Hadron Structure from Lattice QCD

Robert Edwards
Jefferson Lab

- DWF Valence on a Asqtad light sea
- Nucleon Properties:
  - Form Factors
  - Momentum and Spin Fraction
  - GPDs
- Pion Form Factor
- Radiative transitions - Hybrid meson decays

LHP Collaboration

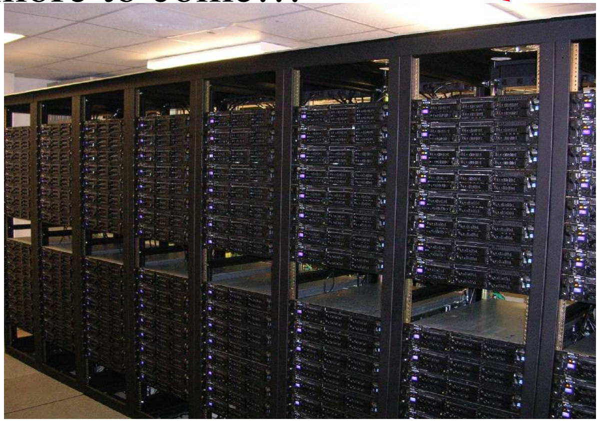
- Dru Renner *University of Arizona*
- Richard Brower, James Osborn *Boston University*
- Rebecca Irwin, Michael Ramsey-Musolf *CalTech*
- Robert Edwards, David Richards *JLab*
- Bojan Bistrovic, Jonathan Bratt, Patrick Dreher, Oliver Jahn, John Negele, Kostas Orginos, Andrew Pochinsky, Dmitry Sigaev *MIT*
- Matthias Burkardt, Michael Engelhardt *New Mexico State University*
- George Fleming *Yale*
- Constantia Alexandrou, Antonios Tsapalis Cyprus, Wolfram Schroers *DESY*, Philippe de Forcrand *EHT-Zurich*, Philipp Hägler *Vrije Universiteit*

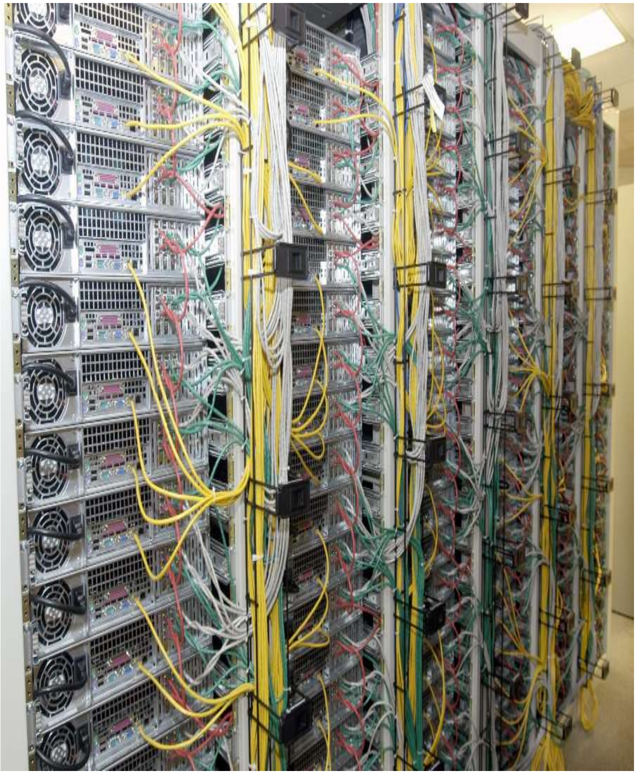
SciDAC Initiative

- DOE Scientific Discovery through Advanced Computing Initiative: develop software/hardware infrastructure for next generation computers
- U.S. Lattice QCD Collaboration consists of 64 senior scientists. Research closely coupled to DOE's experimental program:
  - Weak Decays of Strongly Interacting Particles
    - Babar (SLAC)
    - Tevatron B-Meson program (FNAL)
    - CLEO-c program
  - Quark-Gluon Plasma
    - RHIC (BNL)
  - Structure and Interactions of Hadrons
    - Bates, BNL, FNAL, JLab, SLAC
- The JLab-MIT led LQCD Hadron Physics Collaboration includes 25 theorists from 14 institutions.
- Evolved into USQCD Collaboration

Lattice QCD Hardware

- JLab SciDAC Clusters:
  - 2002: 128 singles, 0.1 TFlops†
  - 2003: 256 singles, 0.37 TFlops†
  - 2004: 384 singles, 0.65 TFlops†
    - 2005-9: more to come…





First teraflops LQCD machine to achieve $1 per MFlops!

† Sustained performance on lattice QCD code

- Other resources: BNL QCDOC and FNAL Cluster

State of the Art

- Current resources enable us to begin exploiting these advances in full QCD
  - Employ computationally efficient full QCD lattices generated by MILC Collaboration using “staggered” fermions for sea quarks
  - Employ DWF for valence quarks
- Pion masses down to less than 300 MeV to yield
  - Moments of GPD’s and PDF’s
  - Elastic form factors
  - Resonance spectrum of baryons and mesons
- Approach requires introducing multiplicity of pions and approximation to fermion determinant

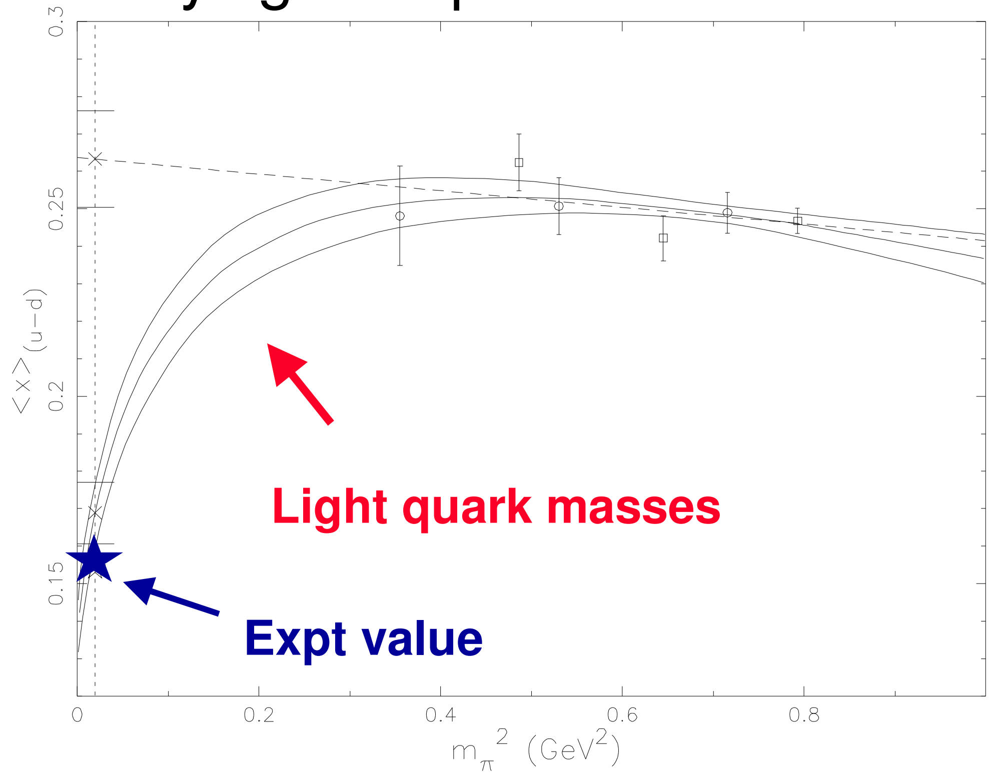
Next step: fully consistent treatment of chiral symmetry for both sea and valence quarks

Onward to light pion masses….

- Achievable precision limited by lightest pion mass

Flavor-non-singlet momentum fraction in nucleon.

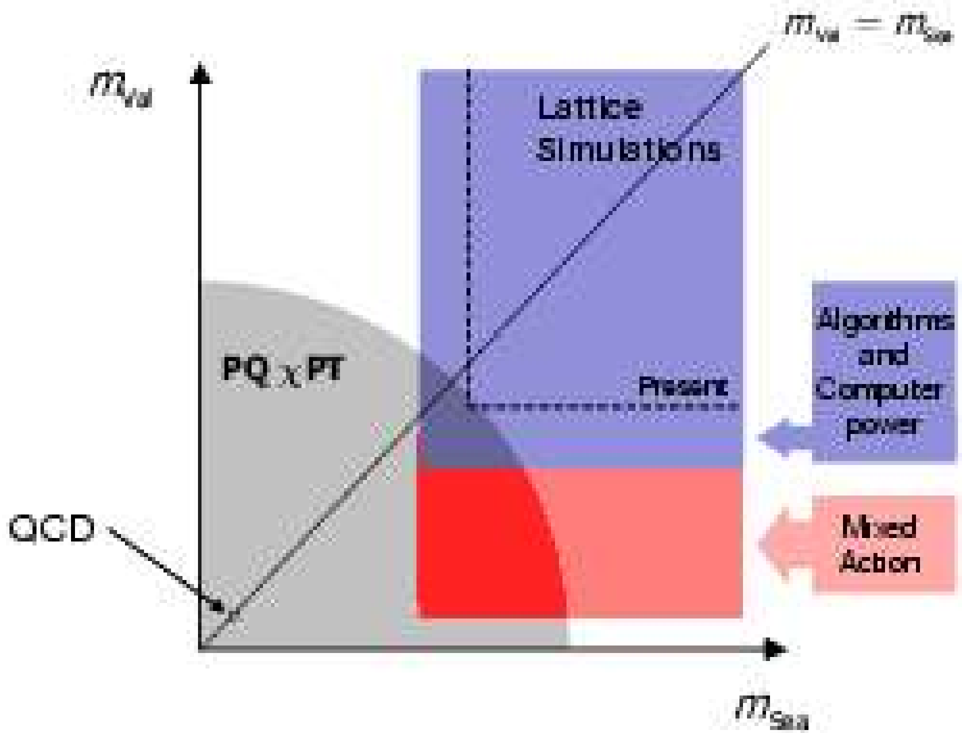
Detmold *et al*, PRL87, 172001 (2001)



- Exact chiral symmetry at Finite Spacing *a*
  - Domain-wall fermions (DWF) in five dimensions

Partially Quenched Chiral Perturbation Theory

- Full QCD expensive!
  - Leverage off cheap(er) valence calcs
- Correct low-energy constants, *in principle*
- Must be in domain of validity
- Extend partially quenched χPT to include O(a) terms
  - Mixed actions



