Exploring the spectrum of QCD using a space-time lattice

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New Theoretical Tools for Nucleon Resonance Analysis
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Outline

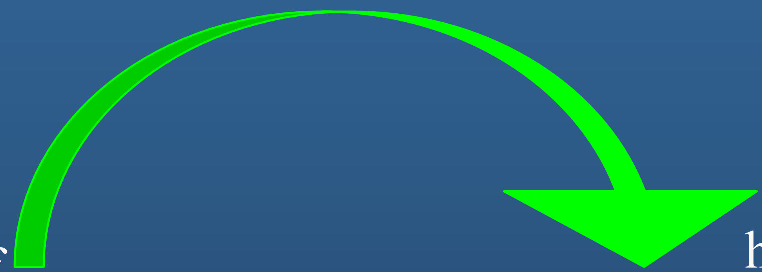
- spectroscopy is a powerful tool for distilling key degrees of freedom
- calculating spectrum of QCD → introduction of space-time lattice
  - spectrum determination requires extraction of excited-state energies
  - discuss how to extract excited-state energies from Monte Carlo estimates of correlation functions in Euclidean lattice field theory
- applications:
  - Yang-Mills glueballs
  - heavy-quark hybrid mesons
  - baryon and meson spectrum (work in progress)

Monte Carlo method with space-time lattice

- introduction of space-time lattice allows Monte Carlo evaluation of path integrals needed to extract spectrum from QCD Lagrangian

L_{QCD}

Lagrangian of QCD



hadron spectrum, structure, transitions

- tool to search for better ways of calculating in gauge theories
  - what dominates the path integrals? (instantons, center vortices,…)
  - construction of effective field theory of glue? (strings,…)

Energies from correlation functions

- stationary state energies can be extracted from asymptotic decay rate of temporal correlations of the fields (in the imaginary time formalism)
- evolution in Heisenberg picture $\phi(t) = e^{Ht}\,\phi(0)\,e^{-Ht}$ ($H =$Hamiltonian)
- spectral representation of a simple correlation function
  - assume transfer matrix, ignore temporal boundary conditions
  - focus only on one time ordering

$$\langle 0|\,\phi(t)\phi(0)\,|0\rangle = \sum_n \langle 0|e^{Ht}\phi(0)\,e^{-Ht}\,|n\rangle\langle n|\phi(0)|0\rangle$$
$$= \sum_n \left|\langle n|\phi(0)\,|0\rangle\right|^2 e^{-(E_n-E_0)t} = \sum_n A_n e^{-(E_n-E_0)t}$$

insert complete set of energy eigenstates (discrete and continuous)

- extract A_1 and $E_1 - E_0$ as $t \to \infty$

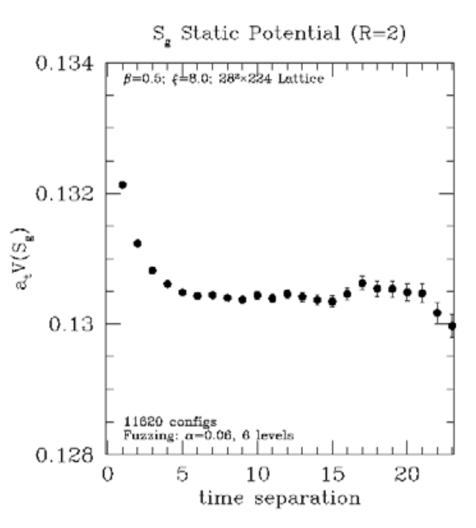
  (assuming $\langle 0|\phi(0)\,|0\rangle = 0$ and $\langle 1|\phi(0)\,|0\rangle \neq 0$)

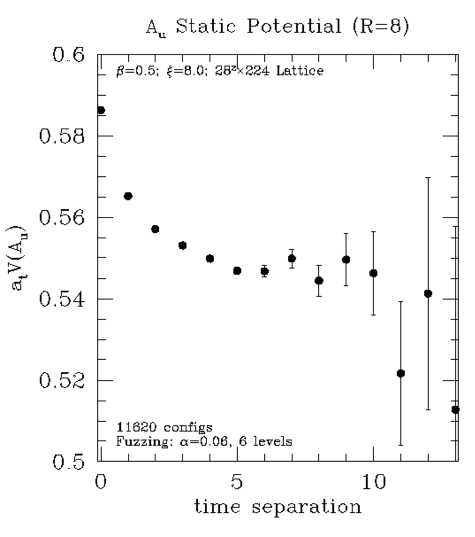
Effective mass

- the "effective mass" is given by $m_{\text{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$
- notice that (take $E_0 = 0$)

$$\lim_{t\to\infty} m_{\text{eff}}(t) = \ln\left(\frac{A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \cdots}{A_1 e^{-E_1(t+1)} + \cdots}\right) \to \ln e^{-E_1} = E_1$$

- the effective mass tends to the actual mass (energy) asymptotically
- effective mass plot is convenient visual tool to see signal extraction
  - seen as a plateau
- plateau sets in quickly for good operator
- excited-state contamination before plateau





Reducing contamination

- statistical noise generally increases with temporal separation t
- effective masses associated with correlation functions of simple local fields do not reach a plateau before noise swamps the signal
  - need better operators
  - better operators have reduced couplings with higher-lying contaminating states
- recipe for making better operators
  - crucial to construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing
  - spatially extended operators
  - use large *set* of operators (variational coefficients)

Principal correlators

- extracting excited-state energies described in
  - C. Michael, NPB **259**, 58 (1985)
  - Luscher and Wolff, NPB **339**, 222 (1990)
- can be viewed as exploiting the variational method
- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta^+(0) | 0 \rangle$ one defines the N *principal correlators* $\lambda_\alpha(t,t_0)$ as the eigenvalues of

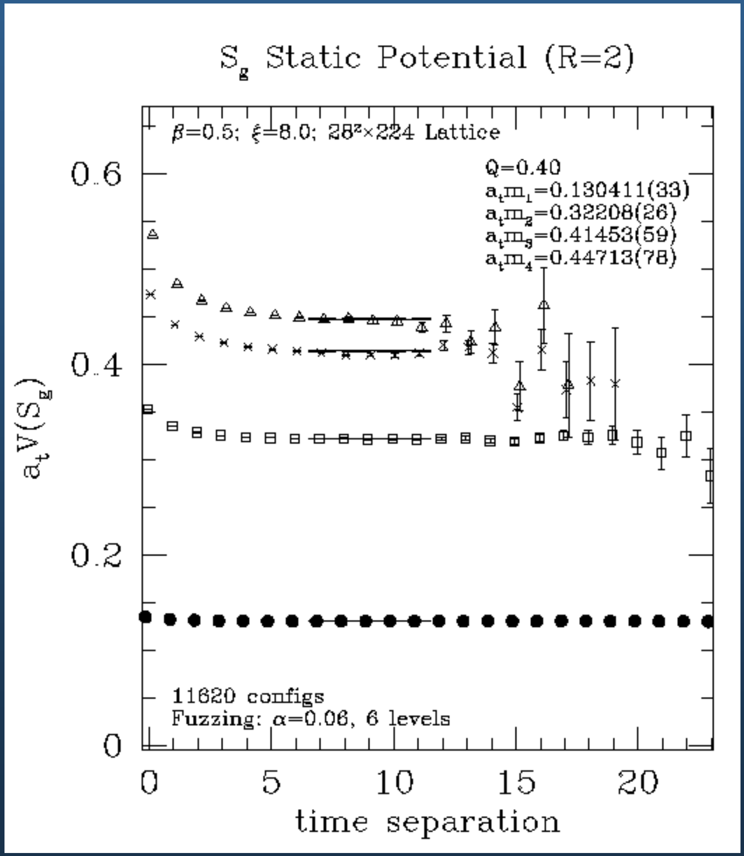
$$C(t_0)^{-1/2}\, C(t)\, C(t_0)^{-1/2}$$

where t_0 (the time defining the "metric") is small

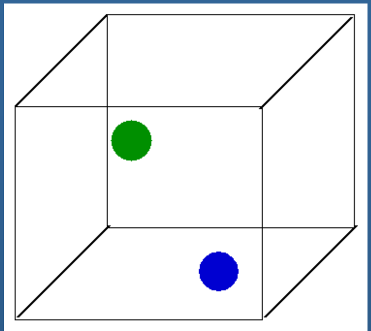
- can show that $\lim_{t\to\infty} \lambda_\alpha(t,t_0) = e^{-(t-t_0)E_\alpha}(1+e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m_\alpha^{\mathrm{eff}}(t) = \ln\left(\dfrac{\lambda_\alpha(t,t_0)}{\lambda_\alpha(t+1,t_0)}\right)$ now tend (plateau) to the N lowest-lying stationary-state energies

Principal effective masses

- just need to perform single-exponential fit to each principal correlator to extract spectrum!
  - can again use sum of two-exponentials to minimize sensitivity to t_{min}
- note that principal effective masses (as functions of time) can cross, approach asymptotic behavior from below
- final results are independent of t_0, but choosing larger values of this reference time can introduce larger errors



Unstable particles (resonances)



- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron, …
- scattering phase shifts → resonance masses, widths (in principle) deduced from finite-box spectrum
  - B. DeWitt, PR **103**, 1565 (1956) (sphere)
  - M. Luscher, NPB**364**, 237 (1991) (cube)
- more modest goal: "ferret" out resonances from scattering states
  - must differentiate resonances from multi-hadron states
  - avoided level crossings, different volume dependences
  - know masses of decay products → placement and pattern of multi-particle states known
  - resonances show up as extra states with little volume dependence

Resonance in a toy model (I)

- O(4) non-linear σ model (Zimmerman et al, NPB(PS) **30**, 879 (1993))

$$S = -2\kappa \sum_{x} \sum_{\mu=1}^{4} \Phi_a(x)\Phi_a(x+\hat{\mu}) + J \sum_{x} \Phi^4(x), \qquad \sum_{a=1}^{4} \Phi_a^2(x) = 1$$

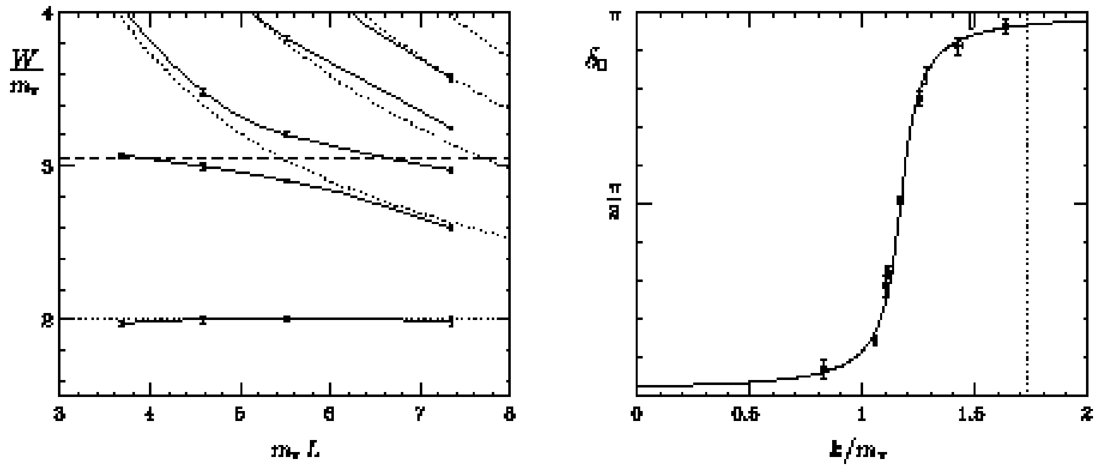


Figure 2. *Two-particle energy spectrum*

Figure 3. *Scattering phase shift δ_0 in the isospin 0 channel*

Resonance in a toy model (II)

- coupled scalar fields: (Rummukainen and Gottlieb, NPB**450**, 397 (1995))

$$S = \frac{1}{2}\int d^4x\left(\left(\partial_\mu\phi\right)^2 + m_\pi^2\phi^2 + \lambda\phi^4 + \left(\partial_\mu\rho\right)^2 + m_\pi^2\rho^2 + \lambda_\rho\rho^4 + g\rho\phi^2\right)$$

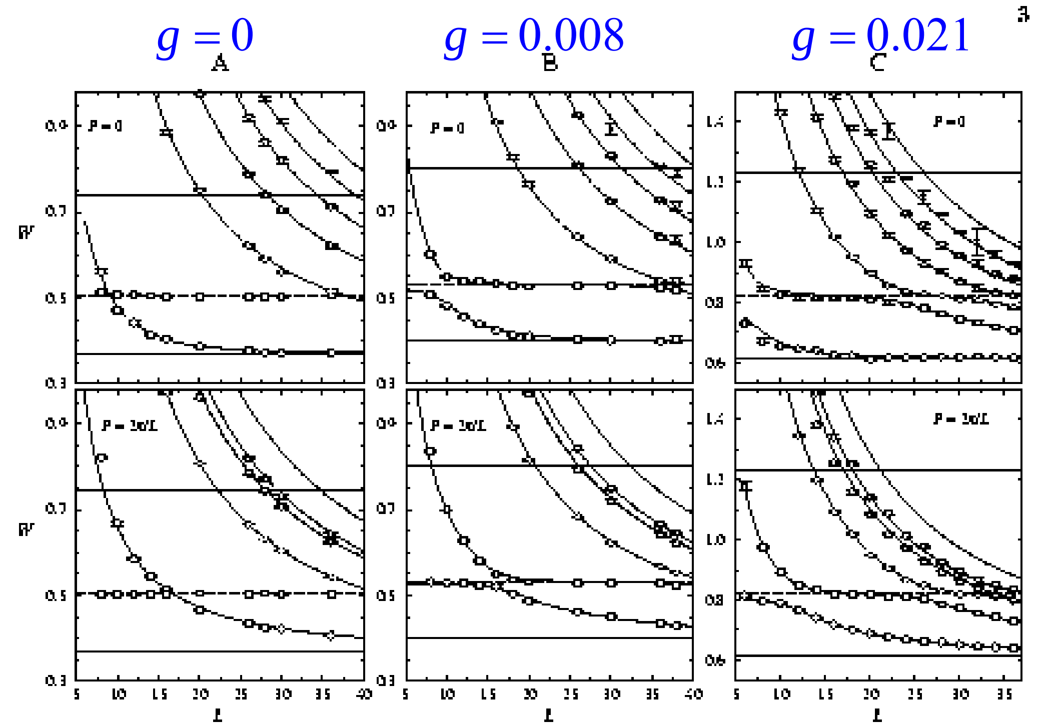
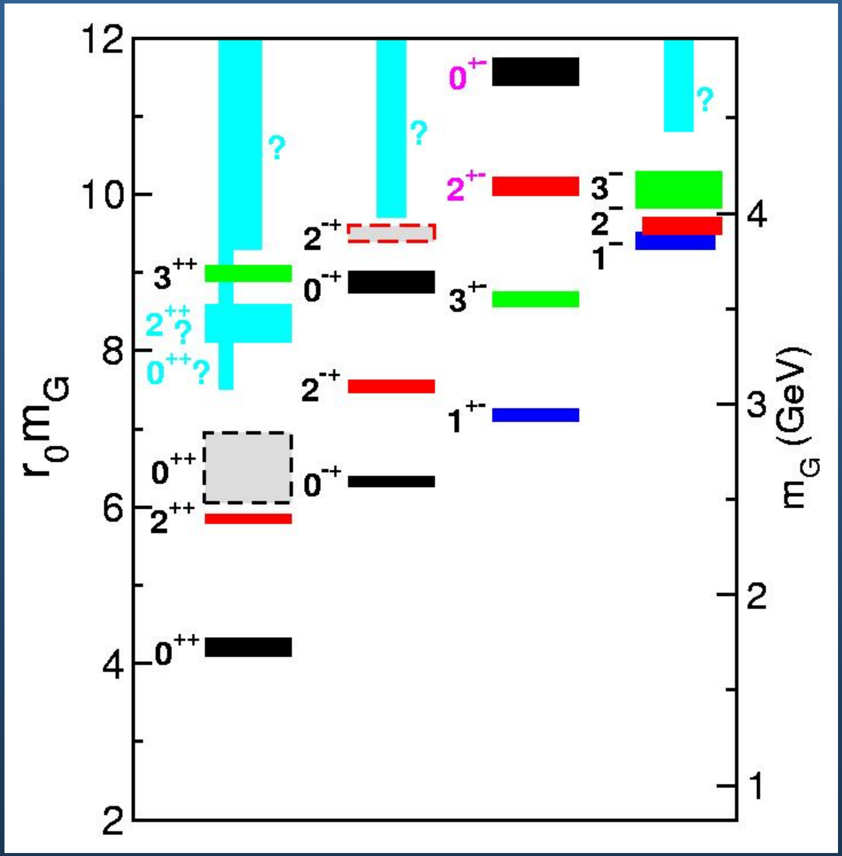


Figure 2. The center of mass energy levels for sectors $\vec{P}=0$ (top row) and $\vec{P}=2\pi/L$ (bottom) for cases A, B and C (see table 1).

