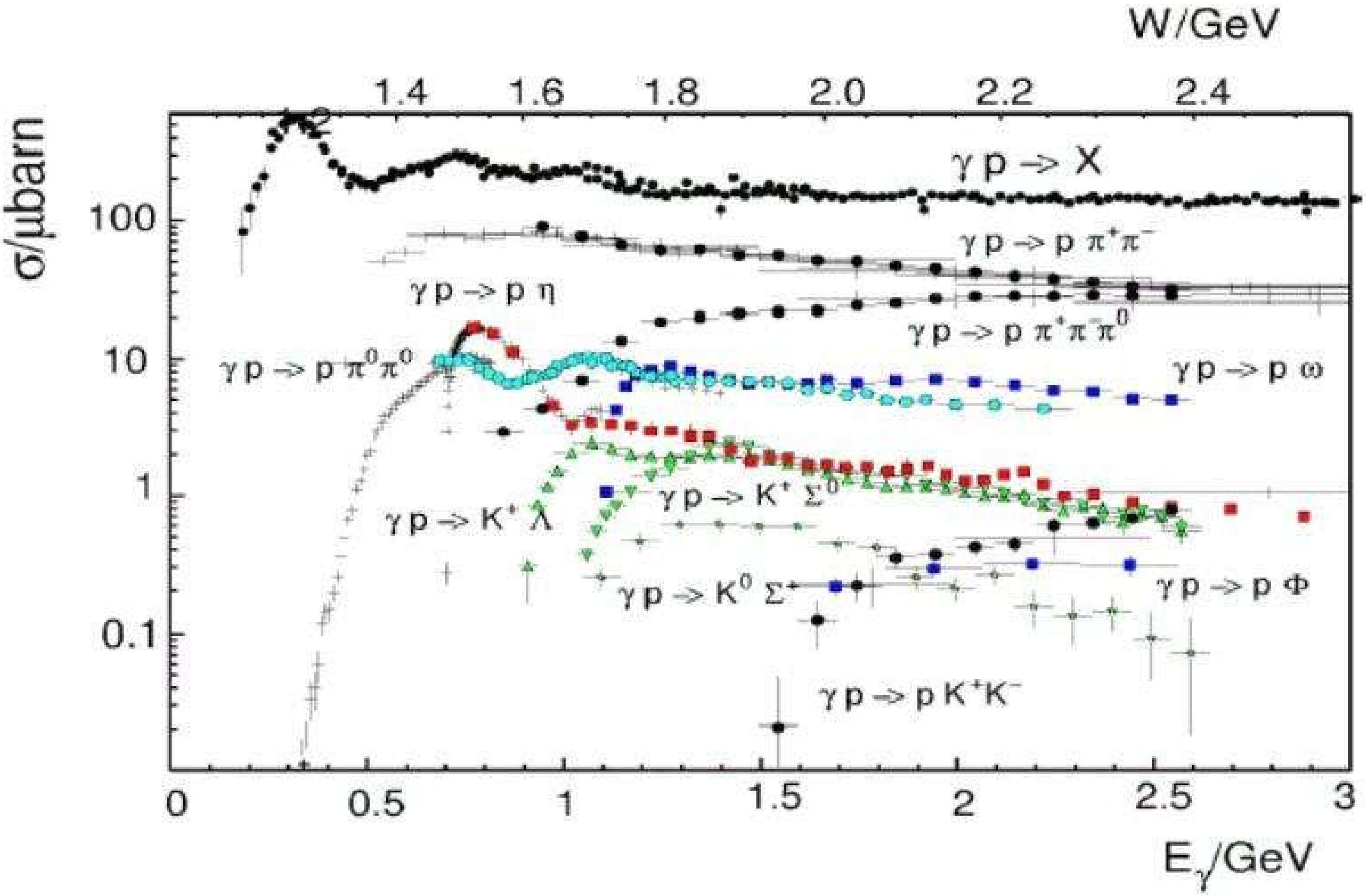
Reaction Models of Electromagnetic Meson Production in the Nucleon Resonance Region

T.-S. Harry Lee

Argonne National Laboratory

Data of γp reaction cross sections



- Challenge :

  Extensive data of electromagnetic production of
  π, η, K, ω, ϕ, and $\pi\pi N$ (ρN, $\pi\Delta$)

$$\downarrow\downarrow$$

  Understand the structure of nucleon resonances (N^*)

$$\downarrow\downarrow$$

  Understand non-perturbative QCD :

  - Confinement of constituent quarks
  - Chiral dynamics of meson cloud of baryons

- Traditional practice:

  Amplitude Analyses of data

  $\rightarrow$

  Extract N^* parameters

  No interpretation of N^* parameters

- Theoretical task :

  Develop Dynamical Reaction Models

  $\rightarrow$

  Extract N^* parameters

  Also attempt to interpret N^* parameters in terms of QCD :

  - Hadron Models (now)
  - Lattice QCD (near future)

Question :

Why do we need reaction models ??

(difficult and complicated)

$\rightarrow$

Answer :

Many-year's experiences in nuclear physics

Herman Feshbach :

"Much of the present-day understanding of $nuclear\ structure$ has been gained from the study of $nuclear\ reactions$. For this purpose it is necessary to understand the dynamics of $nuclear\ reactions$, while at the same time methods must be developed that permit the extraction of $nuclear\ structure$ information."

$\rightarrow$

Develop reaction models

to separate $reaction\ mechanisms$ from $nuclear\ structure$

(Optical potentials, DWIA, Coupled-channel, multiple-scattering etc.)

$\rightarrow$

Replace $nuclear \rightarrow hadron$

Similar situation in the study of N^* structure

$\rightarrow$

Develop reaction models

to separate $reaction\ mechanisms$ from $hadron\ structure$

Question :

What could be wrong in using the information from amplitude analyses (Particle Data Group) to test theoretical models of N^* structure ?

- Analyses are often guided by numerical simplicity in many-parameters fits

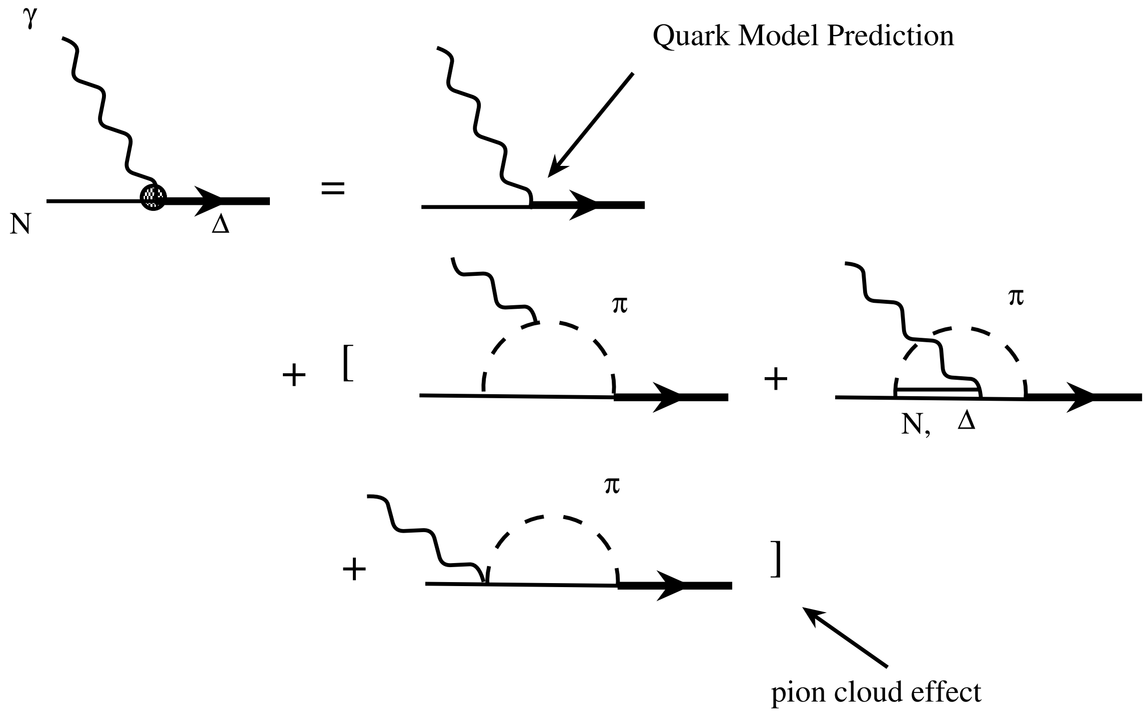
$\rightarrow$

Answer:

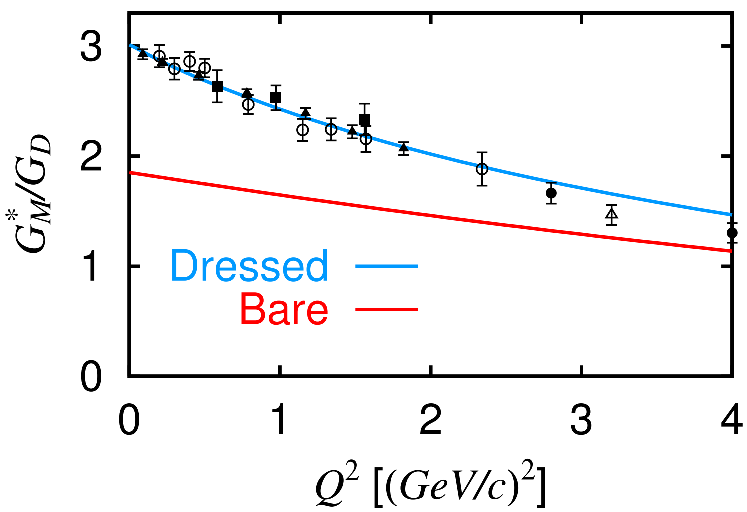
Results from the study of N-Δ transitions in the past 10 years

In a dynamical reaction model of $\gamma N \rightarrow \pi N$:

$$\bar{\Gamma}_{\gamma N \rightarrow \Delta} \quad = \quad \Gamma_{\gamma N \rightarrow \Delta} + \bar{\Gamma}_{\pi N \rightarrow \Delta} G_{\pi N}(E) v_{\gamma \pi}$$