Bär, Rupak, Shoresh, 2002, 2003

Hybrid Lattice Action

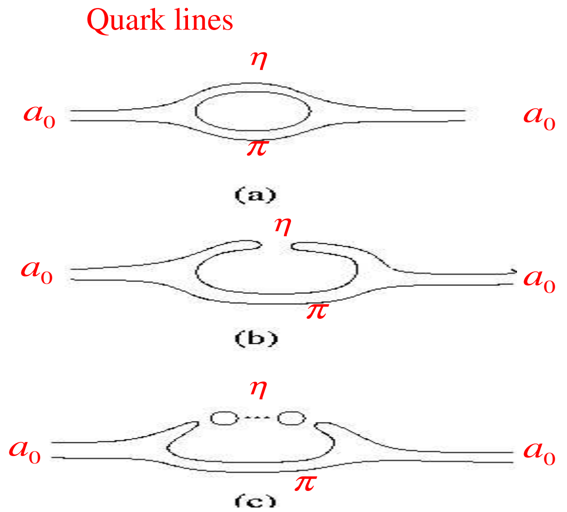
- MILC Asqtad lattices – staggered sea quarks
- Conventional DWF valence quarks (L_5=16) computed on HYP-smeared lattices

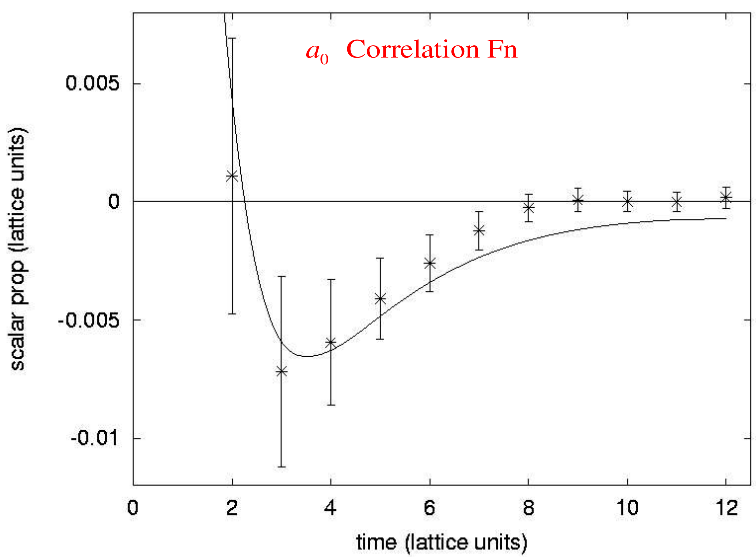
PRD64, 034504

- Parameter matching – tune DWF pion to match Asqtad Goldstone pion mass m_π
- a = 0.125 fm, two volumes at lightest pion mass

$am_{u/d}^{\text{asqtad}}$	L/a	L	$am_{u/d}^{\text{DWF}}$	m_π^{DWF}	#
		fm		MeV	
0.05	20	2.52	0.0810	775	425
0.04	"	"	0.0644	696	350
0.03	"	"	0.0478	605	564
0.02	"	"	0.0313	498	486
0.01	"	"	0.0138	359	656
0.01	28	3.53	0.0138	359	270

"Decay" in Quenched Approximation



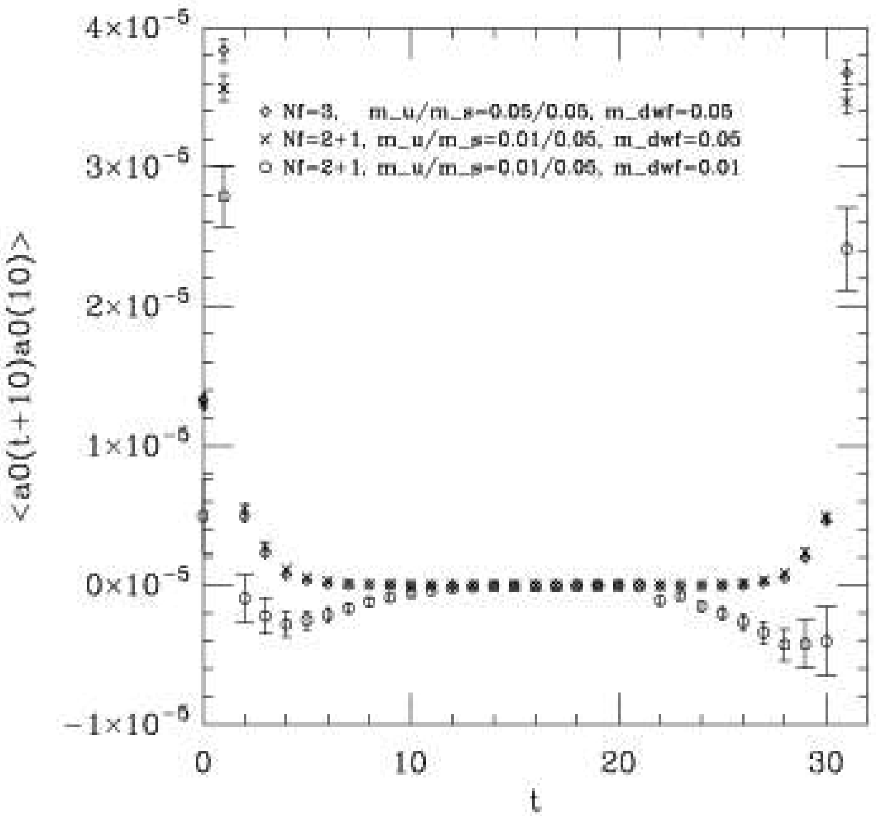


- Dramatic behavior in isotriplet scalar particle $a_0 \to \eta\pi$ intermediate state
- Loss of positivity of a_0 propagator from missing bubble insertions
- Quenched a_0 has double pole in χPT
- Also appears in $m_{\eta'}$

Bardeen, Duncan, Eichten, Thacker, 2000

Partially Quenched Singularity

- Non-positivity of a_0 correlator
- (Partially) Quenched singularity (still) present at $m_{\pi,\ valence}a = m_{\pi,\ sea}a$
- Suggests not single *staggered* pion in chiral loops – taste breaking not neglible
- Need complete partial χPT
  - Vary valence and sea masses
  - Theory under development…



DIS and GPD's for Nucleons

- Measures light-cone correlation functions

$$O(x) = \int \frac{d}{4} e^{i\,x} \left(- \frac{\quad}{2} n \right) nPe^{-ig \int_{/2}^{/2} d\ n.A(\ n)} \left(\frac{\quad}{2} n \right)$$

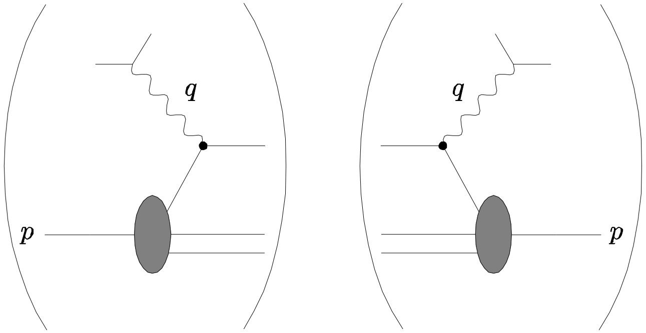
- DIS gives diagonal matrix element

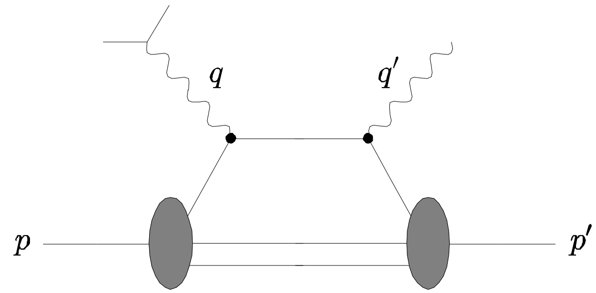
$$¿P|O(x)|P>=q(x)$$

- DVCS gives off-diagonal matrix element

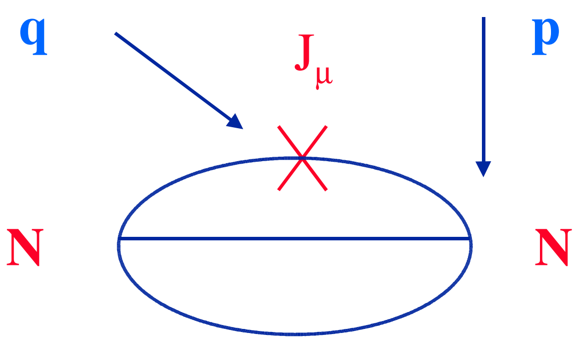
$$¿P'|O(x)|P> = < \quad > H(x, \quad ,t) + \frac{i}{2m} < \quad > E(x, \quad ,t)$$

$$= P - P', t = {}^2, \quad = -n. \quad /2$$





Moments of Structure Functions and GPD's



- Generates tower of twist-two operators
- Expand *O(x)* around light-cone

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$

- Diagonal matrix element

$$\langle P|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}q(x)$$

- Off-diagonal matrix element

$$\begin{aligned}\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle &\simeq \int dx\, x^{n-1}[H(x,\xi,t),E(x,\xi,t)] \\ &\longrightarrow A_{ni}(t),B_{ni}(t),C_n(t)\end{aligned}$$

Moments…..

- GPDs and GFFs are related through Mellin Transformation:

$$\int_{-1}^{1} dx\ x^{n-1} H^q(x,\xi,Q^2) = \sum_{i=0,\ \text{even}}^{n-1} A^q_{ni}(Q^2)(-2\xi)^i + \delta_{n,\ \text{even}} C^q_n(Q^2)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E^q(x,\xi,Q^2) = \sum_{i=0,\ \text{even}}^{n-1} B^q_{ni}(Q^2)(-2\xi)^i - \delta_{n,\ \text{even}} C^q_n(Q^2)(-2\xi)^n$$

- Generalized Form Factors related to familiar quantities
  - $A^q_{10}(Q^2) = F^q_1(Q^2),\ B^q_{10}(Q^2) = F^2_2(Q^2)$
  - $\tilde{A}^q_{10}(Q^2) = G^q_A(Q^2),\ \ \tilde{B}^q_{10}(Q^2) = G^q_P(Q^2),$
  - $J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right),\ \frac{1}{2}\Delta\Sigma^q = \tilde{A}^q_{10}(0)$
  - $L^q = J^q - \frac{1}{2}\Delta\Sigma^q$
  - $\left\langle x^{n-1}\right\rangle_q = A^q_{n0}(0),\ \left\langle x^{n-1}\right\rangle_{\Delta q} = \tilde{A}^q_{n0}(0),\ \left\langle x^{n-1}\right\rangle_{\delta q} = A^q_{Tn0}(0)$

Renormalization of Operators

- One-loop renormalization of operators in boosted perturbation theory:

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.68	0.971	1.07
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.765	0.964	0.99
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.821	0.987	0.989
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.986	0.968	0.929
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.972	0.962	0.925
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.206	0.982	0.898
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	8.78×10^{-3}	2.88×10^{-3}	1.26×10^{-3}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.191	0.98	0.898
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.375	0.989	0.876
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.018	0.991	0.945
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.967	0.973	0.983
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.931	0.937	0.947

B. Bistrovic, PhD Thesis, MIT 2005

Table 11.17: Full $\overline{MS}$ to lattice renormalization coefficients for $M = 1.7$ and 1-loop expression for g. By chiral symmetry matrix elements are the same (except for parity) with and without γ_5, and this is indicated by the $[\gamma_5]$ notation where the upper parity arises in the absence of γ_5.

