Soft-QCD Modeling

of Di-quark Correlations and Mesons for the Study of the Nucleon and its Resonances

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Collaborators

- Pieter Maris, Univ of Pittsburgh
  - Much of this not possible without him
- Mandar Bhagwat, Kent State University
- Mike Pichowsky, Kent State University
- Craig Roberts, Argonne National Laboratory

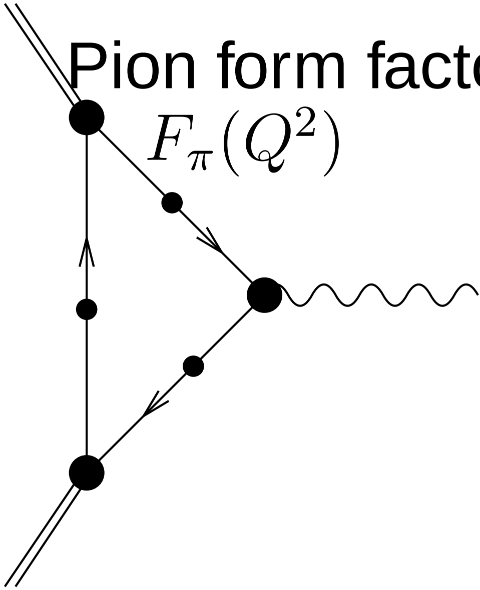
Topics

- Context of QCD modeling of hadron observables via DSEs
  - Ladder-rainbow truncation and symmetry constraints
  - Ordering/organization of mechanisms
  - Comparison with lattice-QCD
- Summary of results for meson observables, decays, transitions, form factors $\cdots$
- Comment on t-channel “meson exchange”
- Diquarks—qq correlations in baryons: results for Arne Hoell’s work—see Friday talk
- Limitations of ladder-rainbow—some consequences from 3-gluon coupling
- Summary

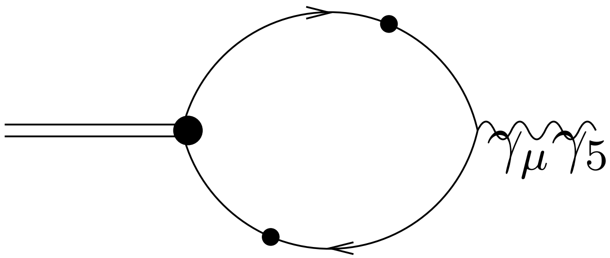
To Calculate Meson Observables

Pion form factor

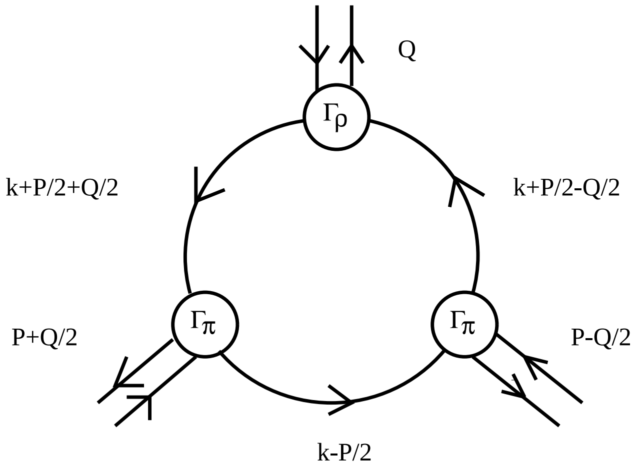
$F_\pi(Q^2)$



decay constant f_π



Strong decay $\rho \to \pi\pi$



Lattice-QCD and DSE-based modeling

- Lattice: $\int D\bar{q}qG\ \mathcal{O}(\bar{q},q,G)\ e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space
  - Discretize, finite vol, modify and improve $\mathcal{L}(x)$
  - Monte-Carlo
  - Large time limit $\Rightarrow$ nearest hadronic mass shells
  - Ch. extrap., contin. and vol. limits, CPU intensive
- EOMs: DSEs $\int D\bar{q}qG\ \frac{\delta}{\delta q(x)}\ e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space
  - Truncate: replace high n-point fns by phenomenology
  - Conv discretized integral eqns, modest CPU needs
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass shells
- In the middle: a clearing house for dominant mechanisms?

Organization chosen for DSE-based modeling

- EOMs: DSEs $\int D\bar{q}qG \; \frac{\delta}{\delta q(x)} \, e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)} = 0$
- Euclidean metric, p-space, covariant, no 3-space reduction
- Truncate to minimum 2-point, 3-point fns; IR phenomenology for ignorance
- Insist on preserving 1-loop QCD renorm group in UV
- Analtyic contin in external hadronic P^2 to mass shells
- Constraints for truncation: vector WTI, axial vector WTI
  E.g.
  $$-iP_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 \tfrac{\tau}{2} + \gamma_5 \tfrac{\tau}{2} S^{-1}(k_-) \\ -2\, m_q(\mu)\, \Gamma_5(k;P)$$
- $\Rightarrow$ kernels of DSE$_q$ and $K_{\rm BSE}$ are related

Organization chosen for DSE-based modeling (2)

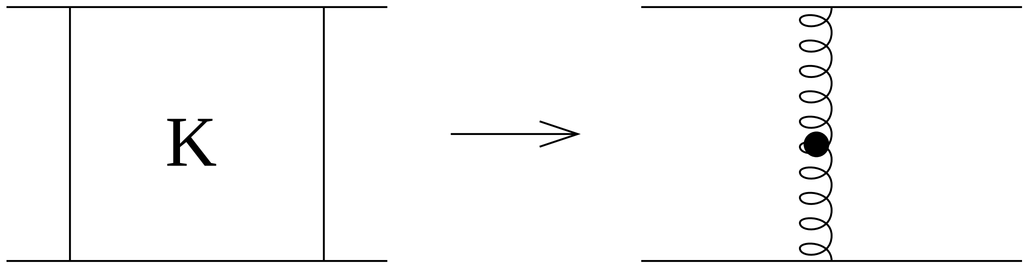
- Constraints for truncation: vector WTI, axial vector WTI
  - E.G. at 2,3-point fn level:
  - Rainbow DSE, ladder BSE, and IA for $F_\pi(Q^2)$ are symm-matched set
  - $\Rightarrow F_\pi(Q^2=0) = Q_\pi = 1$, always
  - $\Rightarrow$ leading asymptotic $F_\pi(Q^2)$ phys content present
  - Hopefully the interpolation can't go too wrong
  - Present IR phenomenology: 1 param to fit $\langle \bar{q}q \rangle_\mu$
  - Goldstone nature of ps octet, and phys masses from explicit ch symm breaking, will always be correct—indep of model details
- A systematic symm-preserving correction scheme is available

Organization chosen for DSE-based modeling (3)

- DSE approach emphasizes p^2-depn of q-masses, connecting constituent to current masses
  e.g. DCSB$\Rightarrow$ π: $\Gamma^0_\pi(p^2) = i\gamma_5\ \left[\frac{1}{4}\mathrm{tr}S_0^{-1}(p^2)\right]\ / f^0_\pi + \cdots$
- Can compare intermediate quantities with lattice-QCD for important guidance
- Present DSE organization emphasizes a certain ∞ sub-class of multiple gluon components
- Weakness: present choice of truncation may not be efficient for all processes
- Efficiency of description of observables is final guide

Ladder-Rainbow Model

- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement



- $K_{\rm BSE} \to -\gamma_\mu \frac{\lambda^a}{2} 4\pi \alpha_{\rm eff}(q^2) D^{\rm free}_{\mu\nu}(q) \gamma_\nu \frac{\lambda^a}{2}$
- $\frac{Z^2_{1\rm F}(\mu,\Lambda)}{Z^2_2(\mu,\Lambda)\, Z_3(\mu,\Lambda)} \to \left[\frac{\alpha_s(\Lambda^2)}{\alpha_s(\mu^2)}\right]$

$$\alpha_{\rm eff}(q^2) \underset{UV}{\longrightarrow} \alpha_s^{1-\rm loop}(q^2) = \frac{12\,\pi}{(11 N_c - 2 N_f)\ln(q^2/\Lambda^2)}$$

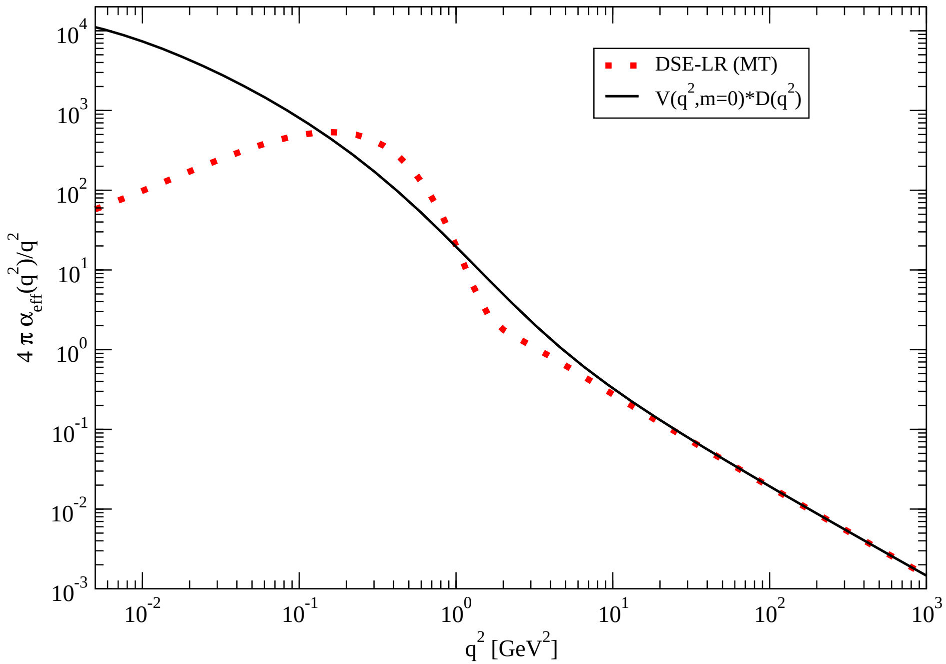
- first term in a systematic expansion

Ladder-Rainbow Model

- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement
- long-range part (IR, low-k^2) of interaction kernel fixed by $\langle \bar{q}q \rangle_{\mu=1\ \mathrm{GeV}} = -(240\mathrm{MeV})^3$
- single model parameter:

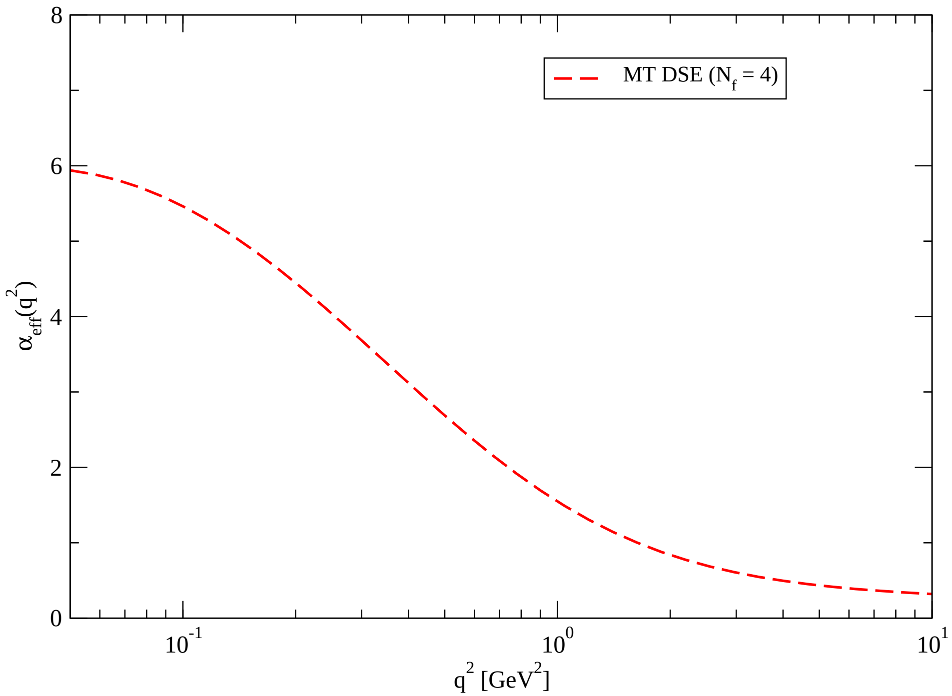
gluon mass scale $\sim$ 700 MeV

PM & P.C. Tandy, PRC60, 055214 (1999)



