Light Quark Confinement And The Trajectory Of The Pseudoscalar Meson

STEWART V. WRIGHT

Physics Division

Argonne National Laboratory

Nonperturbative Methods in QCD

- Green function methods

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- Light front relativistic quantum mechanics

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Dyson–Schwinger equations

Contemporary Reviews

- **Dyson–Schwinger Equations: Density, Temperature and Continuum Strong QCD**

  C.D. Roberts and S.M. Schmidt, `nucl-th/0005064`
  Prog. Part. Nucl. Phys. 45 (2000) S1

- **The IR behaviour of QCD Green's functions: Confinement, DCSB and hadrons ...**

  R. Alkofer and L. von Smekal, `hep-ph/0007355`
  Phys. Rept. 353 (2001) 281

- **Dyson–Schwinger Equations: A Tool for Hadron Physics**

  P. Maris and C.D. Roberts, `nucl-th/0301049`
  Int. J. Mod. Phys. E12 (2003) pp. 297–365

Dyson–Schwinger Equations

Dyson–Schwinger Equations

- Nonperturbative, continuum approach to QCD

Dyson–Schwinger Equations

- Nonperturbative, continuum approach to QCD
- Simplest level: Generating tool for perturbation theory

Dyson–Schwinger Equations

- Nonperturbative, continuum approach to QCD
  - Hadrons are composites of Quarks and Gluons

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Dyson–Schwinger Equations

- Nonperturbative, continuum approach to QCD
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    - Quantitative and Qualitative importance of:
      - Dynamical Chiral Symmetry Breaking

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    - Quantitative and Qualitative importance of:
      - · Dynamical Chiral Symmetry Breaking
      - · Quark and Gluon confinement

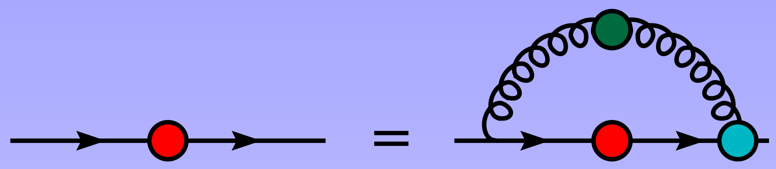
    $\Rightarrow$ Understanding Infrared (long range) behaviour of $\alpha_s(Q^2)$

Maris, Roberts *Phys. Rev.* **C56** 3369

Maris, Tandy *Phys. Rev.* **C60** 055214

Dressed Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

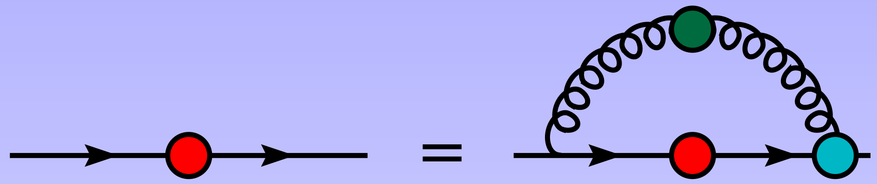


- Weak coupling expansion: Reproduces every diagram in Perturbation Theory
- Perturbation Theory:

$$\begin{aligned} M(p^2) &= m_0 \left(1 - \frac{3\alpha}{4\pi} \ln\left[\frac{p^2}{m_0^2}\right] + \mathcal{O}(\alpha^2) \right) \\ &\stackrel{m\to 0}{\longrightarrow} 0 \end{aligned}$$

A Challenge

- Infinitely Many Coupled Equations



A Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean **Green** Functions)
  - Same VEVs measured in Lattice QCD simulations

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  - Weak coupling expansion
    $\Rightarrow$ Perturbation Theory
    Not useful for nonpertubative problems

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- "Many Body" physics approach

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  - Rainbow–Ladder ($\sim$ Hartree–Fock) truncation. Widely explored

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A Challenge

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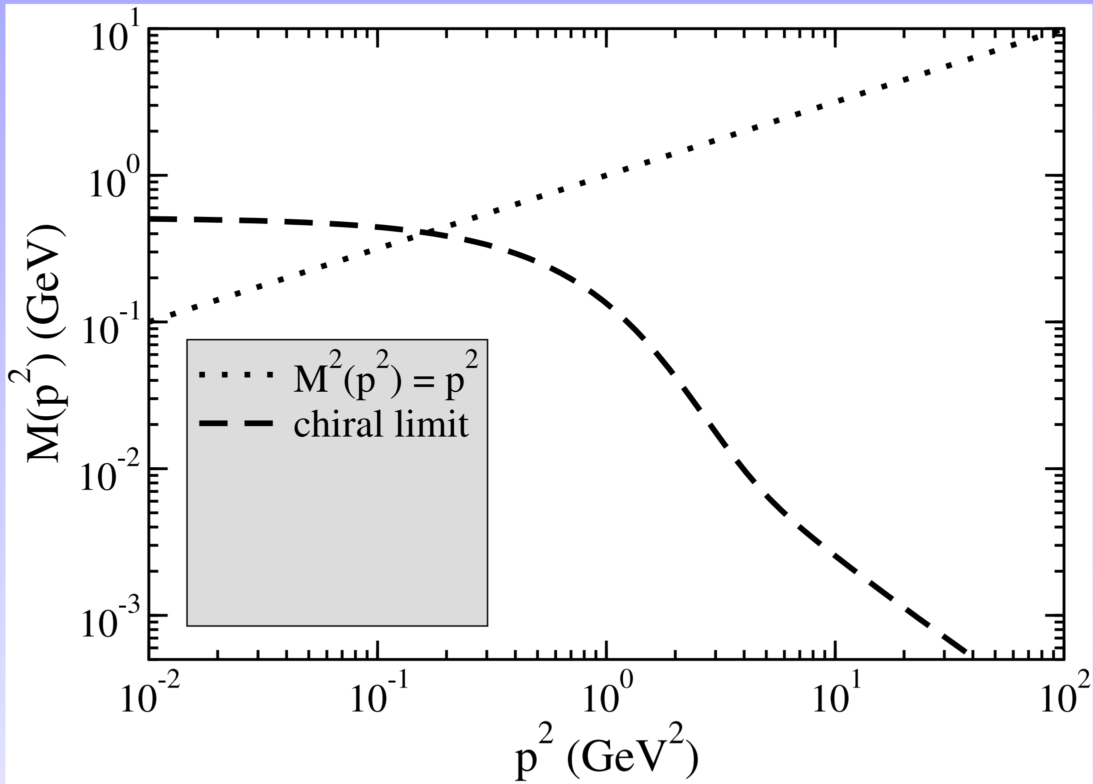
  - Systematically improvable.

Nonperturbative Quark Propagator

QCD gap equation has solution $M(p^2) \neq 0$ for $m_q = 0$



Adapted from P. Maris & C.D. Roberts, PRC56, 3369 (1997) [nucl-th/9708029]

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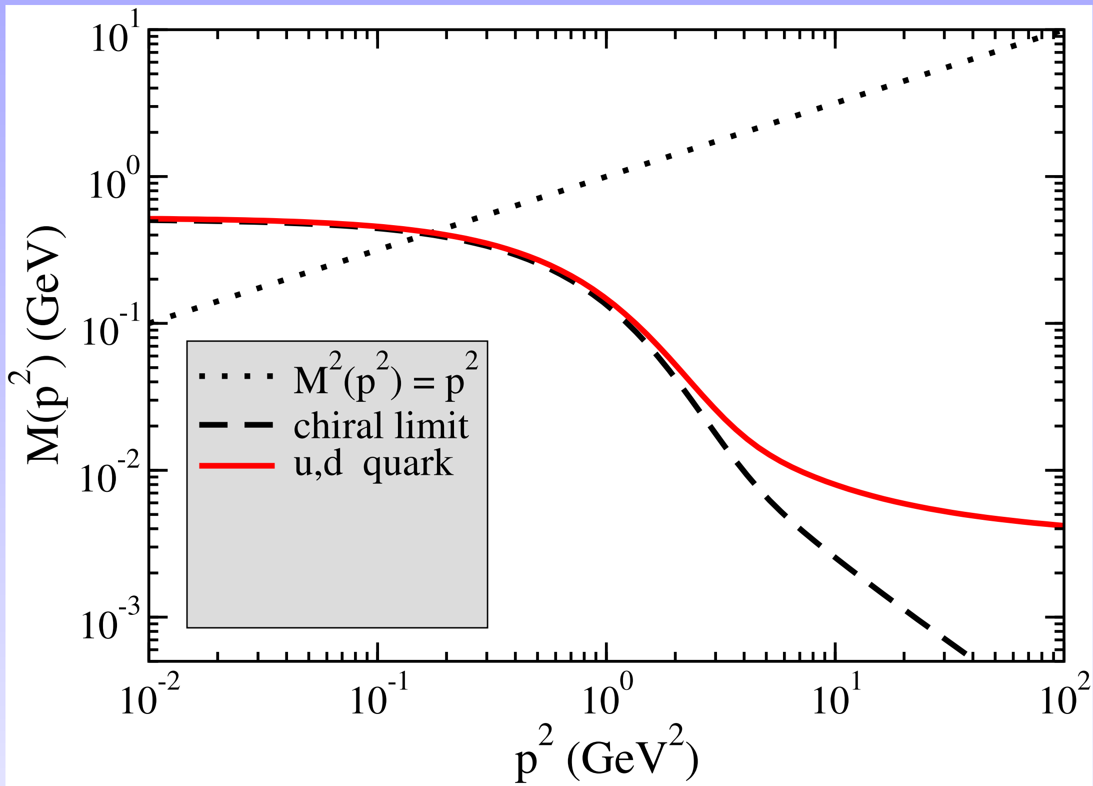

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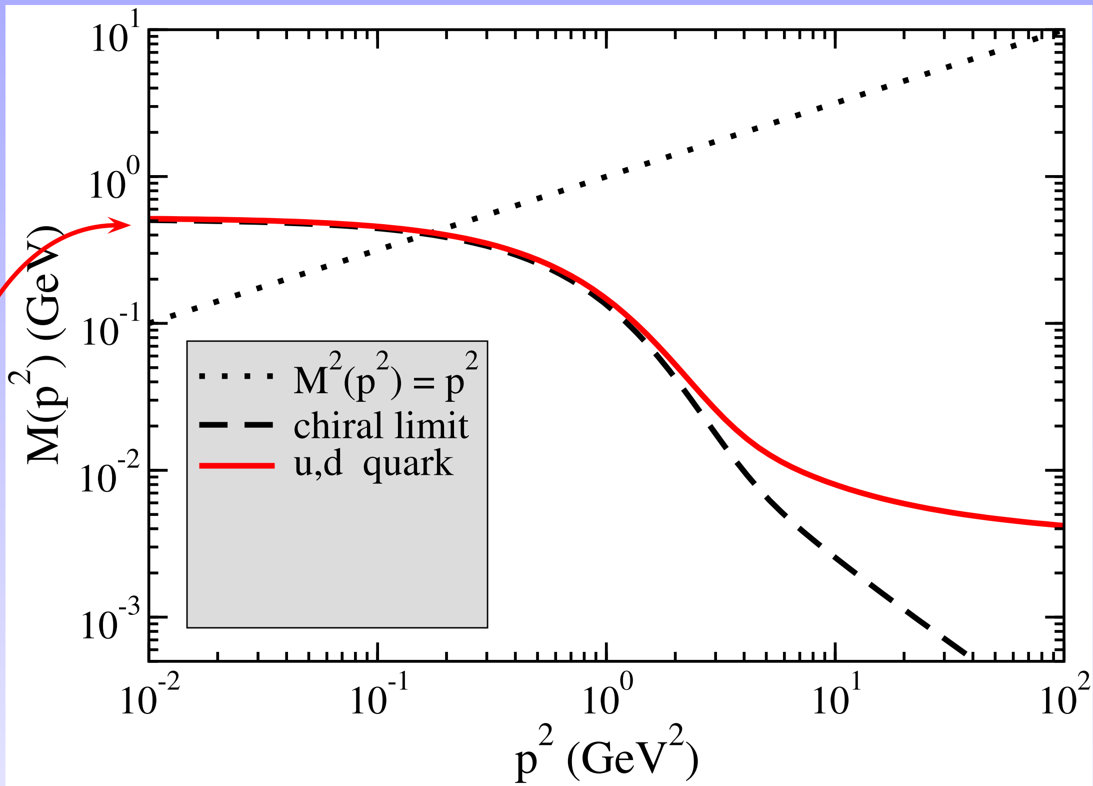

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- If $m_q \neq 0$, $M(p^2)$ describes the evolution from a current quark mass at high energies to a constitutent-like quark mass at low energies.

Nonperturbative Quark Propagator

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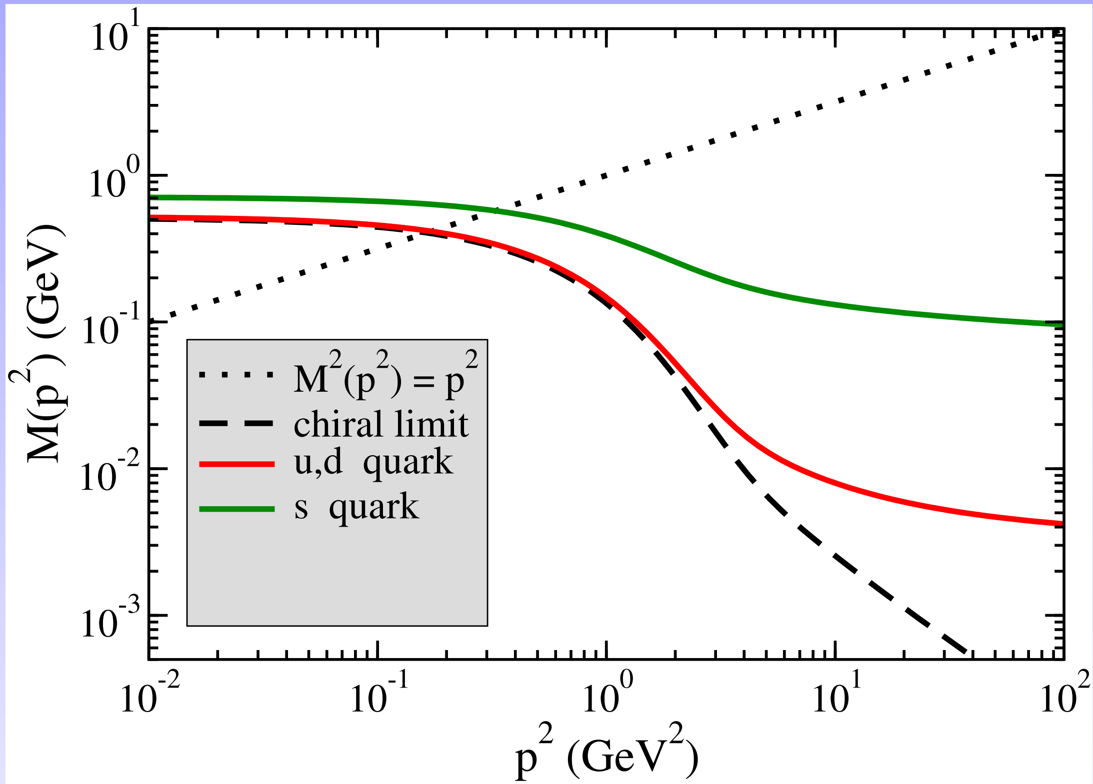

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$M(p^2)$ connects nonperturbative QCD with perturbative QCD

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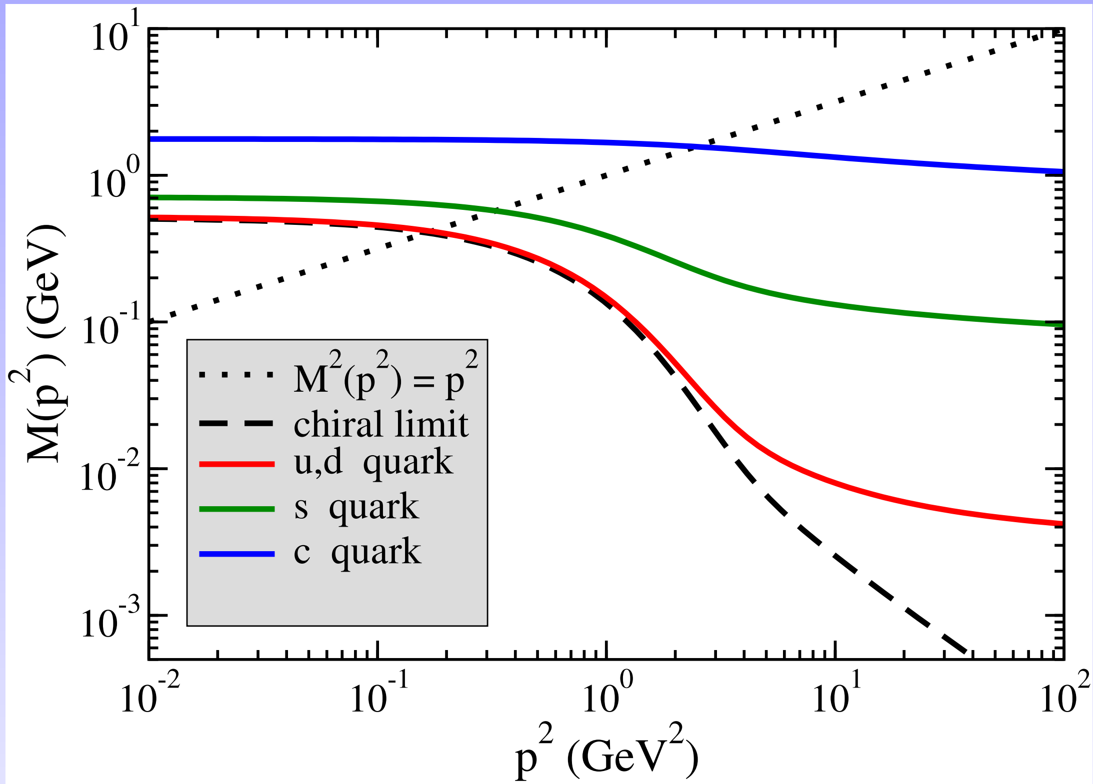

