On Nucleon Strong and Electroweak Form Factors

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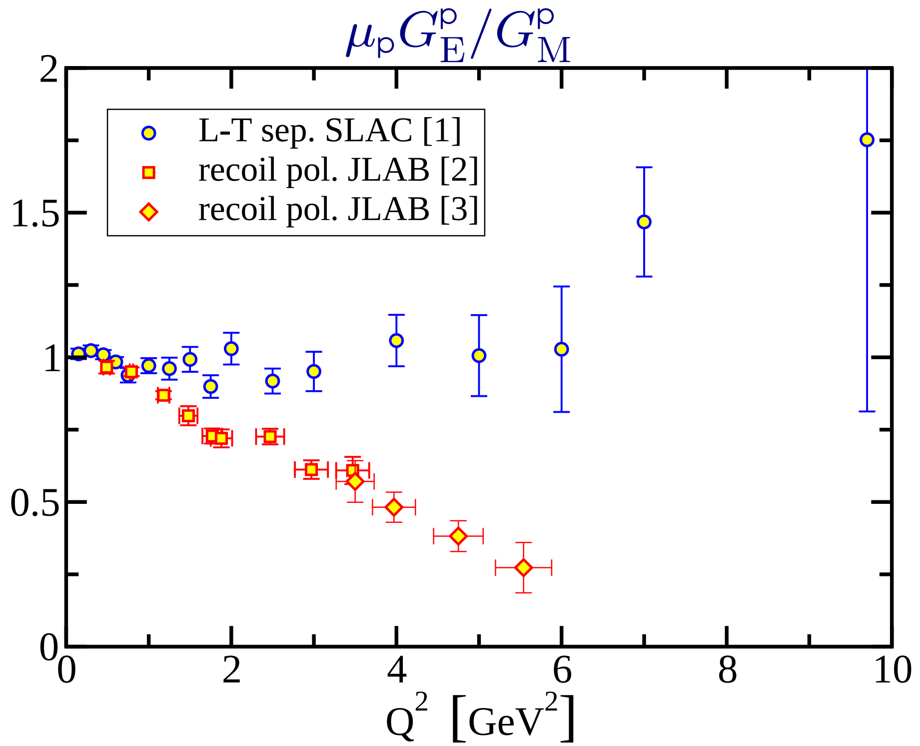
Peter Tandy	Kent State University, Ohio
Stewart Wright	Argonne National Lab., Illinois

Introduction: Nucleon Electromagnetic Form Factors

current debate on nucleon electromagnetic form factors

J. Arrington, nucl-ex/0305009.
H.H. Matevosyan, G.A. Miller, A.W. Thomas, nucl-th/0501044. →
A.V. Afanasev, S.J. Brodsky, C.E. Carlson, Yu-Chun Chen, M. Vanderhaeghen, nucl-th/0502013. →



[1] proton elastic form factor from Rosenbluth separation (Long.–Transv. separation)

[1] R.C. Walker et al., PRD **49**, 5671 (1994).

[2,3] proton elastic form factor from polarization transfer

[2] M.K. Jones et al., PRL **84**, 1398 (2000).
[3] O. Gayou et al., PRL **88**, 092301 (2002).

Rosenbluth separation

$$\frac{d\sigma}{d\Omega} \frac{\varepsilon(1+\tau)}{\sigma_{\mathrm{Mott}}} = \tau \left(G_M^p(Q^2)\right)^2 + \varepsilon \left(G_E^p(Q^2)\right)^2$$

Polarization Transfer

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l} \frac{(E_e + E_e') \tan(\theta_e/2)}{2M}$$

Poincaré Covariant Faddeev Equation for the Nucleon

Poincaré Covariant Nucleon Faddeev Equation

- truncation of three body problem:

C. J. Burden, R. T. Cahill and J. Praschifka, Austral. J. Phys. **42**, 147 (1989).

⇝ neglect irreducible 3–quark interactions

⇝ approximate quark–quark scattering matrix

by sum over pseudoparticles (diquarks)

Dirac–scalar + Dirac–axial–vector + . . .

$(qq)_{J^P}$	$(ud)_{0^+}$	$(ud)_{1^+}$	$(ud)_{0^-}$	$(ud)_{1^-}$
m_{qq} (GeV)	0.74	0.95	1.50	1.47

C.J. Burden, Lu Qian, C.D. Roberts, P.C. Tandy, M.J. Thomson,
PRC **55**, 2649 (1997); nucl-th/9605027.

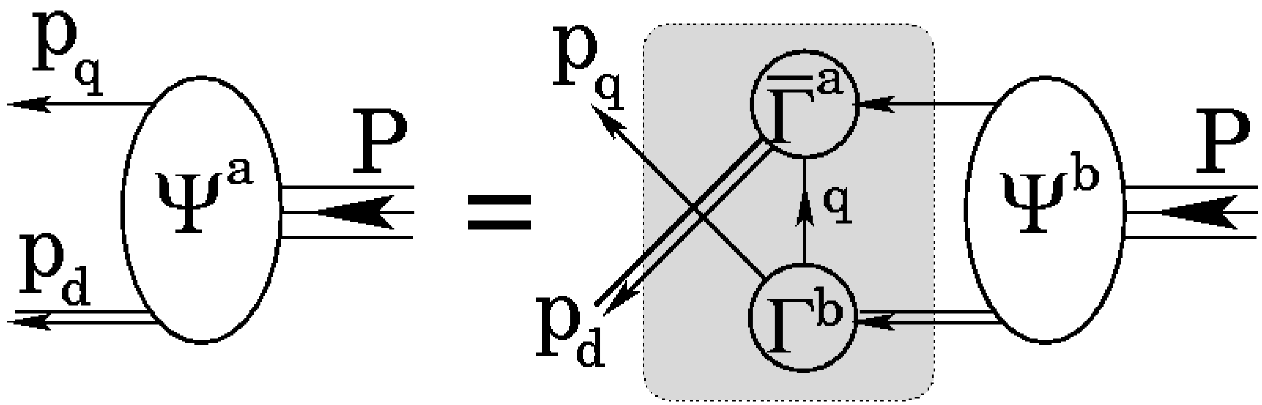
⇝ keep only lightest diquark correlations,

pseudoscalar and axial–vector to describe N and Δ

Covariant Nucleon Faddeev Equation

- covariant Faddeev equation

M. Oettel, L. von Smekal, R. Alkofer Comput. Phys. Commun. **144**, 63 (2002); hep-ph/0109285



Faddeev amplitudes ⟶

⇝ nucleon: boundstate of dressed, confined quarks and nonpointlike confined diquarks

⇝ exchange kernel leads to iterated exchange of diquark and quark

⇝ keep full covariant structure (8 components)

- here focus on the nucleon and the Δ

Need expressions for dressed quantities in Faddeev Equation (quarks, diquarks, diquark Bethe–Salpeter amplitudes)

Dressed Quark Propagator

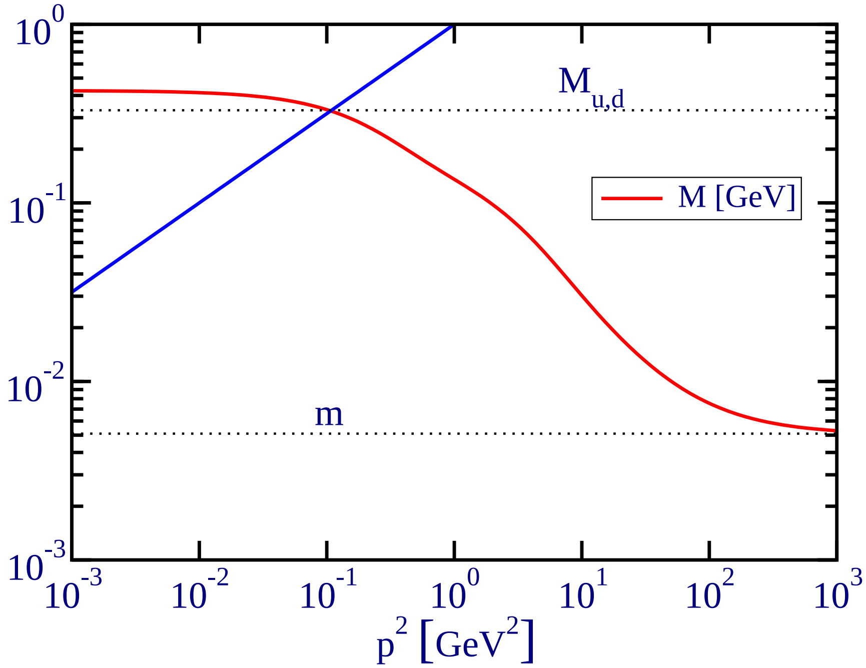
- consistent solution of Gap–equation numerically very demanding

  ⇝ use efficacious parametrization for

  dressed quark propagator $S(p)$ parametrization ⟶

  C.J. Burden, C.D. Roberts, M.J. Thomson, Phys. Lett. B**371**, 163 (1996).

  ⇝ quark propagator fixed to light-meson observables



⇝ current quark mass $m = 5.1$ MeV

⇝ constituent quark mass

$M_{\rm u,d} = 0.33$ GeV

⇝ realization of

dynamical chiral symmetry breaking

⇝ applied in numerous studies of meson properties

C.D. Roberts, S.M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).
R. Alkofer, L. von Smekal, Phys. Rep. **353**, 281 (2001).
M.B. Hecht, C.D. Roberts, M. Oettel, A.W. Thomas, S.M. Schmidt, P.C. Tandy, PRC **65**, 055204 (2002).

Diquark Properties

- diquark propagators (scalar: $J^P = 0^+$ and axial–vector: $J^P = 1^+$)
  M.B. Hecht et al. PRC **65**, 055204 (2002).

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2}\mathcal{F}(K^2/\omega_{0^+}^2) \; ; \quad \mathcal{F}(x) = \frac{1-\mathrm{e}^{-x}}{x}$$

$$\Delta_{\mu\nu}^{1^+}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right)\frac{1}{m_{1^+}^2}\mathcal{F}(K^2/\omega_{1^+}^2)$$

⇝ pole free on timelike axis;

⇝ free–particle–like inside the baryon:

$$\frac{\mathrm{d}}{\mathrm{d}k^2}\Delta_{J^P}^{-1}(k^2)\Big|_{k^2\to 0} = \frac{m_{J^P}^2}{2\omega_{J^P}^2} = 1$$

⇝ $l = 1/m$ propagation length of diquark correlation inside the baryon

- diquark Bethe–Salpeter amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a C i\gamma_5 i\tau_2 \mathcal{F}(k^2/\omega_{0^+}^2)$$

$$t^i\Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a i\gamma_\mu t^i \mathcal{F}(k^2/\omega_{1^+}^2)$$

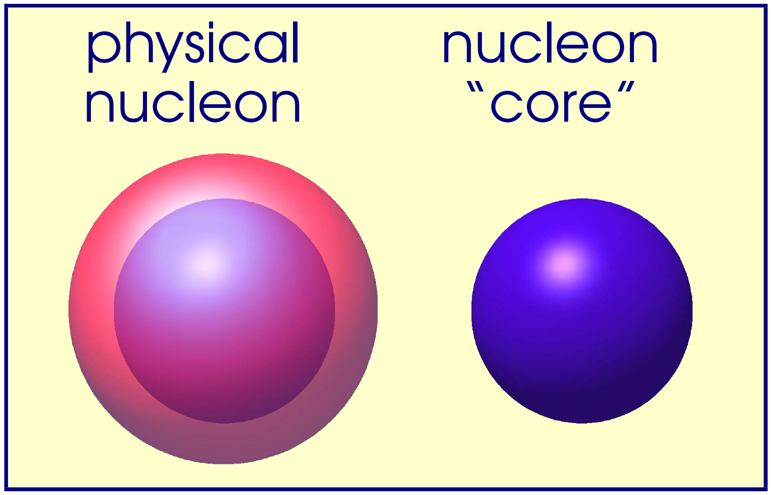
⇝ simple forms

⇝ $\mathcal{N}^{J^P}$: canonical normalization

two free parameters $\mathbf{m_{0^+}}$ and $\mathbf{m_{1^+}}$

Pion Dressing

- 2 open parameters m_{JP} naturally fixed by nucleon (M_N) and Delta mass (M_Δ)



- Which are the correct values for M_N and M_Δ?