$\gamma N \to \Delta$ Magnetic Dipole $G_M(Q^2)$

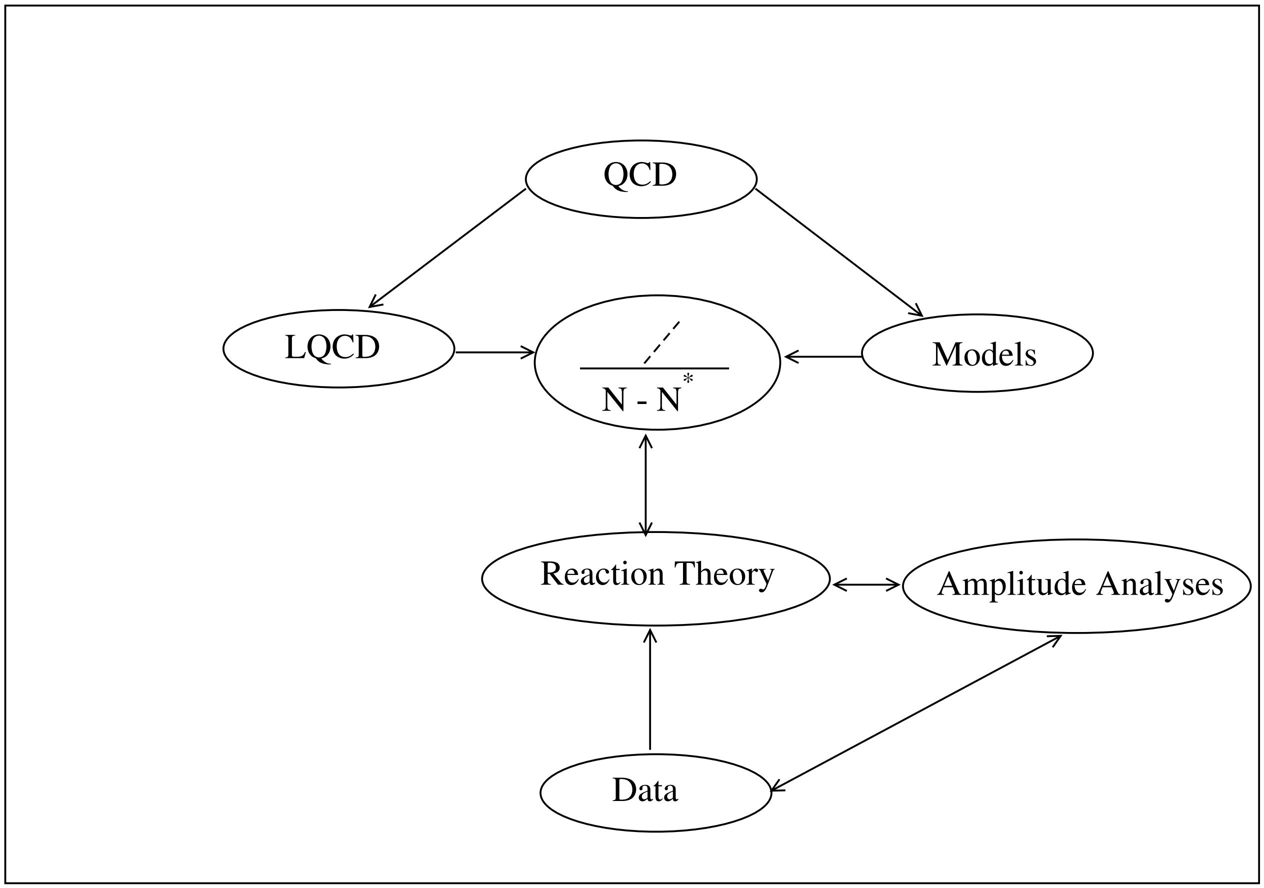


- Pion cloud has a very large effect on G_M; about 40% at $Q^2 = 0$
- At $Q^2 = 0$, $G_M^{bare} \sim G_M^{SU(6)} \sim 2$.

  $\rightarrow$

  Resolve a *long-standing* puzzle !!

Possible Plan



This talk:

- Review the current reaction models of meson production reactions
- Report the status of a reaction model being developed by Argonne-Osaka-Schizuoka collaboration

  (A. Matsuyama, T. Sato, T.-S. H. Lee)

General Consideration

Question :

What is the structure of N^* ?

Theoretical guidence:

Chiral symmetry in QCD is broken spontaneously

$\rightarrow$

N^* can be described in terms of constituent quarks and meson clouds

$\rightarrow$

$$\mid N^* >=\mid N^*_0 > + \mid N^*_0\pi > + \mid N^*_0\pi\pi > + \cdots$$

$$\mid N^*_0 >=\mid qqq >$$

$q =$ constituent quarks

$\rightarrow$

A tractable reaction model is based on:

- each quark core (N_0^*) is treated as an elementary field ψ_B
- N_0^* structure is defined by form factors

data $\rightarrow$ [form factors] $\leftarrow$ hadron models/Lattice QCD

$\rightarrow$

Starting Interaction Lagrangian :

$$L_I = \sum_{B,B'} \sum_M \bar{\psi}_{B'}[\Gamma^0 \phi_M]\psi_B$$

- ψ_B =bare $N, \Delta, N_1^*, N_2^* \cdot \cdot$
- $\phi_M = \gamma, \pi, \eta, K, \omega, \phi \cdots$
- Γ^0 must be consistent with chiral symmetry :

  $L_I \sim \bar{\psi}_N[\gamma_5 \gamma^\mu \partial_\mu \phi_\pi]\psi_N + \cdots$

Approximations:

- Ladder Bethe-Salpeter equations
  - I. Afnan and collaborators
  - N. Kaiser, E. Oset, M. Lutz et al
- Three-dimensional ladder Bethe-Salpeter equations
  - Julich coupled-channel model
  - F. Gross and Y.Suyra
  - Dubna-Mainz-Taipei (MAID-DMT) model (S.N. Yang)
  - V. Pascalutsa, J. Tjon, G. L. Caia, L. Wright
  - Many earlier πN models
- Unitary transformation method
  - T. Sato, A. Matsuyama, and T.-S. H. Lee
  - M. Fuda
  - B. Julia-Diaz W.-T. Chiang, B. Saghai, F. Tabakin, T.-S. H. Lee

Focus on :

Formulation based on unitary transformation method

$\rightarrow$

- Derive most of the current reaction models

Method of Unitary Transformation

- Start with a Lagrangian $L(x)$ of relativistic quantum field theory
- Apply the canonical quantization to define Hamiltonian density

$$h(x) = \sum_B \pi_B(x)\partial_0\psi_B(x) + \sum_M \pi_M(x)\partial_0\phi_M(x) - L(x)$$

- Define Hamiltonian in Fock-space

$$H = \int d\vec{x} h(\vec{x}, t=0)$$

- Apply unitary transformation to derive $H_{eff} = U^\dagger H U$ which leads to

Soluble few-body scattering equations

Example : $L_I = \bar{\psi}_{B'}\Gamma^0\phi_M\psi_B$ ($B = N, \Delta, M = \pi, \gamma$)

$\rightarrow$ Hamiltonian :

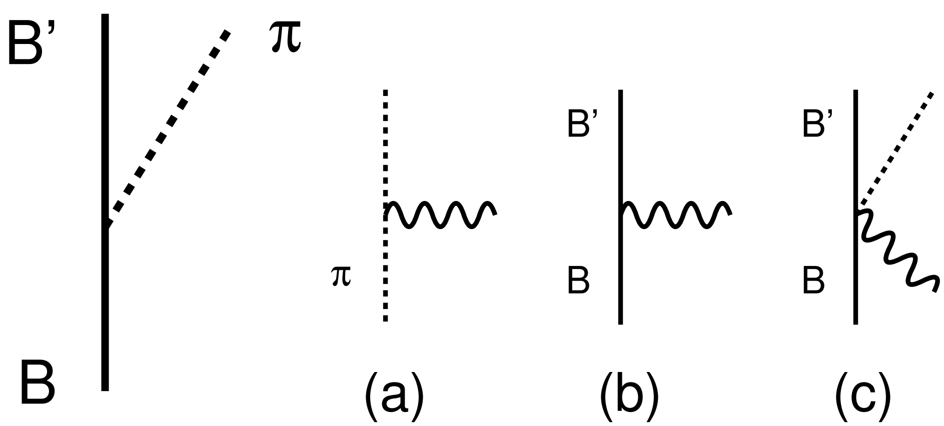
$$H = H_0 + H_I + H_{em} \ ,$$

with

$$H_I = \sum_{B,B'} \Gamma^0_{\pi B',B}$$

$$H_{em} = \int d\boldsymbol{x} A \cdot J$$

$$J^\mu = J^\mu_\pi + J^\mu_{B',B} + J^\mu_{B',B,\pi},$$



- Decompose interaction term :

$$H_I = H_1^P + H_1^Q \, ,$$

Physical process: $\Delta \leftrightarrow \pi N$

$$H_1^P = \Gamma^0_{\pi N, \Delta}$$

Unphysical process : $N \leftrightarrow \pi N$, $N \leftrightarrow \pi \Delta$, $\Delta \leftrightarrow \pi \Delta$

$$H_1^Q = \Gamma^0_{\pi N, N} + \Gamma^0_{\pi \Delta, N} + \Gamma^0_{\pi \Delta, \Delta}$$

- Introduce unitary transformations

$$U_n = \exp(iS_n)$$
$$S_n \propto (H_I)^n$$

$\rightarrow$

$$H^{(n)} = U_n^\dagger U_{n-1}^\dagger \cdot\cdot H \cdot\cdot U_{n-1} U_n$$
$$= H_{eff}(g^1, \cdot\cdot\cdot, g^n) + \sum_{m>n} [H^P(g^m) + H^Q(g^m)]$$

$H_{eff}(g^1, \cdot\cdot\cdot, g^n)$: no unphysical processes ($H_I \propto g$)

Consider n=2 :

$$H = H_{eff}(g^1, g^2) = H_0 + V$$
$$V = v^{bg} + \sum_{N_i^*}[\Gamma_i + \Gamma_i^\dagger]$$

v^{bg} : Non-resonant $MB \rightarrow M'B'$

Γ_i : $N^* \rightarrow MB$

$\rightarrow$

Scattering amplitude

$$T(E) = V + V\frac{1}{E - H + i\epsilon}V$$

Main Feature : V is energy-independent and hermitian

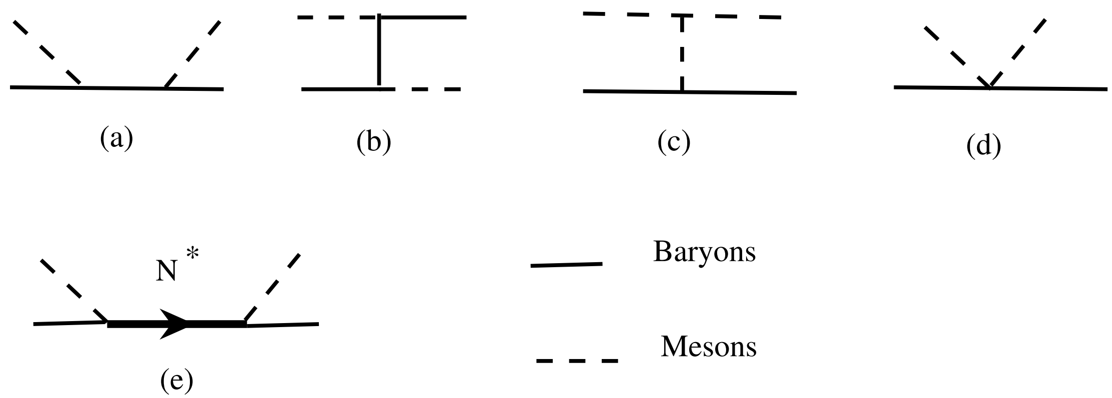
$\rightarrow$

Unitarity condition is trivially satisfied.

n=2 interactions:

v^{bg} : Non-resonant

$v^R = \frac{\Gamma_i^\dagger \Gamma_i}{E - M_{N_i^*}}$: Resonant



$\rightarrow$

Can derive

- Unitary Isobar Models :

  MAID

  Jlab/Yerevan UIM

- Multi-channel K-matrix models :

  SAID

  Giessen

  Kent State University (KSU)

- Carnegie-Mellon Berkeley (CMB) Model

- Dynamical reaction models

Starting point :

- Relation between scattering T and K matrix :

$$T(E) = V + V\left[\frac{P}{E - H_0} - i\pi\delta(E - H_0)\right]T(E)$$

$$K(E) = V + V\frac{P}{E - H_0}K(E)$$

P : the principal-value integration.