Yang-Mills SU(3) Glueball Spectrum

- pure-glue mass spectrum known
  - still needs some "polishing"
  - improve scalar states
- "experimental" results in simpler world (no quarks) to help build models of gluons
- mass *ratios* predicted, overall scale is not
- mass gap with $1 million bounty (Clay mathematics institute)
- glueball structure
  - constituent gluons vs flux loops?

C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509 (1999)



$r_0^{-1} = 410(20)$ MeV, states labeled by J^{PC}

Glueballs (bag model)

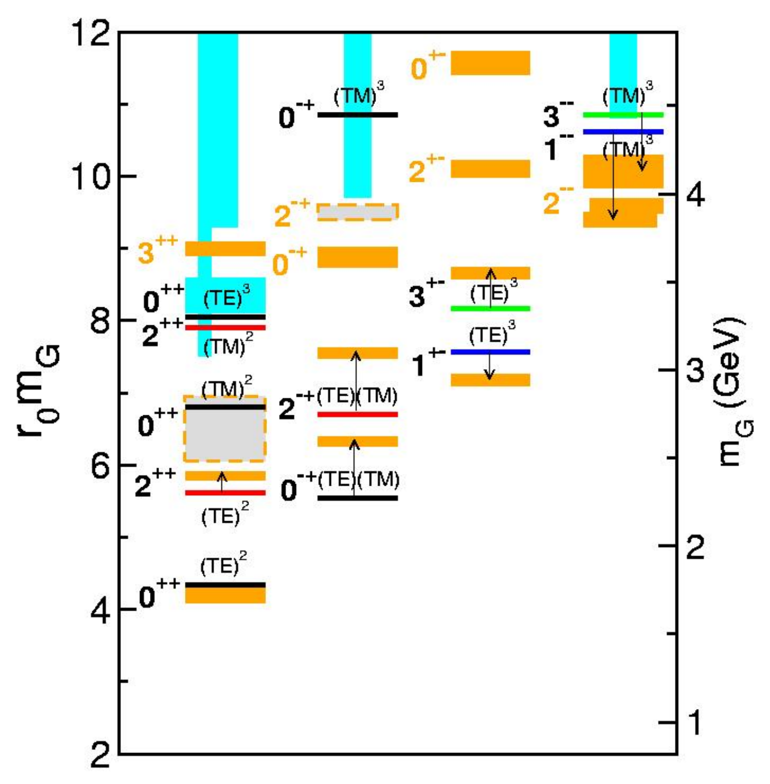
- qualitative agreement with bag model
  - constituent gluons are TE or TM modes in spherical cavity
  - Hartree modes with residual perturbative interactions
  - center-of-mass correction

	1983	1993
	light baryon spectroscopy	static-quark potential
α_s	1.0	0.5
$B^{1/4}$	230 MeV	280 MeV

- recent calculation using another constituent gluon model shows qualitative agreement

Szczepaniak, Swanson, PLB577, 61 (2003)

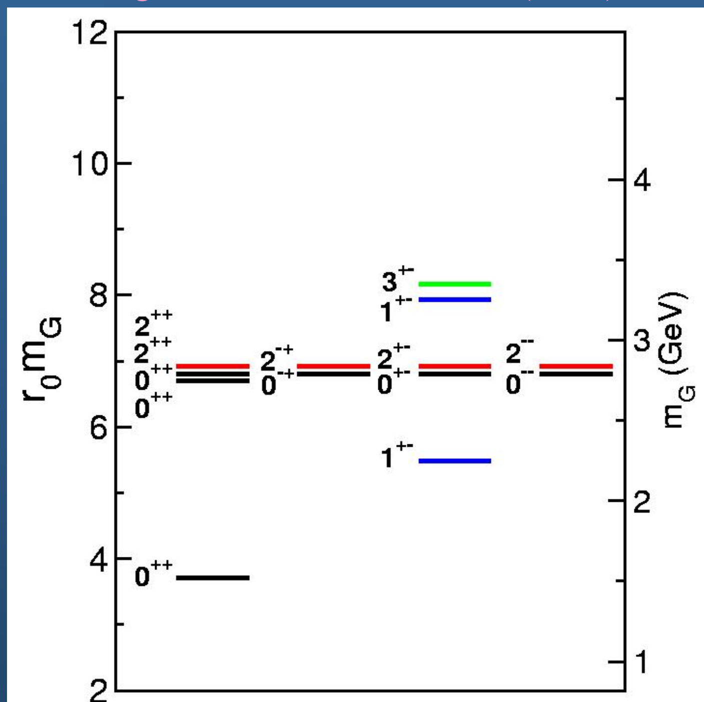
Carlson, Hansson, Peterson, PRD27, 1556 (1983);
J. Kuti (private communication)

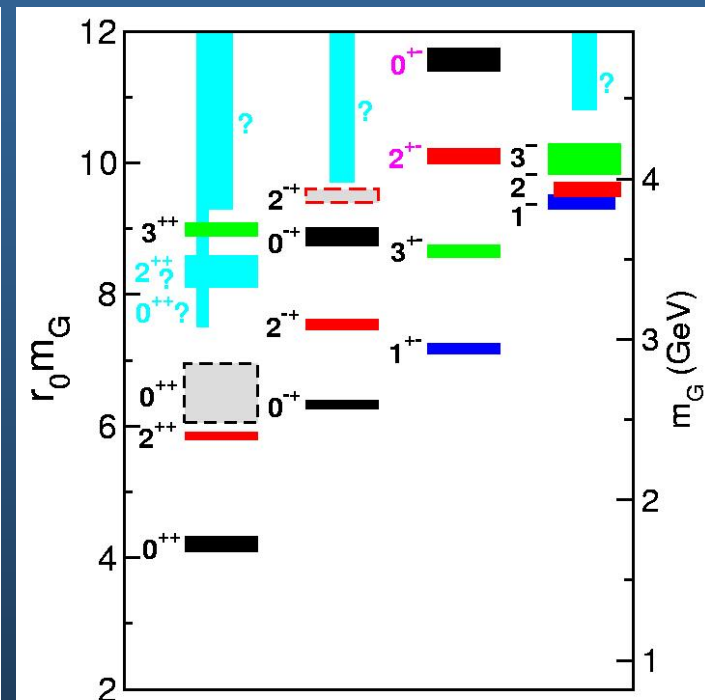


Glueballs (flux tube model)

- disagreement with one particular string model

Isgur, Paton, PRD**31**, 2910 (1985)

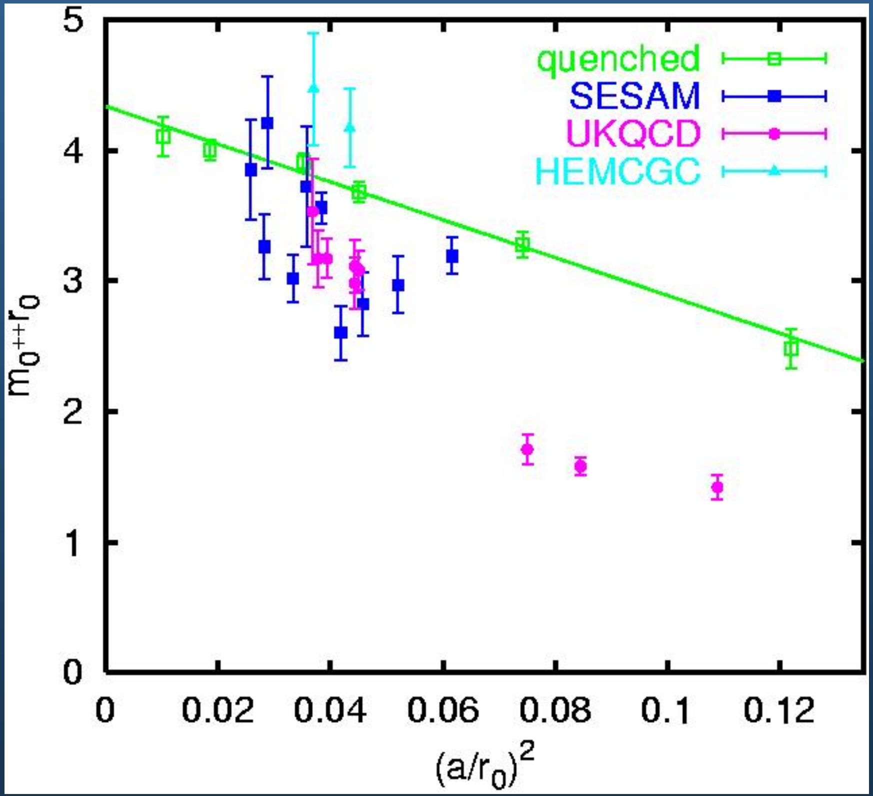




- future comparisons:
  - with more sophisticated string models (soliton knots)
  - AdS theories, duality

Inclusion of quark loops

- scalar glueball results 2002
  - quark masses near strange
- still exploratory
- difficult to get adequate statistics
- light quarks problematic
- mixing, resonances
  - no correlation matrices



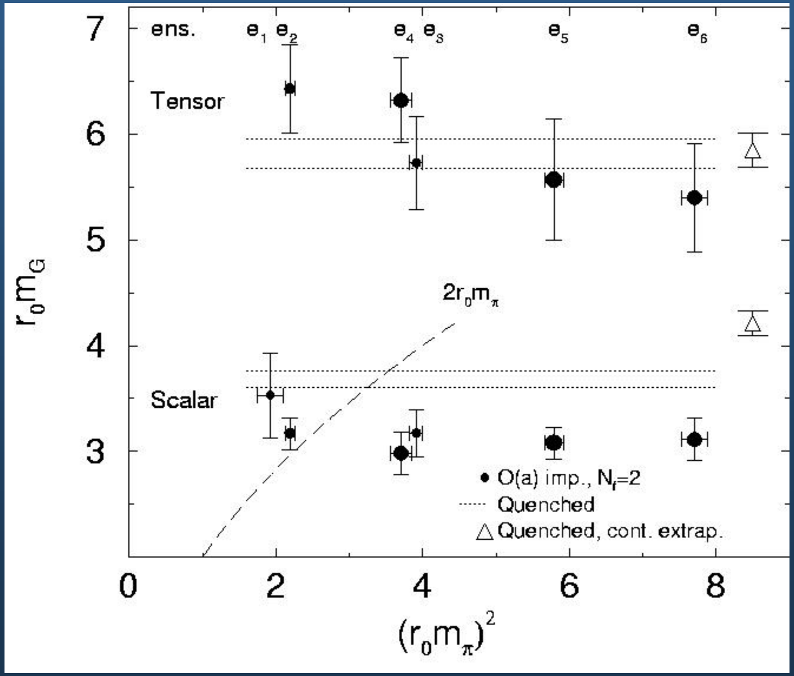
SESAM: PRD**62**, 054503 (2000)

UKQCD: PRD**65**, 014508 (2002)

HEMCGC: PRD**44**, 2090 (1991)

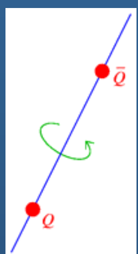
Unquenched masses

- *un*quenched analysis (Hart, Teper, PRD65, 034502 (2002))
- Wilson gauge, clover fermion action $N_f = 2, \quad a \approx 0.1\,\mathrm{fm}, \quad m_q \geq \frac{1}{2} m_s$
- tensor glueball mass same as pure-gauge
- scalar mass suppression: 0.85 of pure-gauge
  - not finite volume effect
  - independent of quark mass! → lattice artifact (another "curve ball")
  - most likely explanation: fermion action adds "adjoint piece"
- quarkonium states ignored

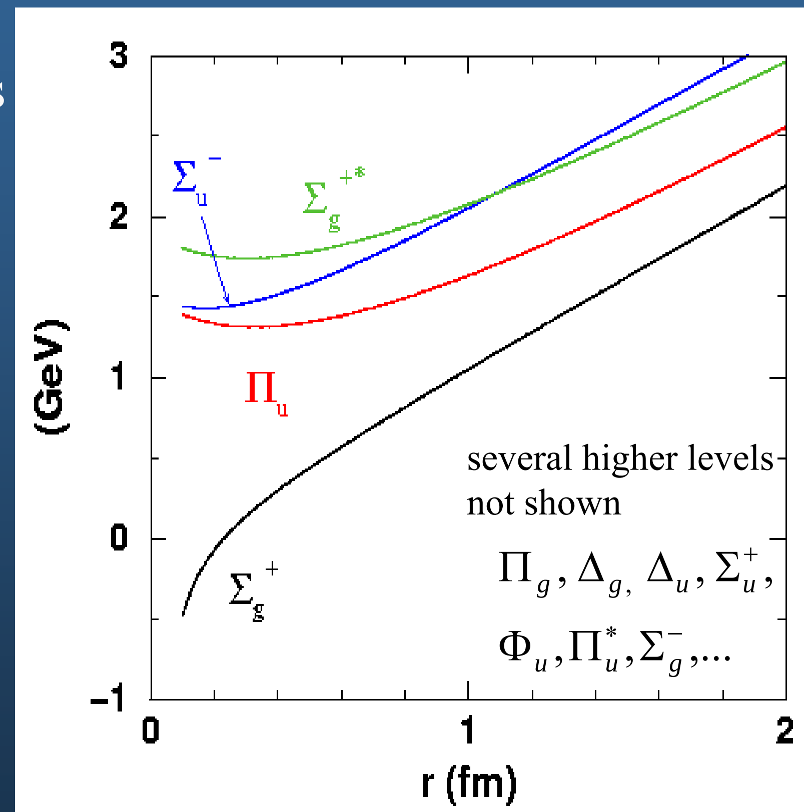


Excitations of static quark potential

- gluon field in presence of static quark-antiquark pair can be *excited*
- classification of states: (notation from molecular physics)
  - magnitude of glue spin projected onto molecular axis
    $\Lambda = 0,1,2,\ldots$
    $= \Sigma, \Pi, \Delta, \ldots$



