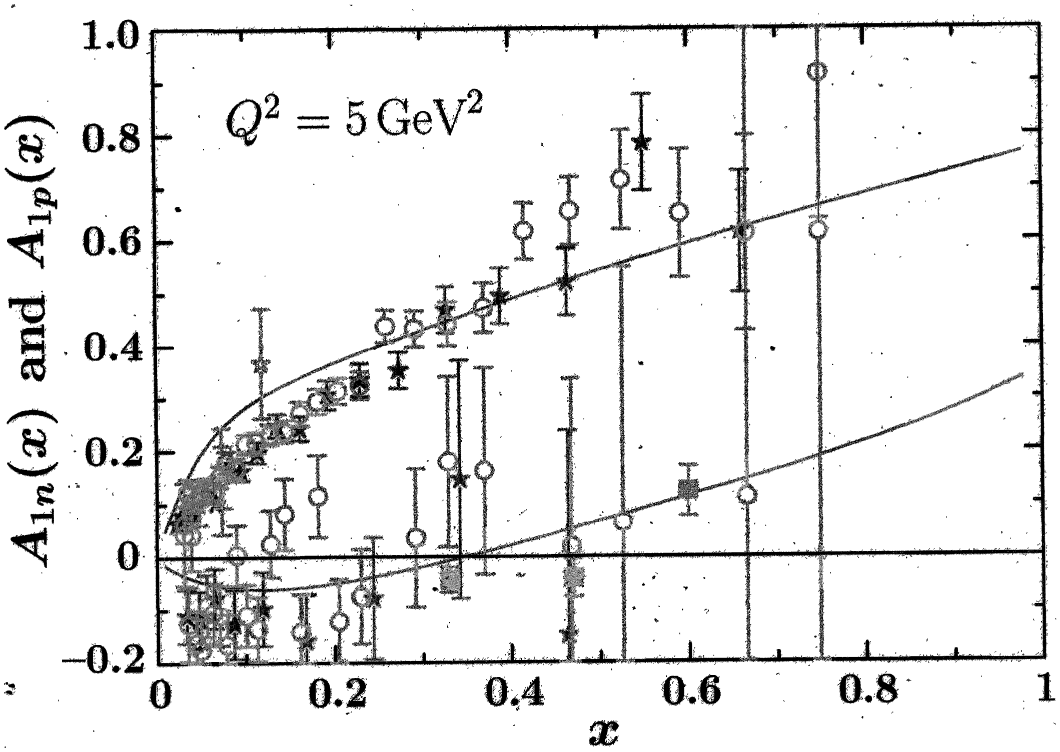






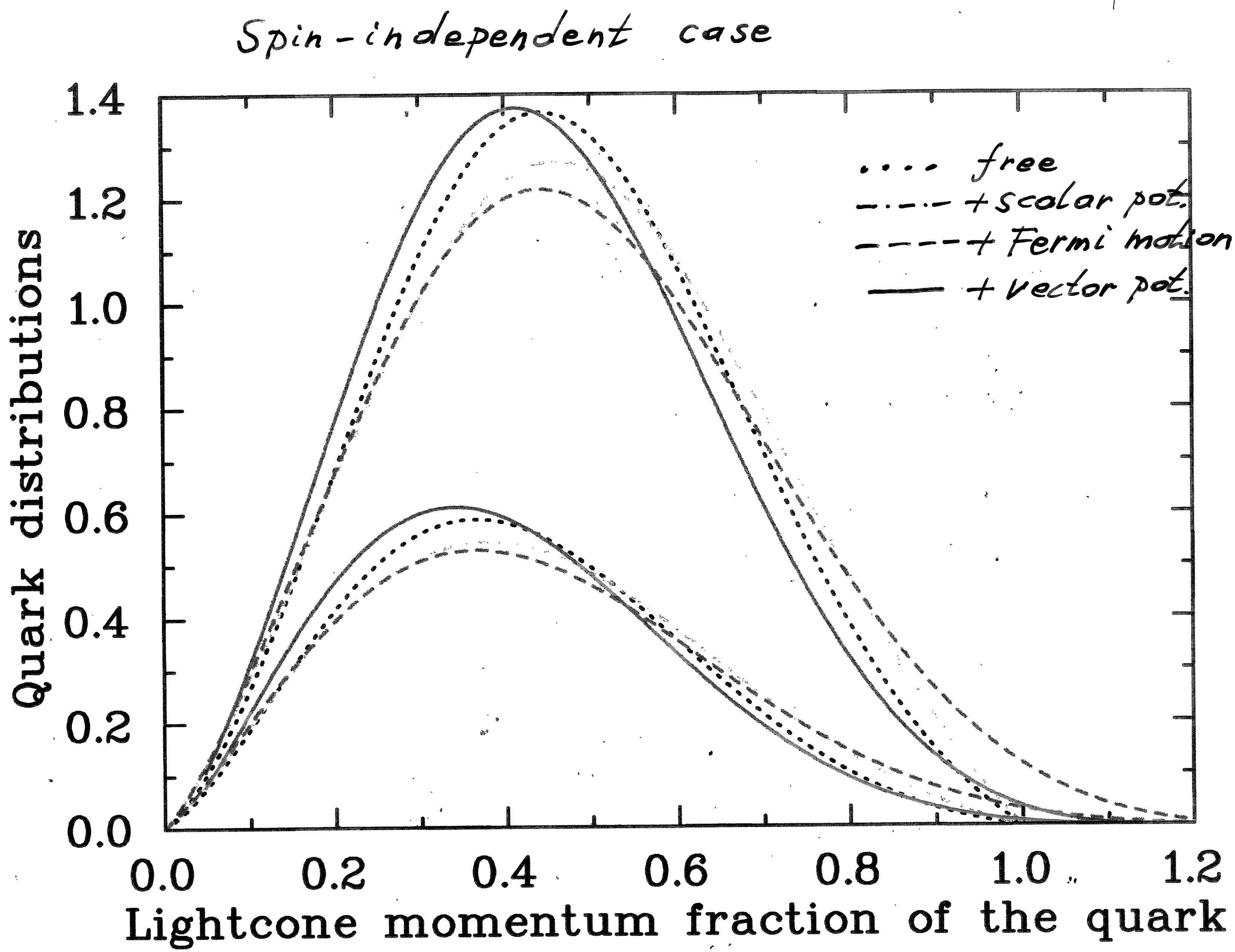
Data : Hermes, JLab





Exp: Hermes, SLAC, SMC, JLab

Shaded areas: J. Blumlein et al
NPB 636 ('02) 225.



Spin-independent case
Quark distributions
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Lightcone momentum fraction of the quark
.... free
-.-.- + scalar pot.
---- + Fermi motion
—— + vector pot.

How the ordinary EMC effect is described in this model:

Binding effects on level of nucleons cannot explain the EMC effect, but ...

<u>Mean vector field has direct effect on $q_A(x_A)$:</u>

$$\boxed{q_A(x_A) = \frac{\epsilon_F}{E_F} q_{A0}\left(x'_A = \frac{\epsilon_F}{E_F} x_A - \frac{V_0}{E_F}\right)}$$

- $\epsilon_F = \sqrt{k_F^2 + M_N^2} + 3V_0 \equiv E_F + 3V_0$ ... Fermi energy of nucleon. (V_0 ... mean vector field acting on a quark.)