Ladder-Rainbow Model

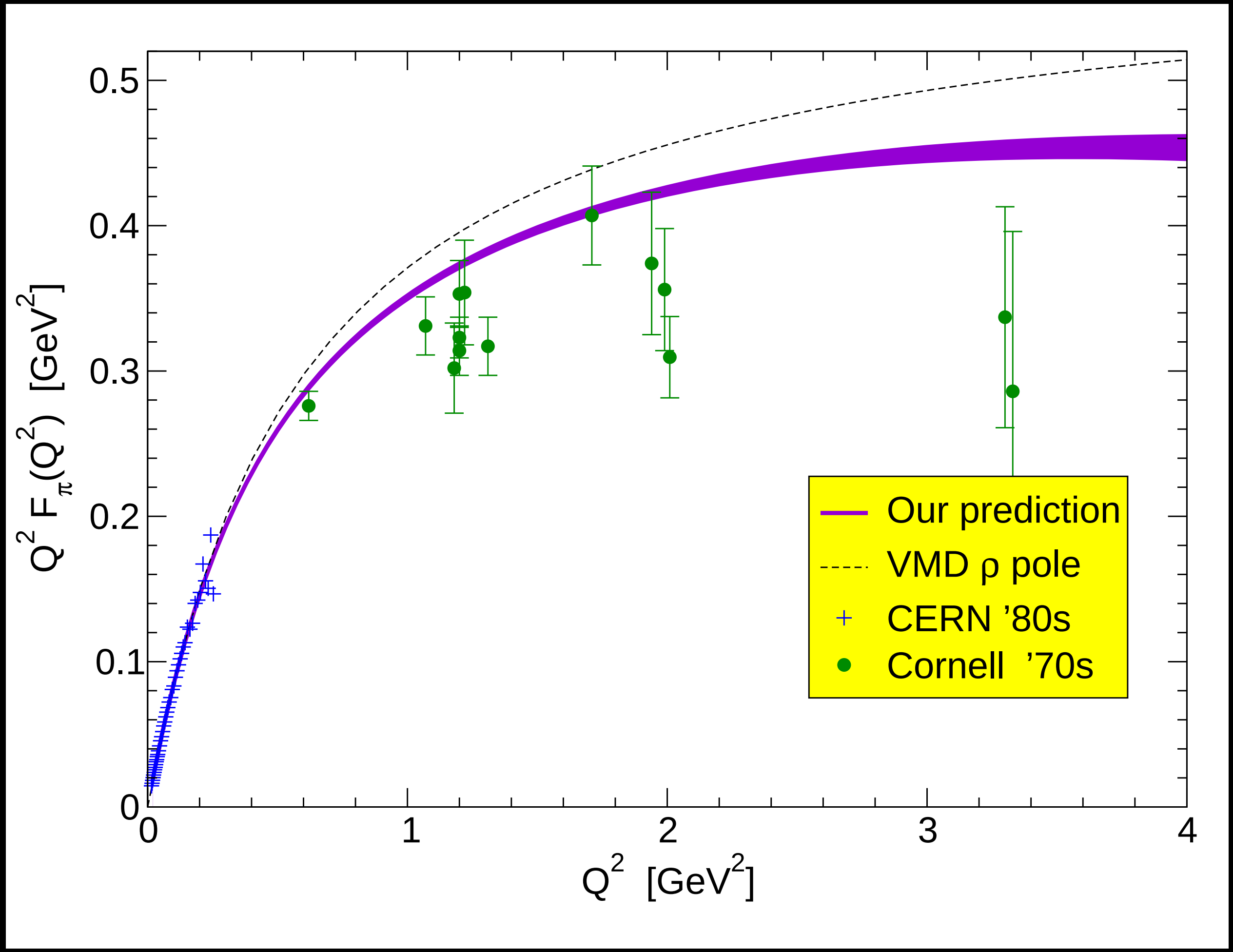
- short-range part of interaction kernel fixed by pQCD —-one-gluon exchange with 1-loop renormalization group improvement
- long-range part (IR, low-k^2) of interaction kernel fixed by $\langle \bar{q}q \rangle_{\mu=1\ \mathrm{GeV}} = -(240\mathrm{MeV})^3$
- Effective running coupling



Ladd-Rainb Model: Performance and Limitations

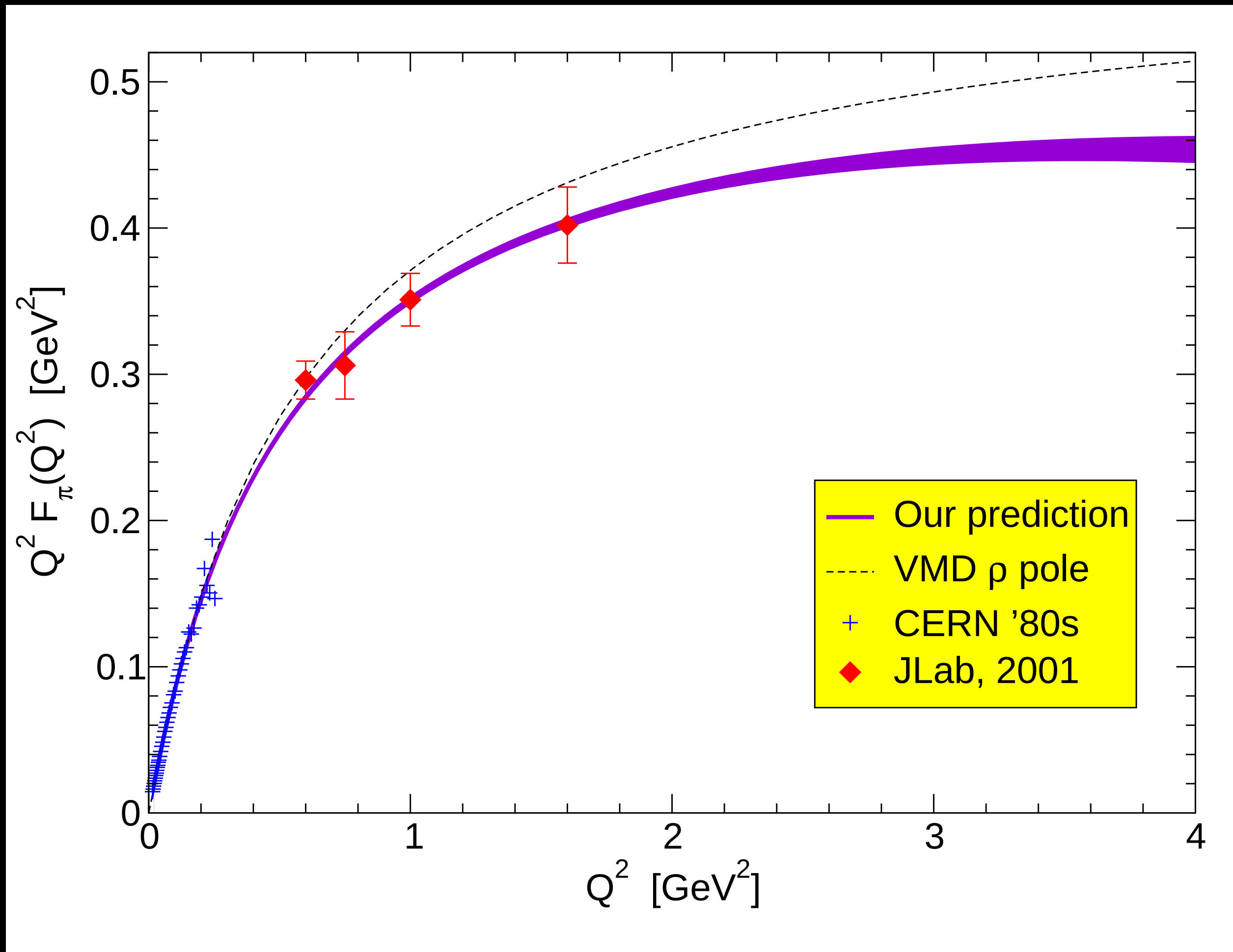
- Corrections are small in ps and vect meson channels
  Bender, et al. PLB380, 7 (96); Bender, Detmold, et al. PRC65, 065203 (02);
  Bhagwat, Höll, Krassnigg, Roberts & Tandy, PRC70, 035205 (2004)
- 1-parameter for $\langle \bar{q}q \rangle_\mu \Rightarrow$
- $M_\rho, M_\phi, M_{K^\star}$ to 5%; $f_\rho, f_\phi, f_{K^\star}$ to 10%
- Em form factors $Q^2 < 5$ GeV^2 good, but chiral loops not in
- Strong decays, em transition form factors satisfactory
- Limitations–corrections to ladder-rainbow needed:
  - Present LR model too attractive for axial vectors M_{a_1}, M_{b_1}
  - Chiral loops have to be added
  - Need extension to non-Abelian axial anomaly and $\eta - \eta'$
  - Etc

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

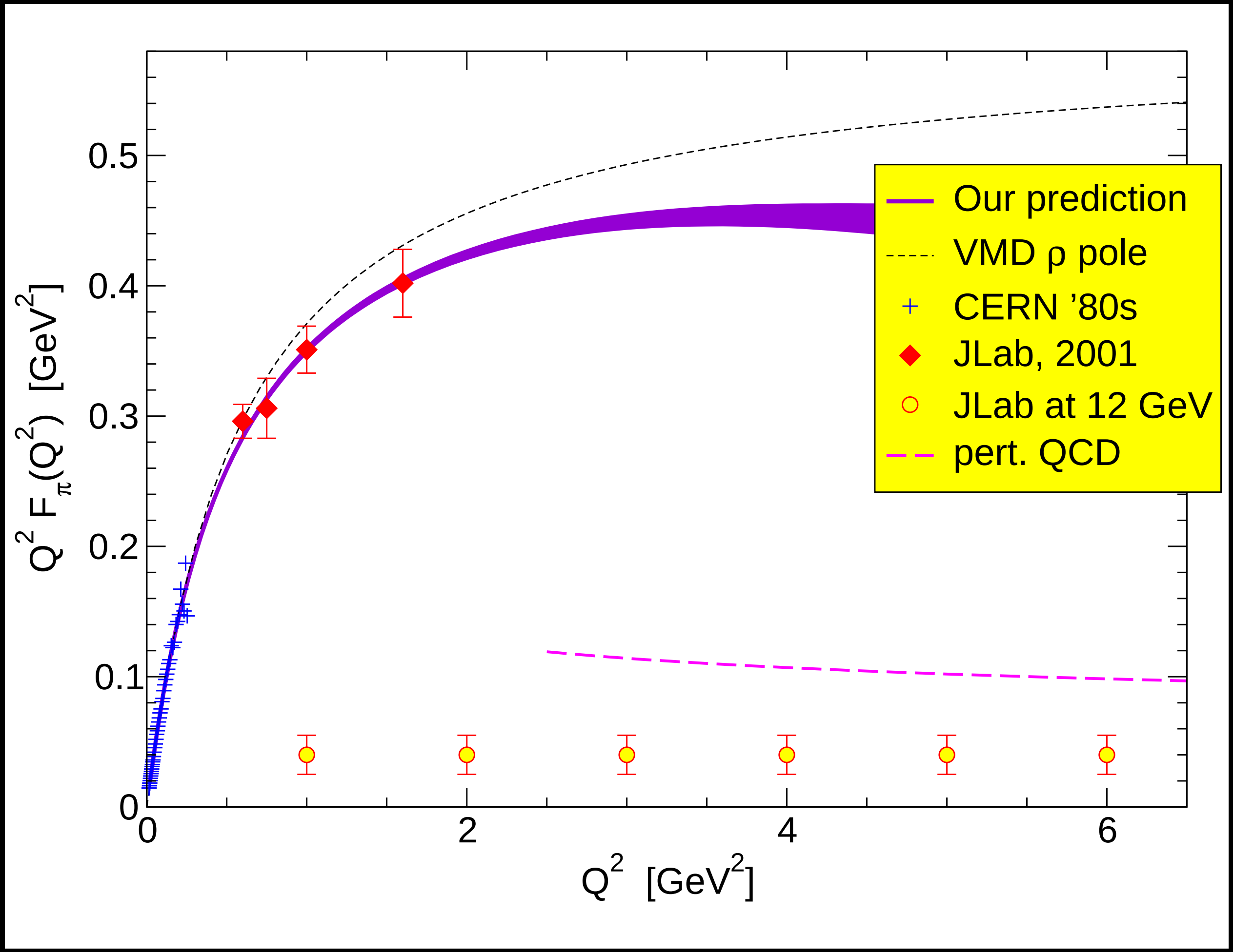
Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	7.6 $\cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu 3})$	5.2 $\cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

