Chiral SU(3) Coupled-Channel Model and Hadronic Resonances

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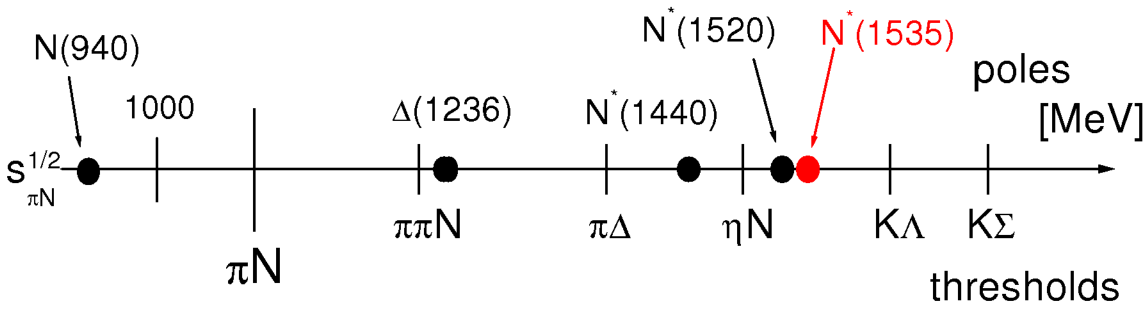
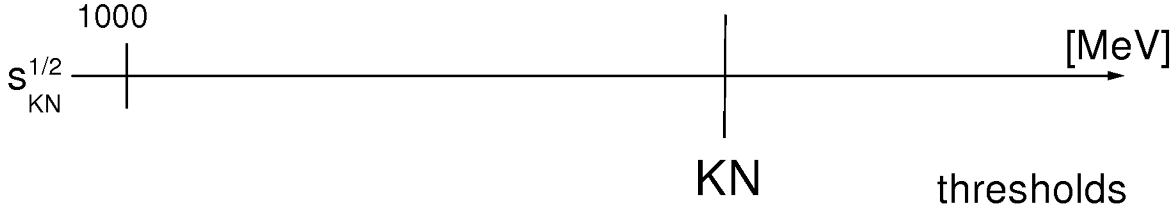
- Introduction
- Bethe-Salpeter Equation
- SU(3) Meson-Nucleon Scattering
- Hadronic Resonances

World of Meson-Nucleon Interaction

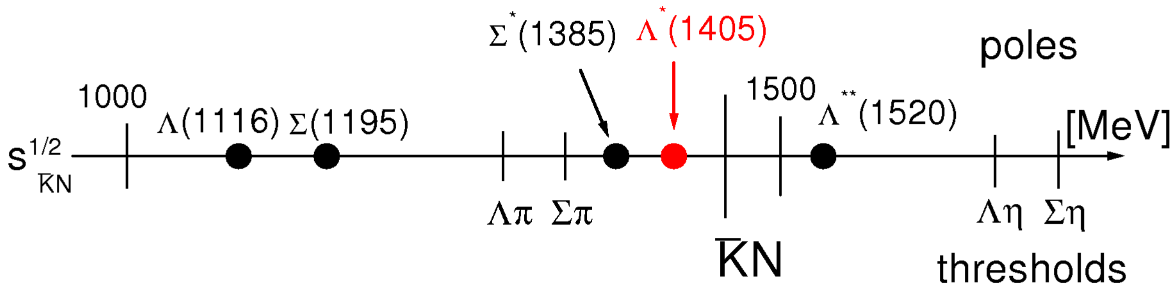
pion–nucleon



kaon–nucleon

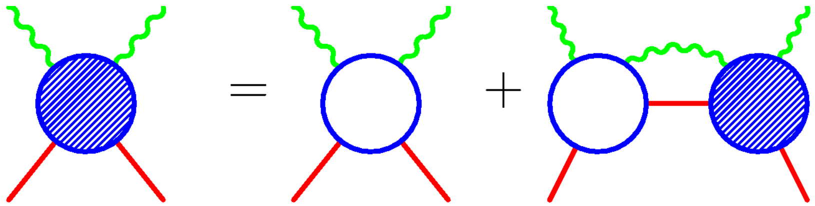


antikaon–nucleon



Coupled-channel Bethe-Salpeter equation

✔ **Scattering amplitude:**



✔ **Compact notation:**

$$T = K + K \cdot G \cdot T \quad \text{with}$$

– K – interaction kernel
– G – two-particle propagator

✔ **Strategy:** chiral and large-N_c expansion of effective interaction V

On- and off-shell scattering amplitudes

✔ **Unique covariant projection operators:** $\mathcal{Y}^{(J,P)}(\bar{q},q;w)$ with $w^2=s$

- preserve total angular momentum J and parity P
- regularity in q (initial meson) and $\bar{q}$ (final meson)
- defined for any off-shell kinematics

$$\left(\mathcal{Y}^{(J,P)}\cdot G\cdot \mathcal{Y}^{(J',P')}\right)(\bar{q},q;w) \overset{!}{=} \delta_{J,J'}\,\delta_{P,P'}\,J^{(J,P)}(\sqrt{s}\,)\,\mathcal{Y}^{(J,P)}(\bar{q},q;w)$$

$\longrightarrow$ divergent loop-functions $J^{(J,P)}(\sqrt{s}\,)$ $\longleftarrow$ $\overline{\mathrm{MS}}_\chi$ regularization

✔ **On-shell reduction:**

$$T \;=\sum_{J,P} M^{(J,P)}(\sqrt{s}\,)\,\mathcal{Y}^{(J,P)}+T_L+T_R+T_{LR}$$

- $T_{L/R}$ vanish if initial/final states are on shell
- $T_L+T_R+T_{LR}$ defines off-shell part of two-body amplitude

Solution of Bethe-Salpeter Equation

✔ **Effective interaction kernel V:**

$$[1 - K \cdot G\,]^{-1} \cdot K \quad \leftarrow \text{on-shell} \rightarrow \quad [1 - V \cdot G\,]^{-1} \cdot V$$

defined by:

$$\begin{aligned} K &= V + (1 - V \cdot G) \cdot V_L + V_R \cdot (1 - G \cdot V) \\ &+ (1 - V \cdot G) \cdot V_{LR} \cdot (1 - G \cdot V) - V_R \cdot \frac{1}{1 - G \cdot V_{LR}} \cdot G \cdot V_L \,. \end{aligned}$$

note: V_L and V_R vanish for on-shell kinematics

✔ **Algebraic solution of Bethe-Salpeter equation**

- $V = \sum_{J,P} V^{(J,P)}(\sqrt{s}\,)\,\mathcal{Y}^{(J,P)} \quad \Longrightarrow \quad M^{(J,P)} = \left(1 - V^{(J,P)}\,J^{(J,P)}\right)^{-1} V^{(J,P)}$
- to leading order: $T_L = V_L \cdot \frac{1}{1-G\cdot V}\,, \qquad T_R = \frac{1}{1-G\cdot V} \cdot V_L\,, \qquad T_{LR} = V_{LR}$