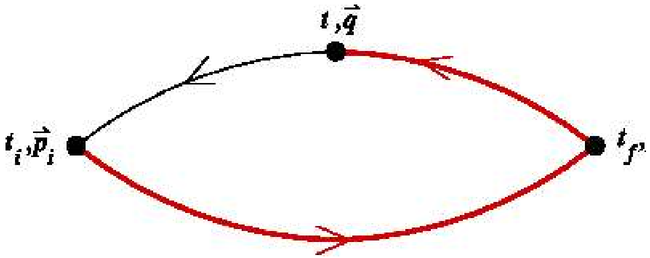
HYP Constants close to unity

Anatomy of a Matrix Element Calculation

$J_{f,i}^{\dagger}$: Current with desired quantum numbers of state A,B

$$\langle \mathbf{T\,J_f(t_f)}\mathcal{O}\mathbf{(t)J_i^{\dagger}(t_i)}\rangle = \sum_{m,n}\langle 0|J|n\rangle\langle n|\mathcal{O}|m\rangle\langle m|J^{\dagger}|0\rangle e^{-E_n(t_2-f)-E_m(t-t_i)}$$

$$\rightarrow \langle 0|J^{\dagger}|B\rangle\langle B|\mathcal{O}|A\rangle\langle A|J|0\rangle e^{-E_f(t_f-t)}e^{-E_i(t-t_i)}$$

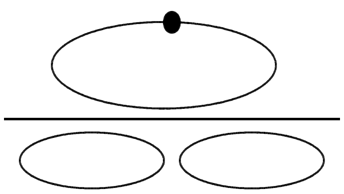


Want $|\langle 0|J^{\dagger}|n\rangle|^2 \sim \delta_{n,0}$ for best plateau

Normalize:

$$\langle \mathbf{TJ(t_f)J^{\dagger}(t_i)}\rangle = \sum_n |\langle 0|J^{\dagger}|n\rangle|^2 e^{-E_n(t_f-t_i)}$$

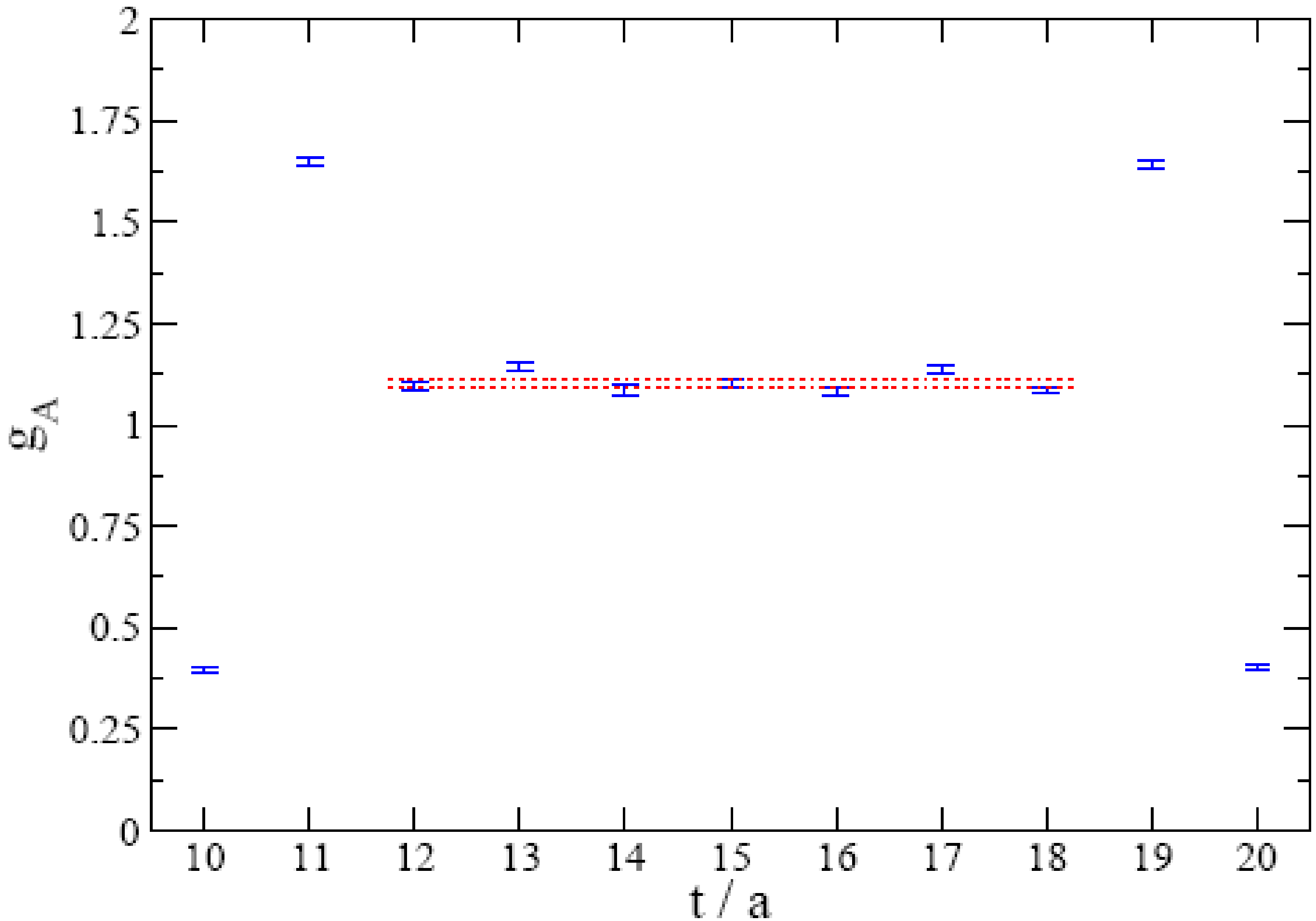
$$\rightarrow |\langle 0|J^{\dagger}|N\rangle|^2 e^{-E_0(t_f-t_i)}$$

Compute ratio: $$\langle \mathbf{B}|\mathcal{O}|\mathbf{A}\rangle = \frac{\langle \mathbf{J_f}\mathcal{O}\mathbf{J_i^{\dagger}}\rangle}{\langle \mathbf{J_fJ_f^{\dagger}}\rangle\langle \mathbf{J_iJ_i^{\dagger}}\rangle} =$$ 

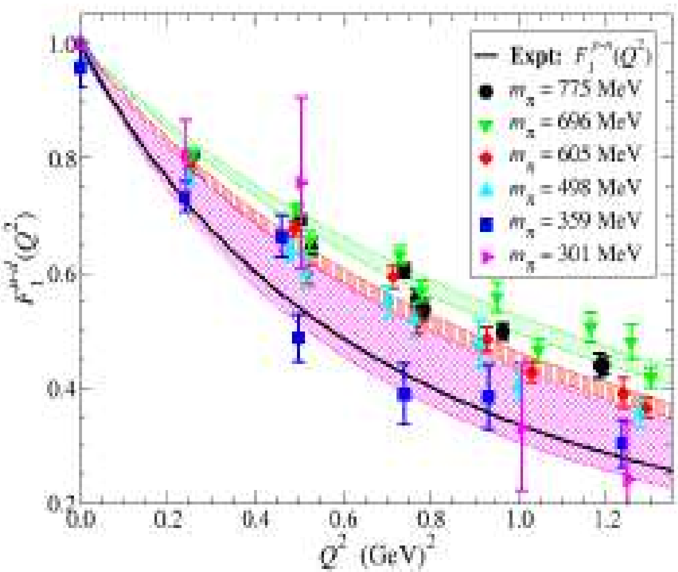
Need propagator from $t \rightarrow t_f$

Example plateau: g_A

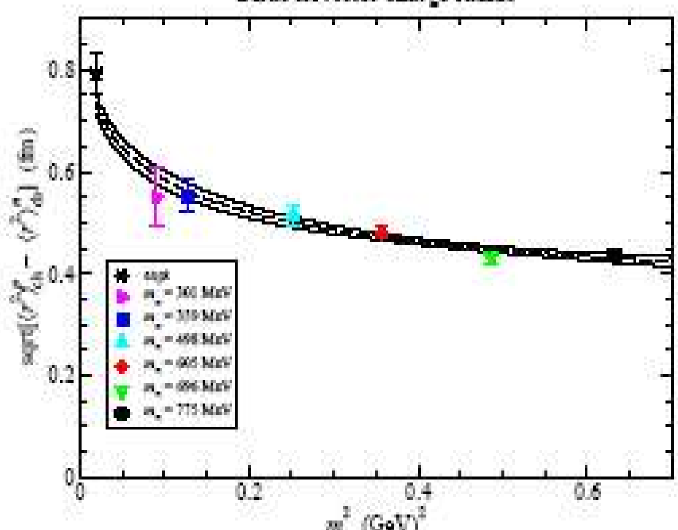
Time plot of current insertion: Nucleon source, t=10, nucleon sink, t=20