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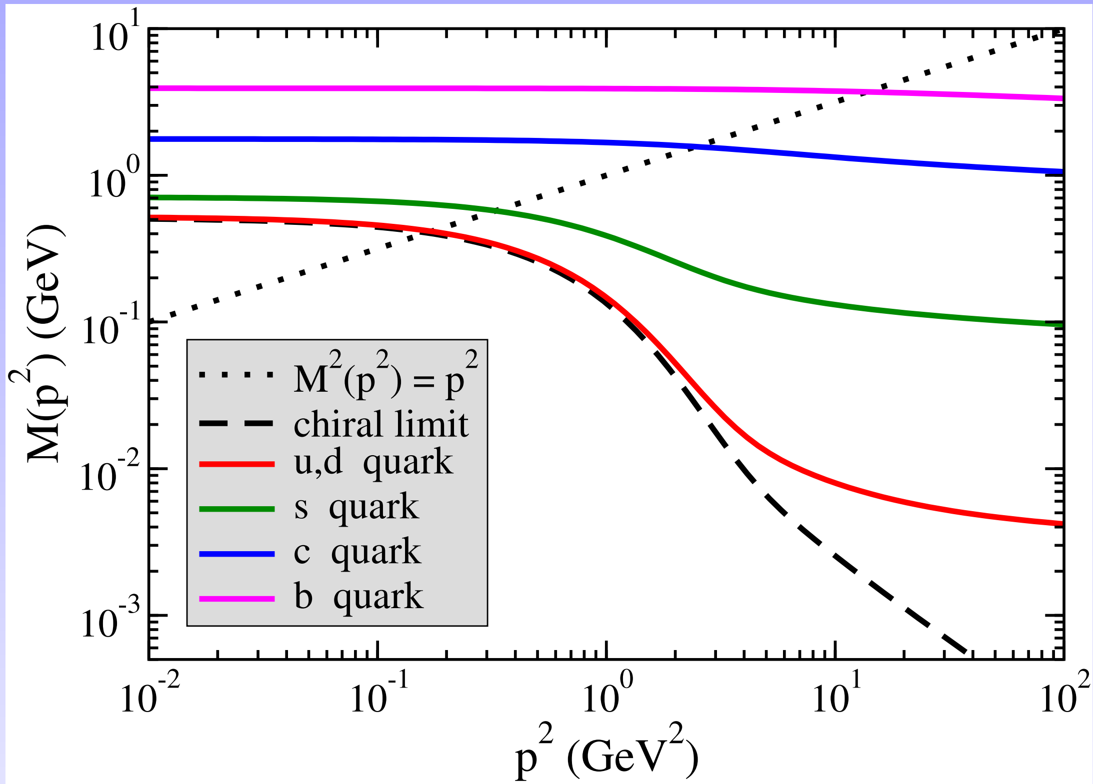

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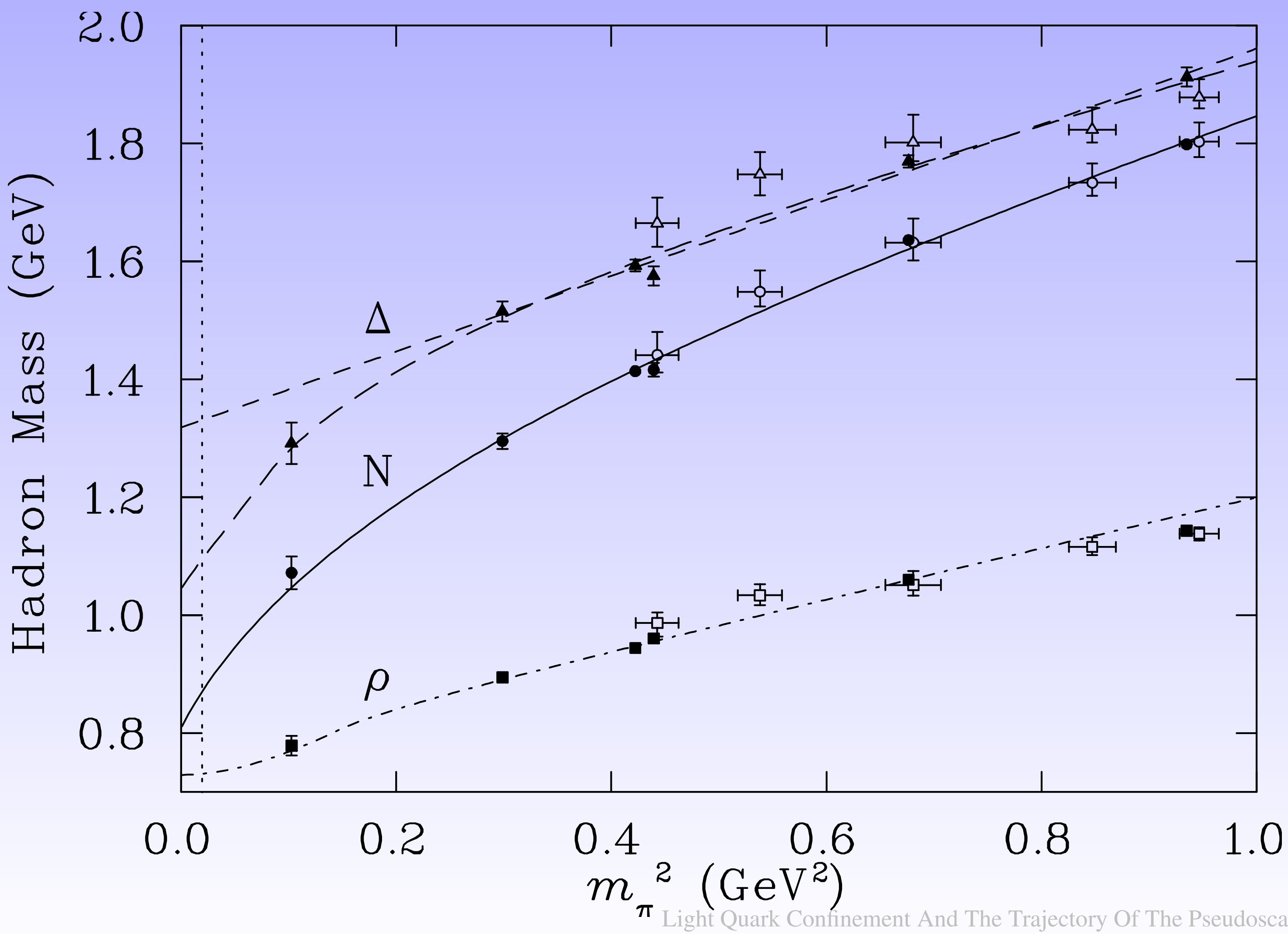
. . . On the Lattice

Leinweber, Thomas, Tsushima, SVW

`Phys. Rev.` **D61**`,074502; Phys. Rev.` **D64**`,094502`



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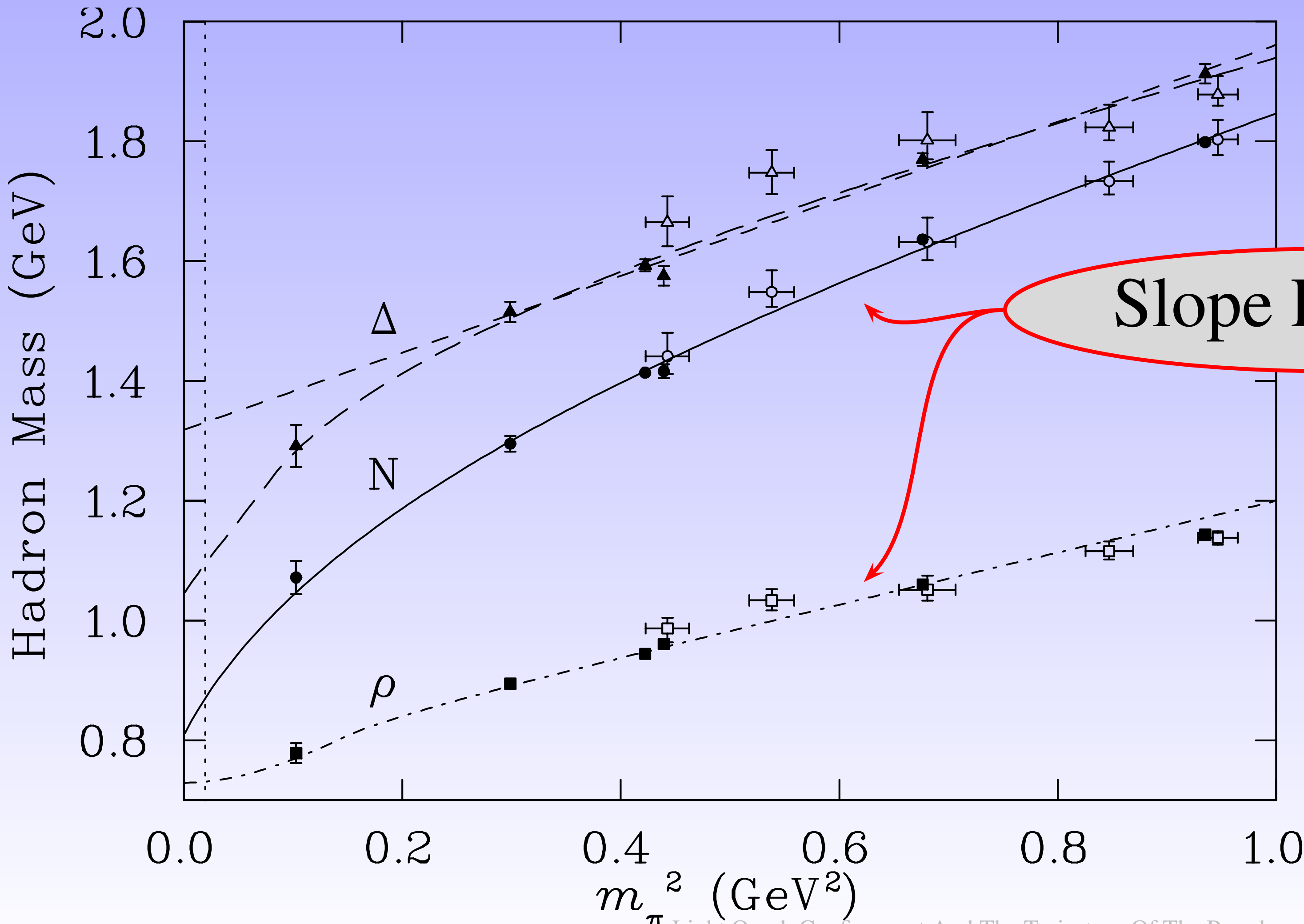
Phys. Rev. **D61**, 074502; Phys. Rev. **D64**, 094502



Slope Ratio 3:2

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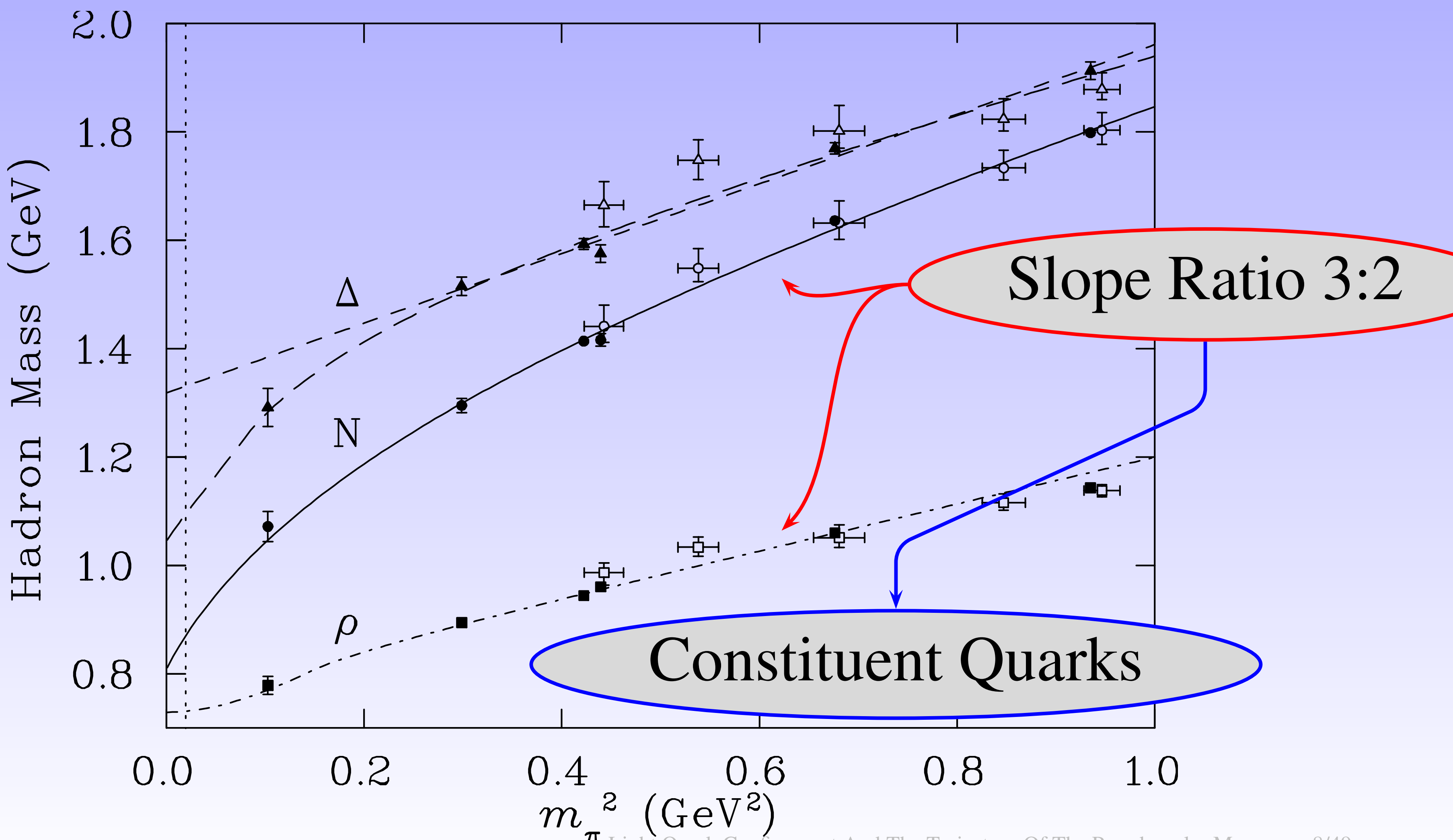
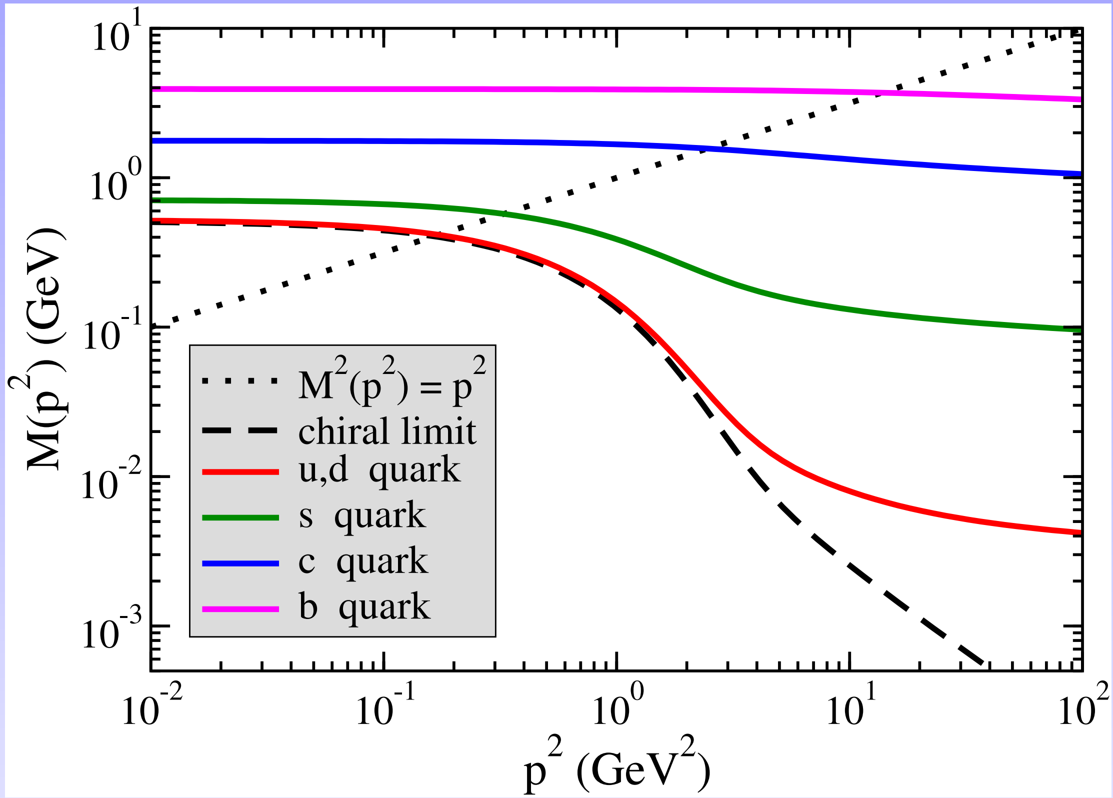


Nonperturbative Mass Function



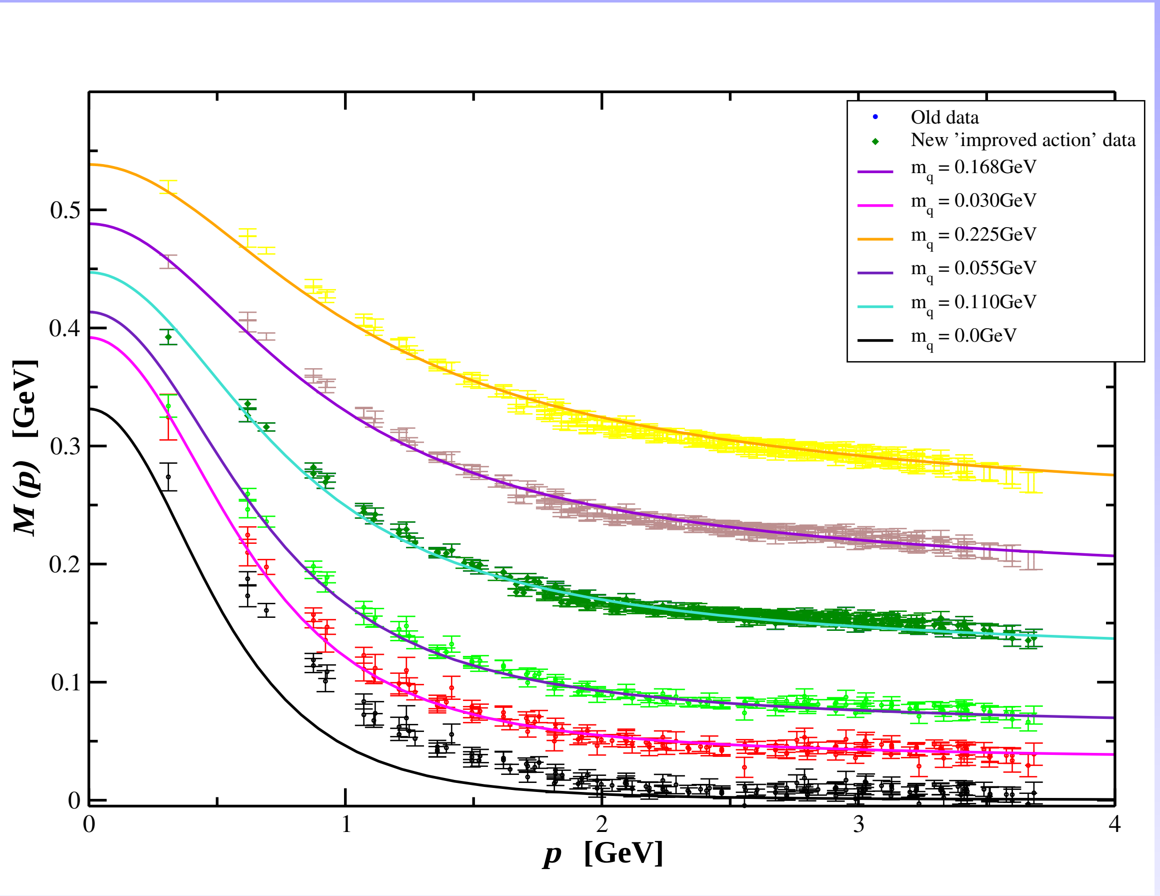
Chiral Symmetry and its dynamical breaking is important property for light Quarks

Comparison with the Lattice

- One parameter model provides persuasive results.