✔ **No dependence on choice of fields**

- **divergent loop functions** :

$$I_{M,N}(\sqrt{s}) = i \int \frac{d^4 l}{(2\,\pi)^4}\, \frac{1}{l^2 - m_{M,N}^2}\,, \quad I_{MB}(\sqrt{s}) = -i \int \frac{d^4 l}{(2\,\pi)^4}\, \frac{1}{l^2 - m_M^2}\, \frac{1}{(w-l)^2 - m_N^2}$$

- **renormalization scheme**

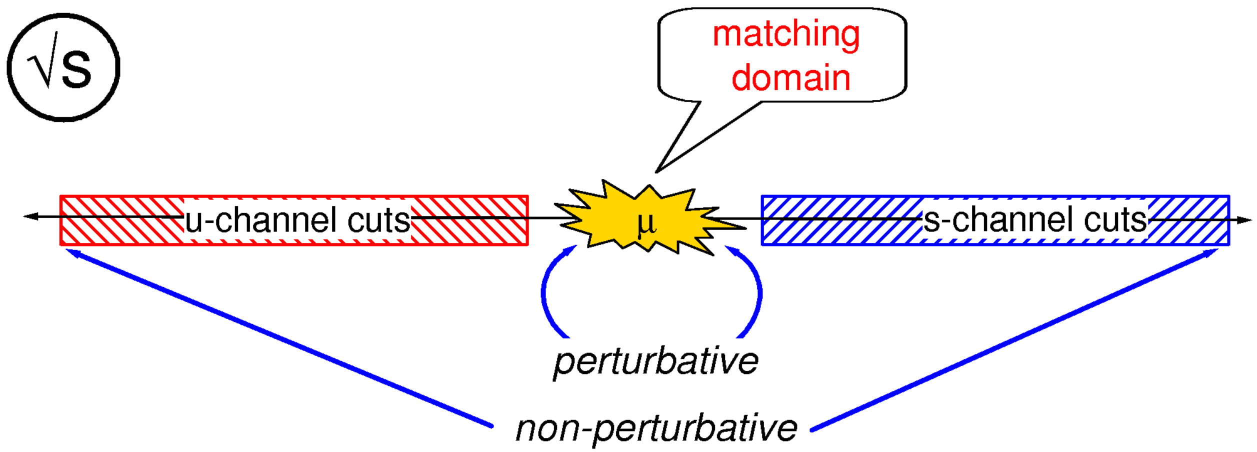
regularization: loop functions respect chiral counting:

$$I_N \sim Q^3\,,\; I_\pi \sim Q^2 \text{ and } I_{\pi N} \sim Q$$

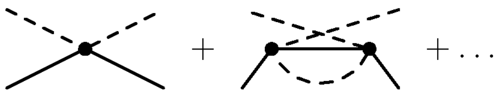
strategy: move all tadpole contributions into Bethe-Salpeter kernel

- **subtraction point** $\mu_S \longrightarrow$ approximate crossing symmetry

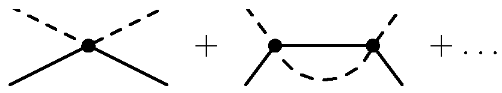
Gluing of s- and u-channel unitarized amplitudes



✔ **Approximated crossing symmetry:**



$$\Longrightarrow T_{\mathrm{u-chan.}}(\mu) \sim T_{\mathrm{s-chan.}}(\mu) \Longleftarrow$$



$$T = \begin{cases} T_{\mathrm{s-chan.}} & \text{if} \quad \sqrt{s} > \mu \\ T_{\mathrm{u-chan.}} & \text{if} \quad \sqrt{s} < \mu \end{cases}$$

✔ **Renormalization condition:** $T^{(J,P)}(\sqrt{s} = \mu) = V^{(J,P)}(\sqrt{s} = \mu)$

✔ **Natural matching point:** e.g., for πH-scattering $\rightarrow \mu \sim m_H$

Meson-Nucleon Scattering

Chiral $SU(3)$ interaction terms

✔ **Large N_c ground states**

- Goldstone boson octet $\Phi_{[8]} = \left(\pi\,, K\,, \overline{K}\,, \eta\right)$
- Baryon octet $B_{[8]} = \left(N\,, \Sigma\,, \Lambda\,, \Xi\right)$
- Baryon decuplet $B_{[10]} = \left(\Delta\,, \Sigma^*\,, \Xi^*\,, \Omega\right)$

✔ **Systematic approximation strategy:**

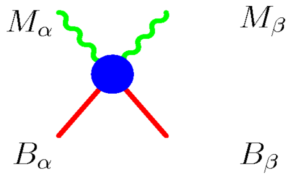
expand in *powers* of the small current quark masses, momenta and $1/N_c$

$$\frac{m_{\rm quark}}{\Lambda_{\chi SB}} \ll 1\,, \qquad \frac{1}{N_c} \ll 1$$

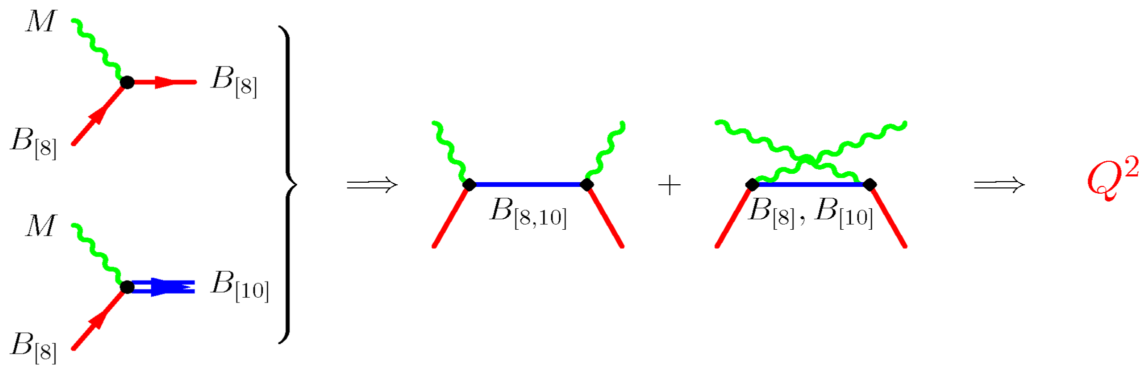
- heavy fields: $M_{[8,9,10]} \sim \Lambda_{\chi SB}$ but $M_{[10]} - M_{[8]} \sim \frac{1}{N_c}$
- light Goldstone bosons: $m_{[8]} \sim m_{\rm quark}^{1/2}$

Diagrams to chiral order Q^3

Q:



$\propto \frac{\not{q}}{f^2}$ Weinberg-Tomazawa term



Local background terms

Q^2 :