$\rightarrow$

$$T(E) = K(E) - T(E)[i\pi\delta(E - H_0)]K(E)$$

$\rightarrow$

Lead to on-shell relations between T and K

Approaches :

- Start with $V = v^{bg} + v^{R}$:

$$T_{a,b}(k_a, k_b, E) = V_{a,b}(k_a, k_b) + \sum_c \int dk \frac{V_{a,c}(k_a, k)T_{c,b}(k, k_b)}{E - E_{M_c}(k) - E_{B_c}(k) + i\epsilon}$$

a,b = πN, γN, ηN, ωN, KY, $\rho N, \pi\Delta, \sigma N$ (represent $\pi\pi N$)

– Need off-shell information

– Equations for Dynamical Models

- Start with K matrix:

  A matrix relation in partial-wave representation:

$$T_{a,b}(E) = \sum_c [(1 + iK(E))^{-1}]_{a,c} K_{c,b}(E)$$

  a,b = πN, γN, ηN, ωN, KY, $\rho N, \pi\Delta, \sigma N$ (represent $\pi\pi N$)

  – Need only on-shell information

  – Equations for K-matrix Models

Derivations

- Unitary Isobar Model (UIM) :
  - start with K matrix
  - channels : γN, πN (or ηN)

$\rightarrow$

$\gamma N \rightarrow \pi N$ amplitude :

$$
\begin{aligned}
T_{\pi N,\gamma N} &= [1 + iK_{\pi N,\pi N}(E)]^{-1} K_{\pi N,\gamma N}(E) \\
&= e^{i\delta_{\pi N}} cos\delta_{\pi N} K_{\pi N,\gamma N}(E) \\
&\sim e^{i\delta_{\pi N}} cos\delta_{\pi N} V_{\pi N,\gamma N}
\end{aligned}
$$

$V_{\pi N,\gamma N}$ = Tree-diagrams

$\delta_{\pi N}$: πN phase shifts

$\rightarrow$

Satisfy Watson Theorem in $W < 1.3$ GeV

– MAID and Jlab/Yerevan UIM :

1. Include of N^* by using Walker's parameterization
2. Unitarize the total amplitude

$\rightarrow$

$$T_{\pi N,\gamma N}(UIM) = e^{\delta} cos\delta[v^{bg}_{\pi N,\gamma N}] + \sum_{N_i^*} T^{N_i^*}_{\pi N,\gamma N}(W)$$

$$T^{N_i^*}_{\pi N,\gamma N}(E) = f_{\pi N}(W)\frac{\Gamma^{tot} M_i e^{i\Phi_i}}{M_i^2 - W^2 - iM_i\Gamma^{tot}} A_{\gamma N}(W)$$

Φ_i : Unitarization Phase

Results from MAID and JLab/Yerevan UIM:

1. Successful in extracting Δ parameters
2. Can fit pion production data up to W=2 GeV
3. More will be discussed in I. Aznauryan's talk

Comments:

Coupled-channel effects are not treated explicitly

[$\gamma N \rightarrow (\pi\Delta, \rho N \cdot\cdot) \rightarrow \pi N$ is neglected]

$\rightarrow$

The extracted N^* parameters in the second and third resonance regions need to be verified

- Multi-channel K-matrix models
  - SAID :

    Consider γN, πN, $\pi\Delta$(all inelastic channels)

    $\rightarrow$

$$
\begin{aligned}
T_{\gamma N,\pi N}(SAID) &= A_I(1 + iT_{\pi N,\pi N}) + A_R T_{\pi N,\pi N} \\
A_I &= K_{\gamma N,\pi N} - \frac{K_{\gamma N,\pi\Delta} K_{\pi N,\pi N}}{K_{\pi N,\pi\Delta}} \\
A_R &= \frac{K_{\gamma N,\pi\Delta}}{K_{\pi N,\pi\Delta}}
\end{aligned}
$$

Actual analysis:

$$A_I = v^{bg}_{\gamma N,\pi N} + \sum_{n=0}^{M} \bar{p}_n z Q_{l_\alpha+n}(z)$$

$$A_R = \frac{m_\pi}{k_0}(\frac{q_0}{k_0})^{l_\alpha} \sum_{n=0}^{N} p_n (\frac{E_\pi}{m_\pi})^n$$

$\bar{p}_n, p_n$: fitting parameters

N^* parameters are extracted by fitting the resulting amplitudes to a Briet-Wigner parameterization at $W \rightarrow M^*$

Results from SAID :

$*$ determine $\pi N \to \pi N$ amplitudes

$*$ determine $\gamma N \to \pi N$ multipole amplitudes

$*$ extract N^* parameters

Comments :

Its many-parameter parameterization of the non-resonant amplitudes need to be justified theoretically

Coupled-channel effects are not treated explicitly

$\rightarrow$

The extracted N^* parameters in the second and third resonance regions need to be verified

– Giessen Model :

Approximation : $K = V =$ Tree-daigrams

$\rightarrow$

$$T_{a,b}(Giessen) = \sum_c [(1 + iV(E))^{-1}]_{a,c} V_{c,b}(E)$$

Results:

$*$ Fit both πN and γN reaction data with channels: γN, πN, σN, ηN, $K\Lambda$, $K\Sigma$ and ωN.

$*$ Indentify N^* :$P_{31}(1750)$, $P_{13}(1900)$, $P_{33}(1920)$, $D_{13}(1950)$

Comments :

Multiple-scattering effects in K matrix is neglected

$\rightarrow$

The extracted N^* parameters in the second and third resonance regions need to be verified

For deriving:

- Carnegie-Mellon Berkeley (CMB) Model
- Kent State University (KSU) model
- Dynamical models

Apply two-potential scattering formulation

for $V = v^{bg} + \frac{\Gamma^\dagger_{N*}\Gamma_{N*}}{E - M^0_{N*}}$

$\rightarrow$

$$T(E) = t^{bg}(E) + \frac{\bar{\Gamma}^\dagger_{N*}(E)\bar{\Gamma}_{N*}(E)}{E - M^0_{N*} - \Sigma_{N*}(E)}$$

$$t^{bg} = v^{bg} + v^{bg}G(E)t^{bg}(E)$$

Resonances are detrmined :

$$\bar{\Gamma}_{N^*} = \Gamma_{N^*} + \Gamma_{N^*} G(E) t^{bg}(E)$$

$$\Sigma_{N^*}(E) = \Gamma^{\dagger}_{N^*} G(E) \bar{\Gamma}_{N^*}$$

Main feature :

- Non-resonant effects on resonance parameters are identified

For multi -channels multi -resonances case:

$$T_{a,b}(E) = t^{bg}_{a,b}(E) + \sum_{N^*_i, N^*_j} \bar{\Gamma}^\dagger_{N^*_i,a}(E)[\hat{G}(E)]_{i,j}\bar{\Gamma}_{N^*_j,b}(E)$$

$$t^{bg}_{a,b} = v^{bg}_{a,b} + \sum_c v^{bg}_{a,c} G_c(E) t^{bg}_{c,b}(E)$$

$$\bar{\Gamma}_{N^*,a} = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t^{bg}_{b,a}$$

$$[\hat{G}(E)^{-1}]_{i,j}(E) = (E - M^0_{N^*_i})\delta_{i,j} - \Sigma_{i,j}(E)$$

$$\Sigma_{i,j}(E) = \sum_a \Gamma^\dagger_{N^*,a} G_a(E) \bar{\Gamma}_{N^*_j,a}$$

$a, b = \gamma N$, πN, ηN, ωN, KY, $\pi\Delta$, ρN, σN (for $\pi\pi N$)

- Carnegie-Mellon Berkeley (CMB) Model

Set : $v^{bg}_{a,b}(E) = \frac{\Gamma^\dagger_{L,a}\Gamma_{L,b}}{E-M_L} + \frac{\Gamma^\dagger_{H,a}\Gamma_{H,b}}{E-M_H}$ (separable form)

$\rightarrow$

$$V = v^{bg} + v^R = \sum_{i=N^*_i,L,H} \frac{\Gamma^\dagger_{i,a}\Gamma_{i,b}}{E - M_i} = Separable$$

$\rightarrow$

$$\begin{aligned}
T_{a,b}(E) &= \sum_{i,j} \Gamma^\dagger_{i,a} G_{i,j}(E) \Gamma_{j,b} \\
G(E)^{-1}_{i,j} &= (E - M^0_i)\delta_{i,j} - \Sigma_{i,j}(E) \\
\Sigma_{i,j}(E) &= \sum_a \int k^2 dk \frac{\Gamma^\dagger_{i,a}(k)\Gamma_{j,a}(k)}{E - E_{M_a}(k) - E_{B_a}(k) + i\epsilon}
\end{aligned}$$

With appropriate variable changes : $s = E^2$

$\rightarrow$

CMB's dispersion relations :

$$
\begin{aligned}
\Sigma_{i,j}(s) &= \sum_c \gamma_{i,c}\Phi_c(s)\gamma_{j,c} \\
Re[\Phi_c(s)] &= Re[\Phi_c(s_0)] + \frac{s - s_{th,c}}{\pi}\int_{s_{th}}^{\infty}\frac{Im[\Phi_c(s')]}{(s'-s)(s'-s_0)}ds'
\end{aligned}
$$

$\rightarrow$

CMB model is analytic

Recent applications/extensions of CMB model :

- Zagreb : M. Batinic, A. Svarc and collaborators

  Consider three channels : πN, ηN, $\sigma(\pi\pi)N$

- PITT-ANL : T. Varana, S. Dytman, T.-S. H. Lee

  Consider up to eight channels:
  πN, ηN, $\pi\Delta$, ρN, $\sigma(\pi\pi)N$, $\pi N^*(1440)$, $K\Lambda$, γN

Results:

- N^* in S_{11} channel is better understood
- The interplay between channel coupling and N^* excitation has been better understood
- Some extracted N^* parameters are very different from PDG values

  $\rightarrow$

  be careful in using PDG's values to test hadron models

Comments :

Its separable non-resonant amplitudes need to be justified theoretically

$\rightarrow$

The extracted N^* parameters in the second and third resonance regions need to be verified

- Kent State University (KSU) model

Start with

$$T(E) = t^{bg}(E) + \frac{\bar{\Gamma}^{\dagger}_{N*}\bar{\Gamma}_{N*}}{E - M^0_{N*} - \Sigma_{N*}(E)}$$

One can derive exactly the distorted-wave form

$$\begin{aligned} S(E) &= 1 + 2iT(E) \\ &= \omega^{(+)T}[1 + 2i\frac{\Gamma^{\dagger}_{N*}(E)\Gamma_{N*}}{E - M^0_{N*} - \Sigma_{N*}(E)}]\omega^{(+)} \end{aligned}$$

where

$$\omega^{(+)} = 1 + G(E)t^{bg}(E) \tag{1}$$

$\rightarrow$

S-matrix :

$$S(E) = \omega^{(+)T} R(E) \omega^{(+)}$$

$$R(E) = 1 + 2iT^R(E)$$

$$T^R(E) = \frac{\Gamma^{\dagger}_{N^*}(E)\Gamma_{N^*}(E)}{E - M^0_{N^*} - \Sigma_{N^*}(E)}$$

KSU separable parameterization:

$$T^R(E) = \frac{K}{1+iK} \sim x \frac{\Gamma/2}{E - M - i\Gamma/2}$$

$$\omega^{(+)} = B_1 B_2 \cdot \cdot B_n$$

$$B_i \sim e^{iX\Delta_i}$$

Γ, x and X are parameters in the fit

Results from KSU:

- fits to E_{0+} of $\gamma N \to \pi N$
- being applied to study kaon production

Comments :

Its separable parameterization of non-resonant amplitude need to be justified theoretically

$\rightarrow$

The extracted N^* parameters in the second and third resonance regions need to be verified

Dynamical Models

Two equivalent approaches:

- Solve dynamical equations with $V = v^{bg} + v^{R}$ directly :

$$T_{a,b}(E) = V_{a,b} + \sum_c V_{a,c} G_c(E) T_{c,b}(E)$$

$a, b, c = \pi N, \gamma N, \eta N, \pi\Delta \cdots$

Recent works :