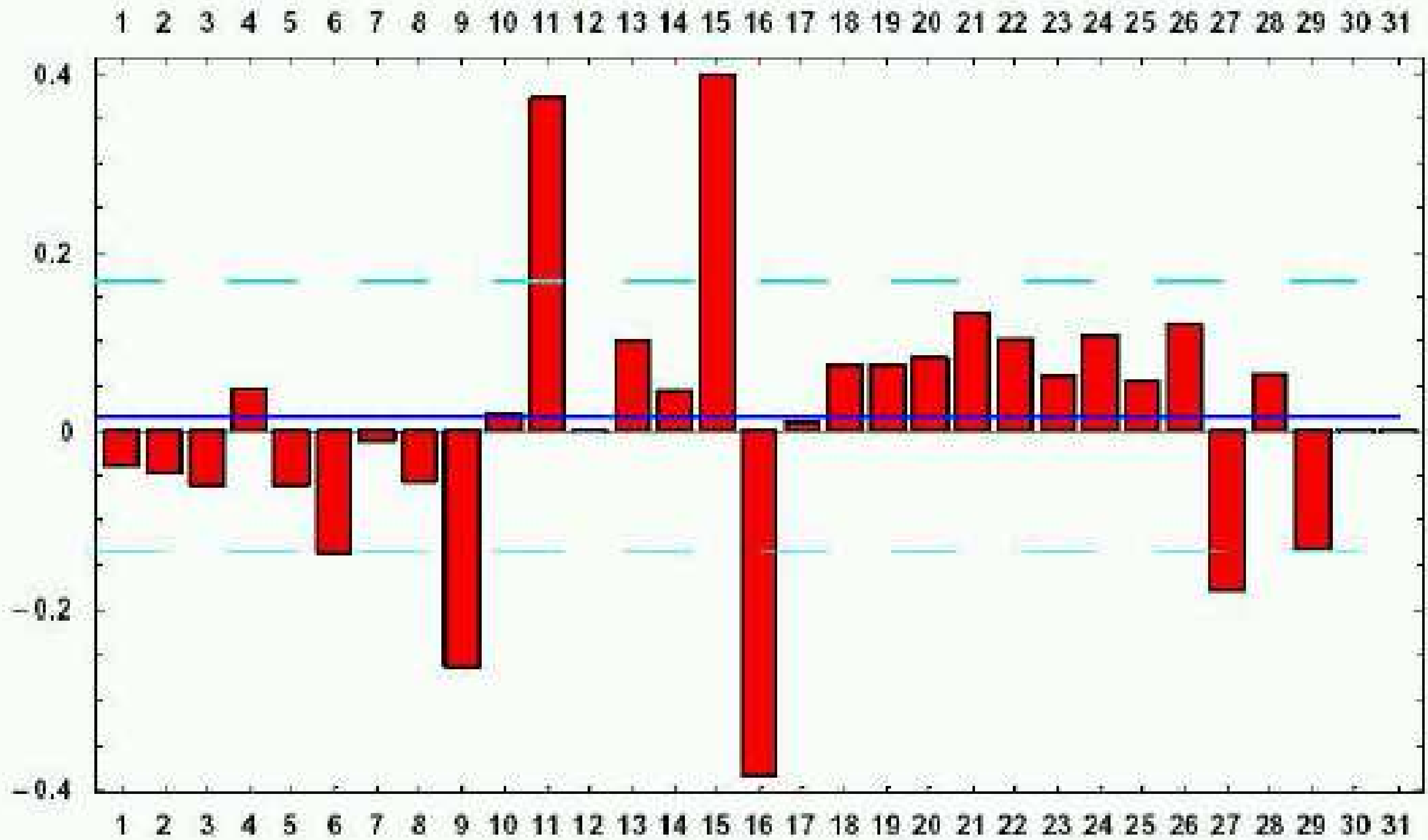
Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

Light meson sector is well understood



Relative Error, Predictions of Maris and Tandy Model

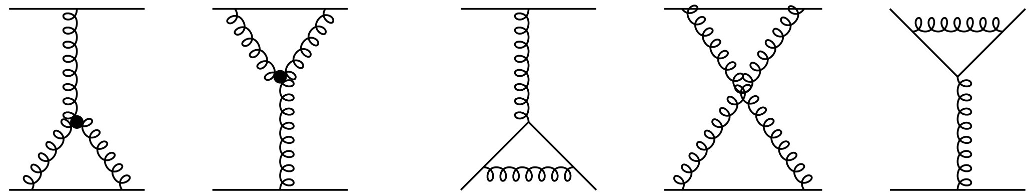
All tabulated quantities in nu-th/0301049

<error> = 1.6%, Sqrt[<error^2>] = 15%

Beyond Ladder-Rainbow

- Preservation of AV-WTI: dressing of $\bar{q}\gamma q$ vertex in DSE
  $\Rightarrow$ corrections to ladd-rainb K_{BSE}
- qq mesons: Feyn diagrammatic $\Sigma \Rightarrow K_{\mathrm{BSE}}$
- If Σ incl:

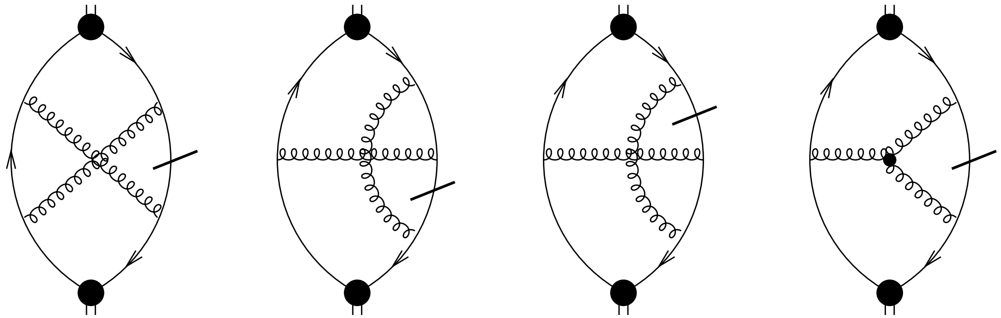
- $K_{\mathrm{BSE}} = -\frac{\delta\Sigma}{\delta S}$ incl:



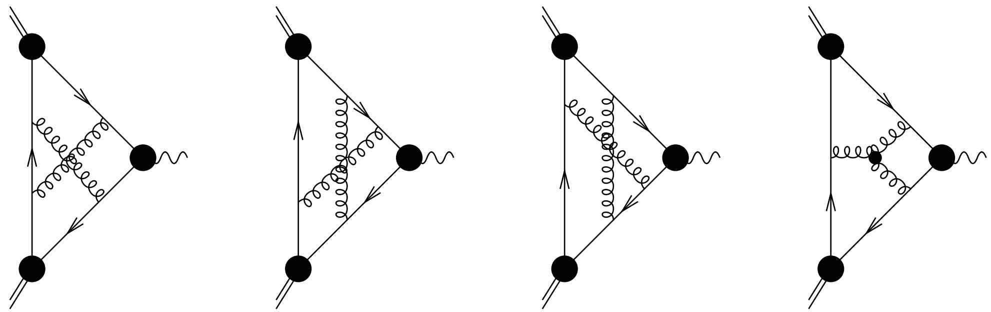
- $\frac{\delta}{\delta S}$ [closed q-loop] $\Rightarrow$ annihilation kernel
  for flavor singlets, e.g. $\eta - \eta'$

Beyond ladder-rainbow ⇒ beyond IA

Corrections to the ladd-rainb truncation ⇒ corresp corrections to the BSE norm condition



⇒ (via vector Ward Id) corresp corrections to IA for $F_\pi(Q^2 = 0)$:



and so on $\cdots$. Different organizations may absorb some into a wavefn.

DSE/BSE kernel from Lattice Gluon Propagator

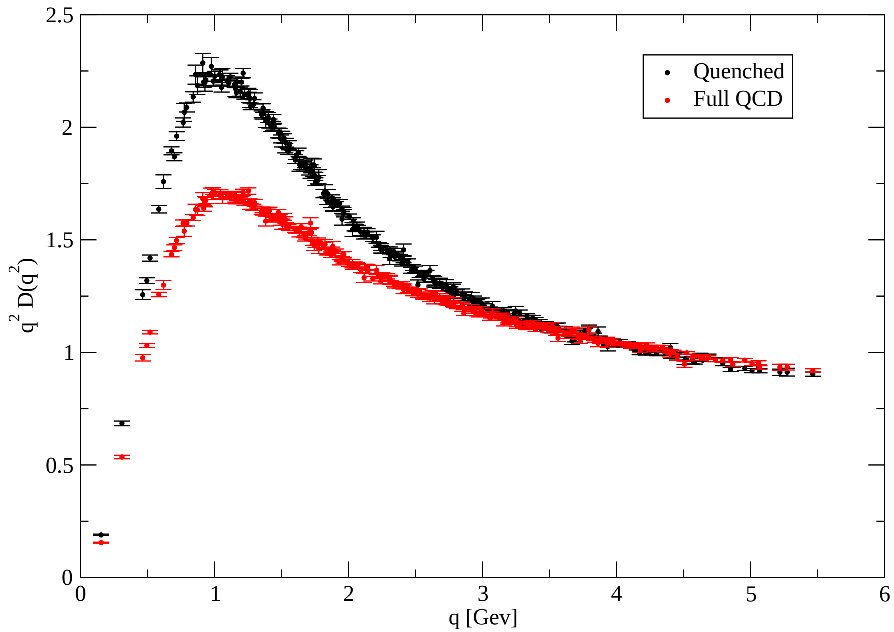
——Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (03)

- Qu-lattice $D_{\rm gluon}(q)$

  Leinweber, Bowman et al

  PRD60, hep-lat/9811027

- Find $\Gamma^{\rm eff}_\nu(q,p)$ so DSE produces $S_{\rm latt}(p)$ from $D_{\rm latt}(q)$