- $q_{A0}(x'_A)$ ... distribution without direct effect of vector field (i.e., only scalar field and Fermi motion).
- Qualitative explanation: Including the vector field, we have

  $x_A = k/(P/A) = k/\epsilon_F \,, \qquad (k \equiv k_-, \; P \equiv P_- = P_0/\sqrt{2}, \text{ etc})$

  while without the vector field we have

$$x'_A = k'/(P'/A) = k'/E_F \,.$$

  Here the quark (light cone) momentum is shifted by V_0 ($\Rightarrow k = k' + V_0$), and the total (light cone) momentum per nucleon (=chemical potential) by $3\,V_0$ ($\Rightarrow \epsilon_F = E_F + 3V_0$). Then

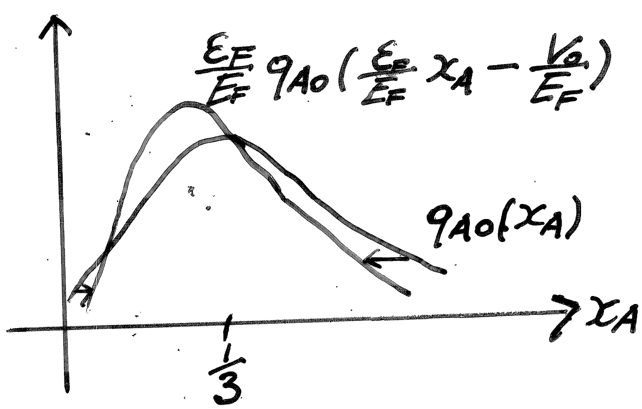
$$x'_A = \frac{k'}{P'/A} = \frac{k - V_0}{E_F} = \frac{k}{\epsilon_F}\frac{\epsilon_F}{E_F} - \frac{V_0}{E_F} = x_A \frac{\epsilon_F}{E_F} - \frac{V_0}{E_F}$$

• This rescaling of Bjorken variable implies:

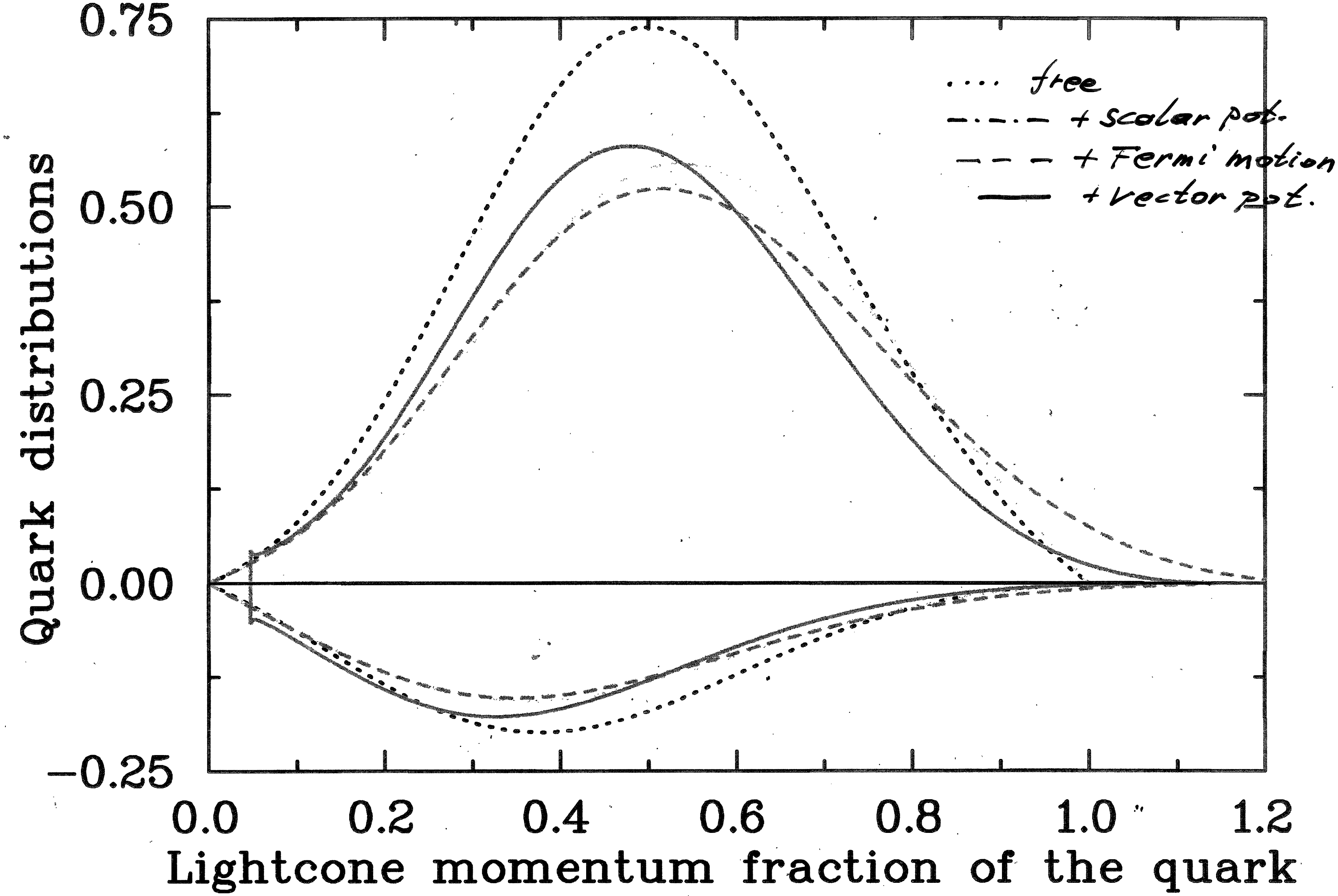
$$x_A > \frac{1}{3} \Rightarrow x'_A > x_A\,, \qquad x_A < \frac{1}{3} \Rightarrow x'_A < x_A$$

and we can expect the following behavior consistent with the EMC effect:

- Depletion of $R(x)$ for large and intermediate x
- Enhancement of $R(x)$ for smaller x.