  - Julich Model :πN
  - Fuda et al. :πN, γN
  - DMT Model : πN, γN, ηN (S.N. Yang's talk)
  - Ohio-Utrecht Model : πN, γN (V. Pascalutsa's talk)
  - Chiral SU(3) models : KY, ωN, γN, πN (M. Lutz's talk)

- Use two-potential formulation to identify resonant mechanism

$$T_{a,b}(E) = t^{bg}_{a,b}(E) + \sum_{N^*_i,N^*_j} \bar{\Gamma}^{\dagger}_{N^*_i,a}[D^{-1}(E)]_{i,j}\bar{\Gamma}_{N^*_j,b}$$

$$t^{bg}_{a,b}(E) = v^{bg}_{a,b} + \sum_c v^{bg}_{a,c} G_c(E) t^{bg}_{c,b}(E)$$

$$\bar{\Gamma}_{N^*,a} = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t^{bg}_{b,a}(E)$$

Recent Works

- Sato-Lee Model : πN, γN
- Yoshimoto et al. : πN, ηN, $\pi\Delta$
- Oh et al.: γN, ωN
- Julia-Diaz et al. : γN, KY, πN (B. Julia-Diaz's talk)
- Lee, Matsuyama, Sato (2004) : $\pi N, \eta N, \gamma N$

DMT model (S.N. Yang's talk)

Ohio-Utrecht Model (V. Pascalutsa's talk)

Kaon production (B. Julia-Diaz's talk)

Chiral SU(3) models (M. Lutz's and E. Kolomeitsev's talk)

Julich's Coupled-channel Model

O. Krehl, C. Hanhart, S. Krewald, J. Speth (2000)

- Channels : $\pi N, \eta N, \sigma N, \pi\Delta, \rho N$.
- V : meson-exchange, s-channel N^*
- fit : πN amplitudes up to 1.9 GeV
- Main result:

  P_{11} is due to meson-baryon coupled-channel effects

Comments :

1. It does not satisfy $\pi\pi N$ unitarity condition
2. Need to check its predictions of $\pi N \to \pi\pi N$ and γN cross sections

$\to$

Question on the nature of P_{11} is still open

Coupled-channel study of N^* in S_{11}

(T.-S. H. Lee, A. Matsuyama, T. Sato (2004))

- Channels : $\pi N, \eta N, \gamma N$
- Non-resonant int. : tree-diagram of chiral Lagrangian

  $v_{\pi N,\pi N}$, $v_{\pi N,\eta N}$, $v_{\eta N,\eta N}$

  $v_{\gamma N,\pi N}$, $v_{\gamma N,\eta N}$

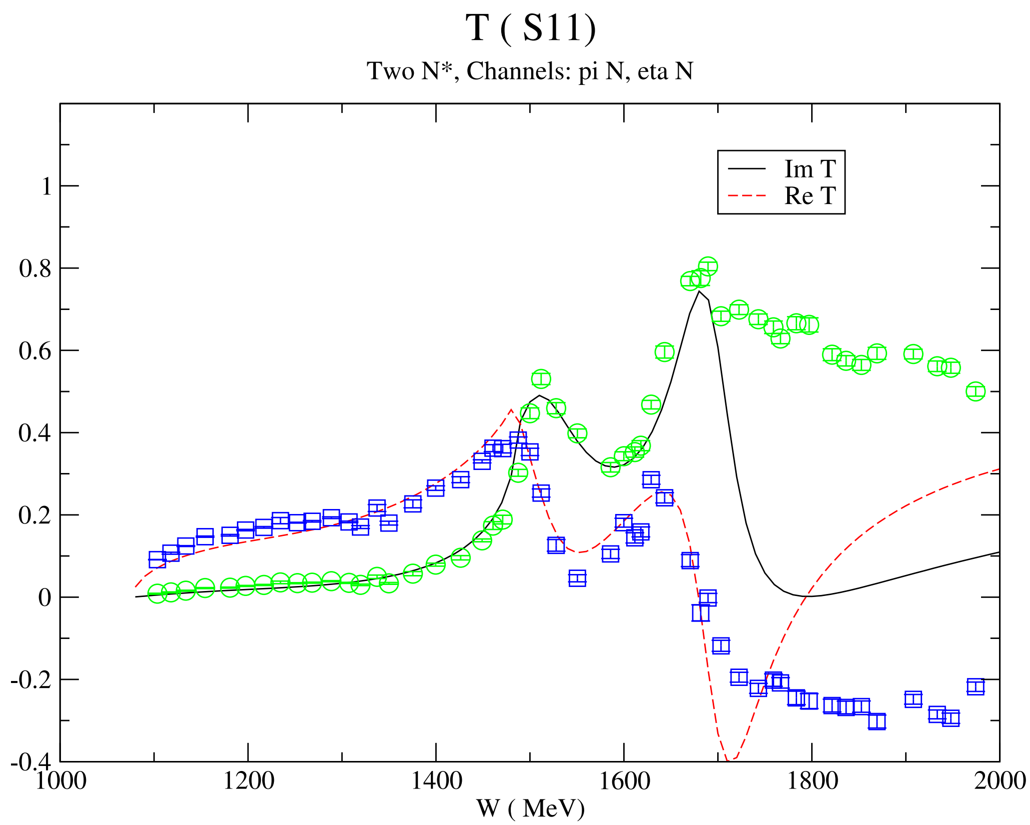
- 2 N^* : Related to constituent quark model

- Finding :
  1. Can describe the data only up to W= 1.7 GeV
  2. Meson cloud effect on $\gamma N \rightarrow N^*$ is about 20%
  3. Bare helicity amplitude is close to quark model

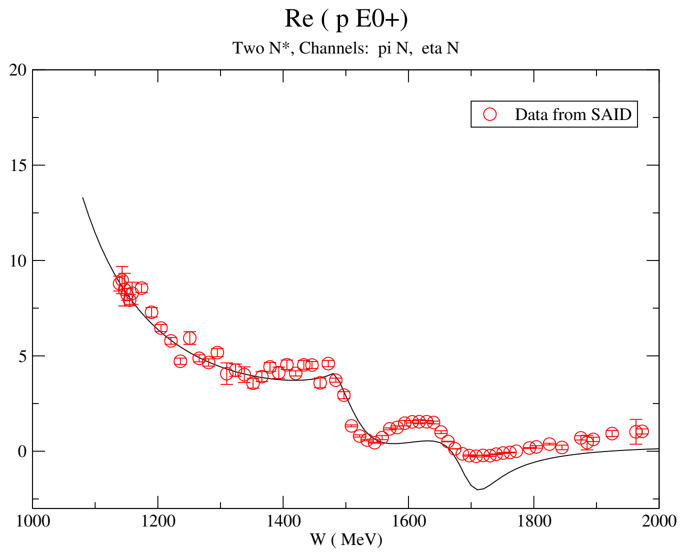
$\rightarrow$

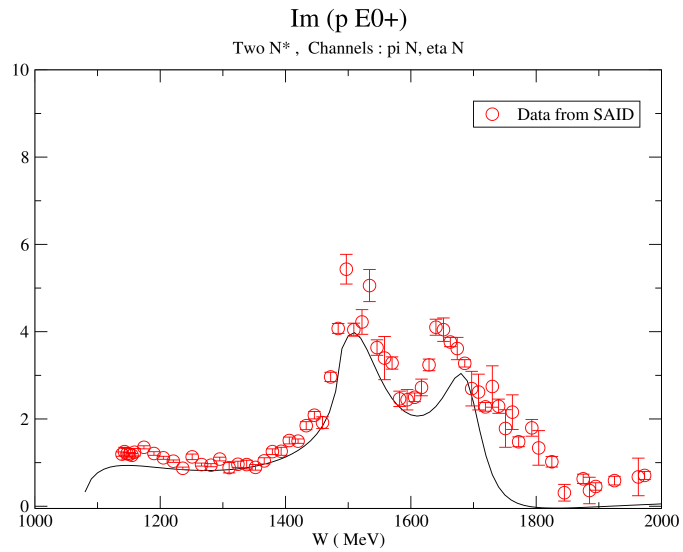
Consistent with the finding in the study of Δ excitation

$\pi N \to \pi N$ amplitude

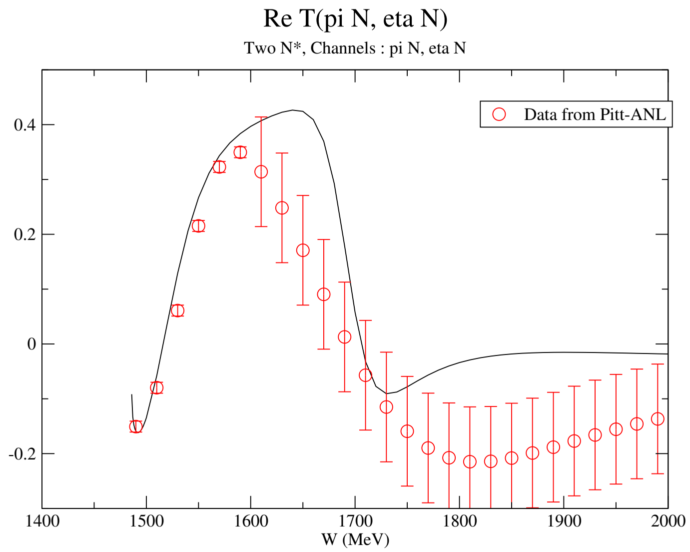


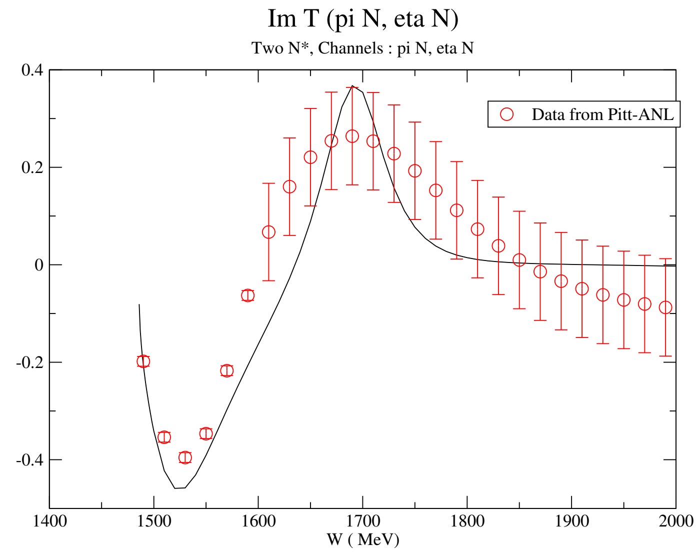
$\gamma N \to \pi N$ amplitude





$\pi N \rightarrow \eta N$ amplitude





	M_R	Γ_R	$\frac{\Gamma_\pi}{\Gamma_R}(\%)$	$A_{1/2}$
Coupled-channel model	1538	122	36	61.24
			Bare	(77.64)
Capstick				76

Comments :

To explore N^* from all πN and γN data up to $W = 2.5$ GeV, we need to include coupling with $\pi\pi N$ channel

$\rightarrow$

Develop coupled-channel model with $\pi\pi N$ channels

Argonne-Osaka-Shizuoka collaboration

A. Matsuyama, T. Sato, T.-S. H. Lee (in progress)

Coupled-channel model with $\pi\pi N$

$\rightarrow$

Apply second-order unitary transformation method to derive $H_{eff}(g^2, g^3)$

g : strong coupling constant of H_I

$$H'' = U_2^\dagger [H'] U_2$$

$$\begin{aligned} H' &= U_1^\dagger H U_1 \\ &= H_{eff}(g^2) + H_I'^Q(g^2) + H_I'^P(g^3) + H_I'^Q(g^3) + \cdots \end{aligned}$$