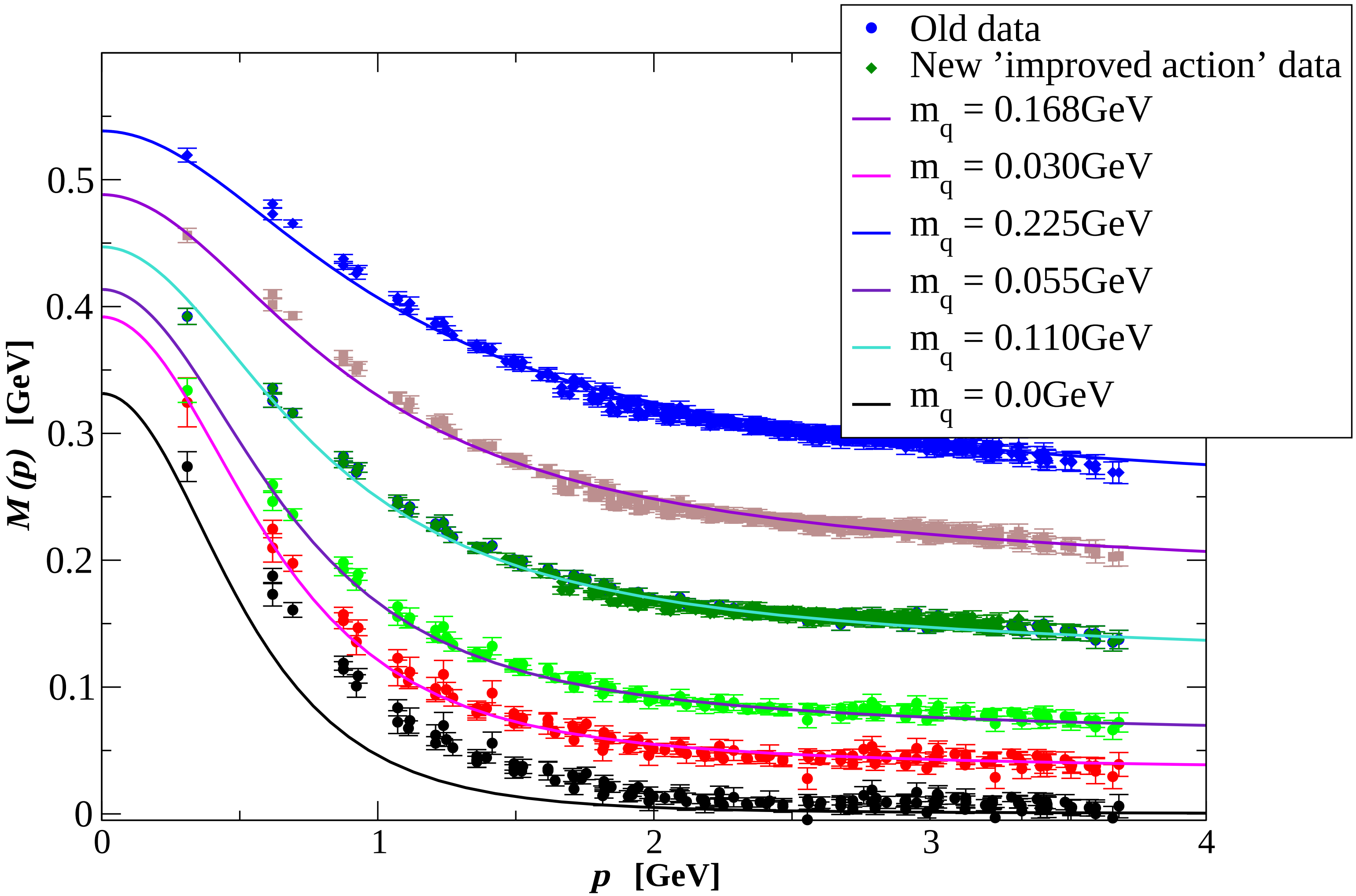


$$g^2\gamma_\mu D(p-q)\,Z_{1F}(\mu,\Lambda)\,\Gamma_\nu(q,p) \to \gamma_\mu g^2 D(p-q)\gamma_\nu\,V(p-q)$$

UV limit: $$g^2\,D(k^2)\,V(k^2) \to \frac{4\pi\alpha_s^{1-\rm loop}(k^2)}{k^2}$$

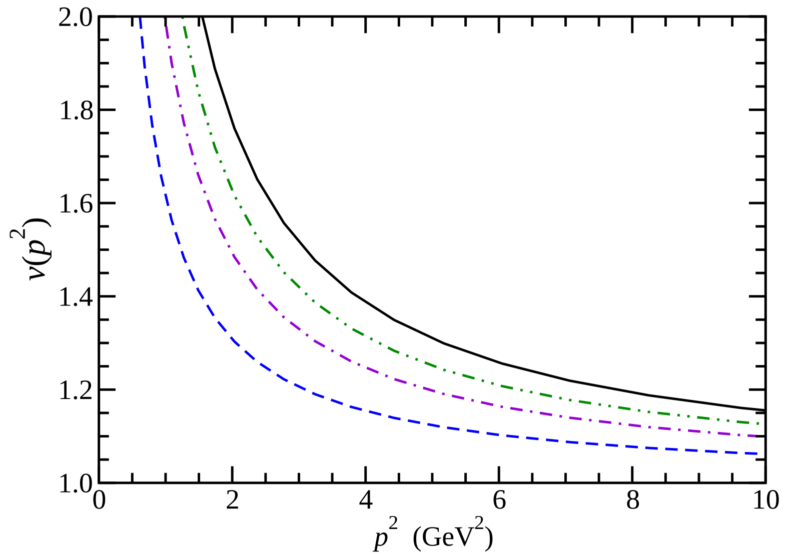
Qu-lattice $S(p), D(q)$ mapped to a DSE kernel

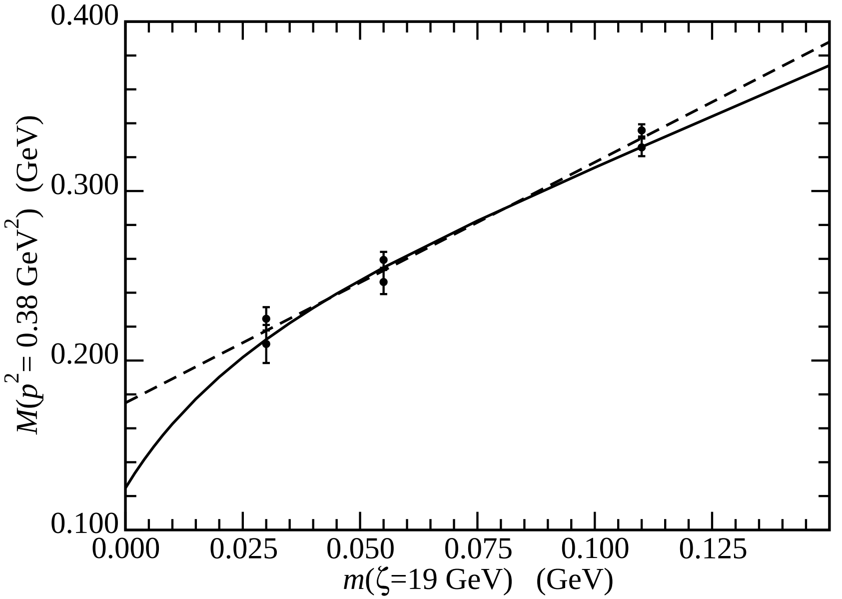
$$S(p) = Z(p)\,[i\,\not{p} + M(p)]^{-1}$$



Lattice-assisted DSE Results

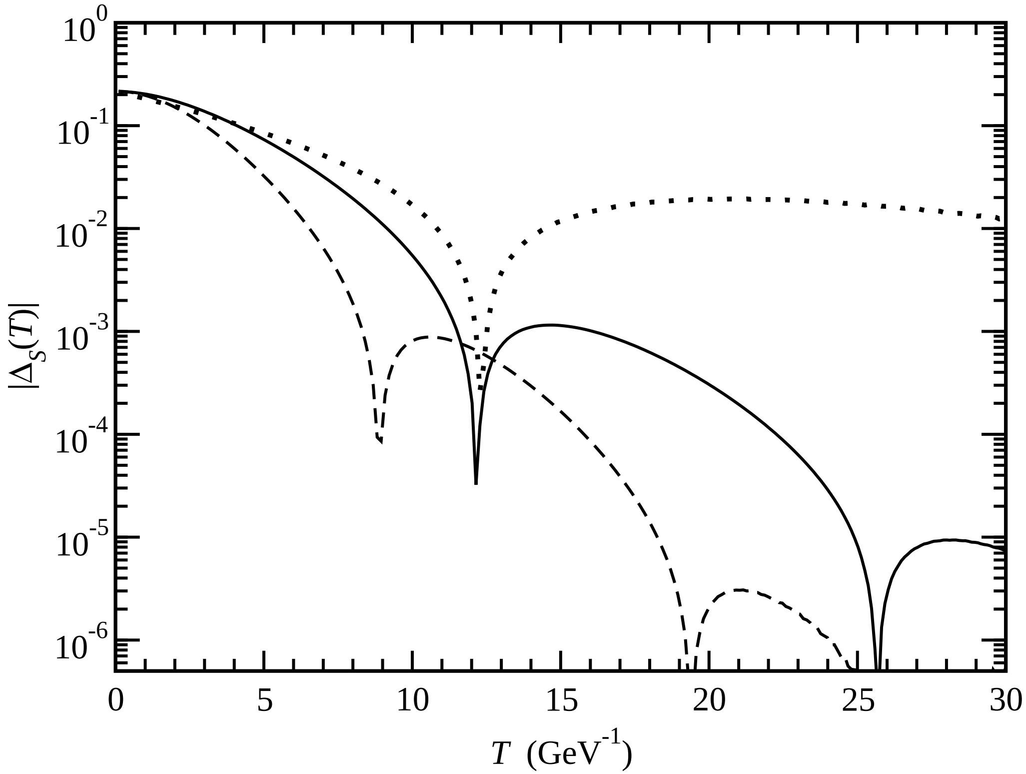
- Evident vertex enhancement
- Curvature in low m_q depn
- $M^{\rm IR}(p^2)$ 40% below linear
- Chiral Extrapolation
- $\langle \bar q q\rangle^{\rm qu-lat}_{\mu=1\ {\rm GeV}} = -(190\ {\rm MeV})^3$
- $\langle \bar q q\rangle^{\rm qu-lat} \approx \langle \bar q q\rangle^{\rm expt}/2$
- f_π 30% low





