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N → Δ transition form factors in Lattice QCD

Outline

- Introduction
- $\gamma N \rightarrow \Delta$ matrix element on the lattice
- Quenched results
- Full QCD results
- Conclusions

Introduction

Quantum ChromoDynamics is the fundamental theory of strong interactions. It is formulated in terms of quarks and gluons with only parameters the coupling constant and the masses of the quarks.

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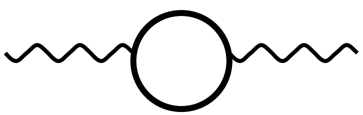


Self energy diverges → QCD + regularization →renormalization

defines a physical theory

Produces the rich and complex structures of strongly interacting matter in our Universe

Solving QCD

- At large energies, where the coupling constant is small, perturbation theory is applicable
→ has been successful in describing high energy processes

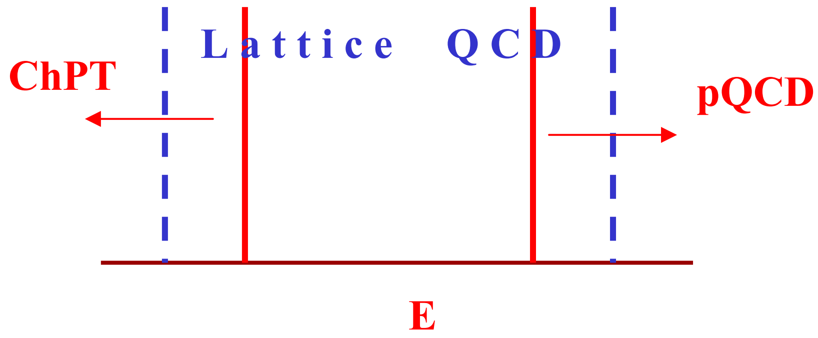
- At very low energies chiral perturbation theory becomes applicable

- At energies ~ 1 GeV the coupling constant is of order unity → need a non-perturbative approach

Present analytical techniques inadequate

→ Numerical evaluation of path integrals on a space-time lattice

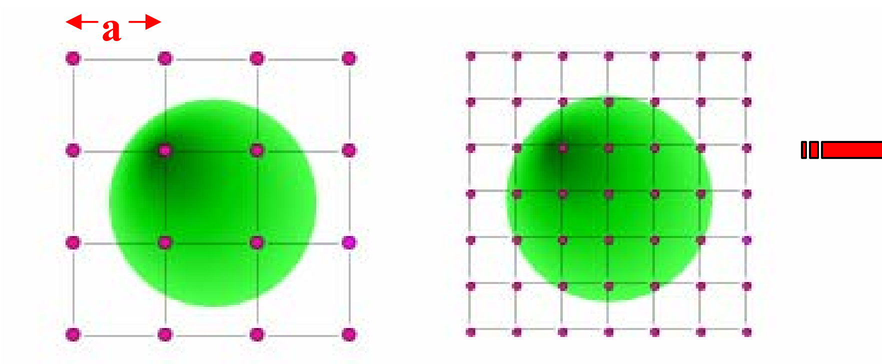
→ Lattice QCD – a well suited non-perturbative method that uses directly the QCD Langragian and therefore no new parameters enter



Lattice QCD

Lattice QCD is a discretised version of the QCD Lagrangian with only parameters the coupling constant and the masses of the quarks

- **Finite lattice spacing a**: is determined from the coupling constant and gives the length/energy scale with respect to which all physical observables are measured



→ must take a → 0 to recover continuum physics

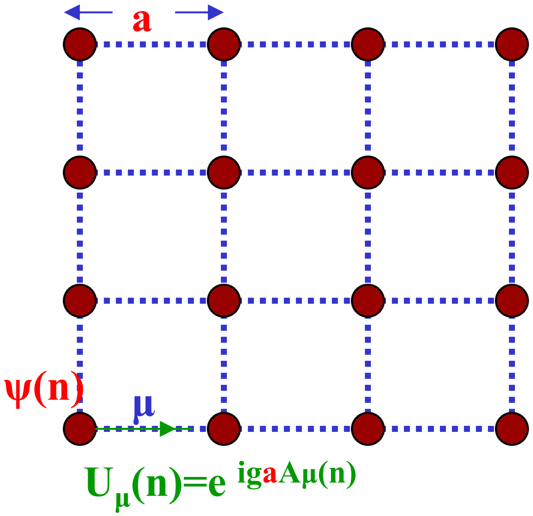
- **specify the bare quark mass m_q**: is taken much larger than the u and d quark mass → extrapolate to the chiral limit

- **Wick rotation into Euclidean time:** $e^{\int x\, L} \to e^{\int \tau\, H}$

limits applicability to lower states

- **must be solved numerically** on the computer using similar methods to those used in Statistical Mechanics → Finite volume: must take the spatial volume to infinity

Observables in Lattice QCD



$$S_F = \sum_{nk} \bar{\psi}(n) D_{nk} \psi(k)$$

$$S = S_g[U] + S_F[U,\bar{\psi},\psi]$$

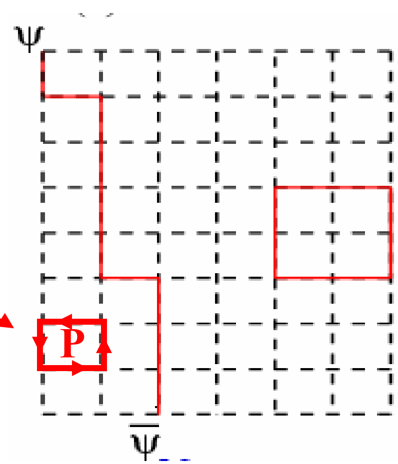
Depends on coupling, $\beta=6/g^2$ constant and quark masses m

$$am = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

$$<\hat{O}> = \frac{\int DU\, D\bar{\psi}\, D\psi\, O[U,\bar{\psi},\psi]\, e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}{\int DU\, D\bar{\psi}\, D\psi\, e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}$$

gauge invariant quantities

smallest possible



Integrating out the Grassmann variables is possible since

$$\langle \hat{O} \rangle = \frac{\int DU \{\det D\}\, O[U, D^{-1}]\, e^{-S_g[U]}}{\int DU\ \{\det D\}\, e^{-S_g[U]}} = \prod_n \int dU_n \frac{1}{Z} \{\det D(U)\}\, e^{-S_g[U]}\ O[U, D^{-1}]$$

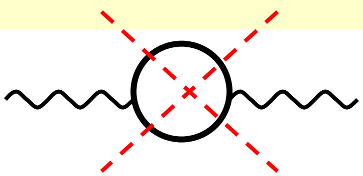
Sample with M.C.

Sea-quarks and valence are described by the same Dirac operator

1. Generate a sample of N independent gauge fields U
2. Calculate propagator D^{-1} for each U

$\hat{}$ — $\sum$ $\{\ \}$ $\in \frac{1}{Z}$ → $\sum$

Quenched approximation: set det D=1



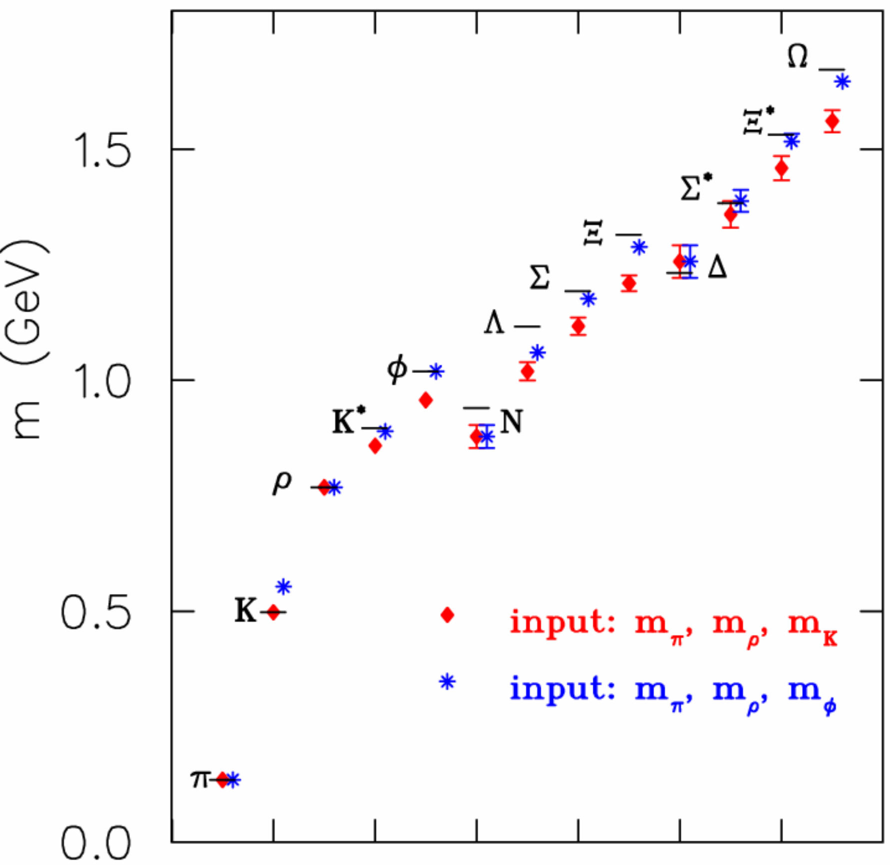
omit pair creation

→ Easy to simulate since $S_g[U]$ is local: use M. C. methods from Statistical Mechanics like the Metropolis algorithm

$\hat{}$ — $\sum$ $\in \frac{1}{Z}$

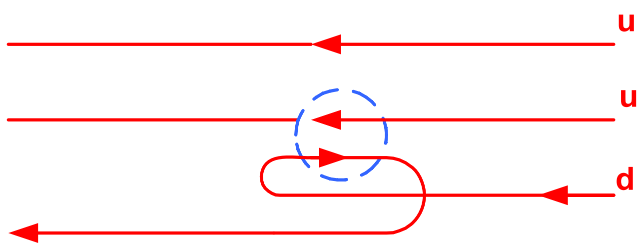
For a lattice of size 32^3 x 64 we have 10^8 gluonic variables

Precision results in the quenched approximation

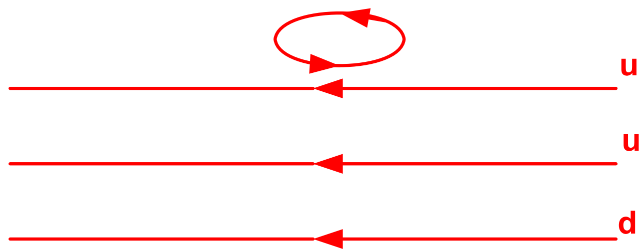


CP-PACS, Aoki *et al.*, PRD 67 (2003)

- Lattice spacing a → 0
- Chiral extrapolation
- Infinite volume limit

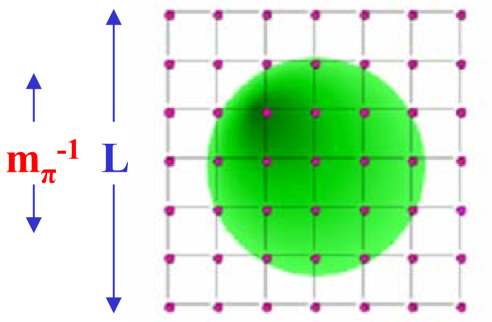


Included in the quenched approximation



Not included in the quenched approximation

Computational aspects



- Lattice spacing **a** small enough to have continuum physics
- Quark mass small to reproduce the physical pion mass
- Lattice size large enough so that $\leq 4 -$
- Fermion determinant – Full QCD

