Implementations of Gauge Invariance in Photo- and Electroproduction Calculations

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New Theoretical Tools for Nucleon Resonance Analysis

Old Theoretical Tool

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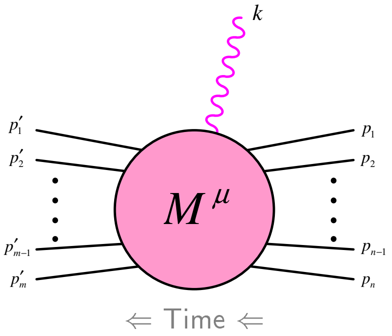
Gauge Invariance

[1] J.C. Ward, Phys. Rev. **78**, 182 (1950)
[2] Y. Takahashi, Nuovo Cimento **6**, 371 (1957)
[3] E. Kazes, Nuovo Cimento **13**, 1226 (1959)

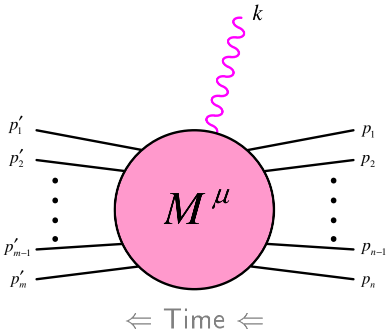
Gauge Invariance



Physical condition:

Current Conservation: $k_\mu M^\mu = 0$ all hadrons on-shell

Gauge Invariance



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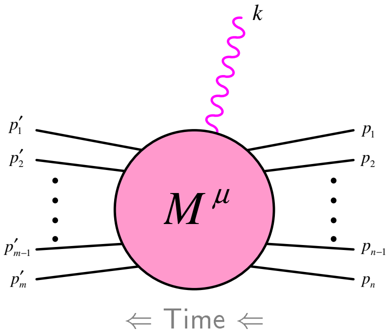
Gauge Invariance



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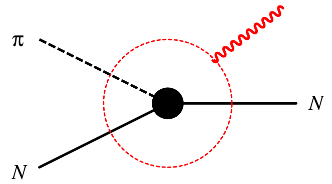
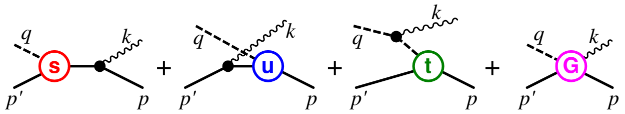
Not good enough for microscopic approaches!

Example: $\gamma N \to \pi N$

Attaching a Photon to the πNN Vertex





M_s^μ | M_u^μ | M_t^μ | M_{int}^μ

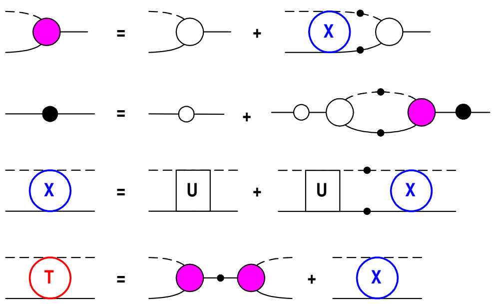
s-channel | u-channel | t-channel | Interaction current

Total photoproduction amplitude

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$

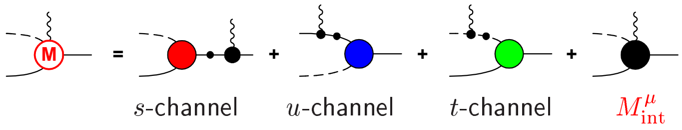
Pions, Nucleons, and Photons

Hadronic scattering: $\pi N \to \pi N$

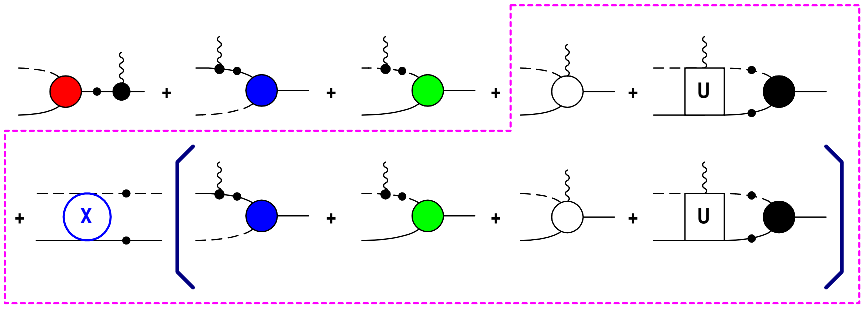


Photoproduction: $\gamma N \to \pi N$

Amplitude:

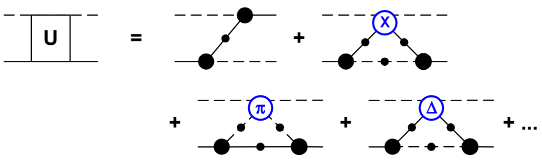


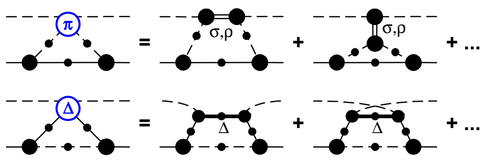
Theory: PRC **56**, 2041 (1997)



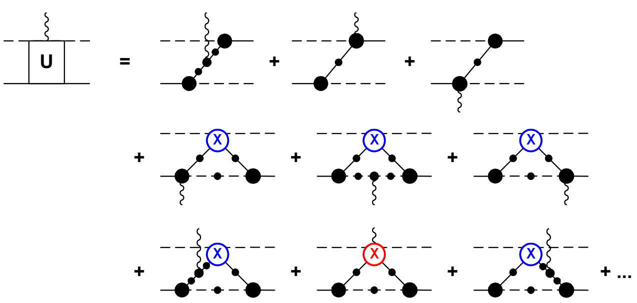
Interaction Current M^{μ}_{int}

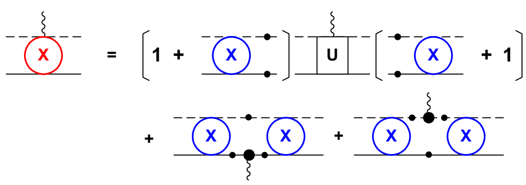
Hadronic Driving Terms



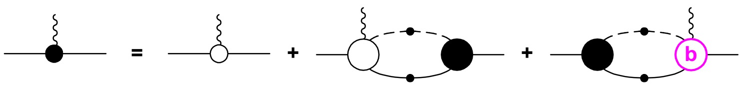


Electromagnetic Couplings to Driving Terms





Dressed Nucleon Current



This is VERY complicated!

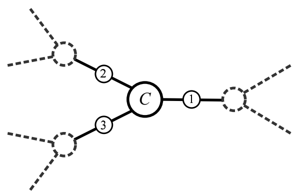
- The full theory is gauge-invariant as a matter of course.
- In practice, truncations are necessary.
- Truncations usually destroy gauge invariance.

Prescriptions must be found that restore gauge invariance in practical applications.

Q: So why is it not okay to simply subtract terms to create a transverse current?

A: Because such currents are always transverse — on- and off-shell. This creates unacceptable inconsistencies in microscopic approaches.

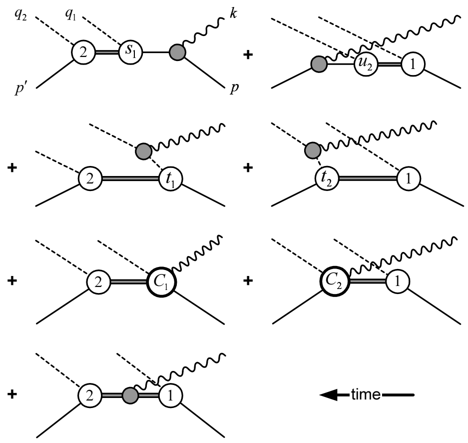
To show this consider



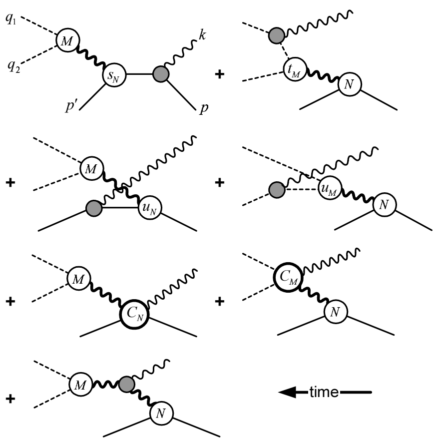
and attach photon in all places of the internal 3-point function...