- self energy corrections to M_N in the cloudy bag model

  B. C. Pearce and I. R. Afnan, Phys. Rev. **C 34**, 991 (1986).

$$\delta M_N = -300 \text{ to } -400\,\text{MeV}$$

- corrections due to π exchange between quark and diquark

  N. Ishii, Phys. Lett. **B 431**, 1 (1998).

$$\delta M_N = -150 \text{ to } -300\,\text{MeV}$$

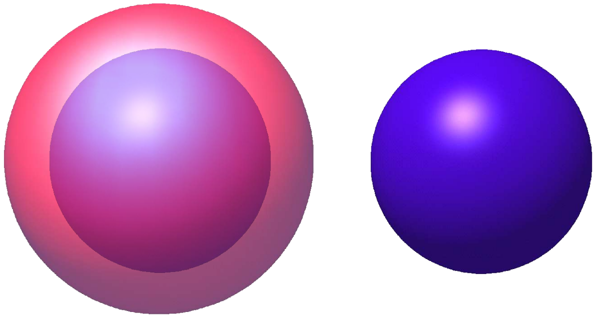
Parameter Fixing

- use two parameter sets **A** and **B**

  set A: fit to experimental masses M_N and M_Δ

  set B: fit to "inflated" masses M_N and M_Δ

  allowing for pion corrections



set	M_N	M_Δ	m_{0^+}	m_{1^+}
A	**0.94**	**1.23**	0.63	0.84
B	**1.18**	**1.33**	0.79	0.89
A*	**1.15**	**—**	0.63	—
B*	**1.46**	**—**	0.79	—

[all units in GeV]

- axial–vector diquark provides significant binding to the nucleon
- extra binding must be matched by "pion cloud"

Nucleon Electromagnetic Form Factors

Nucleon Electromagnetic Current

- nucleon current J_μ parametrized
  by Dirac and Pauli form factor (F_1 and F_2)

$$J_\mu(P',P) = ie\bar{u}(P')\left(\gamma_\mu F_1(Q^2) + \frac{\sigma_{\mu\nu}q_\nu}{2M_N}F_2(Q^2)\right)u(P)$$

- normalization of F_1 and F_2

$$F_1(Q^2=0)=1$$

$$F_2(Q^2=0)=\kappa \quad \text{(nucleon anomalous magnetic moment)}$$

- F_1 and F_2 are related to Sachs form factors G_E and G_M
  R.G. Sachs, Phys. Rev. **126**, 328 (1952).

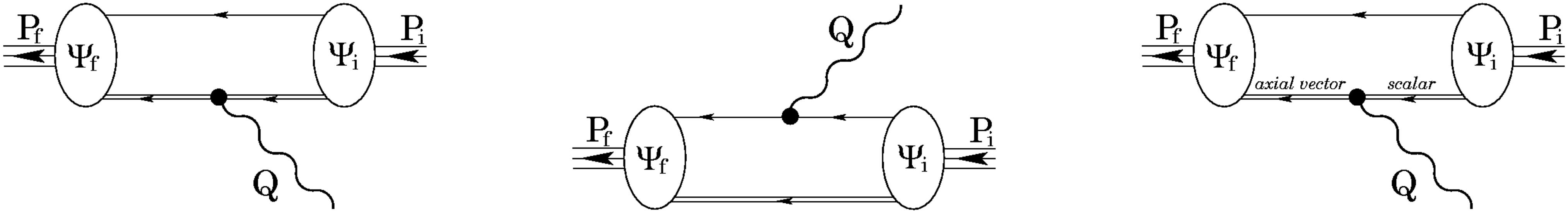
$$G_E(Q^2) = F_1 - \frac{Q^2}{4M_N^2}F_2 \quad ; \quad G_M(Q^2) = F_1 + F_2$$

- normalization: $G_E(0) = 1 \quad ; \quad G_M(0) = \mu = 1+\kappa$

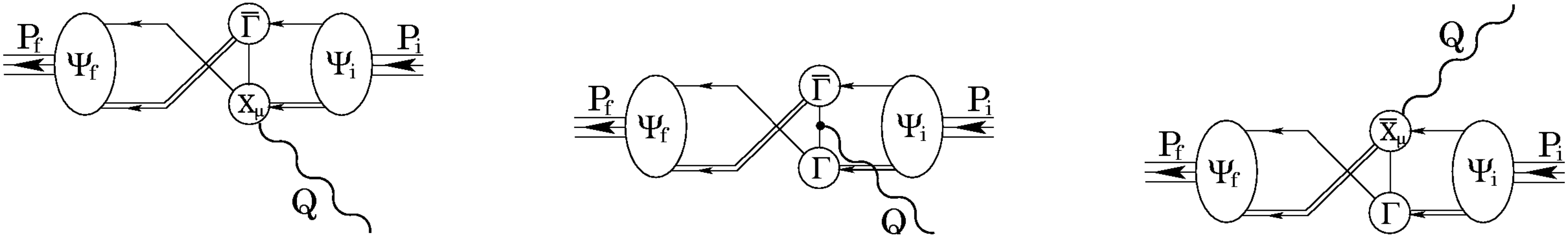
  $\rightsquigarrow$ $G_\mathrm{E}, G_\mathrm{M} \to$ Breit Frame charge and current density

Contributions to the Nucleon Current

1–loop diagrams



2–loop diagrams



- same dressed propagators and BS amplitudes as in Faddeev equation
- boost of Faddeev amplitudes $\Psi_i, \ \Psi_f$ simple due to covariance
- dressed photon vertices, constrained by Ward–Takahashi identity (WTI)

> if **every** dressed photon vertex is constrained by WTI
> $\Longrightarrow$ nucleon current conservation

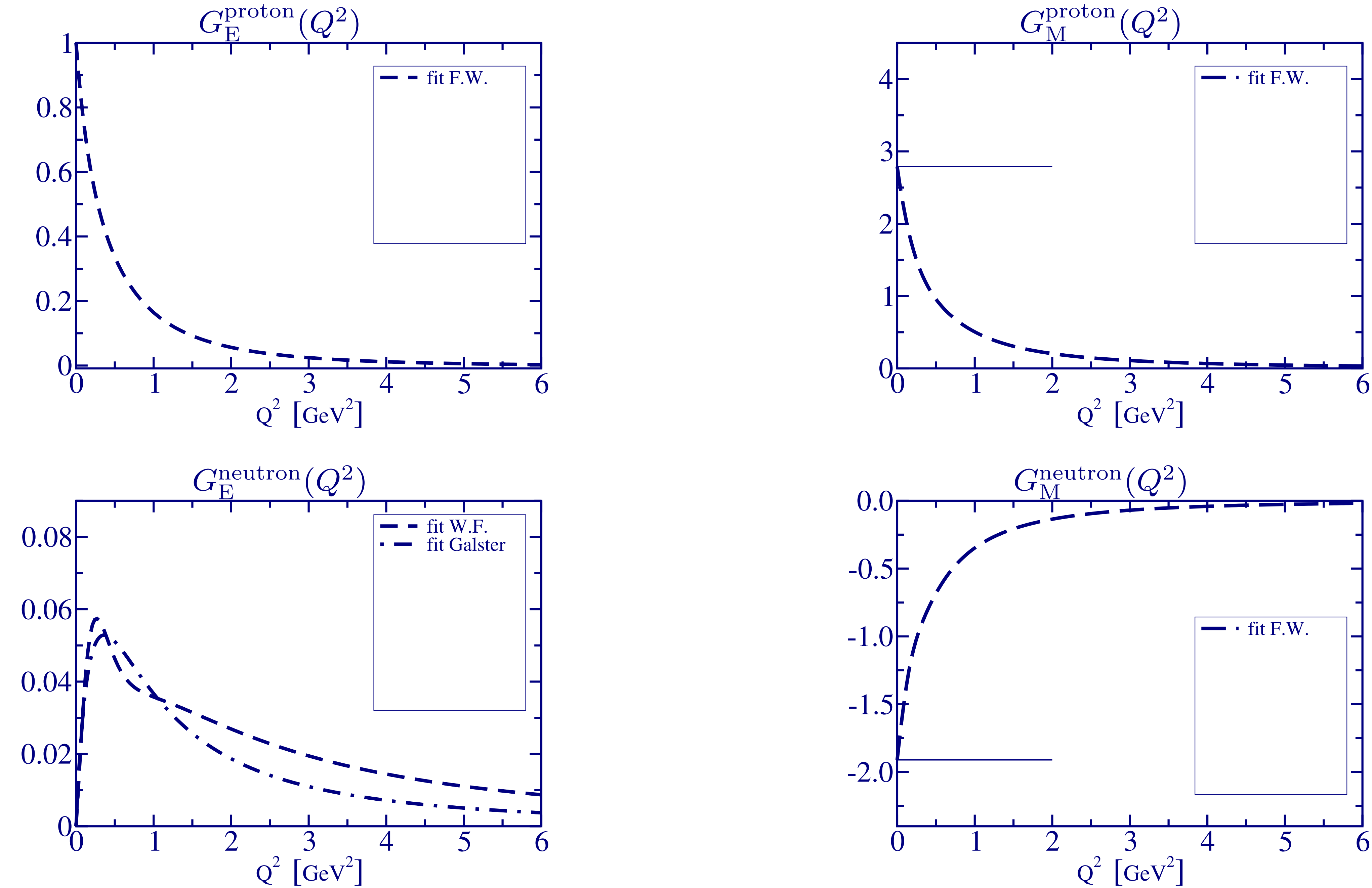
M. Oettel, M.A. Pichowsky, L. von Smekal, Eur. Phys. J. **A 8**, 251 (2000).

Summary Photon Vertices

- all dressed photon vertices constrained by WTI
- reproduce point–like forms for the vertices
- constraints due to asymptotic form
- introduction of three parameters to specify vertices:
  - ⇝ transition strength scalar ↔ axial–vec. diquark ($\kappa_{\mathcal{T}} = 2$)
  - ⇝ static magnetic dipole moment of axial–vec. diquark ($\mu_{1^+} = 2$)
  - ⇝ static electric quadrupole moment of axial–vec. diquark ($\chi_{1^+} = 1$)

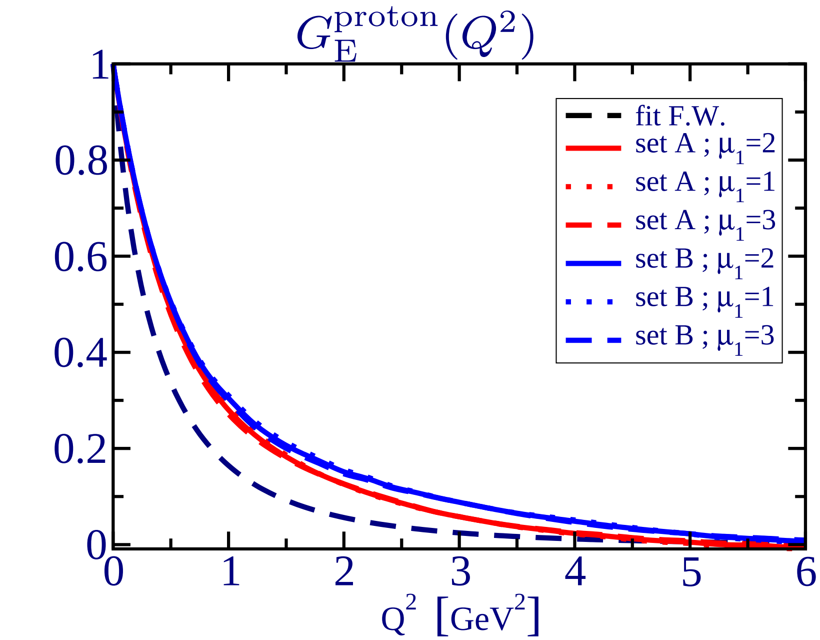
R. Alkofer, A. Höll, M. Kloker, A. Krassnigg, C.D. Roberts, nucl-th/0412046.

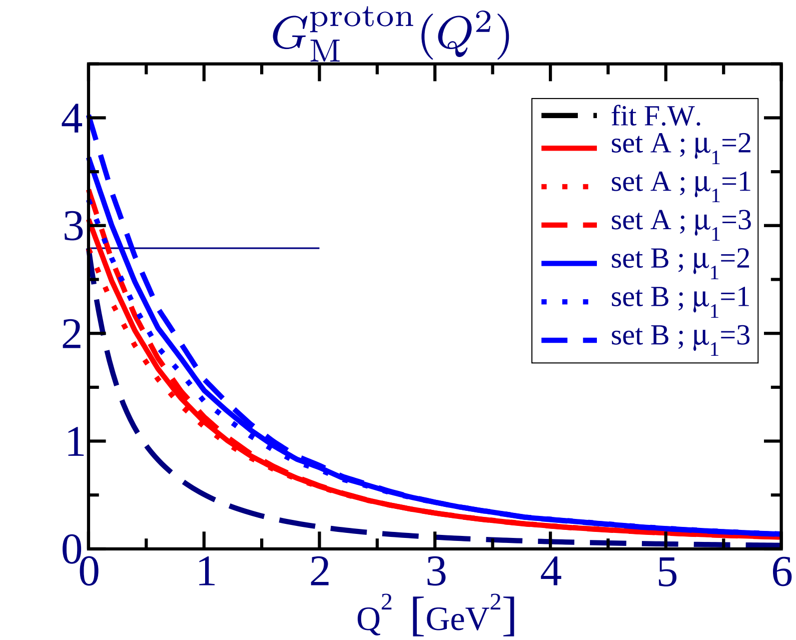
Results: Nucleon Electromagnetic Form Factors



Dashed Curves: phenomenological fit of data: J. Friedrich, Th. Walcher, Eur. Phys. J. **A 17**, 607 (2003).