L (fm)	m_π (MeV)
1.6	560
2.5	360
4.5	200
6.4	140

most quenched calculations reaching ~300 MeV pions

Computational cost ~ $(m_q)^{-4.5} \sim (m_\pi)^{-9}$

Currently we are starting to do full QCD by including the fermionic determinant but we are still limited to rather heavy pions → understand dependence on quark mass

Physical results require terascale machines

Motivation

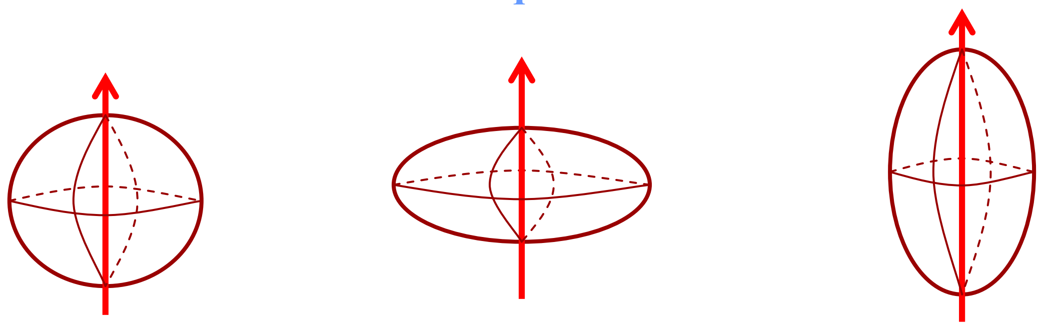
Study Hadron Deformation

Spin defines an orientation → hadron can be deformed with respect to its spin axis

- **Deformation of the Δ by evaluating** $\left\langle \Delta, J = J_z = J = \frac{3}{2} \right| \underbrace{(3z^2 - r^2)}_{Q_{20}} \left| \Delta, J = J_z = J = \frac{3}{2} \right\rangle$

Q_{20} quadrupole operator

extract the wave function squared from current-current correlators



Sphere: $Q_{20}=0$ **Oblate: $Q_{20} < 0$** **Prolate: $Q_{20} > 0$**

- **For spin 1/2 : Quadrupole moment not observable**

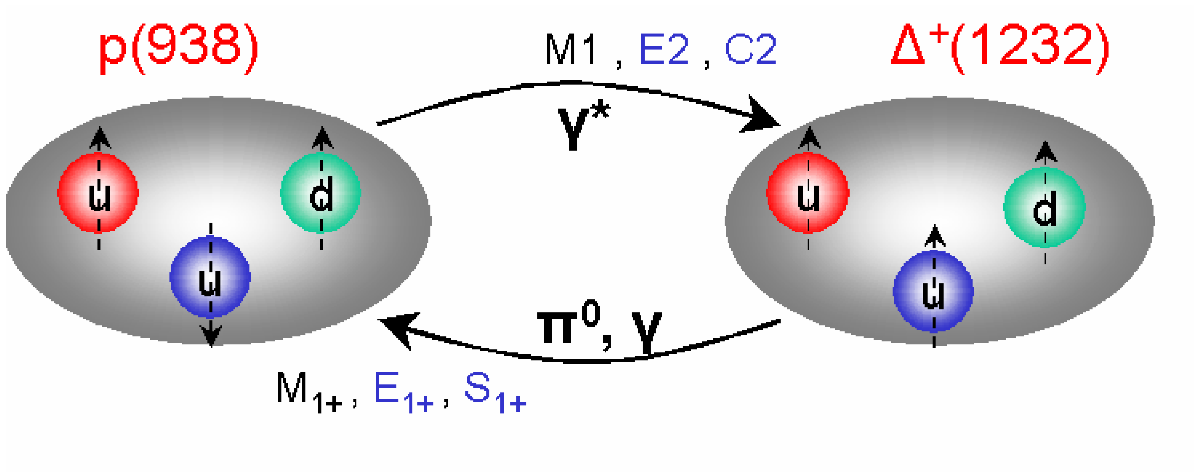
$$Q_{20} = \frac{(2J-1)J}{(J+1)(2J+3)} Q_0$$

observable (Q_{20}); intrinsic (with respect to the body fixed frame) (Q_0)

→ **Make direct connection to experiment**

Transition matrix elements: Look for quadrupole strength in γN→Δ(1232) with real or virtual photons → non-zero C2 and E2 multipoles signal deformation

$\gamma N \rightarrow \Delta$



• **Spin-parity selection rules allow a magnetic dipole, M1, an electric quadrupole, E2, and a Coulomb quadrupole, C2, amplitude.**

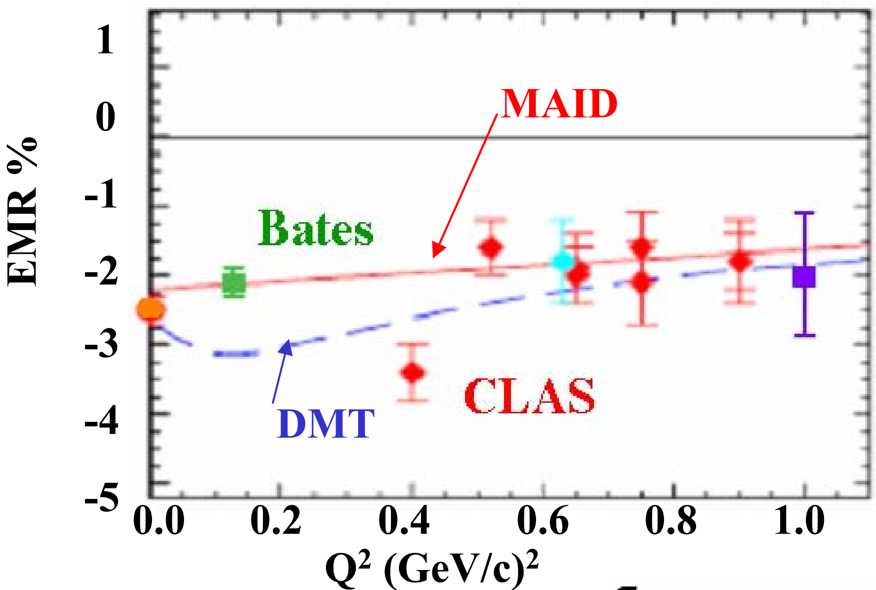
Three transition form factors, Γ_{M1}, Γ_{E2} and Γ_{C2}

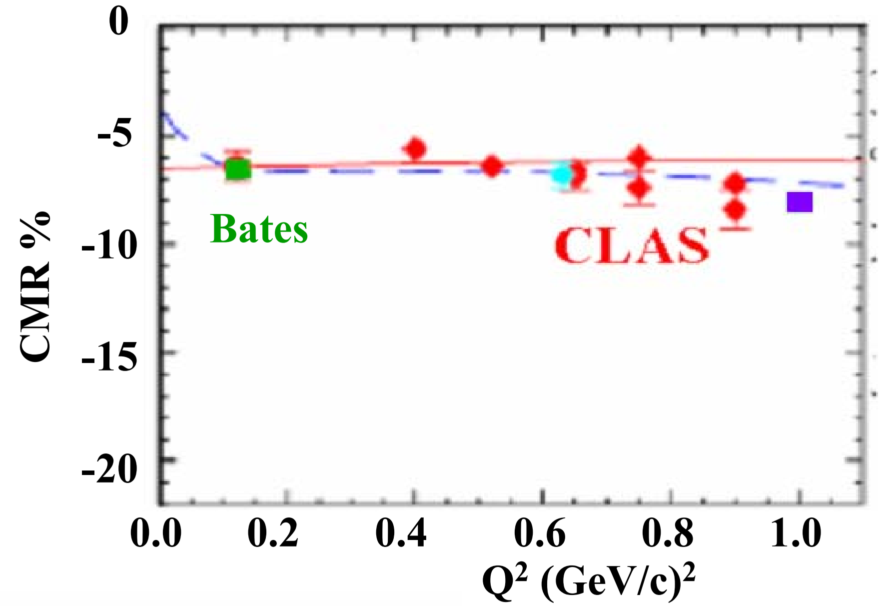
→ non-zero E2 and C2 multipoles signal deformation

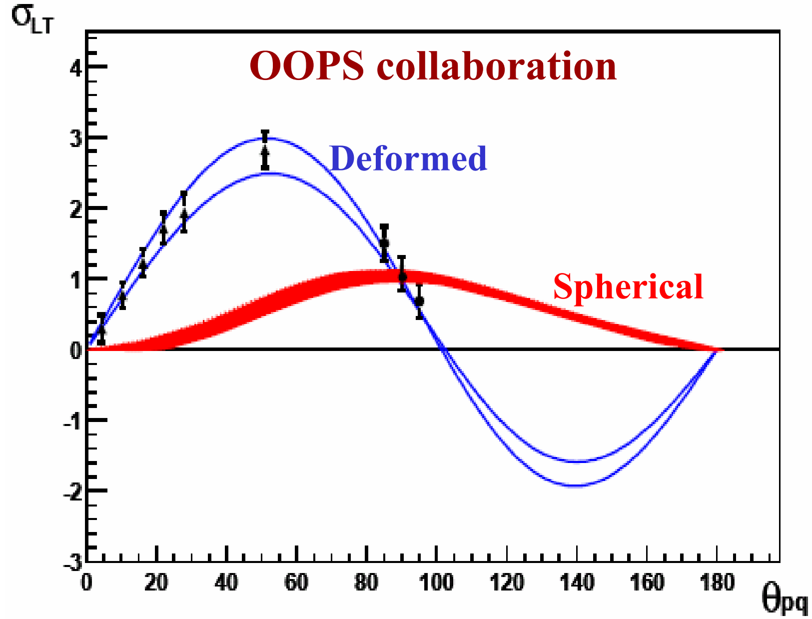
• **Experimentally the measured quantities are given usually in terms of the ratios E2/M1 or C2/M1 known as EMR (or R_{EM}) and CMR (or R_{SM}):**

$$\frac{G}{G} \quad \text{and} \quad \frac{G}{G}$$

Deformed nucleon?







C.N. Papanicolas, "Nucleon Deformation" Eur. Phys. J. A18, 141 (2003)

Ground state

In Quantum Mechanics to find the ground state of a system with a Hamiltonian H one can evolve an initial trial state $|\varphi_{trial}>$ in Euclidean time:

w_0 w_1

$\hat{}$ φ $\sum$ $\hat{}$ φ φ φ L

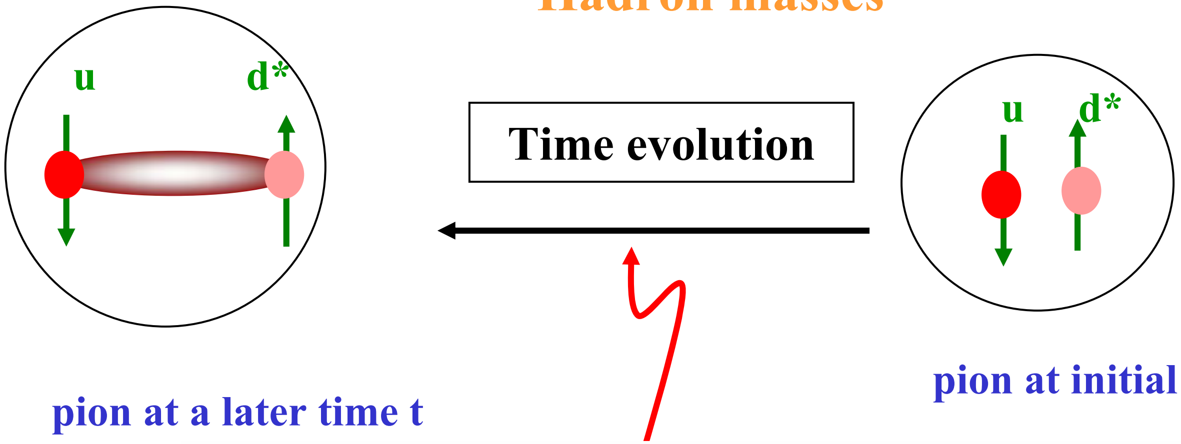
$[$ w' K $]$ $\xrightarrow{(E_1-E_0)T>>1}$ w