Σ-terms $\propto m_K^2\,,\; m_\pi^2$

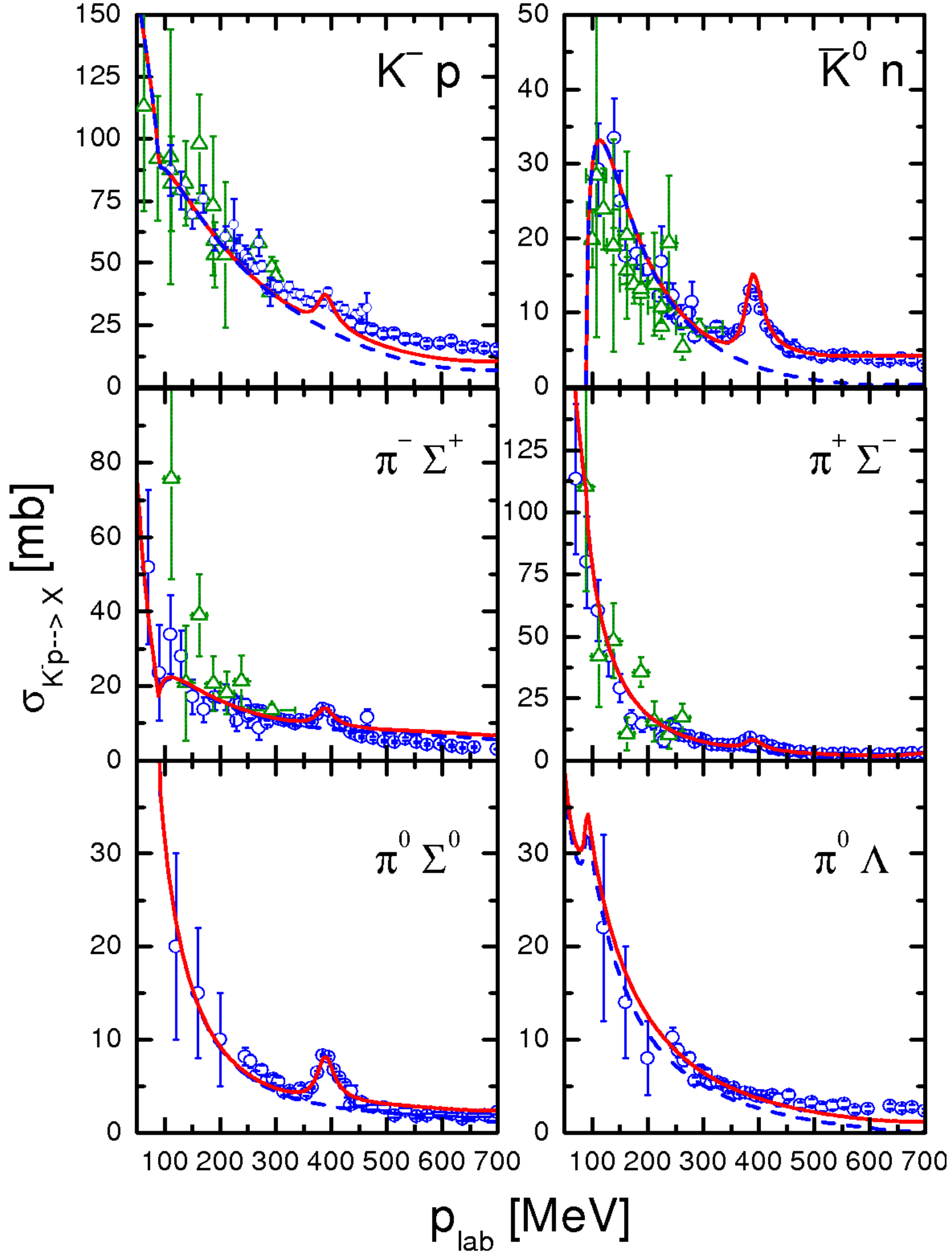
s-wave range, p-wave terms $\propto (\bar{q}\cdot q)$ (11 parameter)

Q^3 :

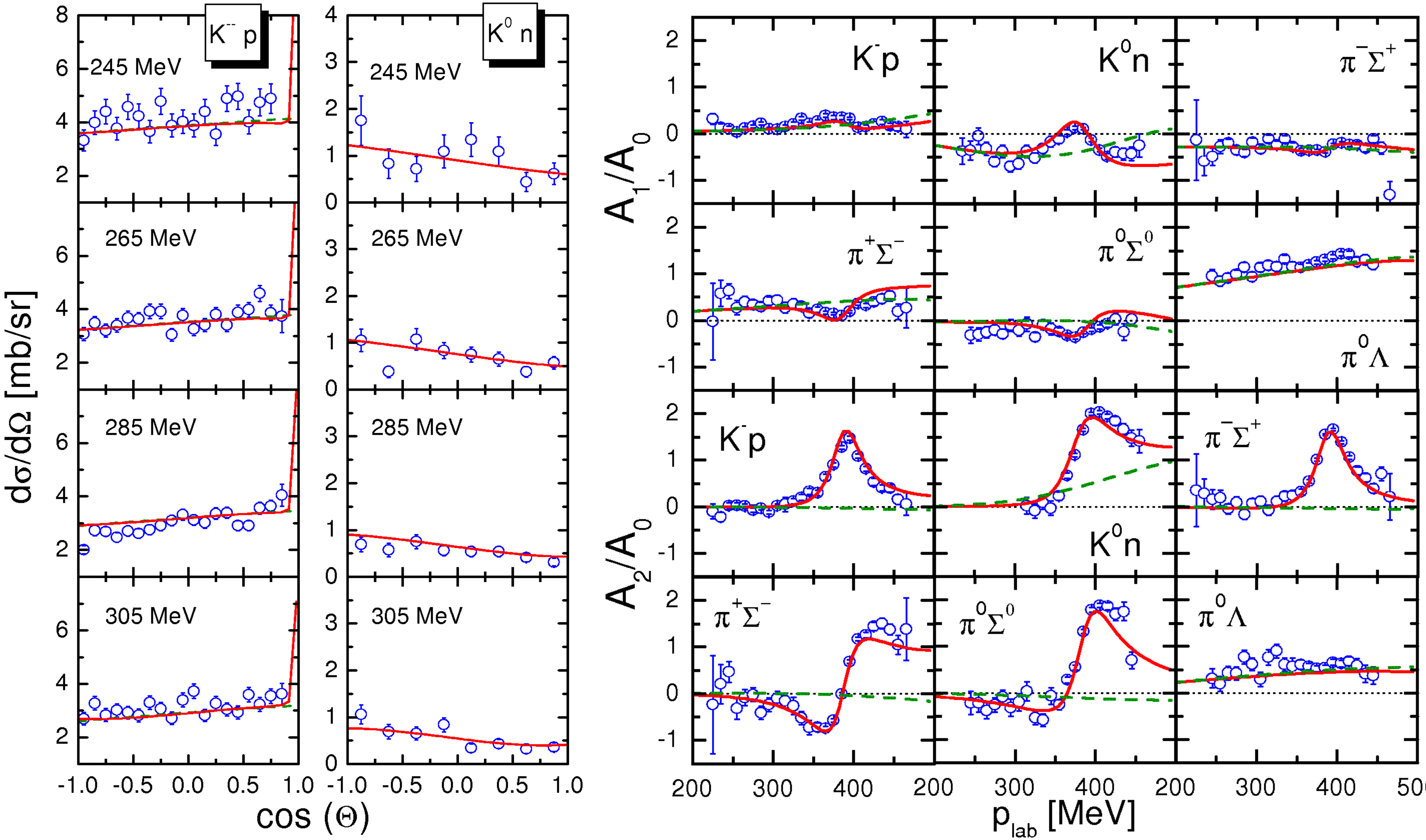
local 2-body background terms (10 parameters)