Nucleon Isovector F_1 (or A_{10}) Form Factor



Dirac isovector charge radius

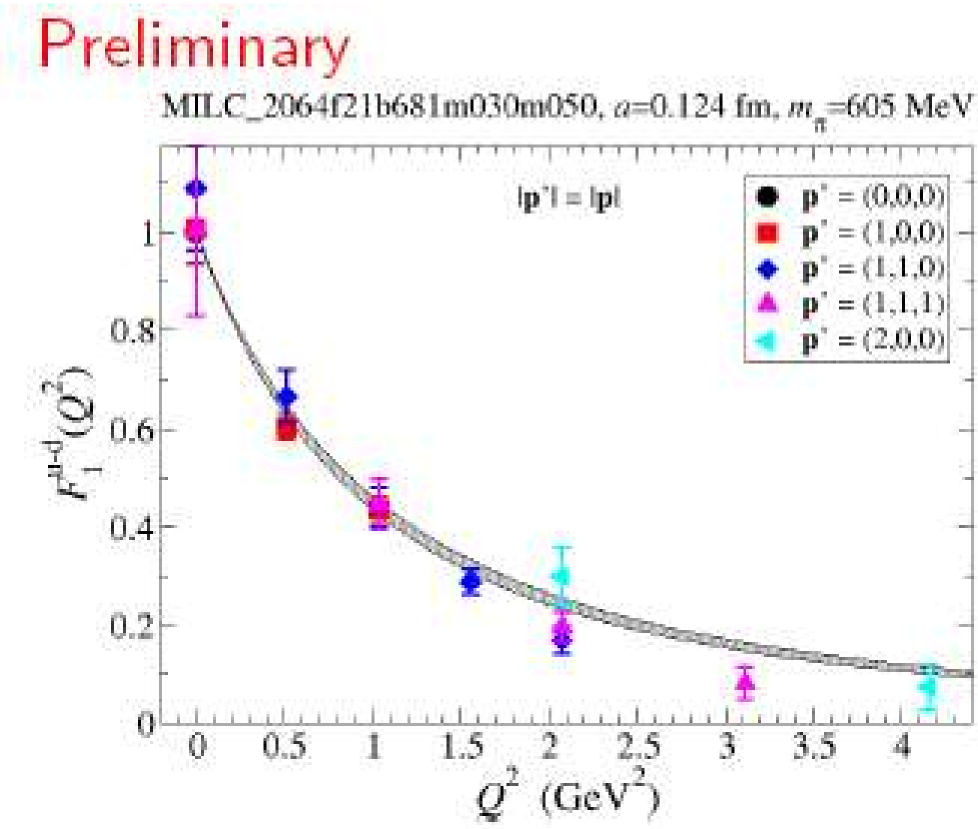


- Charge radii $\langle r^2 \rangle^p_{ch}$ and $\langle r^2 \rangle^n_{ch}$
- Dirac charge radius $\langle r^2 \rangle^{u-d}_{ch}$ using *dipole ansatz* and $Q^2 \leq 1$ GeV^2
- Chiral extrap. Using LNA and LA terms and finite-range regulator (PRL 86, 5011)

$$\langle r^2 \rangle^{u-d}_{ch} = a_0 - 2\frac{(1+5g_A^2)}{(4\pi f_\pi)^2}\frac{1}{2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

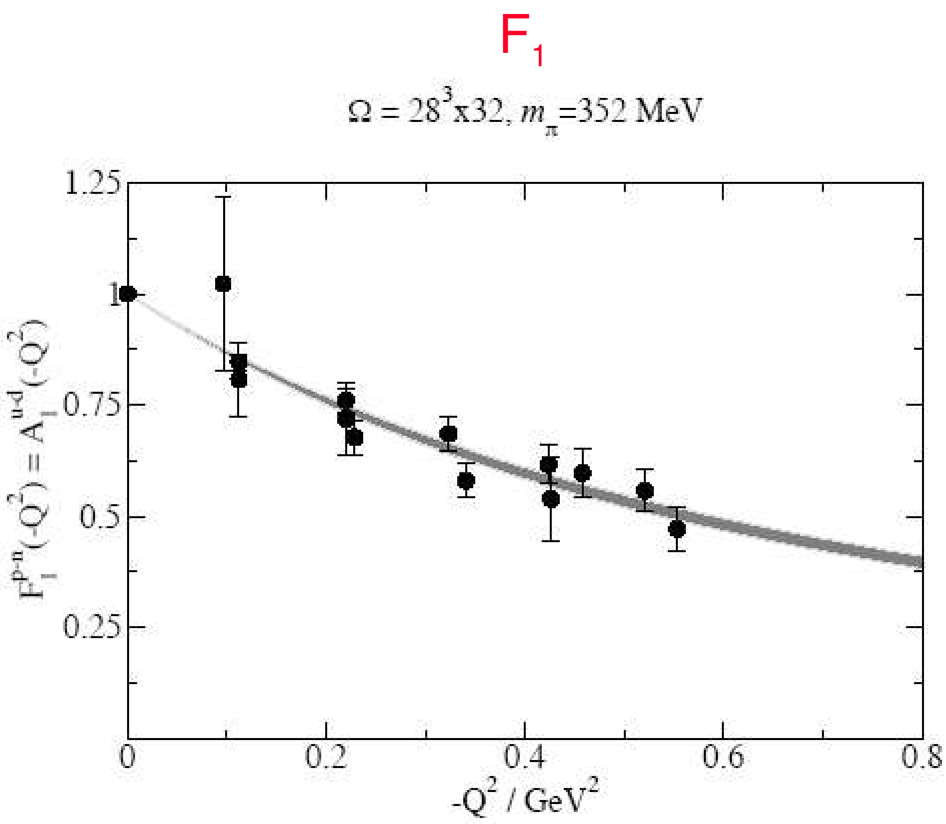
- Best fit: $\Lambda \approx 740$ MeV
- As the pion mass approaches the physical value, the size approaches the correct value

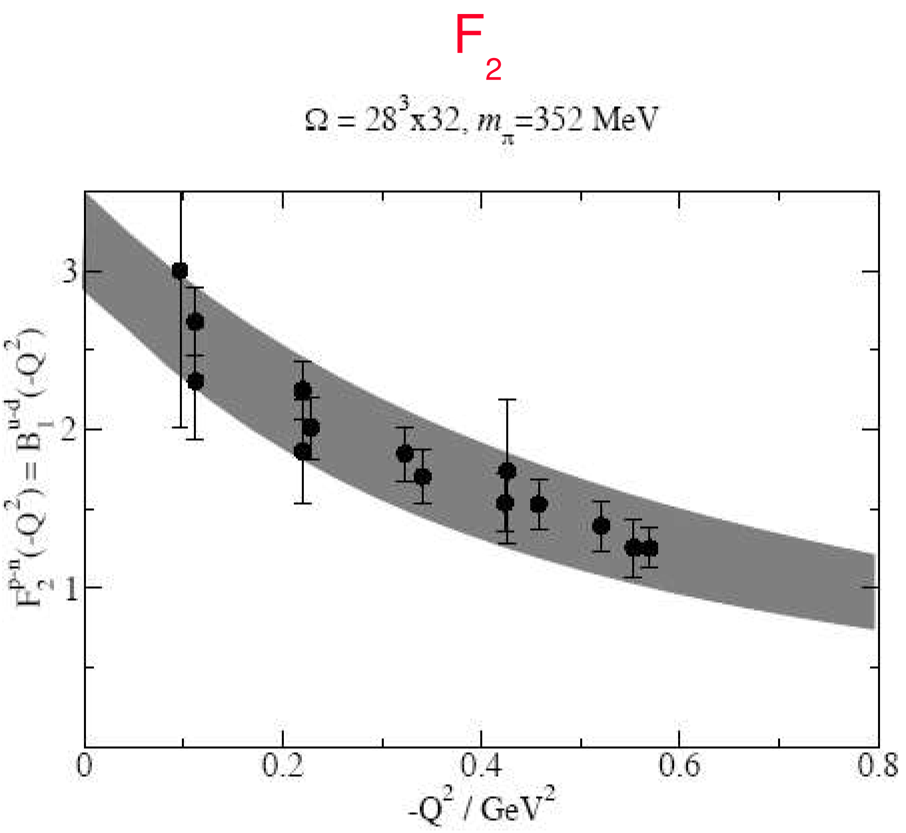
Nucleon Isovector F_1 at Higher Q^2



- Breit frame at $m_\pi \approx 600$ MeV
- Need more statistics as Q^2 increases
- Both polarization transfer experiments and lattice data consistent with dipole *ansatz* up to 4 GeV^2

