The DMT Dynamical Model for Electromagnetic Production of Pions

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DMT dynamical model

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Motivation

> Meson-exchange πN model below 400 MeV

> DMT dynamical model for pion e.m. production

> Extension to higher energies

> Conclusions

Motivation

QCD \iff Hadronic phenomena

- Iow energies ChPT
- high energies, high momentum transfer— pQCD
- medium energies
 - LQCD

Phenomenology : hadron models, reaction theory

Aim: To construct a coupled-channel dynamical model to study πN scattering and pion e.m. production

• low energies:

e.m. threshold pion production, low energy theorems and compare with ChPT

• medium energies:

 $N! \Delta$ transition form factors, resonance parameters

 high energy and high momentum transfer: transition to pQCD region?

Threshold π^{0} electromagnetic production

Photoproduction

	HBChPT $O(p^4)$	Dispersion relation	Exp. $(10^{-3} / m_{\pi})$
$E_{0^+} \Big _{\pi^0 p}$	-1.1	-1.22	-1.33±0.88±0.03
$\begin{bmatrix} E_{0^+} \end{bmatrix}_{\pi^+ n}$	-0.43	-0.56	-0.45±0.06±0.02

• LET (Gauge Inv. + PCAC) $E_{0+}(\pi^0 p) = -2.3 \times 10^{-3}/m_{\pi}$

$$E_{0+}(\pi^0 p) = -\frac{2.3 \times 10^{-3}}{m_{\pi}} (1 + O(\mu))$$

ChPT; The above expansion in $\mu = m_{\pi}/m_{N}$ converges slowly Electroproduction <u>HBChPT</u> a low energy effective field theory respecting the symmetries of QCD, in particular, chiral symmetry perturbative calculation crossing symmetric

<u>DMT</u> Lippman-Schwinger type formulation with potential constructed from chiral effective lagrangian unitarity loops to all orders

What are the predictions of DMT?

 $\gamma N \leftrightarrow \Delta$ transition

- In a symmetric SU(6) quark model, the e.m. excitation of the A could proceed only via M1 transition
- ➤ If the △ is deformed, then the photon can excite a nucleon into a △ through electric E2 and Coulomb C2 quadruple transitions

At Q² = 0, recent experiments give, R_{EM} = E2/M1 ' -2.5%, (Q_{N!} = -0.108fm² < 0, oblate),</p>

indication of a deformed Δ

Resonance parameters

For example, in the case of the four-star resonance $S_{11}(1535)$,

- Breit-Wigner mass (MeV): 1520 –1555 (1/4 1535)
- Breit-Wigner width (MeV): 100 200 (1/4 150)

 120 ± 20 , Hoehler'79 (π N ! π N) 270 ± 50 , Cutkovsky, '79 (π N ! π N)66, Arndt'95 (π N ! π N) 151 ± 27 , Manley'92 (π N ! π N, $\pi\pi$ N)151 - 198, Mosel'98 (π N ! π N, $\pi\pi$ N, η N, K Σ)

- Pole position: (1495-1515, 90-250)
- $A_{1/2}^{p}$ (10⁻³ GeV^{-1/2}):

60	, Arndt	'96 (γN!πN)
120	,Krusche	'97 (γN! ηN)

Simultaneous analysis of $(\pi N \mid \pi N, \gamma N \mid \pi N, \gamma N \mid \pi N)$

$\gamma^* N \leftrightarrow \Delta$ transition and pQCD

pQCD predicts that as Q² !1,

- hadronic helicity conservation: A_{1/2} À A_{3/2}
- scaling: $A_{1/2}^{\Delta}: Q^{-3}, A_{3/2}^{\Delta}: Q^{-5}, S_{1/2}^{\Delta}: Q^{-3}$

$$\begin{split} R_{EM} &= E_{1+}^{(3/2)} / M_{1+}^{(3/2)} \to 1, \\ R_{SM} &= S_{1+}^{(3/2)} / M_{1+}^{(3/2)} \to const. \end{split}$$

What region of Q² corresponds to the transition from nonperturbative to pQCD description?