Results: Nucleon Electromagnetic Form Factors



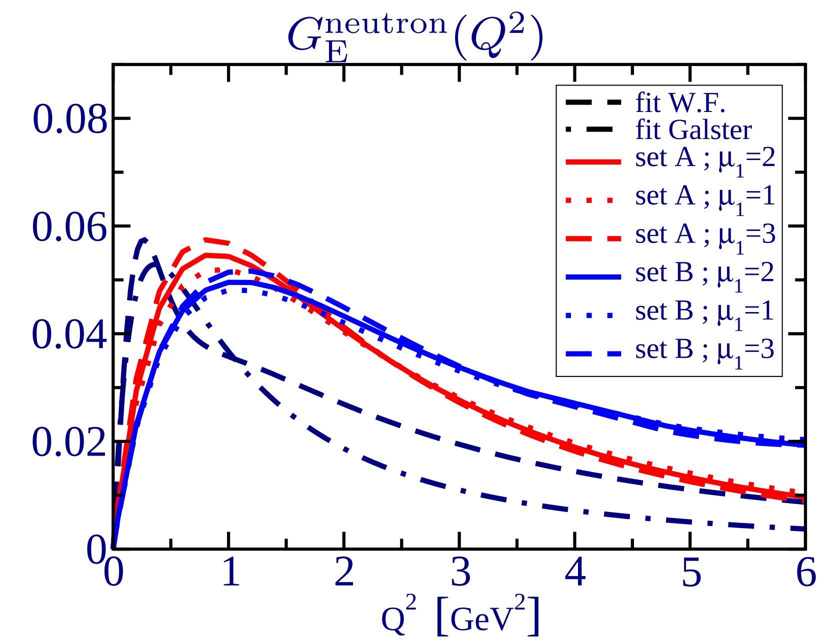


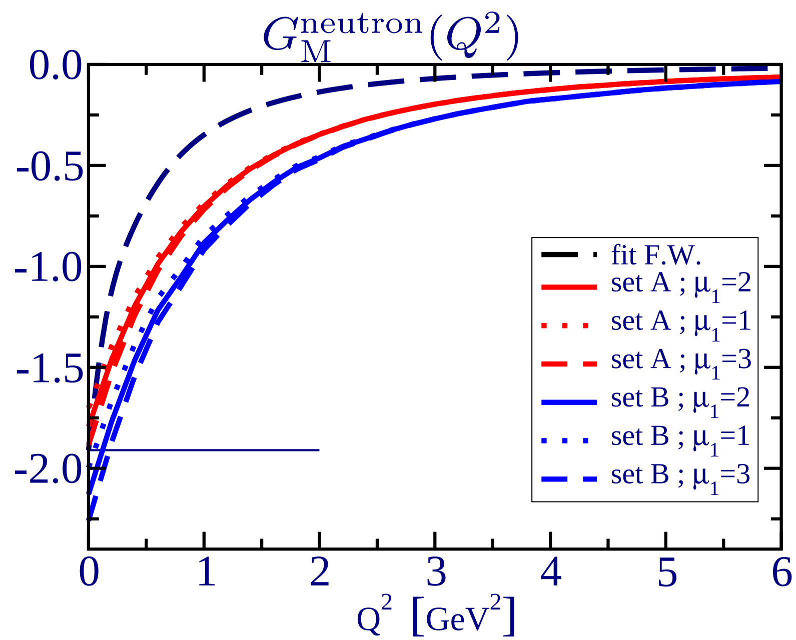
set A

set B

$$\chi_{1+} = 1$$

$$\kappa_{\mathcal{T}} = 2$$





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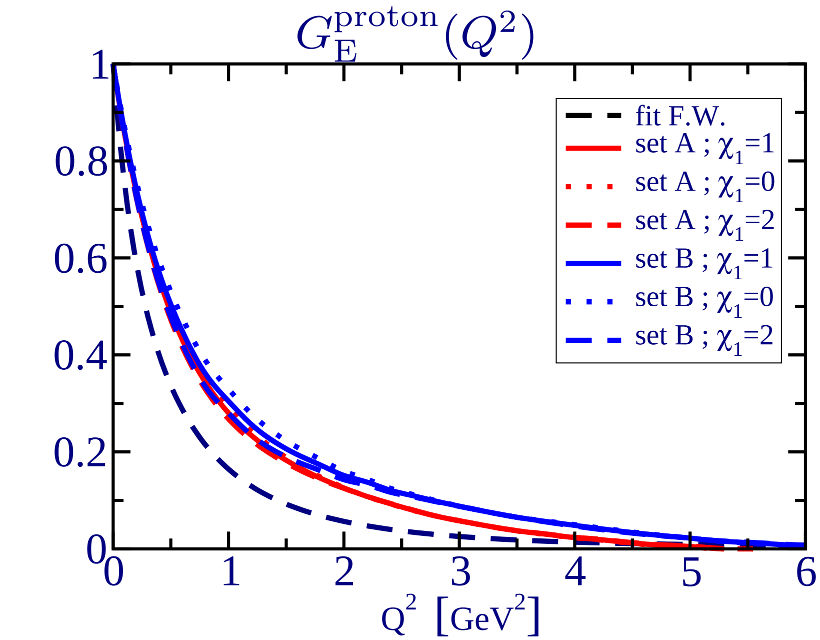
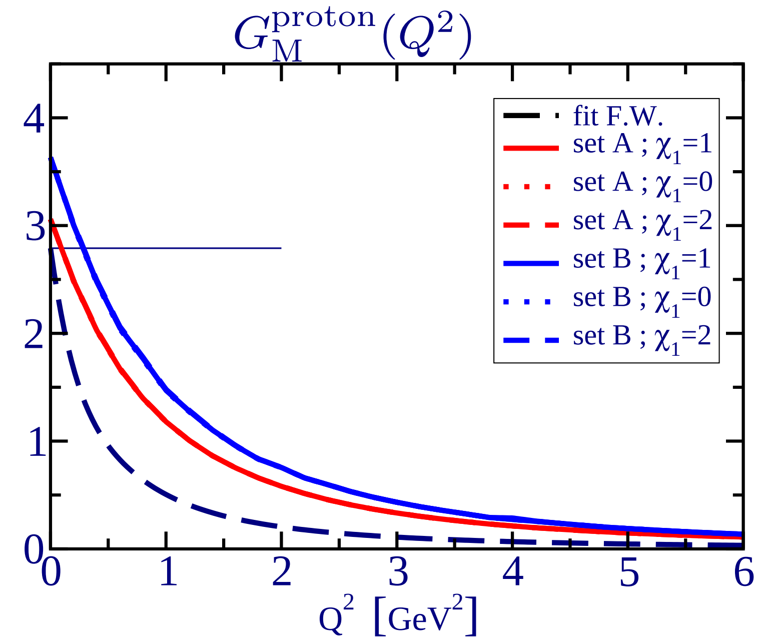


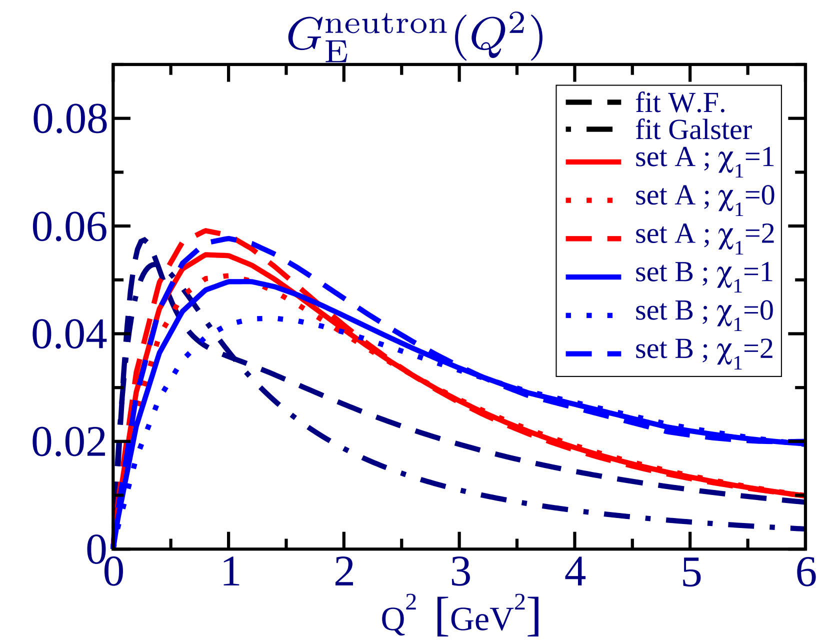
set A

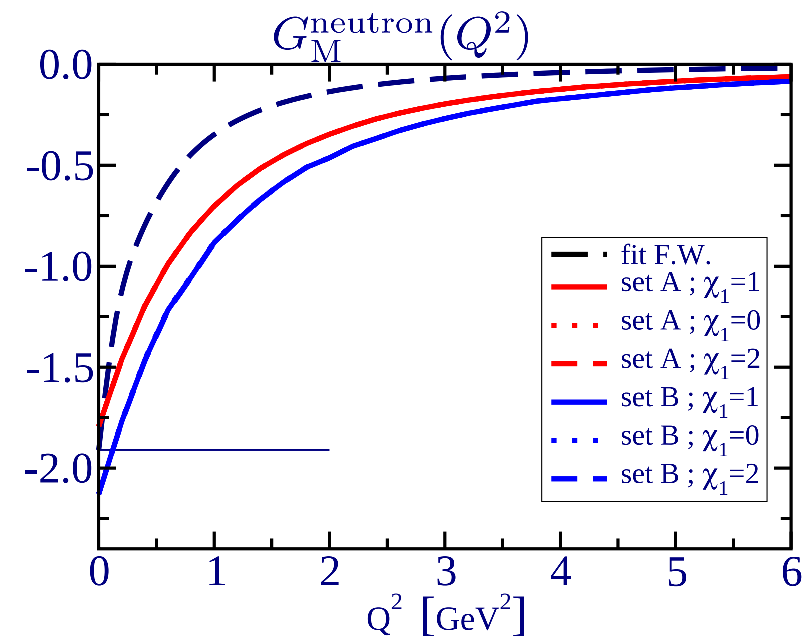
set B

$$\mu_{1+} = 2$$

$$\kappa_T = 2$$







Dashed Curves: phenomenological fit of data: J. Friedrich, Th. Walcher, Eur. Phys. J. **A 17**, 607 (2003).

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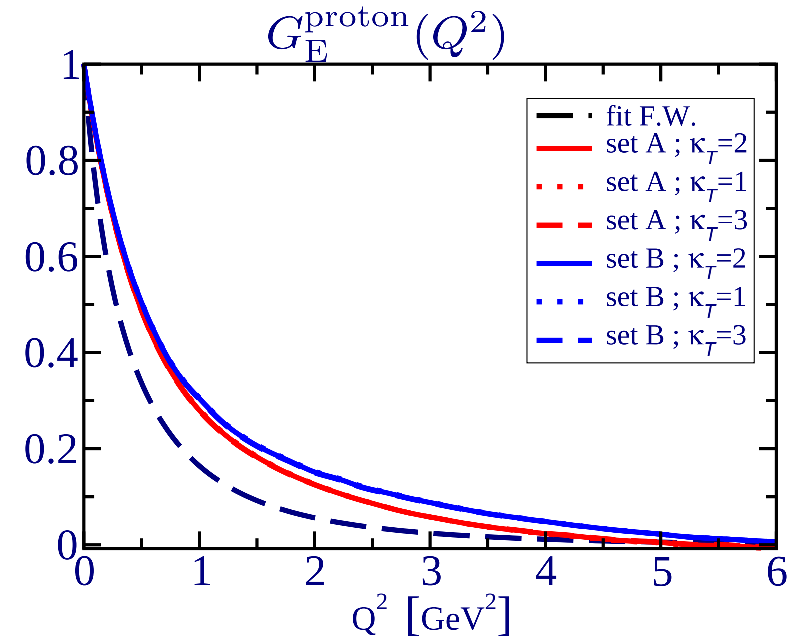
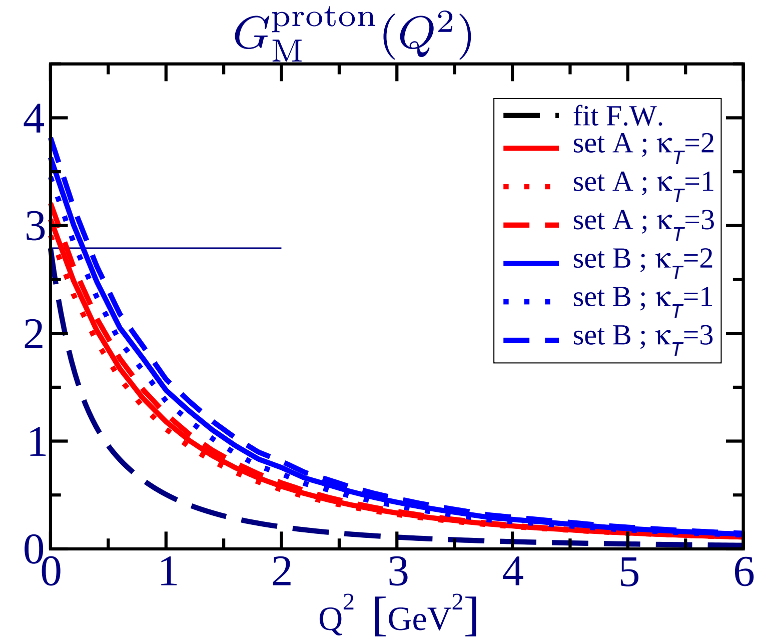
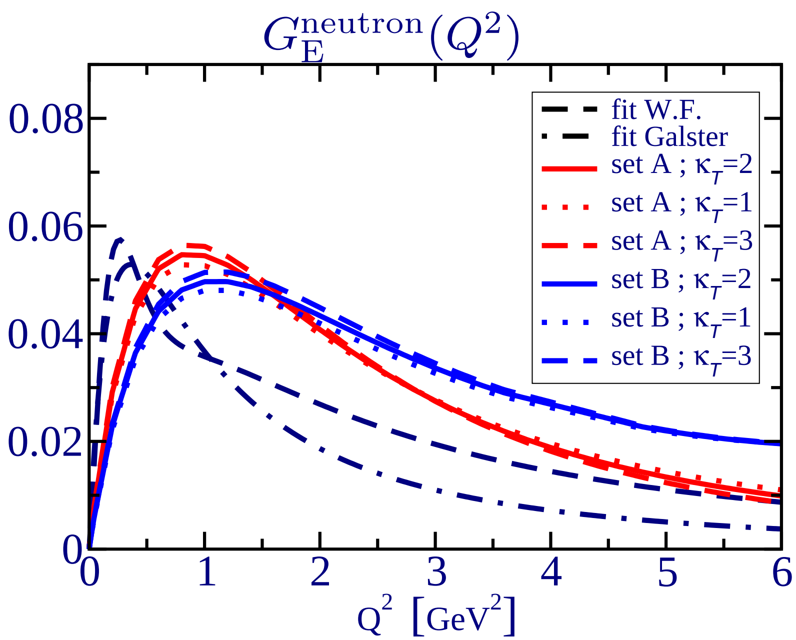