  - charge conjugation + parity about midpoint
    $\eta = g$ (even)
    $= u$ (odd)
  - chirality (reflections in plane containing axis) Σ^+, Σ^-
    $\Pi, \Delta, \ldots$ doubly degenerate (Λ doubling)



Juge, Kuti, Morningstar, PRL 90, 161601 (2003)

Initial remarks

- viewpoint adopted:
  - the nature of the confining gluons is best revealed in its

    excitation spectrum
- robust feature of any bosonic string description:
  - $N\pi / R$ gaps for large quark-antiquark separations
- details of underlying string description encoded in the fine structure
- study different gauge groups, dimensionalities
- several lattice spacings, finite volume checks
- very large number of fits to principal correlators → web page interface set up to facilitate scrutinizing/presenting the results

String spectrum

- spectrum expected for a non-interacting bosonic string at large R
  - standing waves: $m = 1,2,3,\ldots$ with circular polarization $\pm$
  - occupation numbers: n_{m+}, n_{m-}
  - energies E
  - string quantum number N
  - spin projection Λ
  - CP η_{CP}

$$E = E_0 + N\pi / R$$

$$N = \sum_{m=1}^{\infty} (n_{m+} + n_{m-})$$

$$\Lambda = \left| \sum_{m=1}^{\infty} (n_{m+} - n_{m-}) \right|$$

$$\eta_{CP} = (-1)^N$$

String spectrum (*N=1,2,3*)

- level orderings for *N=1,2,3*

$N=0$:	Σ_g^+	$\vert 0\rangle$	
$N=1$:	Π_u	$a_{1+}^\dagger \vert 0\rangle$	$a_{1-}^\dagger \vert 0\rangle$
$N=2$:	$\Sigma_g^{+\prime}$	$a_{1+}^\dagger a_{1-}^\dagger \vert 0\rangle$	
	Π_g	$a_{2+}^\dagger \vert 0\rangle$	$a_{2-}^\dagger \vert 0\rangle$
	Δ_g	$(a_{1+}^\dagger)^2 \vert 0\rangle$	$(a_{1-}^\dagger)^2 \vert 0\rangle$
$N=3$:	Σ_u^+	$(a_{1+}^\dagger a_{2-}^\dagger + a_{1-}^\dagger a_{2+}^\dagger)\vert 0\rangle$	
	Σ_u^-	$(a_{1+}^\dagger a_{2-}^\dagger - a_{1-}^\dagger a_{2+}^\dagger)\vert 0\rangle$	
	$\Pi_u^\prime$	$a_{3+}^\dagger \vert 0\rangle$	$a_{3-}^\dagger \vert 0\rangle$
	$\Pi_u^\prime$	$(a_{1+}^\dagger)^2 a_{1-}^\dagger \vert 0\rangle$	$a_{1+}^\dagger (a_{1-}^\dagger)^2 \vert 0\rangle$
	Δ_u	$a_{1+}^\dagger a_{2+}^\dagger \vert 0\rangle$	$a_{1-}^\dagger a_{2-}^\dagger \vert 0\rangle$
	Φ_u	$(a_{1+}^\dagger)^3 \vert 0\rangle$	$(a_{1-}^\dagger)^3 \vert 0\rangle$

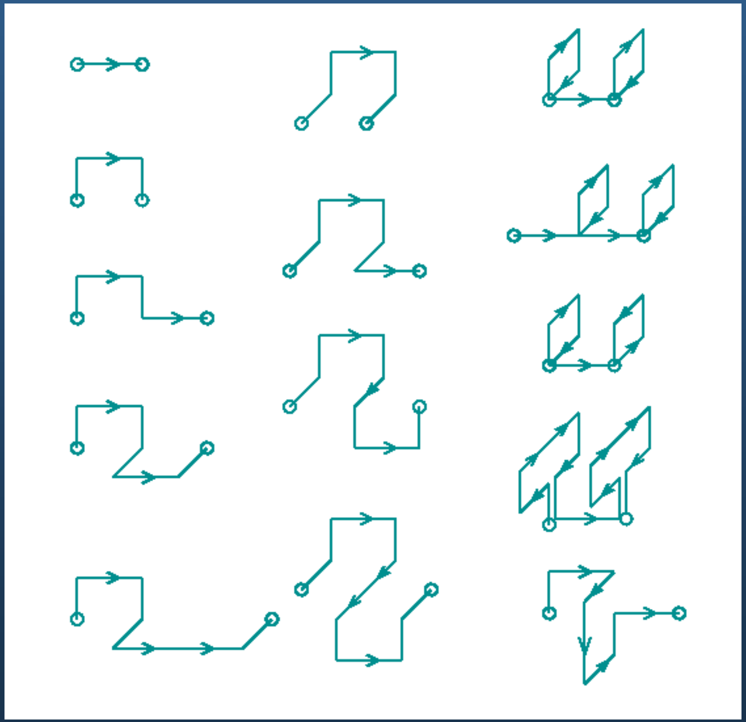
String spectrum (*N*=4)

- *N*=4 levels

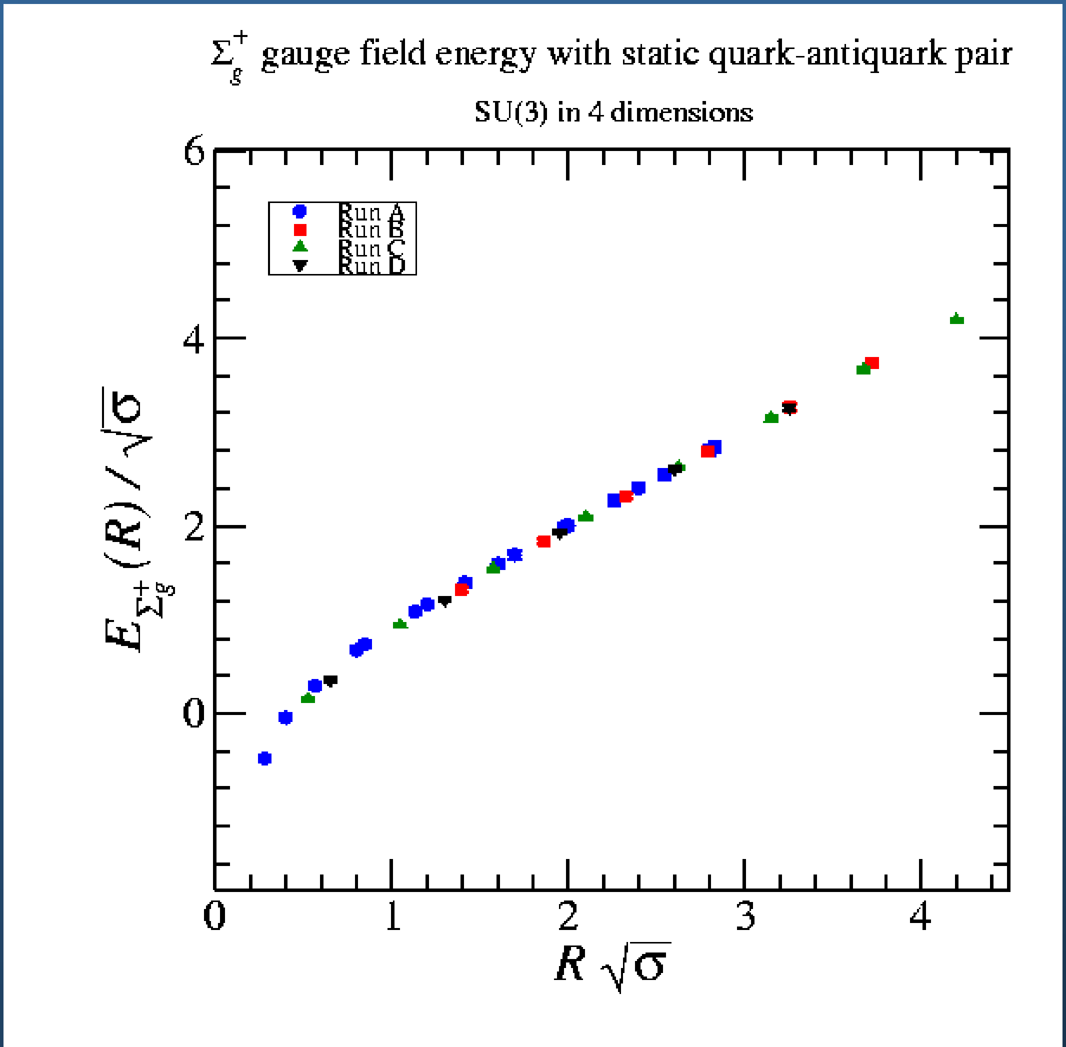
$N=4$:	$\Sigma_g^{+\prime\prime}$	$a_{2+}^\dagger a_{2-}^\dagger \vert 0\rangle$	
	$\Sigma_g^{+\prime\prime}$	$(a_{1+}^\dagger)^2(a_{1-}^\dagger)^2\vert 0\rangle$	
	$\Sigma_g^{+\prime\prime}$	$(a_{1+}^\dagger a_{3-}^\dagger + a_{1-}^\dagger a_{3+}^\dagger)\vert 0\rangle$	
	Σ_g^{-}	$(a_{1+}^\dagger a_{3-}^\dagger - a_{1-}^\dagger a_{3+}^\dagger)\vert 0\rangle$	
	$\Pi_g^\prime$	$a_{4+}^\dagger\vert 0\rangle$	$a_{4-}^\dagger\vert 0\rangle$
	$\Pi_g^\prime$	$(a_{1+}^\dagger)^2 a_{2-}^\dagger\vert 0\rangle$	$(a_{1-}^\dagger)^2 a_{2+}^\dagger\vert 0\rangle$
	$\Pi_g^\prime$	$a_{1+}^\dagger a_{1-}^\dagger a_{2+}^\dagger\vert 0\rangle$	$a_{1+}^\dagger a_{1-}^\dagger a_{2-}^\dagger\vert 0\rangle$
	$\Delta_g^\prime$	$a_{1+}^\dagger a_{3+}^\dagger\vert 0\rangle$	$a_{1-}^\dagger a_{3-}^\dagger\vert 0\rangle$
	$\Delta_g^\prime$	$(a_{2+}^\dagger)^2\vert 0\rangle$	$(a_{2-}^\dagger)^2\vert 0\rangle$
	$\Delta_g^\prime$	$(a_{1+}^\dagger)^3 a_{1-}^\dagger\vert 0\rangle$	$a_{1+}^\dagger(a_{1-}^\dagger)^3\vert 0\rangle$
	Φ_g	$(a_{1+}^\dagger)^2 a_{2+}^\dagger\vert 0\rangle$	$(a_{1-}^\dagger)^2 a_{2-}^\dagger\vert 0\rangle$
	Γ_g	$(a_{1+}^\dagger)^4\vert 0\rangle$	$(a_{1-}^\dagger)^4\vert 0\rangle$

Generalized Wilson loops

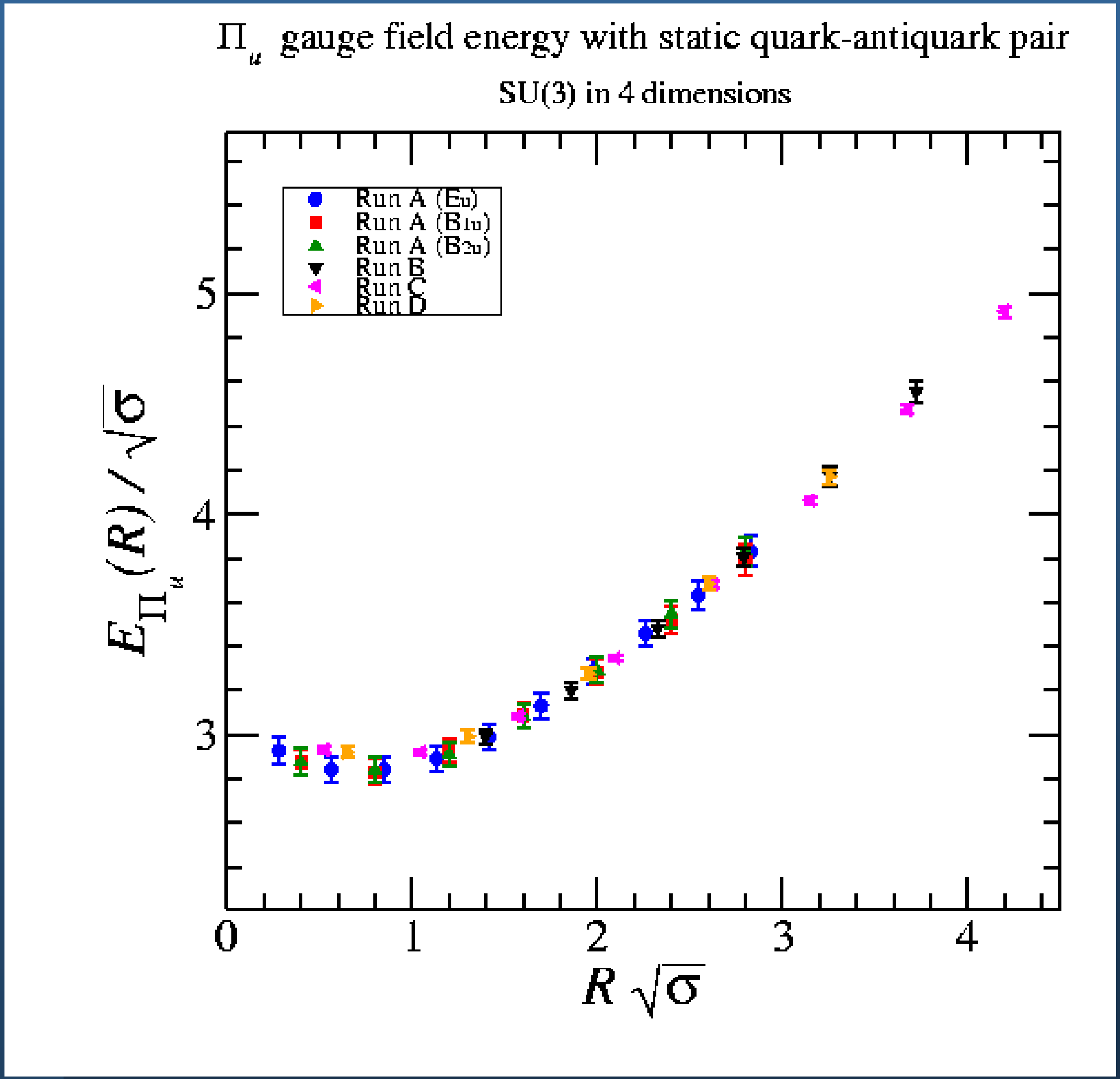
- gluonic terms extracted from generalized Wilson loops
- large set of gluonic operators → correlation matrix
- link variable smearing, blocking
- anisotropic lattice, improved actions