Depend on initial trial state $|\varphi_{trial}>$

Overlap with $<\varphi_{trial}|$ → $|^2$

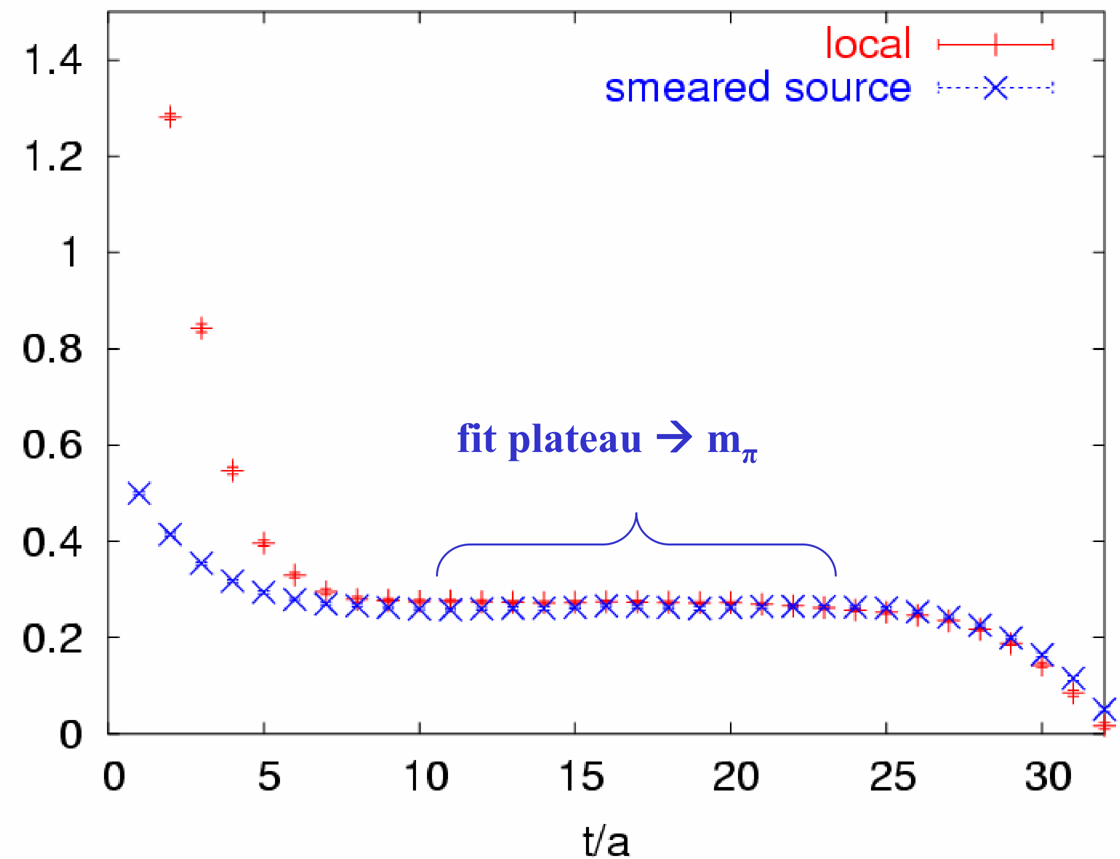
extract ground state information before signal is buried in the statistical noise

Hadron masses



pion at a later time t

pion at initial time t=0



Jp to corrections of
ɔrder $\exp(-3m_\pi t)$

Take overl

Correlator :

Projects to zer

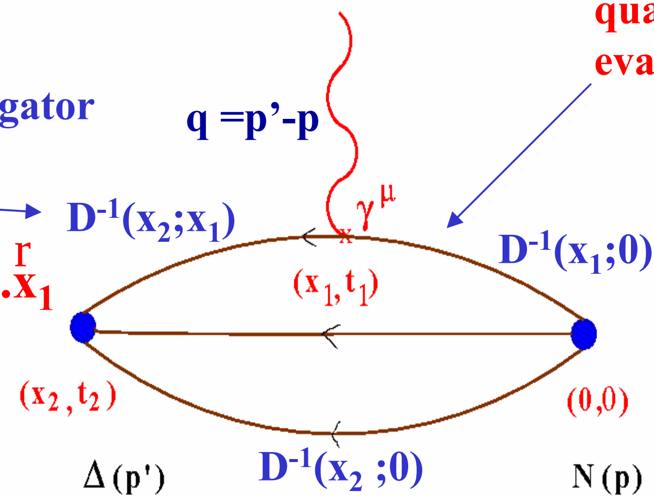
Three-point functions

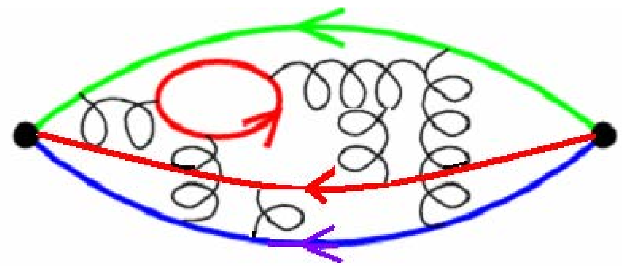
Matrix elements like <Δ| j^μ |N>:

all to all propagator

quark line with photon needs to be evaluated → sequential propagator

$$\sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}'.\vec{x}_2} e^{i\vec{q}.\vec{x}_1}$$





Projects momentum p' to Δ → sum first: fixed sink technique

Projects to momentum transfer q for the photon → sum first: fixed current technique

Use fixed sink technique → obtained all q^2 values at once

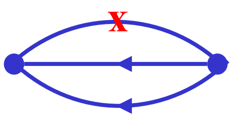
Since we worked on a finite lattice the momentum takes discrete values 2πk/L, k=1,…,N

$\gamma N \rightarrow \Delta$ matrix element on the Lattice

$|\varphi_J\rangle = \bar{J}\,|0\rangle$ **Trial initial state with quantum numbers of the nucleon**

$\langle\psi_{J'}| = \langle 0|\,J'_\sigma$ **Trial final state with quantum numbers of the Delta**

$$\langle T J'_\sigma(t_2)\hat{O}(t_1)\bar{J}(0)\rangle = \sum_{m,n}\langle\psi_{J'}|n\rangle\langle n|\hat{O}|m\rangle\langle m|\varphi_J\rangle e^{-E_n(t_2-t_1)}\,e^{-E_m t_1}$$

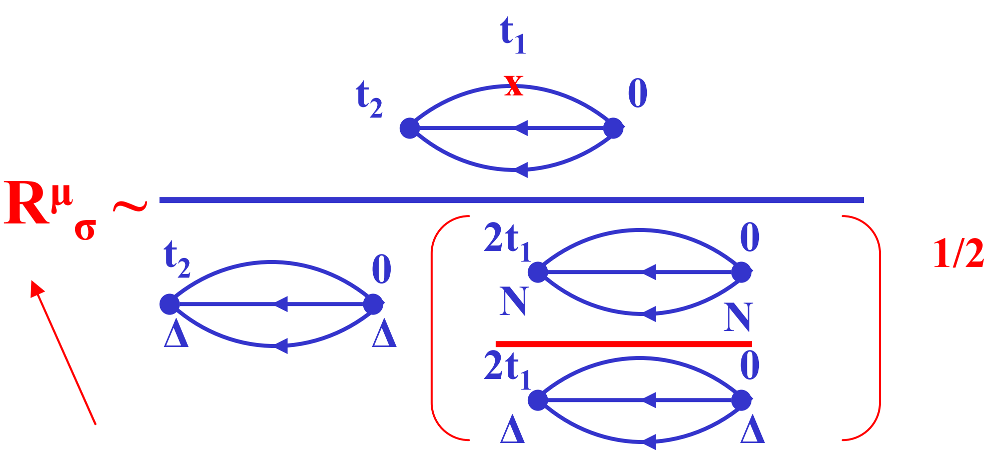


$$\xrightarrow[t_1 \quad 1]{t_2-t_1 \quad 1} \langle\psi_{J'}|\Delta\rangle\langle N|\varphi_J\rangle\langle\Delta|\hat{O}|N\rangle e^{-E_\Delta(t_2-t_1)}\,e^{-E_N t_1}$$

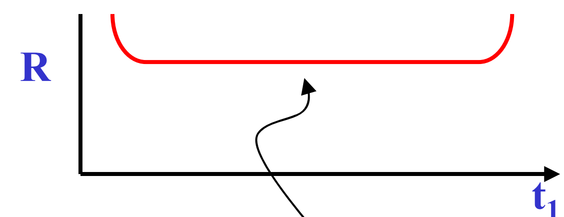
Normalize with

$$\langle T J(2t_1)\bar{J}(0)\rangle = \sum_n |\langle\varphi_J|n\rangle|^2 e^{-2E_n t_1} \xrightarrow[t_1 ?\ 1]{} |\langle\varphi_J|N\rangle|^2 e^{-2E_N t_1}$$

$$\langle T J'_\sigma(2t_1)\bar{J}'_\sigma(0)\rangle = \sum_n |\langle\psi_{J'}|n\rangle|^2 e^{-2E_n t_1} \xrightarrow[t_1 ?\ 1]{} |\langle\psi_{J'}|\Delta\rangle|^2 e^{-2E_\Delta t_1}$$



time dependence cancels in the ratio



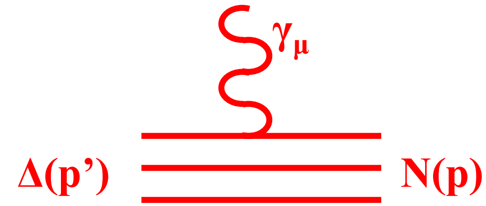
Fit in the plateau region to extract

$$\langle\Delta(p')|\,j_\mu\,|N(p)\rangle$$

M1, E2, C2

$\gamma N \rightarrow \Delta$ matrix element

$$\langle \Delta(p') \,|\, j_\mu \,|\, N(p) \rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta E_N}\right)^{1/2} \bar{u}_\sigma(p') \mathcal{O}^{\sigma\mu} u(p)$$



$$\mathcal{O}^{\sigma\mu} = G_{M1}(q^2) K^{\sigma\mu}_{M1} + G_{E2}(q^2) K^{\sigma\mu}_{E2} + G_{C2}(q^2) K^{\sigma\mu}_{C2}$$

M1 E2 C2

Sach´s form factors

- At the lowest value of q^2 the form factors were evaluated in both the quenched and the unquenched theory but with rather heavy sea quark masses. The main conclusion of this comparison is that quenched and unquenched results are within statistical errors.

C.A., Ph. de Forcrand, Th. Lippert, H. Neff, J. W. Negele, K. Schilling. W. Schroers and A. Tsapalis, PRD 69 (2004)

- Quenched results are done on a lattice of spatial volume 32^3 at β=6.0 (~3.2 fm)^3 using smaller quark masses (smallest gives $m_\pi/m_\rho = 1/2$) up to q^2 of 1.5 GeV^2

C. A., Ph. de Forcrand, H. Neff, J. W. Negele, W. Schroers and A. Tsapalis, PRL 94 (2005)

- MILC configurations and DWF

C. A., R. Edwards, G. Koutsou, Th. Leontiou, H. Neff, J.W. Negele, W. Schroers and A. Tsapalis, Lattice 2005

Technicalities

The form factors are extracted from the ratio $^{\mu}_{\sigma}\left(\quad \right) =$

$$\longrightarrow \Pi_{\sigma}\left(\quad \right)$$

where σ is the spin index of the Δ field and the projection matrices Γ are given by

We consider kinematics where the Δ is produced at rest i.e.