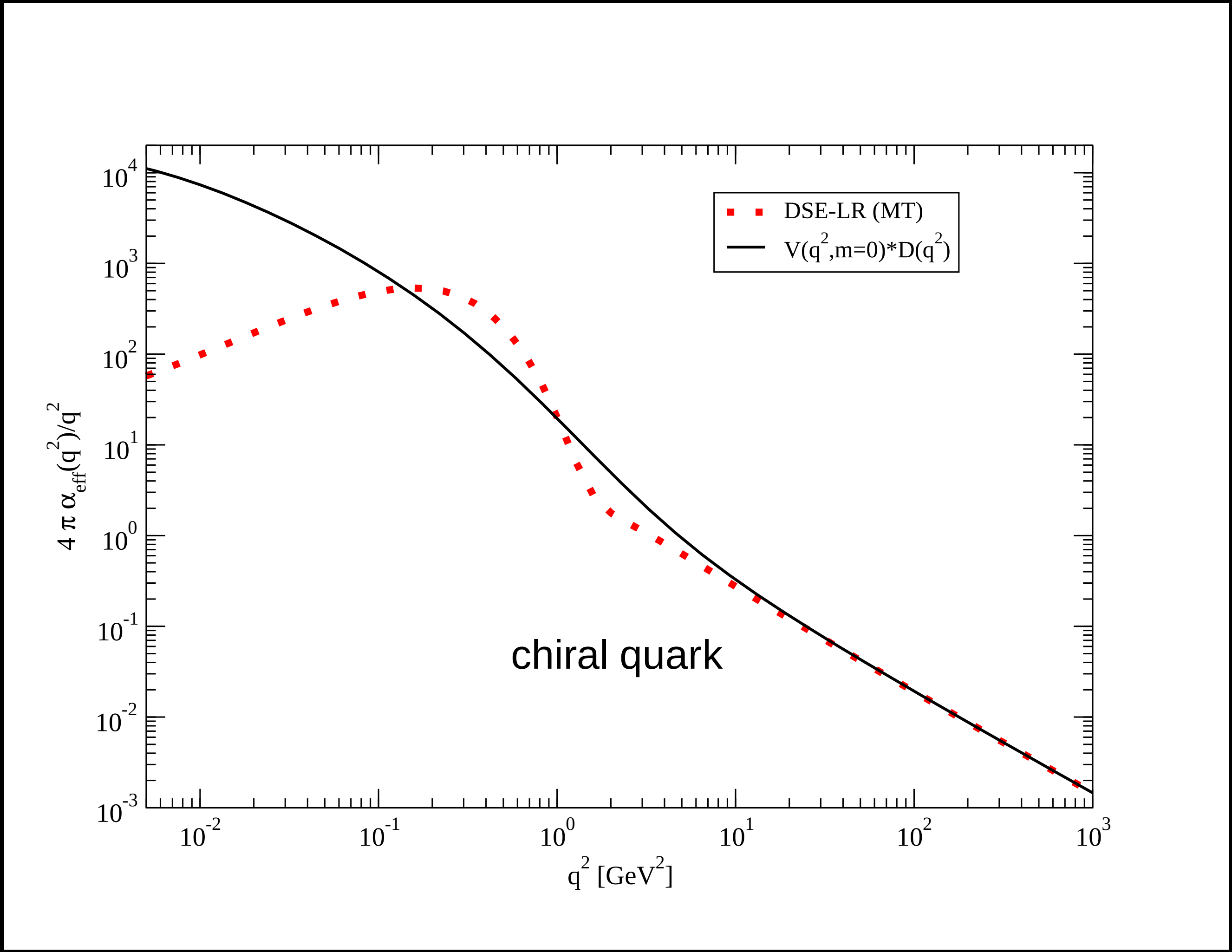
Lattice-assisted DSE Results

- Confinement/positivity analysis (Osterwalder-Schrader axiom No. 3)
- Fourier transf $\sigma_S(p_4, \vec{p} = 0)$ to Eucl time T



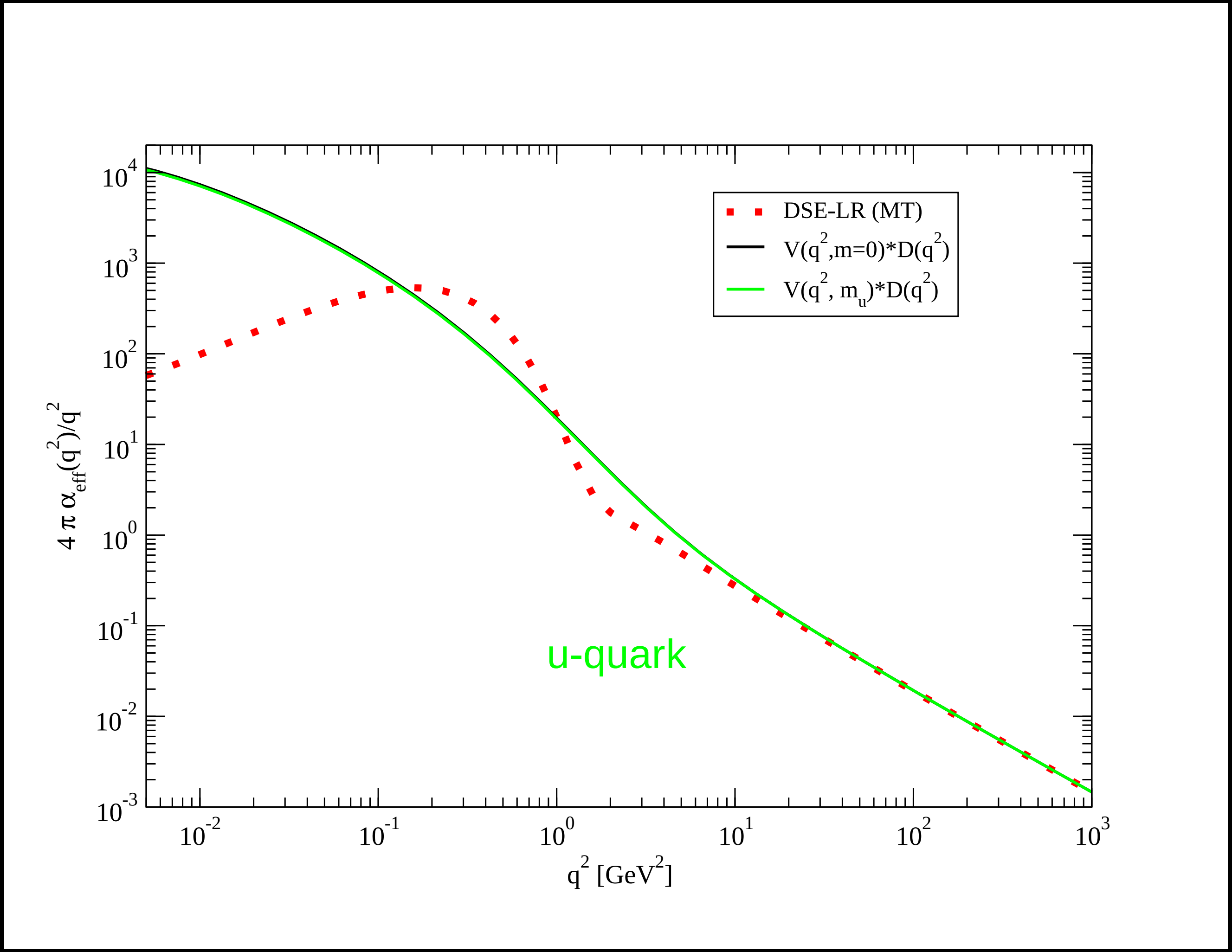
solid = lattice prop, dashed = MT DSE, dotted = cc pole eg

Quenched lattice $\Rightarrow m_q$ Depn of DSE Kernel



Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

Quenched lattice $\Rightarrow m_q$ *Depn of DSE Kernel*



Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

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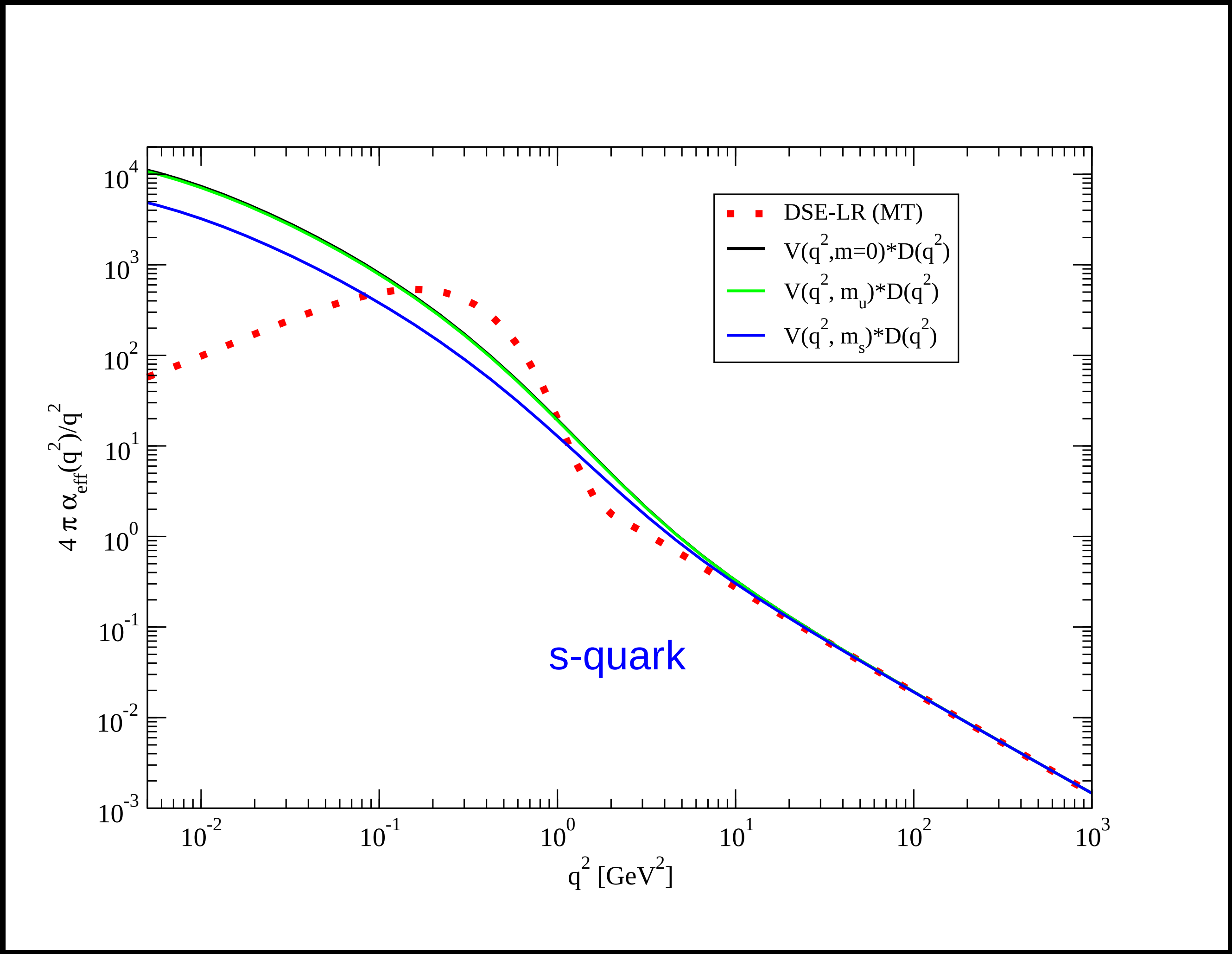