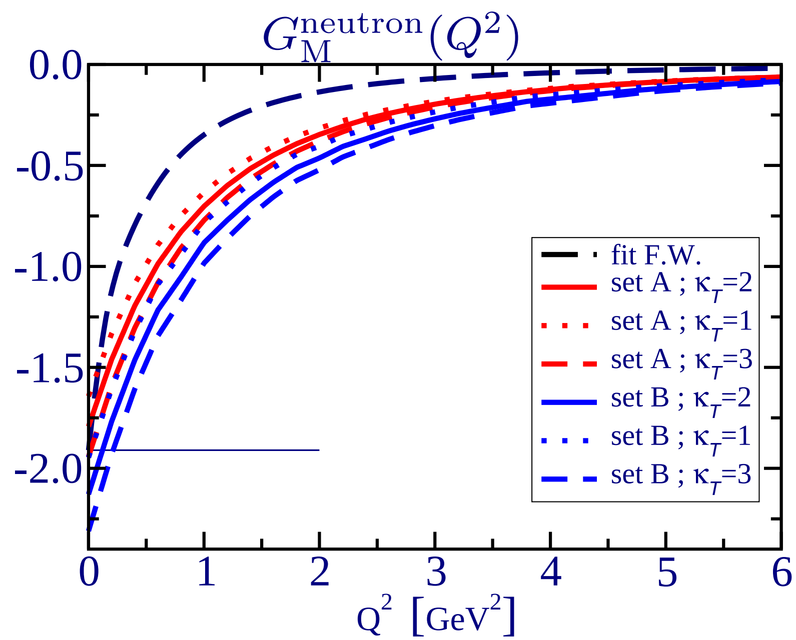
set A

set B

$$\mu_{1+} = 2$$
$$\chi_{1+} = 1$$

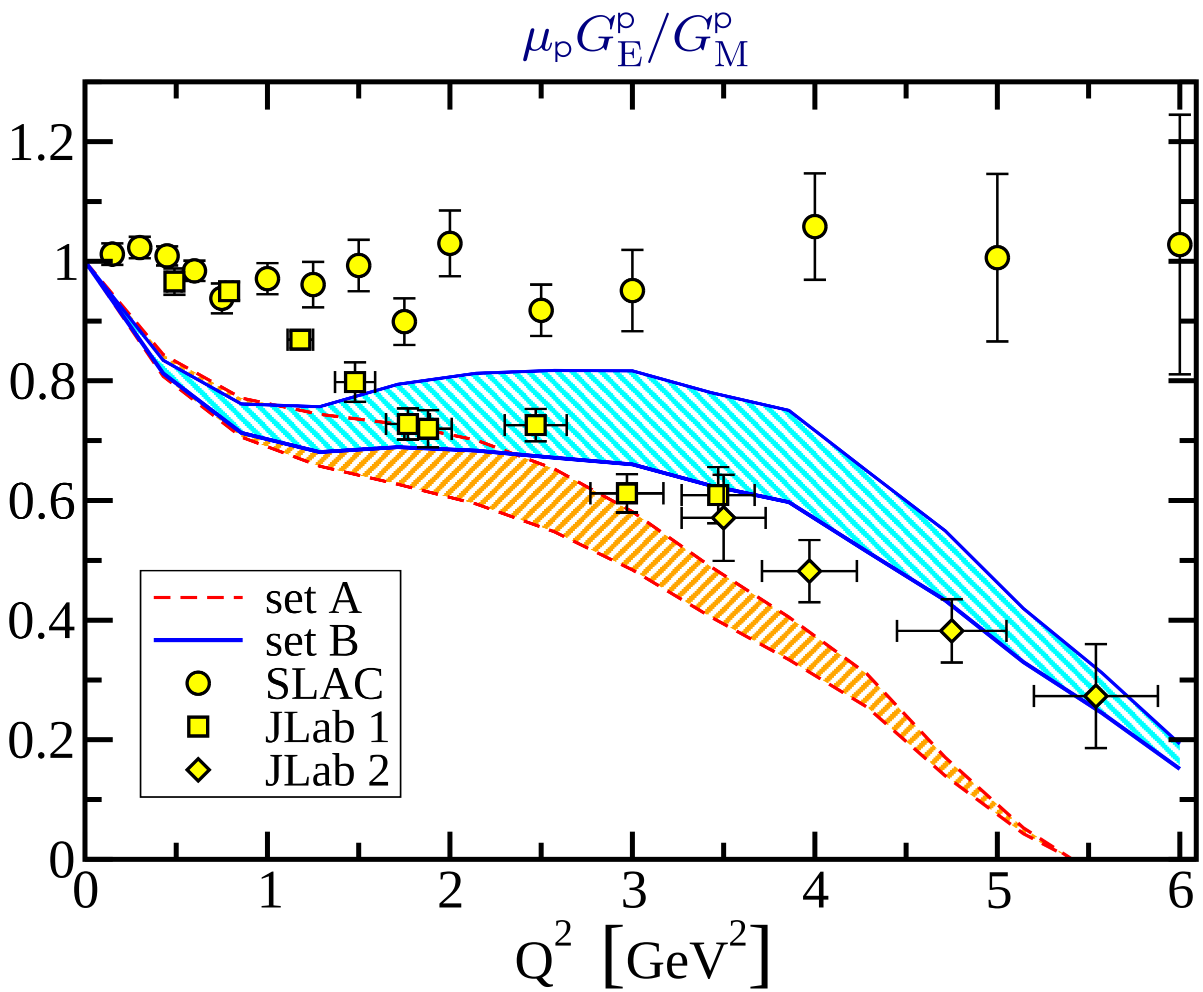






Dashed Curves: phenomenological fit of data: J. Friedrich, Th. Walcher, Eur. Phys. J. **A 17**, 607 (2003).

Results: Proton EM. Form Factor Ratio G_E^p / G_M^p



SLAC : R.C. Walker et al., PRD **49**, 5671 (1994).
JLab 1: M.K. Jones et al., PRL **84**, 1398 (2000).
JLab 2: O.Gayou et al., PRL **88**, 092301 (2002).

small Q^2– behaviour:
$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

⇝ determined by electric and magnetic charge radius r_p and r_p^μ

Results: Proton EM. Form Factor Ratio G_E/G_M

- small Q^2 behaviour: $$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$
- look at proton electric and magnetic charge radius r_p and r_p^μ

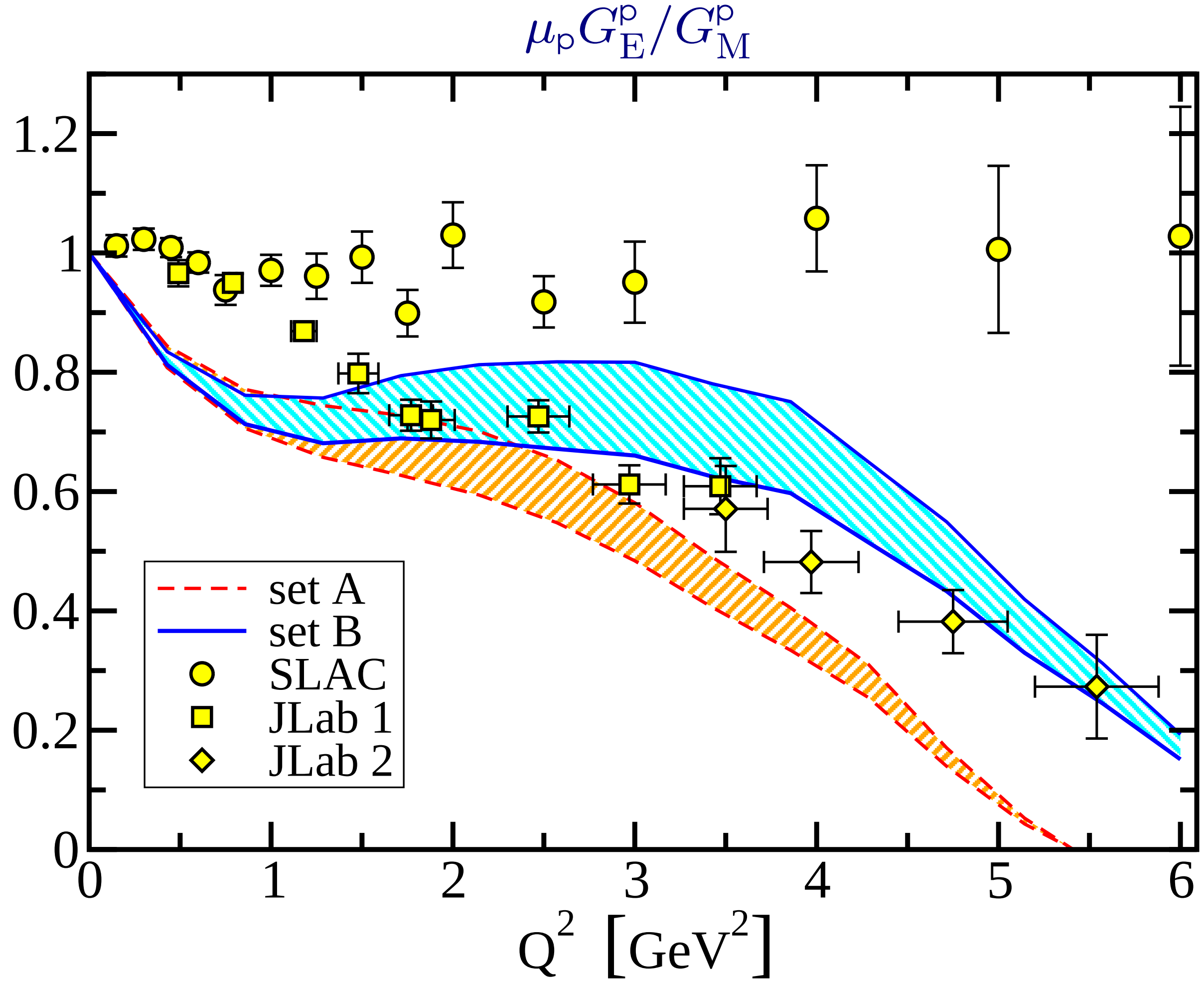
	r_p	r_p^μ
$q-(qq)$ core	0.595	0.449
$+\pi$-loop correction	0.762	0.761
experiment	0.847	0.836

"$q-(qq)$ core": nucleon core with $\mu_{1^+}=2$, $\chi_{1^+}=1$, $\kappa_T=2$

$+\pi$-loop correction: chiral loop corrections (1-loop);
regularisation parameter-dependent part (λ_R)
J.D. Ashley, D.B. Leinweber, A.W. Thomas, R.D. Young, Eur. Phys. J. **A 19**, 9 (2004).
effective for $p^2 < \lambda_R^2 = (0.305\ \text{GeV})^2$ i.e. $R > 0.64$ fm

chiral corrections provide necessary corrections to proton form factor ratio at small Q^2

Results: Proton EM. Form Factor Ratio G_E^p/G_M^p



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JLab 2: O.Gayou et al., PRL **88**, 092301 (2002).