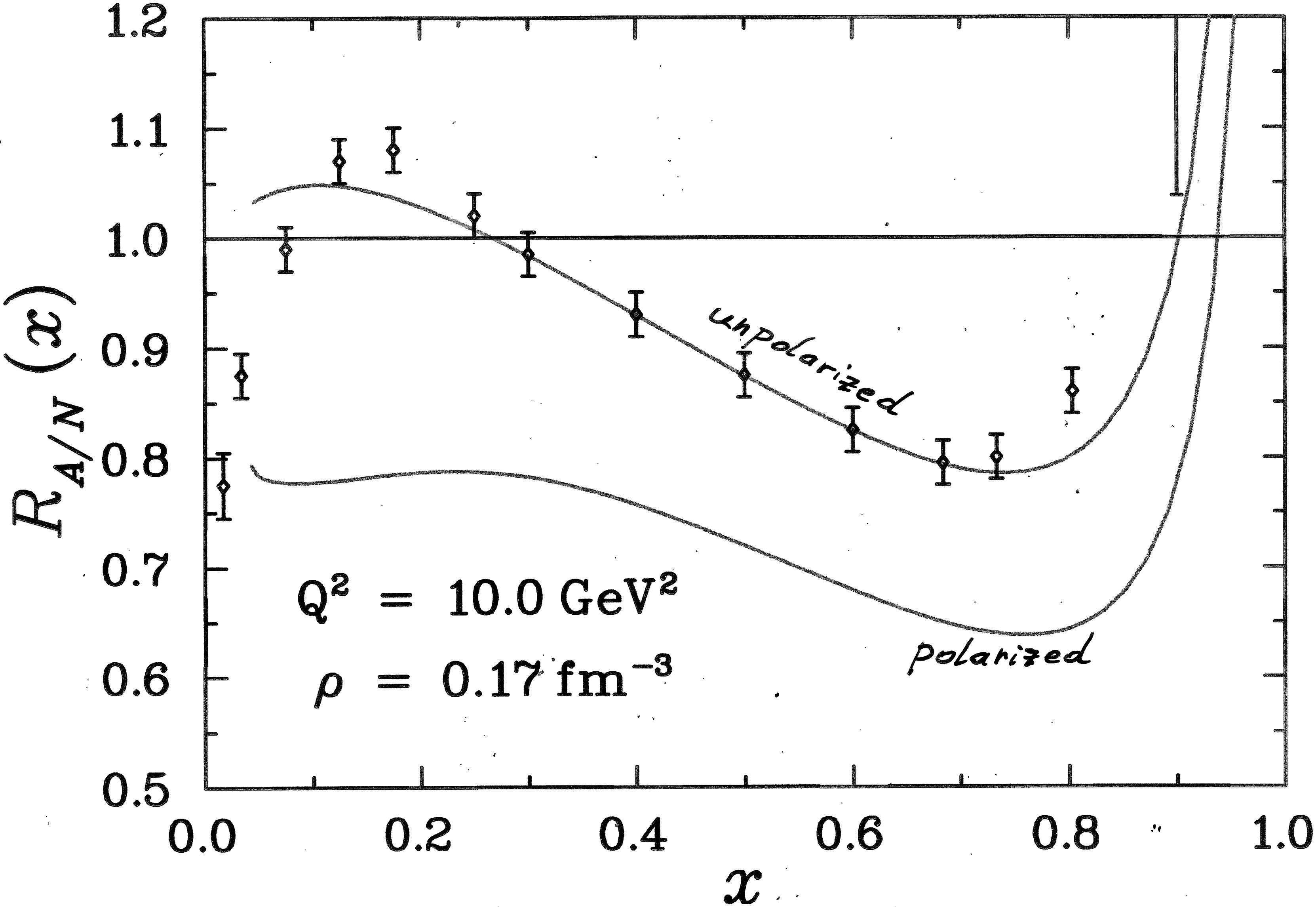


Spin dependent case , $Q_0^2 = 0.16$ GeV^2

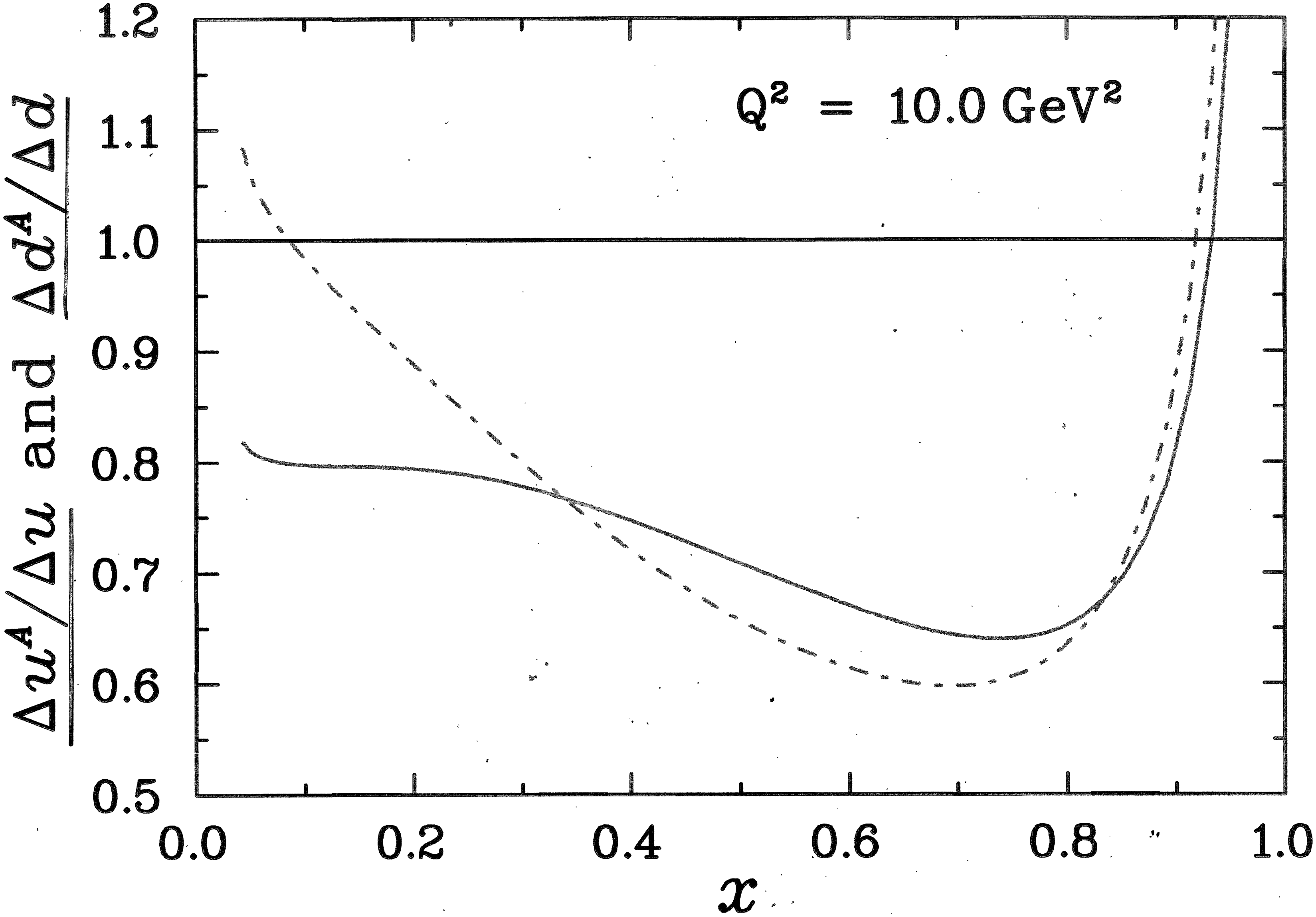