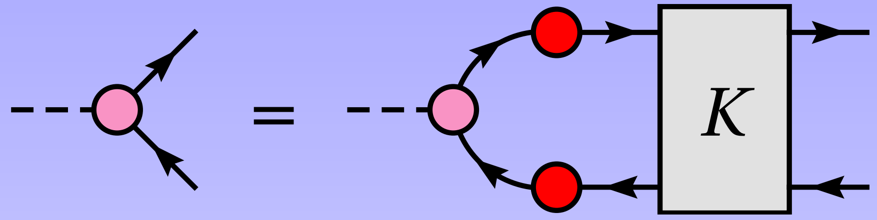
DSE: Bhagwat, Pichowsky, Roberts, Tandy nucl-th/0304003

Lattice: Bowman, Heller, Leinweber, Williams hep-lat/0209129



Hadrons: Bound States (BSE)

Bethe–Salpeter Equation



Hadrons: Bound States (BSE)

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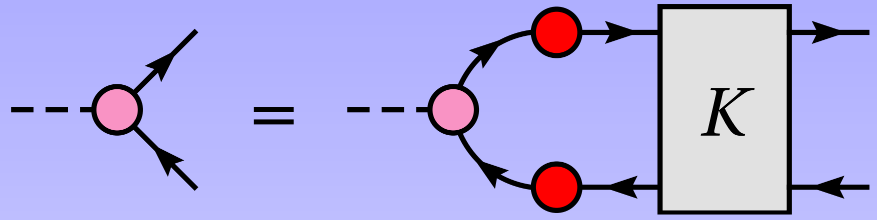


- Axial-Vector Ward–Takahashi identity relates kernel of BSE and DSE

My interest

- Coming from a (pseudo-) Lattice background

My interest

- Coming from a (pseudo-) Lattice background
  - I *like* observables

My interest

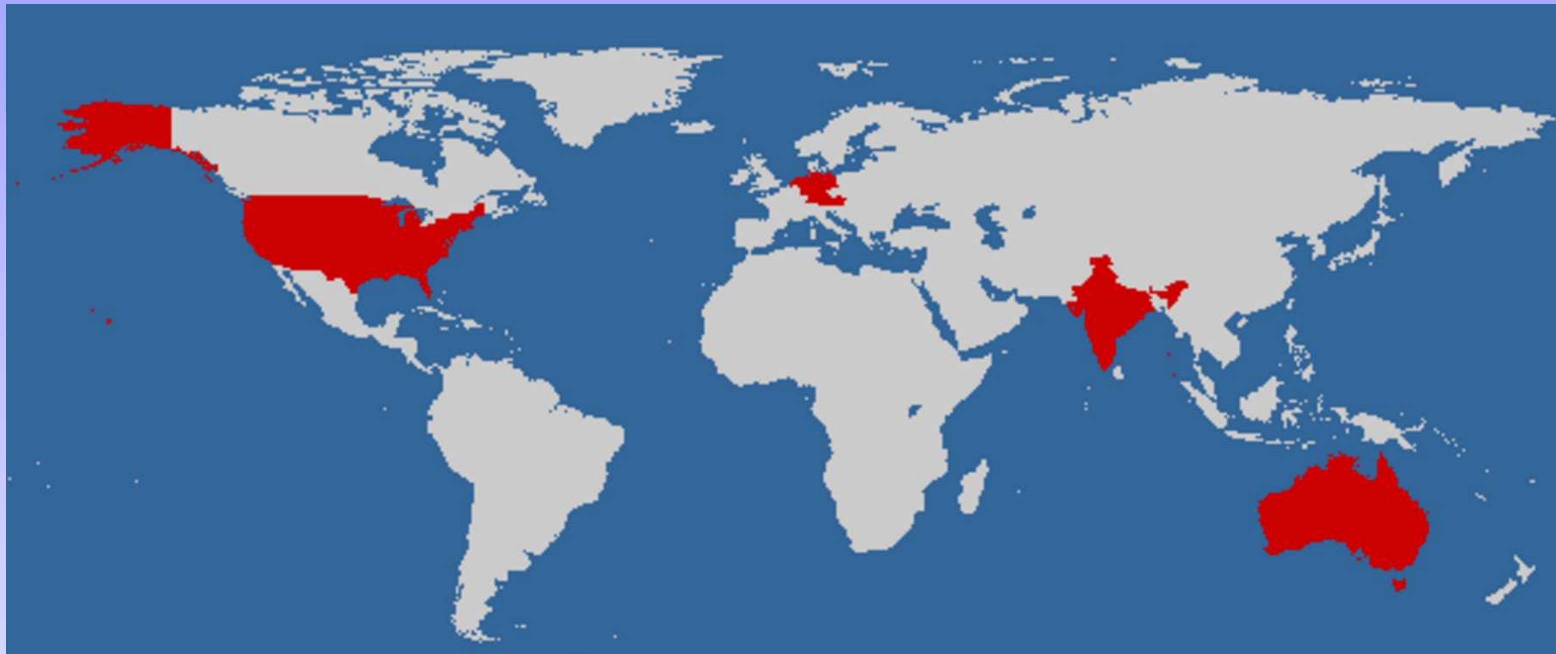
- Coming from a (pseudo-) Lattice background
  - I *like* observables
- Ground state mesons thoroughly investigated

  $\Rightarrow$ Excited States

Why?

SPIRES: Almost 70% of papers with “*excited meson(s)*” in the title published in the last 10 years.

Collaborators



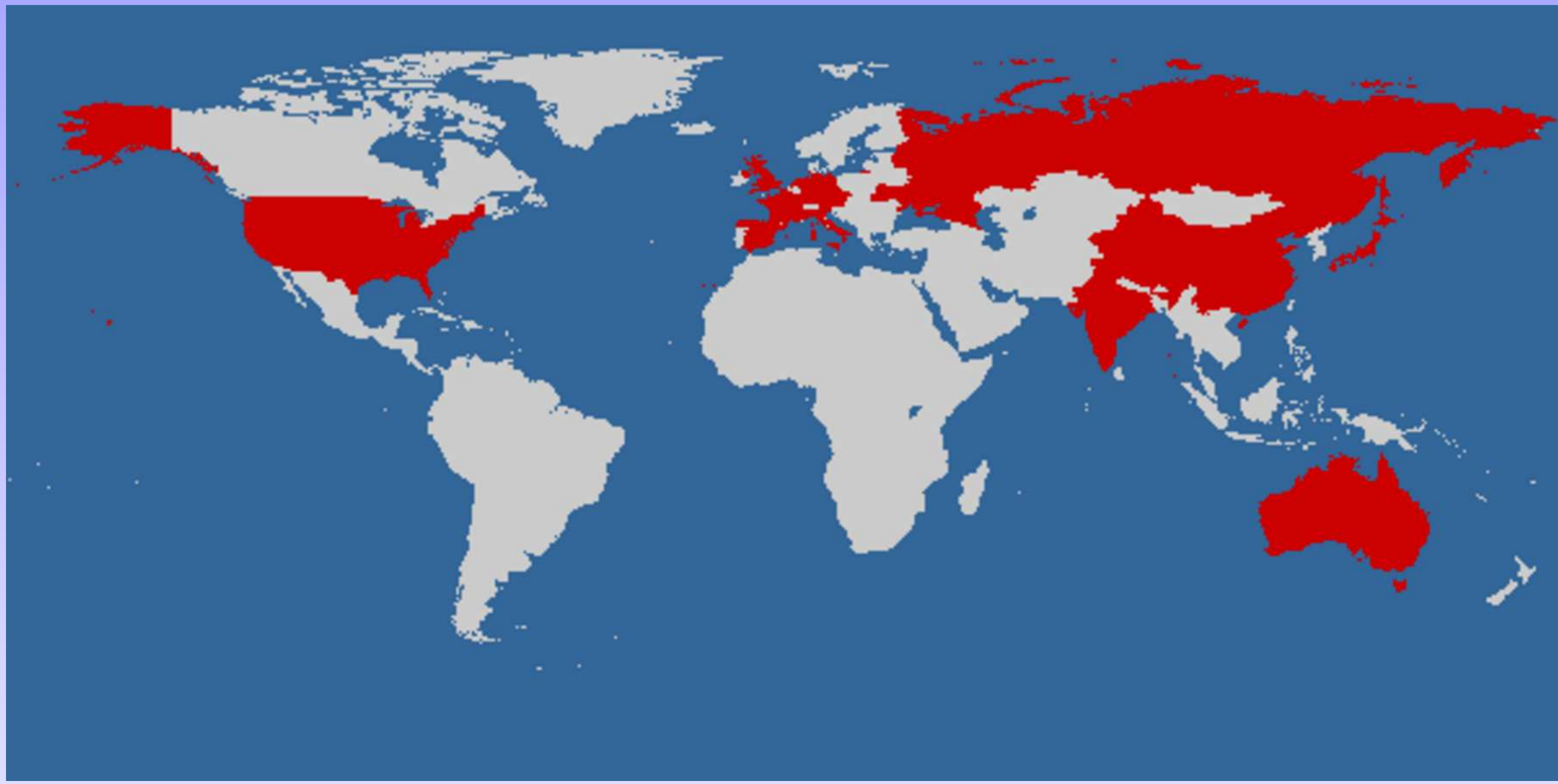
ARGONNE: Craig Roberts, Arne Höll

PITTSBURG UNIV: Pieter Maris

GRAZ UNIV: Andreas Krassnigg

KENT STATE UNIV: Peter Tandy
(Mandar Bhagwat)

Recent DSE Publications



Last 25 papers listed on SPIRES with “*DSE*” in the title…

This Study: Pions

- Solve inhomogeneous vertex DSE.

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- Solve inhomogeneous vertex DSE.

- Numerical solution exists for both TIMELIKE and SPACELIKE momentum.

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- Solve inhomogeneous vertex DSE.
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- Poles in timelike region $\rightarrow$ bound states

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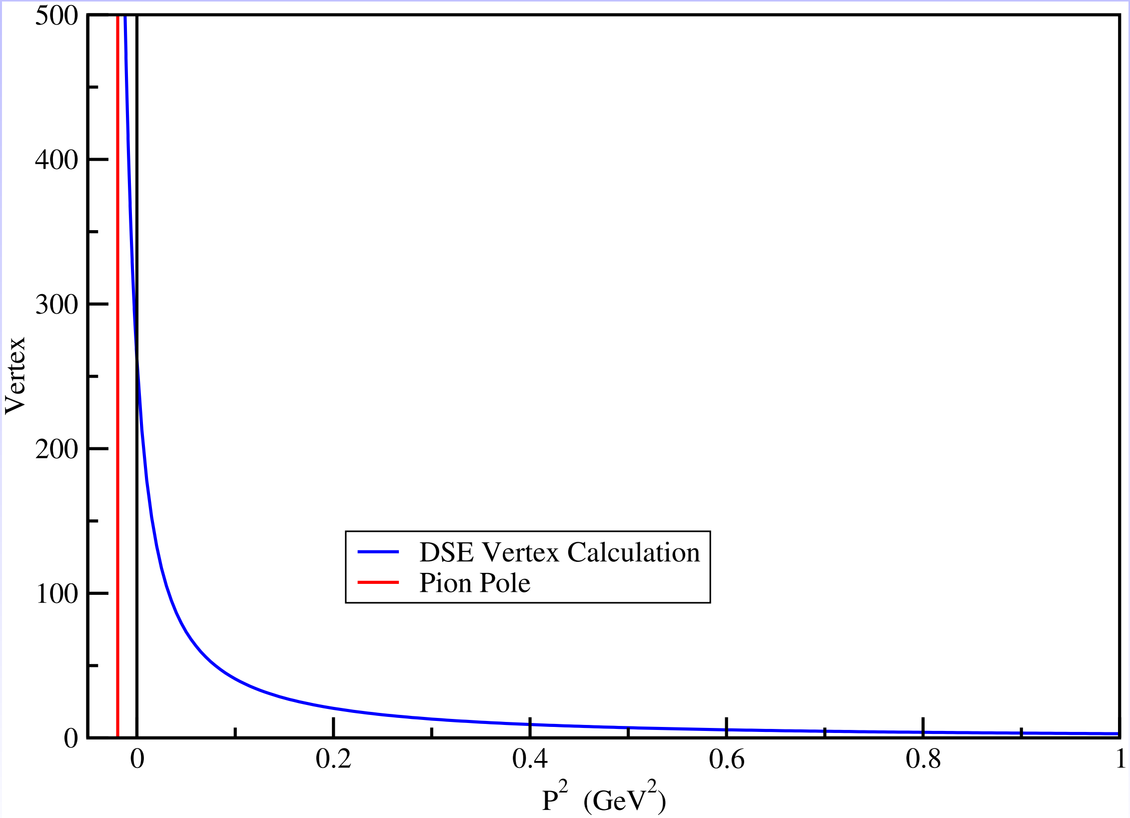
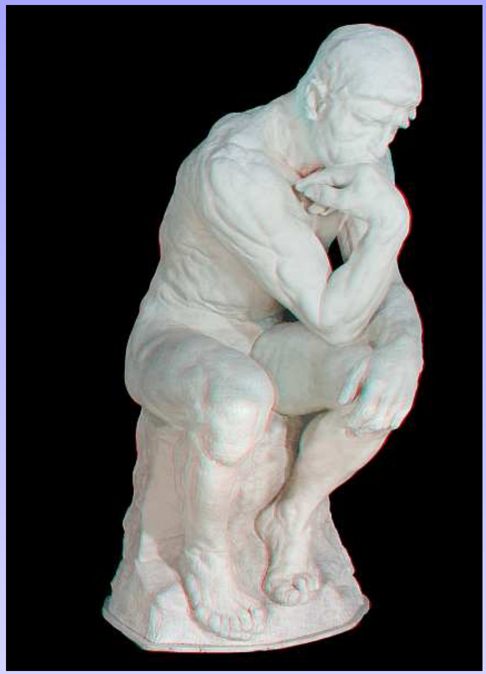