- probing: "pion cloud" at small Q^2 ; "nucleon core" at large Q^2
- zero crossing at $Q^2_{(0)} \approx 5.4\,\mathrm{GeV}^2$ (**set A**) ; $Q^2_{(0)} \approx 6.5\,\mathrm{GeV}^2$ (**set B**) **[1]**
- quenched lattice extrapolations: $Q^2_{(0)} \approx 5.8 \ldots 6.5\,\mathrm{GeV}^2$ **[2]**

[1] A. Höll, R. Alkofer, M. Kloker, A. Krassnigg, C.D. Roberts, S.V. Wright, nucl-th/0501033.
[2] H.H. Matevosyan, G.A. Miller, A.W. Thomas, arXiv:nul-th/0501044.

Strong and Weak Nucleon Form Factors

Pseudoscalar and Pseudovector Current

- pseudoscalar current J_5^a

$$J_5^a(P_f, P_i) = i g_{\pi NN}(Q^2) \bar{u}(P_f) \tau^a \gamma_5 u(P_i)$$

- pseudovector current $J_5^{a,\mu}$

$$J_5^{a,\mu}(P_f, P_i) = \bar{u}(P_f) \frac{\tau^a}{2} \left[i\gamma^\mu \gamma_5 g_A(Q^2) + Q^\mu \gamma_5 g_P(Q^2) \right] u(P_i)$$

- need to construct conserved pseudoscalar and pseudovector current
  - ⇝ follow electromagnetic form factor calculation
  - ⇝ construction of currents constraint by WTI
  - ⇝ restrict to chiral limit here (for simplicity)

- current conservation relates form factors in the soft limit

$$g_A(0) = f_\pi \frac{g_{\pi NN}(0)}{M_n} \qquad \text{(Goldberger-Treiman relation)}$$

Note: no chiral seagull terms included so far

⇝ currents not conserved exactly

Pseudoscalar and Pseudovector Quark Vertices

for details see: J.C.R. Bloch, C.D. Roberts, S.M. Schmidt, Phys. Rev. C**61**, 065207 (2000).

pseudoscalar vertex:

- use leading amplitude of pion Bethe–Salpeter amplitude (BSA) Γ_π^a

$$\Gamma_\pi^a(k;Q) = i\tau^a\gamma_5 E_\pi(k;Q) + \dots$$

- chiral limit: $E_\pi(k;Q=0) = B_0(k^2)/\mathcal{N}_\pi^0$
- model for off–shell BSA:

$$\Gamma_\pi^a(k;Q) = i\tau^a\gamma_5 \frac{B_0(k+Q/2)+B_0(k-Q/2)}{2\mathcal{N}_\pi^0}$$

pseudovector vertex:

- WTI preserving dressed quark–axial–vector vertex

$$\Gamma_5^{a,\mu}(k;Q) = \gamma_5 \frac{\tau^a}{2}\left[\gamma^\mu \Sigma_A(k_+^2,k_-^2) + 2k^\mu \not{k}\Delta_A(k_+^2,k_-^2) + 2i\frac{Q^\mu}{Q^2}\Sigma_B(k_+^2,k_-^2)\right]$$

with $\Sigma_f(p^2,q^2) = \dfrac{f(p^2)+f(q^2)}{2}$, $\Delta_f = \dfrac{f(p^2)-f(q^2)}{p^2-q^2}$, $k_\pm = k \pm \dfrac{Q}{2}$

⇝ same vertices used for quark exchange contribution

Pseudoscalar and Pseudovector Diquark Vertices

details given in: M. Oettel, R. Alkofer, L. von Smekal, Eur. Phys. J. A**8**, 553 (2000).

- model diquark vertices:

  ⇝ Lorentz structure from quark loop and effective strength

axial–vector–diquark vertices

$$\Gamma^{\alpha\beta}_{5,ax}(p',p) = \frac{\kappa^{\mathbf{5}}_{\mathbf{ax}}}{2M_N}\frac{m_q}{f_\pi}\epsilon^{\alpha\beta\mu\nu}(p'+p)^\mu Q^\nu$$

$$\Gamma^{\mu\alpha\beta}_{5,ax}(p',p) = \frac{\kappa^{\mathbf{5}}_{\mu,\mathbf{ax}}}{2}\epsilon^{\mu\alpha\beta\nu}(p'+p)^\mu + 2f_\pi\frac{Q^\mu}{Q^2}\Gamma^{\alpha\beta}_{5,ax}(p',p)$$

scalar to axial–vector transition vertices

$$\Gamma^{\beta}_{5,sa}(p',p) = -i\kappa^{\mathbf{5}}_{\mathbf{sa}}\frac{m_q}{f_\pi}Q^\beta$$

$$\Gamma^{\mu\beta}_{5,sa}(p',p) = iM_N\kappa^{\mathbf{5}}_{\mu,\mathbf{sa}}\delta^{\mu\beta} + 2f_\pi\frac{Q^\mu}{Q^2}\Gamma^{\beta}_{5,sa}(p',p)$$

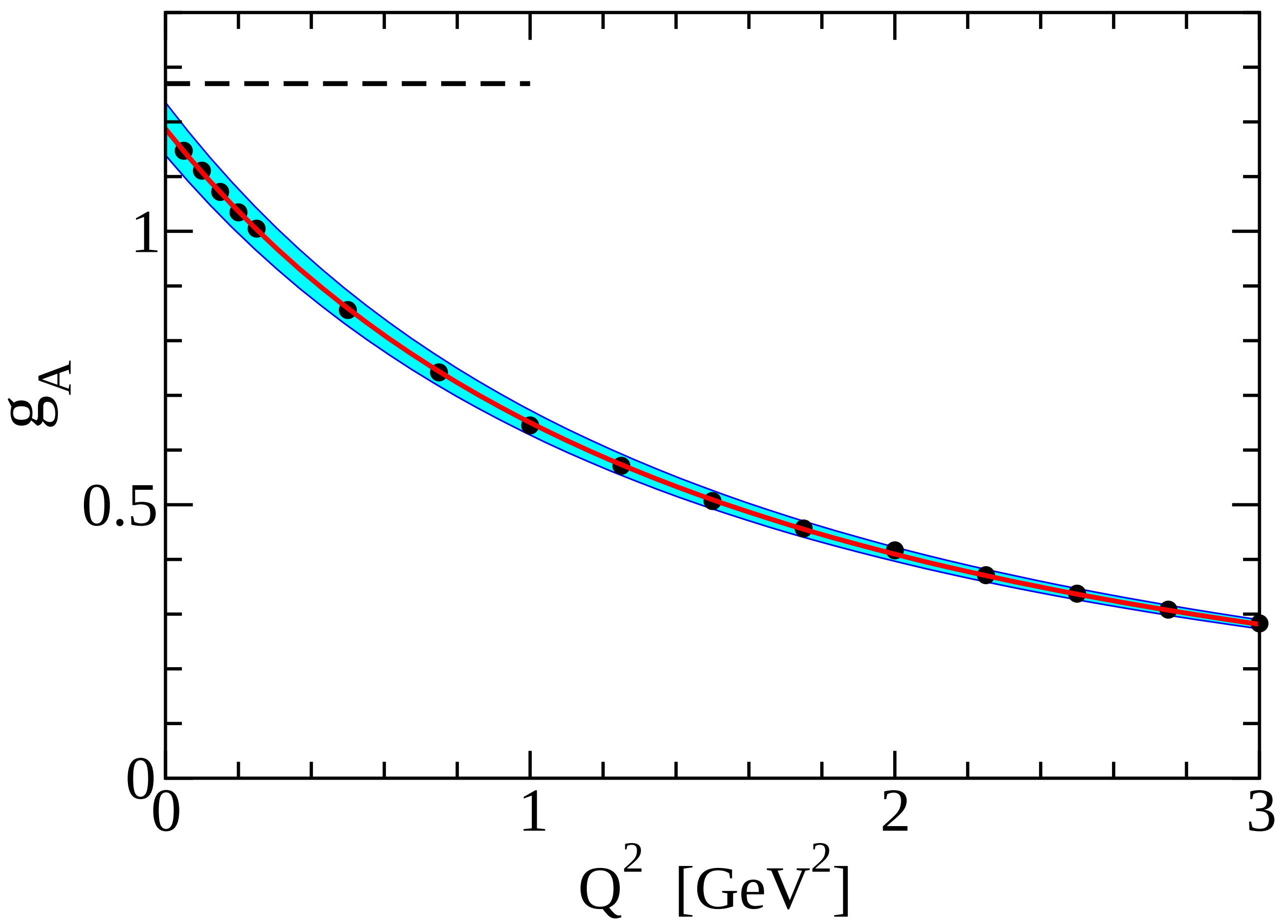
⇝ 4 parameters specify details of the vertices (vary parameters by $\pm 10\%$)

⇝ at $Q^2 > 0$ more general structure possible

Axial–Vector Nucleon Form Factor g_A

- axial–vector nucleon current $J_a^{5\mu}$

$$J_a^{5\mu}(P',P) = \bar{u}(P')\gamma_5 \frac{\tau^a}{2} \left[i\gamma^\mu g_A(Q^2) + Q^\mu g_P(Q^2) \right] u(P)$$



- Faddeev calculation

— dipole fit
mass $m = 1.69$ GeV

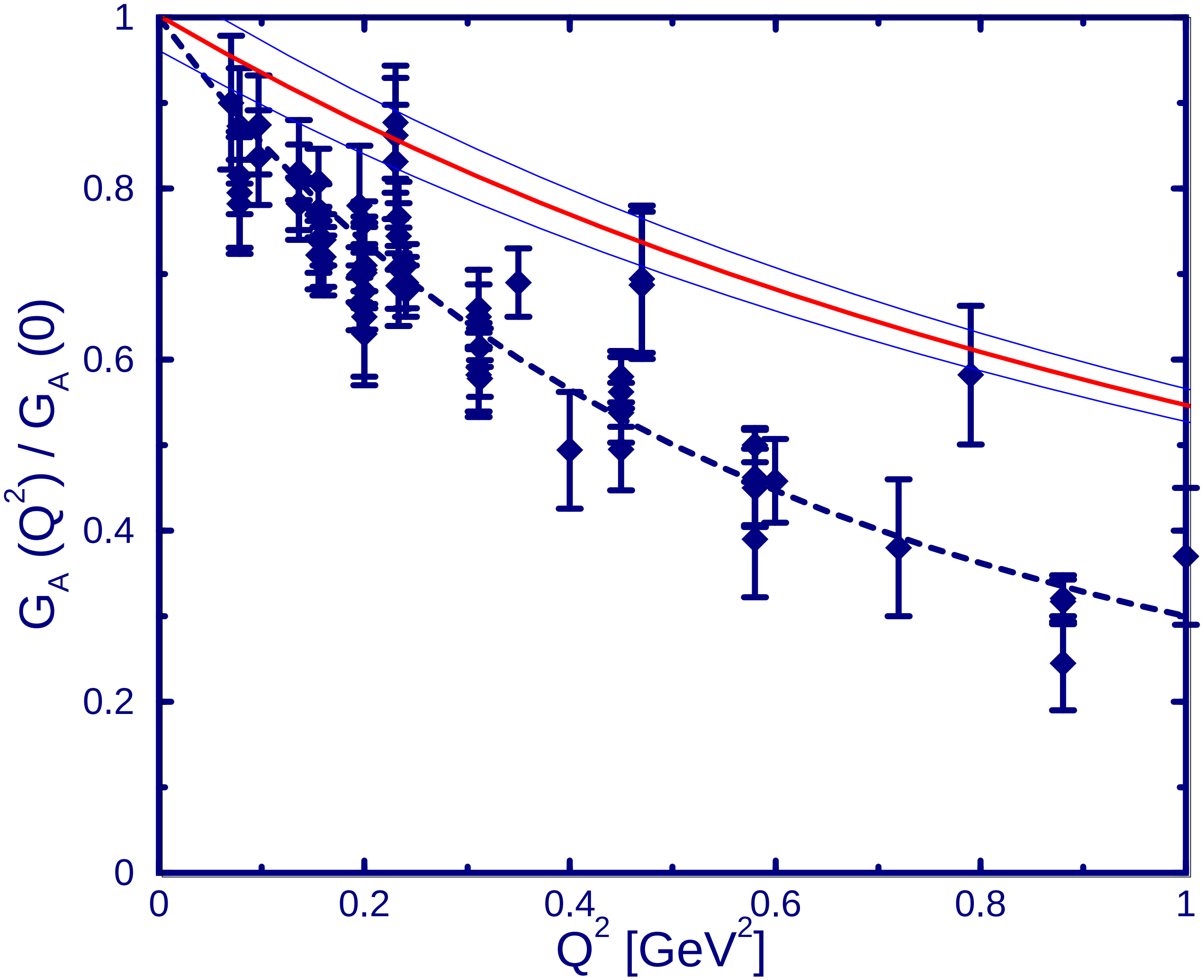
parametric dependence on diquark vertices

dipole mass:
$m = 1.68 \ldots 1.70$ GeV

experiments **(1)**
neutrino scattering:
$m = (1.026 \pm 0.021)$ GeV
pion electroproduction:
$m = (1.069 \pm 0.016)$ GeV

(1) see: V. Bernard, L. Elouadrhiri, U.-G. Meissner, J. Phys. G **28**, R1 (2002).