Note :

SL model : $H_{eff}(g^2)$ in $\Delta \oplus \gamma N \oplus \pi N$

Evaluate $H_{eff}(g^2, g^3)$ in $MB \oplus \pi\pi N$

$\rightarrow$

Coupled-channel model with $\pi\pi N$

$$\begin{aligned} H_{eff} &= H_0 + H_I \\ H_I &= \Gamma_V + v_{2\leftrightarrow 2} + v_{2\leftrightarrow 3} + v_{3\leftrightarrow 3} \end{aligned}$$

with

$$\Gamma_V = \sum_{N^*} \sum_{MB} [\Gamma_{N^* \leftrightarrow MB}] + h_{\rho\leftrightarrow\pi\pi} + h_{\sigma\leftrightarrow\pi\pi}$$

- MB: $\gamma N, \pi N, \eta N, \pi\Delta, \rho N, \sigma N$
- Γ_V : bare vertex interactions

- Non-resonant interactions

$$v_{2\leftrightarrow 2} \quad = \quad \sum_{MB,M'B'} v_{MB,M'B'} + v_{\pi\pi} \tag{2}$$

$v_{MB,M'B'}$: meson-baryon interactions

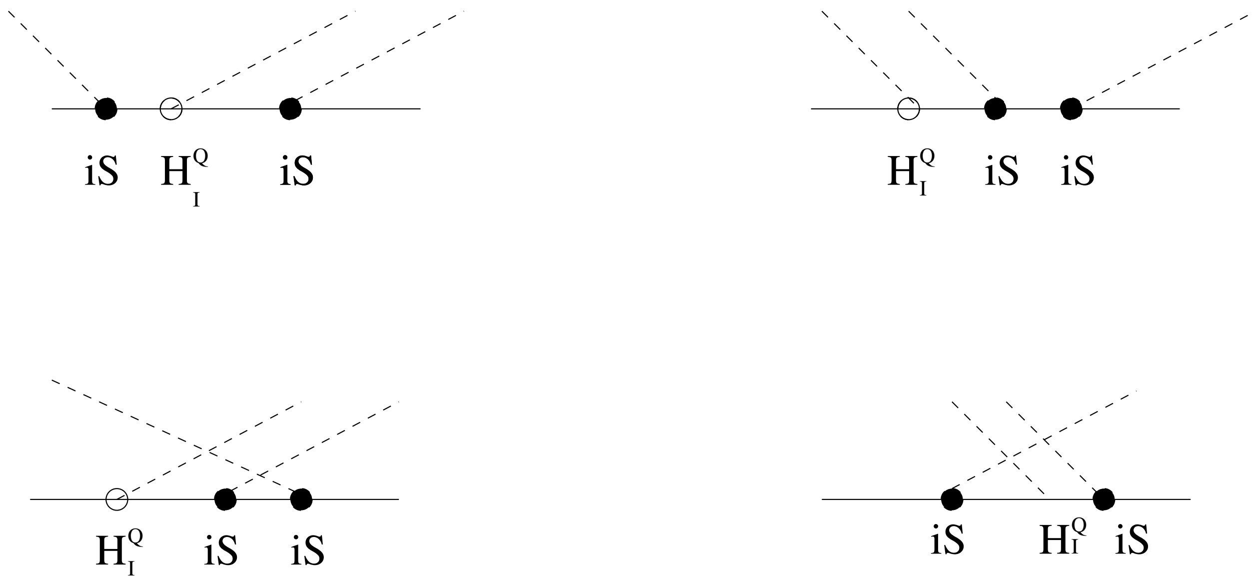
$v_{\pi\pi}$: $\pi\pi$ interaction.

$v_{2\leftrightarrow 3}$: $\pi N \leftrightarrow \pi\pi N$

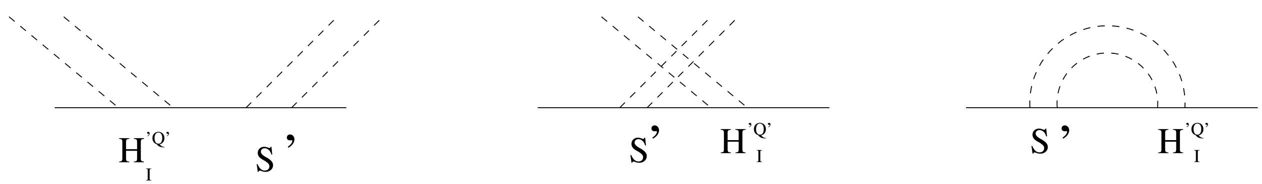
$v_{3\leftrightarrow 3}$: $\pi\pi N \leftrightarrow \pi\pi N$

- $v_{\pi\pi}$, $h_{\rho\leftrightarrow\pi\pi}$, $h_{\sigma\leftrightarrow\pi\pi}$: from $\pi\pi$ scattering

$v_{2\leftrightarrow 3}$:



$v_{3\leftrightarrow 3}$:



$\rightarrow$

$$
\begin{aligned}
T_{\gamma N,\pi\pi N}(E) &= T_{\gamma N,\pi\Delta}(E)G_{\pi\Delta}\Gamma_{\Delta,\pi N} \\
&+T_{\gamma N,\rho N}(E)G_{\rho N}h_{\rho,\pi\pi} \\
&+T_{\gamma N,\sigma N}(E)G_{\sigma N}h_{\sigma,\pi\pi}
\end{aligned}
$$

where

$$
\begin{aligned}
T_{\gamma N,MN}(E) &= t_{\gamma N,MN}(E) + \sum_{N^*} \frac{\bar{\Gamma}_{\gamma N \to N^*}(E)\bar{\Gamma}_{N^* \to MN}(E)}{E - M^0_{N^*} - \Sigma_{N^*}(E)} \\
G_{MB}(E) &= \frac{1}{E - E_B - E_M - \Sigma(E)}
\end{aligned}
$$

$\Sigma(E)$ = Self-energy of the unstable Δ, ρ, and σ

The dressed vertices are

$$\bar{\Gamma}_{N^* \to MB} = \Gamma_{N^* \to MB} + \sum_{M'B'} \Gamma_{N^* \to M'B'} G_{M'B'}(E) X_{M'B',MB}$$

$$\Uparrow$$

$$Bare$$

Non-resonant amplitudes are:

$$\begin{aligned} X_{MB,M',B'}(E) &= Z_{MB,M'B'}(E) \\ &+ \sum_{M''B''} Z_{MB,M''B''}(E) G_{M''B''}(E) X_{M''B'',M'B'}(E) \end{aligned}$$

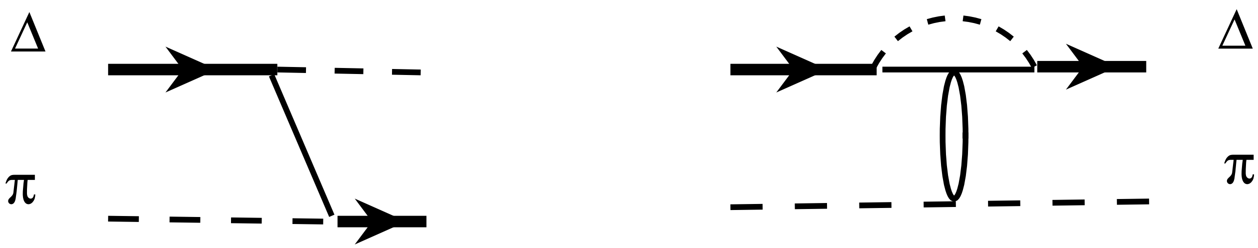
The channel-coupling interactions are:

$$Z_{MB,M'B'} \quad = \quad = v_{MB,M'B'}(E) + Z^{(\pi\pi N)}{}_{MB,M'B'}(E) \tag{3}$$

Three-body $\pi\pi N$ cut is in

$$Z^{(\pi\pi N)}{}_{MB,M'B'}(E) = Z^{(E)}_{MB,M'B'}(E) + Z^{(I)}_{MB,M'B'}(E)$$

Example:



$$Z^{(E)}_{MB,M'B'}(E) = < MB \mid \Gamma_V \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} \Gamma_V \mid B'M' >$$

$$Z^{(I)}_{MB,M'B'}(E) = < MB \mid \Gamma_V \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} t_{\pi\pi N,\pi\pi N}(E) \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} \Gamma_V \mid M'B' >$$

$\rightarrow$

Apply the Spline-function expansion method to solve the $\pi\pi N$ coupled-channel equations on real-axis

* The method was developed in $NN \rightarrow \pi NN$ studies by Matsuyama and Lee (1986)

First Results from Unitary $\pi\pi N$ calculations

Objetive: explore the starting parameters

- $h_{\rho,\pi\pi}$ and $h_{\sigma,\pi\pi}$ from fitting $\pi\pi$ phase shifts (by Johnstone and Lee)
- $v_{2,2}$ are calculated using the coupling constants of Julich's model