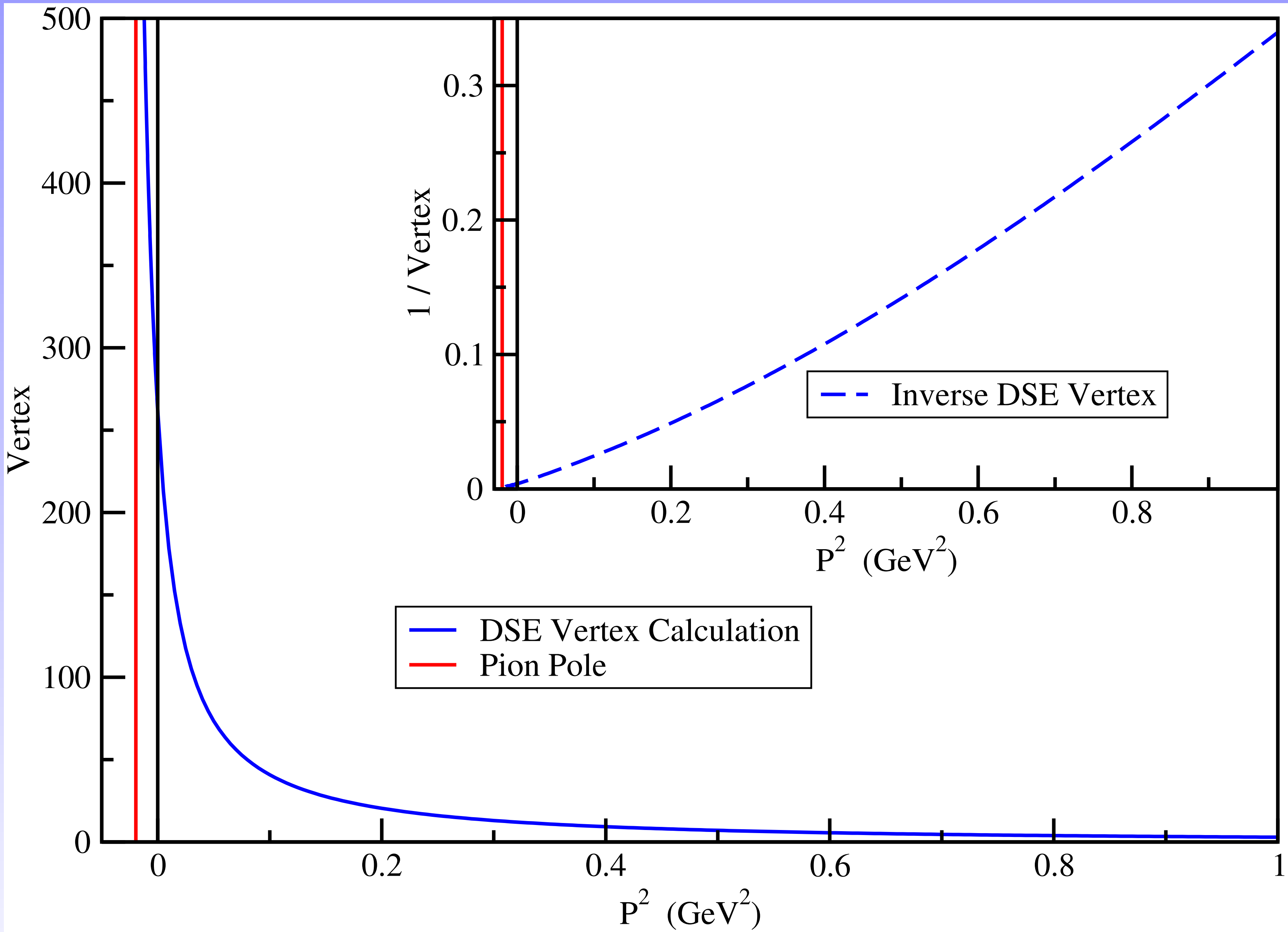
Numerical Reality



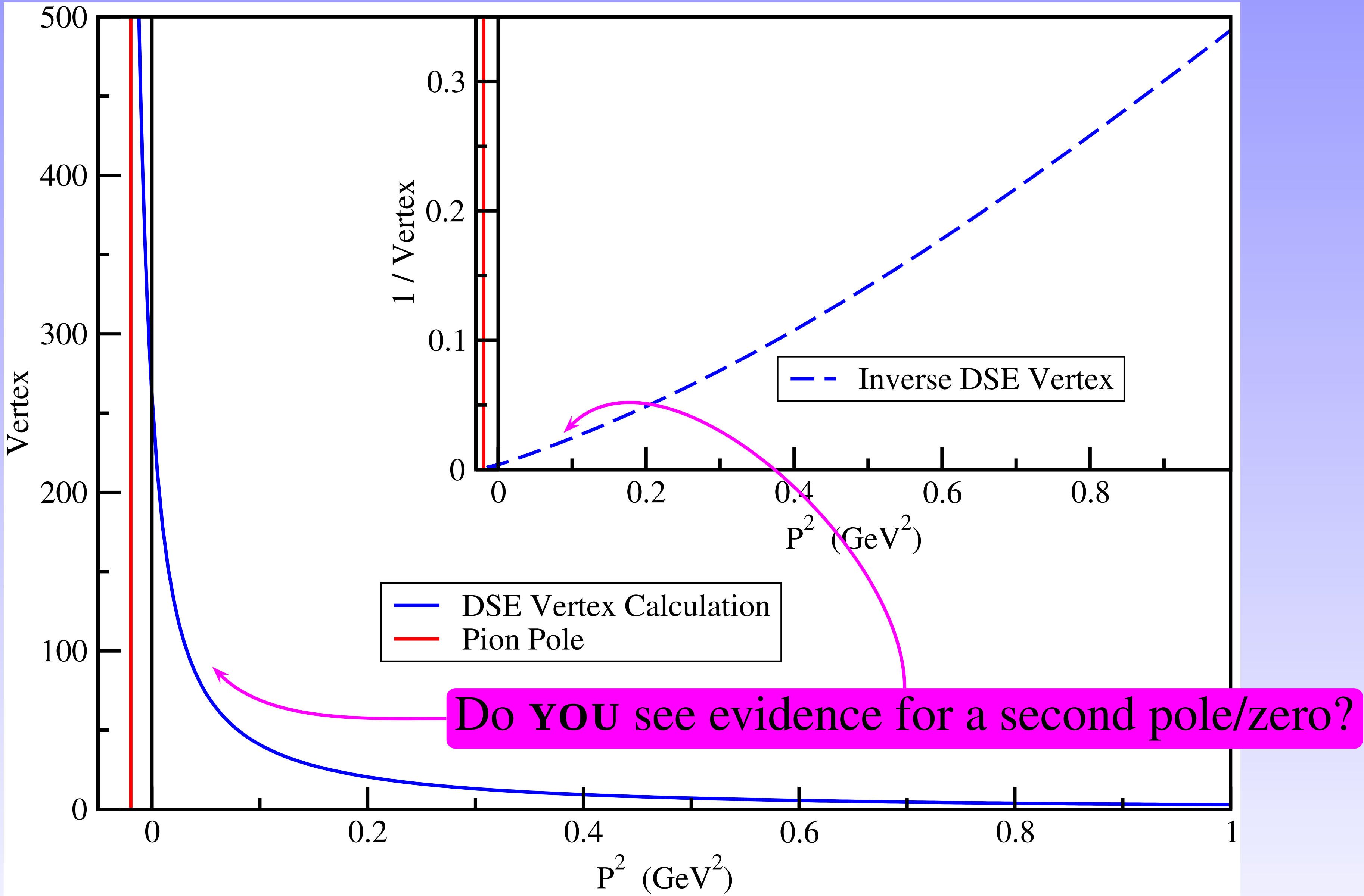
☹ Finding exact pole location is HARD

☺ Finding zeros is EASY

So. . . invert the data!



Vertex
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
P^2 (GeV^2)
DSE Vertex Calculation
Pion Pole
1 / Vertex
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
P^2 (GeV^2)
Inverse DSE Vertex


500
400
300
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0
Vertex
0
0.2
0.4
0.6
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1
P^2 (GeV^2)
0.3
0.2
0.1
0
1 / Vertex
0
0.2
0.4
0.6
0.8
P^2 (GeV^2)
Inverse DSE Vertex
DSE Vertex Calculation
Pion Pole
Do YOU see evidence for a second pole/zero?


SLIGHT
DETOUR

What do we know about calculating excited states?

What do we *think* we know?

How many excited states can one expect to find when you have some vertex correlator?

Simple Model Investigation

Naïvely the spectral model of a 3-point function is a sum of bound states.

Make a simple model for a vertex:

- A sum of monopoles

$$V(\vec{P}) = a + \sum_i \frac{c_i}{\vec{P}^2 + m_i^2}$$

- m_i : Mass of bound state i
- c_i : Residue of bound state i

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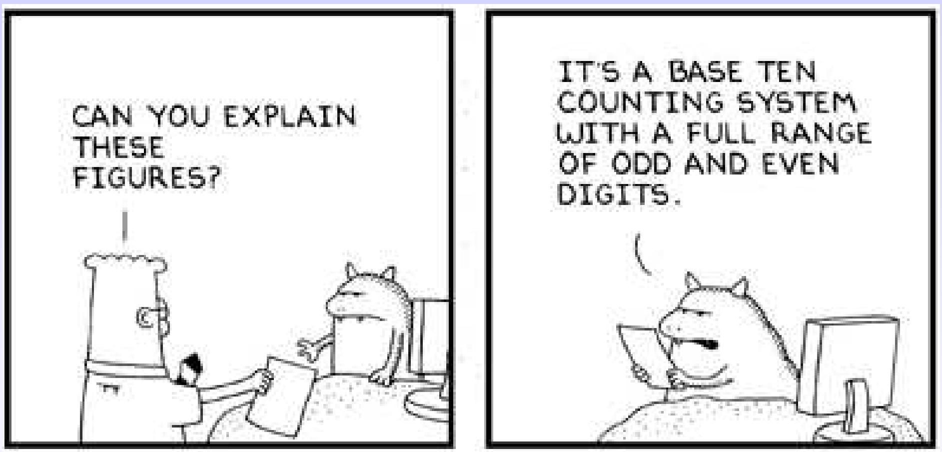
Use some physics

What do we know about m_i and c_i?

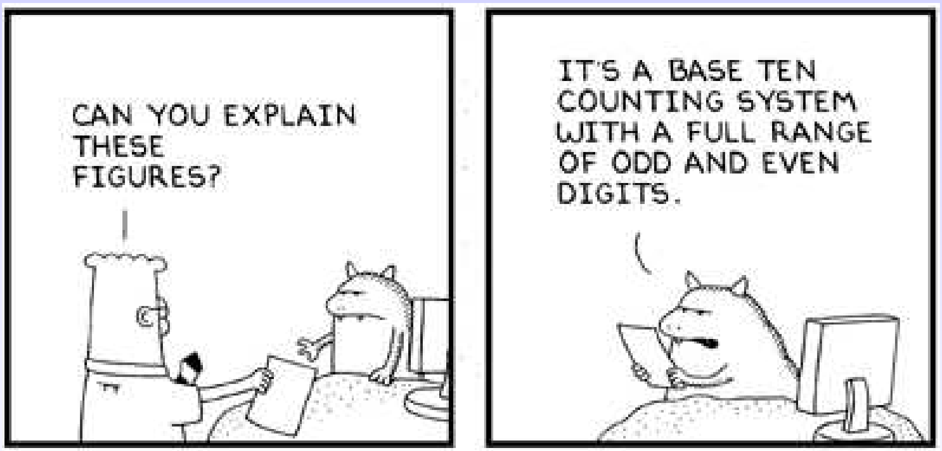


Use some physics

What do we know about m_i and c_i?

Mass

- Masses of the bound states
- PDG publishes these for the REAL WORLD



Use some physics

What do we know about m_i and c_i?

Mass

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- PDG publishes these for the REAL WORLD

Residue

- Related to the decay constant of the bound state, i.e. f_π
- We <u>**know**</u> that these alternate in sign

`Höll, Krassnigg, Roberts nucl-th/0406030`

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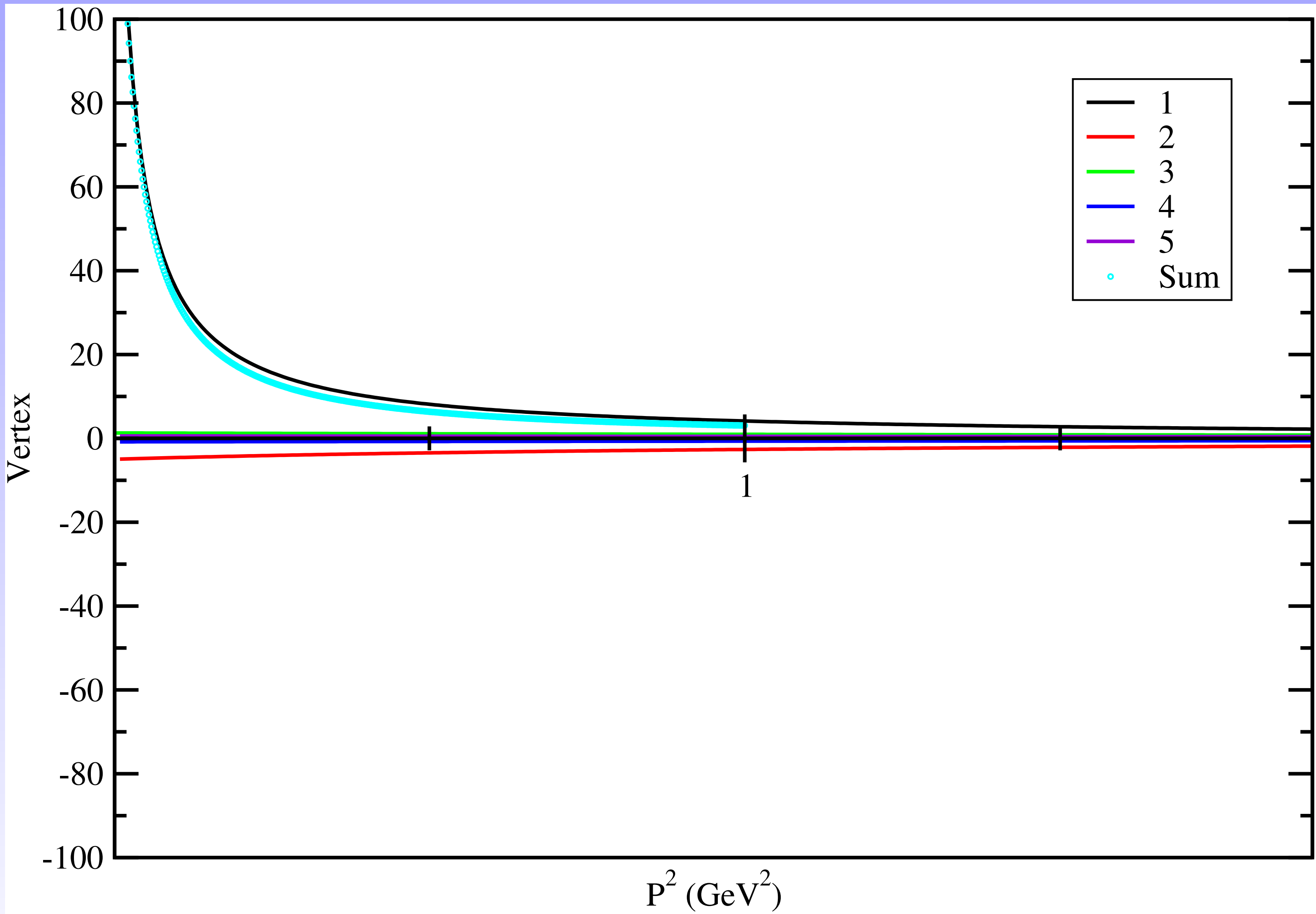
Visualise the problem

Mass	Residue
0.14	4.23
1.06	-5.6
1.72	3.82
2.05	-3.45
2.2	2.8

Masses are motivated by the PDG.

Residues are all of the same magnitude.

The Simple Model



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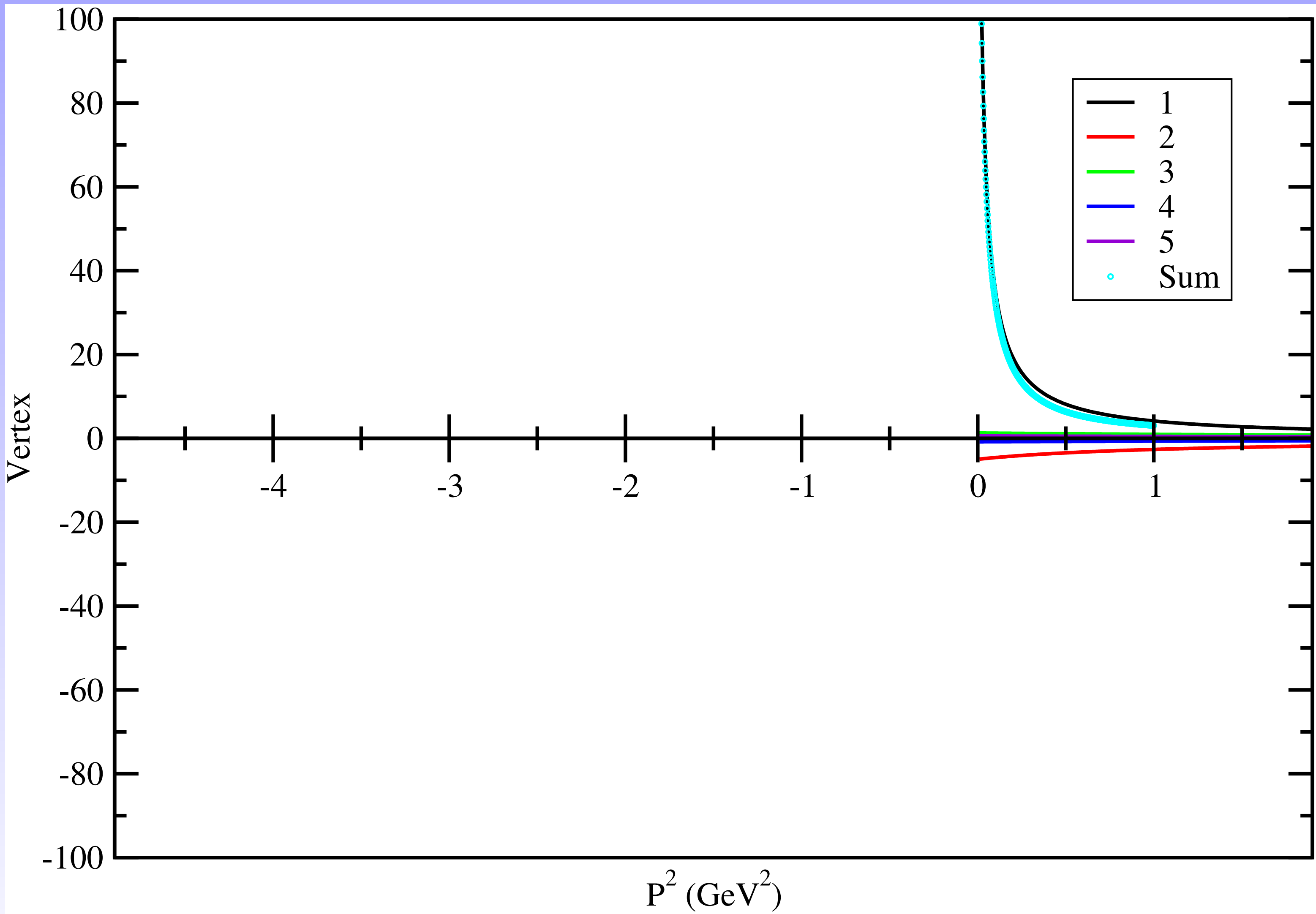

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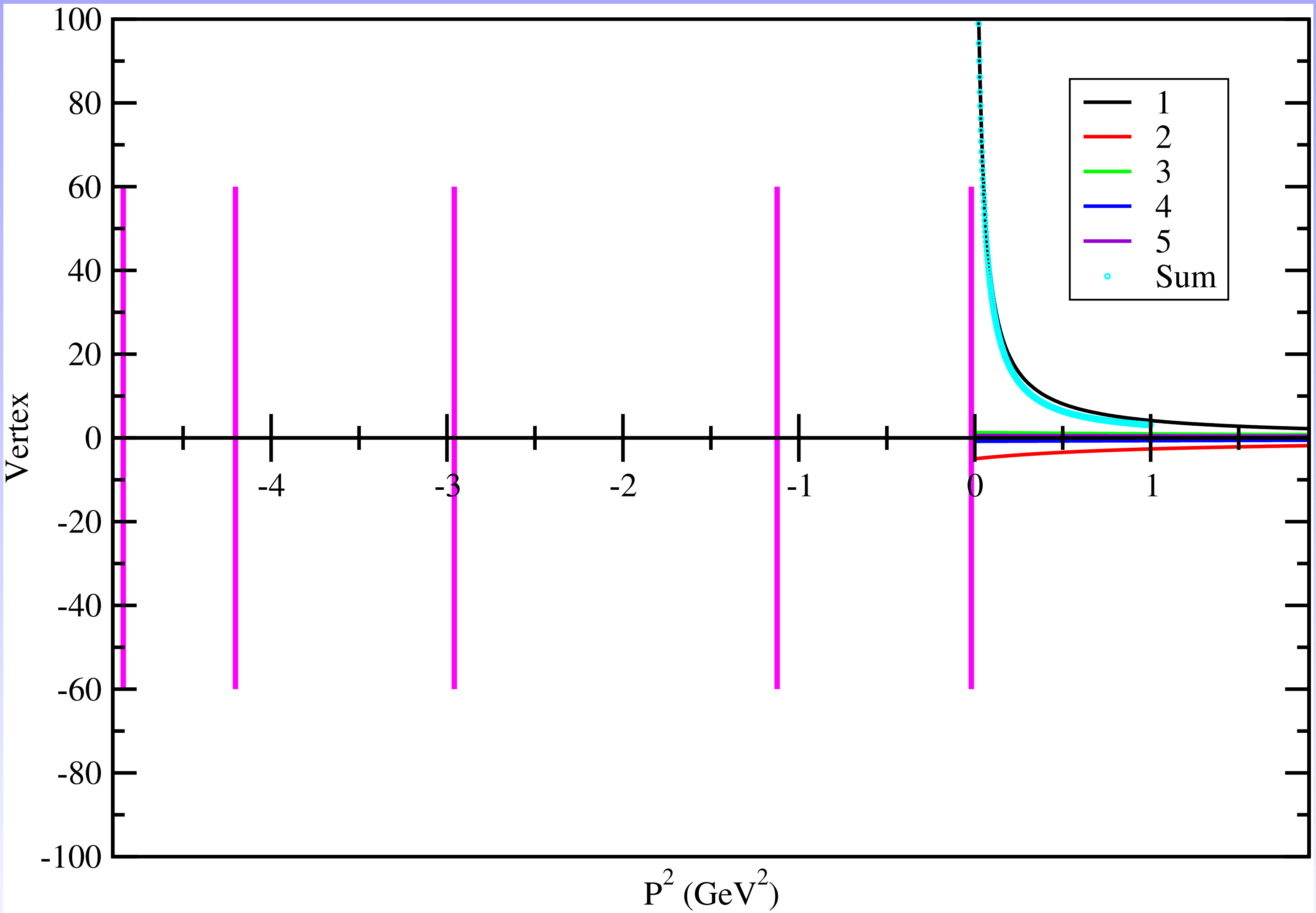