Axial–Vector Nucleon Form Factor g_A



g_A given by point coupling

Q^2-behaviour governed by Faddeev amplitudes (similar to e.m. form factors)

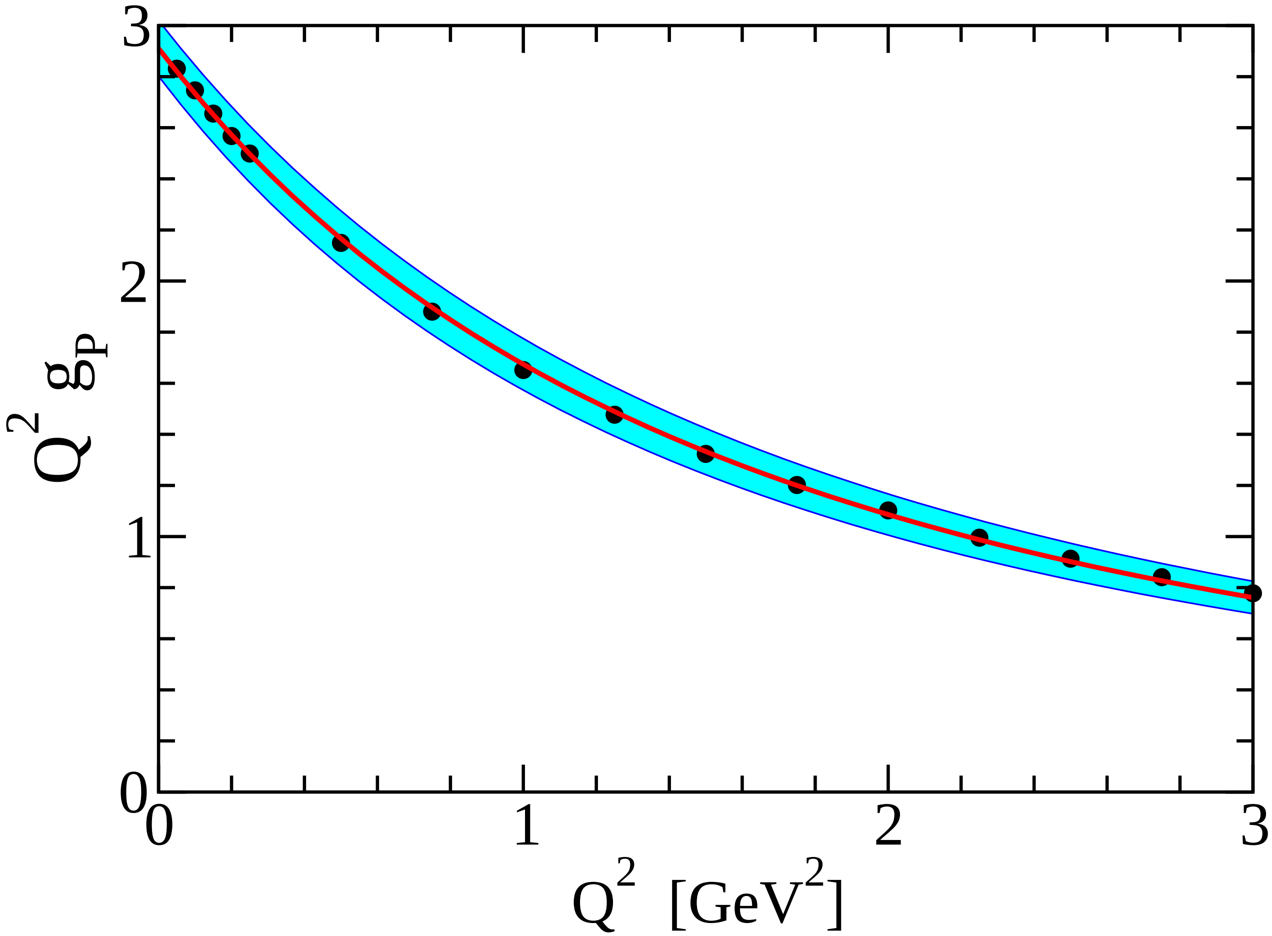
g_A from pion electroproduction in the threshold region ($m = 1.1$ GeV dashed curve)

figure published in: V. Bernard, L. Elouadrhiri, U.-G. Meissner, J. Phys. G **28**, R1 (2002).
overlayed with Faddeev results (colored curves)

Induced Pseudoscalar Nucleon Form Factor g_P

- axial–vector nucleon current $J_a^{5\mu}$

$$J_a^{5\mu}(P', P) = \bar{u}(P')\gamma_5 \frac{\tau^a}{2} \left[i\gamma^\mu g_A(Q^2) + Q^\mu g_P(Q^2) \right] u(P)$$



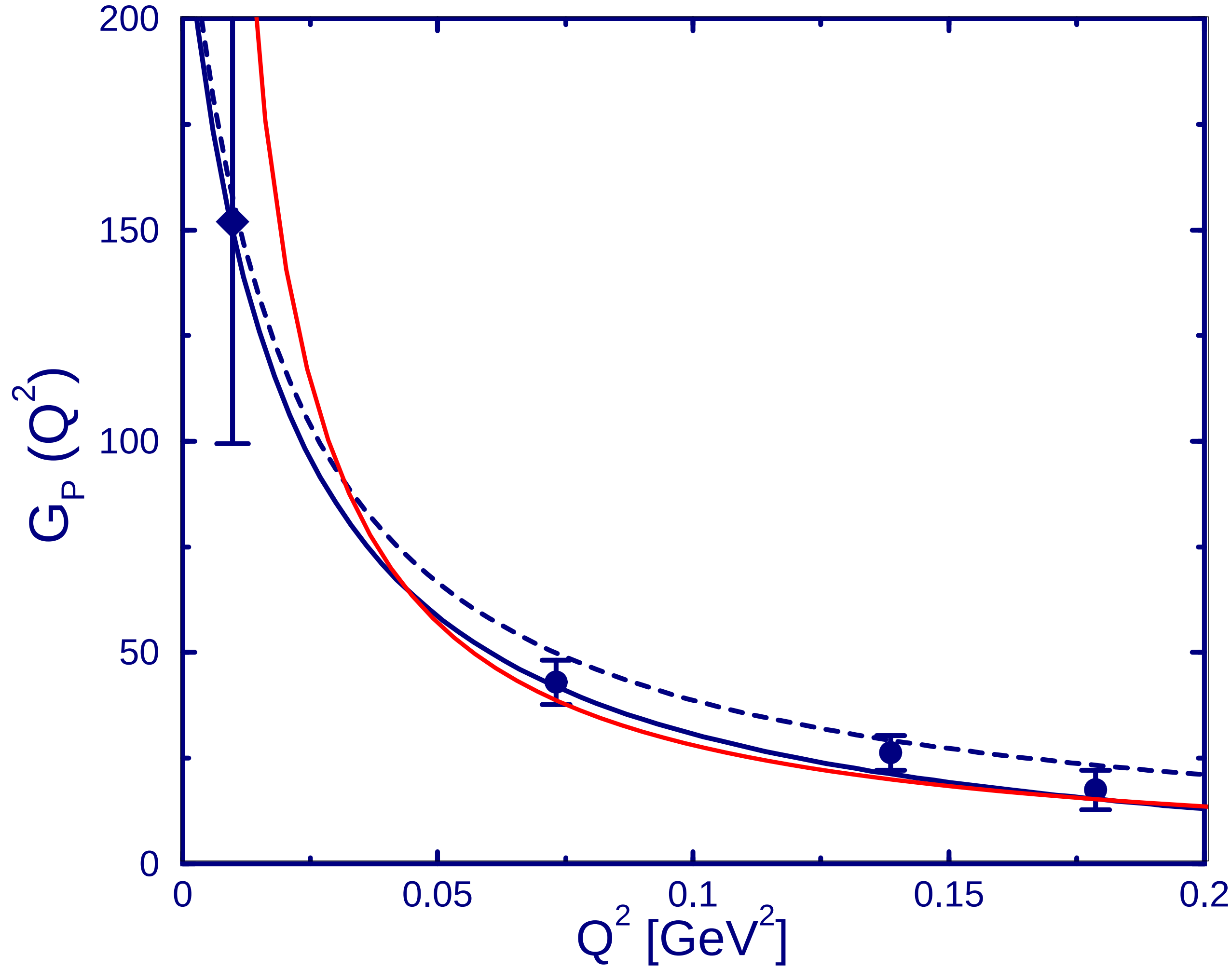
- Faddeev calculation

— dipole fit
mass $m = 1.77$ GeV

parametric dependence on diquark vertices

dipole mass:
$m = 1.72 ... 1.82$ GeV

Axial–Vector Nucleon Form Factor g_A



coupling to non–pointlike pion

Q^2–dependence governed by pion pole contribution at $Q^2 = 0$

NOTE:
pion pole is shifted to $Q^2 = -m_\pi^2$ away from the chiral limit

g_P from pion electroproduction (solid circles);

average for ordinary muon capture at $Q^2 = 0.88\ M_\mu^2$ (solid diamond);

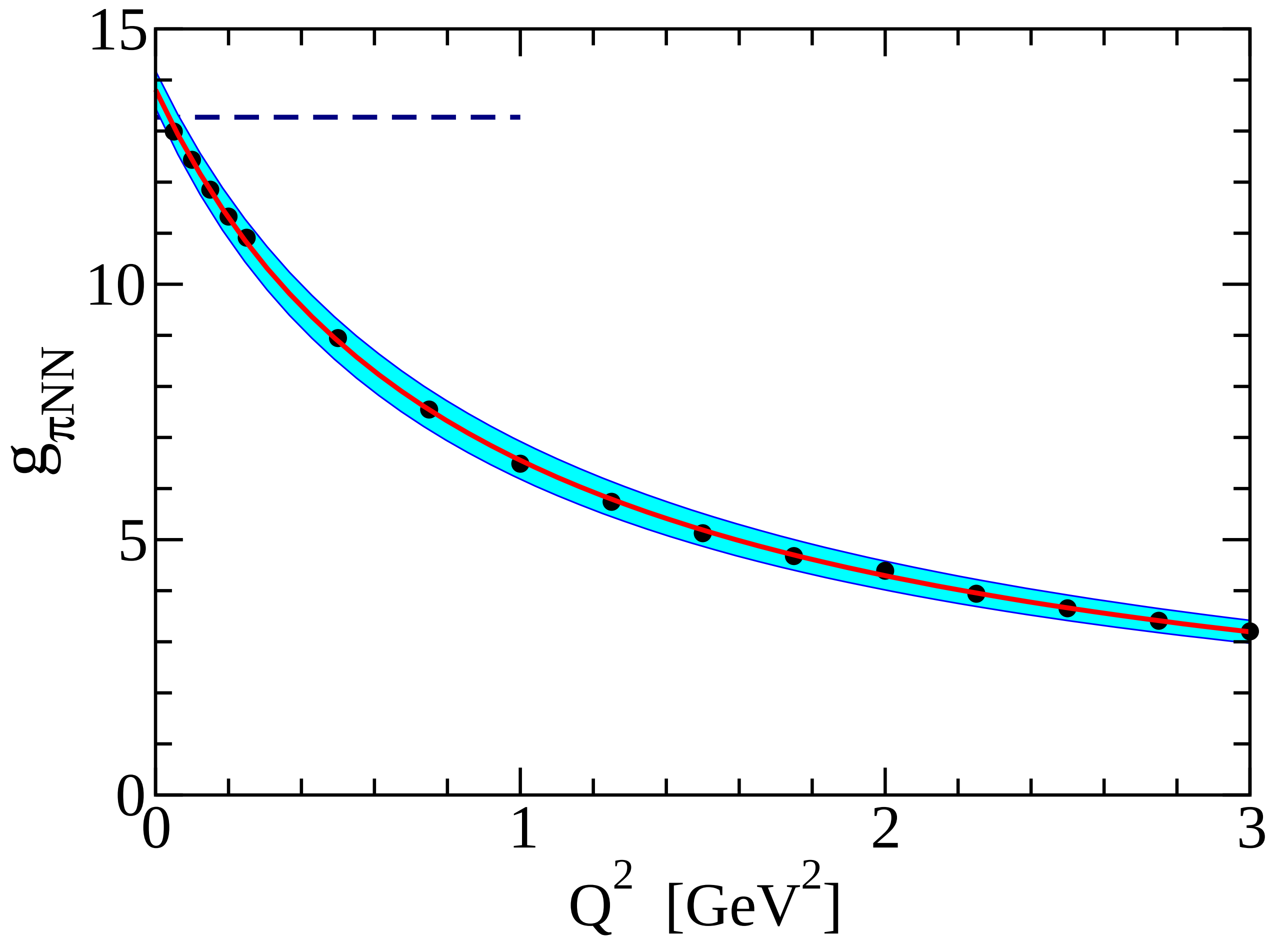
current algebra (dashed line); NLO chiral perturbation theory (solid line)

figure published in: V. Bernard, L. Elouadrhiri, U.-G. Meissner, J. Phys. G **28**, R1 (2002).
overlayed with Faddeev results (colored curves)

Pion–Nucleon Form Factor $g_{\pi NN}$

- pion–nucleon current J_a^5

$$J_a^5(P', P) = i g_{\pi NN}(Q^2)\bar{u}(P')\gamma_5 \tau^a u(P)$$



- Faddeev calculation

monopole fit
mass $m = 0.95$ GeV

parametric dependence
on diquark vertices

monopole mass:
$m = 0.92 \ldots 0.98$ GeV

pionic radius of nucleon:
$r_{\pi NN} = 0.49 \ldots 0.52$ fm
$r_{\pi NN} \sim 0.3$ fm **(1)**

(1) inferred from R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

Summary

Results: internal consistent picture of nucleon em. structure

⇝ "physical nucleon" = "nucleon core" + "pion cloud"

⇝ large Q^2: good "nucleon core" description $M_N = 1.18\,\text{GeV}$ $M_\Delta = 1.46\,\text{GeV}$

⇝ small Q^2: provides room for missing "pion cloud"

first results for strong and weak nucleon form factors

A. Höll, R. Alkofer, M. Kloker, A. Krassnigg, C.D. Roberts, S.V. Wright, nucl-th/0501033.
R. Alkofer, A. Höll, M. Kloker, A. Krassnigg, C.D. Roberts, nucl-th/0412046.