Meson-exchange π N model below 400 MeV

Bethe-Salpeter equation

$$T_{\pi N}=B_{\pi N}+B_{\pi N}G_0T_{\pi N},$$

where

 $B_{\pi N}$ = sum of all irreducible two-particle Feynman amplitues G_0 = relativistic free pion-nucleon propagator

can be rewritten as \Rightarrow

$$T_{\pi N} = \mathcal{B}_{\pi N} + \mathcal{B}_{\pi N} \mathcal{O}_0 T_{\pi N},$$

with

$$B_{\pi N} = B_{\pi N} + B_{\pi N} (G_0 - B_0) B_{\pi N}.$$

Three-dimensional reduction

- Choose a $\mathcal{G}_0(k, P)$ such that
- 1. $T_{\pi N} = B_{\pi N}^{L} + B_{\pi N}^{L} G_{0}^{L} T_{\pi N}^{L}$ becomes three-dimensional
- 2. G_0^{L} can reproduce πN elastic cut

Cooper-Jennings reduction scheme

Choose $B_{\pi N}$ to be given by









C.T. Hung, S.N. Yang, and T.-S.H. Lee, Phys. Rev. C64, 034309 (2001)

Dynamical model for γ **N** ! π **N**

To order e, the t-matrix for $\gamma N \mid \pi N$ is written as

$$t_{\gamma\pi}(E) = \mathbf{v}_{\gamma\pi} + \Sigma_{\kappa} \mathbf{v}_{\gamma k} g_{k}(E) t_{kN}(E),$$

where,

$$\mathbf{v}_{\mathbf{k}}$$
 = transition potential,
 $t_{\mathbf{k}N}(E) = \mathbf{k} \mathbf{N}$ t-matrix,
 $g_k(E) = \frac{1}{E - H_0}$.



(1)

Multipole decomposition of (1) gives the physical amplitude in channel $\alpha = (\xi, l_{\pi}, j)$, (with η N intermediate states neglected)

$$t_{\gamma\pi}^{(\alpha)}(q_E,k;E+i\varepsilon) = \exp(i\delta^{(\alpha)})\cos\delta^{(\alpha)}$$

$$\times \left[v_{\gamma\pi}^{(\alpha)}(q_E,k) + P \int_{0}^{\infty} dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E,q';E) v_{\gamma\pi}^{(\alpha)}(q',k)}{E - E_{\pi N}(q')} \right]$$

where

 $\delta^{(\alpha)}, R^{(\alpha)} : \pi N$ scattering phase shift and reaction matrix in channel α k=| k|, q_E : photon and pion on-shell momentum





If the transition potential $v_{\gamma\pi}$ consists of two terms,

$$v_{\gamma\pi}(E) = v_{\gamma\pi}^{B}(E) + v_{\gamma\pi}^{R}(E),$$

where

$$v_{\gamma\pi}^{B}$$
 = background transition potential
 $v_{\gamma\pi}^{R}$ = contribution of a bare resonance *R*



then one obtains

$$t_{\gamma\pi}(E) = t^{B}_{\gamma\pi}(E) + t^{R}_{\gamma\pi}(E),$$





with

$$t_{\gamma\pi}^{B}(E) = v_{\gamma\pi}^{B} + \sum_{k} v_{\gamma k}^{B} g_{k}(E) t_{k\pi}(E)$$
$$t_{\gamma\pi}^{R}(E) = v_{\gamma\pi}^{R} + \sum_{k} v_{\gamma k}^{R} g_{k}(E) t_{k\pi}(E)$$

both t^B and t^R satisfy Fermi-Watson theorem, respectively.

DMT Model



$$t_{\gamma\pi}^{B,\alpha} = \exp(i\delta^{(\alpha)})\cos\delta^{(\alpha)} \begin{cases} v_{\gamma\pi}^{B,\alpha}(W,Q^2) + P \int_0^\infty dq' \frac{q'^2 R_{\pi N}^{(\alpha)}(q_E,q';E) v_{\gamma\pi}^{B,\alpha}(q',k)}{E - E_{\pi N}(q')}, & DM \\ v_{\gamma\pi}^{B,\alpha}(W,Q^2), & MAID \end{cases}$$

in the dynamical approach the background is differently defined:



In DMT, we approximate the resonance contribution $A^{R}_{\alpha}(W,Q^{2})$ by the following Breit-Winger form

$$A_{\alpha}^{R}(W,Q^{2}) = \overline{A}_{\alpha}^{R}(Q^{2}) \frac{f_{\gamma R}(W)\Gamma_{R}M_{R}f_{\pi R}(W)}{M_{R}^{2} - W^{2} - iM_{R}\Gamma_{R}}e^{i\phi}, \quad \overbrace{\overline{A}_{\alpha}^{R}}^{I\phi}$$

with

 $f_{\pi R}$ = Breit-Winger factor describing the decay of the resonance R $\Gamma_{\rm R}({\rm W})$ = total width

 M_R = physical mass

 $\phi(W)$ = to adjust the phase of the total multipole to be equal to the corresponding π N phase shift $\delta^{(\alpha)}$.