Example: Two-pion production

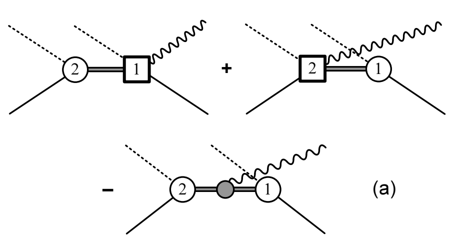
Basic Two-pion Production Mechanisms



(a)



(b)



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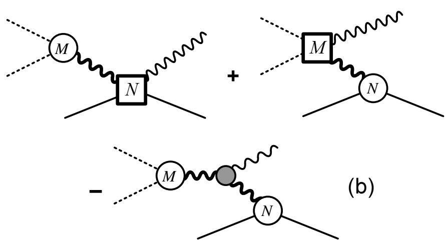
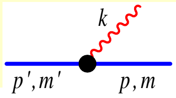


(b)

Ward–Takahashi Identities for the Nucleon and the Pion Currents

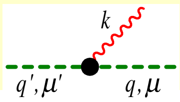


$$k_\mu \Gamma_N^\mu(p',p) = S_N^{-1}(p')Q_N - Q_N S_N^{-1}(p)$$ **nucleon**

Form factors: $f_n^{ij} = f_n^{ij}(p'^2, p^2; k^2)$

$$\Gamma_N^\nu(p',p) = \gamma^\nu Q_N + \left[t^\nu f_1^{00} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{00}\right] + \frac{\not{p}' - m}{2m}\left[t^\nu f_1^{10} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{10}\right]$$
$$+ \left[t^\nu f_1^{01} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{01}\right]\frac{\not{p} - m}{2m} + \frac{\not{p}' - m}{2m}\left[t^\nu f_1^{11} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{11}\right]\frac{\not{p} - m}{2m}$$

$t^\nu = \gamma^\nu k^2 - k^\nu \not{k}$

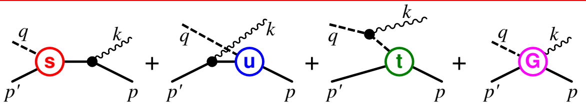


$$k_\mu \Gamma_\pi^\mu(q',q) = \Delta_\pi^{-1}(q')Q_\pi - Q_\pi \Delta_\pi^{-1}(q)$$ **pion**

$$\Gamma_\pi^\nu(q',q) = (q'+q)^\nu Q_\pi + \left[(q'+q)^\nu k^2 - k^\nu k\cdot(q'+q)\right] Q_\pi f(q'^2, q^2; k^2)$$

Generalized Ward–Takahashi Identity for the Pion-Production Current

$$k_\mu M^\mu = -[F_s\tau]S_{p+k}Q_N S_p^{-1} + S_{p'}^{-1}Q_N S_{p'-k}[F_u\tau] + \Delta^{-1}_{p-p'+k}Q_\pi \Delta_{p-p'}[F_t\tau]$$



Equivalently:

$$k_\mu M^\mu_{\text{int}} = -[F_s\tau]Q_N + Q_N[F_u\tau] + Q_\pi[F_t\tau]$$ ★★★★★

Note:

Charge conservation: $-\tau Q_N + Q_N\tau + Q_\pi\tau = 0$

Prescriptions to Restore Gauge Invariance

Gross/Riska

Absorb hadronic form factors in propagator and construct single-hadron currents to satisfy the appropriate WT identities.

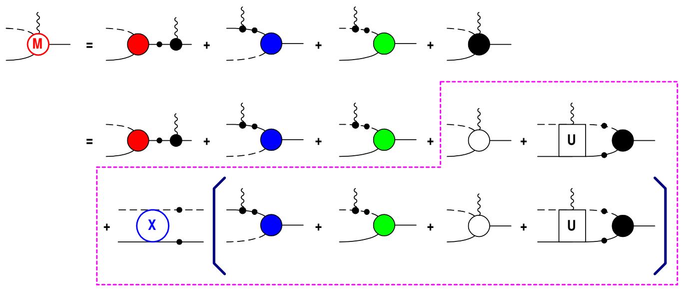
Ohta

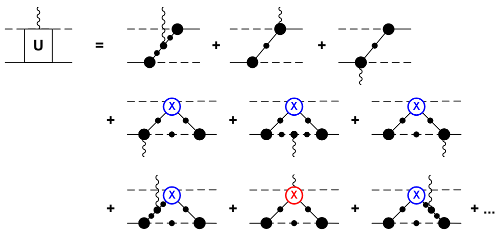
Construct contact current based on minimal substitution of πNN form factor. Basically removes hadronic structure from non-transverse contributions. Not applicable to explicit final-state interactions.