- solution of Poincaré covariant Faddeev equation
  - ⇝ kernel adjusted to nucleon and Δ mass (no free parameters)
- calculation of nucleon form factors
  - ⇝ use realistic covariant Faddeev amplitudes
  - ⇝ all model vertices well constrained (WTI, asymptotics, ...)
  - ⇝ Q^2 dependence of strong and weak f.f. needs to be improved

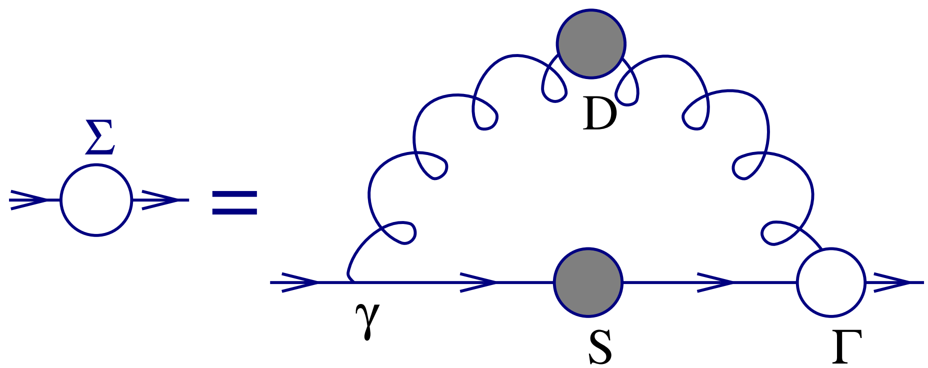
Additional Material

The QCD Gap Equation

- Gap Equation determines dressing of quark 2–point function $S(p)$

$$S_0(p) = \frac{1}{i\gamma \cdot p + m_0} \xrightarrow{\text{dressing}} \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} = S(p)$$

- dressing functions Z and M
  given by self energy Σ:



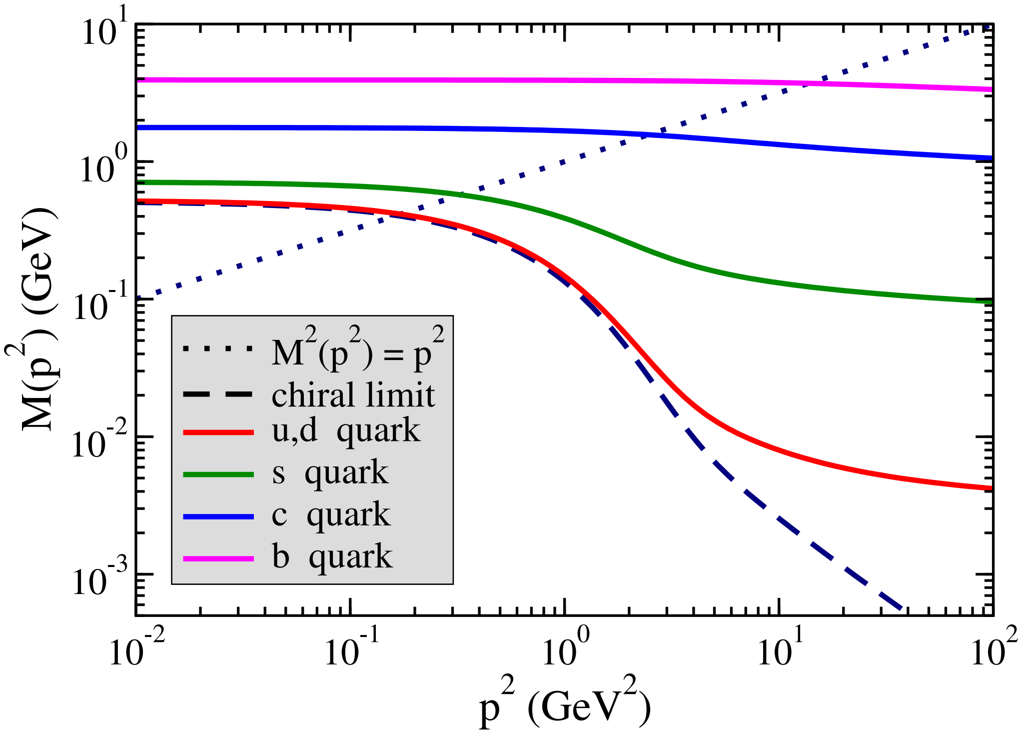
- mass function M in perturbation theory

$$M(p^2) = m_0 \left(1 - \frac{3\alpha}{4\pi} \ln\left[\frac{p^2}{m_0^2}\right] + \mathcal{O}(\alpha^2)\right)$$

perturbatively **no** dynamical chiral symmetry breaking
in the chiral limit $m_0 \longrightarrow 0$

Nonperturbative Mass Function $M(p^2)$

- nonerturbative solution of **QCD** Gap equation
  P. Maris, C. D. Roberts, nucl-th/9708029.
  P. Maris, C. D. Roberts, nucl-th/9710062.

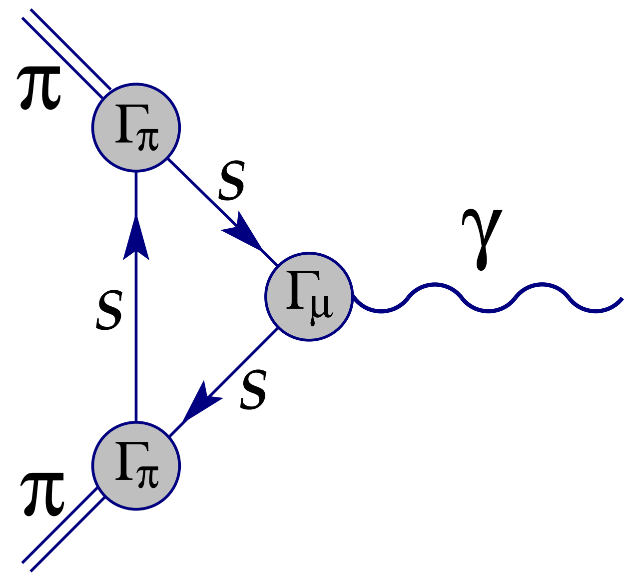


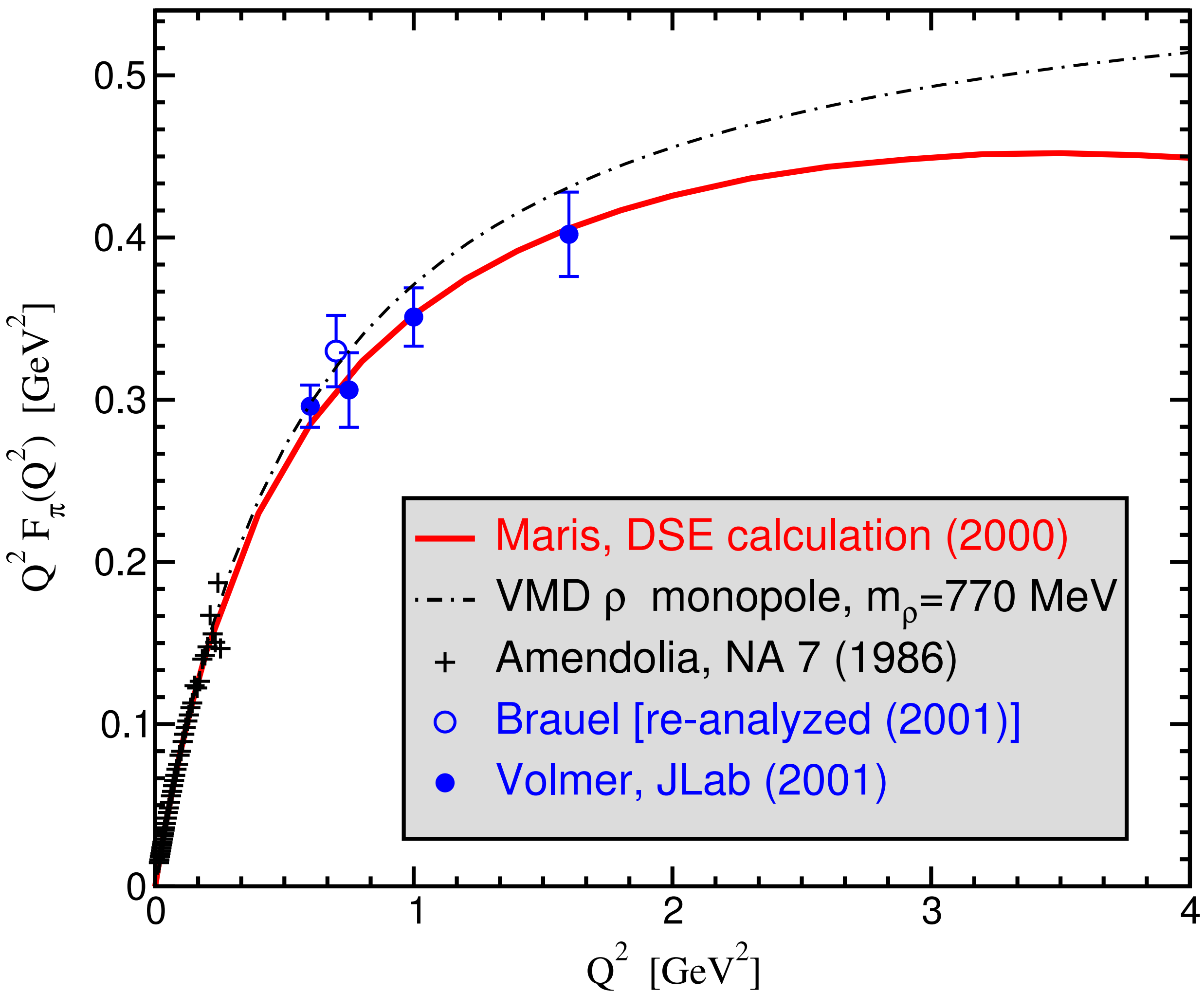
flavor	**u,d**	**s**	**c**	**b**	**t**
M_E/m_0	$\approx \mathbf{100}$	$\approx \mathbf{10}$	$\mathbf{1.2\ldots2.2}$	$\mathbf{1.1\ldots1.2}$	$\to \mathbf{1}$

chiral symmetry and its dynamical breaking
is an **important** property for light quarks

Some Results • Pion Form Factor

- pion electromagnetic form factor
  P. Maris, P.C. Tandy, PRC**61**, 045202 (1999); [nucl-th/9910033].
  P. Maris, P.C. Tandy, PRC**62**, 055205 (2000); [nucl-th/0005015].
  - ⋆ solution of gap–equation, pion BSE, vector BSE (ρ–pole shows up in Γ_μ)





⇝ calculated DSE–results published in 2000

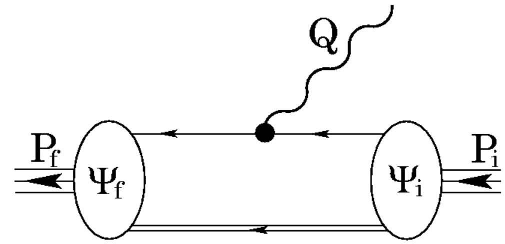
⇝ experimentally confirmed in 2001

Dressed Photon Vertices

- quark–photon vertex: Ball Chiu *Ansatz* ⟶

  J.S. Ball, T.-W. Chiu, Phys. Rev. **D 22**, 2542 (1980).