What combination of Δ index σ , current direction μ and projection matrices Γ is optimal?

Using the fixed sink technique we require an inversion for each combination of σ index and projection matrix Γ_i whereas we obtain the ratio R_σ for all current directions μ and momentum transfers q=p'-p without extra inversions.

Optimal choice of matrix elements Π_σ

The form factors, Γ_{M1}, Γ_{E2} and Γ_{C2}, can be extracted by appropriate choices of the current direction, Δ spin index and projection matrices Γ_j

For example the magnetic dipole can be extracted from

Π G →

kinematical factor

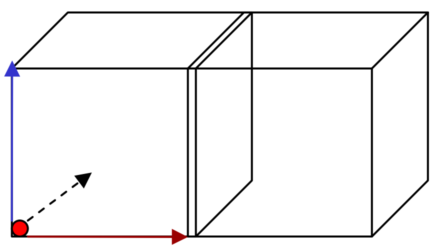
This means there are six statistically different matrix elements each requiring an inversion.

e.g. If σ=3 and μ=1 then only momentum transfers in the 2-direction contribute.

However if we take the more symmetric combination:

μ μ μ μ

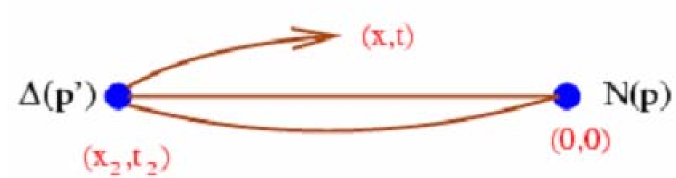
$= [\quad] G$



Smallest lattice momentum vectors:
2π/L {(1,0,0) (0,1,0) (0,0,1)}
contributing to the same Q^2

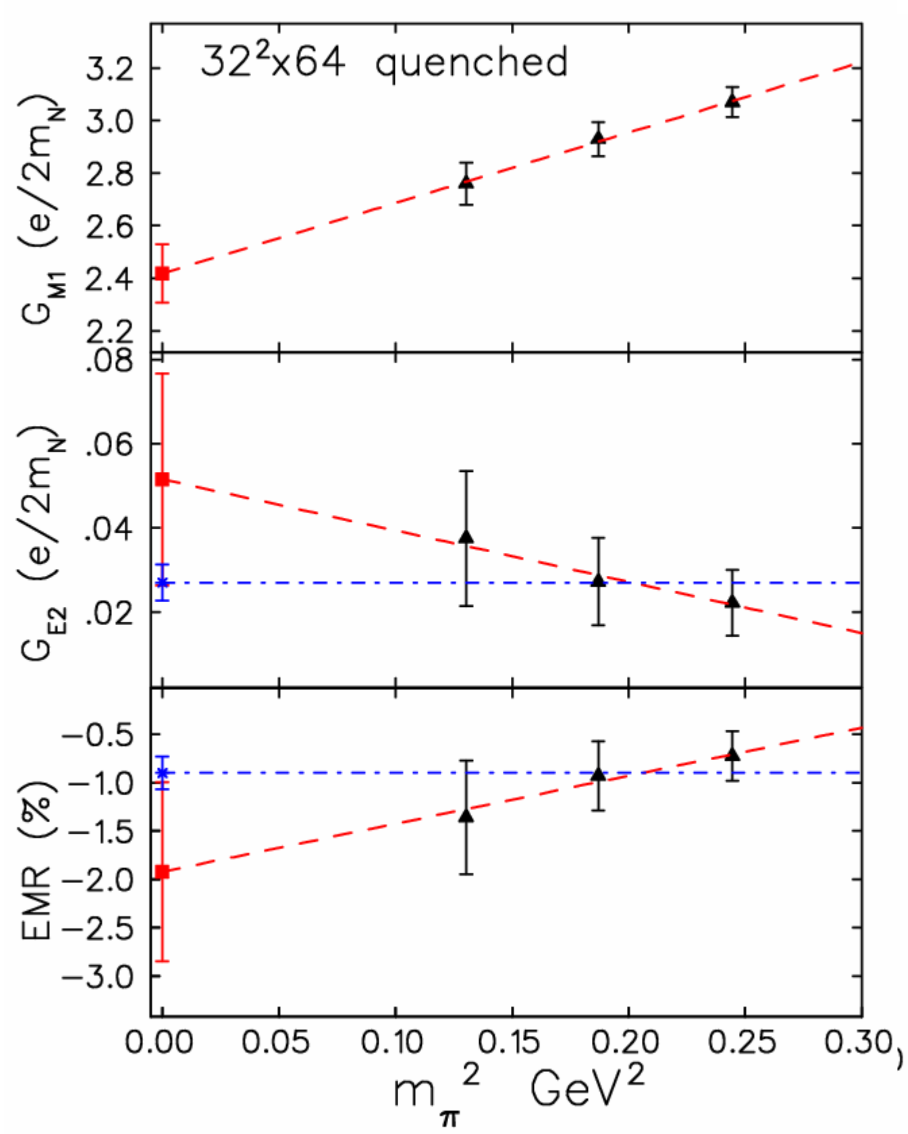
→ all lattice momentum vectors in all three directions, resulting in a given Q^2, contribute

→ This combination requires only one inversion if the appropriate sum of Δ interpolating fields is used as a sink.



Linear extrapolation

Lowest value of Q^2



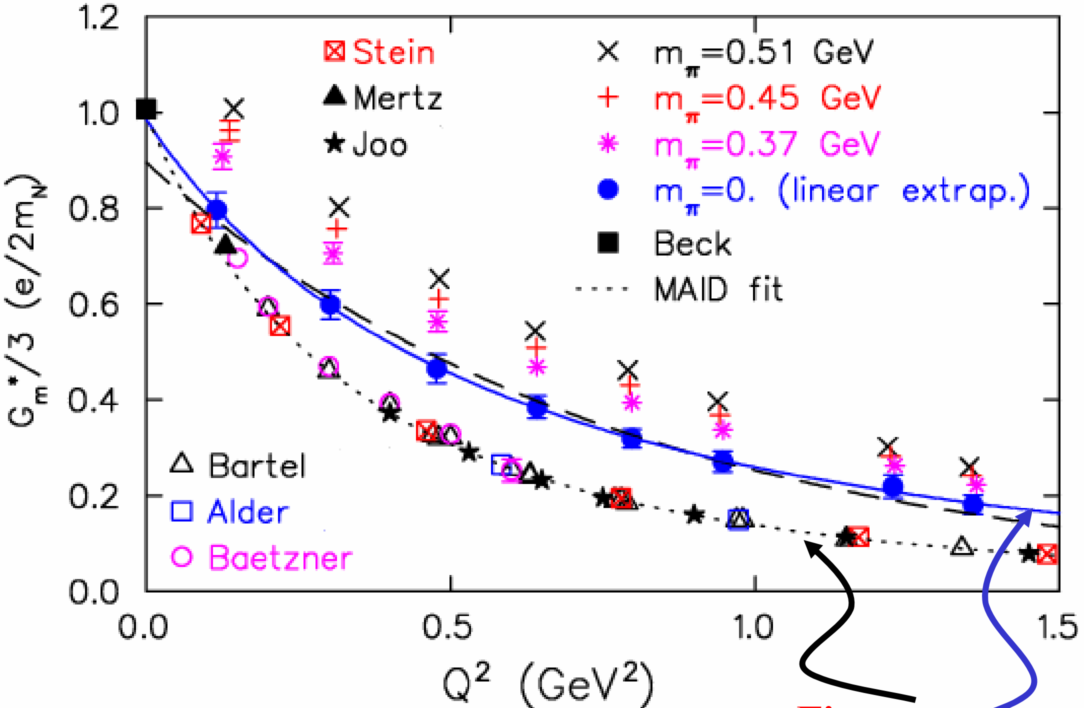
We perform a linear extrapolation in m_π^2.
Chiral logs are expected to be suppressed due to the finite momentum carried by the nucleon .

Systematic error due to the chiral extrapolation?

Overconstrained analysis

Results are obtained in the quenched theory for β=6.0 (lattice spacing a~0.1 fm) on a lattice of 32^3x64 using 200 configurations at κ=0.1554, 0.1558 and 0.1562 (m_π/m_ρ=0.64, 0.58, 0.50)

M1



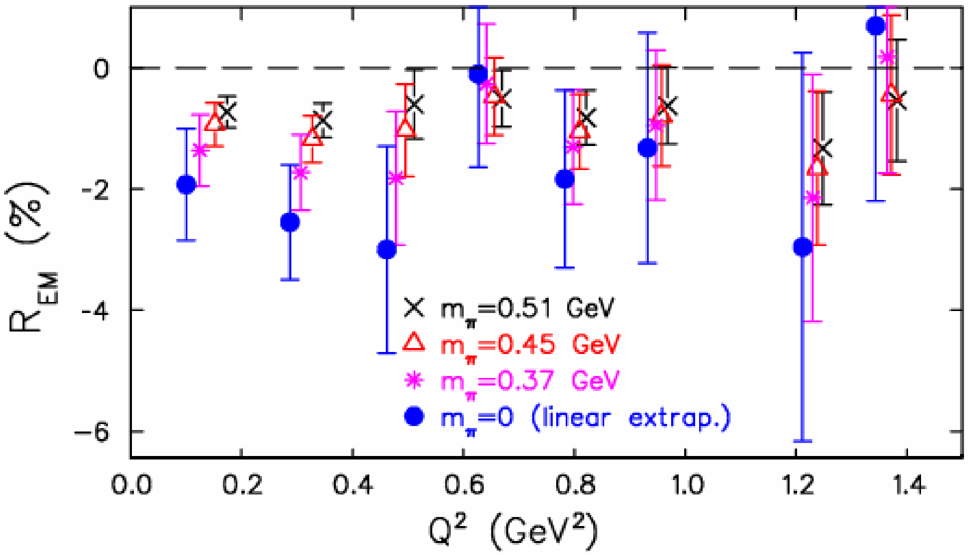
Linear extrapolation in m_π^2 to obtain results at the chiral limit

Fits to

Proton electric form factor

G G G G (0) (α)