Note that

 $\overline{A}_{\alpha}^{R}(Q^{2}) = \begin{cases} \text{bare, DMT} \\ \text{dressed, MAID} \end{cases}$











 $\gamma p \rightarrow \pi^0 p$



Threshold values of E_{0+} (in units of $10^{-3}/m_{\pi}$) for different channels predicted by DMT ($Q^2 = 0$)

	Tree	1-loop	2-loop	Full	ChPT	Exp.
πр	-2.26	-1.06 (53.1%)	-1.01 (2.2%)	-1.00	-1.1	-1.33±0.11
πn	27.72	28.62 (3.2%)	28.82 (0.7%)	28.85	28.2±0.6	28.3±0.3
πn	0.46	2.09 (354.3%)	2.15 (13.0%)	2.18	2.13	
πр	-31.65	-32.98 (4.2%)	-33.27 (0.9%)	-33.31	-32.7±0.6	-31.8±1.9

	DMT	HBChPT
chiral symmetry	yes	yes
crossing symmetry	no	yes
unitarity	yes	no
counting	$loop(g_{\pi N})$	chiral power

Threshold values of $E_{0+}(10^{-3}/m_{\pi})$ at $Q^2 = 0.05(GeV/c)^2$ for

different channels predicted by DMT

	Tree	1-loop	2-loop	Full	ChPT	Exp.
πр	-0.19	0.96	1.02	1.03	0.27	0.57±0.11
Πn	23.92	24.86	25.09	25.12		
Πn	1.53	3.00	3.07	3.09		
πр	-26.42	-27.69	-27.96	-28.00		

Threshold values of $E_{0+}(10^{-3}/m_{\pi})$ at $Q^2 = 0.1(\text{GeV/c})^2$ for

different channels predicted by DMT

	Tree	1-loop	2-loop	Full	ChPT	Exp.
πр	1.30	2.37	2.43	2.44	1.42	0.58±0.18
Πn	20.89	21.88	22.10	22.13		
Πn	2.27	3.57	3.64	3.66		
πр	-22.33	-23.51	-23.75	-23.79		

H. Merkel et. al., PRL 88 (2002) 012301

Threshold values of $L_{0+}(10^{-3}/m_{\pi})$ at $Q^2 = 0.05(GeV/c)^2$ for

different channels predicted by DMT

	Tree	1-loop	2-loop	Full	ChPT	Exp.
πр	-1.97	-16.7	-1.67	-1.67	-1.55	-
Πn	4.96	5.06	5.10	5.11		1.29±0.02
Πn	0.05	0.54	0.56	0.57		
πр	-7.92	-8.33	-8.42	-8.43		

Threshold values of $L_{0+}(10^{-3}/m_{\pi})$ at $Q^2 = 0.1(GeV/c)^2$ for

different channels predicted by DMT

	Tree	1-loop	2-loop	Full	ChPT	Exp.
πр	-1.32	-1.15	-1.15	-1.15	-1.33	-
Πn	2.32	2.35	2.37	2.38		1.38±0.01
Πn	0.06	0.36	0.37	0.37		
πр	-4.37	-4.61	-4.66	-4.67		



FIG. 3. The total cross section σ_{tot} versus Q^2 , at a value of $\epsilon = 0.8$. The solid (dashed) line is the prediction of ChPT (MAID), data points at $Q^2 = 0$ and 0.1 GeV²/ c^2 from [6,11].

Merkel et. al., PRL 88 (2002) 012301.

 $p(\vec{e}, e'p)\pi^0$ Electroproduction near Threshold E₀=854.5 MeV, Q²=0.05 GeV²/c², ε =0.03, θ =90°, ϕ =90° ϕ exp. data: M. Weis, Mainz 2003

 $\frac{d\sigma_{\nu}}{d\Omega} = \frac{d\sigma_{T}}{d\Omega} + \varepsilon \frac{d\sigma_{L}}{d\Omega} + \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{LT}}{d\Omega} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{d\Omega} \cos2\phi + h\sqrt{2\varepsilon(1-\varepsilon)} \frac{d\sigma_{LT'}}{d\Omega} \sin\phi$





	A _{1/2} (10 ⁻³ GeV ^{-1/2})	A _{3/2}	$Q_{N!\Delta}$ (fm ²)	μ _N ! Δ
PDG	-135	-255	-0.072	3.512
LEGS	-135	-267	-0.108	3.642
MAINZ	-131	-251	-0.0846	3.46
DMT	-134 (-80)	-256 (-136)	-0.081 (0.009)	3.516 (1.922)
SL	-121 (-90)	-226 (-155)	-0.051 (0.001)	3.132 (2.188)