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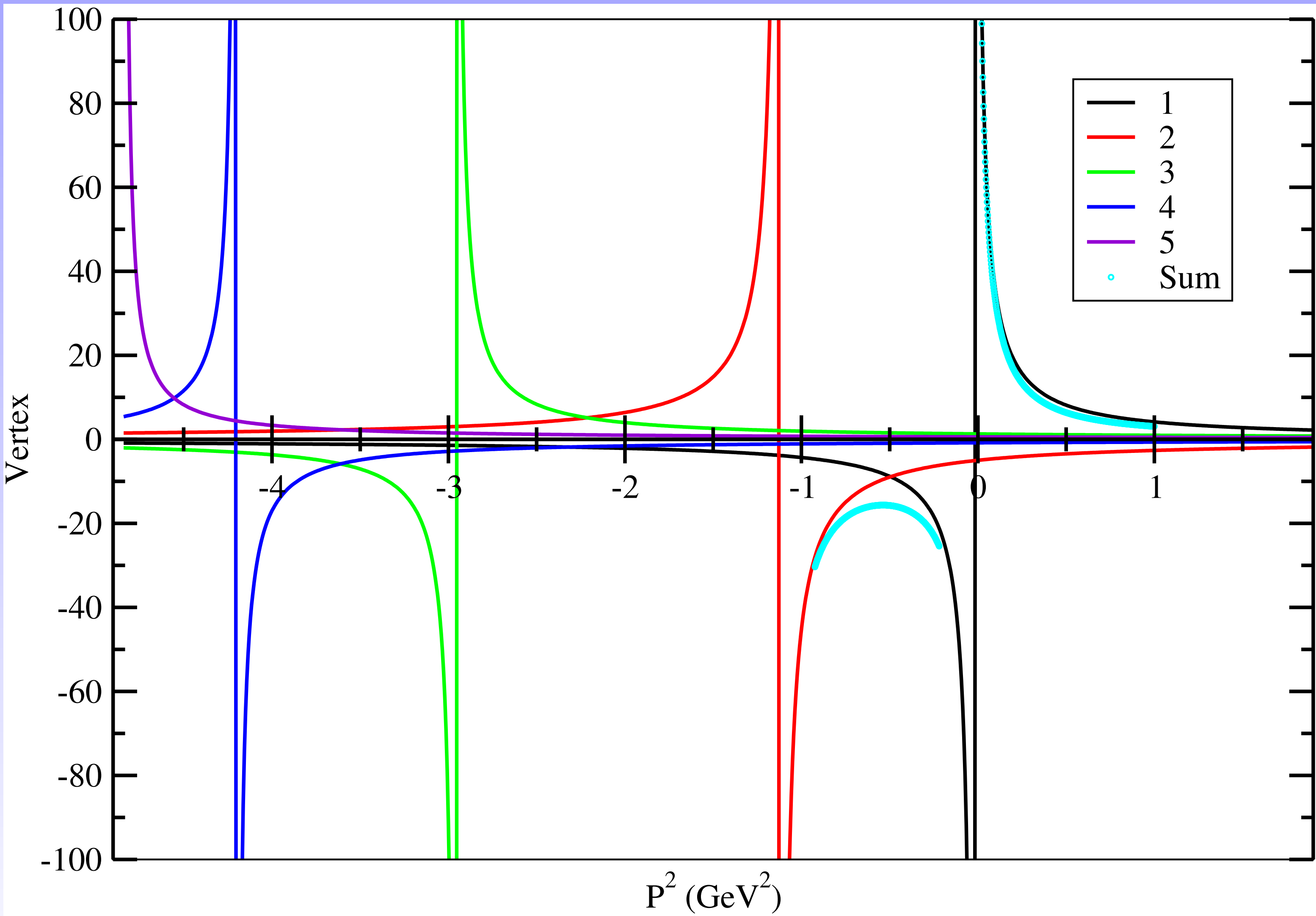
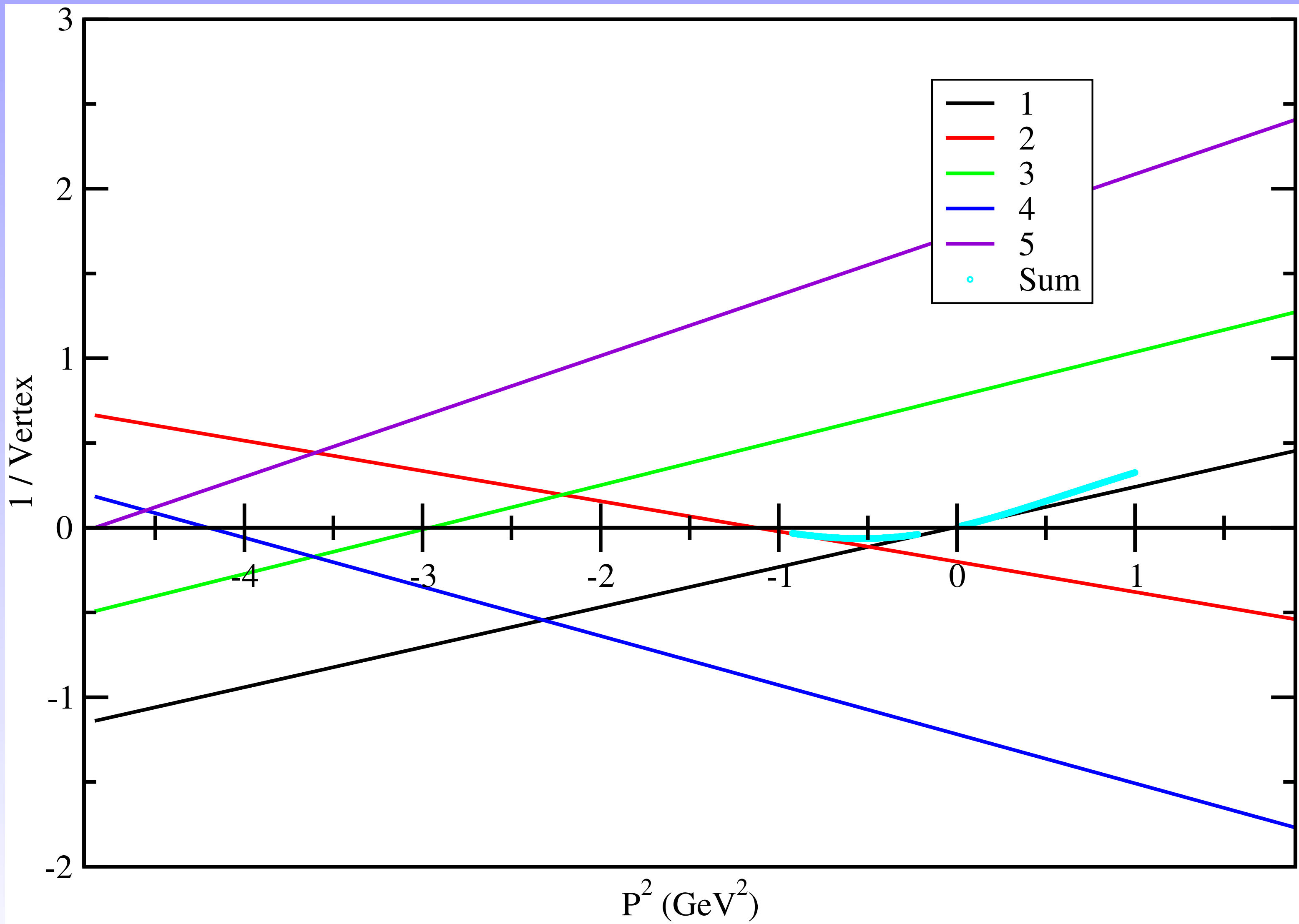


Invert for zeros



Fitting the Data

Assume we know nothing about the source of the data...

Fitting the Data

Assume we know nothing about the source of the data...

Fit a Padé to data:

- We know the data has zeros
- Little else is known about the functional form i.e. what does the background look like?

So choose a generic form:

$$f(P^2) = \frac{a_0 + a_1 P^2 + a_2 P^4 + \ldots + a_n P^{2n}}{1 + a_{n+1} P^2 + a_{n+2} P^4 + \ldots + a_{2n} P^{2n}}$$

Interpreting the solution

The “data” has an unknown number of bound states contributing.

How reliable are the n-zeros we find?

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- The contribution to the vertex from the bound states falls as $1/m^2$
  $\Rightarrow$ Clear hierarchy

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How reliable are the n-zeros we find?

- The contribution to the vertex from the bound states falls as $1/m^2$
  $\Rightarrow$ Clear hierarchy
- At BEST the first $n-1$ solutions would be reliable
- The last solution would contain all the remaining physics

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How?

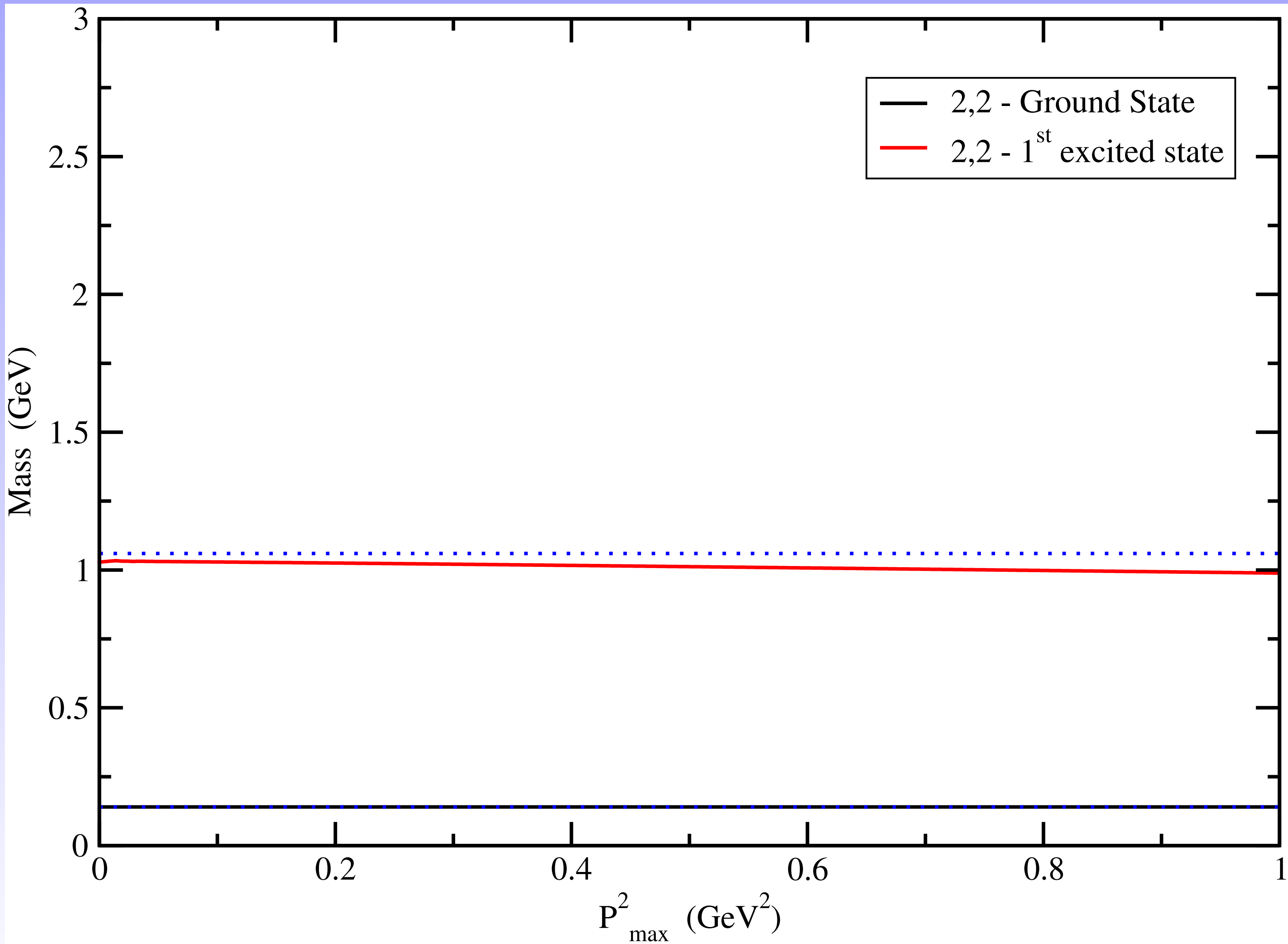
The fitting function:

$$f(P^2) = \frac{a_0 + a_1 P^2 + a_2 P^4 + \ldots + a_n P^{2n}}{1 + a_{n+1} P^2 + a_{n+2} P^4 + \ldots + a_{2n} P^{2n}}$$

Have data for $P^2 \in (0, 1]$ GeV^2.

1. Fit data in range $P^2 = (0, P^2_{max}]$.
2. Increase P^2_{max} and repeat fit.

Results — Masses



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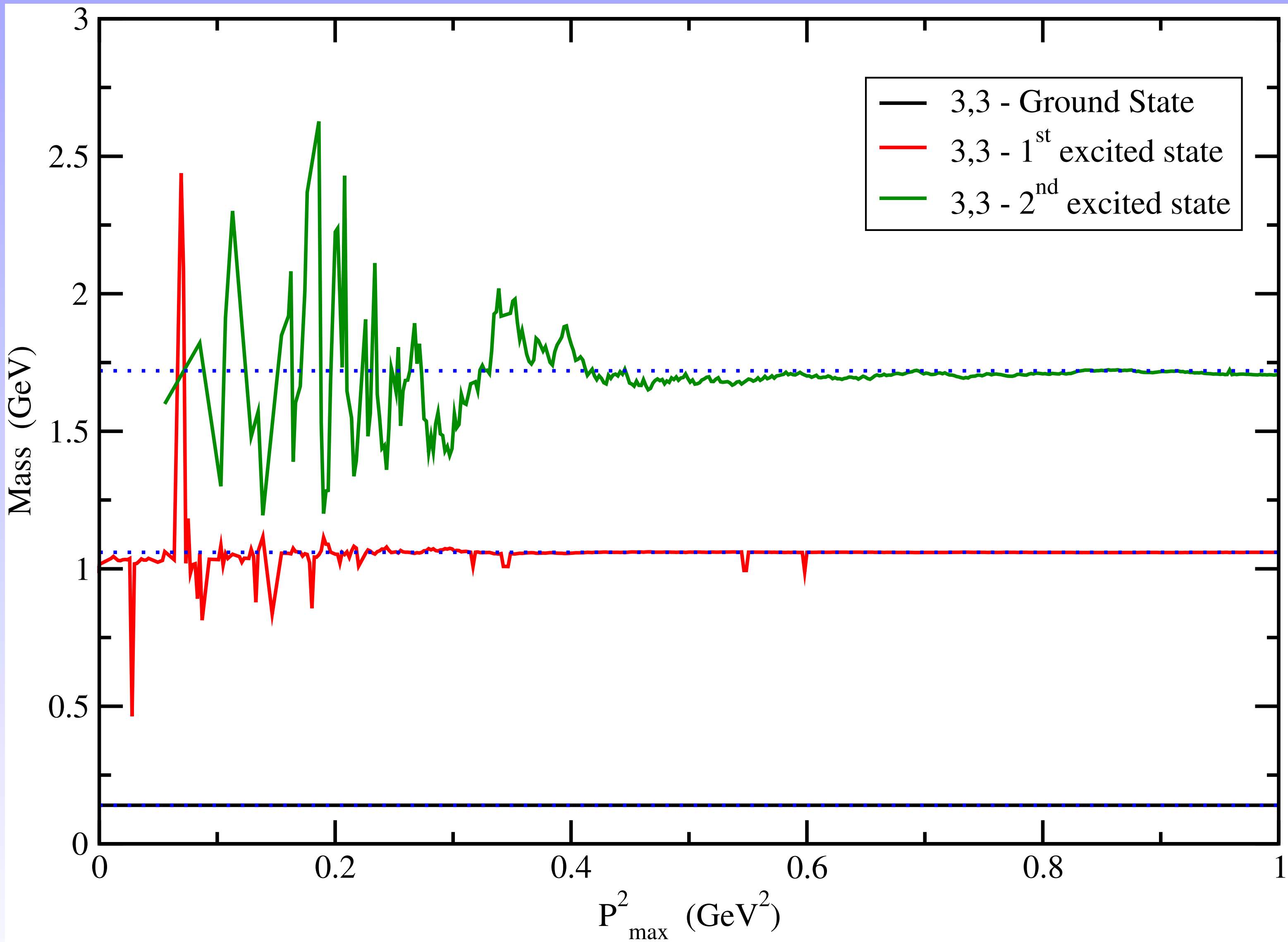

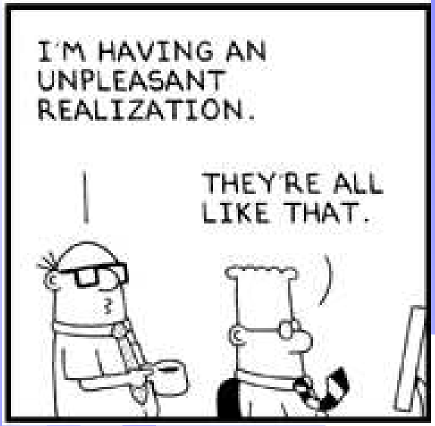

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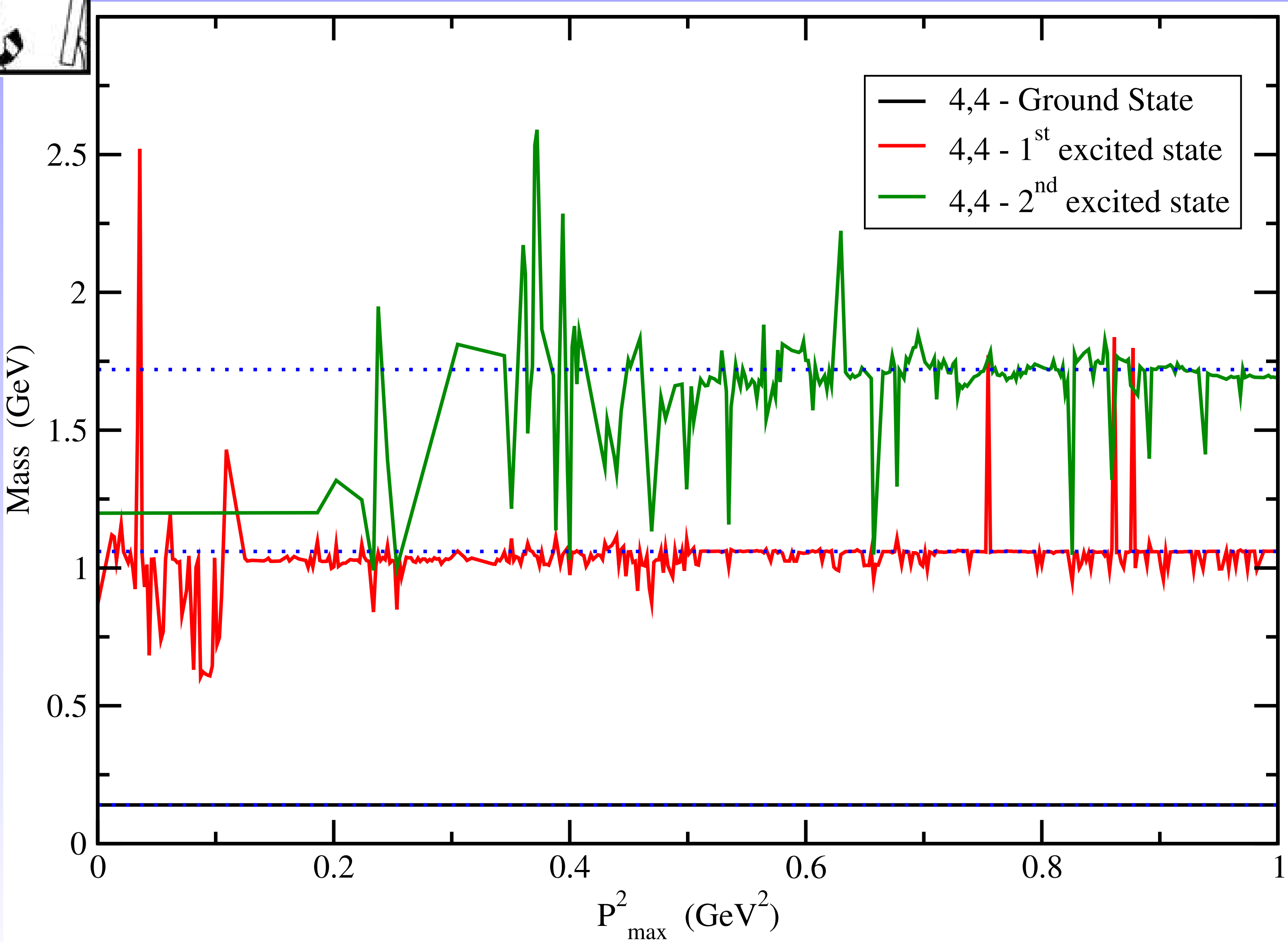