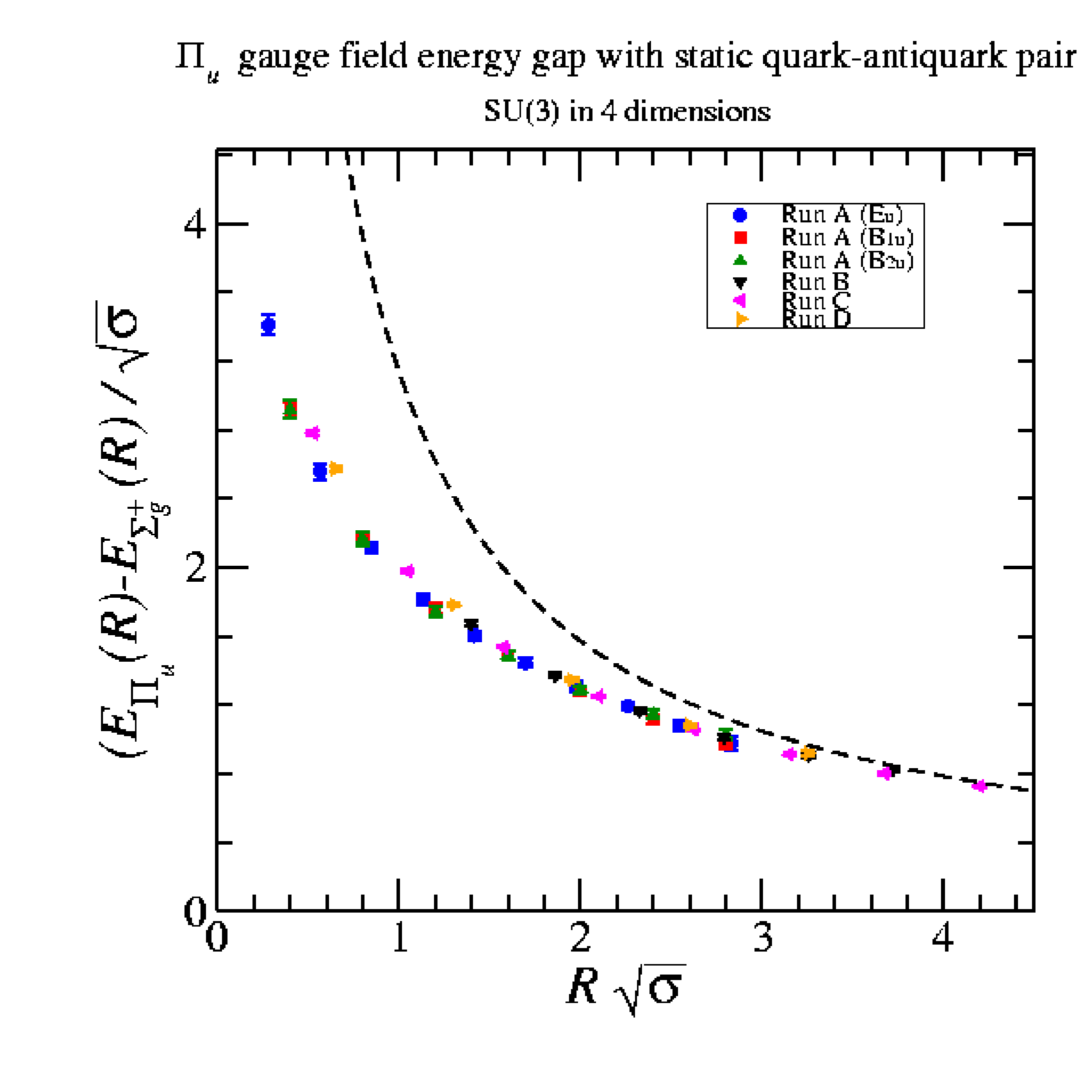
Ground state



First-excited state

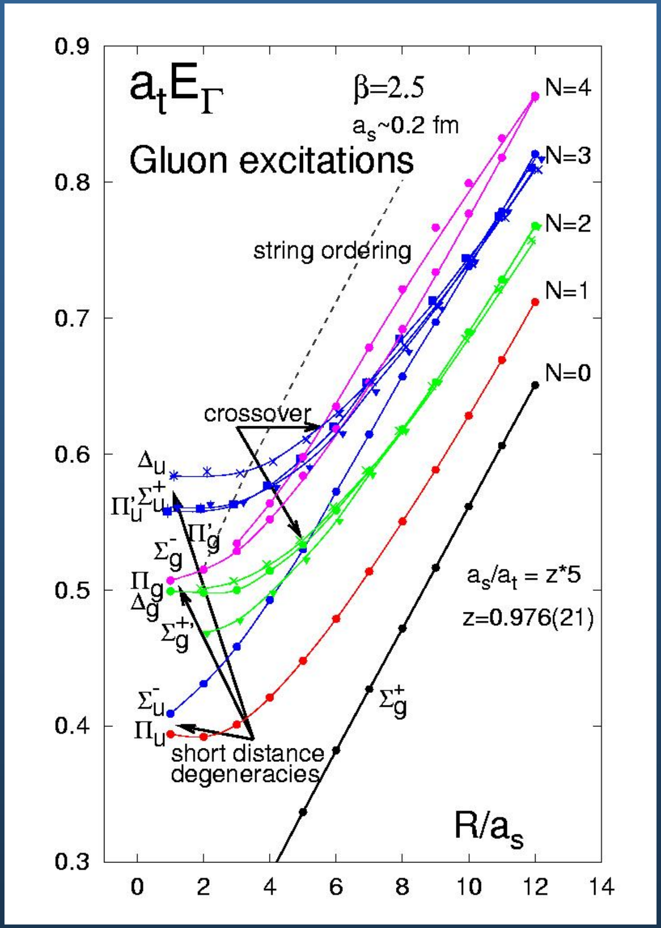


First-excited state gap



Three scales

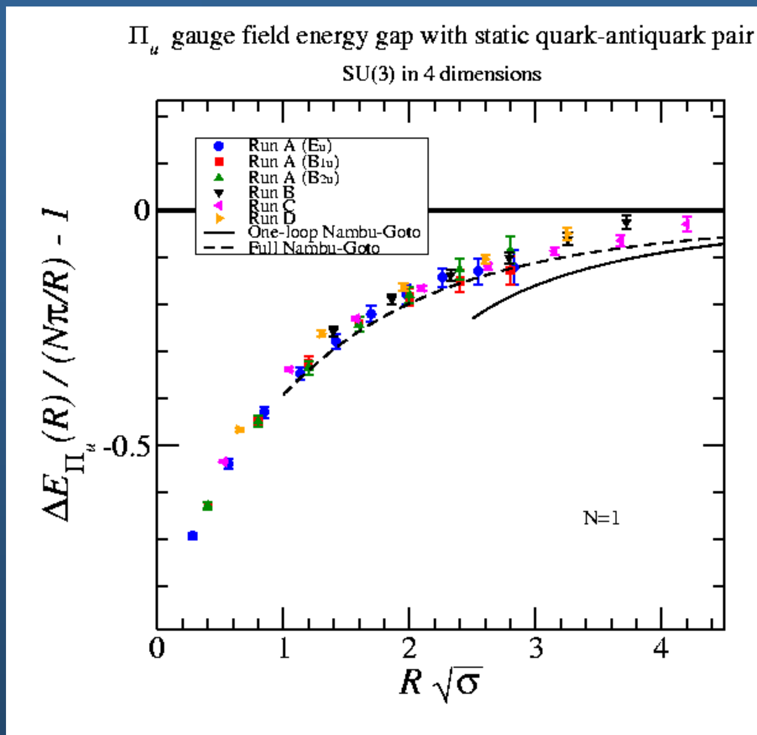
- studied the energies of 16 stationary states of gluons in the presence of static quark-antiquark pair
- small quark-antiquark separations R
  - excitations consistent with states from multipole OPE
- crossover region $0.5\text{fm} < R < 1\text{fm}$
  - dramatic level rearrangement
- large separations $R > 1\text{fm}$
  - excitations consistent with expectations from string models

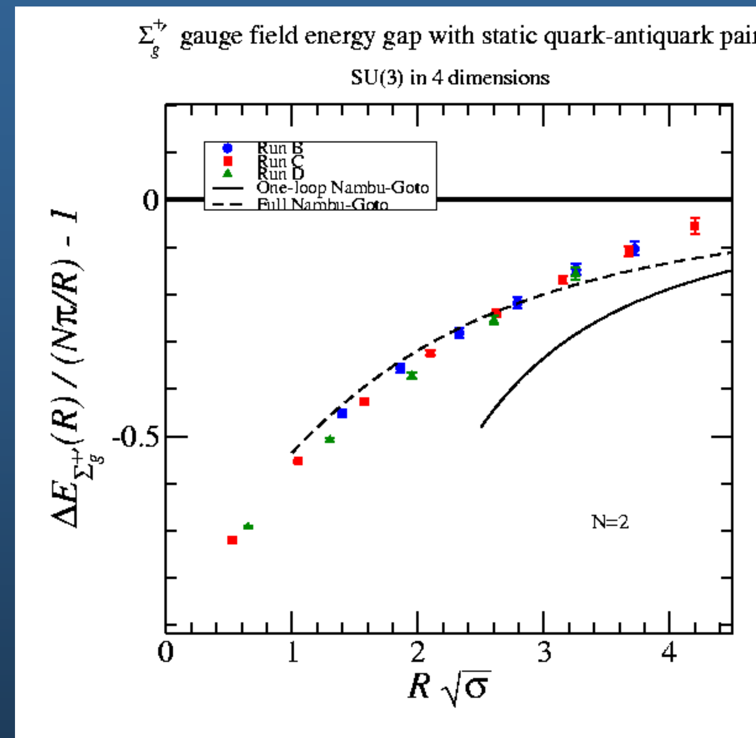


Juge, Kuti, Morningstar, PRL 90, 161601 (2003)

Gluon excitation gaps ($N=1,2$)

- comparison of gaps with $N\pi / R$ and Nambu-Goto





Gluon excitation gaps (N=1,2)

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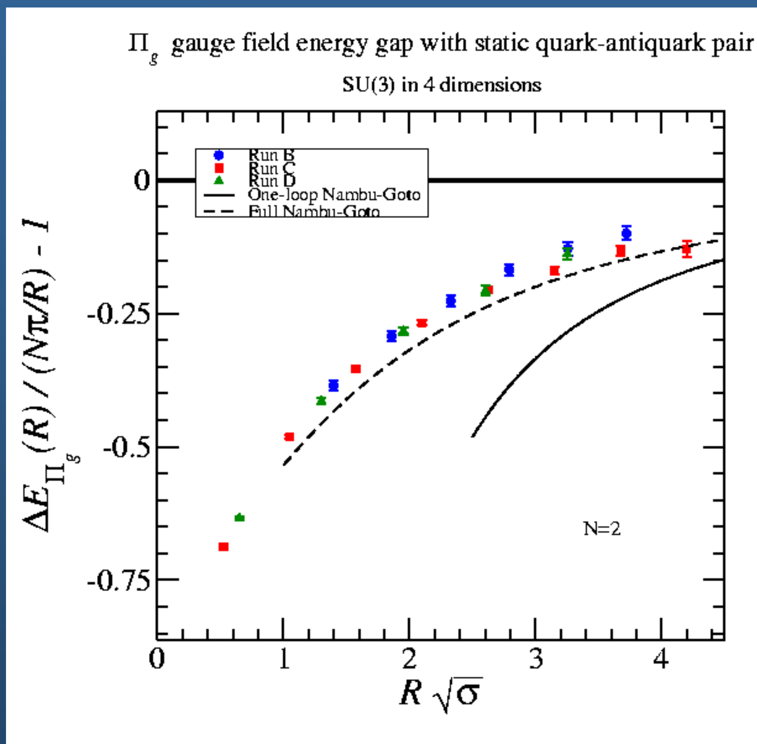
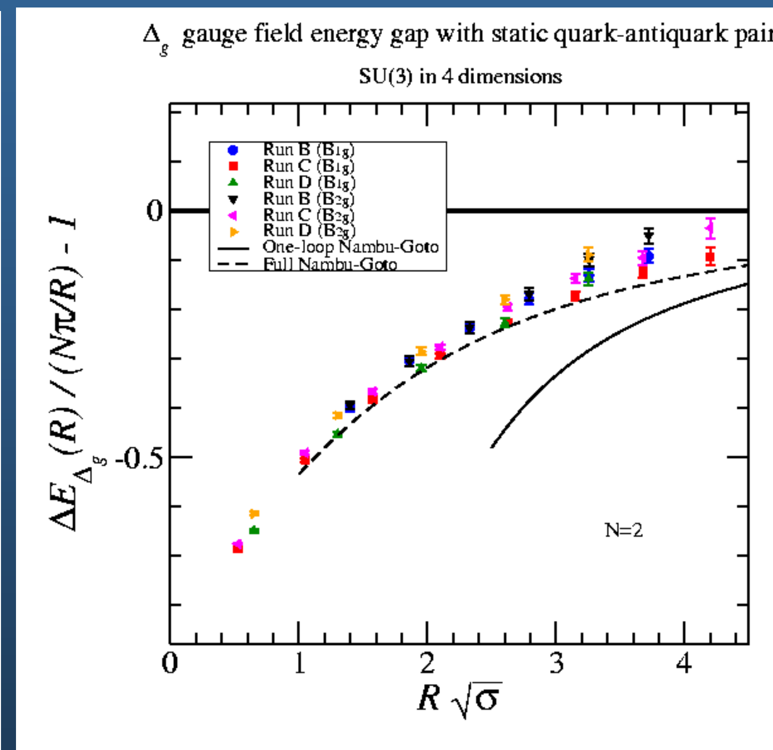
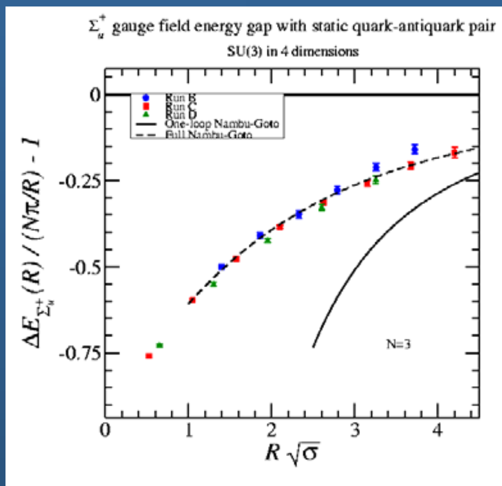
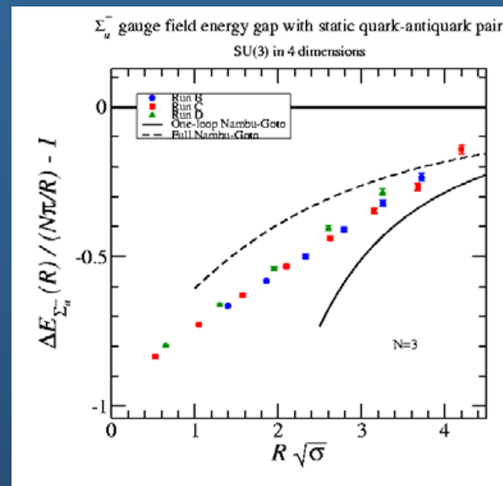