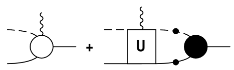
This work

Generalizes Ohta by introducing common form factor for non-transverse contributions only. Allows inclusion of explicit final-state interactions.

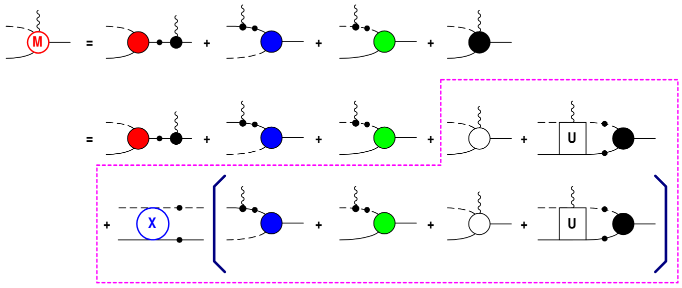
Strategy

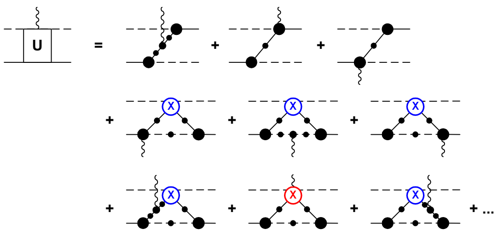


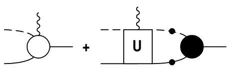


Replace  by phenomenological contact term such that WTI for interaction current is satisfied.

Strategy





Replace  by phenomenological contact term such that WTI for interaction current is satisfied.

Result

$$M^\mu_{\text{int}} = M^\mu_c + T^\mu + XG_0\left[\underbrace{(M^\mu_u - m^\mu_u)}_{\text{transverse}} + \underbrace{(M^\mu_t - m^\mu_t)}_{\text{transverse}} + T^\mu\right]$$

T^μ: undetermined transverse current

with

$$k_\mu M^\mu_c = -F_s e_i + F_u e_f + F_t e_\pi \ .$$

Phenomenological Choice for M_c^μ

$$M_c^\mu = g_\pi \gamma_5 \left\{ \left[\lambda + (1-\lambda) \frac{\not{q} - \beta \not{k}}{m'+m} \right] C^\mu - (1-\lambda) \frac{\gamma^\mu}{m'+m} \left[e_\pi f_t - \beta\, k_\rho C^\rho \right] \right\}$$

λ, β: free parameters

Non-singular auxiliary current:

$$C^\mu = -e_\pi \frac{(2q-k)^\mu}{t-q^2} (f_t - \hat{F}) - e_f \frac{(2p'-k)^\mu}{u-p'^2} (f_u - \hat{F}) - e_i \frac{(2p+k)^\mu}{s-p^2} (f_s - \hat{F}) \ ,$$

with

$$k_\mu C^\mu = e_\pi f_t + e_f f_u - e_i f_s$$

Subtraction function:

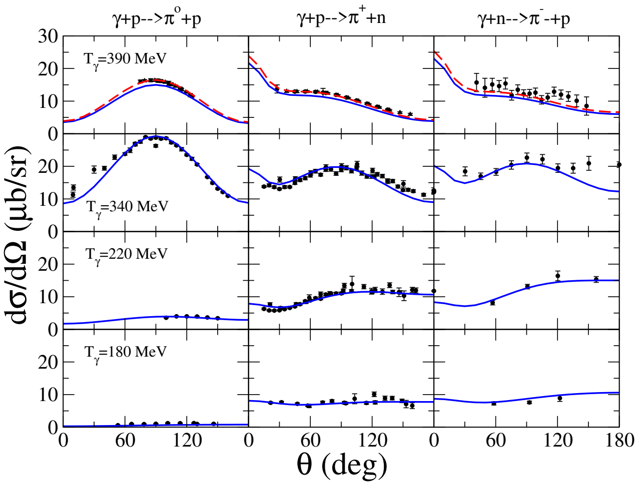
$$\hat{F} = 1 - \hat{h} \left(1 - \delta_s f_s\right) \left(1 - \delta_u f_u\right) \left(1 - \delta_t f_t\right)$$