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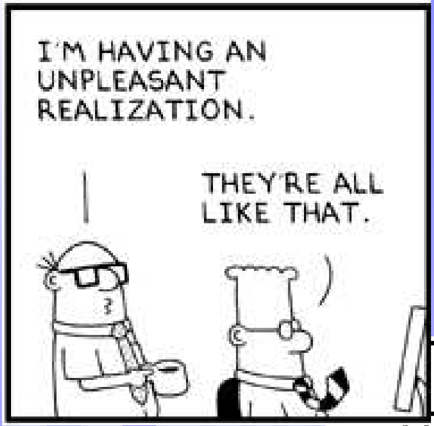
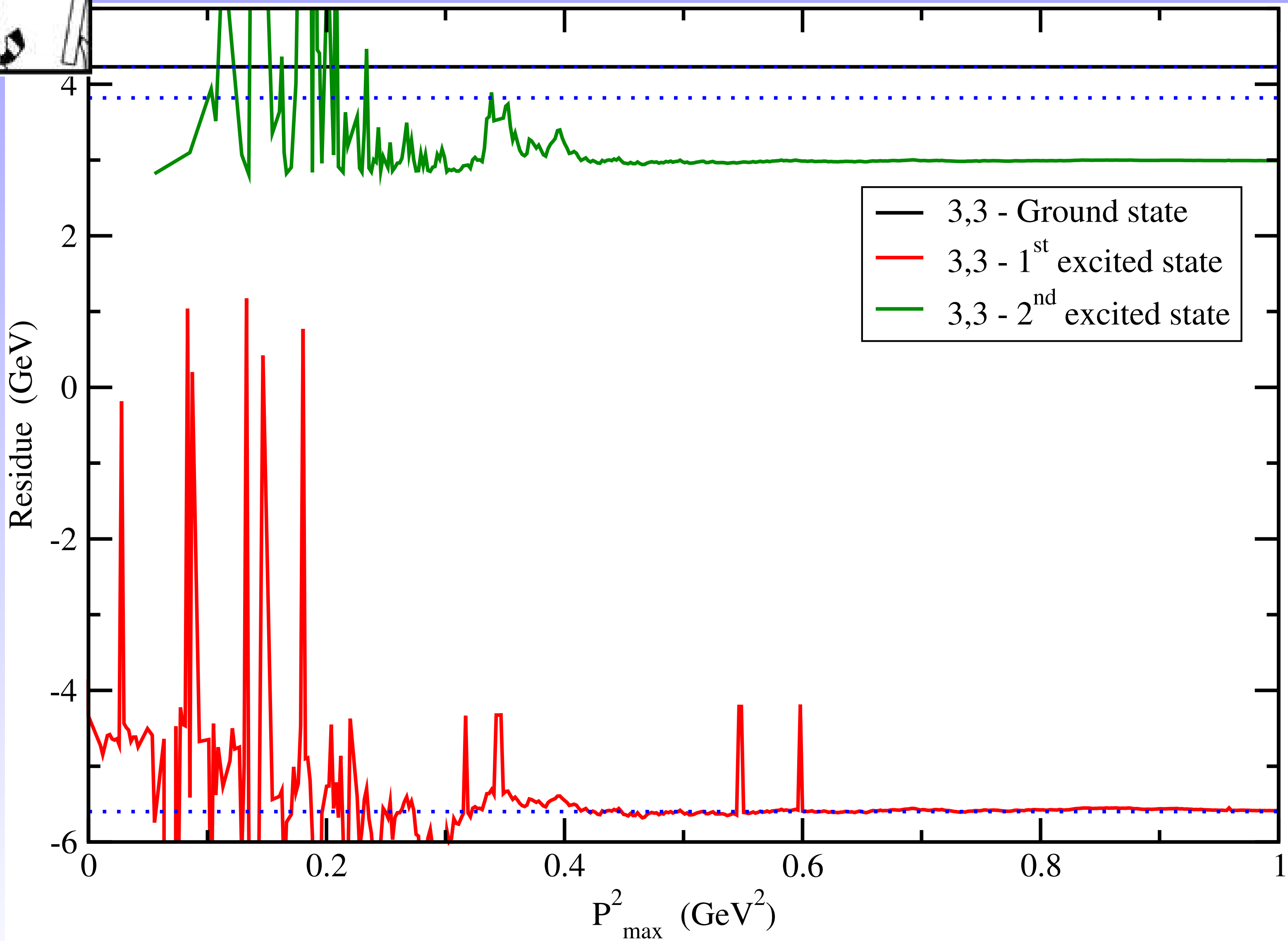


Results — Residues



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The Glass is Half Full

Optimistically there is a chance to get some information about excited states with only spacelike data. . .

	Mass	Residue
Ground State:	☺	☺
First Excited State:	☺	☺
Second Excited State:	☺	☹

Lattice QCD

A similar investigation in the Lattice QCD mindset is relatively easy:

- Momentum space $\rightarrow$ Coordinate space

$$f(p) = a + \sum_i \frac{c_i}{p^2 + m_i^2}$$

Becomes...

$$C(\tau) = a\delta(\tau) + \sum_i \frac{c_i}{2m_i} e^{-m_i \tau}$$

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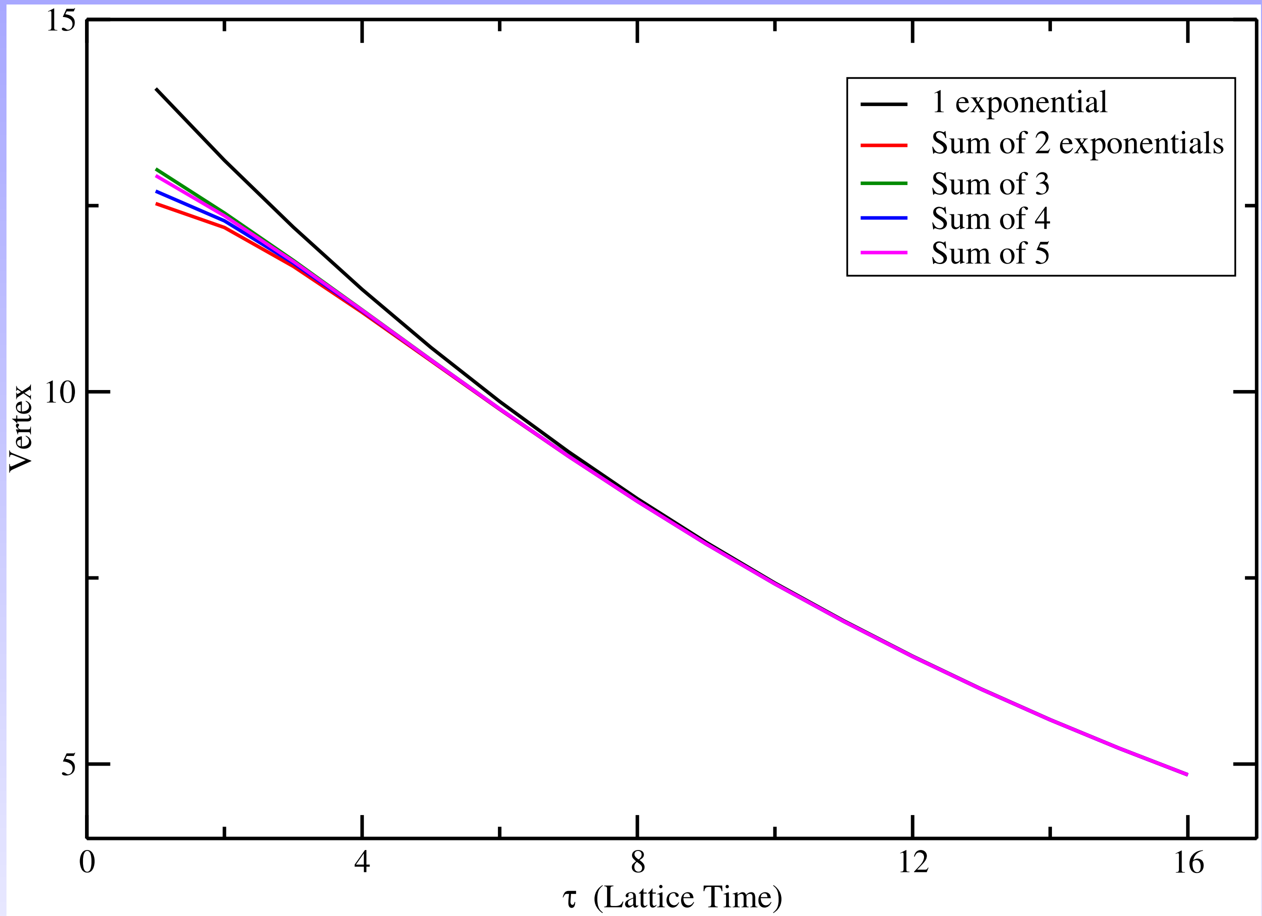
Lattice QCD



15
10
5
Vertex
0
4
8
12
16
τ (Lattice Time)
1 exponential
Sum of 2 exponentials
Sum of 3
Sum of 4
Sum of 5

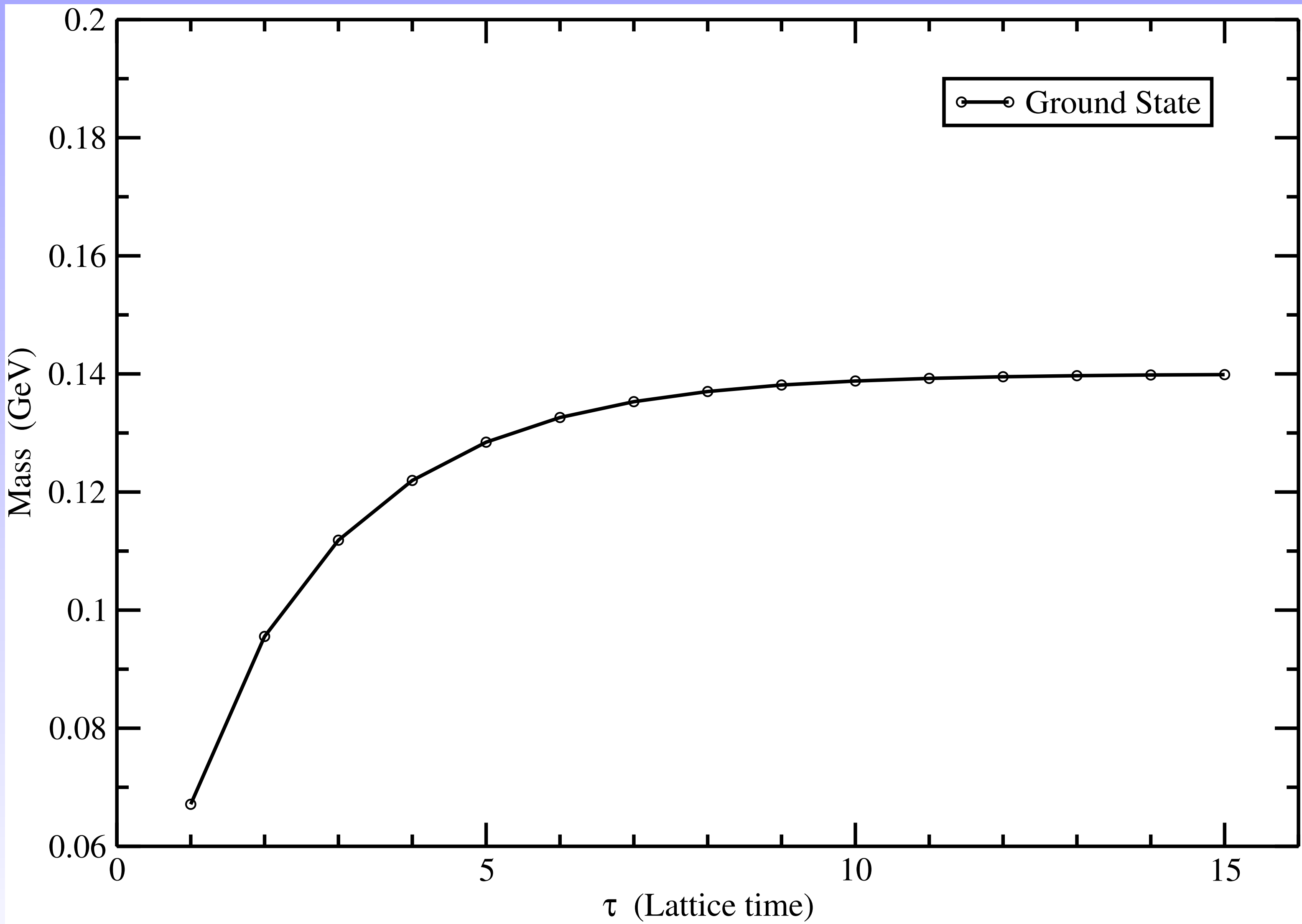
Effective Mass

Simple to extract the ground state effective mass ANALYTICALLY on the lattice.

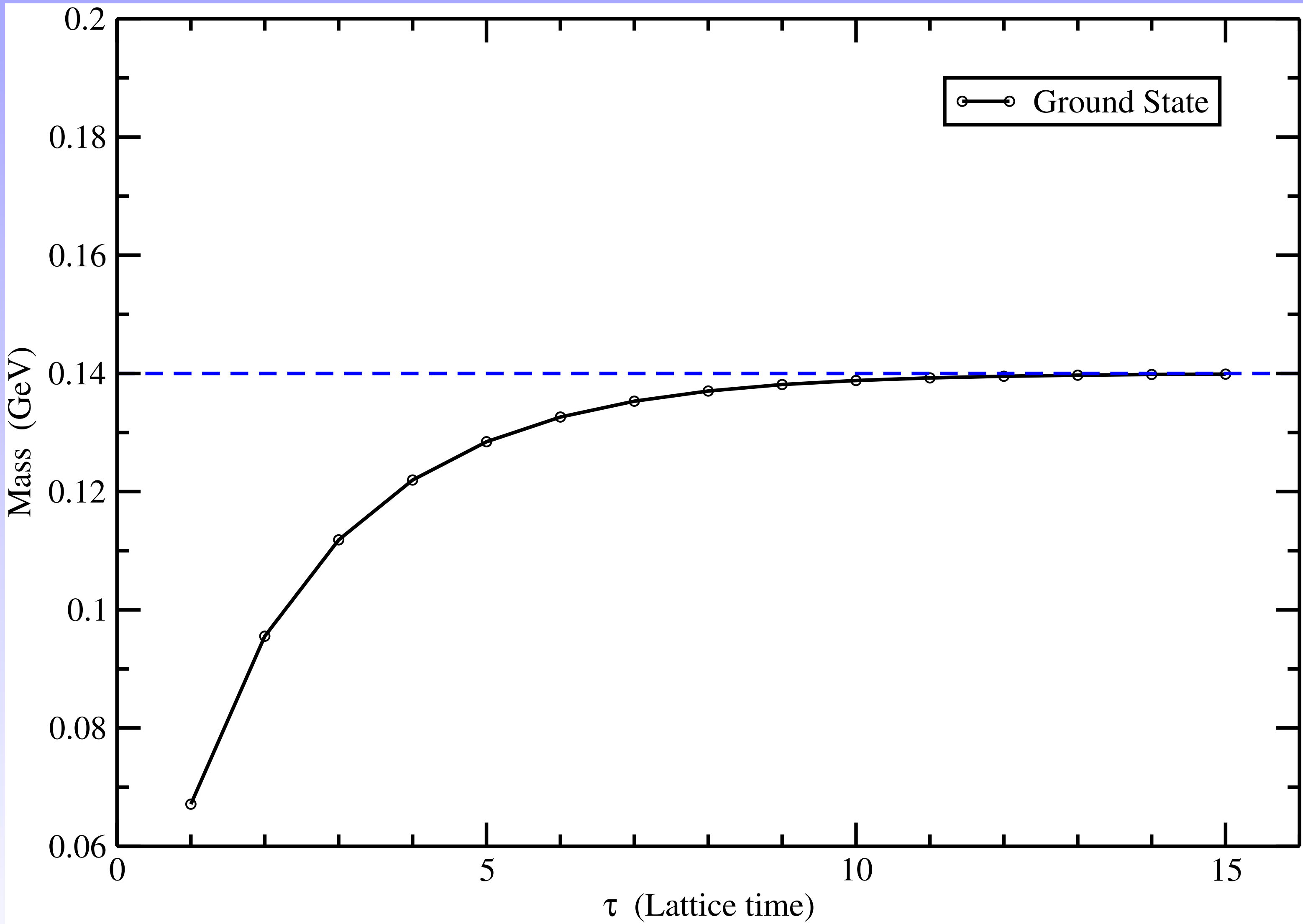
$$m = -\ln\left(\frac{C(t+1)}{C(t)}\right)$$

Left as an exercise for this evening.