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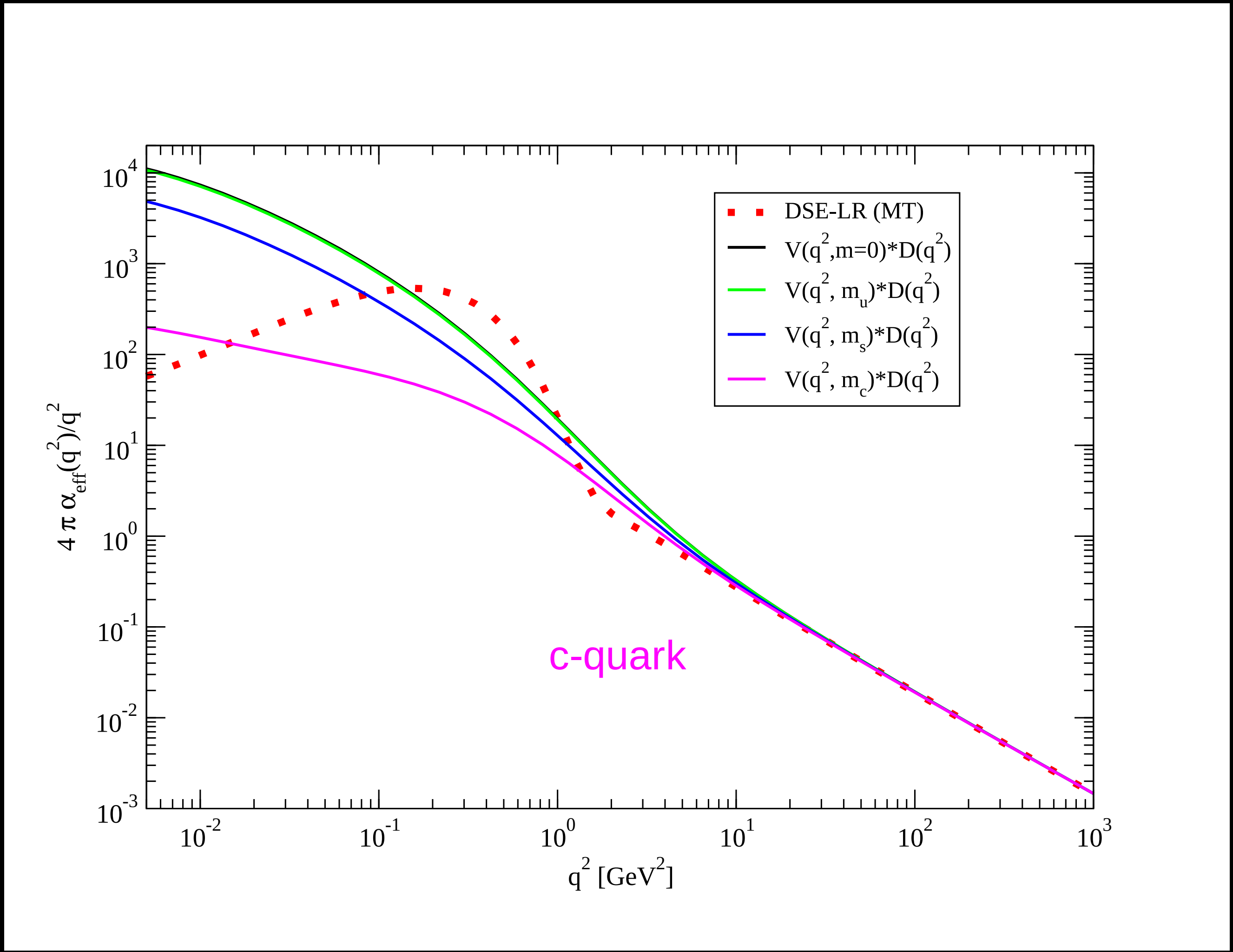

Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

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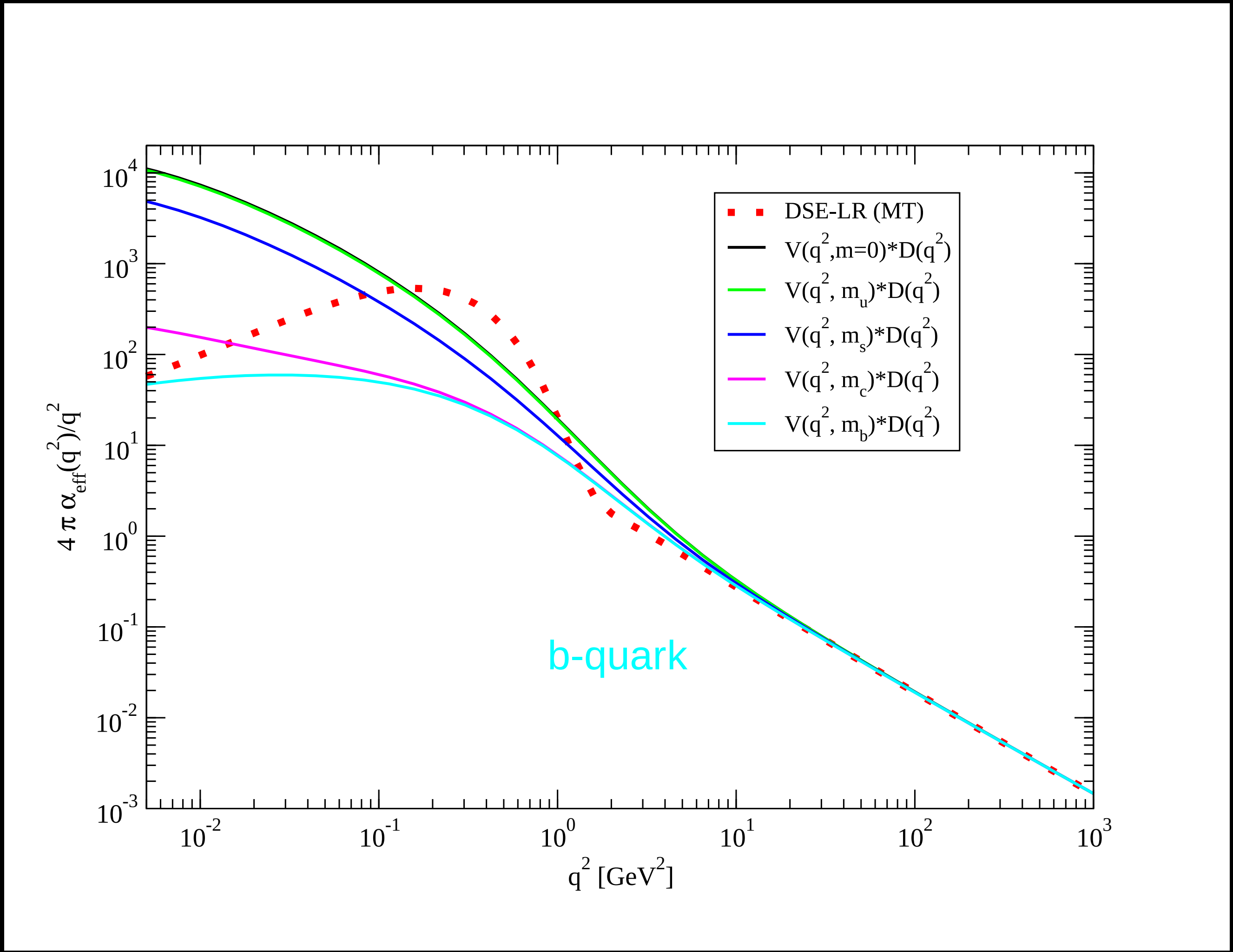
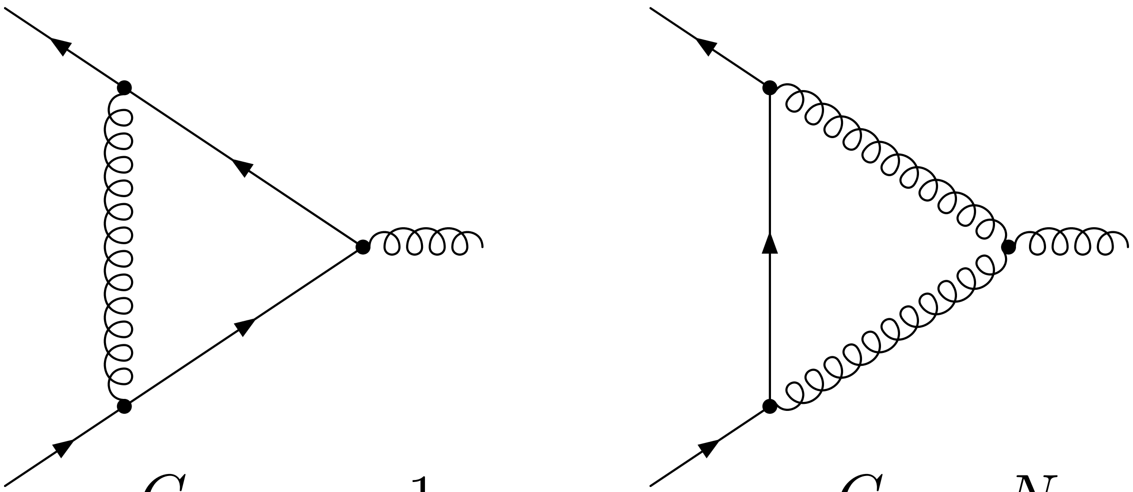


Bhagwat,Pichowsky,Roberts,Tandy, PRC68, 015203 (2003)

Dressed gluon-quark vertex: 1-loop pQCD



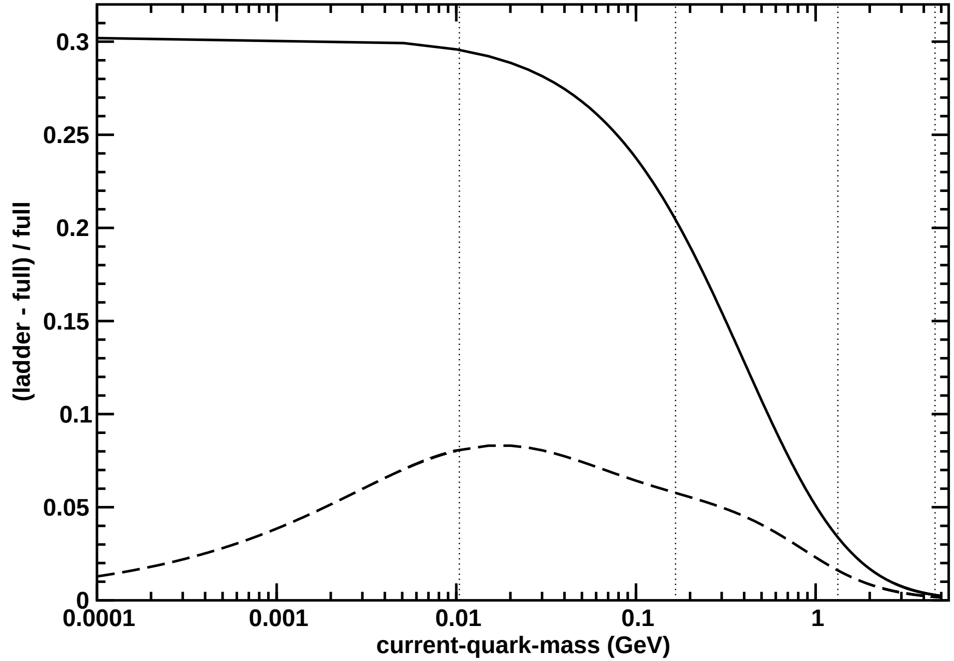
$(C_{\rm F} - \frac{C_{\rm A}}{2}) = -\frac{1}{2N_c}$ (weak repulsive)

$\frac{C_{\rm A}}{2} = \frac{N_c}{2}$ (v. strong attractive)

- Satisfies Slavnov-Taylor Id to $\mathcal{O}(g^3)$
  $ik_\nu \Gamma_\nu = G(k^2)[(1+B)\, S^{-1}(p_+) - S^{-1}(p_-)\,(1+\tilde{B})]$
- (Abelian, QED) color singlet channel: $C_{\rm F} = (N_c^2-1)/2N_c$ (strong attractive)