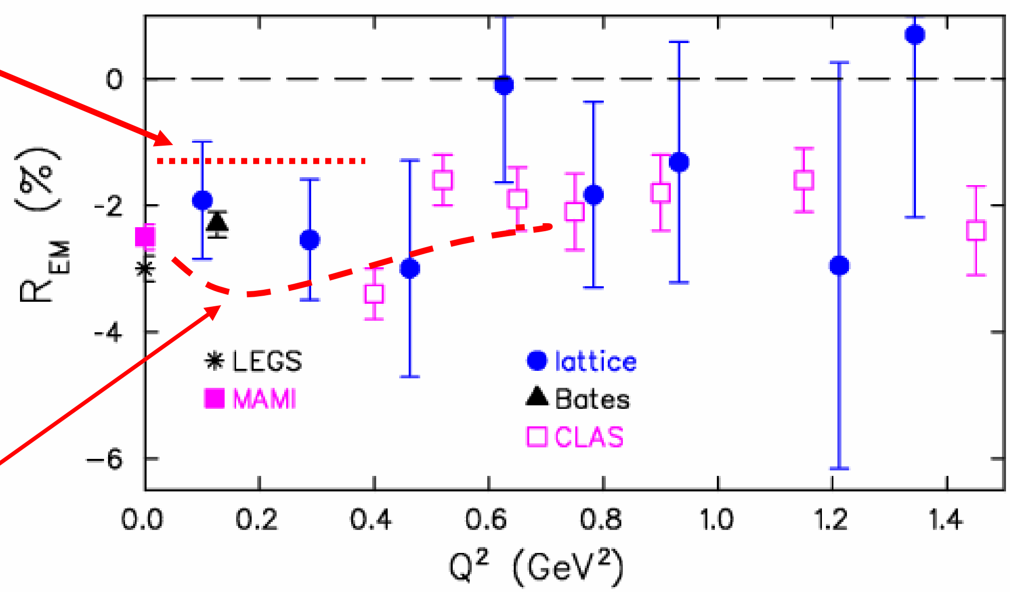
Results: EMR



Linear extrapolation in m_π^2 to obtain results at the chiral limit

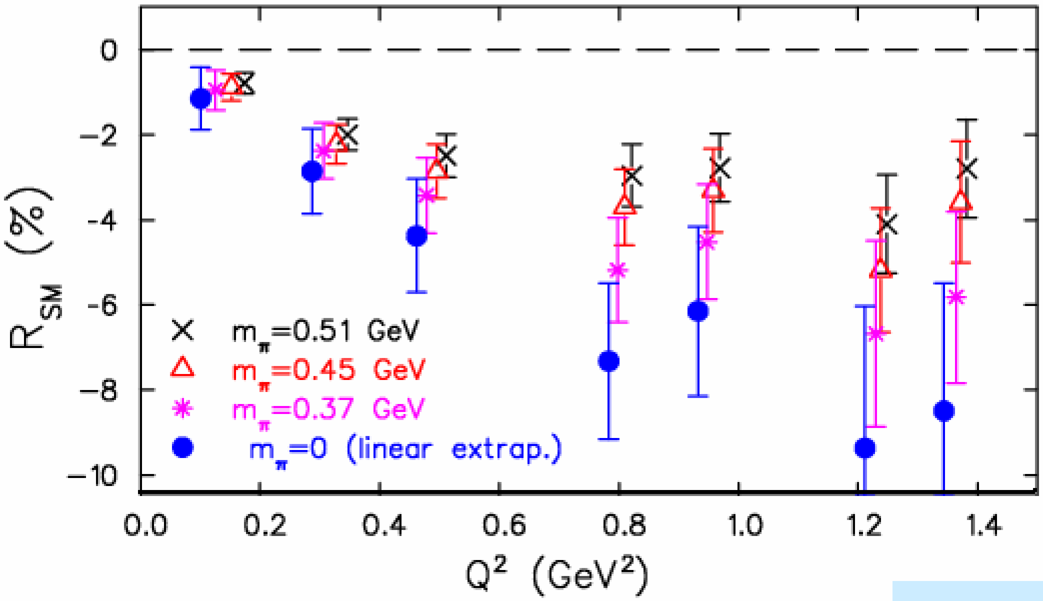
pQCD: **EMR $\xrightarrow{Q^2 \to \infty}$ 1** → **?**

Sato and Lee model with bare vertices



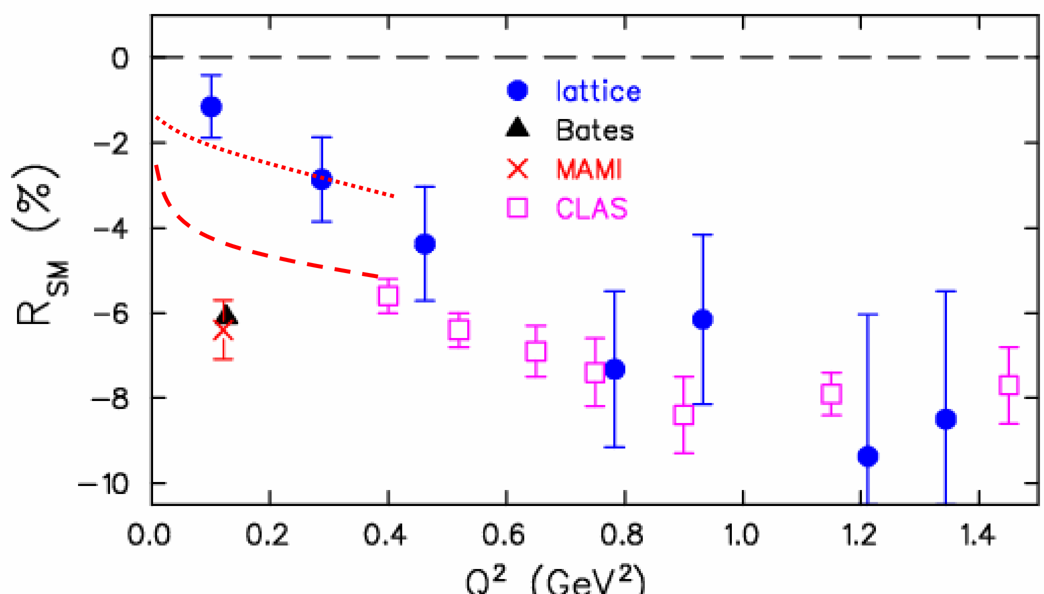
Sato and Lee model with dressed vertices

Results: CMR



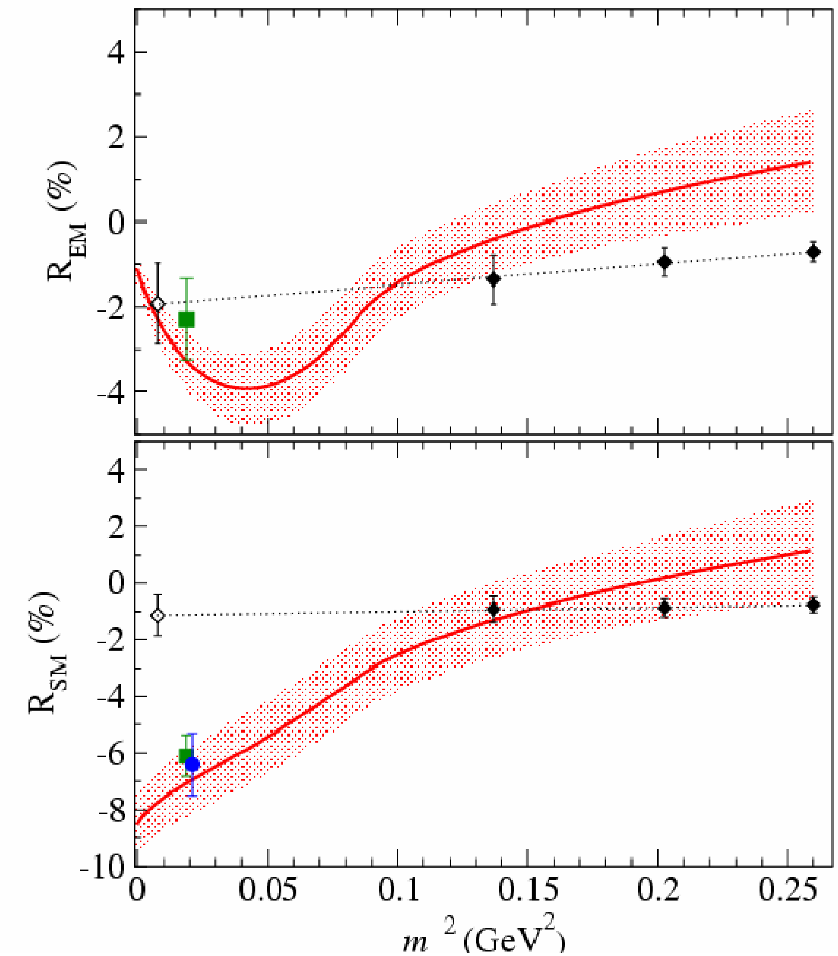
Linear extrapolation in m_π^2 to obtain results at the chiral limit

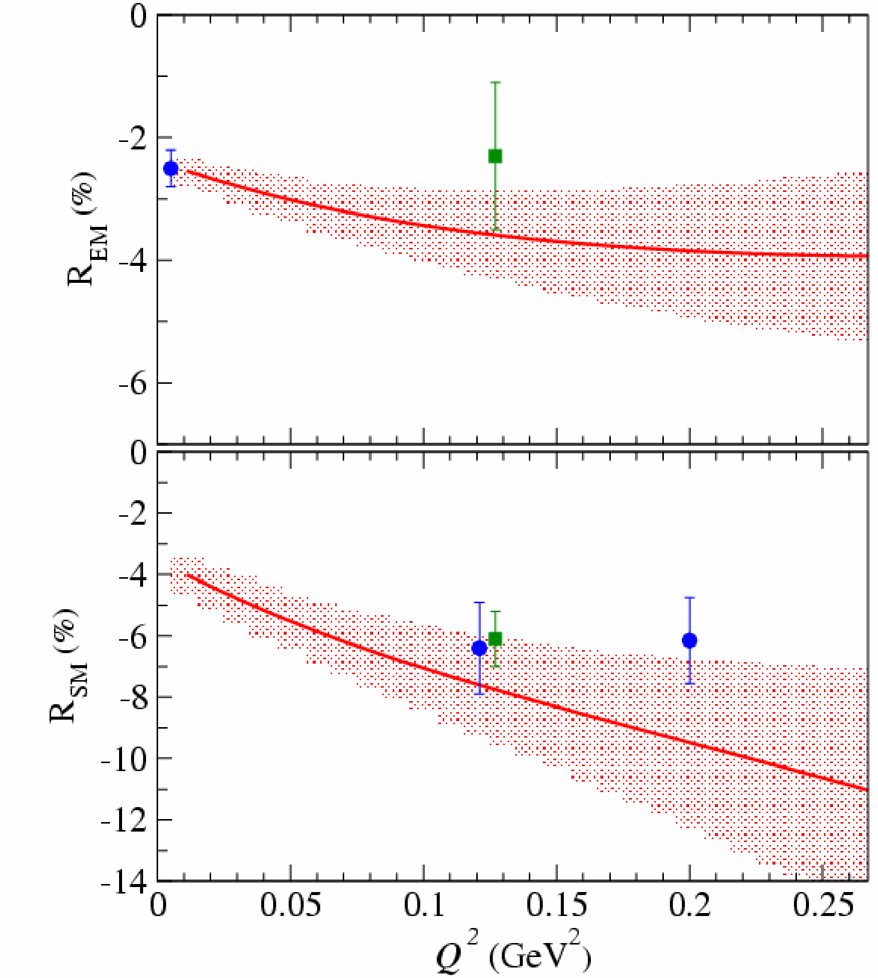
pQCD: **CMR $\xrightarrow{Q^2\to\infty}$ constant**



Discrepancy at low Q^2 where pion cloud contributions are expected to be important

Chiral extrapolation





NLO chiral extrapolation on the ratios using $m_\pi/M\sim\delta^2$, $\Delta/M\sim\delta$. G_{M1} itself not given.

V. Pascalutsa and M. Vanderhaeghen, hep-ph/0508060

1. Dynamical Wilson:

conceptually well-founded

many years of experience

at least so we thought until Lattice 2005

Wilson fermions break chiral symmetry explicitly → fine tuning to get light pions

2. Dynamical Kogut-Susskind (staggered) fermions

Advantages

- fastest to simulate
- Have a residual chiral symmetry giving zero pions at zero quark mass
- MILC collaboration simulates full QCD using improved staggered fermions. The dynamical 2+1 flavor MILC configurations are the most realistic simulations of the physical vacuum to date. ➡ Just download them!