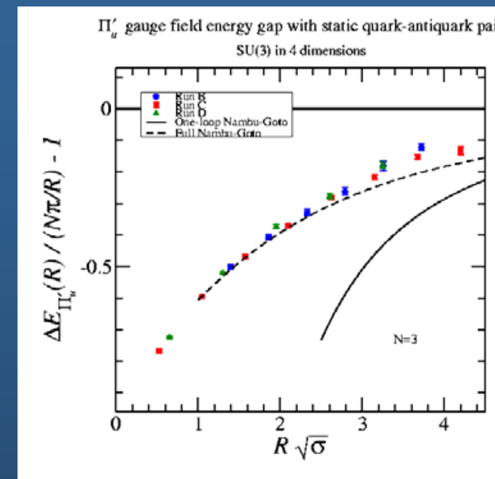


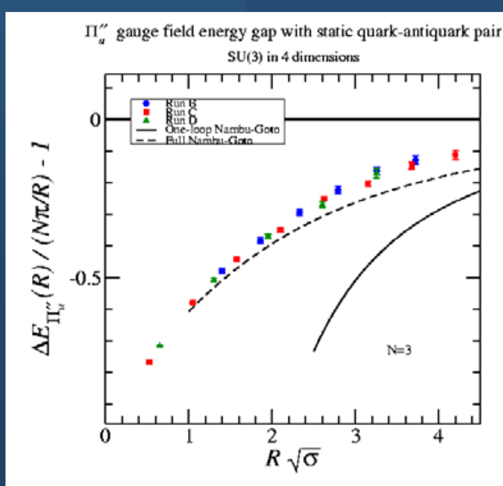
Gluon excitation gaps (N=3)

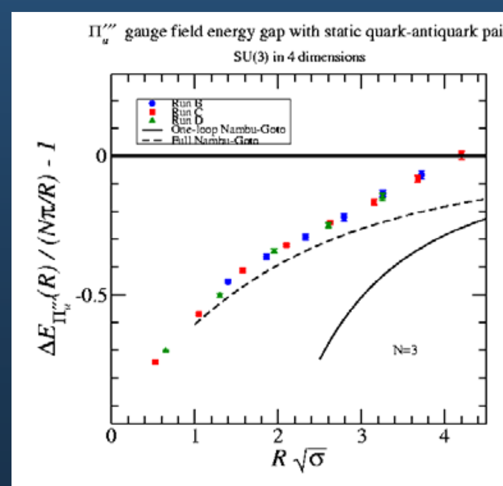
- comparison of gaps with $N\pi / R$ and Nambu-Goto

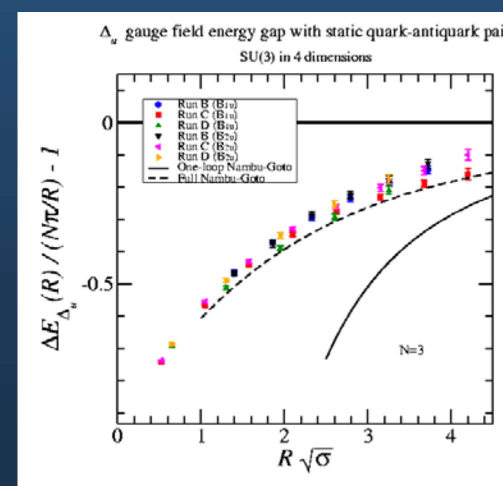






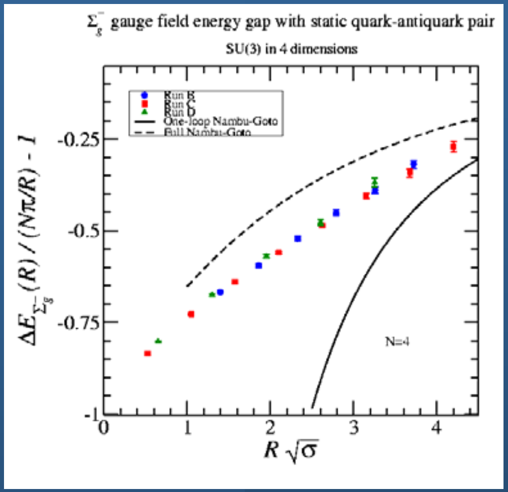


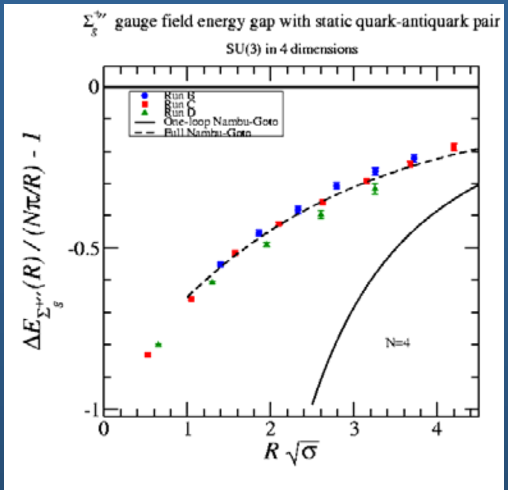


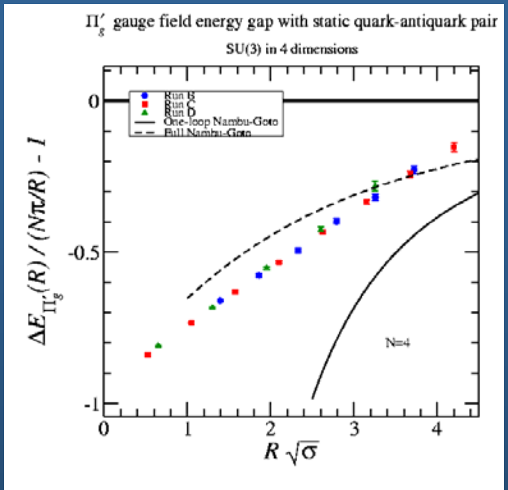


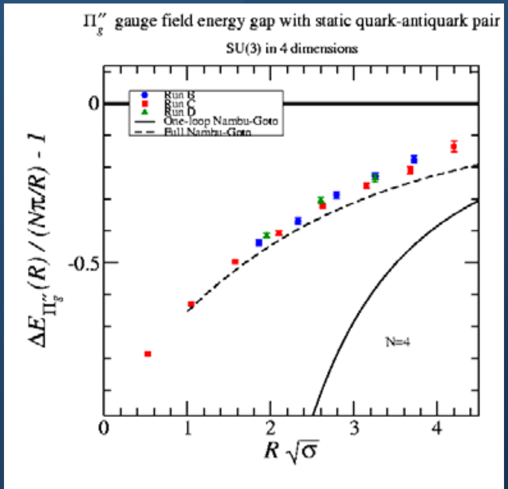
Gluon excitation gaps ($N=4$)

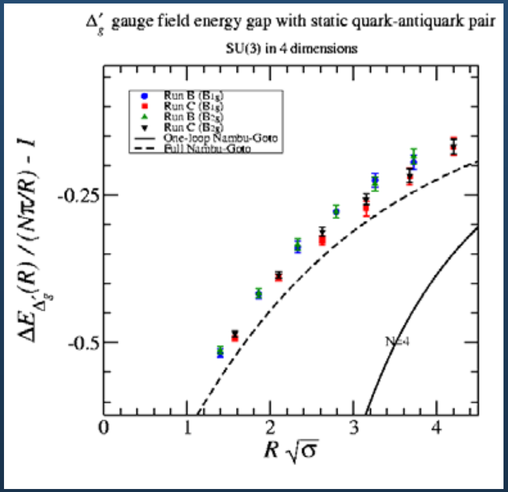
- comparison of gaps with $N\pi / R$ and Nambu-Goto





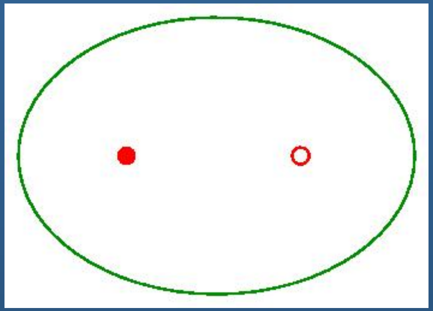


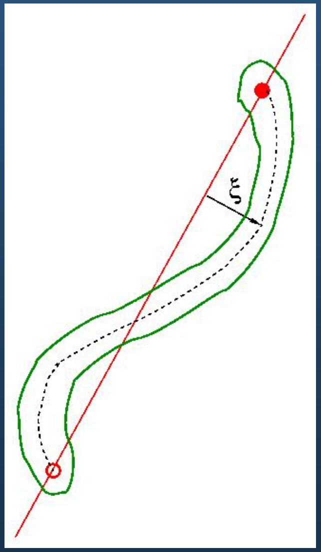




Possible interpretation

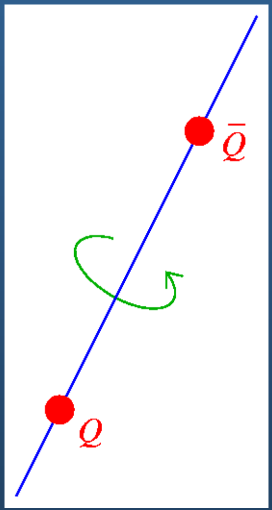
- small *R*
  - strong *E* field of $q\bar{q}$-pair repels physical vacuum (dual Meissner effect) creating a *bubble*
  - separation of degrees of freedom
    - gluonic modes inside bubble (low lying)
    - bubble surface modes (higher lying)
- large *R*
  - bubble stretches into thin tube of flux
  - separation of degrees of freedom
    - collective motion of tube (low lying)
    - internal gluonic modes (higher lying)
  - low-lying modes described by an effective string theory ($N\pi/R$ gaps – Goldstone modes)





Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- early work: Hasenfratz, Horgan, Kuti, Richard (1980), Michael, Griffiths, Rakow (1983)
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - slow heavy quarks ←→ nuclei
  - fast gluon field ←→ electrons
    (and light quarks)



- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - gluons fully relativistic, interacting
  - potentials computed in lattice simulations
- nonrelativistic quark motion described in *leading order* by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$

$$\left\{\frac{p^2}{2\mu}+V_{Q\bar{Q}}(r)\right\}\psi_{Q\bar{Q}}(r)=E\,\psi_{Q\bar{Q}}(r)$$

- conventional mesons from Σ_g^+ ; hybrids from $\Pi_u, \Sigma_u^-, \ldots$

Leading Born-Oppenheimer

- replace covariant derivative $\vec{D}^2$ by $\vec{\nabla}^2$ → neglects retardation
- neglect quark spin effects
- solve radial Schrodinger equation

$$\frac{-1}{2\mu}\frac{d^2u(r)}{dr^2}+\left\{\frac{\langle L_{q\bar{q}}^2\rangle}{2\mu r^2}+V_{q\bar{q}}(r)\right\}u(r)=Eu(r)$$

- angular momentum

$$\vec{J}=\vec{L}+\vec{S} \qquad \vec{S}=\vec{s}_q+\vec{s}_{\bar{q}} \qquad \vec{L}=\vec{L}_{q\bar{q}}+\vec{J}_g$$

- in LBO, L and S are good quantum numbers
- centrifugal term

$$\langle \vec{L}_{q\bar{q}}^2\rangle = L(L+1)-2\Lambda^2+\langle \vec{J}_g^2\rangle$$

$$\langle \vec{J}_g^2\rangle = 0 \quad (\Sigma_g^+)$$
$$= 2 \quad (\Pi_u, \Sigma_u^-)$$

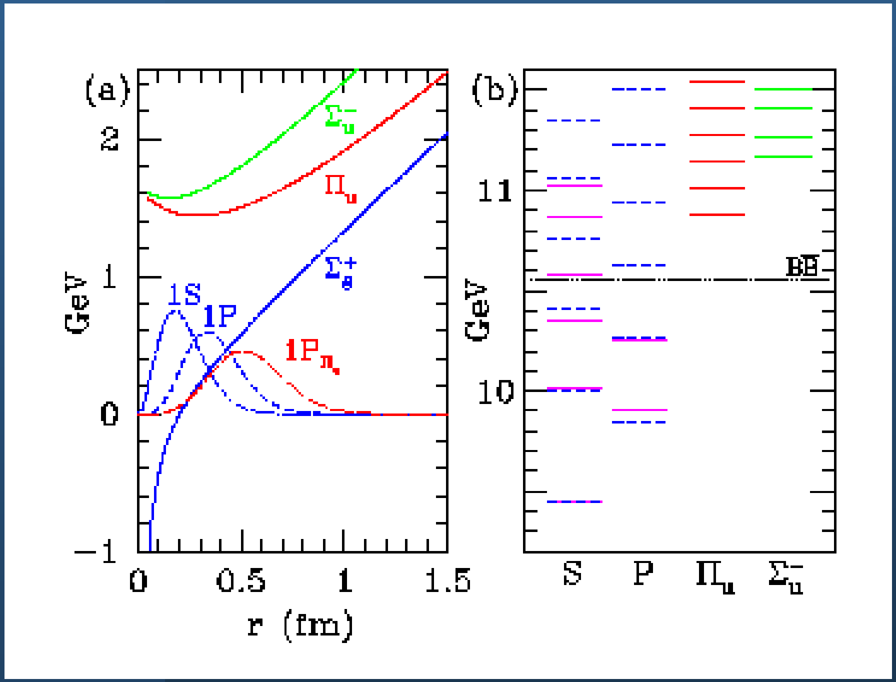
- J^{PC} eigenstates → Wigner rotations

$$|LSJM;\Lambda\eta\rangle+\varepsilon|LSJM;-\Lambda\eta\rangle$$

  - η is CP, $\varepsilon=\pm 1$ for $\Lambda\geq 1$, $\varepsilon=\pm 1$ for $\Sigma^\pm$
- LBO allowed J^{PC} → $P=\varepsilon(-1)^{L+\Lambda+1}$, $C=\eta\varepsilon(-1)^{L+S+\Lambda}$

Leading Born-Oppenheimer spectrum

- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\overline{B}$ threshold
- plethora of hybrid states predicted *when light quark loops ignored*
- but is a Born-Oppenheimer treatment valid?