3-point vertex corrections+pseudo-scalar vertices (23 terms)

K⁻ proton cross sections

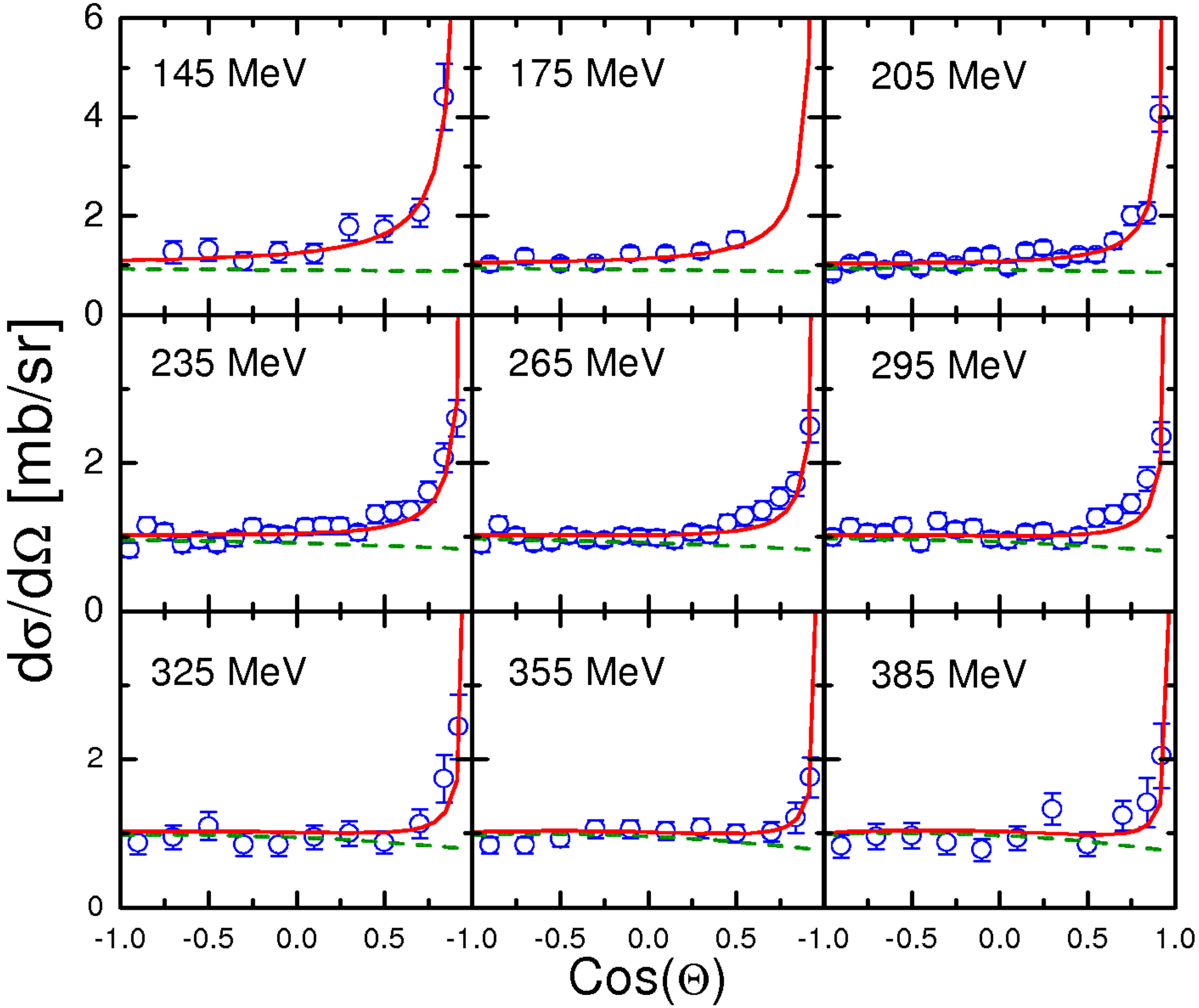


Differential cross sections

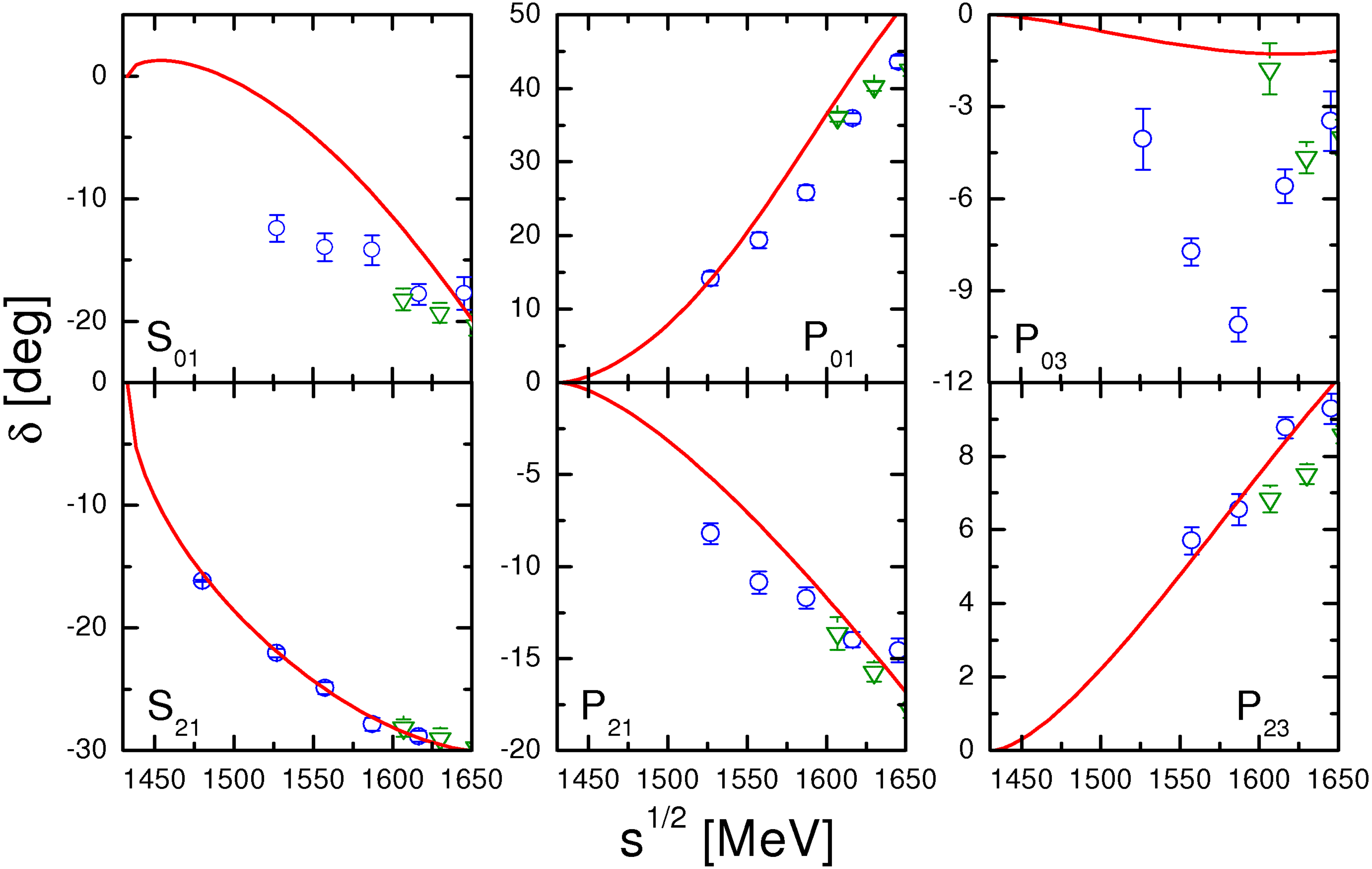


$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = A_0(s) + A_1(s)\,P_1(\cos\theta) + A_2(s)\,P_2(\cos\theta)$$