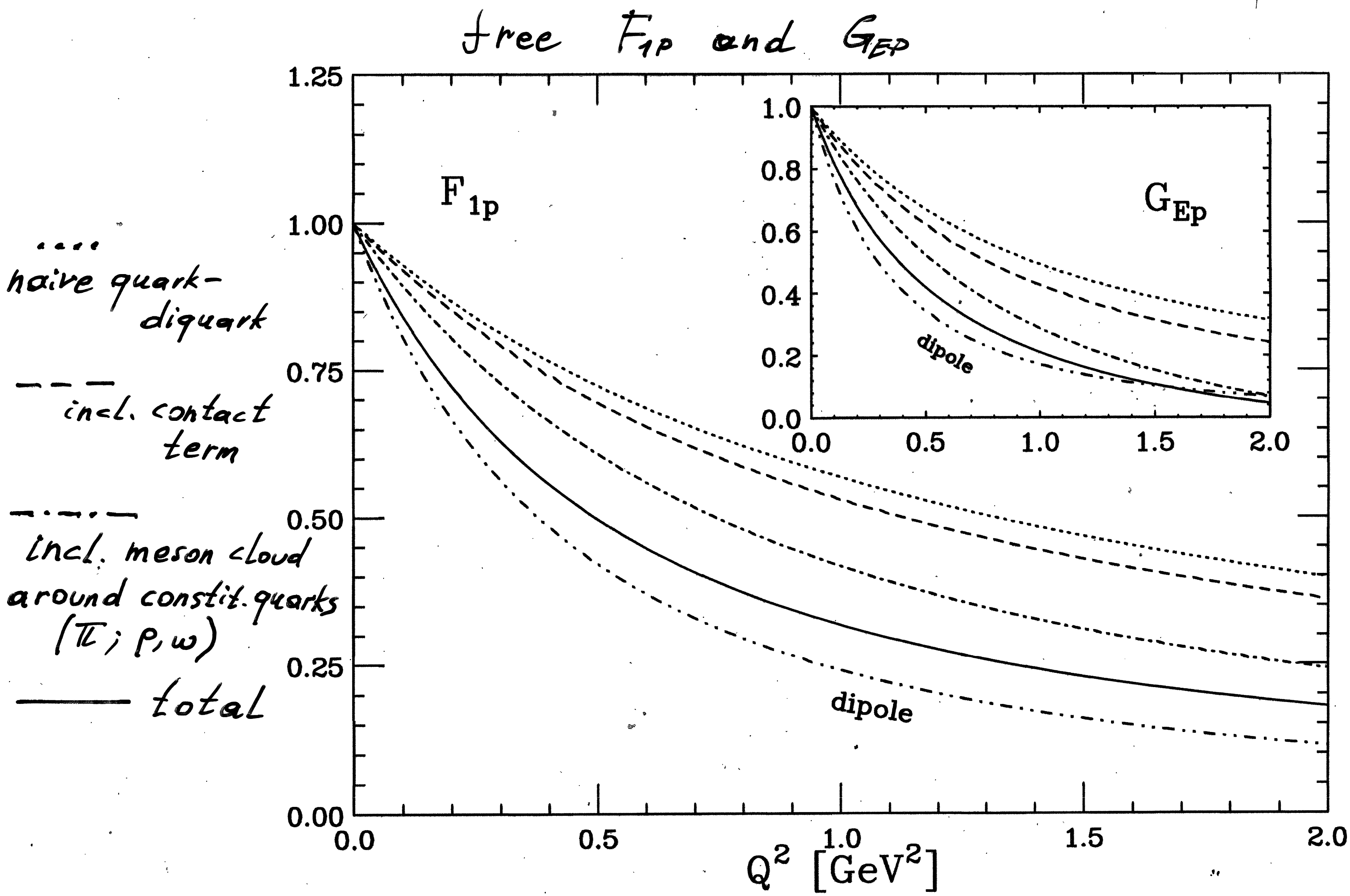
$\Delta u_A - \Delta d_A \equiv g_A^* = 1.013$, $\Delta u_A + \Delta d_A \equiv \Delta\Sigma^* = 0.408$