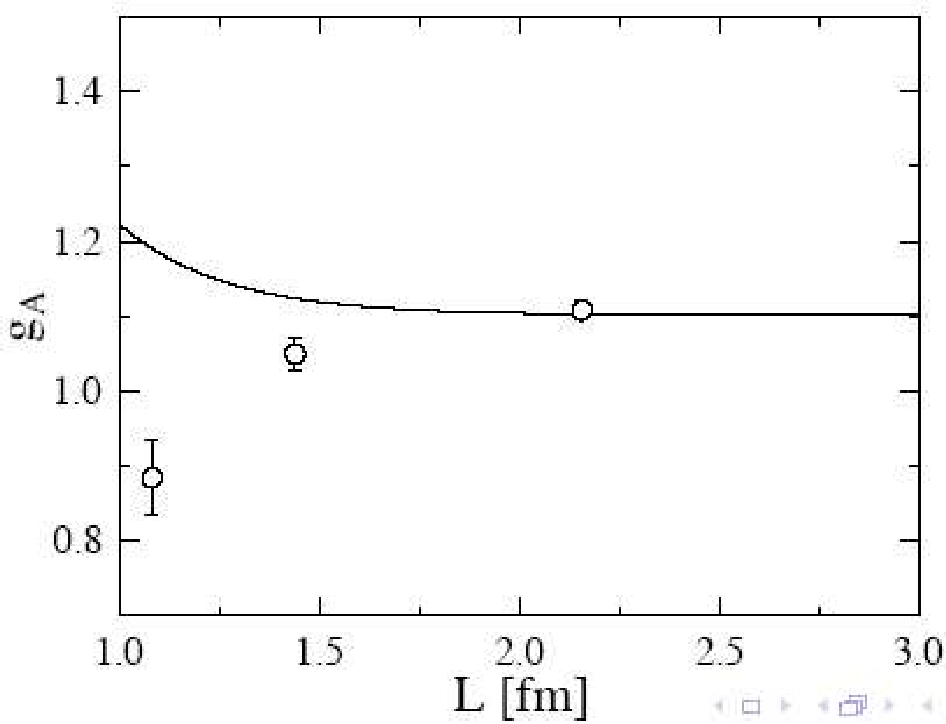
Nucleon Isovector F_1 and F_2





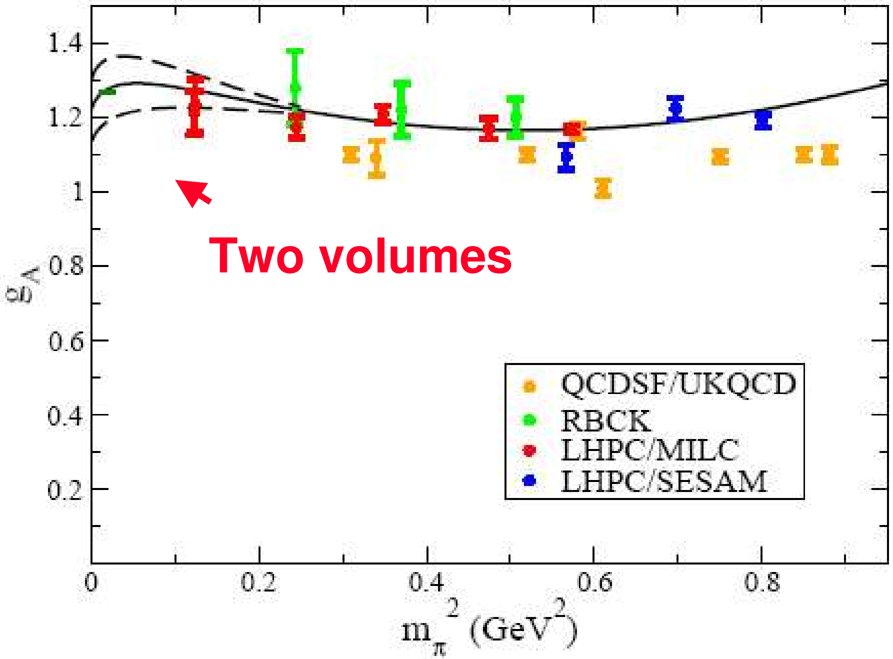
Nucleon Axial Charge g_A in a Finite Volume

- g_A strongly suppressed by finite volume when $m_\pi L < 4$
- hep-lat/0409161 (QCDSF), m_π = 717 MeV, curve is LO χ PT finite volume correction



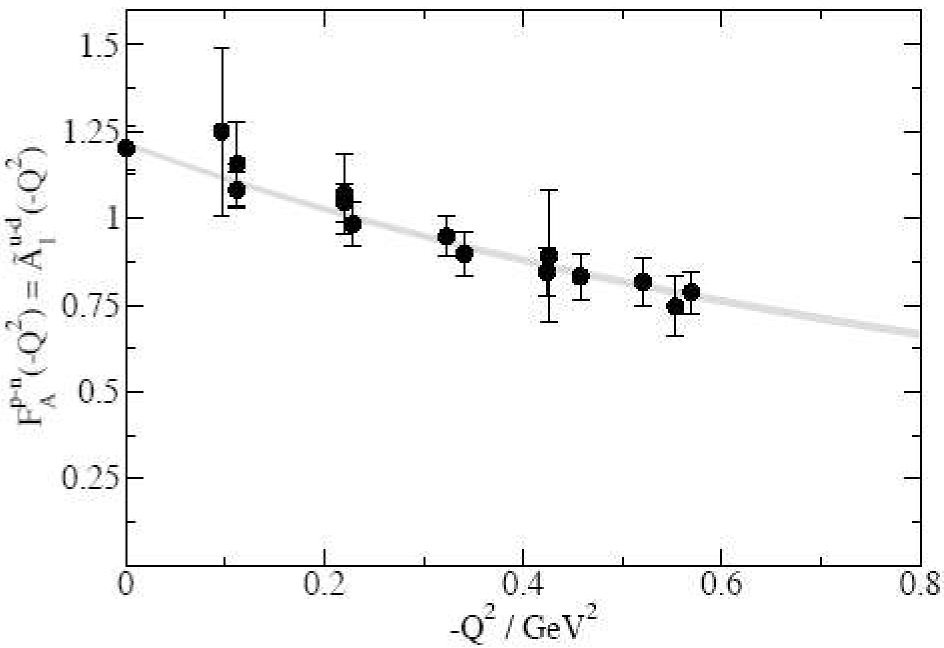
Axial Form Factor

Axial charge: $g_A = \langle 1 \rangle_{\Delta u - \Delta d} = \tilde{A}^{u-d}_{10}(0)$

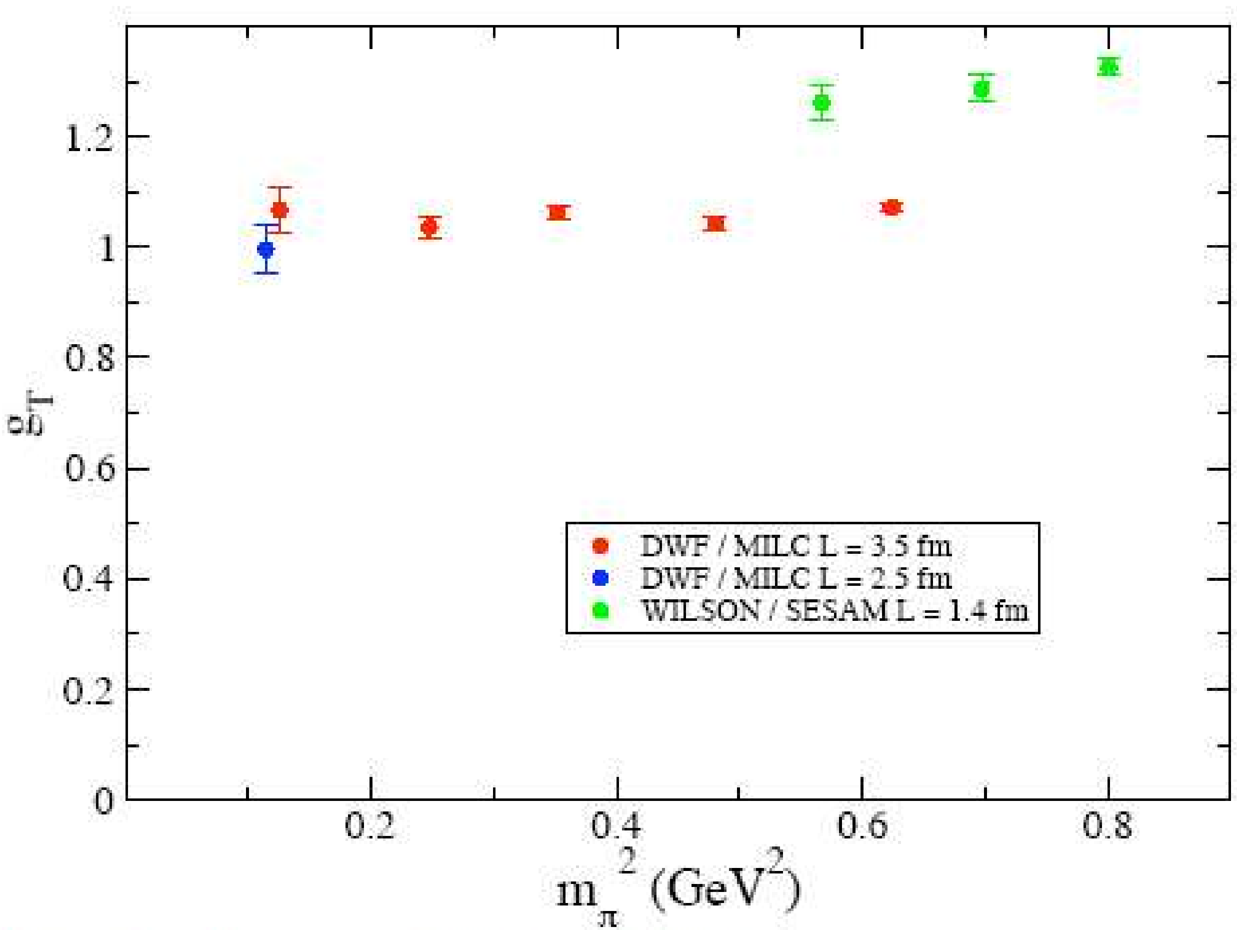


$F^{u-d}_A(Q^2) = \tilde{A}^{u-d}_{10}(Q^2)$

$\Omega = 28^3 x32, m_\pi=352$ MeV

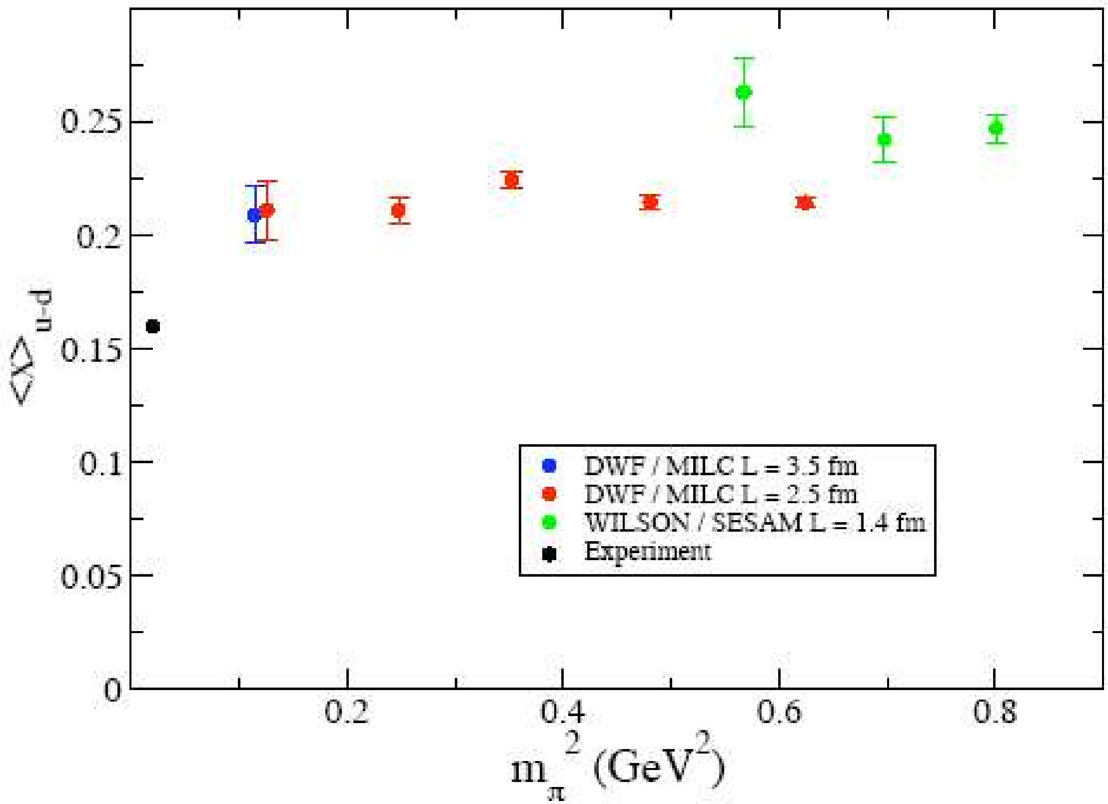


Tensor Charge: $g_T = \langle 1 \rangle_{\delta u - \delta d} = A^{u-d}_{T\,10}(0)$