Ground State extraction



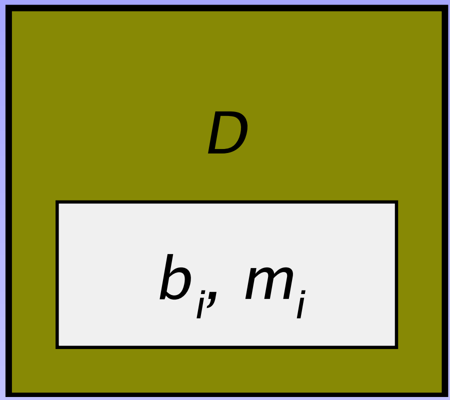
Ground State extraction



Alternate Solution

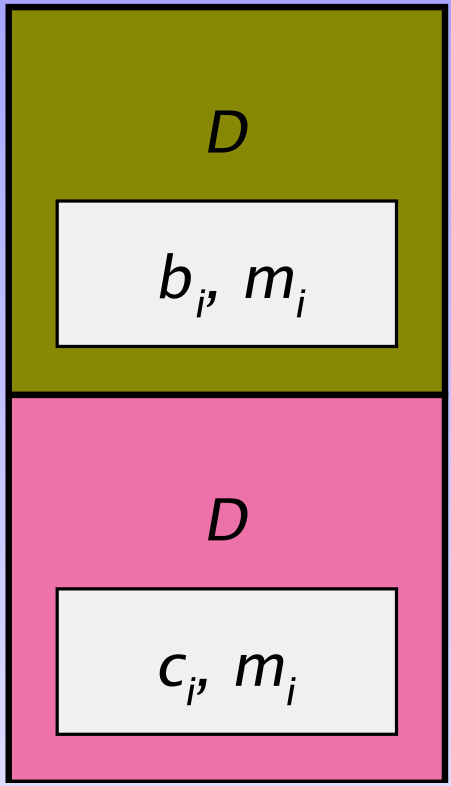
WISDOM: The best (only?) way to get access to higher excited states is through improved operators.

More Operators?



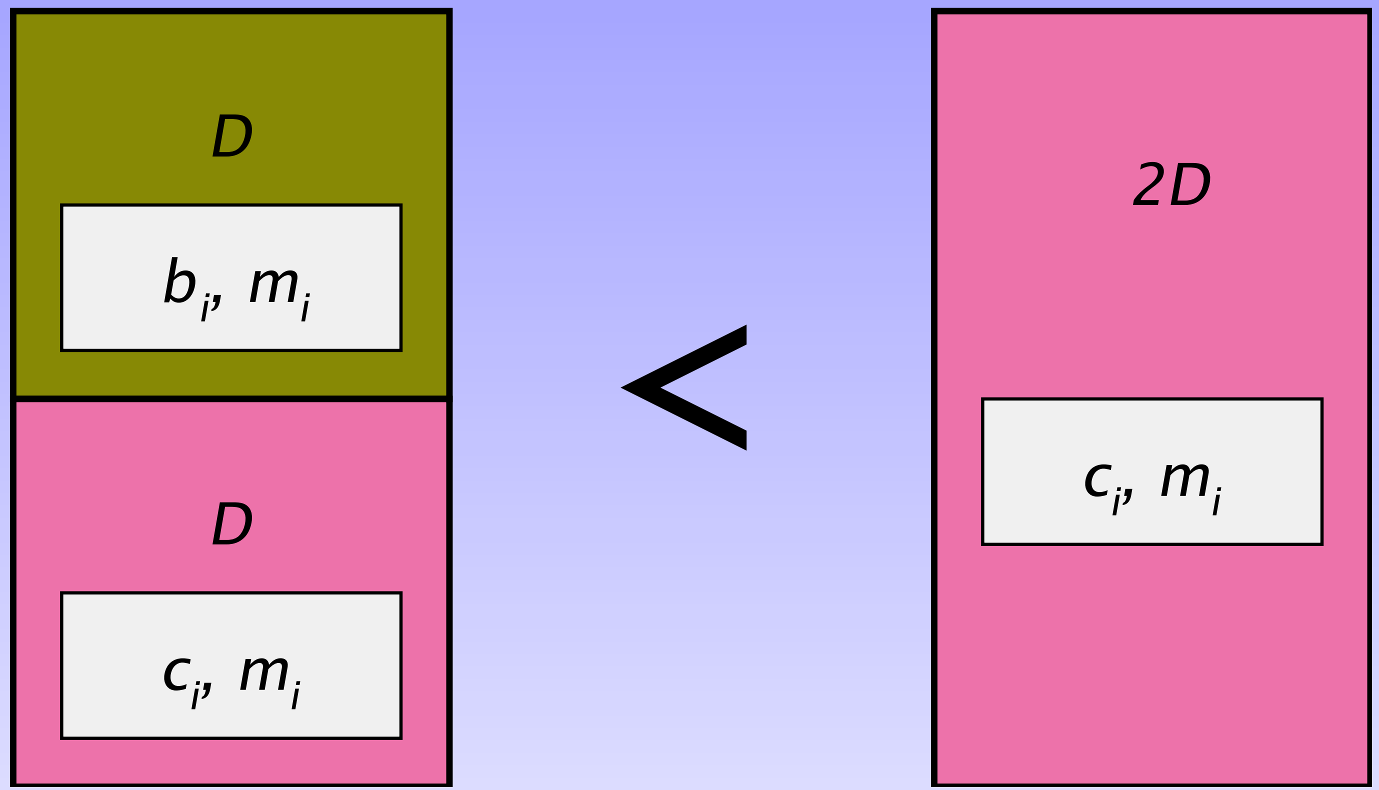
- A data set, size D gives description (b_i, m_i)

More Operators?



- A data set, size D gives description (b_i, m_i)
- A second data set, also size D gives description (c_i, m_i)

More Operators?



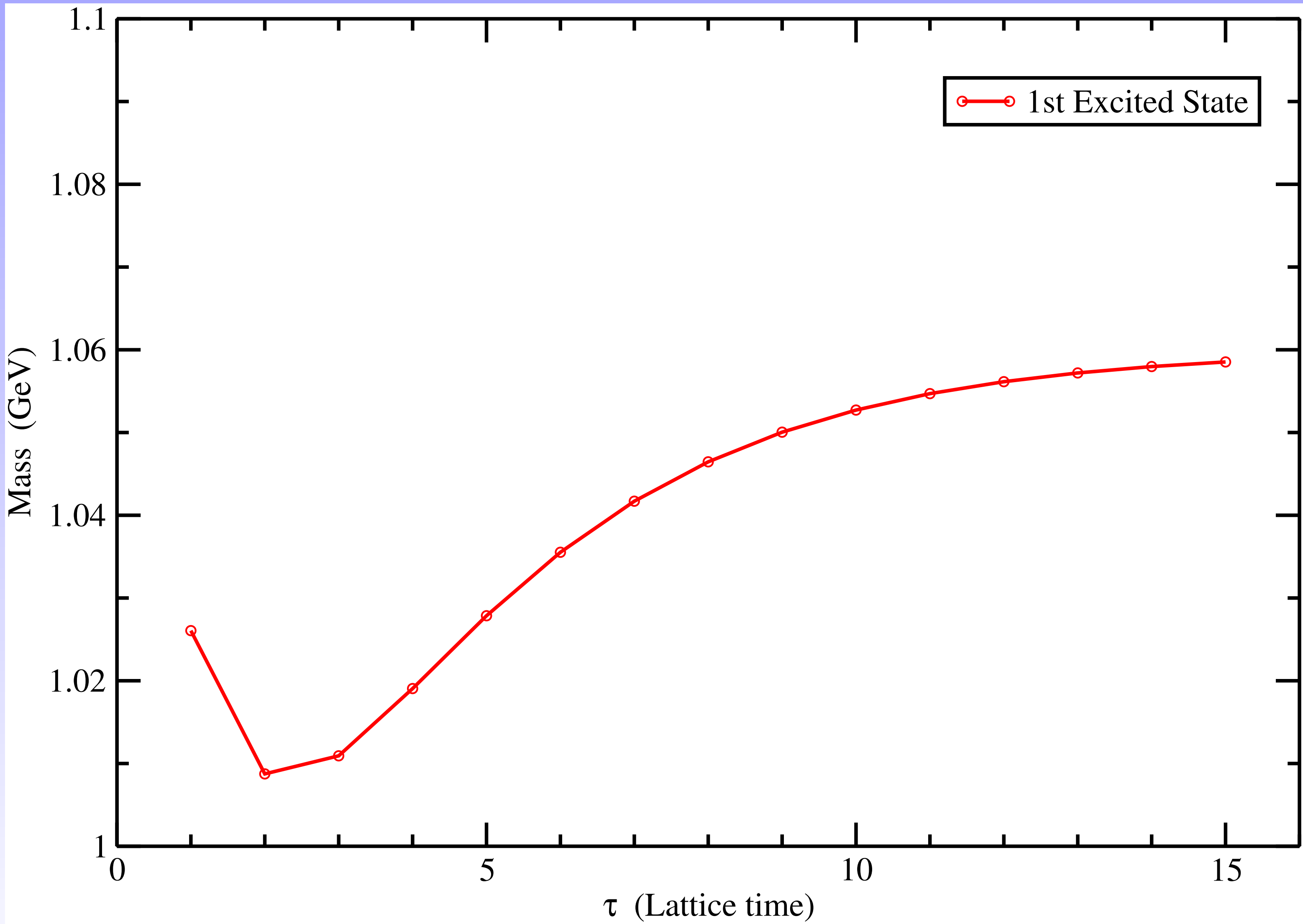
A larger data set, size $2D$ with description (c_i, m_i)
$\Rightarrow$ Provides **more** constraints.

Perfect Operators

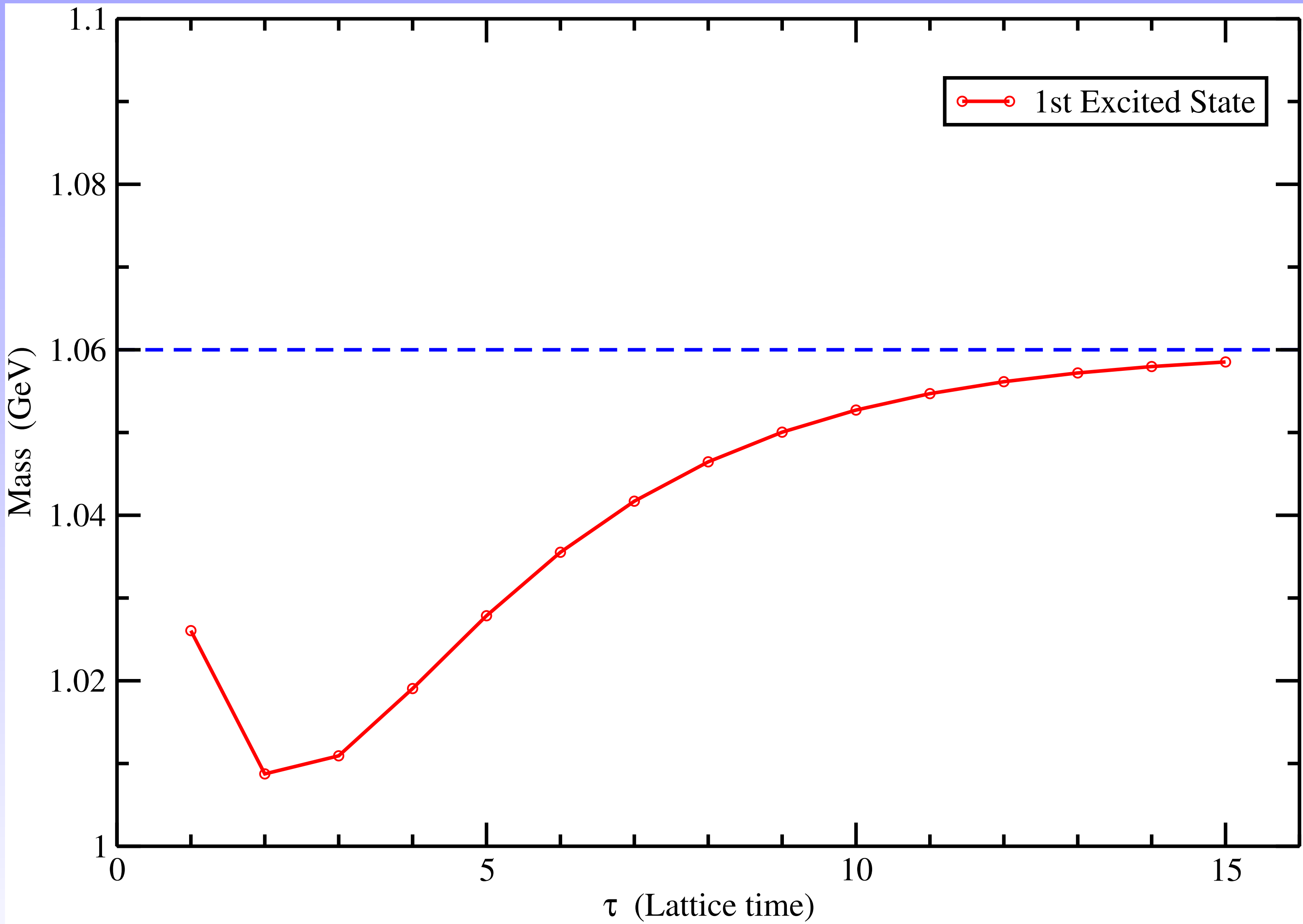
It's my model. . .

. . . and I'll truncate if I want to.

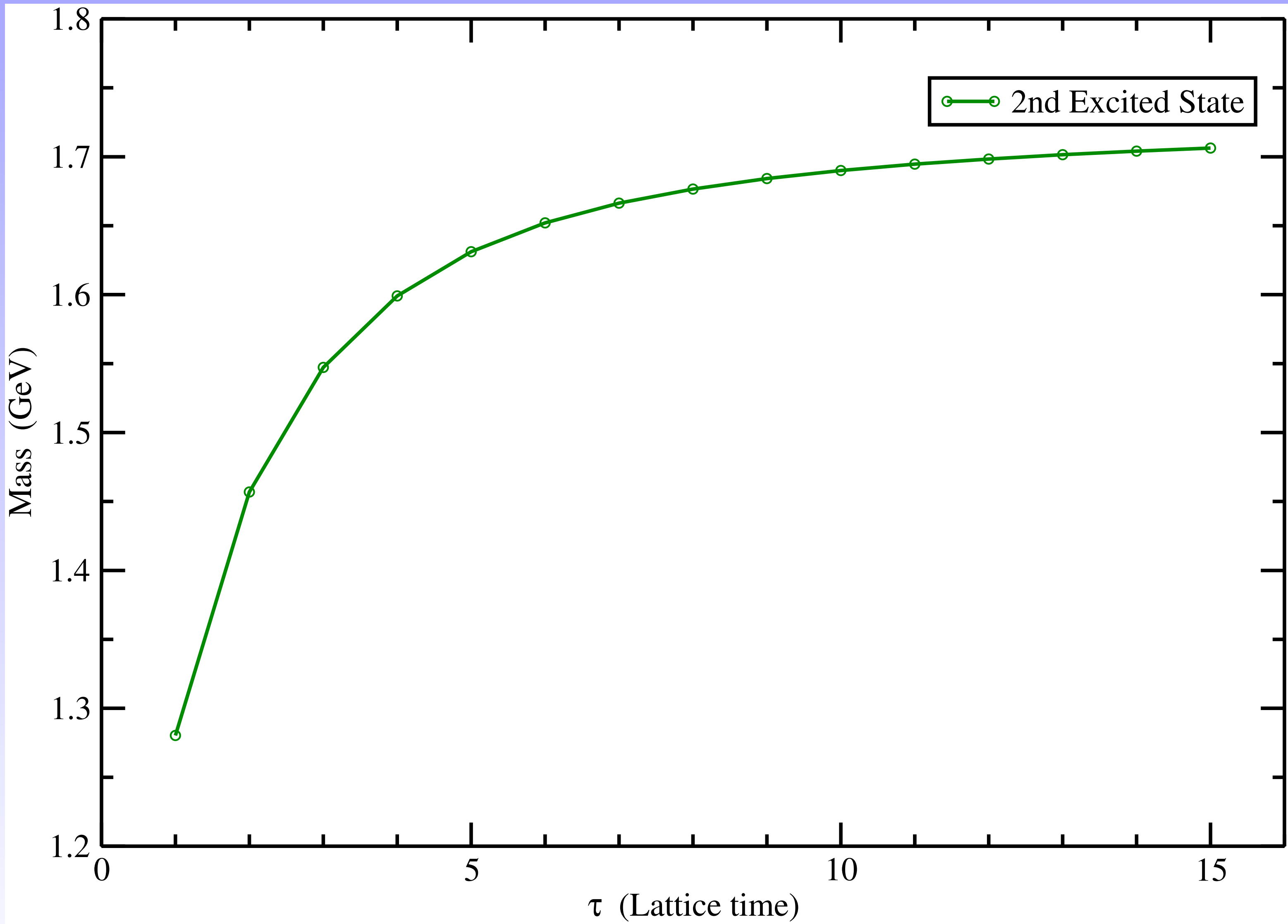
First Excited State extraction



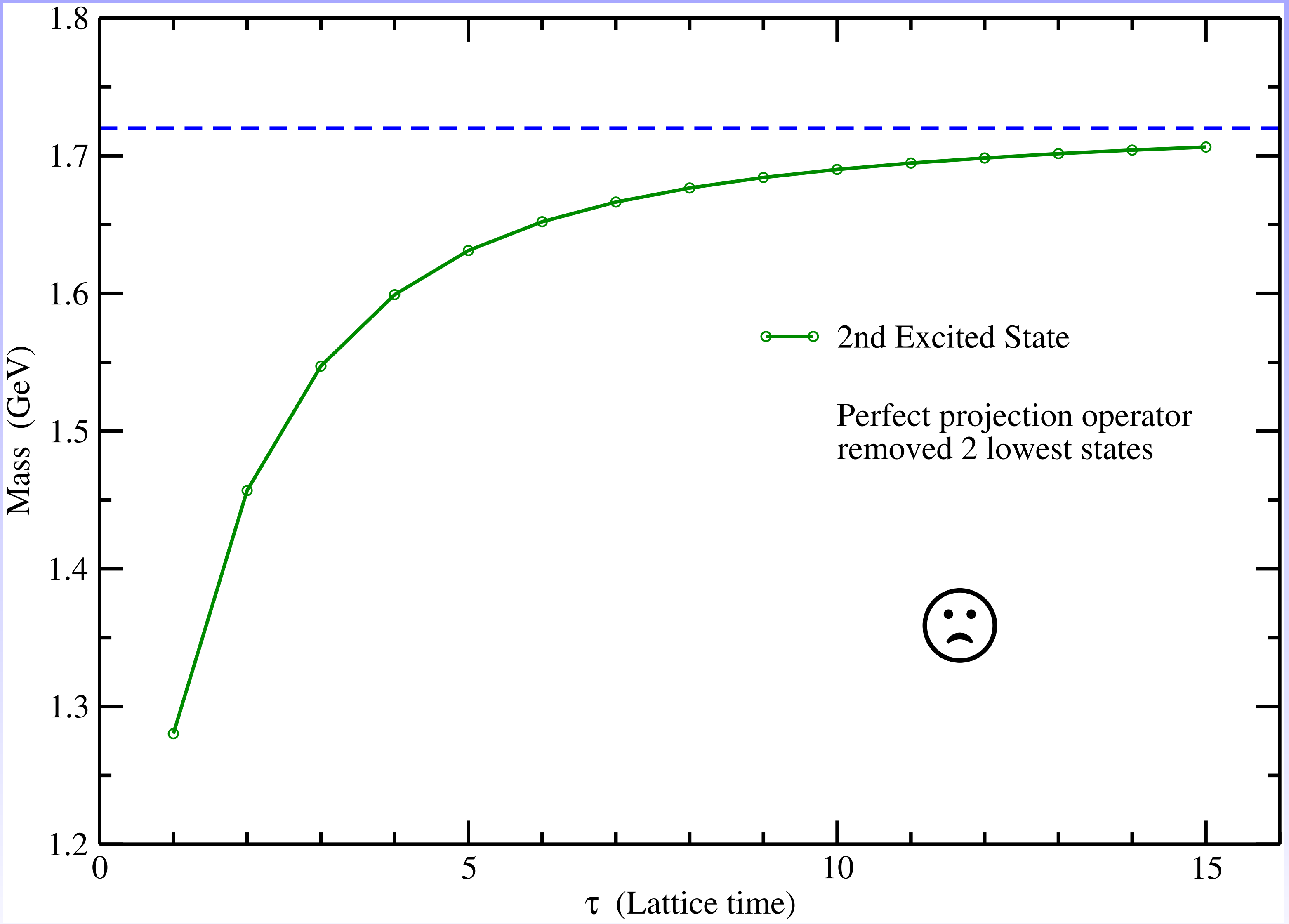
First Excited State extraction



Second Excited State extraction



Second Excited State extraction



Exponential Fits

m_0	m_1	m_2	m_3	m_4	χ^2
0.14	0.89	—	—	—	$< 10^{-06}$
0.14	1.05	1.80	—	—	$< 10^{-10}$
0.14	1.06	1.62	1.97	—	$< 10^{-11}$
0.14	1.06	1.36	1.73	1.94	$< 10^{-11}$
0.14	1.06	1.72	2.05	2.2	Source