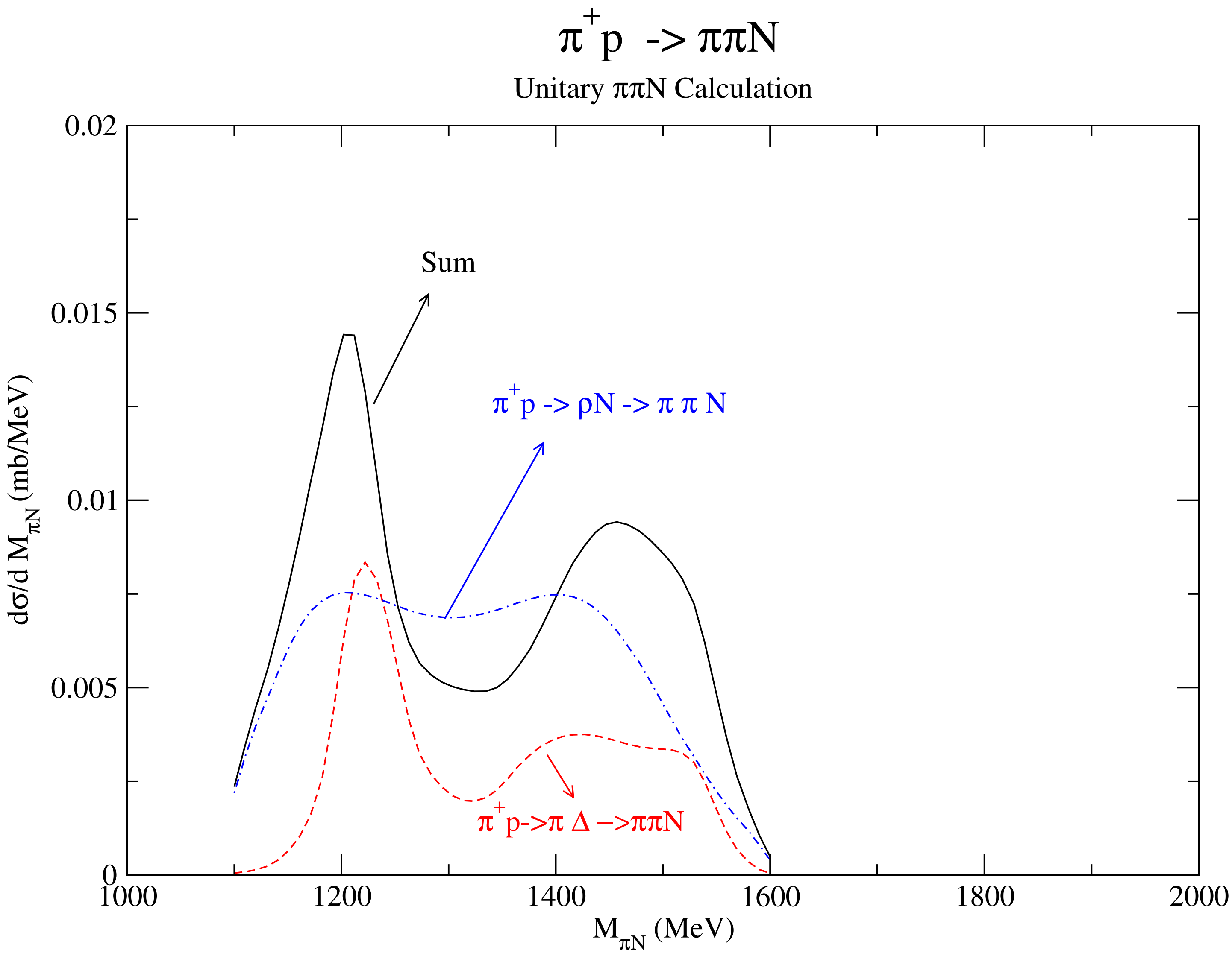
  $v_{a,b}$

  where $a, b = \gamma N,\, \pi N,\, \eta N,\, \rho N,\, \pi\Delta,\, \sigma N$

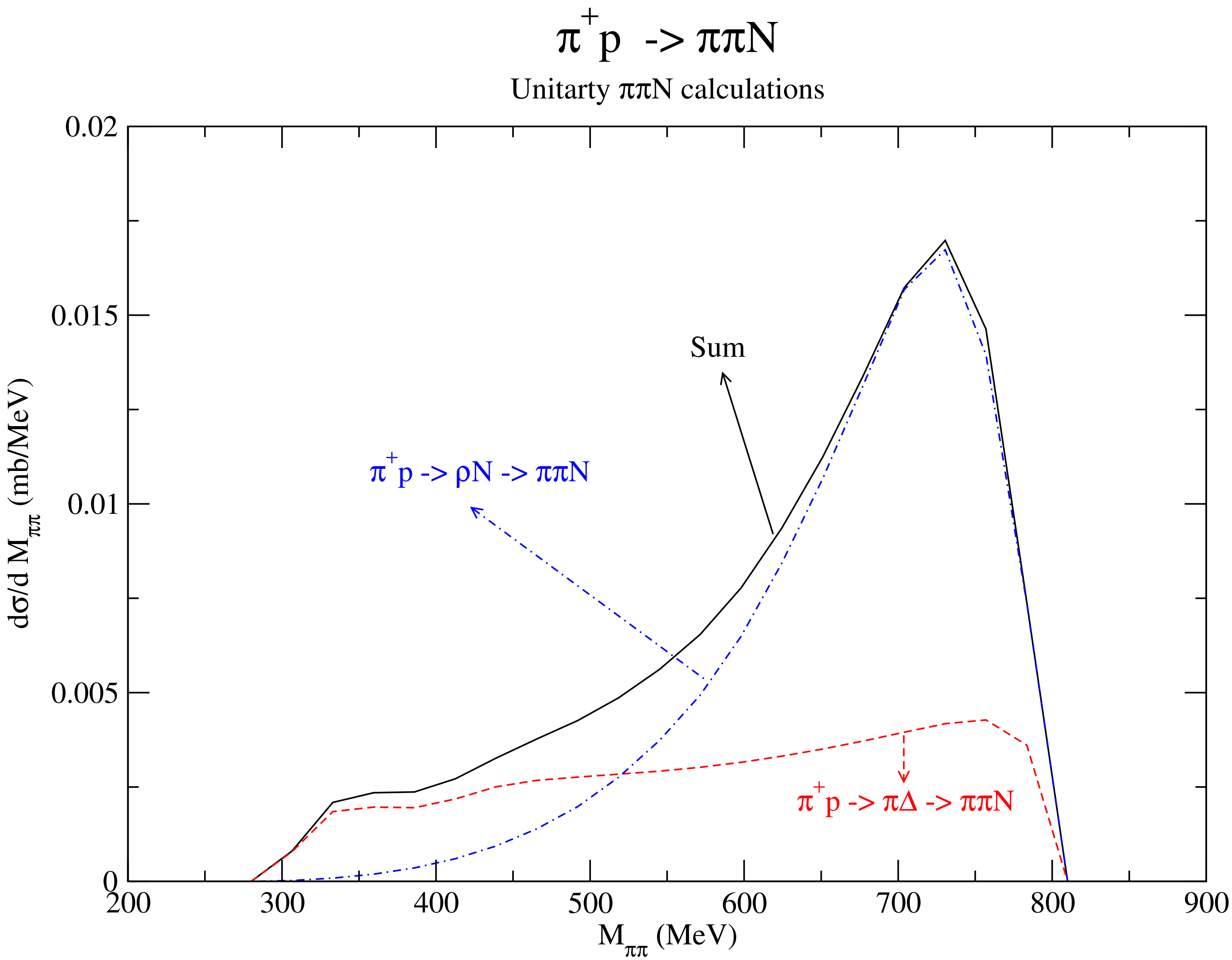
  have been constructed

- $N^* -> \gamma N,\, \pi N,\, \eta N,\, \rho N,\, \pi\Delta$ are taken from the ${}^3P_0$ model of Capstick and Roberts and/or from PDG

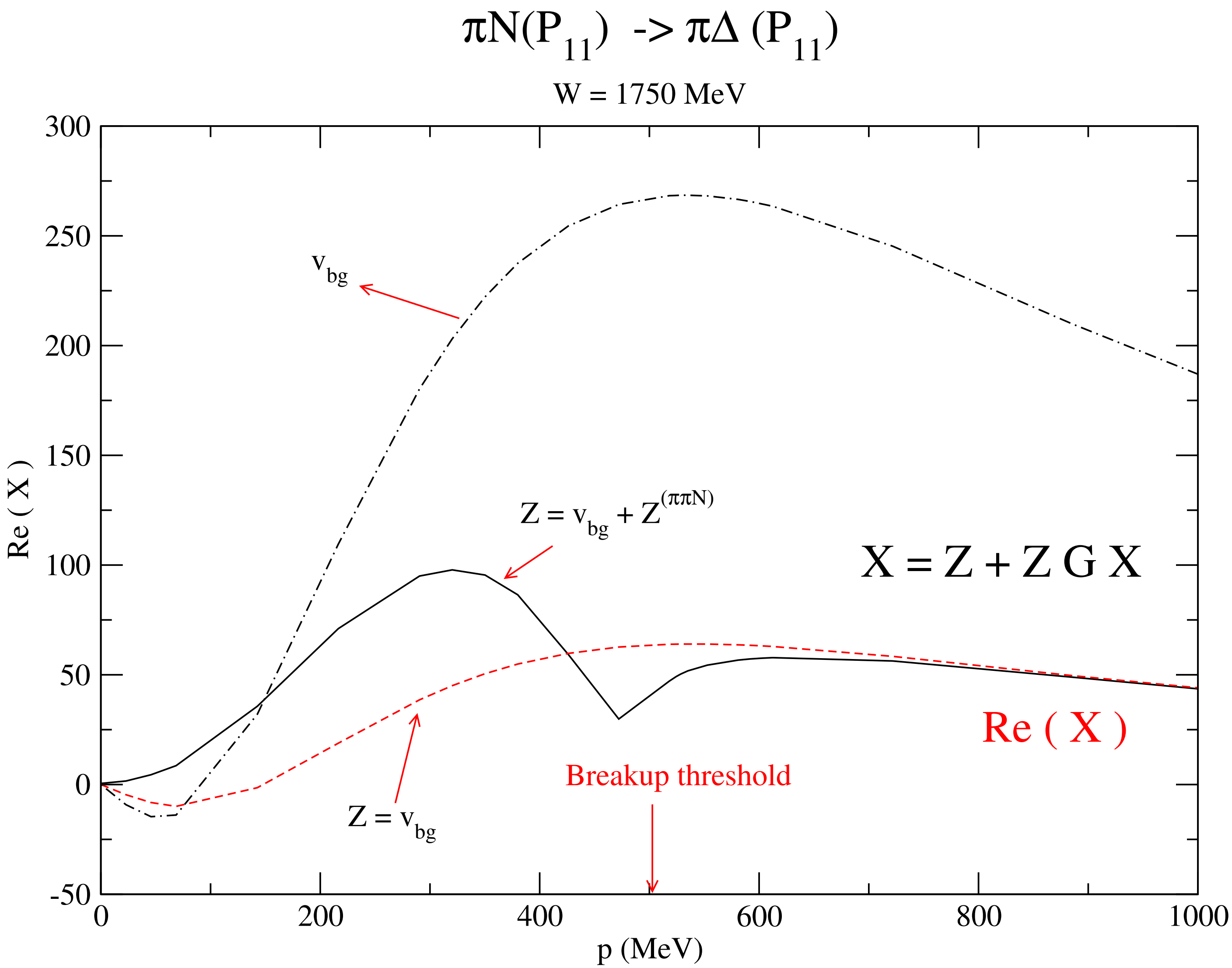
$d\sigma/dM_{\pi N}$ of $\pi^+ p \rightarrow \pi\pi N$



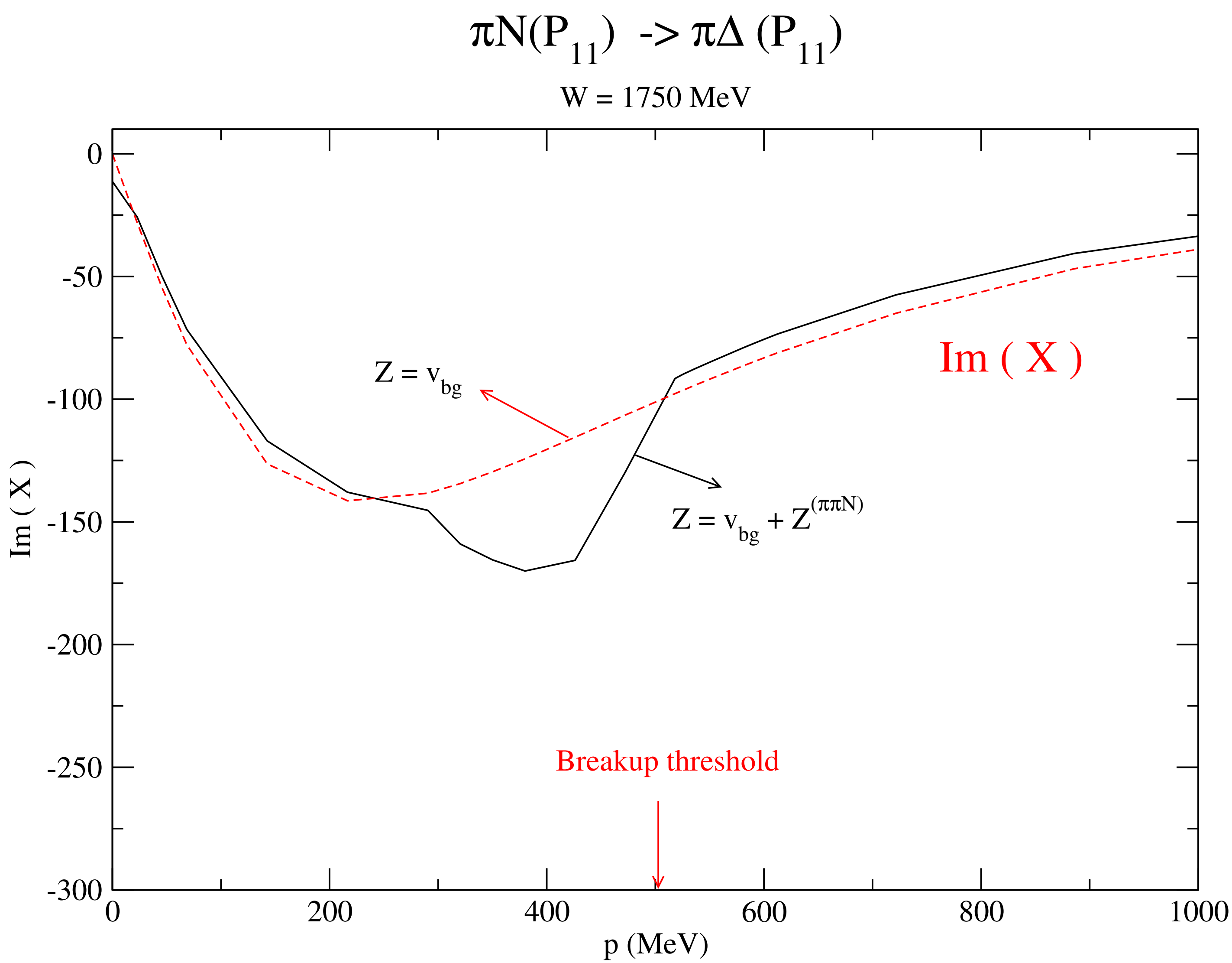
$d\sigma/dM_{\pi\pi}$ of $\pi^+ p \to \pi\pi N$