K^+ proton differential cross sections

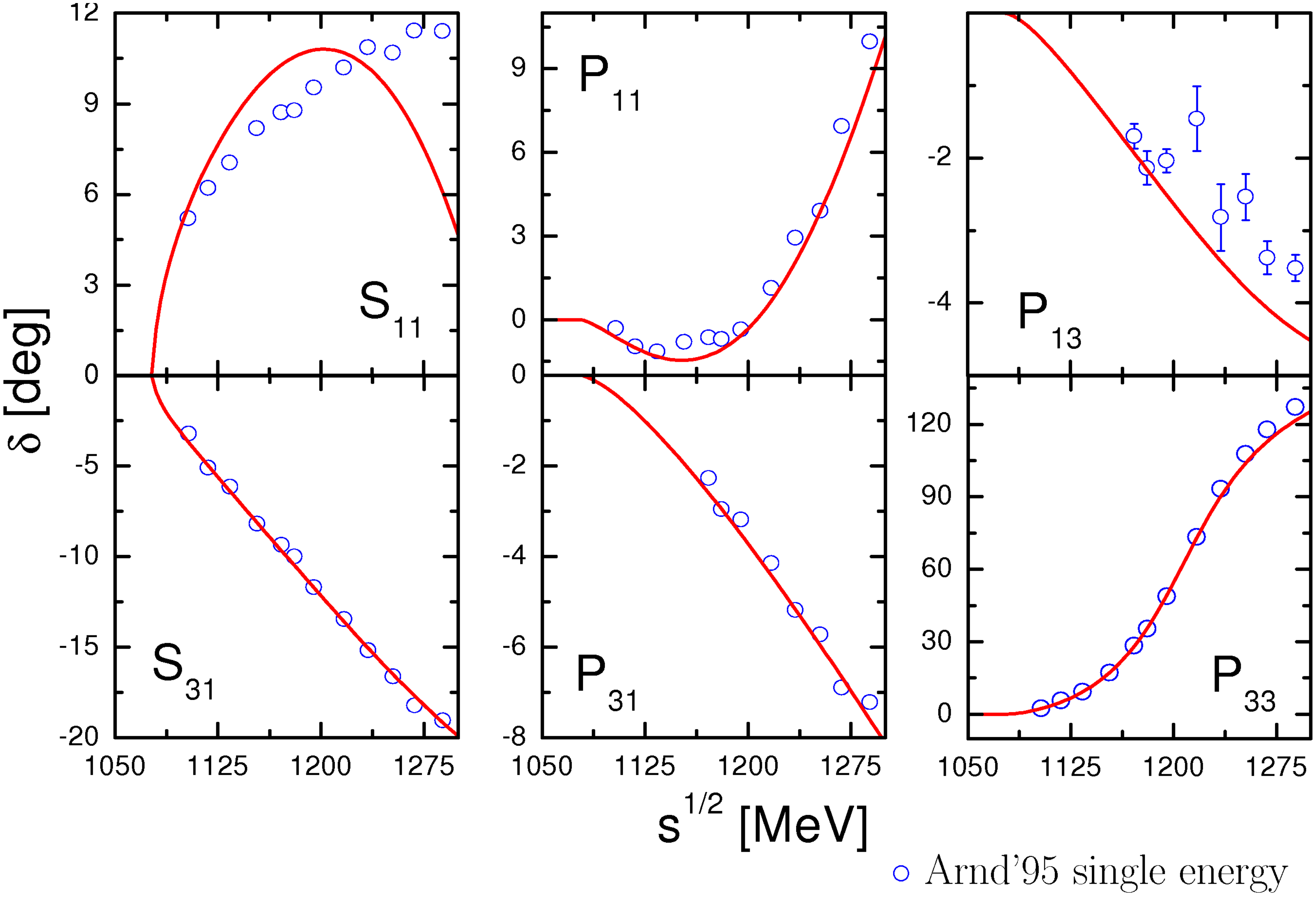


K^+ nucleon scattering: phase shifts



○: Hyslop [PRD 46 (1992) 961]; ▽: Hashimoto [PRC 29 (1984) 1377]

Pion-Nucleon Scattering: phase shifts



Hadronic Resonances

Study of chiral excitations in QCD

✔ **Large-N_c ground states:**

- Goldstone boson octet $\Phi_{[8]} = \Big(\pi\,, K\,, \overline{K}\,, \eta\Big)$
- Baryon octet $B_{[8]} = \Big(N\,, \Sigma\,, \Lambda\,, \Xi\Big)$
- Baryon decuplet $B_{[10]} = \Big(\Delta\,, \Sigma^*\,, \Xi^*\,, \Omega\Big)$

✔ **Chiral excitations of baryons**

- S-wave scattering of Goldstone bosons off baryons with J^P:

$$\mathbf{0}^- + \frac{\mathbf{1}}{\mathbf{2}}^+ \to \frac{\mathbf{1}}{\mathbf{2}}^- \to \mathbf{0}^- + \frac{\mathbf{1}}{\mathbf{2}}^+$$

$$\mathbf{0}^- + \frac{\mathbf{3}}{\mathbf{2}}^+ \to \frac{\mathbf{3}}{\mathbf{2}}^- \to \mathbf{0}^- + \frac{\mathbf{3}}{\mathbf{2}}^+$$

Chiral SU(3) predicts s-wave interaction strength of Goldstone bosons with any hadron !