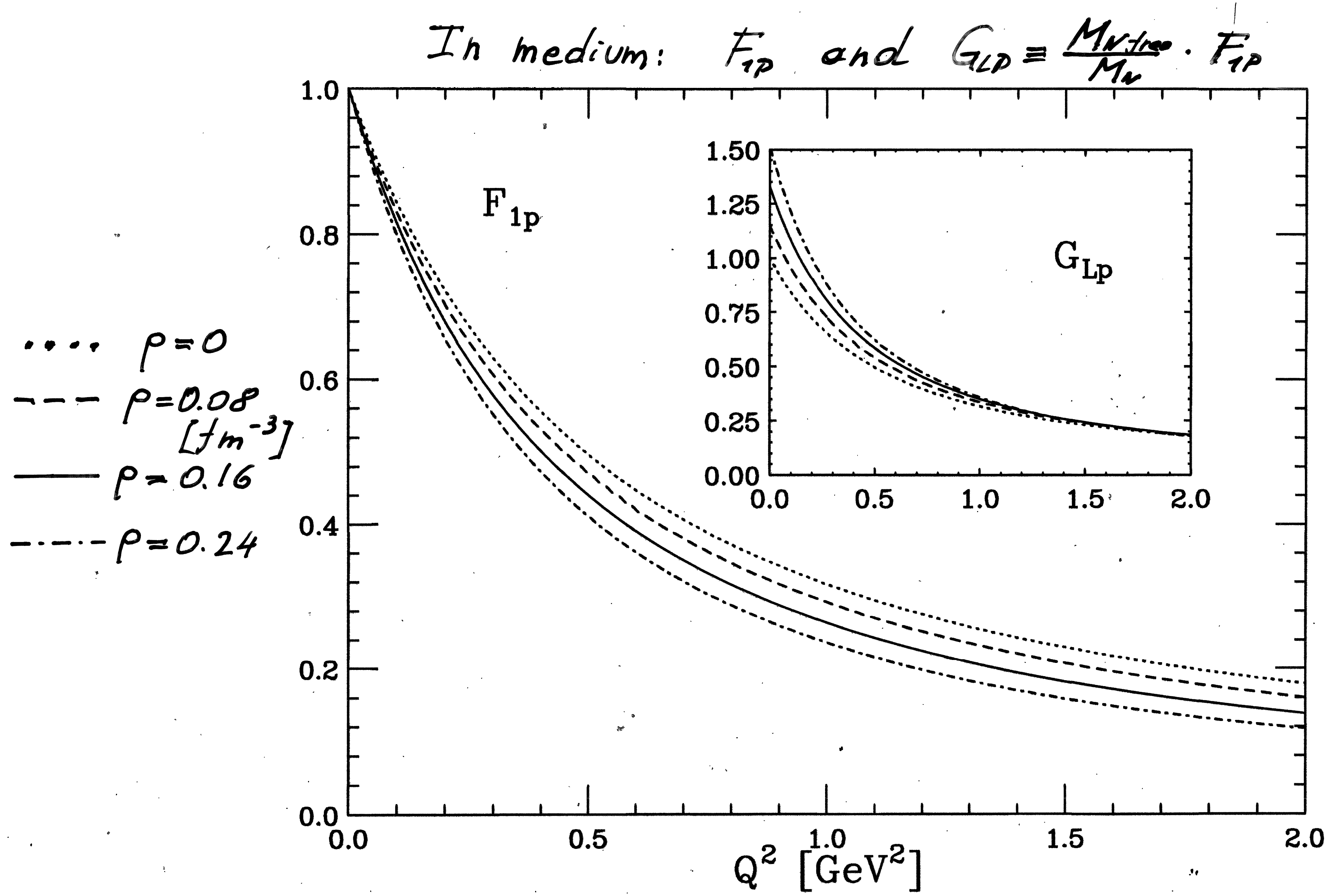
EMC ratio

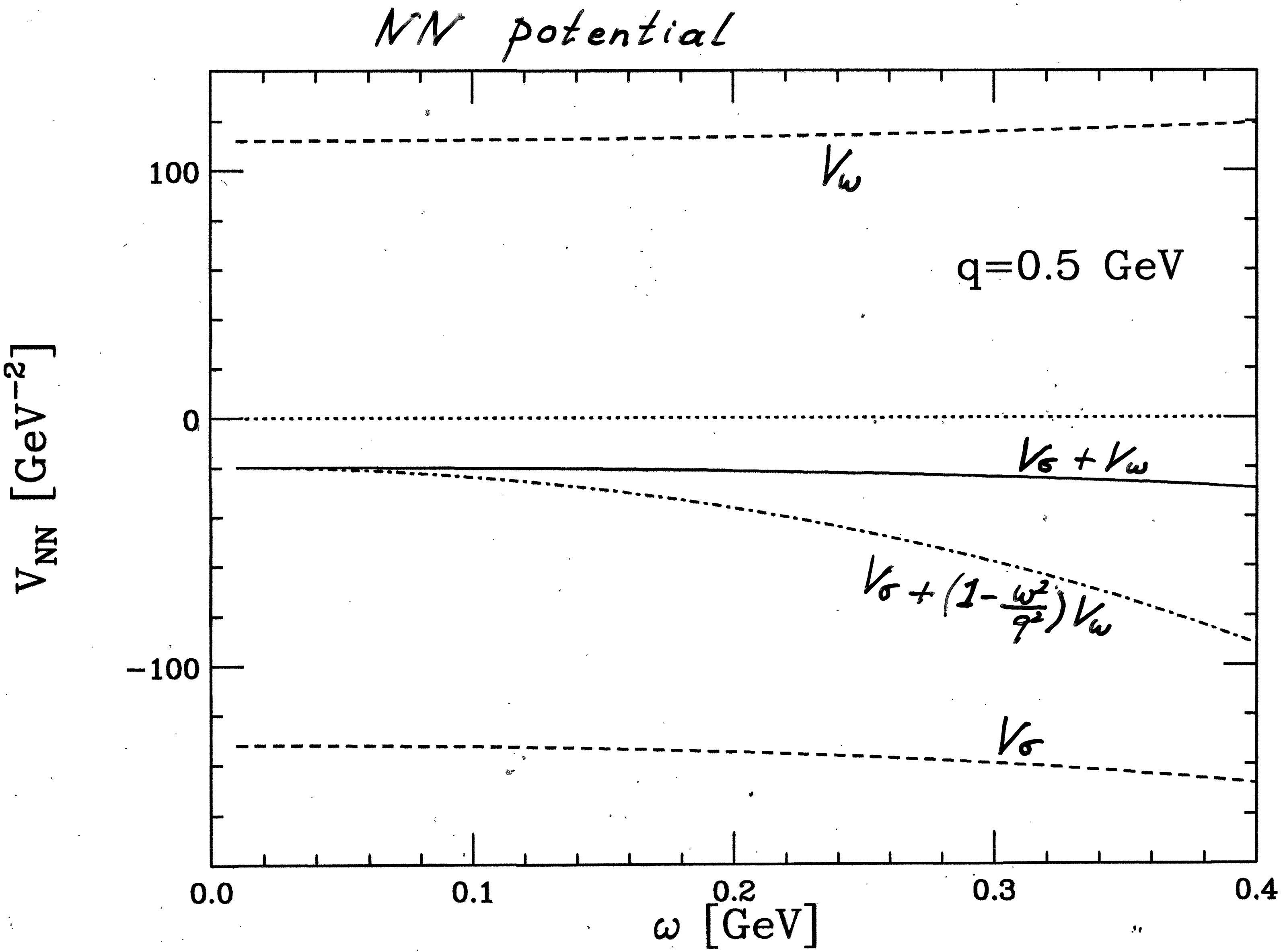


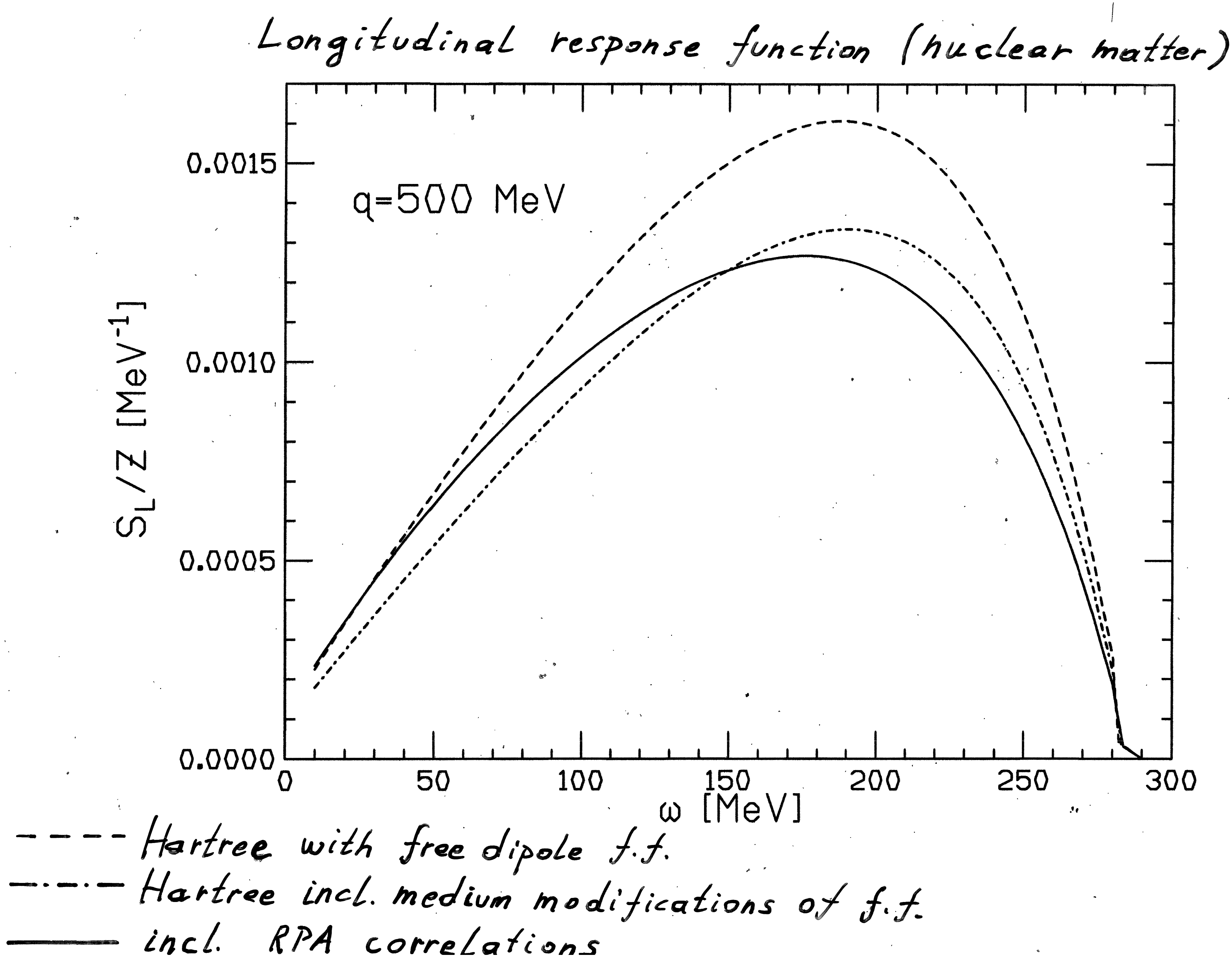
Nucl. matter "data": I. Sick, D.Day; PL B 274 ('92) 16.


$Q^2 = 10.0\ GeV^2$
$\Delta u^A/\Delta u$ and $\Delta d^A/\Delta d$
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
0.0
0.2
0.4
0.6
0.8
1.0
x


free F_{1p} and G_{Ep}
.... naive quark-diquark
– – – incl. contact term
–·–·– incl. meson cloud around constit. quarks (π; ρ, ω)
—— total
F_{1p}
G_{Ep}
dipole
dipole
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.5
1.0
1.5
2.0
0.0
0.2
0.4
0.6
0.8
1.0
Q^2 [GeV^2]


In medium: F_{1p} and $G_{Lp} \equiv \frac{M_{N\,free}}{M_N} \cdot F_{1p}$
$\rho=0$
$\rho=0.08$ [fm^{-3}]
$\rho=0.16$
$\rho=0.24$
F_{1p}
G_{Lp}
Q^2 [GeV^2]