Comparison of our predictions for the helicity amplitudes, $Q_{N!\Delta}$, and $\mu_{N!\Delta}$ with experiments and Sato-Lee's prediction. The numbers within the parenthesis in red correspond to the bare values.

$$\overline{A}^{\Delta}_{\alpha}(Q^2) = X^{\Delta}_{\alpha}(Q^2)\overline{A}^{\Delta}_{\alpha}(0)\frac{k}{k_w}F(Q^2), \quad \alpha = M, E, S$$

 $F(Q^{2}) = (1 + \beta Q^{2})e^{-\gamma Q^{2}}G_{D}(Q^{2}), \quad G_{D}(Q^{2}) = 1/(1 + Q^{2}/0.71)^{2}.$

 β and γ were determined by setting $X_M^{\Delta}(Q^2) \equiv 1$, and fitting $\overline{A}_{\alpha}^{\Delta}(Q^2)$ to the data for G_M^* . $\overline{A}_M^{\Delta}(0)$ and $\overline{A}_E^{\Delta}(0)$ were determined by fitting to the multipoles.

fitting to Jlab $d\sigma/d\Omega$ data of $p(e,e'p)\pi^0$ (751+867 pts) \Rightarrow

 $X_E^{\Delta}(\text{DMT})=1+Q^4/2.4, \quad X_S^{\Delta}(\text{DMT})=1-10Q^2.$



FIG. 1. The virtual photons differential cross sections at $Q^2 = 4.03$ (GeV/c)² and W = 1232 MeV. The full and dashed curves are the results from the MAID and DM analysis, respectively. Data are from Ref. [1].



L-T Interference Structure Functions







FIG. 4. The Q^2 dependence of the bare (dashed curves) and dressed (solid curves) helicity amplitudes $A_{1/2}$ and $A_{3/2}$ (in units of 10^{-3} GeV^{-1/2}) extracted with DM. The dotted curves are the pion cloud contributions.

 $A_{1/2}$ ¹/₄ $A_{3/2}$, hadron helicity conservation not yet observed



FIG. 5. The Q^2 dependence of the $Q^3 A_{1/2}^{\Delta}$ (solid curve), $Q^5 A_{3/2}^{\Delta}$ (dashed curve), and $Q^3 S_{1/2}^{\Delta}$ (dotted curve) amplitudes (in units of 10^{-3} GeV^{n/2}) obtained with DM.

bare $S_{1/2}^{\Lambda}$ and $A_{1/2}^{\Lambda}$ start exhibiting pQCD scaling behavior

Extension to higher energies

Exampled π , η , 2π channels

$$t_{ij}(E) = v_{ij} + \sum_{k} v_{ik} g_{k}(E) t_{kj}(E),$$

(*i*, *j*, *k* = π , η)

> Include resonances R's with couplings to π , η , 2π channels $v_{ii}(E) = v_{ii}^B(E) + v_{ii}^R(E)$ Non-resonant background v_{ij}^B :

$$v_{\pi\pi}^{B}$$
 as given before,
 $v_{\pi\eta}^{B} = v_{\eta\pi}^{B} = v_{\eta\eta}^{B} = 0.$

$$L_{I} = ig_{\pi NR}^{(0)} \overline{R} \tau \cdot \overline{\pi} N + ig_{\eta NR}^{(0)} \overline{R} \eta N + h.c.$$

Resonance contribution v_{ij}^{R} :

$$v_{ij}^{R} = \sum_{n=1}^{N} v_{ij}^{R_{n}}(q',q;E),$$

for N overlapping resonances.

$$v_{ij}^{R_n} = \frac{f(\Lambda_i^{0,0}, q'; E)g_i^{(0)}g_j^{(0)}f(\Lambda_j^{0,0}, q; E)}{E - M_{R_n}^{(0)} + i\frac{1}{2}\Gamma_{2\pi}^{(n)}(E)},$$



 $M_{R_n}^{(0)}$ = bare mass of resonance R_n,

$$\Gamma_{2\pi}^{(n)} = \Gamma_{2\pi}^{0,n} \left(\frac{q_{2\pi}}{q_{0,n}}\right)^{2l+4} \left(\frac{X^2 + q_{0,n}^2}{X^2 + q_{2\pi}^2}\right)^{l+2}$$

= effects of $\pi\pi N$ decay channel







(3,4), red-PDG, blue-DMT



(3,3)



(3,3)



(2,2)



(2,3)



(3,3)



(3,3)



(2,2)



(2,2)



(2,1)



(2,2)



(1,0)



(2,2)



(2,1)



Pole Positions and Residues in [MeV] (preliminary)