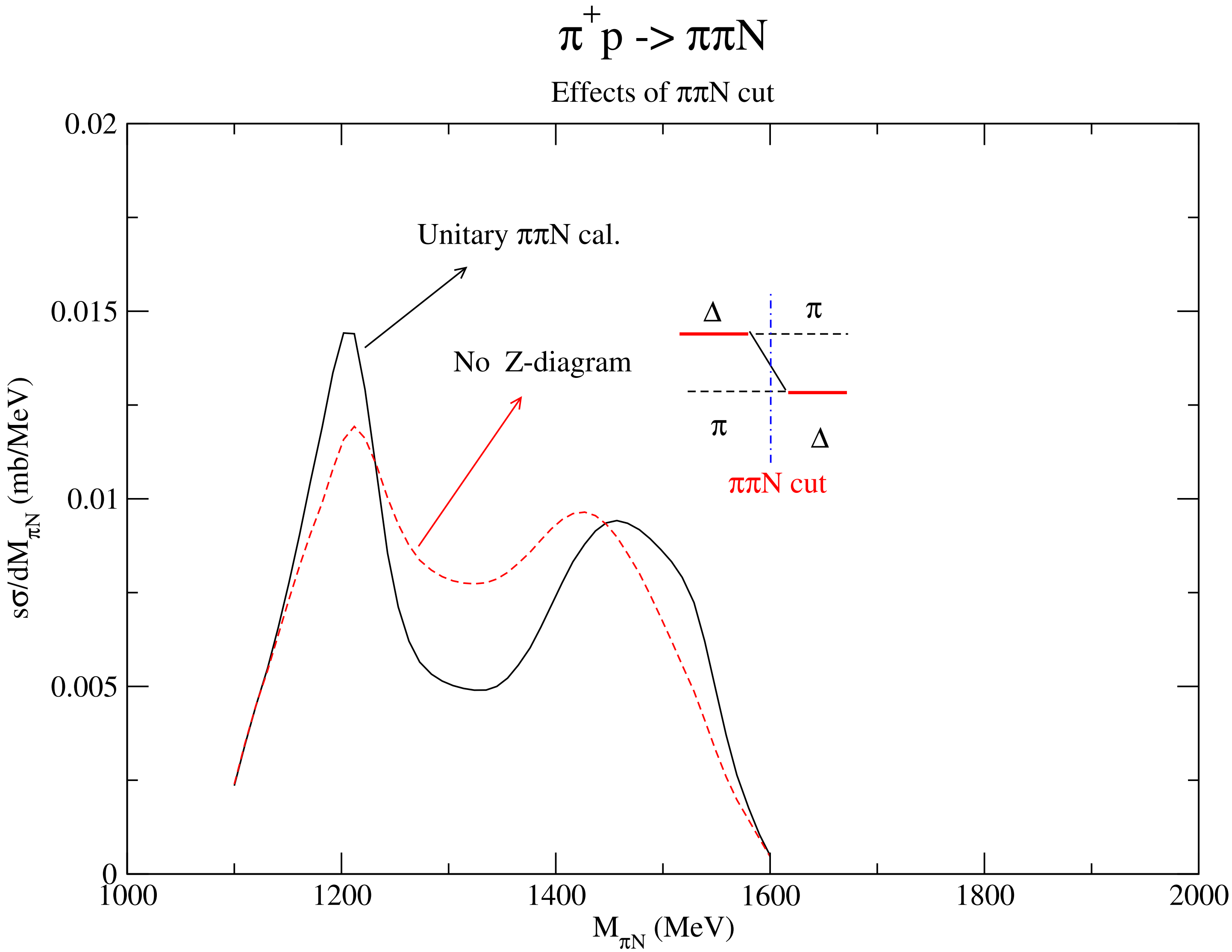
Effect of coupled-channel and $\pi\pi N$ cut on amplitudes



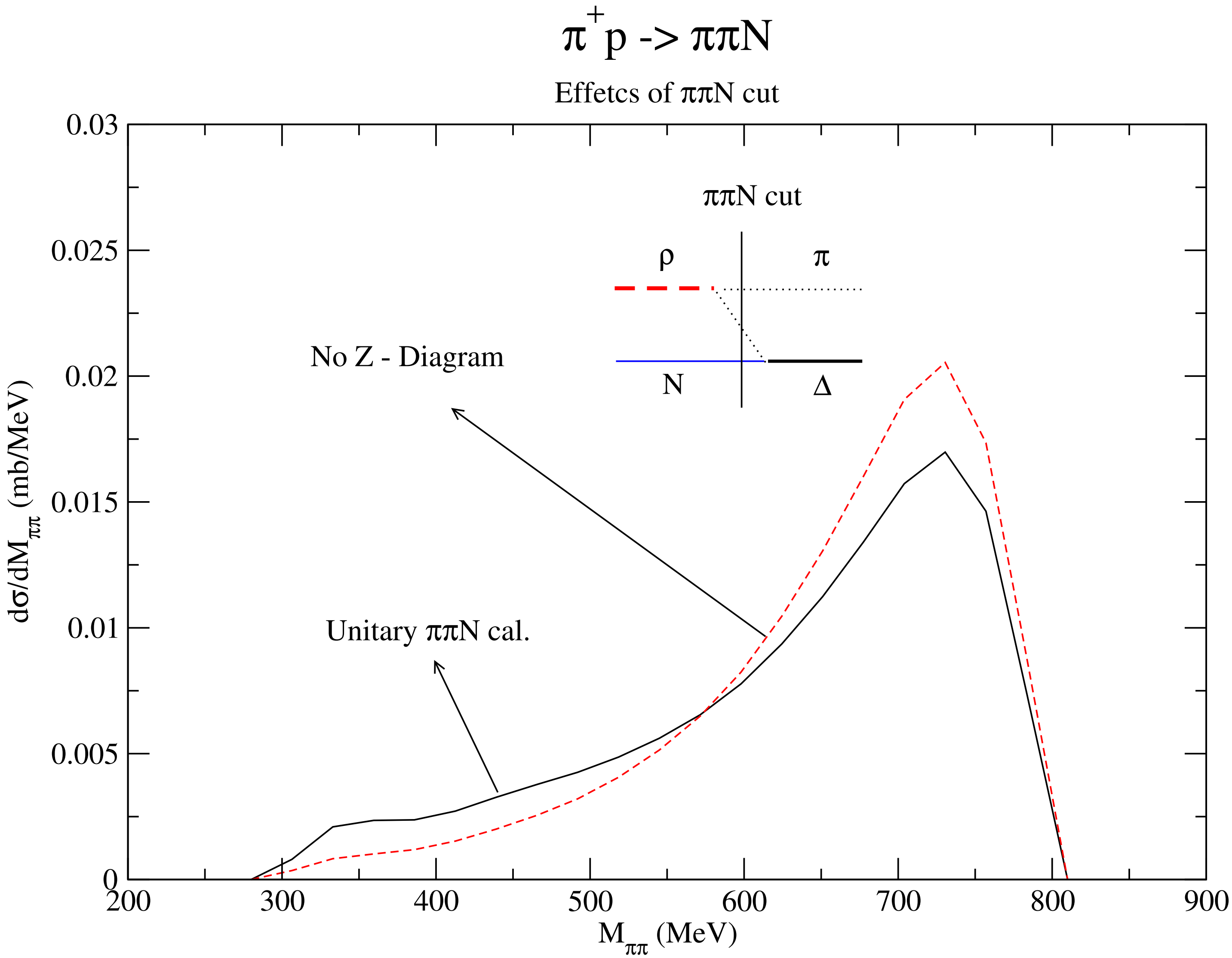
Effect of coupled-channel and $\pi\pi N$ cut on amplitudes