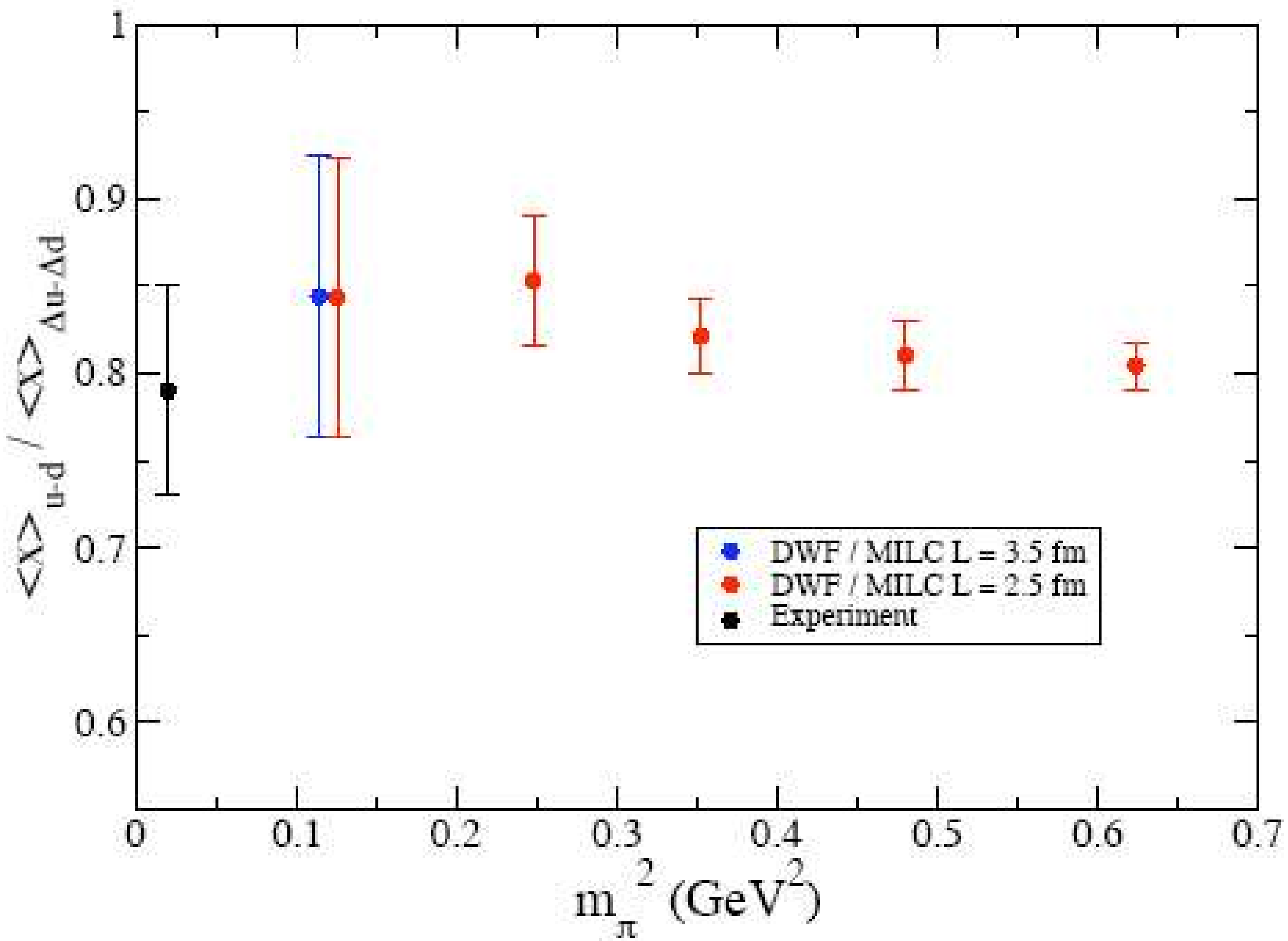


Dru B. Renner, Lattice 2005

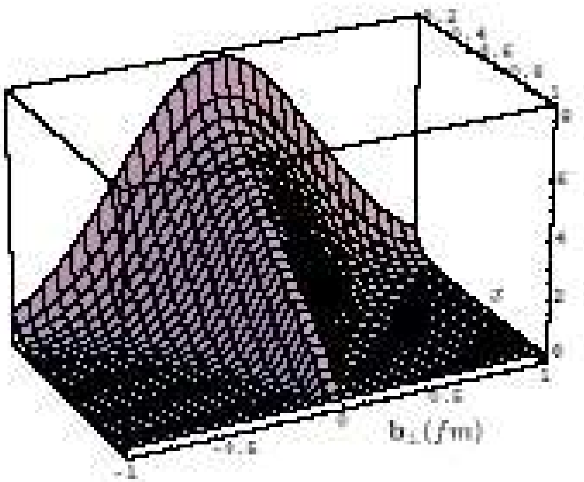
Momentum Fraction: $\langle x \rangle_{u-d} = A^{u-d}_{20}(0)$



Polarized Momentum Fraction: $\langle x \rangle_{\Delta u - \Delta d} = \tilde{A}^{u-d}_{20}(0)$



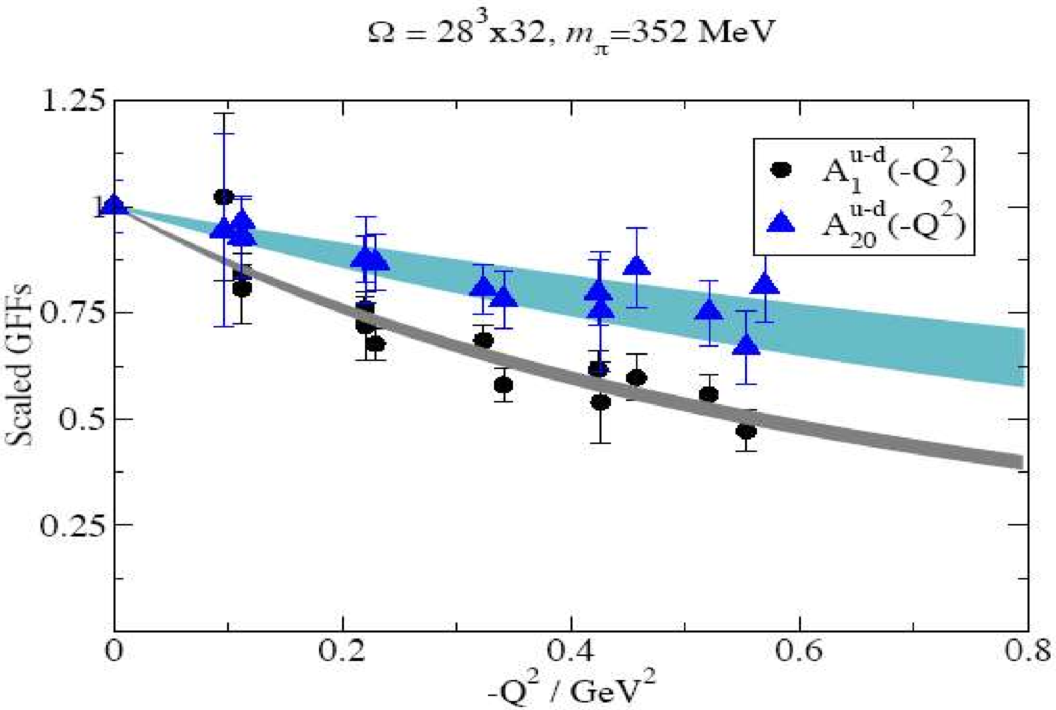
Transverse Quark Distributions



$$A^q_{n0}(-\Delta^2_\perp) = \int d^2 b_\perp e^{i\Delta_\perp \cdot b_\perp} \int_{-1}^{1} x^{n-1} q(x, b_\perp)$$

$$\langle b^2_\perp \rangle^q_{(n)} = -4 \frac{A'^q_{n0}(0)}{A^q_{n0}(0)}$$

$$\lim_{x \to 1} q(x, b_\perp) \propto \delta(b^2_\perp)$$



Higher moments A_{n0} weight $x \sim 1$

Slope of A_{n0} decreases as n increases

Burkardt

Pion Electromagnetic Form Factor: $F_\pi(Q^2)$

- Transition between perturbative and non-perturbative aspects of QCD
  - Asymptotic normalization determined (?)

$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{Q^2} \quad \text{as} \quad Q^2 \to \infty.$$

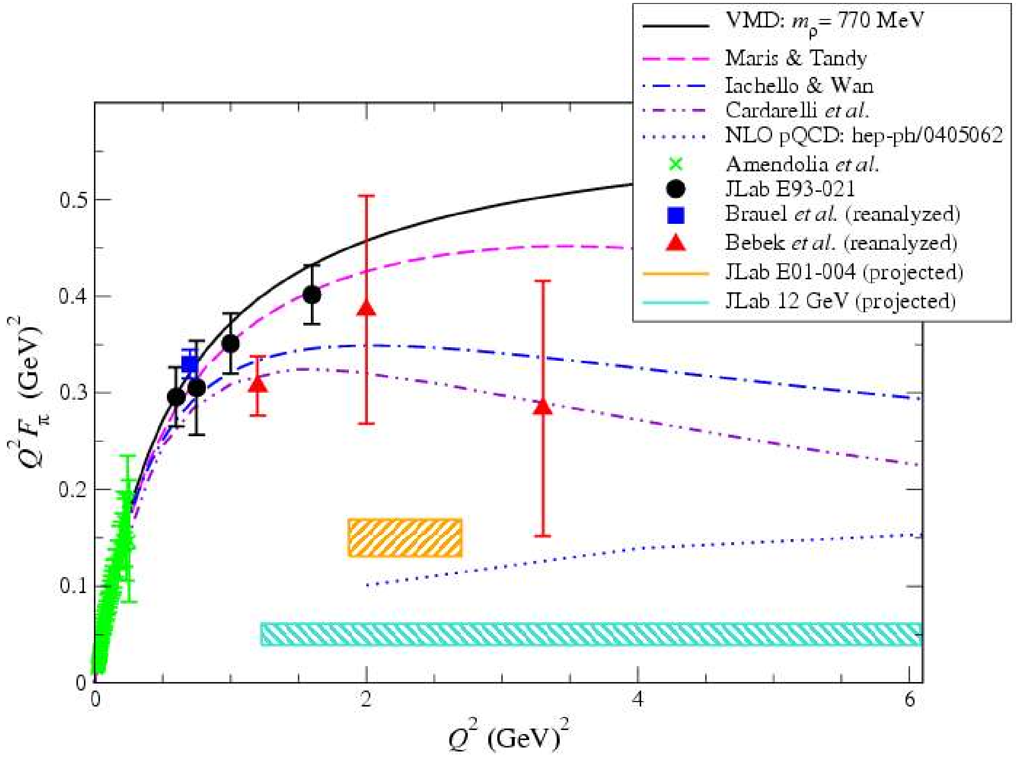
  - Small Q^2, vector-meson dominance provides faithful description

$$F_\pi(Q^2) \approx \frac{1}{1 + Q^2 / m_{\mathrm{VMD}}^2} \quad \text{for} \quad Q^2 \ll m_{\mathrm{VMD}}^2.$$