Disadvantages

- More fermions than in the real theory – extra flavors complicate hadron matrix elements
- Approximation – fourth root of the fermion determinant

3. Chiral fermions

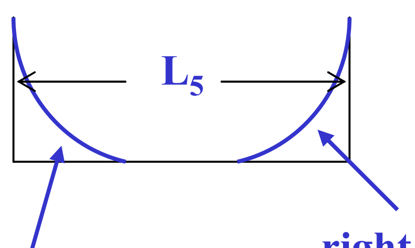
New fermion formulation: Non-perturbative definition of chiral theories

Ginsparg-Wilson relation: $D^{-1}\gamma_5 + \gamma_5 D^{-1} = a\gamma_5$ where D is the fermion matrix

There are two equivalent approaches in constructing a D that obeys the Ginsparg-Wilson relation:

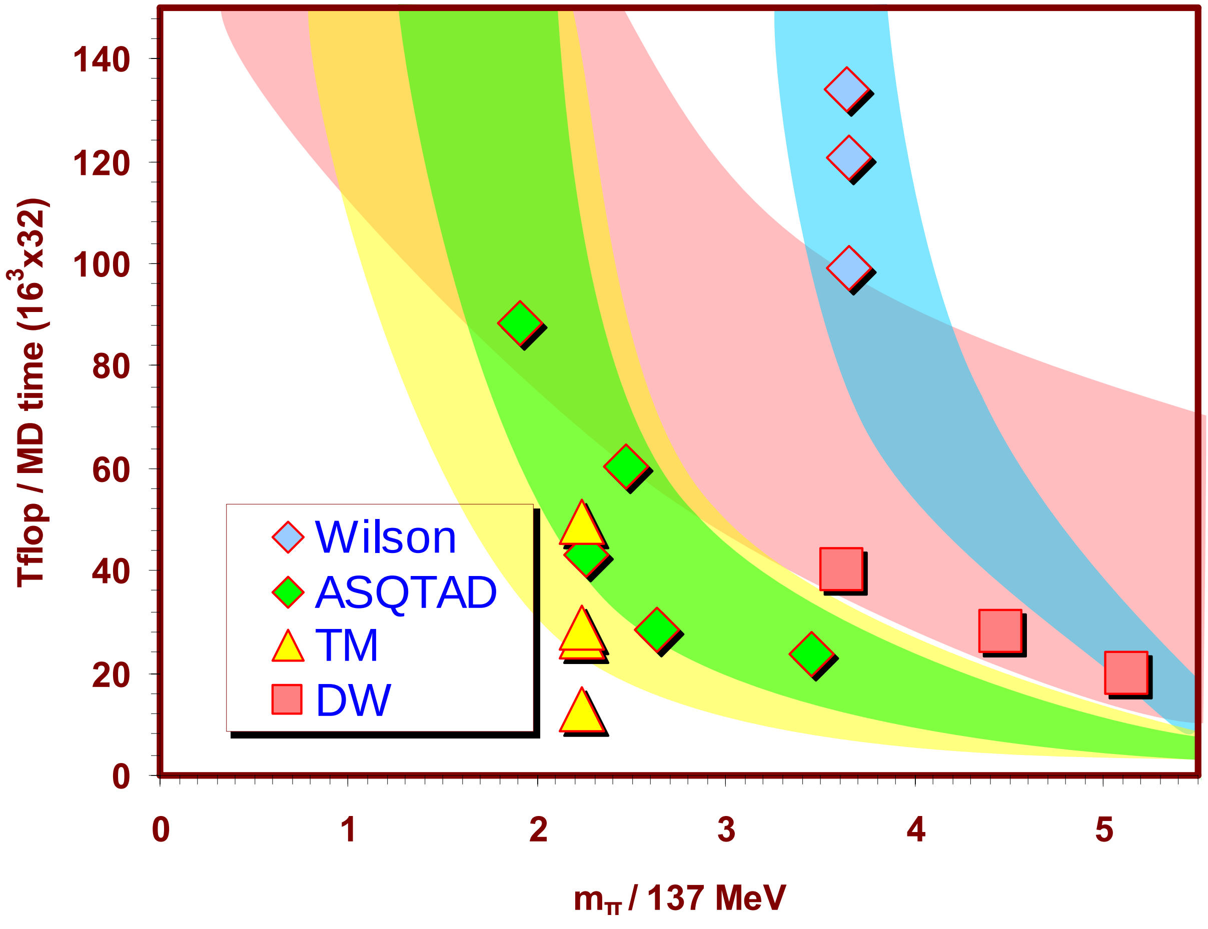
- Domain Wall fermions
- Overlap

PRD 25 (1982)



Dynamical calculations status

A. Kennedy, Lattice 2004



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→ **Hybrid calculation**

A practical option: use different sea and valence quarks

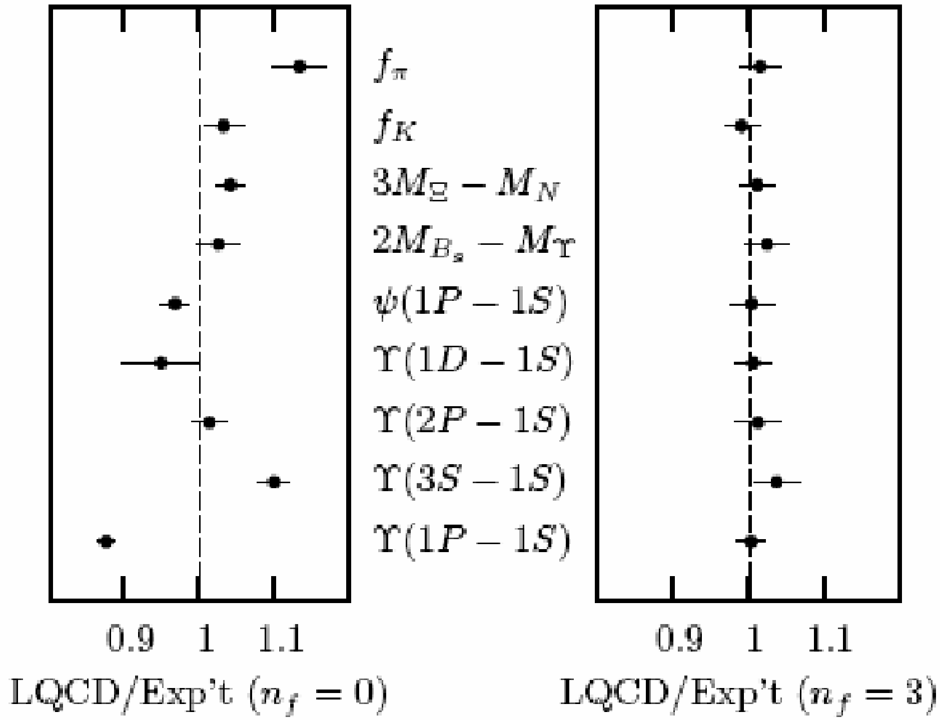
sea → staggered

valence → DWF

A number of physical observables are calculated using MILC configurations and different valence quarks e.g. LHP and HPQCD collaborations to check consistency

Precision results in Full QCD

"Gold plated" observables (hadron with negligible widths or at least below 100 MeV decay threshold and matrix elements that involve only one hadron in the initial and final state

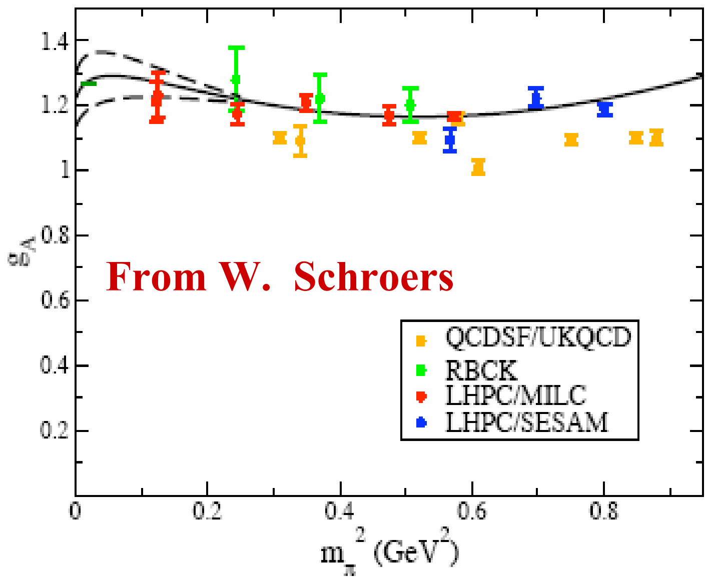


Simulations using staggered fermions, a~0.13 and 0.09 fm

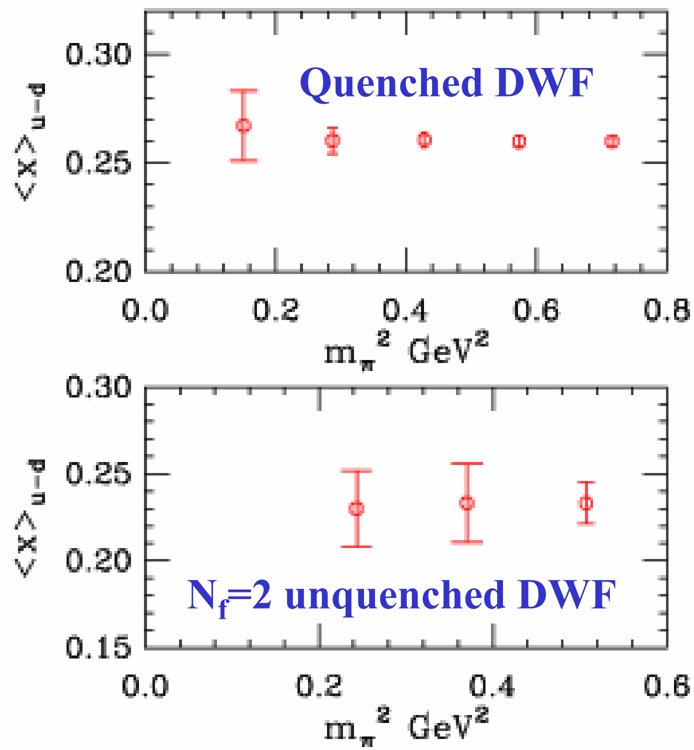
C. Davies et al. PRL 92 (2004)

•**Nucleon structure**

Talk by R. Edwards



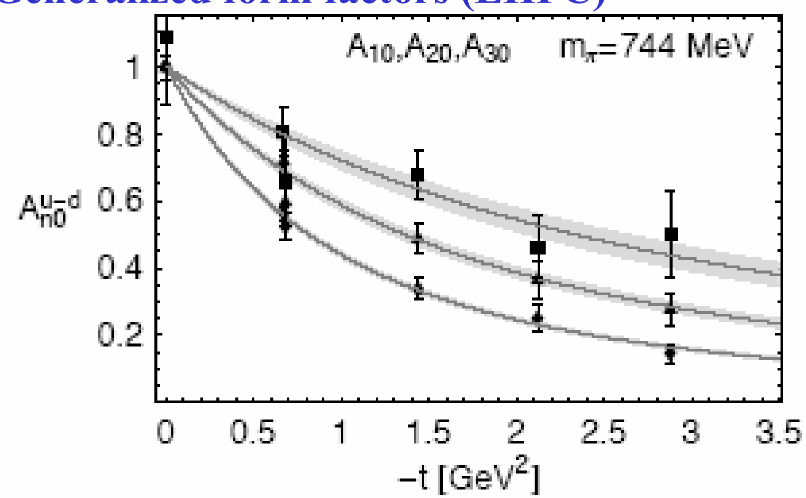
Chiral extrapolation by W. Detmold, *et al.* PRD66 2002: no curvature seen



Still too high
$<x>_{exp}$~0.15

Ohta and Orginos hep/lat0411008

Generalized form factors (LHPC)



Ph. Hägler *et al.* PRL 2004

M. Burkardt, PRD62 (2000)

$$A_{n0}^{q}(-\Delta_\perp^2) = \int d^2b_\perp \, dx \, x^{n-1} \, q(x, b_\perp) \, e^{i \vec{b}_\perp \cdot \vec{\Delta}_\perp}$$

In the forward limit:

$$A_{20}^{q}(0) = < x >_q = \int_0^1 dx \, x \left(q_\uparrow(x) + q_\downarrow(x) \right)$$