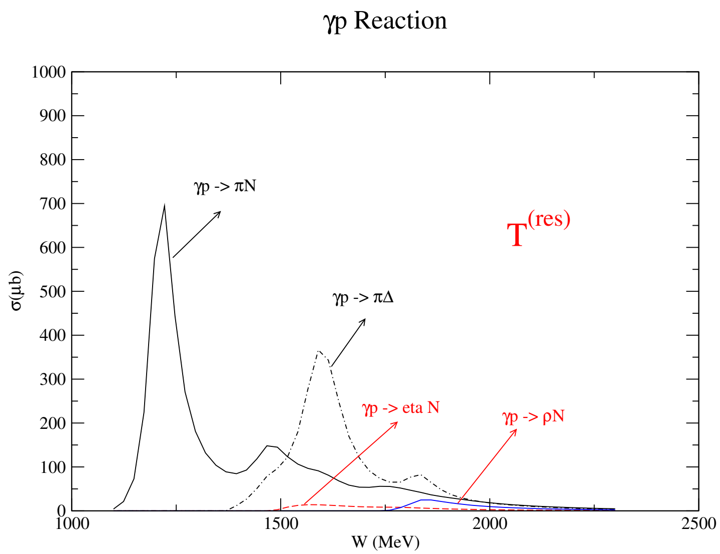
Effect of $\pi\pi N$ cut

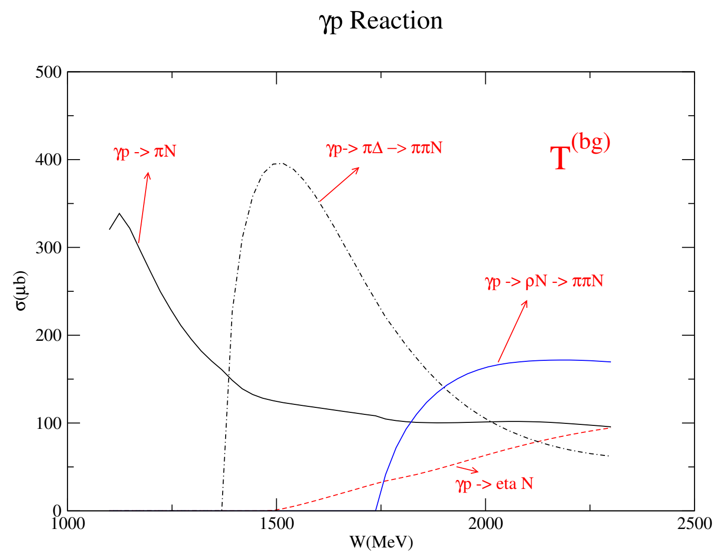


Effect of $\pi\pi N$ cut

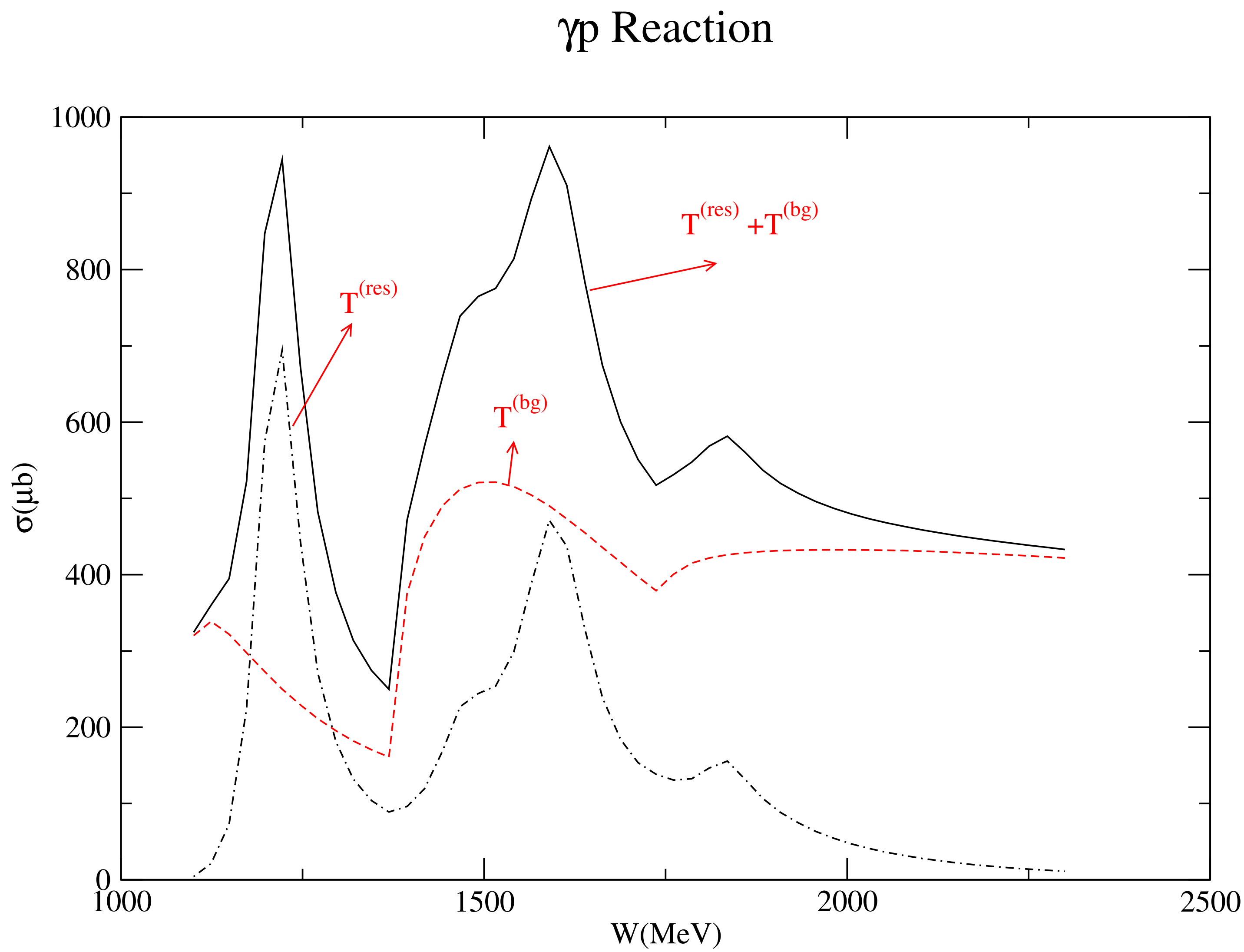


$$\gamma p \to \pi N, \eta N, \pi\pi N(\pi\Delta, \rho N)$$



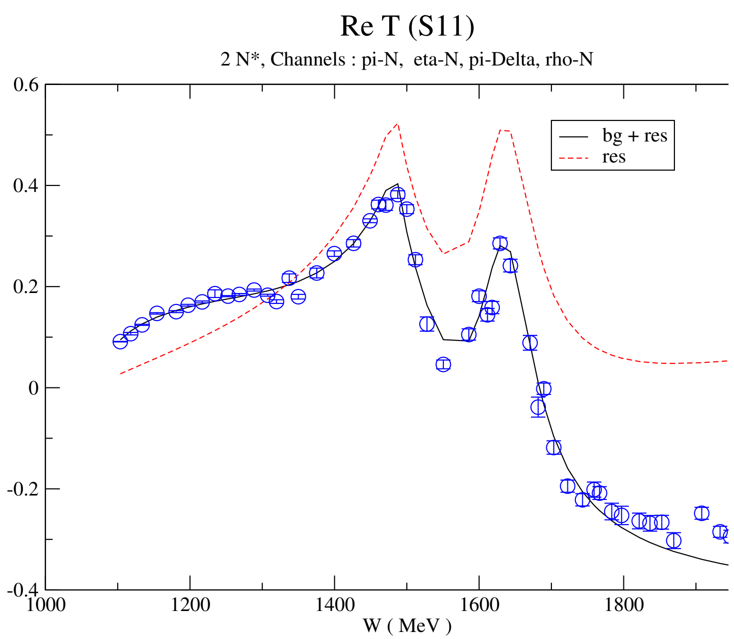


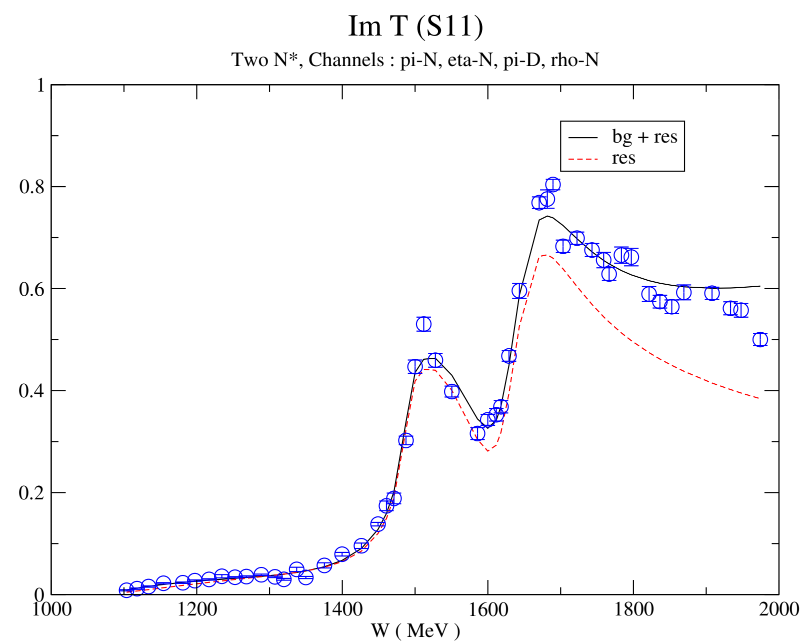
$$\gamma p \to \pi N, \eta N, \pi\pi N(\pi\Delta, \rho N)$$



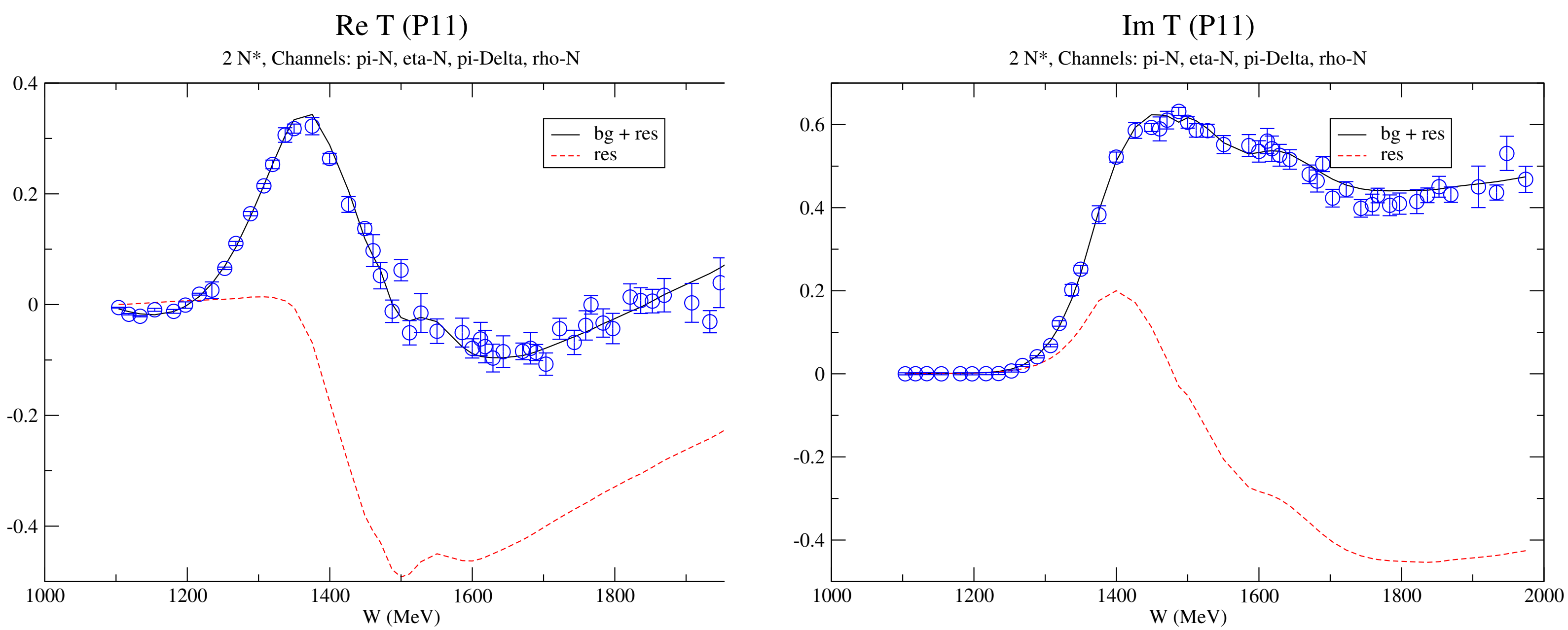
Fits to πN scattering amplitudes

S_{11} :





P_{11} :


Re T (P11)
2 N*, Channels: pi-N, eta-N, pi-Delta, rho-N
bg + res
res
W (MeV)
Im T (P11)
2 N*, Channels: pi-N, eta-N, pi-Delta, rho-N
bg + res
res
W (MeV)

If coupled-channel multiple scattering and $\pi\pi N$ cut structure is neglected

$\rightarrow$

- Moscow/Jlab Isobar Model of $\gamma N \rightarrow \pi\pi N$
  (V. Mokeev's talk)
- Amplitude analyses of $\gamma N \rightarrow \pi\pi N$ of RPI/JLab group

Concluding Remarks

- We have developed a dynamical approach for investigating N^* in πN and γN reactions
- It is tractable and systematic in getting H_{eff} from relativistic quantum field theory
- The model in the Δ region can describe most of the current data
- Numerical methods for solving coupled-channel model including $\pi\pi N$ has been well developed with some preliminary results.
- $\pi\pi N$ unitarity cut is crucial in predicting $\pi\pi N$ production cross sections and identifing "missing" resonances
- Much more work is needed to carry out complete coupled-channel analyses of all meson production data.