LBO degeneracies:

$\Sigma_g^+(S):\quad 0^{-+}, 1^{--}$

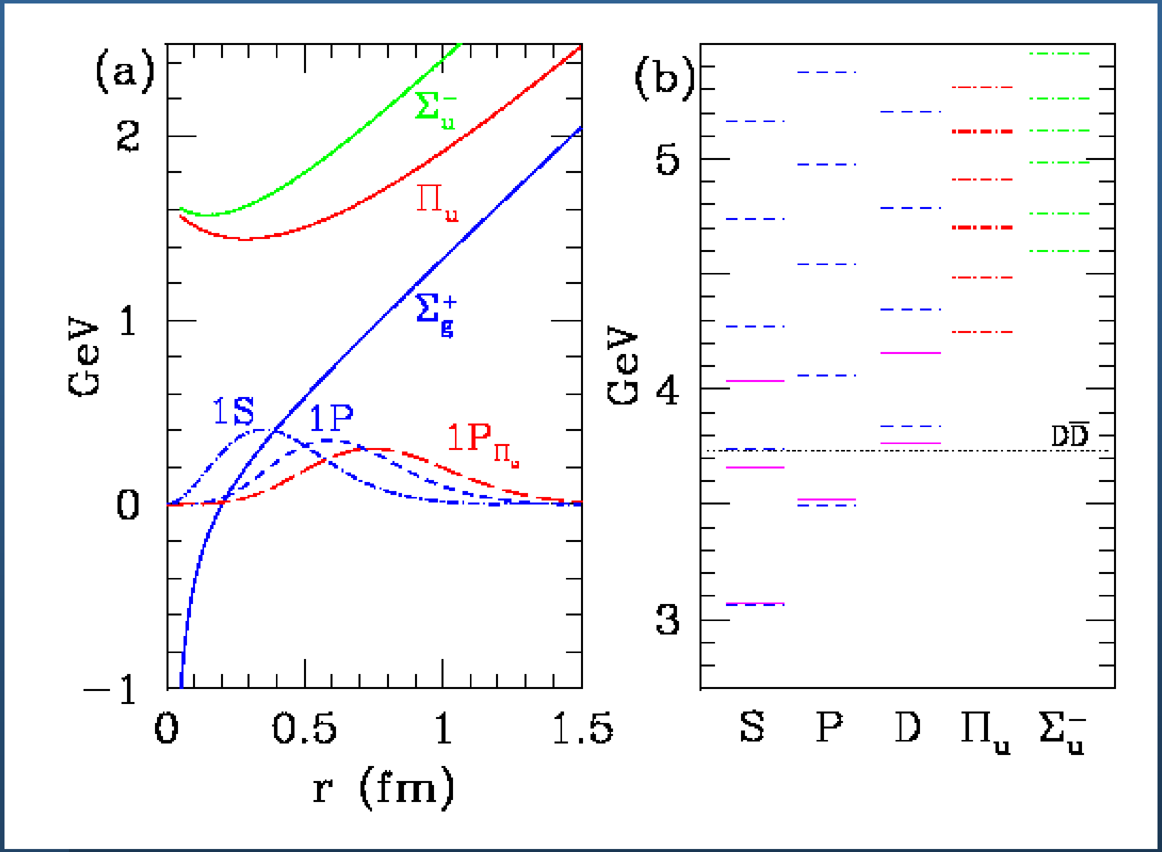
$\Sigma_g^+(P):\quad 0^{++}, 1^{++}, 2^{++}, 1^{+-}$

$\Pi_u(P):\quad 0^{-+}, 0^{+-}, 1^{++}, 1^{--},$
$1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$

Juge, Kuti, Morningstar, Phys Rev Lett **82**, 4400 (1999)

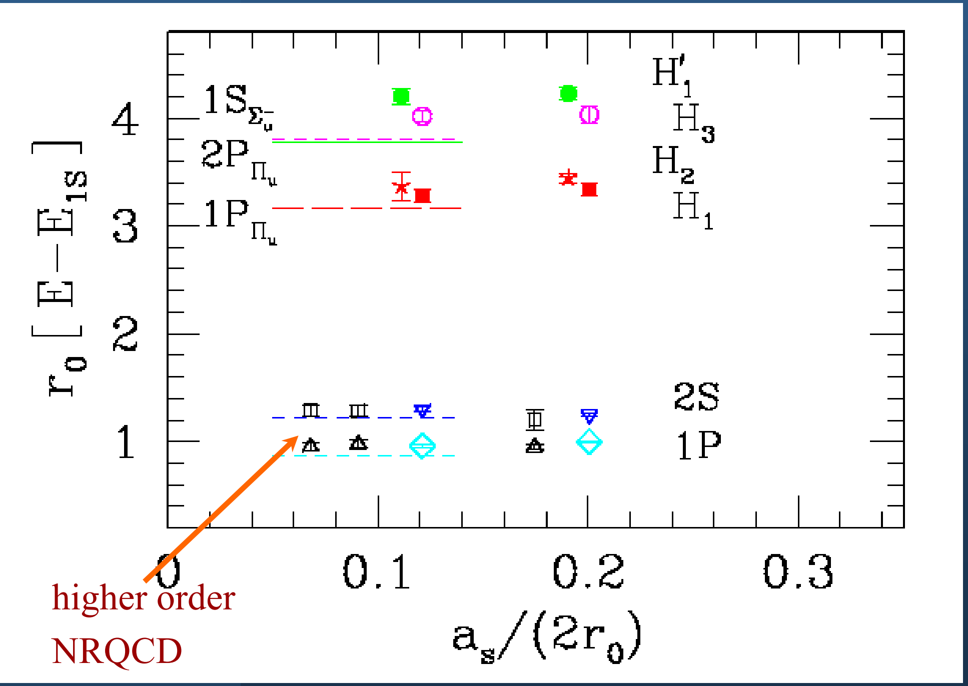
Charmonium LBO

- same calculation, but for charmonium



Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no $\vec{p}^4$, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids



$H_1, H_1' = 1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$

$H_2 = 1^{++}, 0^{+-}, 1^{+-}, 2^{+-}$

$H_3 = 0^{++}, 1^{+-}$

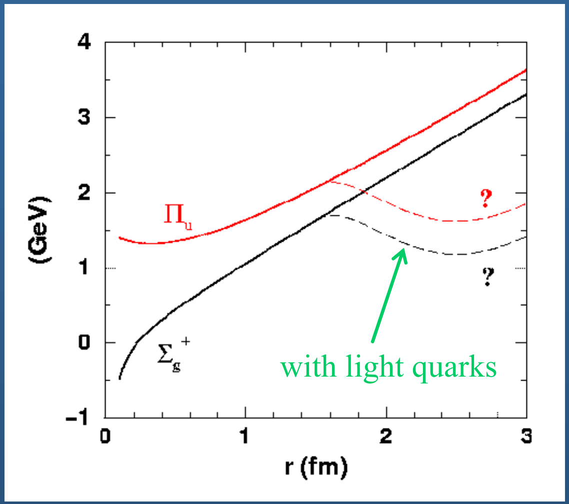
J^{PC}		Degeneracies	Operator
0^{-+}	S wave	1^{--}	$\chi^\dagger\ [\tilde{\Delta}^{(2)}]^p\ \psi$
1^{+-}	P wave	$0^{++}, 1^{++}, 2^{++}$	$\chi^\dagger\ \tilde{\Delta}\ \psi$
1^{--}	H_1 hybrid	$0^{-+}, 1^{-+}, 2^{-+}$	$\chi^\dagger\ \tilde{B}\,[\tilde{\Delta}^{(2)}]^p\ \psi$
1^{++}	H_2 hybrid	$0^{+-}, 1^{+-}, 2^{+-}$	$\chi^\dagger\ \tilde{B}\times\tilde{\Delta}\ \psi$
0^{++}	H_3 hybrid	1^{+-}	$\chi^\dagger\ \tilde{B}\cdot\tilde{\Delta}\ \psi$

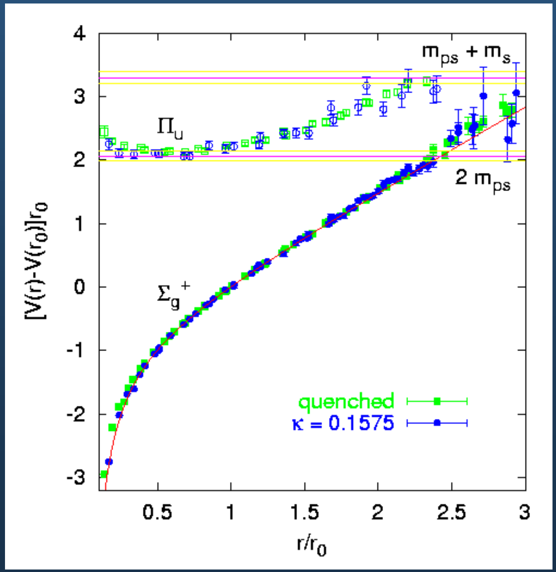
lowest hybrid 1.49(2)(5) GeV above 1S

Light quark spoiler?

- spoil B.O.? → unknown
- light quarks change $V_{Q\bar{Q}}(r)$
  - small corrections at small r
    - fixes low-lying spectrum
  - large changes for $r>1$ fm

    → fission into $(Q\bar{q})(\bar{Q}q)$
- states with diameters over 1 fm
  - most likely *cannot exist* as observable resonances
- dense spectrum of states from pure glue potentials will not be realized
  - survival of a few states conceivable given results from Bali *et al.*
- discrepancy with experiment above $B\bar{B}$
  - most likely due to light quark effects





Baryon blitz (mesons, too)

- charge from the late Nathan Isgur to use Monte Carlo method to extract the spectrum of baryon resonances (Hall B at JLab)
- formed the Lattice Hadron Physics Collaboration (LHPC) in 2000
- current collaborators:
  - Subhasish Basak, Robert Edwards, George Fleming, Adam Lichtl, David Richards, Ikuro Sato, Steve Wallace
- to extract spectrum of resonances
  - need sets of extended operators (correlator matrices)
  - multi-hadron operators needed too
  - deduce resonances from finite-box energies
  - anisotropic lattices $(a_t < a_s)$
  - inclusion of light-quark loops

Operator design issues

- must facilitate spin identification
  - shun the usual method of operator construction which relies on cumbersome continuum space-time constructions
  - focus on constructing operators which transform irreducibly under the symmetries of the lattice
- one eye on maximizing overlaps with states of interest, other eye on minimizing number of quark-propagator sources
- use building blocks useful for mesons, multi-hadron operators as well

Three stage approach

- concentrate on baryons at rest (zero momentum)
- operators classified according to the irreps of O_h

$$G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g, H_u$$

- (1) basic building blocks: smeared, covariant-displaced quark fields

$$(\widetilde{D}_j^{(p)} \widetilde{\psi}(x))_{Aa\alpha} \quad p\text{ - link displacement } (j = 0, \pm 1, \pm 2, \pm 3)$$

- (2) construct elemental operators (translationally invariant)

$$B_i^F(x) = \phi_{ABC}^F \varepsilon_{abc} (\widetilde{D}_j^{(p)} \widetilde{\psi}(x))_{Aa\alpha} (\widetilde{D}_j^{(p)} \widetilde{\psi}(x))_{Bb\beta} (\widetilde{D}_j^{(p)} \widetilde{\psi}(x))_{Cc\gamma}$$

  - flavor structure from isospin, color structure from gauge invariance

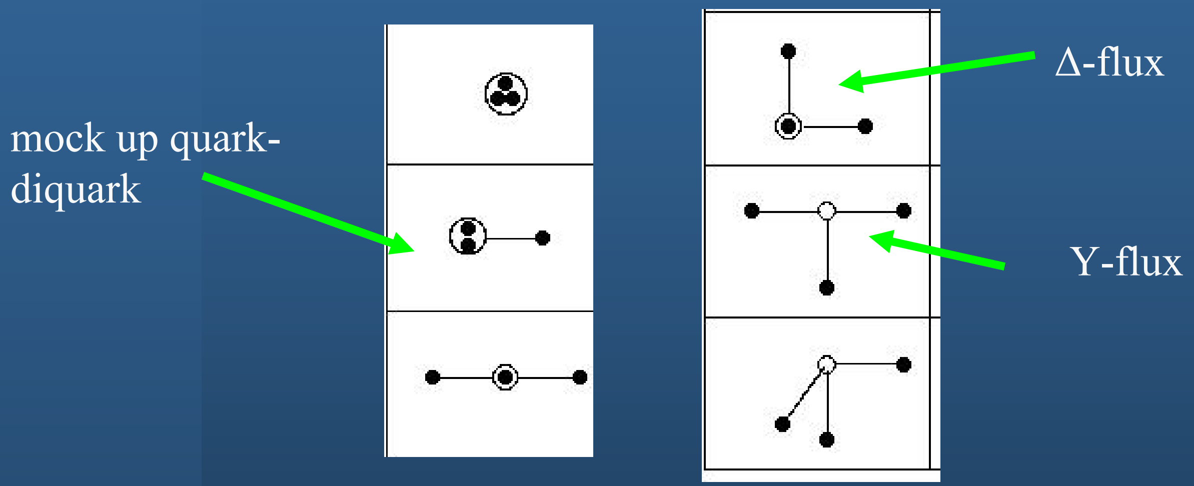
- (3) group-theoretical projections onto irreps of O_h

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R \in O_h^D} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^+$$

  - wrote Grassmann package in Maple to do these calculations

Incorporating orbital and radial structure

- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure



- operator design minimizes number of sources for quark propagators
- useful for mesons, tetraquarks, pentaquarks even!
- can even incorporate hybrid mesons operator (in progress)

Spin identification and other remarks

- spin identification possible by pattern matching

J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3

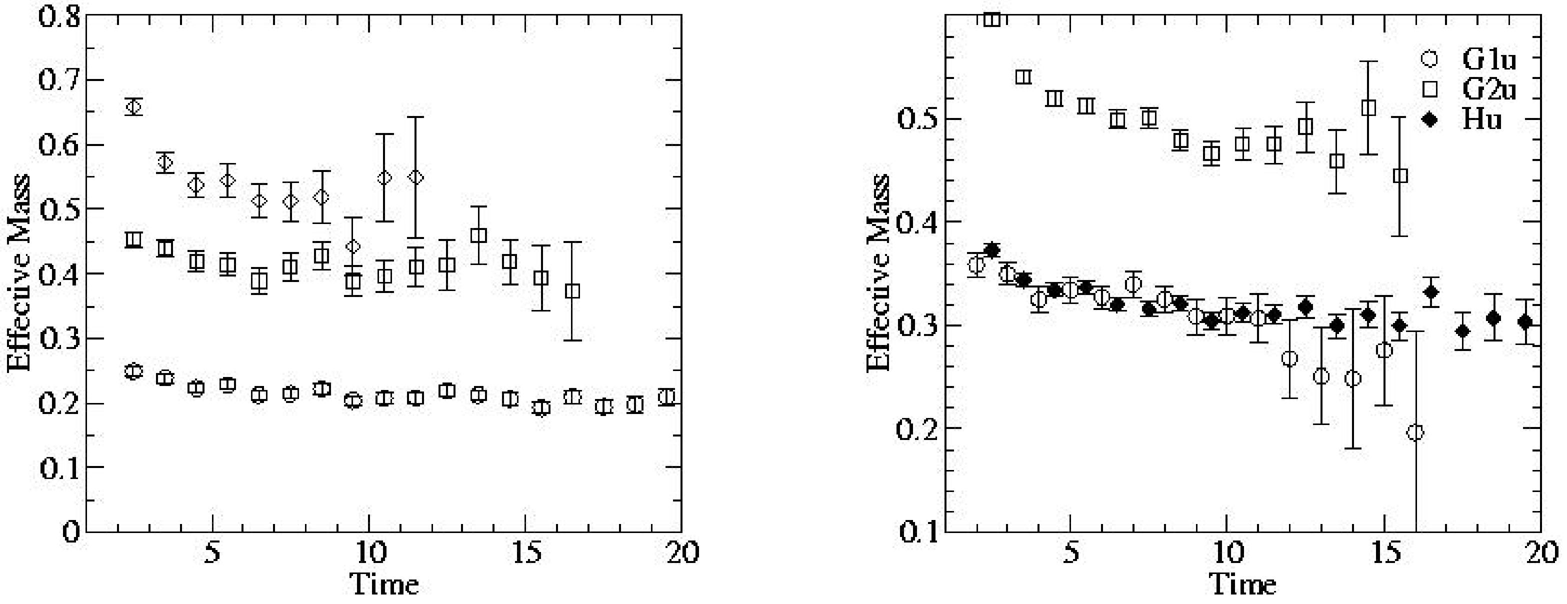
total numbers of operators assuming two different displacement lengths