Baryon resonances (Q^1)

Chiral SU(3) symmetry predicts: $J^P = \frac{1}{2}^-$: $\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \overline{\mathbf{10}} \oplus \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$

✔ **"Heavy" SU(3) limit:** $m_\pi = m_\eta = m_K = 495$ MeV

- resonances turn into bound states
- mass degenerated multiplets formed

✔ **"Light" SU(3) limit:** $m_\pi = m_\eta = m_K = 139$ MeV

- resonances are gone

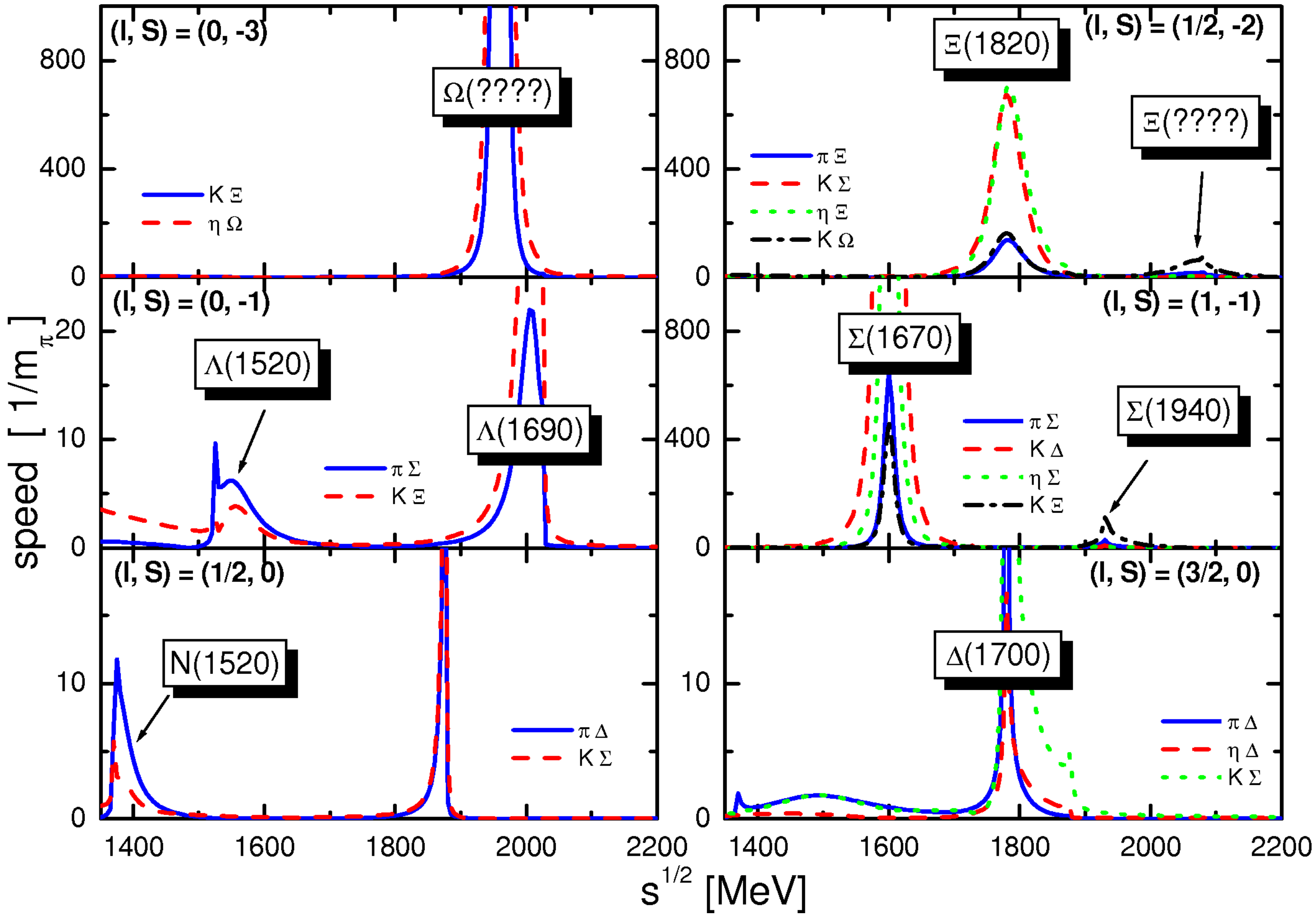
✔ **physical masses:**

$J^P = \frac{1}{2}^-$: strong signals: $N(1535)$,$\Lambda(1405)$, $\Lambda(1670)$ and $\Xi(1690)$
weak signals: $N(1650)$, $\Xi(1620)$ and $\Sigma(1620)$, $\Sigma(1750)$

Phys.Lett. B582,49; Phys. Lett. B585, 243

$J^P = \frac{3}{2}^-$ baryon resonances ($\mathbf{Q}^1$)

Chiral SU(3) symmetry predicts: $\mathbf{8} \otimes \mathbf{10} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$



- decuplet and octet
- bound state in $(0, -3)$-sector
- 27-plet state in $(0, -1)$-sector
- $\Lambda(1520)$–$\Lambda(1690)$ singlet-octet ??

Meson resonances ($\mathbf{Q}^1$)

✔ **Large-N_c ground states:**

- Goldstone boson octet $\Phi_{[8]} = \Big(\pi\,, K\,, \overline{K}\,, \eta\Big)$
- Vector meson nonet $\Phi^{\mu}_{[9]} = \Big(\rho^{\mu}\,, K^{\mu}\,, \overline{K}^{\mu}\,, \omega^{\mu}\,, \phi^{\mu}\Big)$

✔ **Chiral excitations of mesons**

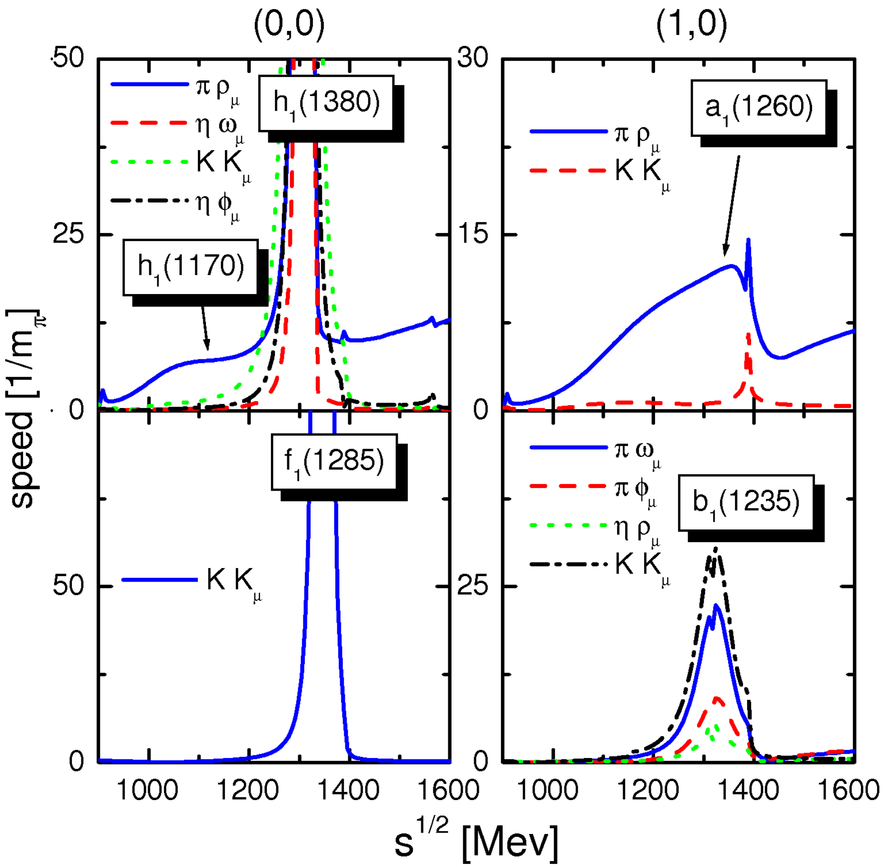
- S-wave scattering of Goldstone bosons off mesons with J^P:

$$\mathbf{0}^- + \mathbf{0}^- \to \mathbf{0}^+ \to \mathbf{0}^- + \mathbf{0}^-$$

$$\mathbf{0}^- + \mathbf{1}^- \to \mathbf{1}^+ \to \mathbf{0}^- + \mathbf{1}^-$$

Chiral SU(3) predicts s-wave interaction strength of Goldstone bosons with any hadron !