Estimate Effect of 3-Gluon Vertex on Mesons

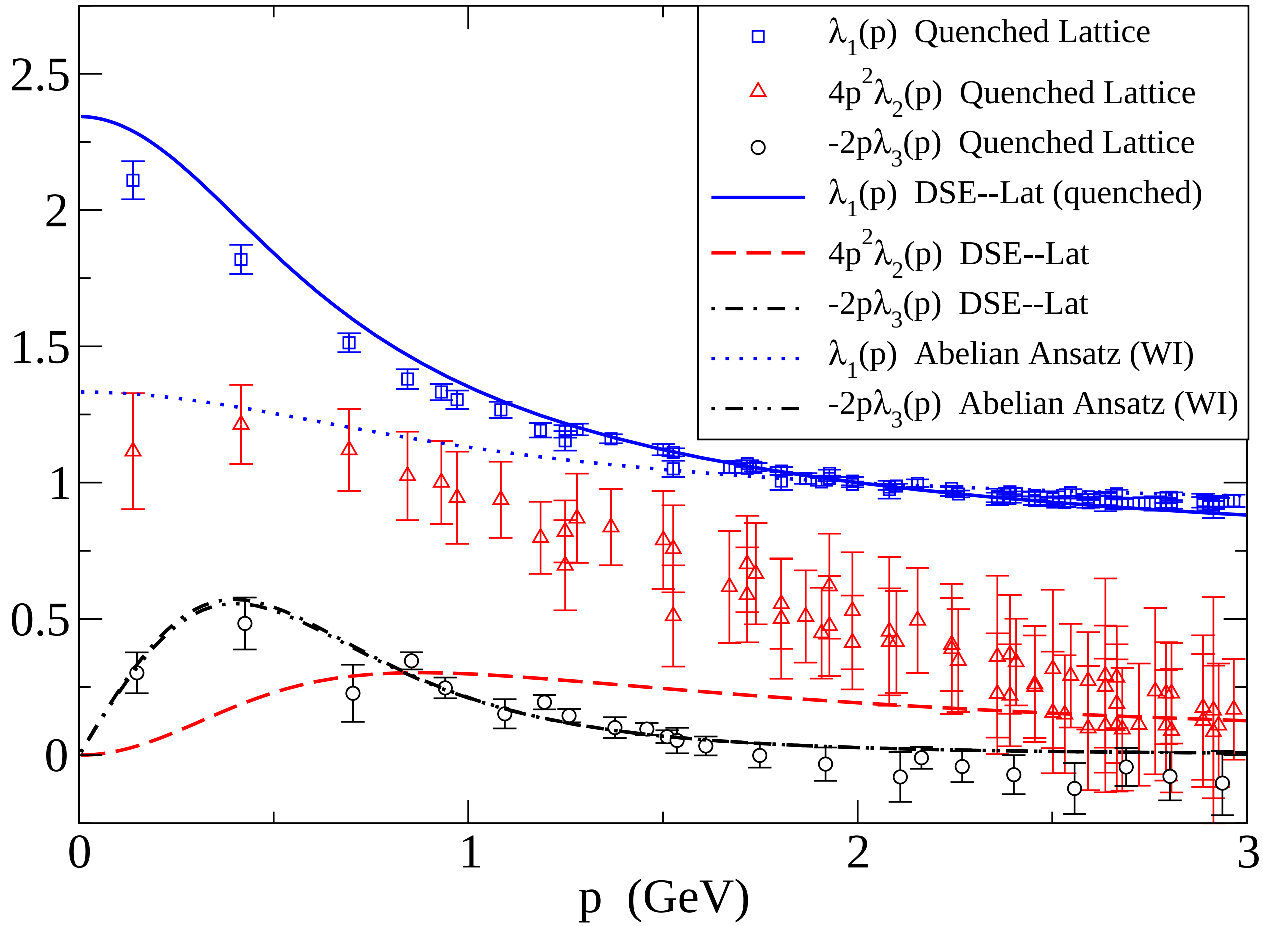
- Enters quark-gluon vertex and $K_{\rm BSE}$, preserves chiral symmetry
- Implemented in DSE$_q$ and meson BSE via (algebraic) MN model
- nucl-th/0403012, Bhagwat, Höll, Krassnigg, Roberts, PCT
- cf Ladder-rainbow:
  30% reduction in $M_{\rm V}$
  minor change in $M_{\rm PS}$



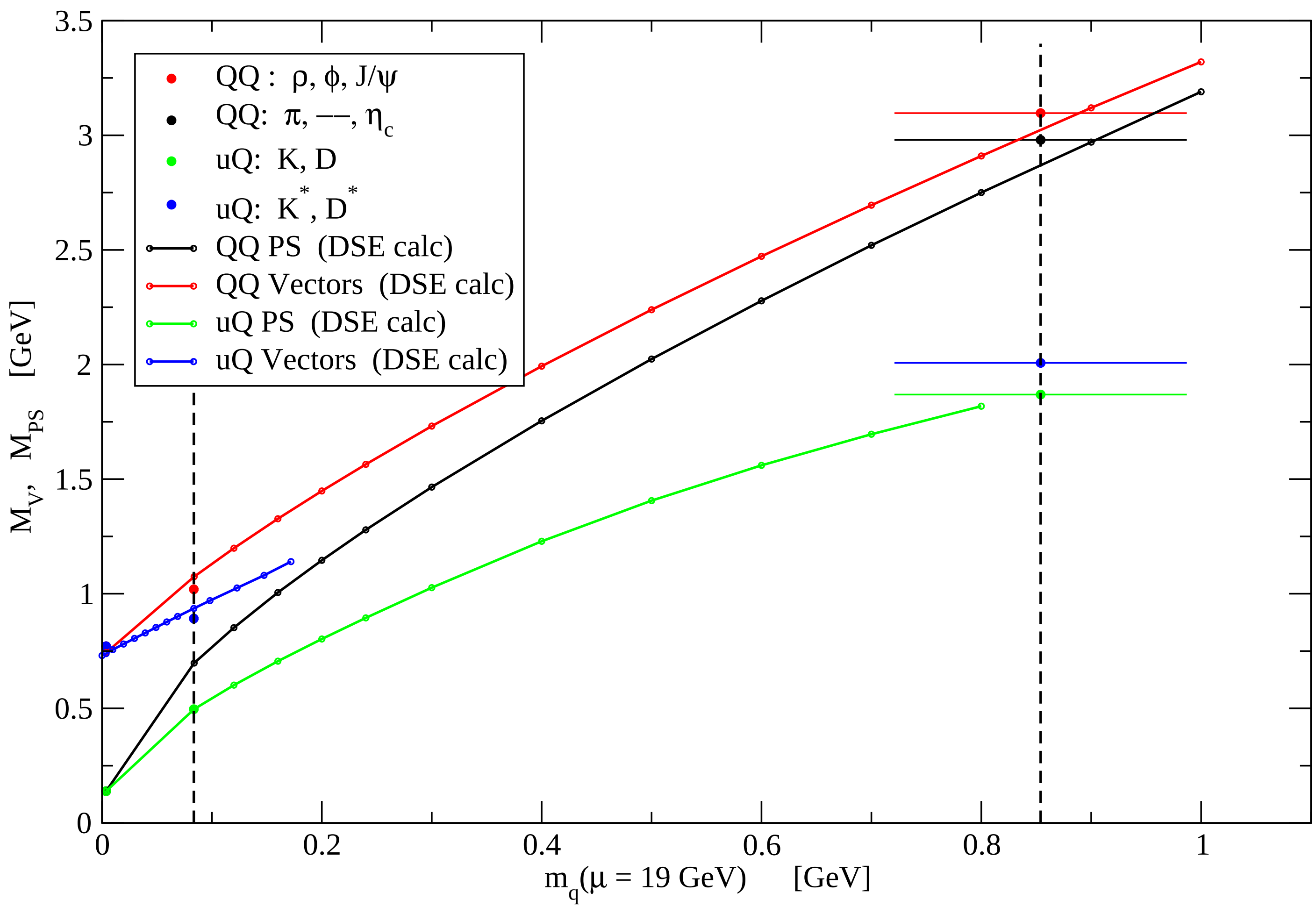
DSE Model for $k = 0$ gluon-quark vertex

$$\Gamma_\nu(p;0) = \gamma_\nu\,\lambda_1(p) - 4p_\nu\, \not{p}\lambda_2(p) - 2ip_\nu\lambda_3(p)$$

WI: $\lambda_1 \sim A, \quad \lambda_2 \sim -A'/2, \quad \lambda_3 \sim B'$



MT Model too attractive at large m_q ?



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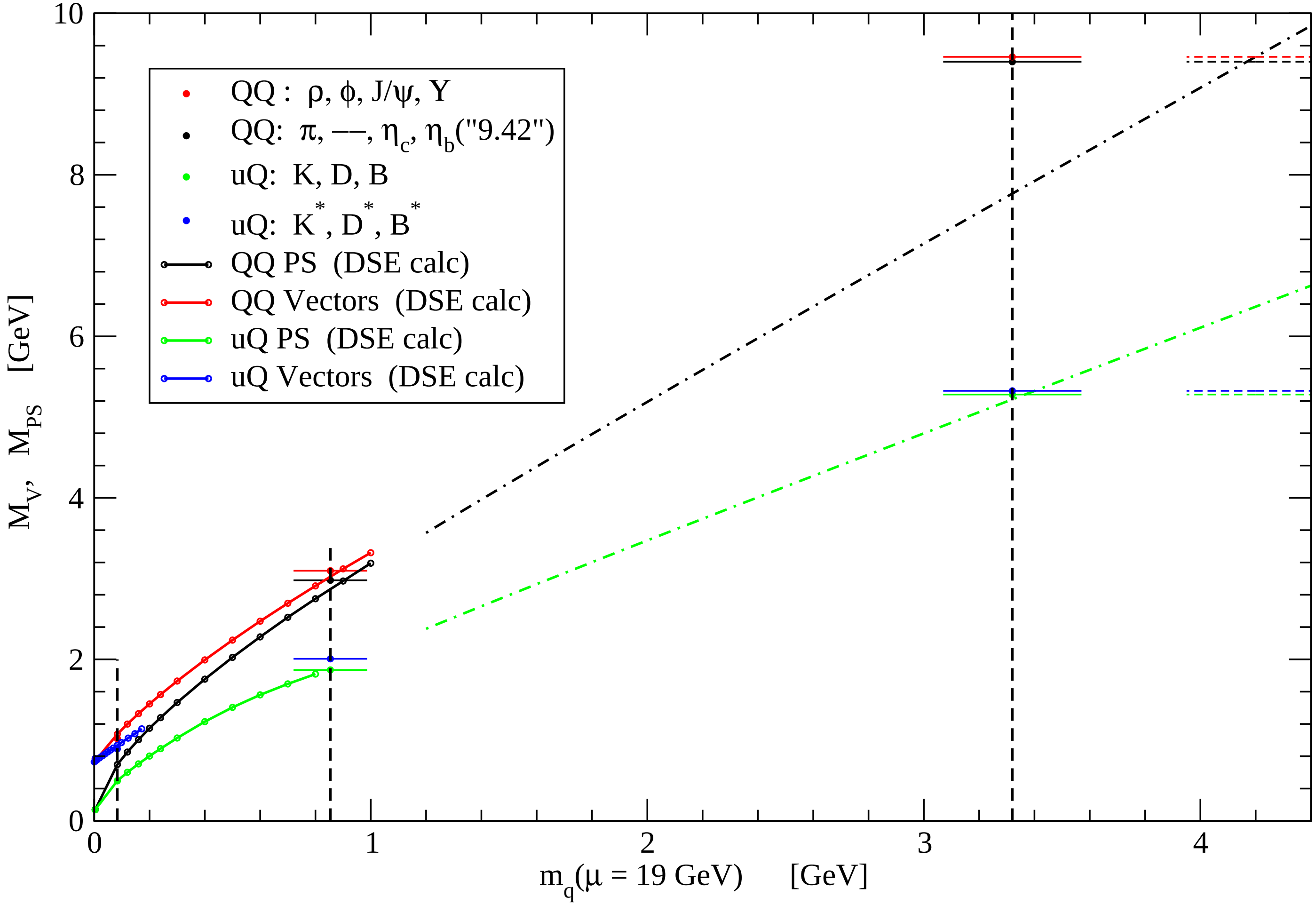
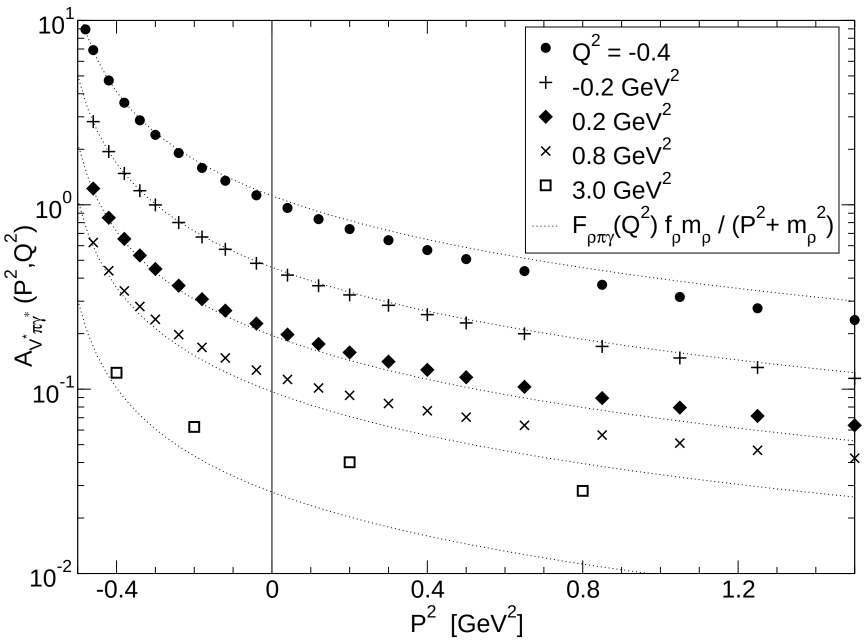