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.5
2.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50


NN potential
100
0
−100
0.0
0.1
0.2
0.3
0.4
V_{NN} [GeV^{-2}]
ω [GeV]
q=0.5 GeV
V_ω
$V_\sigma + V_\omega$
$V_\sigma + (1 - \frac{\omega^2}{q^2}) V_\omega$
V_σ


Longitudinal response function (nuclear matter)
q=500 MeV
S_L/Z [MeV^{-1}]
0.0000
0.0005
0.0010
0.0015
0
50
100
150
200
250
300
ω [MeV]
Hartree with free dipole f.f.
Hartree incl. medium modifications of f.f.
incl. RPA correlations

Summary

Effective chiral quark theories provide a simple tool to incorporate nucleon structure into many-body physics.

In our investigation, we found the following points:

- Binding effects on quark level (i.e., scalar and vector fields acting on quarks in the nucleons) can largely reproduce the EMC effect.
  ($\Leftrightarrow$ Direct effect of vector field on $q_A(x_A)$.)

- Medium modification of spin-dependent structure function ($g_1(x)$) seems to be larger than spin-independent case $\Rightarrow$ Polarized EMC effect may be more pronounced than the ordinary (unpolarized) one.

- Medium modification of form factors leads to an appreciable quenching of the longitudinal response function.

For details, see:

- I.C. Cloet, W. Bentz, A. Thomas: *Nucleon quark distributions in a covariant quark-diquark model*, hep-ph/0504229.
- I.C. Cloet, W. Bentz, A. Thomas: *Spin-dependent structure functions in nuclear matter and the polarized EMC effect*, nucl-th/0504019.
- T. Horikawa, W. Bentz: *Medium modifications of nucleon electromagnetic form factors*, nucl-th/0506021