Domain wall valence quarks using MILC configurations

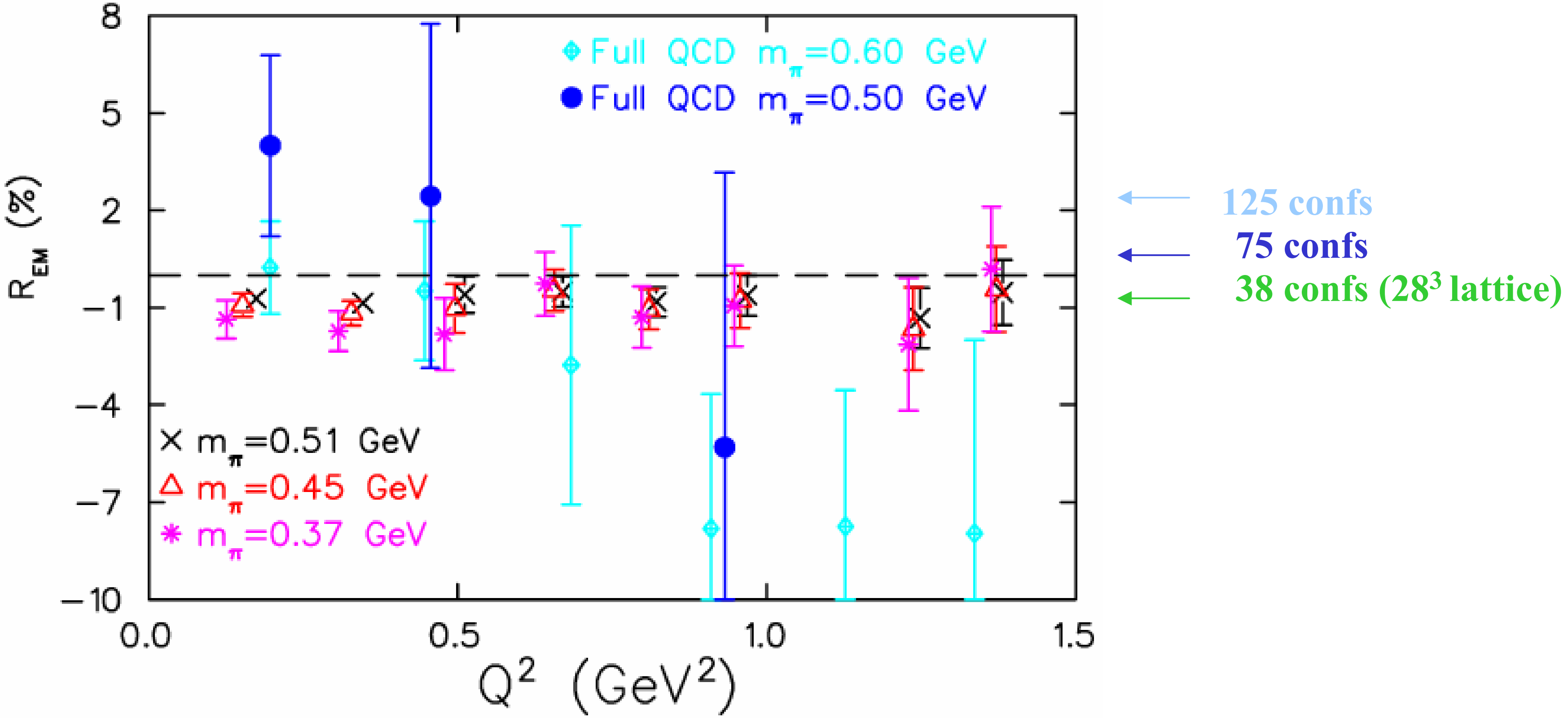
- Fix the size of the 5^{th} dimension: we take the same as determined by the LHP collaboration
- Tune the bare valence quark mass so that the resulting pion mass is equal to that obtained with valence and sea quarks the same.
- The spatial lattice size is 20^3 $(2.5\ \text{fm})^3$ as compared to 32^3 $(3.2\ \text{fm})^3$ used for our quenched calculation.

→ Larger fluctuations

- We use $\bar{\psi}\gamma_\mu \psi$ for the current which is not conserved for finite lattice spacing
→ renormalization constant Z_V which can be calculated

First results in full QCD

Use MILC dynamical configurations and domain wall fermions



EMR and CMR too noisy so far to draw any conclusions

The lightest pion mass that we would like to consider is ~250 MeV. This would require large statistics but it will be close enough to the chiral limit to begin to probe effects due to the pion cloud

Current and future computing power

Three choices:

- Commercial supercomputers : general purpose, rather expensive
- PC Clusters: general purpose, cheaper than commercial but scalability a problem
- custom-design machines (APEnext, QCDOC): optimized for QCD

•Commercial supercomp

-SGI Altrix (LRZ M

16.4 TFlop/s pe

34.6 TFlop/s pe

-BlueGene/L ←

11.2 TFlop/s pe

5.6 TFlop/s pe

T. Wettig, Lattice 2005

• PC clusters

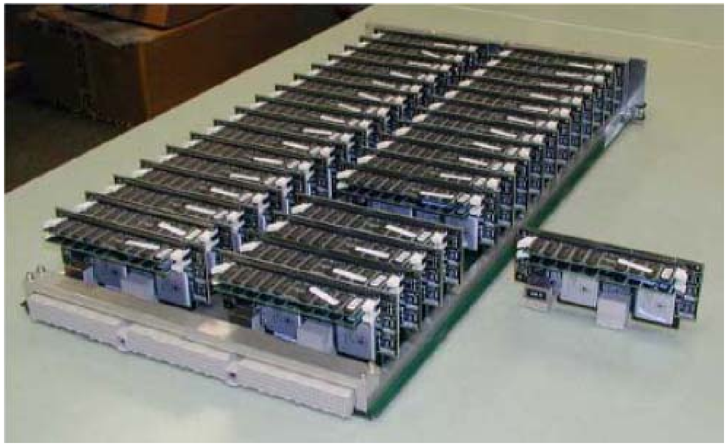
existing lattice QCD clusters with more than 1 TFlop/s peak

	CPU	Network	Peak (TFlop/s)	Name
Wuppertal	1024 Opteron	Gig-E (2d)	3.7	ALICEnext
JLAB	384 Xeon	Gig-E (5d)	2.2	4G
JLAB	256 Xeon	Gig-E (3d)	1.4	3G
Fermilab	260 (520) P4	Infiniband	1.7 (3.4)	Pion
Fermilab	256 Xeon	Myrinet	1.2	W

• Custom-design machines

APEnext

QCDOC



Planned Installations (1 rack=512 nodes =0.66 TFlop/s peak)

- INFN 6 144 nodes (12 racks)
- Bielefeld 3072 nodes (6 racks)
- DESY 1576 nodes (3 racks)
- Orsay 512 nodes (1 rack)

Cost: $0.6 per peak MFlop/s

Planned Installations (1 rack=1024 nodes =0.83 TFlop/s peak)

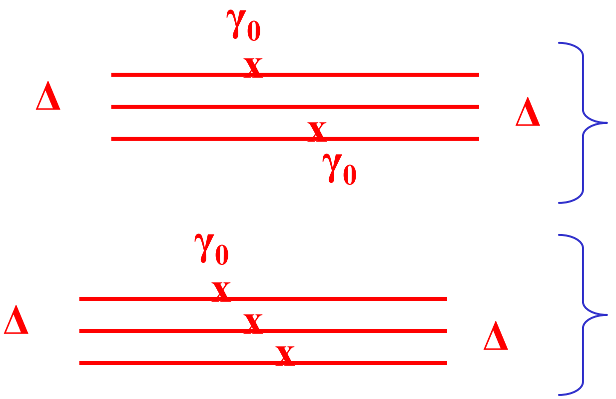
- UKQCD 14 720 nodes
- DOE 14 140 nodes
- RIKEN-BNL 13 308 nodes
- Columbia 2 432 nodes
- Regensburg 448 nodes

Cost: $0.45 per peak MFlop/s

→ Tens of Teraflop/s will be available for lattice QCD

Two- and three- density correlators

They provide information on the spatial distribution of quarks in a hadron



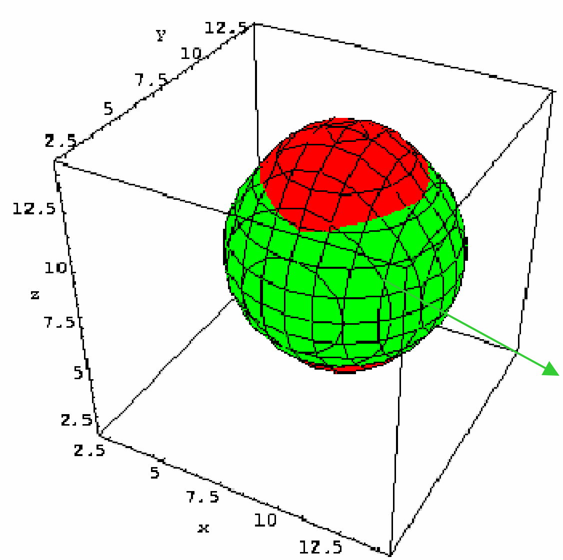
$$j_0(y) = :\bar{u}(y)\,\gamma_0\,u(y):$$

non-relativistic limit → |wave function|^2

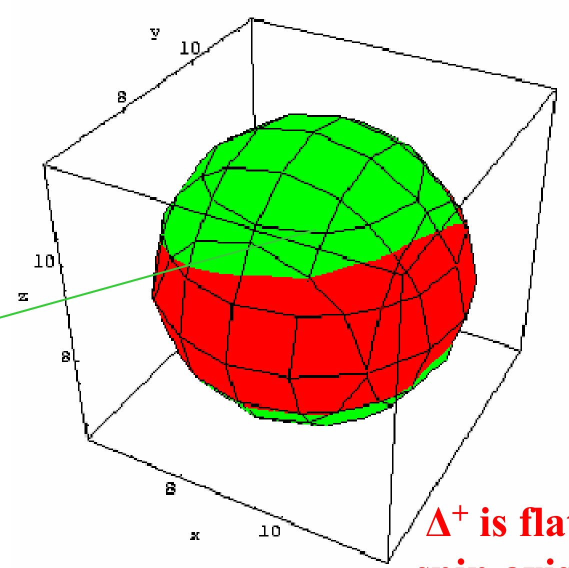
Deformation can be studied

Deformation in full QCD: a feasibility study m_π=760 MeV

C.A., Ph. de Forcrand and A. Tsapalis



sphere



rho is elongated along its spin axis → prolate

Δ^+ is flatten along its spin axis → oblate

Can we improve this calculation?