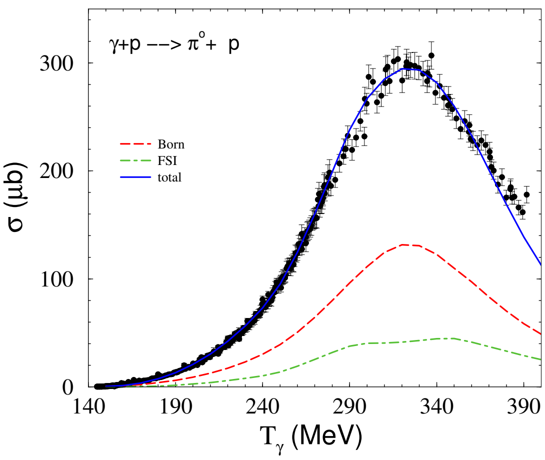
$\delta_x = 0, 1$

The function $\hat{h} = \hat{h}(s,u)$ is free fit function.

[Ohta: $\hat{h} = 0$, $X = 0$, $T^\mu = 0$]

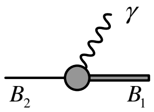
Application



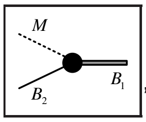


Includes ρ, ω, and a_1 exchanges in the t-channel; Δ in s- and u-chanels; FSI with Jülich πN T-matrix.

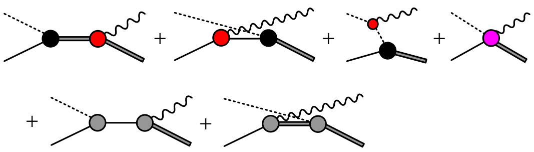
Transition Currents

- Gauge invariance demands that transition currents be transverse. 

  Outline of proof:

  Coupling the photon to MB_1B_2 vertex, 

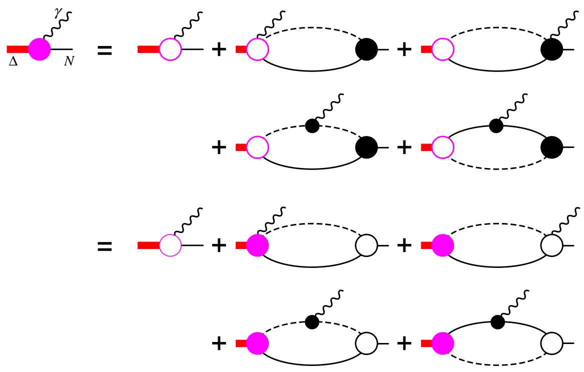
  , produces



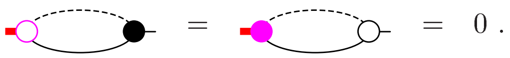
  First line produces correct off-shell relation. Terms in second line must vanish individually. QED

- In a consistent microscopic approach, this should be ensured dynamically. There should be no need to adjust the transversality manually.

Example: $\gamma N \to \Delta$



Transversality of this current is ensured by the fact that $N \to \Delta$ is not possible as a physical process, i.e.



With consistent dynamics, this current satisfies

$$\Gamma^{\beta\mu} = G_1\,\gamma_5\left(k^\beta\gamma^\mu - g^{\beta\mu}\,\not{k}\right) + G_2\,\gamma_5\left(k^\beta P^\mu - g^{\beta\mu}\,k\cdot P\right) + G_3\,\gamma_5\left(k^\beta k^\mu - g^{\beta\mu}\,k^2\right)$$

as a matter of course, where $P = (p + p')/2 = (2p + k)/2$. There is no need for subtractions.

Summary

- Gauge invariance is a fundamental symmetry — without it results become arbitrary.
- Current conservation $k_\mu M^\mu = 0$ necessary, but not sufficient, for gauge invariance of microscopic theories.
- Subtractions $M^\mu = \tilde{M}^\mu - a^\mu \dfrac{k \cdot \tilde{M}}{k \cdot a}$ in general do not ensure gauge invariance.
- Most of the popular dynamical models are not gauge-invariant; despite claims to the contrary.
- Generalized Ward–Takahashi identities are necessary and sufficient.
- Real-world calculations require truncations of dynamical mechanisms that destroy gauge invariance.
- Prescriptions required to restore gauge invariance.
- Transition currents are transverse and this property must be ensured dynamically.