Irrep	Δ, Ω	N	Σ, Ξ	Λ
G_{1g}	221	443	664	656
G_{1u}	221	443	664	656
G_{2g}	188	376	564	556
G_{2u}	188	376	564	556
H_g	418	809	1227	1209
H_u	418	809	1227	1209

- total numbers of operators is huge → uncharted territory
- ultimately must face two-hadron scattering states

Old preliminary results

- principal effective masses for small set of 10 operators



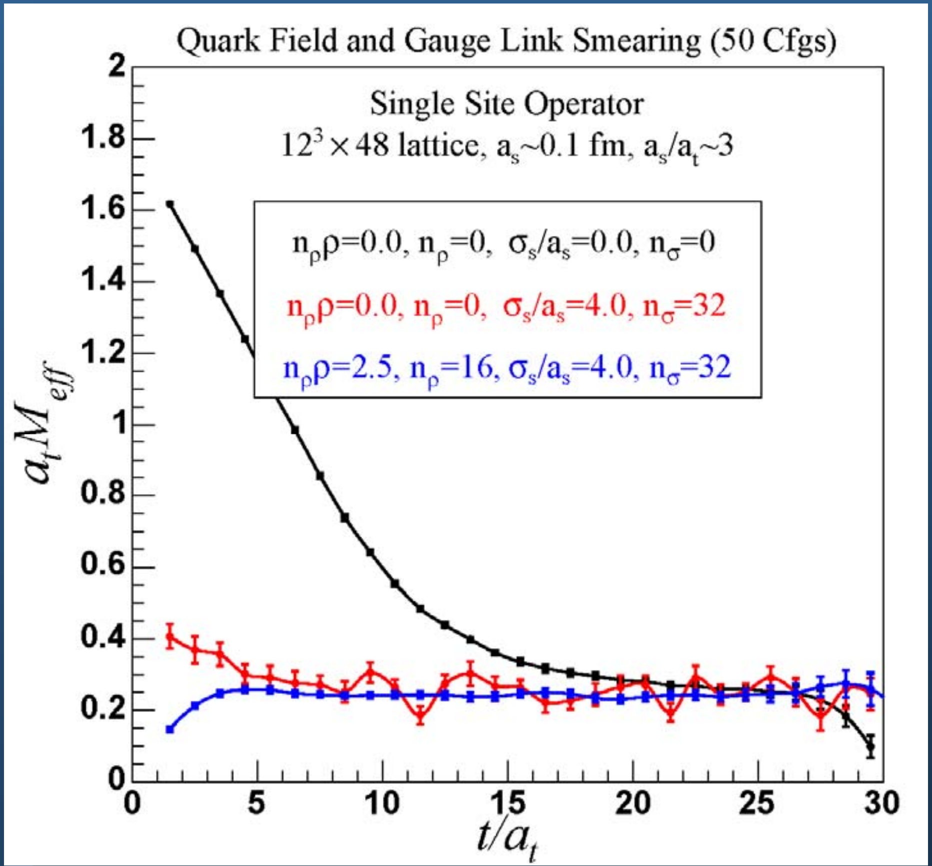
- G_{1g} on left, other irreps on right.

Current status and next step

- Development of software to carry out the baryon computations has been completed and thoroughly tested (at long last!)
  - gauge-invariant three-quark propagators as intermediate step
  - baryon correlators are superpositions of *qqq*-propagator components → superposition coefficients precalculated
  - source-sink rotations to minimize source orientations
- Next step: smearing optimization and operator pruning
  - optimize link-variable and quark-field smearings
  - remove dynamically redundant operators
  - remove ineffectual operators
  - low statistics runs needed
  - monitor progress at http://enrico.phys.cmu.edu

Quark-field smearing

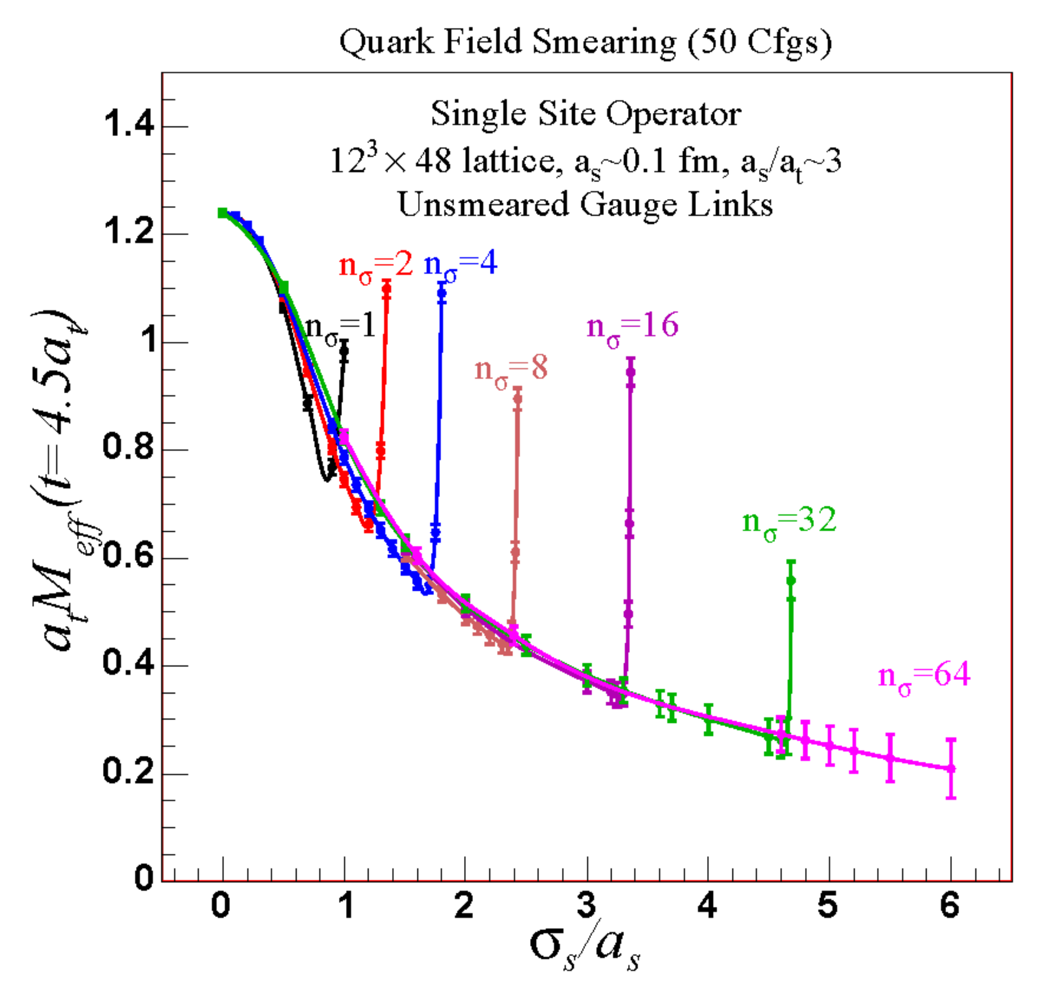
- use smeared quark and gluon fields fields → dramatically reduced coupling with short wavelength modes $\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}^2\right)^{n_\sigma} \psi(x)$



Nucleon G1g channel

Quark-field smearing (2)

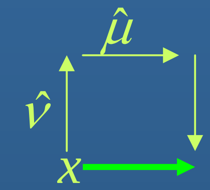
Nucleon G1g channel



Link variable smearing

- link variables: add staples with weight, project onto gauge group
  - define

$$C_\mu(x) = \sum_{\pm(\nu\neq\mu)} \rho_{\mu\nu} U_\nu(x) U_\mu(x+\hat{\nu}) U_\nu^+(x+\hat{\mu})$$



  - common 3-d spatial smearing $\rho_{jk} = \rho, \quad \rho_{4k} = \rho_{k4} = 0$
  - APE smearing $U_\mu^{(n+1)} = P_{SU(3)}\left(U_\mu^{(n)} + C_\mu^{(n)}\right)$
  - or new analytic stout link method (hep-lat/0311018)

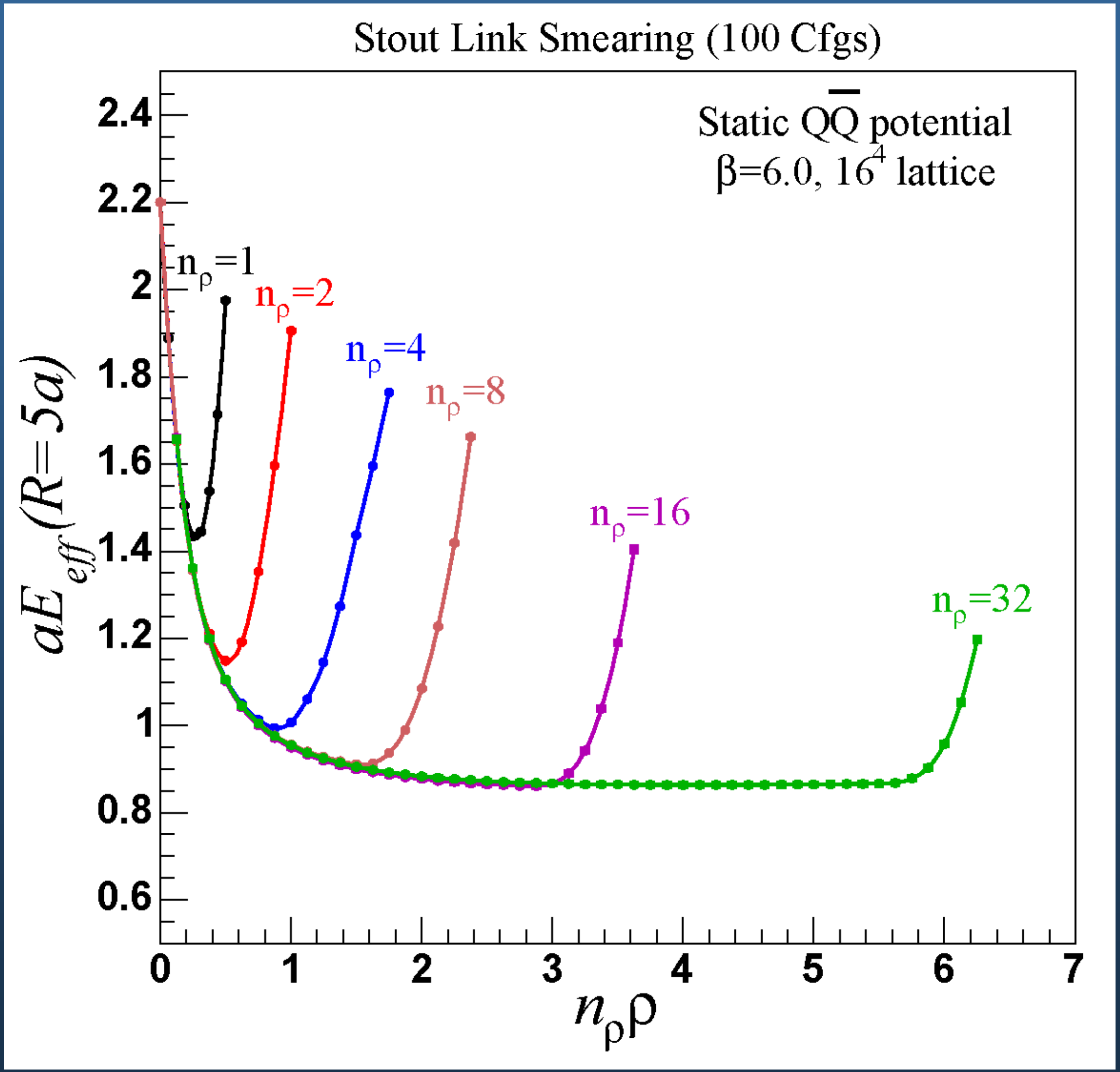
$$\Omega_\mu = C_\mu U_\mu^+$$

$$Q_\mu = \frac{i}{2}\left(\Omega_\mu^+ - \Omega_\mu\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_\mu^+ - \Omega_\mu\right)$$

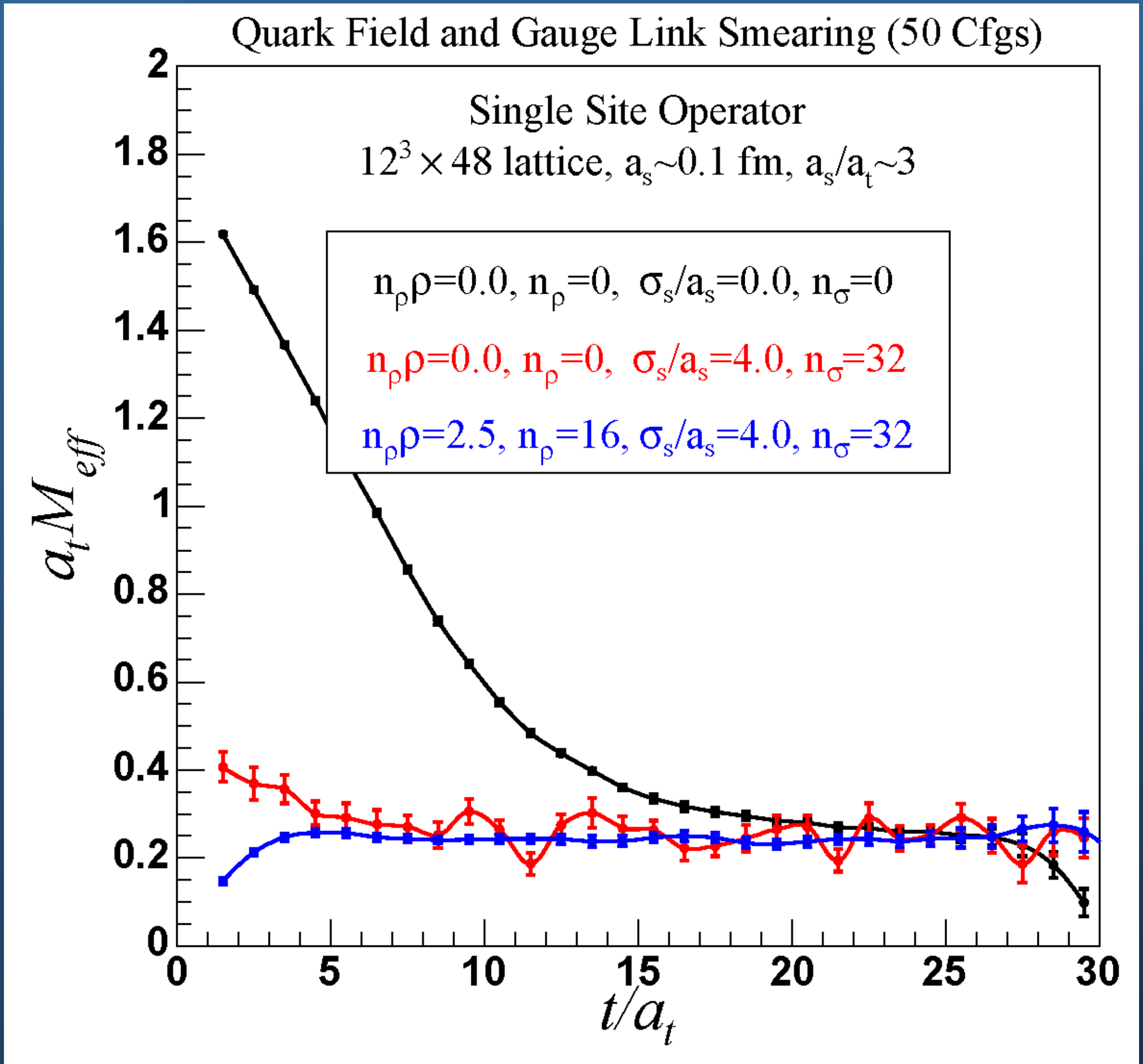
$$U_\mu^{(n+1)} = \exp\left(iQ_\mu^{(n)}\right) U_\mu^{(n)}$$