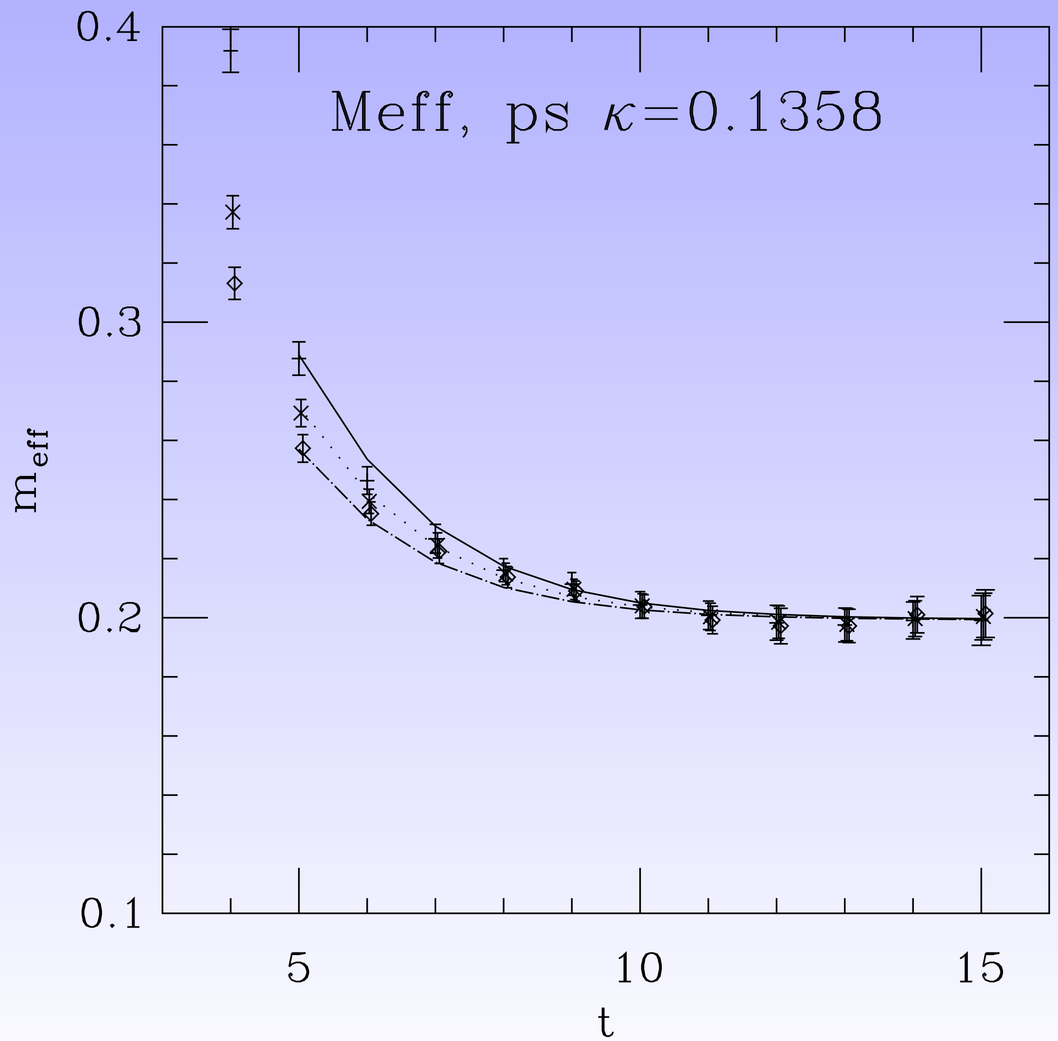
(Numbers in red are within 10%.)

(NOTE: This is χ^2 and not $\chi^2/\mathrm{DoF}\ldots$)

hep-lat/0403007

CR Allton, A Hart, D Hepburn, AC Irving, B Joo, C McNeile, C Michael, SV Wright.

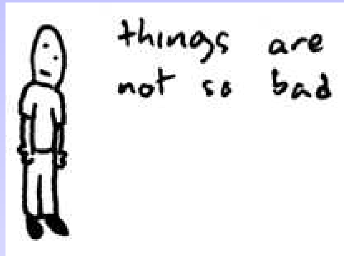
Phys.Rev. D70 (2004) 014501



Return to DSE

Strengths

- Obeys (enough) symmetries of QCD.
- FAST to calculate.
- Able to calculate in *both* Spacelike and Timelike regimes.
- Systematic errors well quantified.



Weaknesses

- Pion loops not included.
- Current implementation restricts maximum accessible meson mass.

Simplest(?) Situation

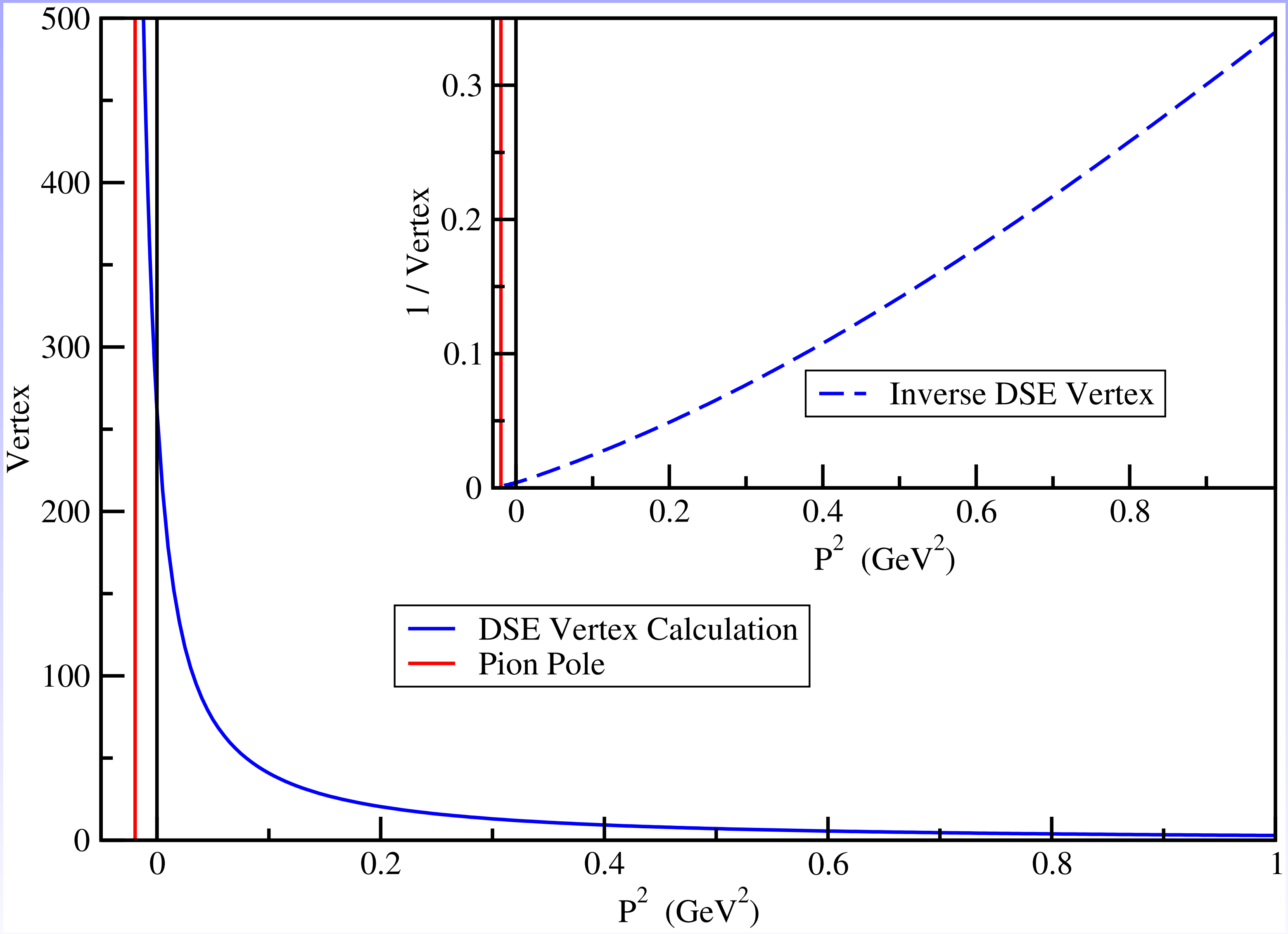
Pseudoscalar
i.e. π

Light Quarks
i.e. $m_q = m_{u,d}$

Spacelike momentum
i.e. $P^2 > 0\,\mathrm{GeV}^2$

DSE Vertex Calculation

ONLY use spacelike data!



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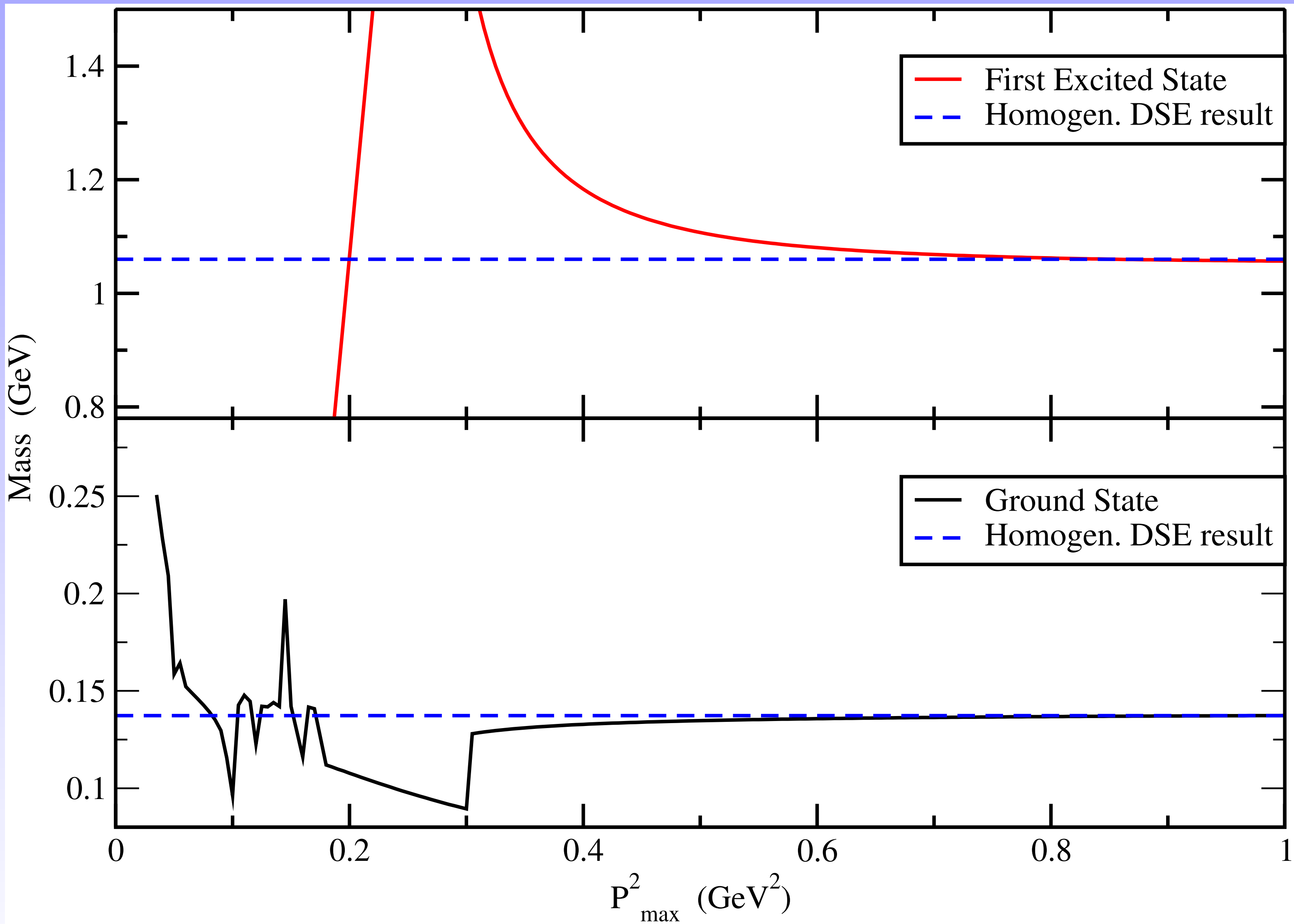
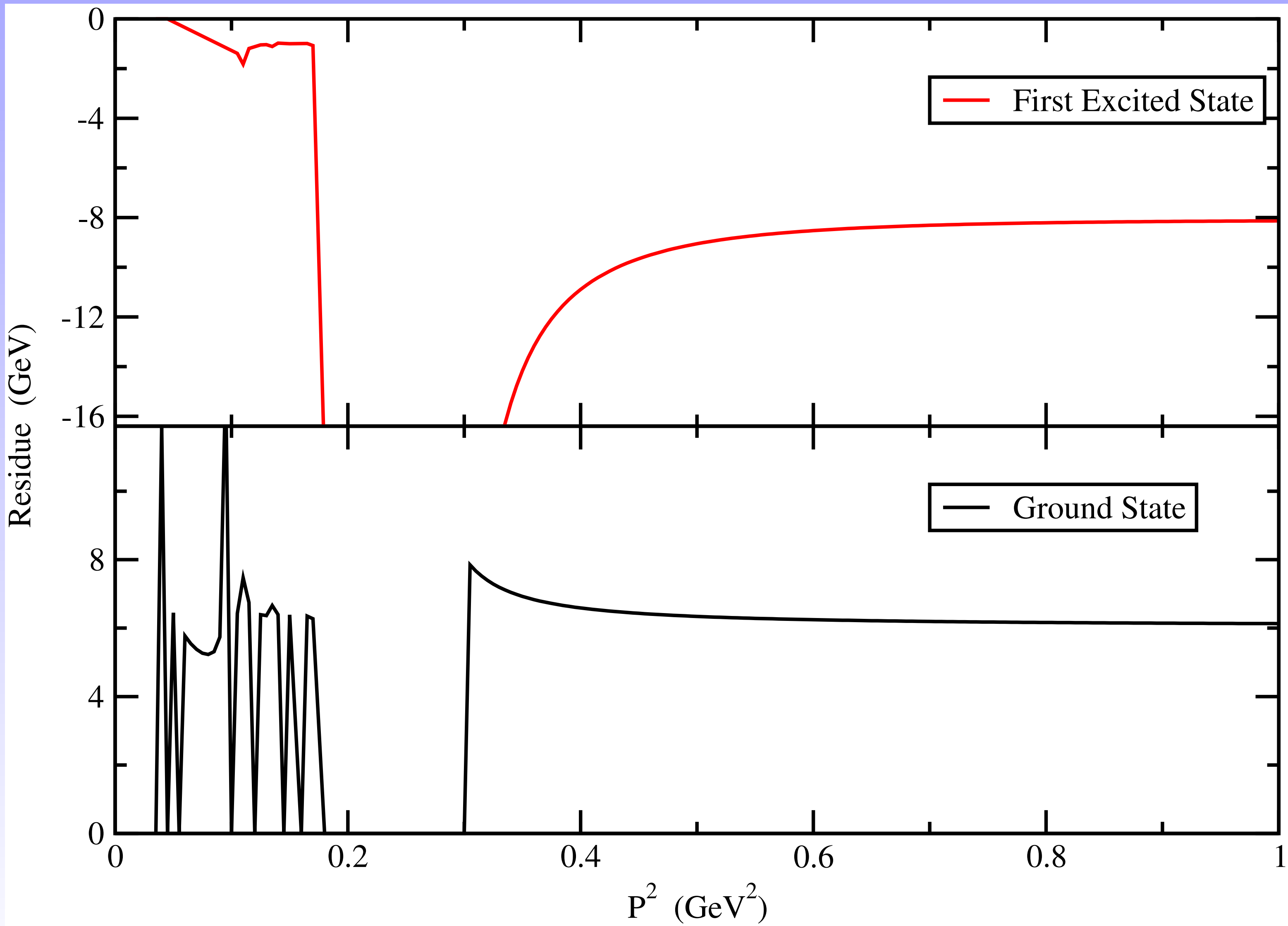
Extracted Masses

Consistent with homogeneous solution



DSE Extracted Residue

Definitive evidence for sign change in residues



In a perfect world:

- Only need to know the function (and all derivatives) at a single point to uniquely determine it.

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- Only need to know the function (and all derivatives) at a single point to uniquely determine it.

In the real world:

- We have FINITE PRECISION calculations.
- Cannot guarantee that there exists a unique solution of a fit of exponentials (or a Padé) to the data to arbitrary precision for:
  — Masses above ~ 1 GeV, OR
  — Second and higher excited states.

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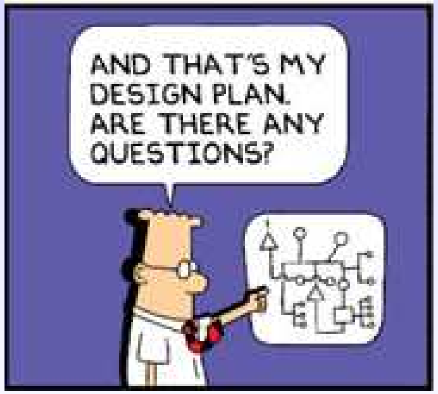
Conclusion

- Even without timelike information the extraction of the ground and first excited states is RELIABLE.
- The sign change in the decay constant is REPRODUCED *without* biasing the fit.
- With inclusion of timelike data higher excited states become ACCESSIBLE.



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