smaller quark mass, larger lattice, chiral fermions, momentum projection

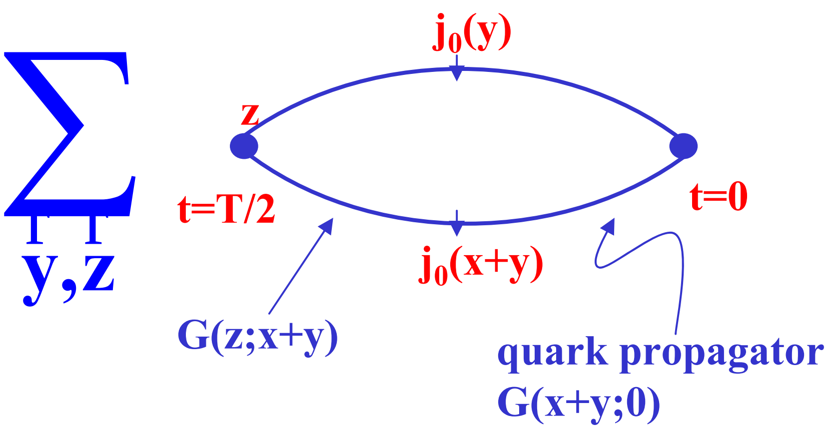
Momentum projection

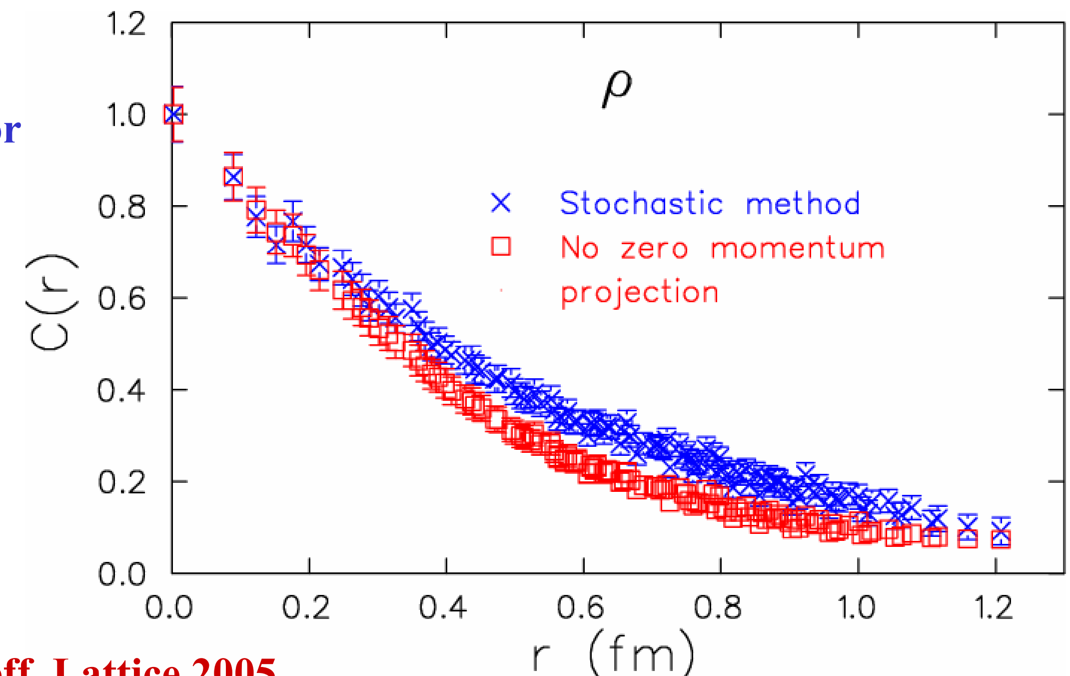
Requires the all to all propagator

We use stochastic techniques to calculate it. We found that the best method is to use the stochastic propagator for one segment and sum over the spatial coordinates of the sink by computing the sequential propagator.

r $\int d^3y$ r r r

$$j_0(y) = :\bar{u}(y)\,\gamma_0\,u(y):$$

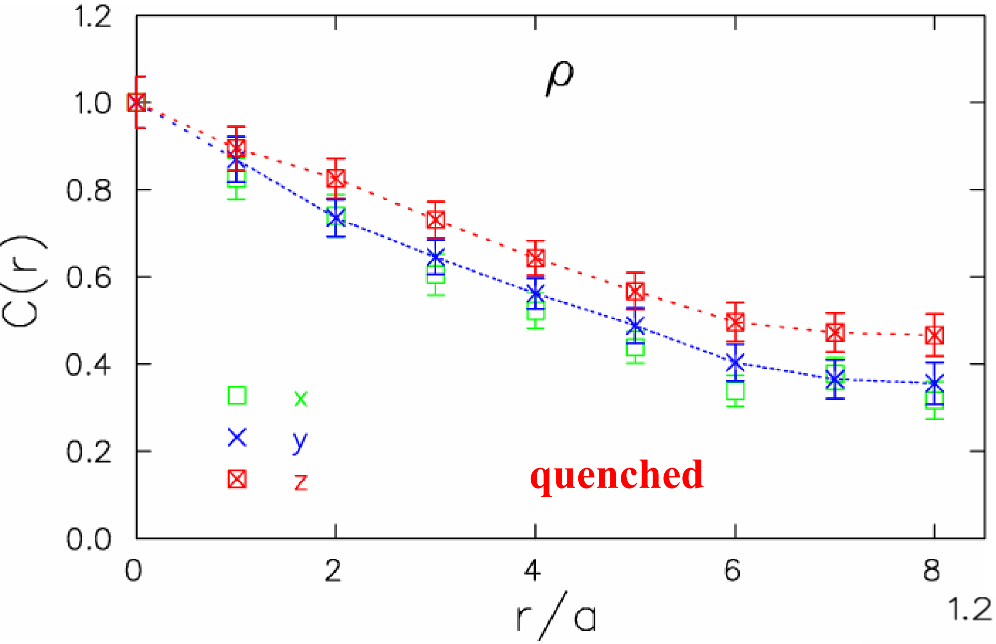


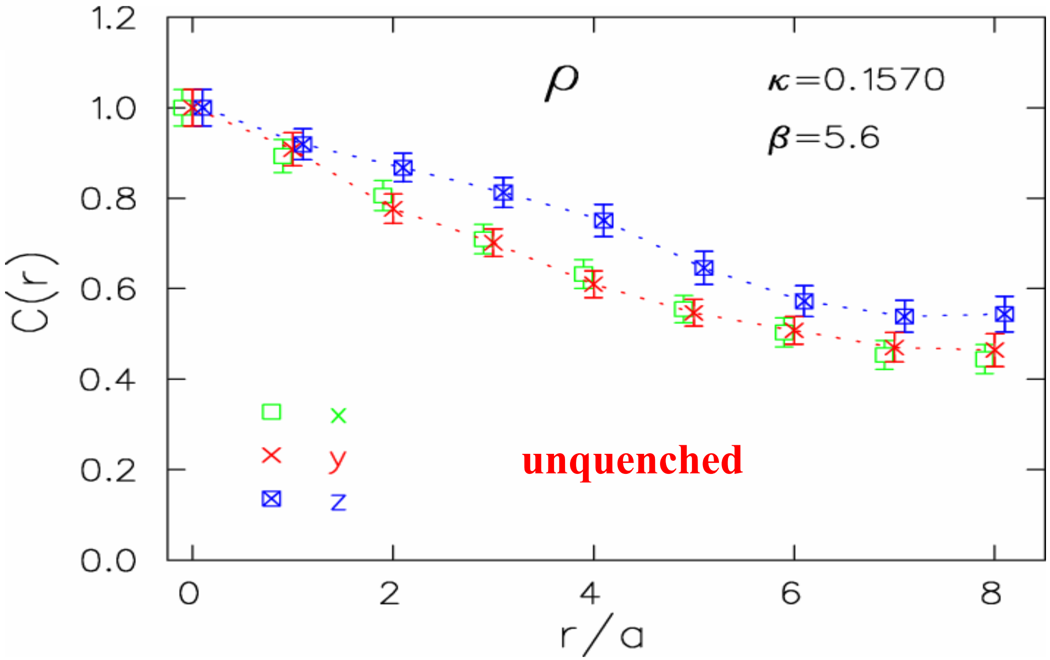


ρ meson distribution is broader

C. A., P. Dimopoulos, G. Koutsou and H. Neff, Lattice 2005

Rho meson asymmetry

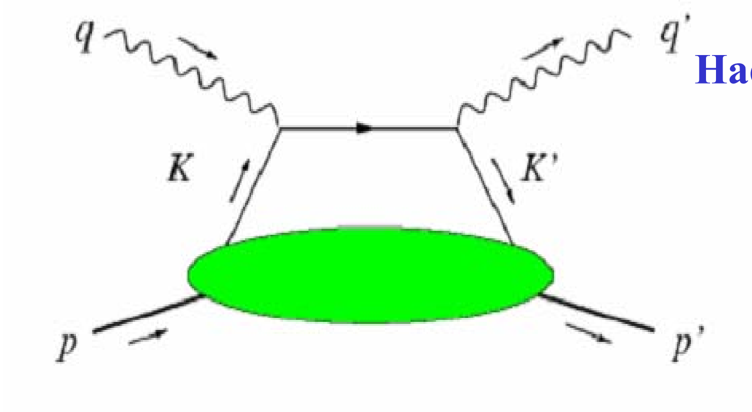




Opens up the way of evaluating other physically interesting quantities like polarizabilites.

Hadron deformation

In addition to $\gamma N \rightarrow \Delta$ deformation can be seen in other processes: Compton scattering, DIS,…



Hadronic tensor of current-current correlation function

$$W_{\alpha\beta}(q^2,\nu) = \left\langle N \left| \int \frac{d^4x}{2\pi} e^{iq.x} J_\alpha(x) J_\beta(0) \right| N \right\rangle_{\text{spin average}}$$

What can be computed on the lattice is

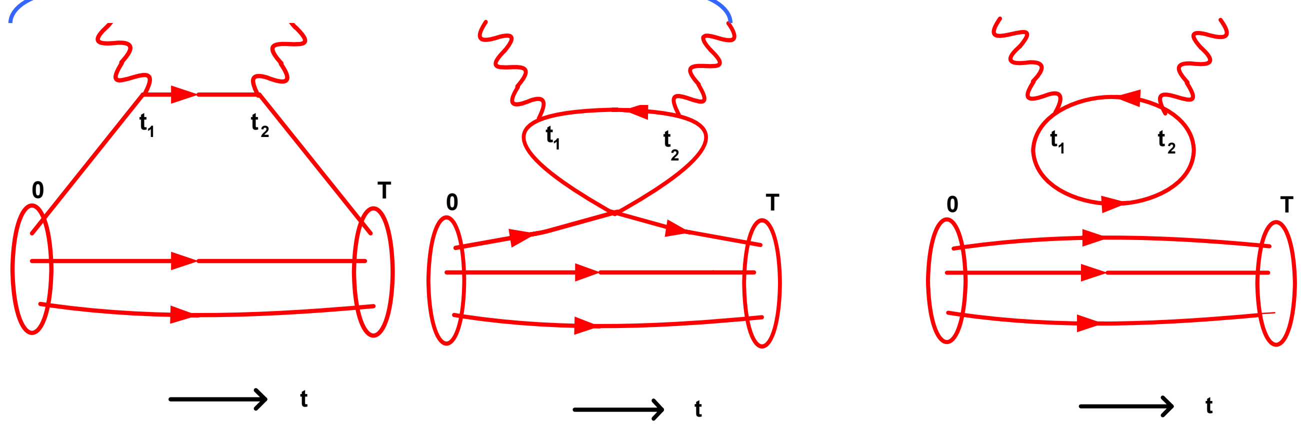
$$\tilde{W}_{\alpha\beta}(q^2,\tau) = \left\langle N \left| \int \frac{d^3x}{2\pi} e^{-i\vec{q}.\vec{x}} J_\alpha(x) J_\beta(0) \right| N \right\rangle_{\text{spin average}}$$

where $\tau = t_2 - t_1$

Included in the quenched approximation

Sea quark contribution

No lattice calculation exists



Conclusions

- Lattice QCD is entering an era where it can make significant contributions in the interpretation of current experimental results.
- A valuable method for understanding hadronic phenomena
  - $\gamma N \rightarrow \Delta : \Gamma_{M1}, \Gamma_{E2}$ and Γ_{C2} calculated in quenched approximation in the range of Q^2 where recent measurements are taken. Results in full QCD are within reach. Non-zero values $\rightarrow$ deformation of the nucleon/Δ
  - Density-density correlators $\rightarrow$ deformation, polarizabilities, form factors
  - N

- Computer technology and new algorithms will deliver 10´s of Teraflop/s in the next five years
  - Provide dynamical gauge configurations in the chiral regime
  - Enable the accurate evaluation of more involved matrix elements