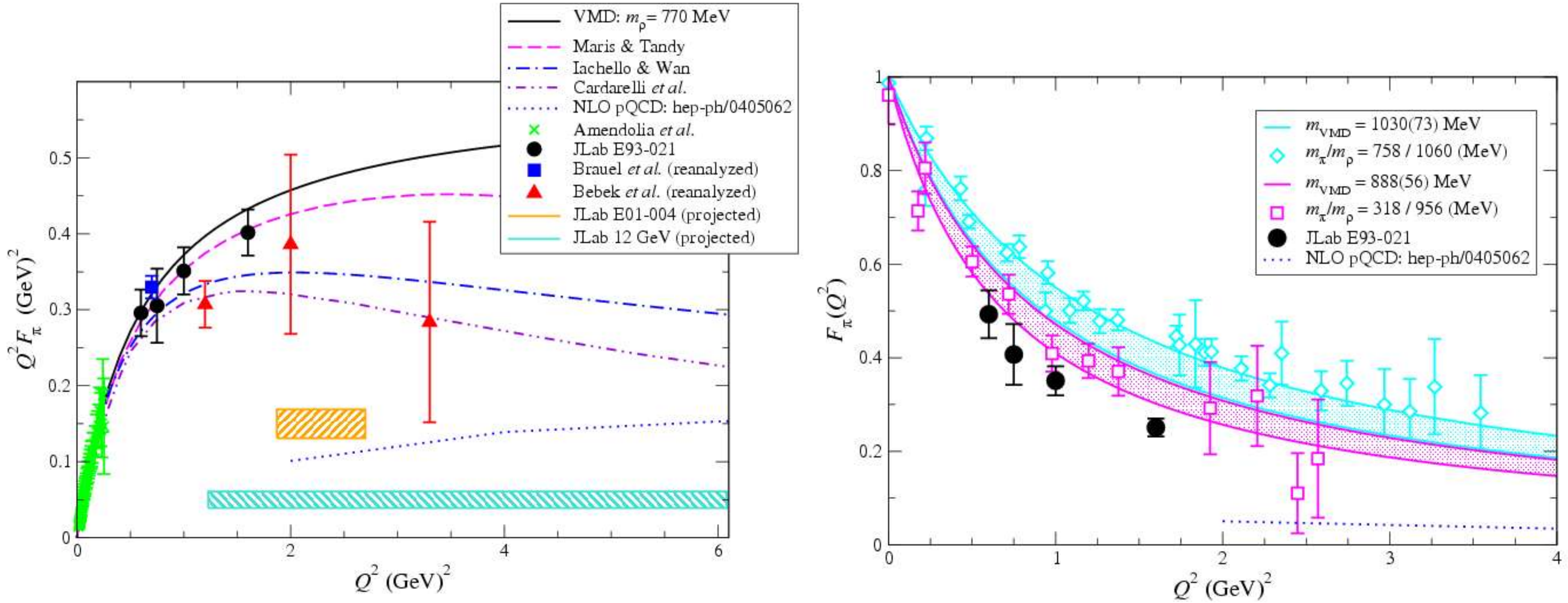
  - Free of disconnected contributions – *straightforward lattice calculation*

Pion Form Factor: Experimental Results

- Existing data fit VMD monopole formulae too well. Where's perturbative QCD?
- Because of simple valence structure, argued that PQCD should apply to lower Q^2 than other hadrons
- PQCD calc. far off.
- Experimental interpretation problematic – must extrapolate to the pion pole
- Use lattice QCD!



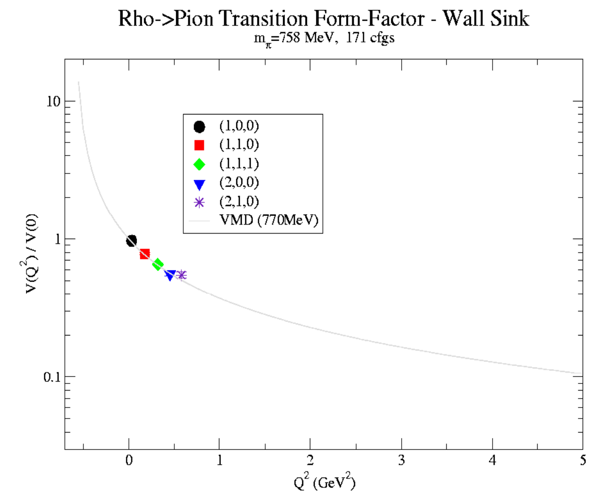
Pion Form Factor –III

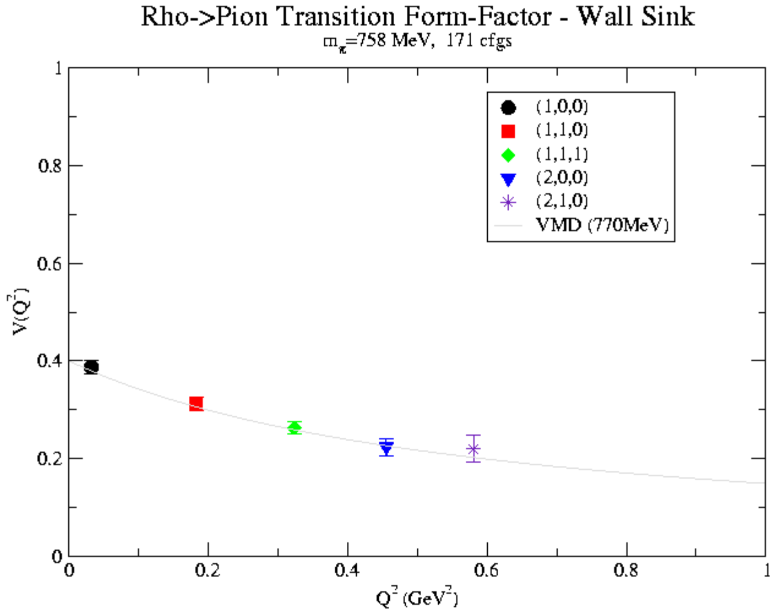


- Pion Form factor over Q^2 commensurate with experiment
- Pion GPDs and transition form factors

Rho→Pion Transition Form-Factor

- Electro-disintegration of deuteron intensively studied
  - Isovector exchange currents identified
  - Isoscalar exchange currents not clear
- $\langle \pi(p_f)|J_\mu|\rho_k(p_i)\rangle \sim 2\, V(Q^2)\, p_f\, p_l / (m_\pi+m_\rho)$
- Extracted coupling rough agreement to expt
- Need higher Q^2 to discern VMD
- First lattice calc!





Maris-Tandy (2002)

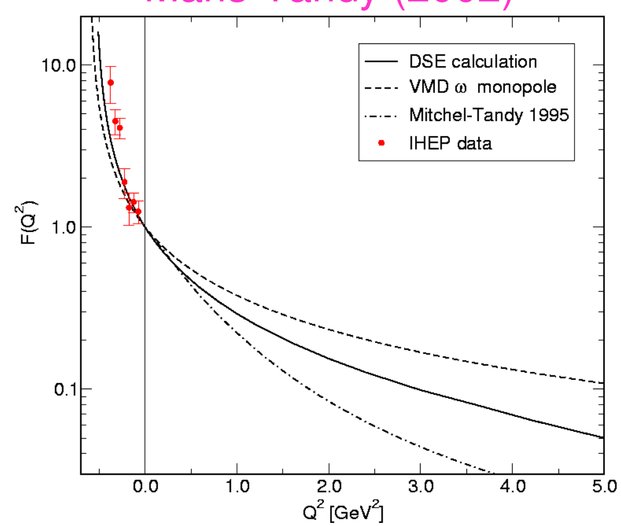


Photo-couplings and Transition FF: H → γ M

- Photo-couplings between hybrid and conventional mesons need to be calculated!

- GlueX proposal to produce hybrid mesons using real photons supported by flux-tube model calculations

  - No suppression of conventional-hybrid photo-couplings for hybrids near 2 GeV

$$\Gamma(\pi^+_{1H} \to a_2^+ \gamma) \sim \mathcal{O}(100)\mathrm{keV}$$
$$\Gamma(b^+_{JH} \to \rho^+ \gamma) \sim \mathcal{O}(1000)\mathrm{keV}$$

Close and Dudek, PRL91, 142001 (2003); PRD 69 034010 (2004)

$$(\mathrm{c.f.}\ \Gamma(b_1^+ \to \rho^+ \gamma) = 230 \pm 60\mathrm{keV})$$

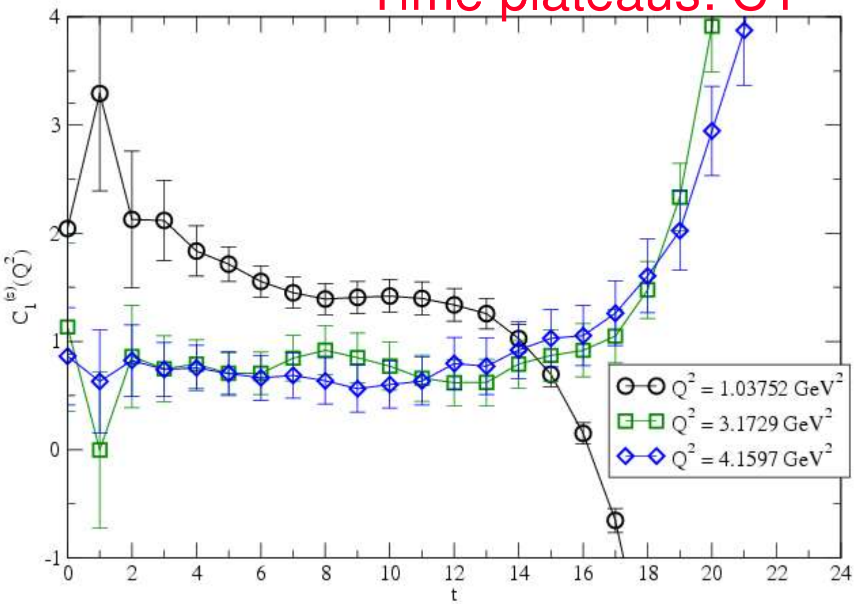
Investigate and attempt to verify prediction using lattice QCD