t-channel $q\bar{q}$ "meson" correlation

$$A_{V^*\pi\gamma^*}(P^2,Q^2) = \frac{f_\rho\, m_\rho}{P^2+m_\rho^2}\, G_{\rho^*\pi\gamma^*}(P^2,Q^2)$$

$$A^{\mathrm{VMD}}{}_{V^*\pi\gamma^*}(P^2,Q^2) \approx \frac{f_\rho\, m_\rho}{P^2+m_\rho^2}\, F_{\rho\pi\gamma^*}(Q^2)$$



- $Q^2 \approx 0,\ P^2 < 5\ \mathrm{GeV}^2$ no P^2 depn in G (VMD)
- Both $P^2 \approx Q^2$ large, $A \sim 1/P^2$, $F \sim 1/P^4, \Rightarrow$ problem
- $Q^2, P^2 > 1\ \mathrm{GeV}^2$, "$\rho$" prop overest P^2 falloff by $>$ 50%
- F would have to grow with P^2 ! meson-exch not relevant !

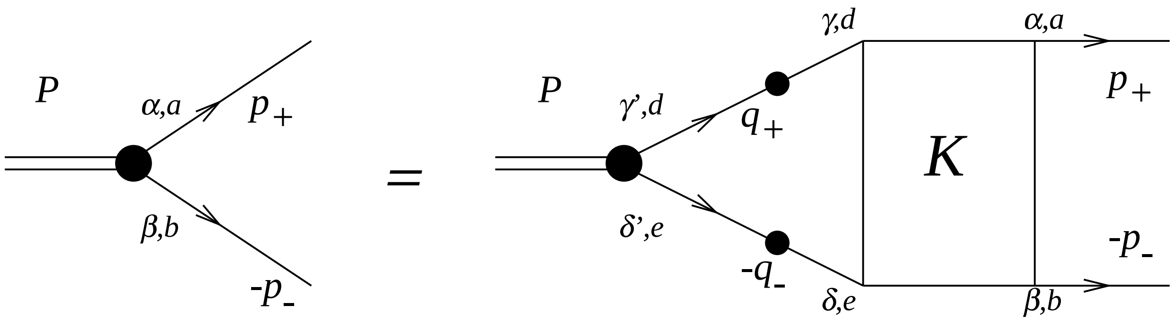
Diquarks-1

- DIQUARK: Correlation of two quarks
- one-gluon exchange in quark-quark channel is
  - attractive in color anti-triplet $\bar{\mathbf{3}}_c$ configuration
  - repulsive in color sextet $\mathbf{6}_c$ configuration
- $\bar{\mathbf{3}}_c$ diquark $+$ $\mathbf{3}_c$ quark $\Rightarrow$ diquark-quark repn of a $\mathbf{1}_c$ baryon
- diquarks are colored, and hence confined
- SU(3) flavor structure: $\mathbf{6}_f$ or $\bar{\mathbf{3}}_f$
  Pauli principle restricts flavor-spin configurations
  - total wave function has to be anti-symmetric
  - diquark $\bar{\mathbf{3}}_c$ is color anti-symmetric
  - diquark is symmetric in space-flavor-spin

Diquarks-2

- DIQUARK BETHE-SALPETER EQN

$$\Gamma_D^{\alpha\beta}(p_+,-p_-)\epsilon_{abc} =$$
$$\int_q^\Lambda K_{ad;be}^{\alpha\gamma;\beta\delta}(p_+,q_+;-p_-,-q_-)S^{\gamma\gamma'}(q_+)\,\Gamma_M^{\gamma'\delta'}(q_+,-q_-)\,\epsilon_{dec}\,S^{\delta\delta'}(-q_-)$$



- Notice that $\Gamma_D(p_+,-p_-)\,C$ satisfies a "meson" BSE, and ladder truncation is

$$\Gamma(p_+,p_-) = -f_c\int_q^\Lambda \mathcal{G}(k^2)\;D_{\mu\nu}^{\rm free}(k)\gamma_\mu\,S(q_+)\,\Gamma(q_+,q_-)\,S(q_-)\,\gamma_\mu$$

- where $f_c = \frac{4}{3}$ for mesons, and $f_c = \frac{2}{3}$ for diquarks

Diquarks-2a

- General decomposition for pseudoscalar mesons Γ_M and scalar diquarks $\Gamma_D\,C$

  $$\Gamma(q_+,q_-) = \gamma_5\big[iE(q^2,q\cdot P) + \not{P}\,F(q^2,q\cdot P) + \not{k}\,G(q^2,q\cdot P) + \sigma_{\mu\nu}\,P_\mu q_\nu\,H(q^2,q\cdot P)\big]$$

  Diquark flavor structure: anti-triplet (anti-symmetric)

- Vector mesons and axial-vector diquarks have eight components

  $\Gamma_\mu(q_+,q_-) = \gamma_\mu^T\;F(q^2,q\cdot P)$, (dominant)

  Diquark flavor structure: sextet (symmetric)

Meson and Diquark masses

P. Maris, FBS **32**, 41 (2002)

	$u\bar{u}, u\bar{d}$, etc			$u\bar{s}, d\bar{s}$, etc			$s\bar{s}$		
meson	π	ρ, ω	σ	K	$K^\star$	κ	0^-	ϕ	0^+
calc.	0.138	0.741	0.671	0.496	0.937	0.893	0.696	1.07	1.08
canonical	0.121	0.875	0.759	0.425	1.08	1.03	0.588	1.24	1.30
separable	0.139	0.736	0.715	0.494	0.854	–		0.950	
lattice	0	0.64					0.88	1.03	
diquark	0^+	1^+	0^-	0^+	1^+	0^-	0^+	1^+	0^-
calc.	0.82	1.02	1.03	1.10	1.30(6)	1.31(4)	1.27	1.44(4)	1.50(4)
canonical	0.74	1.06	1.14(2)	0.94	1.34(4)	1.45(4)	1.12	1.51(4)	1.72(6)
separable	0.74	0.95	1.50	0.88	1.05	–		1.13	
lattice	0.69	0.80					1.19	1.21	

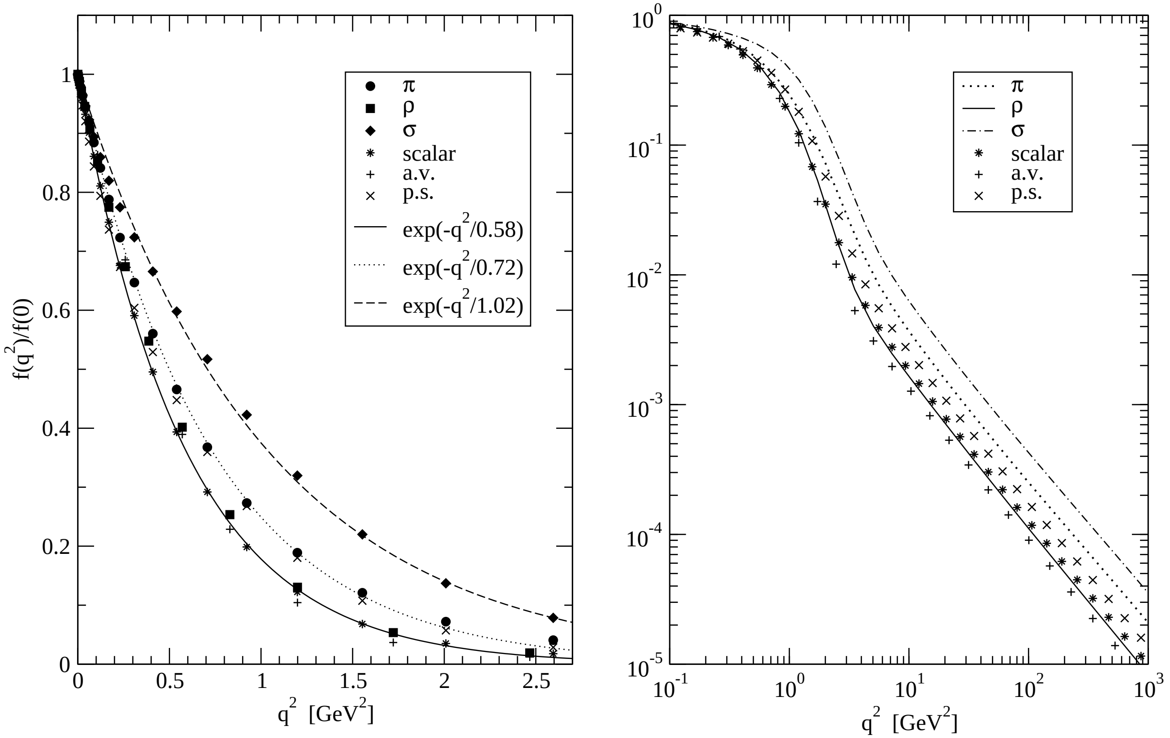
separable model: Burden *et al* PRC55, 2649 (1997)

lattice data: Hess *et al*, PRD58 111502 (1998)

Diquark size-1

P. Maris, FBS **32**, 41 (2002)

- $\langle q^2 \rangle_{\rm diquark} < \langle q^2 \rangle_{\rm meson} \Rightarrow \langle r^2 \rangle_{\rm diquark} > \langle r^2 \rangle_{\rm meson}$



Leading Chebyshev moments of canonical amplitudes in the up/down sector

Diquark size-2

Charge radii in fm^2

meson					diquark				
r^2_π	$r^2_{q\bar{s};q}$	$r^2_{q\bar{s};\bar{s}}$	r^2_{K+}	$r^2_{K^0}$	r^2_{ud}	$r^2_{qs;q}$	$r^2_{qs;s}$	r^2_{us}	r^2_{ds}
0.44	0.46	0.21	0.38	-.08	0.50	0.47	0.30	0.65	0.39

Summary

- Hadron observables and dynamics modeled from QCD -DSEs—covariant, quark confining, DχSB
- Summary of ordering/organization
- $\langle \bar{q}q \rangle_\mu \Rightarrow$ 1 IR parameter
- Propagators compare to lattice-QCD $S(p)$
- Meson observables
- Diquark correlations for baryon studies–see talk by Arne Hoell
- 3-gluon coupling, dressed $\bar{q}\gamma q$ vertex $\Rightarrow$ 30% attraction to ground state vector mesons
- t-channel $q\bar{q}$ correlation “meson propagator”