  ⇝ widely used in meson studies, fulfils WTI

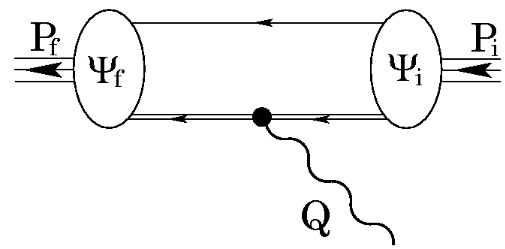


- scalar diquark–photon vertex:

$$\Gamma^{\mu}_{0+}(p',p) = q_{0+}(p+p')^{\mu}\,\frac{\Delta^{-1}_{0+}(p'^2)-\Delta^{-1}_{0+}(p^2)}{p'^2-p^2}$$

  ⇝ well defined limit for elastic scattering ($p'^2 \to p^2$)

  ⇝ WTI fulfilled: $Q^{\mu}\Gamma^{\mu}_{0+} = q_{0+}\left(\Delta^{-1}_{0+}(p'^2)-\Delta^{-1}_{0+}(p^2)\right)$

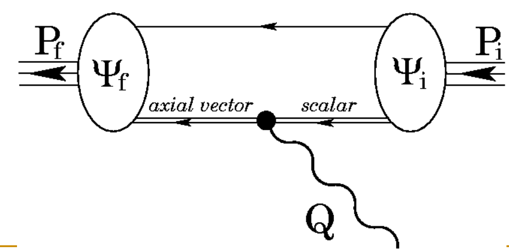


- scalar to axial–vector diquark transition:

$$\Gamma^{\mu\beta}_{\mathsf{SA}}(p',p) = -\Gamma^{\mu\beta}_{\mathsf{AS}}(p',p) = \frac{i}{M_n}\mathcal{T}(p',p)\epsilon_{\mu\beta\rho\lambda}p'^{\rho}p^{\lambda}$$

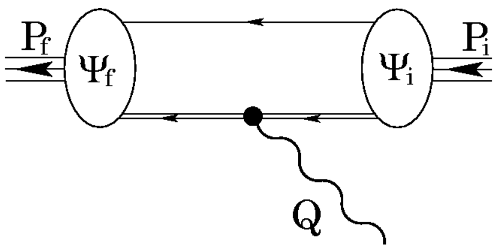
  ⇝ simple *Ansatz:* $\mathcal{T}(p',p) = \kappa_{\mathcal{T}} \approx 2$

  M. Oettel, R. Alkofer, L. von Smekal, Eur. Phys. J. **A 553** (2000).



Dressed Photon Vertices

- axial–vector diquark–photon vertex
  F.T. Hawes, M.A. Pichowsky, Phys. Rev. **C 59**, 1743 (1999).



$$\Gamma^{1^+}_{\mu\alpha\beta}(\ell_1,\ell_2) = -\sum_{i=1}^{3}\Gamma^{[i]}_{\mu\alpha\beta}(\ell_1,\ell_2)$$

- vertex constructed from transverse projectors $T_{\alpha\beta}(\ell) = \delta_{\alpha\beta} - \ell_\alpha \ell_\beta/\ell^2$ and 3 scalar functions F_1, F_2, F_3

$$\Gamma^{[1]}_{\mu\alpha\beta}(\ell_1,\ell_2) = (\ell_1+\ell_2)_\mu \, T_{\alpha\lambda}(\ell_1)\, T_{\lambda\beta}(\ell_2)\, F_1(\ell_1^2,\ell_2^2)$$
$$\Gamma^{[2]}_{\mu\alpha\beta}(\ell_1,\ell_2) = \left[T_{\mu\alpha}(\ell_1)\, T_{\beta\rho}(\ell_2)\, \ell_{1\rho} + T_{\mu\beta}(\ell_2)\, T_{\alpha\rho}(\ell_1)\, \ell_{2\rho}\right] F_2(\ell_1^2,\ell_2^2)$$
$$\Gamma^{[3]}_{\mu\alpha\beta}(\ell_1,\ell_2) = -\frac{1}{2m_{1^+}^2}(\ell_1+\ell_2)_\mu \, T_{\alpha\rho}(\ell_1)\, \ell_{2\rho}\, T_{\beta\lambda}(\ell_2)\, \ell_{1\lambda}\, F_3(\ell_1^2,\ell_2^2)$$

- electric, magnetic and quadrupole form factors $G_{\mathcal{E}}^{1^+}$, $G_{\mathcal{M}}^{1^+}$ and $G_{\mathcal{Q}}^{1^+}$

$$G_{\mathcal{E}}^{1^+}(Q^2) = F_1(Q^2) + \frac{2}{3}\tau_{1^+}\, G_{\mathcal{Q}}^{1^+}(Q^2) \qquad \tau_{1^+} = \frac{Q^2}{4\, m_{1^+}^2}$$
$$G_{\mathcal{M}}^{1^+}(Q^2) = -F_2(Q^2)$$
$$G_{\mathcal{Q}}^{1^+}(Q^2) = F_1(Q^2) + F_2(Q^2) + (1+\tau_{1^+})\, F_3(Q^2)$$

Dressed Photon Vertices

- model *Ansatz* for functions F_1, F_2 and F_3 constrained by:

  R. Alkofer, A. Höll, M. Kloker, A. Krassnigg and C.D. Roberts, nucl-th/0412046.

  ⇝ electric charge normalisation of axial–vector diquarks

  ⇝ static electromagnetic properties

$$G_{\mathcal{E}}^{1^+}(0) = 1 \quad , \quad G_{\mathcal{M}}^{1^+}(0) = \mu_{1^+} \quad , \quad G_{\mathcal{Q}}^{1^+}(0) = -\chi_{1^+}$$

  Note: static properties of pointlike axial–vector $\mu_{1^+} = 2$, $\chi_{1^+} = 1$

  ⇝ current conservation

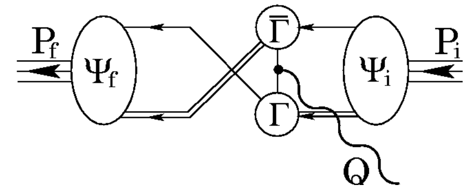
  ⇝ asymptotic form (independent of μ_{1^+} and χ_{1^+})

  S. J. Brodsky and J.R. Hiller, Phys. Rev. **D 46**, 2141 (1992).

$$G_{\mathcal{E}}^{1^+}(Q^2) : G_{\mathcal{M}}^{1^+}(Q^2) : G_{\mathcal{Q}}^{1^+}(Q^2) \overset{Q^2\to\infty}{=} (1 - \frac{2}{3}\tau_{1^+}) : 2 : -1$$

  ⇝ simple form

Dressed Photon Vertices

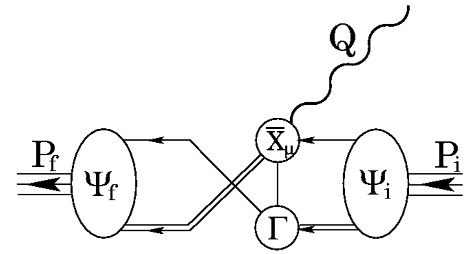


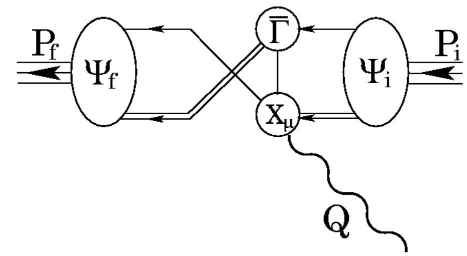
- 2–loop exchange contribution

  Ball Chiu *Ansatz* ⟶

  J.S. Ball, T.-W. Chiu, Phys. Rev. **D 22**, 2542 (1980).

  (same form as 1–loop photon quark vertex)





- 2–loop "seagull" contributions

  M.Oettel, M.A. Pichowsky, L. von Smekal, Eur. Phys. J. **A 8**, 251 (2000).
  R. Alkofer, A. Höll, M. Kloker, A. Krassnigg, C.D. Roberts, nucl-th/0412046.

  ⇝ constrained by WTI (no further parameters)

Summary Photon Vertices

- all dressed photon vertices constrained by WTI
- introduction of three parameters to specify vertices:
  - ⇝ transition strength scalar ↔ axial–vec. diquark ($\kappa_{\mathcal{T}} = 2$)
  - ⇝ static magnetic dipole moment of axial–vec. diquark ($\mu_{1^+} = 2$)
  - ⇝ static electric quadrupole moment of axial–vec. diquark ($\chi_{1^+} = 1$)

Dressed Quark Propagator

- parametrization of scalar σ_S and vector part σ_V of the quark propagator

  M.B. Hecht, C.D. Roberts, M. Oettel, A.W. Thomas, S.M. Schmidt, P.C. Tandy, nucl-th/0201084.

$$S(p) = -i\gamma \cdot p\, \sigma_V(p^2) + \sigma_S(p^2) \qquad \text{with}$$

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}[2(x+\bar{m}^2)] + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)]$$

$$\bar{\sigma}_V(x) = \frac{1-\mathcal{F}[2(x+\bar{m}^2)]}{x+\bar{m}^2}$$

$$\mathcal{F}(x) = \frac{1-\exp\{-x\}}{x}$$

$$x = \frac{p^2}{\lambda^2} \quad , \quad \bar{m} = \frac{m}{\lambda} \quad , \quad \bar{\sigma}_S(x) = \lambda\sigma_S(p^2) \quad , \quad \bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$$

m [MeV]	b_0	b_1	b_2	b_3	λ [GeV]	ϵ
5.1	0.131	2.90	0.603	0.185	0.566	10^{-4}

⇝ all parameters fixed to light meson observables

C.J. Burden, C.D. Roberts, M.J. Thomson, Phys. Lett. **B 371**, 163 (1996).

⇝ realization of Dynamical Chiral Symmetry Breaking

with constituent quark mass $M_{u,d} = 0.33$ GeV

The Ball Chiu *Ansatz* for the Quark–Photon Vertex

- quark–photon vertex: Ball Chiu *Ansatz*
  `J.S. Ball, T.-W. Chiu, Phys. Rev.` **D 22**, `2542 (1980).`
  ⇝ longitudinal part determined by

$$\Gamma^\mu_{\mathrm{BC}}(k,p) = -i\gamma^\mu \frac{A(k)+A(p)}{2} - i(p+k)^\mu \frac{\not k + \not p}{2} \frac{A(k)-A(p)}{k^2-p^2} - (p+k)^\mu \frac{B(k)-B(p)}{k^2-p^2}$$

  ⇝ A and B dressing functions in quark propagator

  ⇝ presently **no** dynamical contribution from $\rho - \omega$ poles
  `M. Oettel, R. Alkofer, Eur. Phys. J.` **A 16**, `95 (2003).`

Decomposition of Faddeev Amplitudes

- decomposition of Faddeev amplitude $\Psi(p,P)$
  into scalar part $\Psi^5(p,P)$ and axial–vector part $\Psi^\mu(p,P)$
  M. Oettel, L. von Smekal, R. Alkofer, Comp. Phys. Comm. **D 144**, 63 (2002).

$$\Psi(p,P) = \begin{pmatrix} \Psi^5(p,P) \\ \Psi^\mu(p,P) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^2 S_i(p^2, \hat{p}\cdot\hat{P})\mathcal{S}_i(p,P) \\ \sum_{i=1}^6 A_i(p^2, \hat{p}\cdot\hat{P})\gamma_5\mathcal{A}_i(p,P) \end{pmatrix}$$

- use covariants $\mathcal{S}_i$ and $\mathcal{A}_i$

$$\mathcal{S}_i = \begin{cases} \mathcal{S}_1 = \Lambda^+ \\ \mathcal{S}_2 = -\frac{i}{p}\not{p}_T\Lambda^+ \end{cases} \quad , \quad \mathcal{A}_i = \begin{cases} \mathcal{A}_1^\mu = -\frac{i}{p}\hat{P}^\mu\not{p}_T\Lambda^+ \\ \mathcal{A}_2^\mu = \hat{P}^\mu\Lambda^+ \\ \mathcal{A}_3^\mu = \hat{p}_T^\mu\hat{\not{p}}_T\Lambda^+ \\ \mathcal{A}_4^\mu = \frac{i}{p}\hat{p}_T^\mu\Lambda^+ \\ \mathcal{A}_5^\mu = \gamma_T^\mu\Lambda^+ - \mathcal{A}_3^\mu \\ \mathcal{A}_6^\mu = \frac{i}{p}\gamma_T^\mu\not{p}_T\Lambda^+ - \mathcal{A}_4^\mu \end{cases}$$

Note: $p_T^\mu = p^\mu - \hat{P}^\mu(p\cdot\hat{P})$

- $\hat{p}\cdot\hat{P}$-dependence in S_i and A_i expanded into Chebyshev polynomials