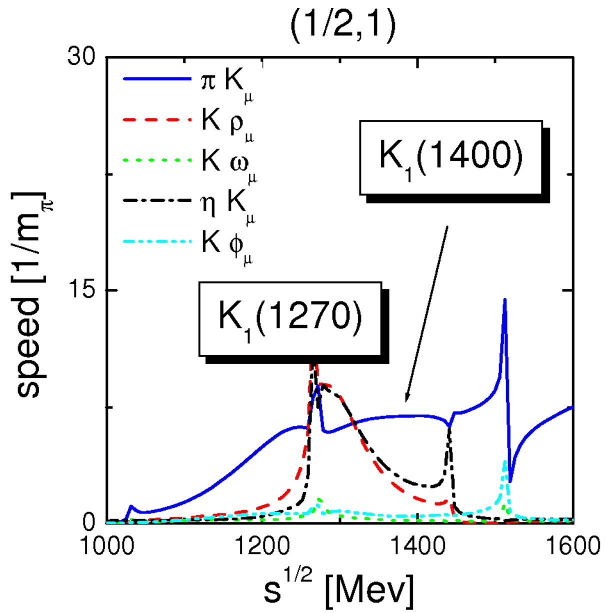
$J^P = 1^+$ meson resonances ($\mathbf{Q}^1$)



✔ **Chiral SU(3) symmetry:**

2 octets and 1 singlet

$$8 \otimes 8 = 27 \oplus \overline{10} \oplus 10 \oplus 8 \oplus 8 \oplus 1$$



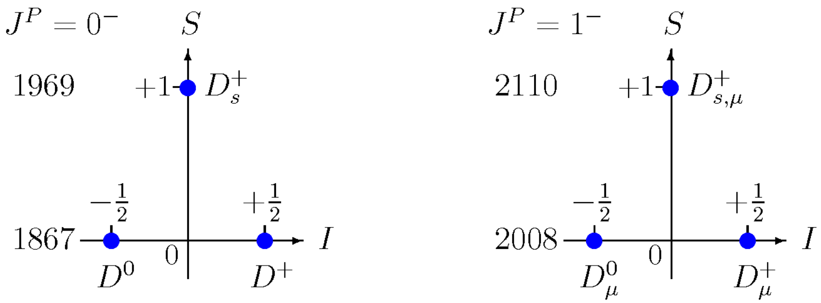
✔ **Predictions:**

- $h_1(1380)$, $f_1(1285)$ and $b_1(1235)$ couples strongly to $K\bar{K}_\mu$–channel
- $h_1(1380) \leftrightarrow (I^G = 0^-)$ state

Nucl. Phys A730, 392

Chiral excitations of open charm

✔ **Heavy-light mesons:**



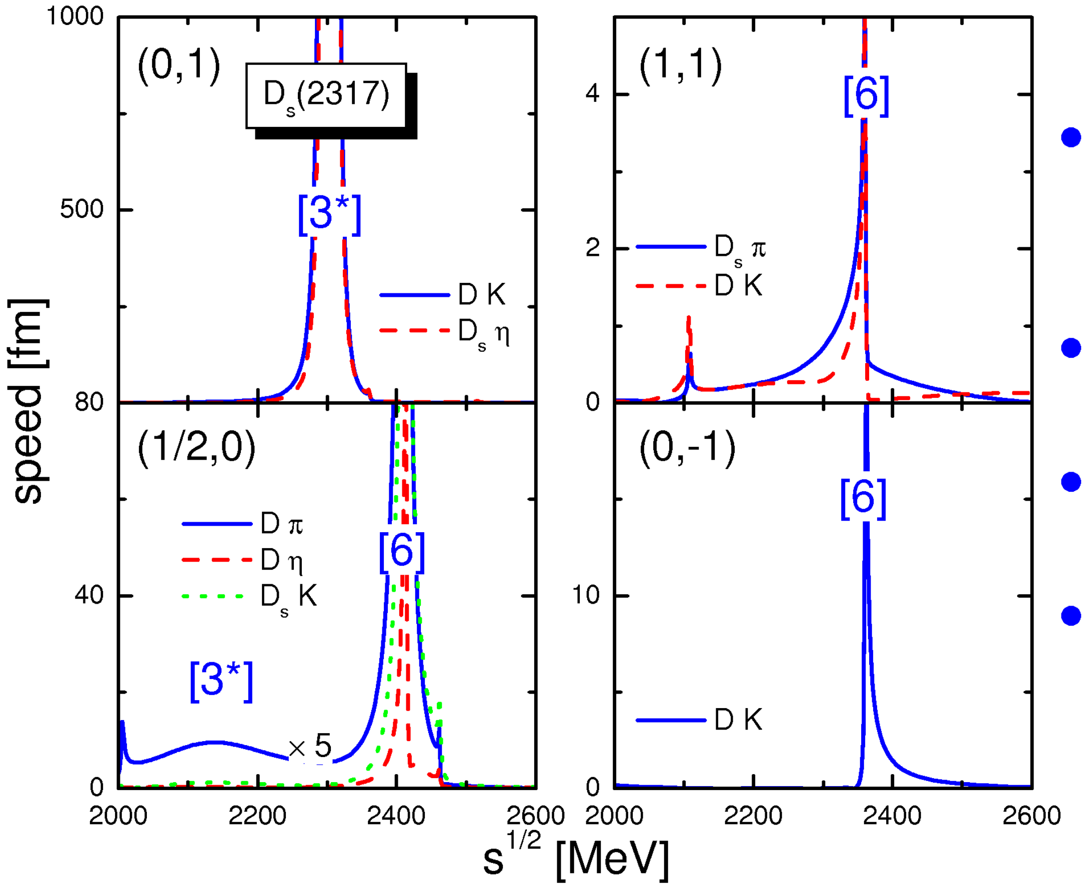
$(c\,\bar{q}_i)$ – SU(3) anti-triplet $[\bar{3}]$

Chiral SU(3) symmetry predicts $\quad 8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \overline{15}$

- attraction in $[\bar{3}]$ and $[6]$ but repulsion in $[\overline{15}]$

$J^P = 0^+$ charmed meson resonances ($\mathbf{Q}^1$)

✔ **Chiral SU(3) predicts:** anti-triplet and sextet states $8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \overline{15}$