	1st Resonance		2nd Resonance		3rd Resonance	
\mathbf{N}^{*}	Wp	Гр	Wp	Гр	Wp	Гр
S ₁₁ (3,4)	1499 (1505, 1501)	67 (170, 124)	1642 (1660, 1673)	97 (160, 82)	2065 (2150±70)	223 (350±100)
S ₃₁ (3 ,3)	1598 (1600, 1585)	136 (115, 104)	1775 (1870±40)	36 (1850±50)	2012 (2140±80)	148 (200±80)
$P_{11}(3,3)$	1366 (1365, 1346)	179 (210, 176)	1721 (1720, 1770)	185 (230, 378)	1869 (2120±40, 1810)	238 (240±80, 622)
P ₁₃ (2,2)	1683 (1700, 1717)	239 (250, 388)	1846 (not listed)	180 (not listed)		
P ₃₁ (2,3)	1729 (1714)	70 (68)	1896 (1855, 1810)	130 (350, 494)	2065	161
$P_{33}(3,3)$	1218 (1210, 1211)	90 (100, 100)	1509 (1600, 1675)	236 (300, 386)	2149 (1900, 1900±80)	400 (300, 300+100)

Red PDG04

Blue Arndt95

Green Cutkovsky80 Orange Vrana00

obtained with speed plot

Pole Positions and Residues in [MeV] (preliminary)

	1st Resonance		2nd Resonance		3rd Resonance	
N*	Wp	Гр	Wp	Гр	Wp	Гр
D ₁₃ (3 ,3)	1516 (1510, 1515)	123 (115, 110)	not seen (1680, not seen)	not seen (100, not seen)	1834 (1880±100)	210 (160±80)
D ₃₃ (2,2)	1609 (1660, 1655)	133 (200, 242)	2070 (1900±100)	267 (200±60)		
D ₁₅ (2,2)	1657 (1660, 1663)	132 (140, 152)	2188 (2100±60)	238 (360±80)		
D ₃₅ (2,1)	1992 (1890, 1913)	270 (250, 246)	not seen (2400±60)	not seen (400±150)		
F ₁₅ (2,2)	1663 (1670, 1670)	115 (120, 120)	1931 (not listed)	62 (not listed)		
F ₃₅ (2,2)	1771 (<mark>1830</mark> , 1832)	190 (280, 254)	2218 (2150±100, 1697)	219 (350±100, 112)		
F ₃₇ (2,1)	1860 (1885, 1880)	201 (240, 236)	2207 (2350±100)	439 (260±100)		

Red PDG04

Blue Arndt95

Green Cutkovsky80

Orange Vrana00

pion cloud effects:





Dashed lines: K-matrix, solid lines: K-matrix + pion clouds









Summary

- The DMT coupled-channel dynamical model gives excellent description of the pion scattering and pion e.m. production data from threshold to first resonance region
 - Excellent agreement with π^0 threshold production data. Two-loop contributions small. For electroproduction, ChPT needs to go at least to $O(p^4)$.
 - DMT predicts $\mu_{N!\Delta} = 3.514 \ \mu_N$, $Q_{N!\Delta} = -0.081 \ fm^2$, and $R_{EM} = -2.4\%$, all in close agreement with the experiments. \Rightarrow dressed Δ is oblate
 - Bare Δ is almost spherical. The oblate deformation of the dressed Δ arises almost exclusively from pion cloud

We have re-analyzed the recent Jlab data for the electroproduction of the ∆(1232) via p(e,e'p)π⁰ with DMT and MAID. In contrast with the previous findings, we find

- At Q²=4.0 (GeV/c)², hadronic helicity conservation is still not yet observed.
- R_{EM}, starting from a small and negative value at the real photon point, actually exhiits a clear tendency to cross zero and change sign as Q² increases.
- S_{1/2} and A_{1/2}, but not A_{3/2}, start exhibiting scaling behavior at about Q² > 2.5 (GeV/c)².
 - the onset of scaling might take place at a lower momentum transfer than that of hadronic helicity conservation.

Extension to 2.2 GeV gives (prelminary)

- Even pole positions and the number of resonance could be sensitive w.r.t. different analysis tools.
- The pion cloud effects for the pion photoproduction are, in general important, in many channels.

Work in progress (completely dynamical)

- extraction of dressed masses, widths, Q² evolution of helicity amplitudes ($\gamma\pi$)
- Further theoretical improvements
 - Consistent treatment for πN and $\gamma \pi$
 - Gauge invariance
 - Sensitivity w.r.t. the πN interaction model
 - Effects of other channels like ρN , $\pi \Delta$, σN et. al.