First tests: V→ S + γ in Charmonium

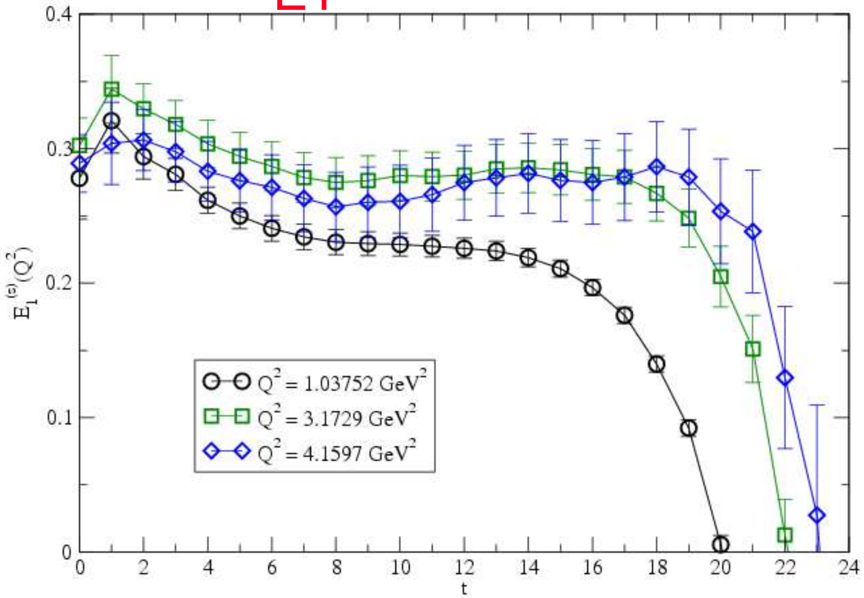
- Many new techniques: test using conventional mesons
  - Chiral fermions at charm scale – automatic O(a) improvement
  - Quenched anisotropic lattices – a_s = 0.1fm, a_t = 0.033 fm
  - Extraction of excited transition form-factors

$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V,\lambda)\rangle = E1(Q^2)K_{E1}(Q^2) + C1(Q^2)K_{C1}(Q^2)$$

Time plateaus: C1



E1



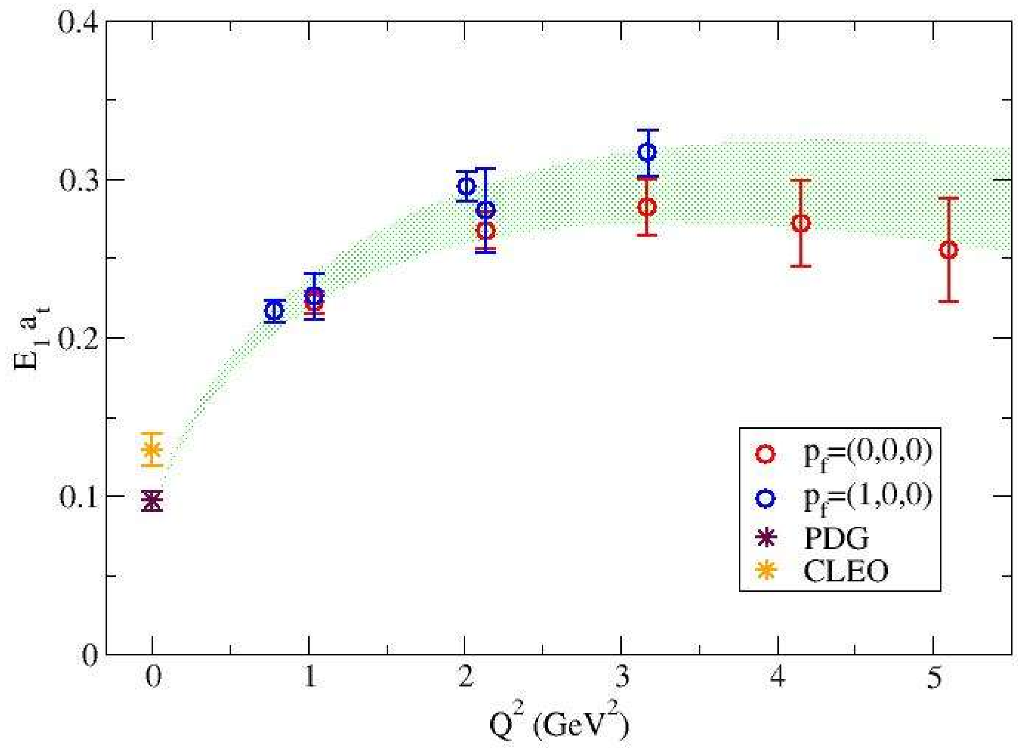
First results: $V \to S + \gamma$ in Charmonium

Lattice calculation of E_1 transition form factor

$$E_1(Q^2) = c \mid \vec{q}(Q^2) \mid e^{-Q^2/16\beta^2}$$

Extrapolate using quark-model wavefunc

$$\Gamma(\chi_{c0} \to J/\psi\,\gamma) = \frac{1}{8\pi}\frac{\mid \vec{q} \mid}{m_S^2}\, 2\,(2e_c)^2 \mid E_1(0) \mid^2$$



Dudek, Edwards, Richards, *Lattice 2005*

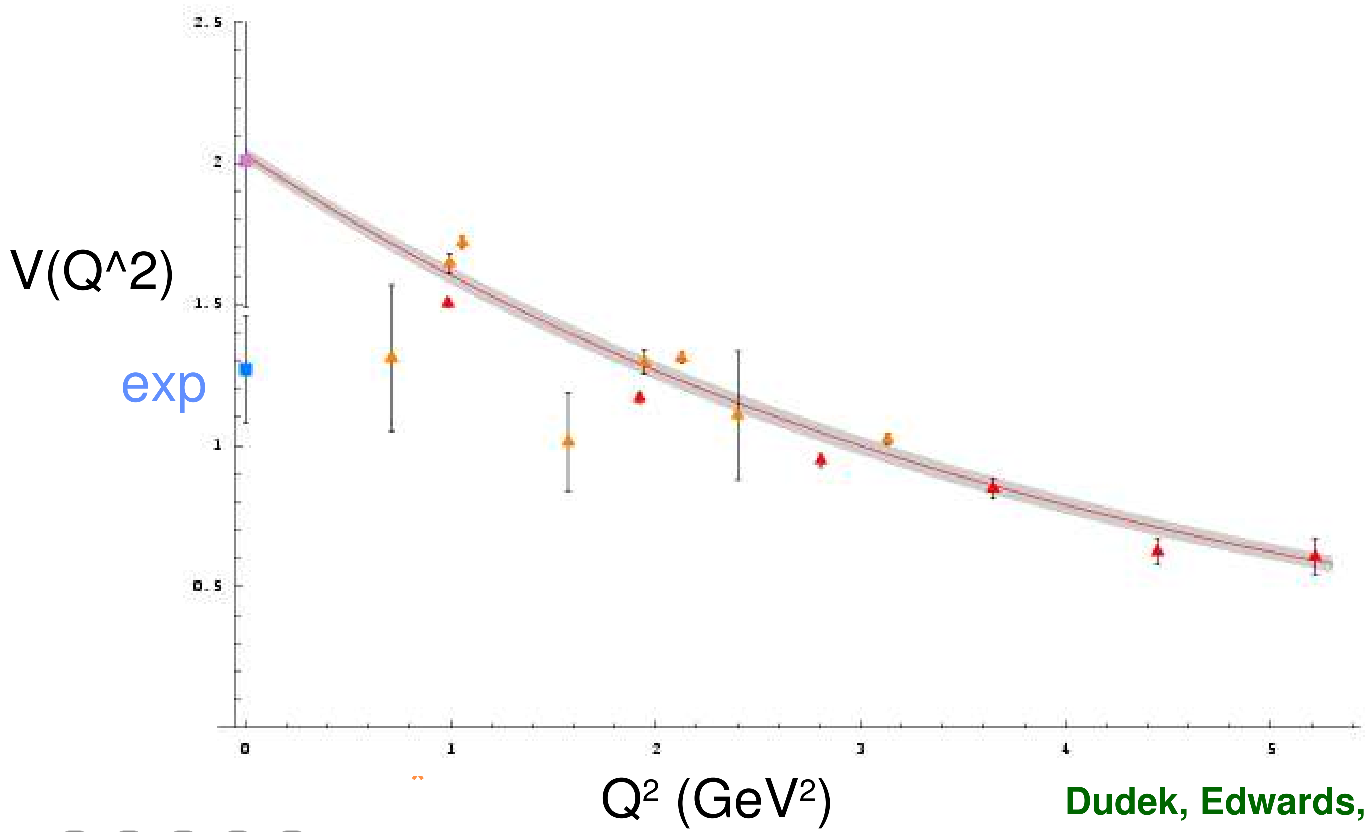
$V \rightarrow P + \gamma$ in Charmonium

Lattice calculation of V→ P transition form factor

$$\langle P(\vec{p}_f)|j^\mu(0)|V(\vec{p}_i,\lambda)\rangle = \frac{2V(Q^2)}{m_\pi + m_\rho}\epsilon^{\mu\alpha\delta\beta} p_{f\alpha} p_{i\delta} \varepsilon_\beta(\vec{p}_i,\lambda)$$

$$V(Q^2) = V(0)e^{-Q^2/16\beta^2}$$

Extrapolate using quark-model wavefunc



Dudek, Edwards, Richards, *Lattice 2005*

Conclusions and Perspective

- Use of hybrid Asqtad/DWF calculation providing entry into pion-cloud regime for range of physical quantities – spectrum, nucleon and meson form factors and transition form factors, moments of PDF's and GPD's…
- Computations being extended both to smaller values of lattice spacing (a=0.9, 0.6 fm), and to lighter m_π = 250 MeV – reveal effects of pion cloud.
- Disconnected diagrams, transition form factors…