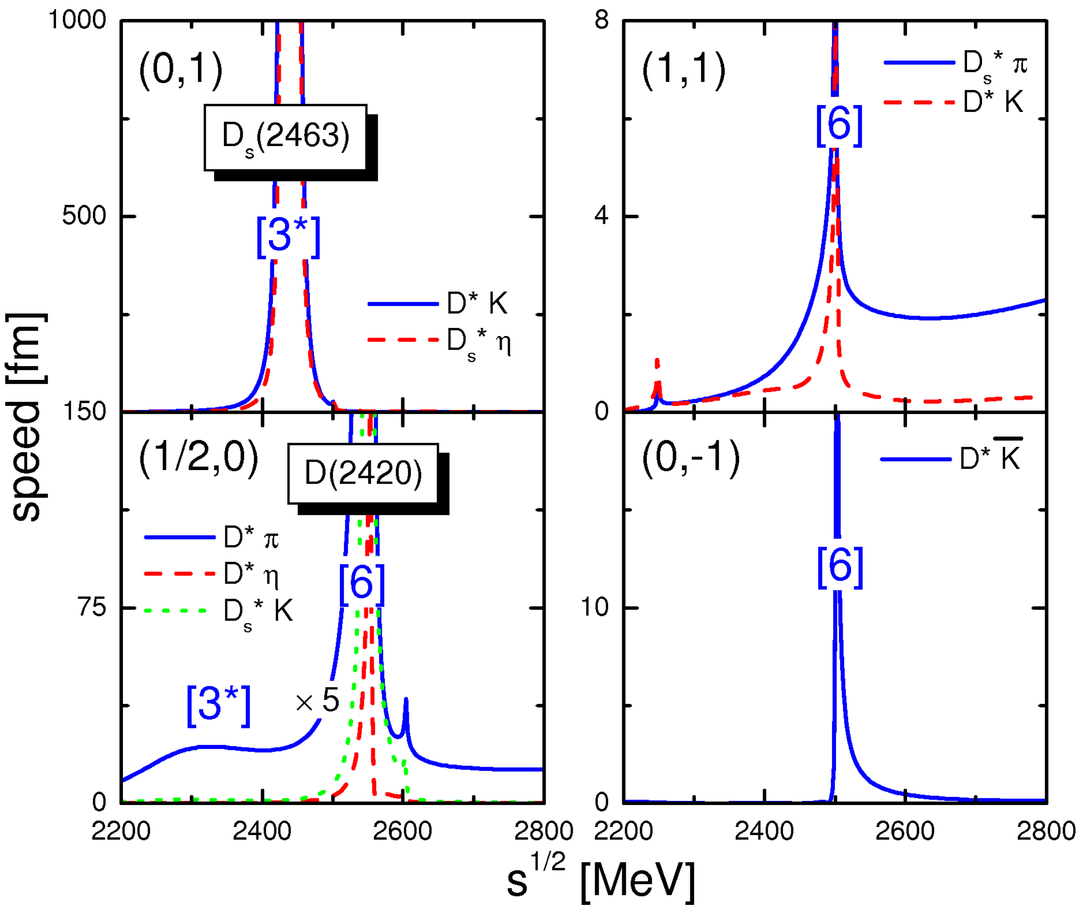


- bound $[\bar{3}]$ in $(0,1)$ at 2303 MeV [BABAR at 2317 MeV]
- broad $[\bar{3}]$ in $(\frac{1}{2},0)$ at 2138 MeV
- narrow $[6]$ in $(\frac{1}{2},0)$ at 2413 MeV
- cusp structures in $(0,-1)$ and $(1,1)$

Phys. Lett. B582, 39

$J^P = 1^+$ charmed meson resonances ($\mathbf{Q}^1$)

✔ **Chiral SU(3) predicts:** anti-triplet and sextet states $8 \otimes \bar{3} = \bar{3} \oplus 6 \oplus \overline{15}$



- bound $[\bar{3}]$ in $(0, 1)$ at 2440 MeV [CLEO at 2463 MeV]
- $M(1^+) - M(0^+) \simeq 140$ MeV
- broad $[\bar{3}]$ in $(\frac{1}{2}, 0)$ at 2325 MeV
- narrow [6] in $(\frac{1}{2}, 0)$ at 2552 MeV [identify with D(2420)]
- cusp structures in $(0, -1)$ and $(1, 1)$

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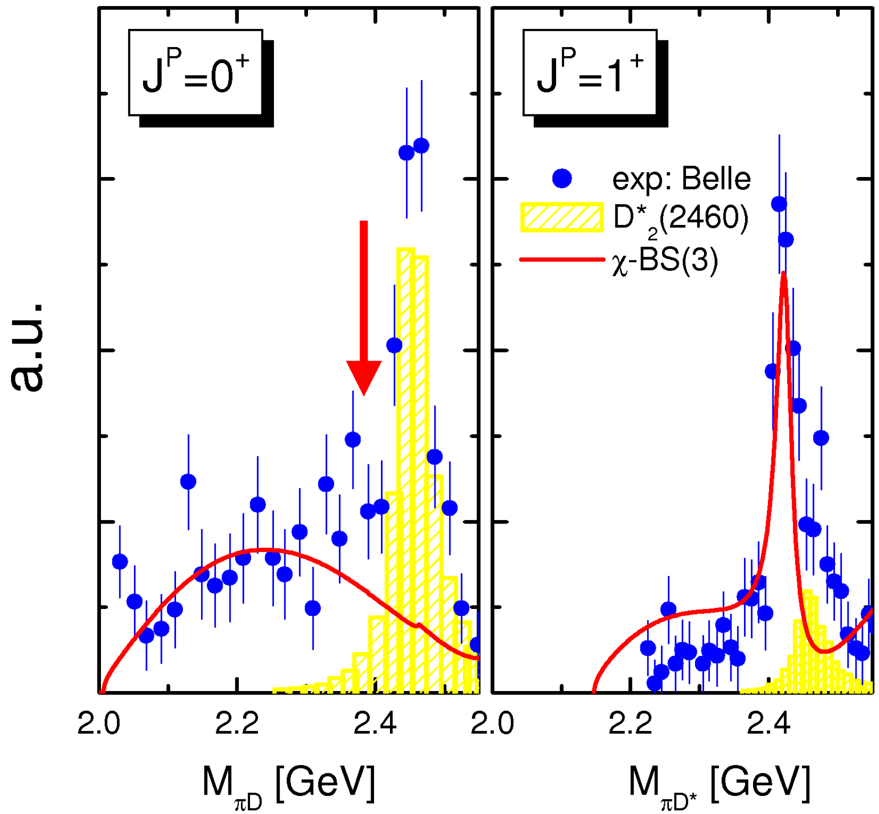
Charmed meson resonance (Q^2)

✔ **Chiral corrections:** 3 parameters tuned to data

(1/2,0)



✔ **"hidden" 0^+-resonance $(I,S) = (\frac{1}{2}, 0)$**

- narrow [6]-state at 2389 MeV
- couples weakly to $\pi D(1867)$ channel

✔ **Is the $D(2420)$ a sextet state ?**

- where is the heavy-quark partner of the 2^+ $D(2460)$?

✔ **$S = -1$ charmed mesons**

- $\overline{K}$ bound at $D(1867)$ and $D_\mu(2008)$
- predict: 2352 MeV $(J^P = 0^+)$ and
  2416 MeV $(J^P = 1^+)$

Nucl. Phys. A733, 142

Summary

Chiral SU(3) coupled-channel dynamics:

- describes meson-nucleon scattering quantitatively
- predicts at leading order baryon and meson resonances
  in light and heavy-light quark sectors

What's next?

Further developments in coupled-channel EFT