  - iterate $U_\mu \rightarrow U_\mu^{(1)} \rightarrow \cdots \rightarrow U_\mu^{(n)} \equiv \widetilde{U}_\mu$

Link variable smearing (2)

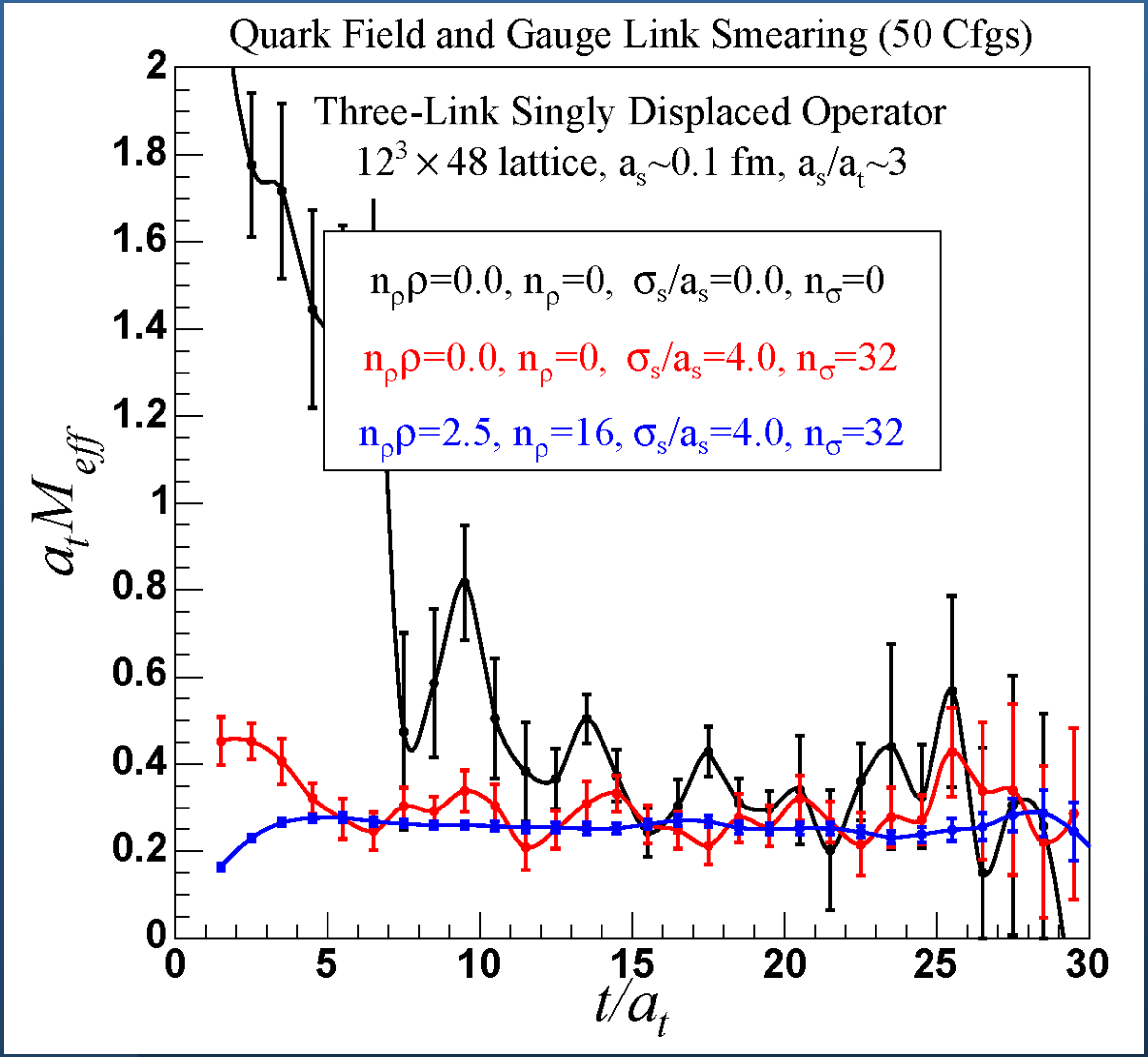


Quark-field with link variable smearing



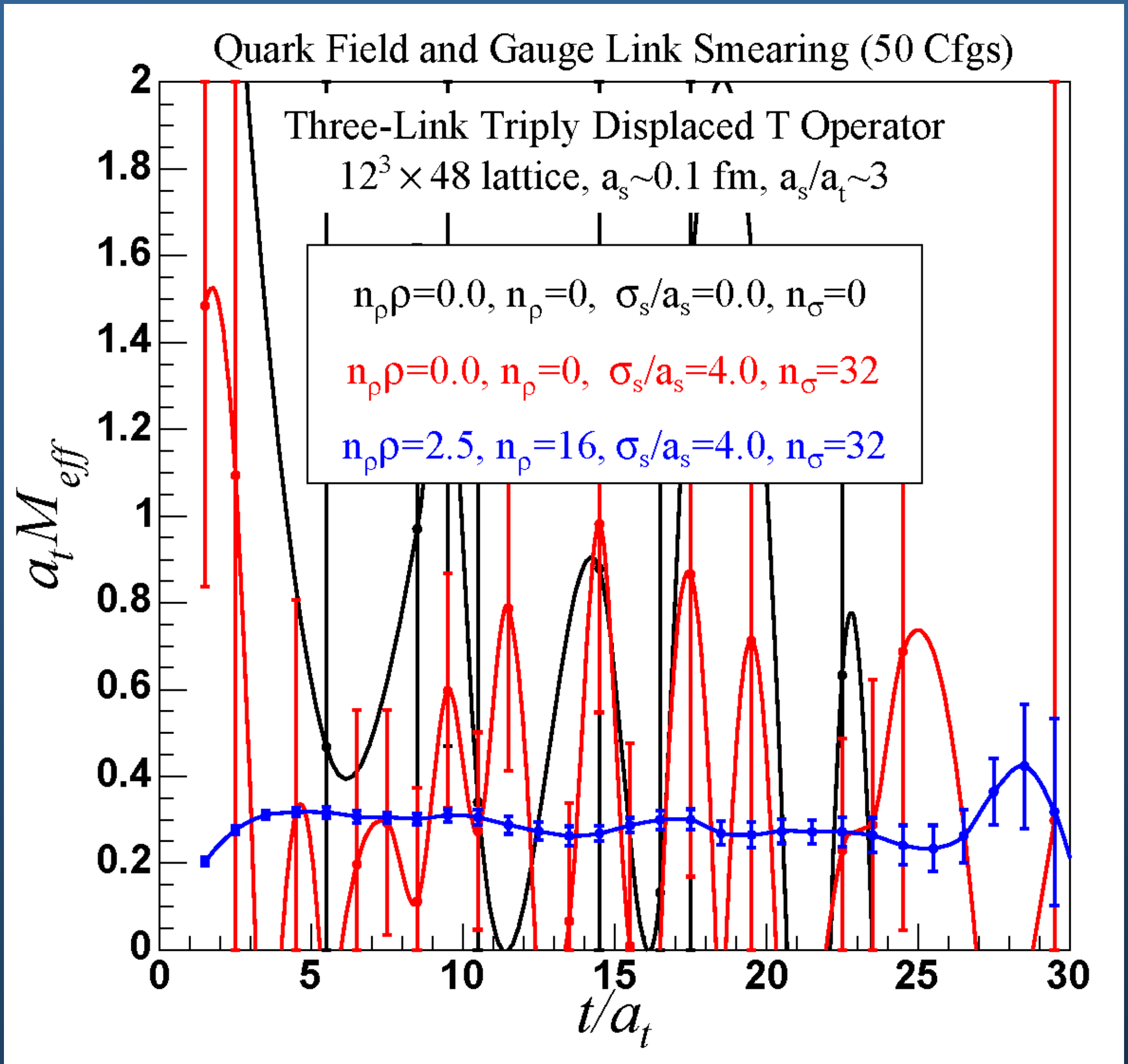
Nucleon
G1g
channel

Quark-field with link variable smearing (2)



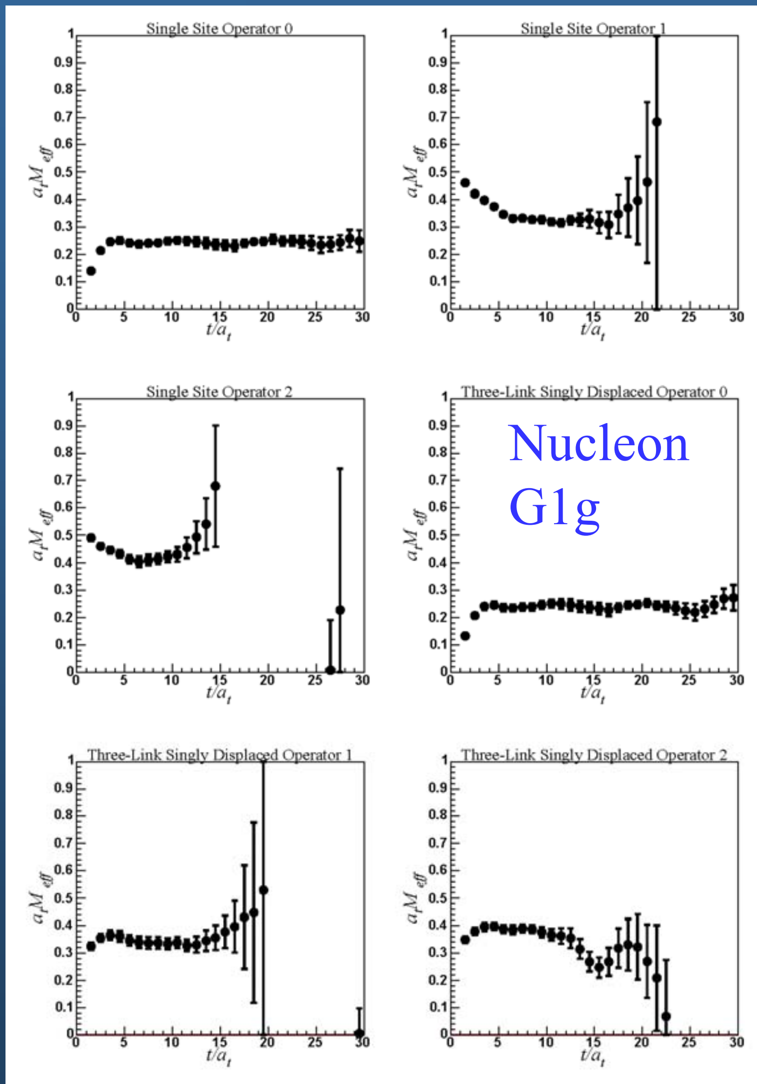
Nucleon
G1g
channel

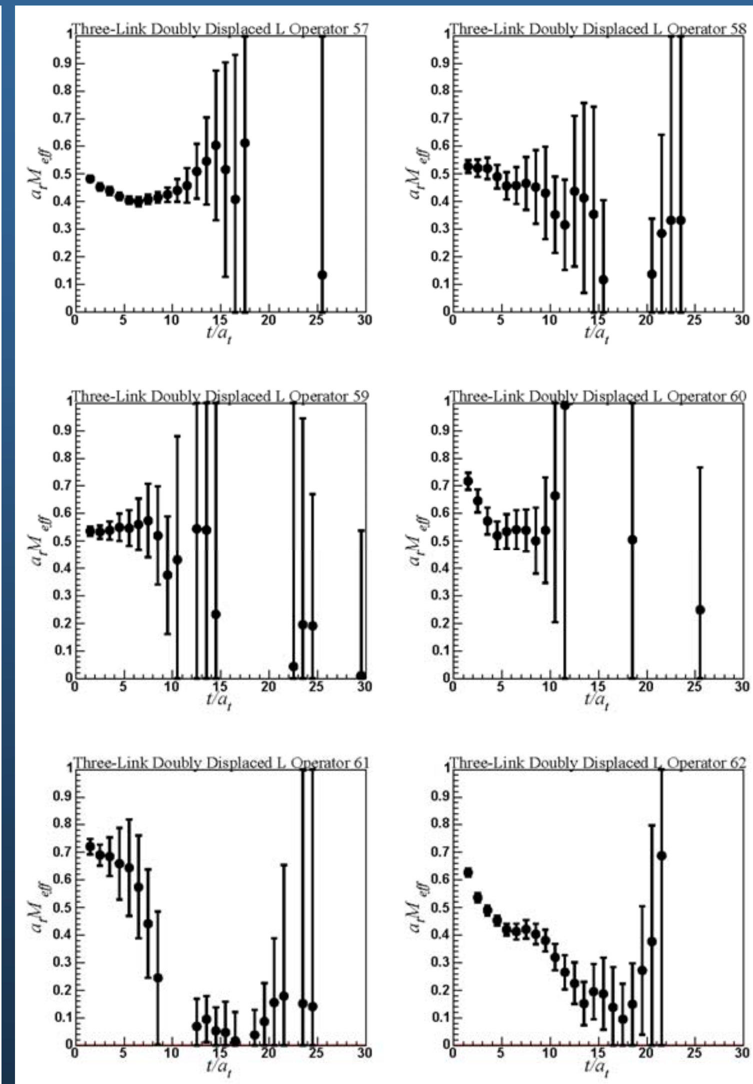
Triply-displaced operator



Nucleon
G1g channel

Operator plethora (pruning needed!)





Summary

- described a few explorations of QCD spectrum using lattice Monte Carlo methods
- glueball mass spectrum in pure gauge theory
- stationary states of gluons in presence of static quark-antiquark pair as a function of separation R
  - unearthed the effective QCD string for $R>1$ fm for the first time
  - tantalizing fine structure revealed→effective string action clues
  - dramatic level rearrangement between small and large separations
- heavy-quark hybrid mesons (Born-Oppenheimer treatment)
- outlined ongoing efforts of LHPC to extracting baryon spectrum with large sets of extended operators
  - applications to mesons (and hybrids), tetraquark, pentaquark