Structure of hadrons on the basis of the Salpeter equation

Bernard Metsch

Tim van Cauteren (U. Gent (B)), Christian Haupt, Matthias Koll, Ulrich Löring, Dirk Merten,

Sascha Migura, Herbert Petry, Jan Ryckebusch (U. Gent (B))

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie)

Rheinische Friedrich-Wilhelms-Universität Bonn

Nußallee 14-16, D-53115 Bonn, GERMANY

e-mail: metsch@itkp.uni-bonn.de

SFB/TR16

I3HP/N5

Description of Hadrons

Goal: Unified description of

- Mass spectra (light quark flavours < 3 GeV, J < 8): Regge-trajectories (M+B), scalar excitations (M+B), (pseudo)scalar mixings (M) , parity doublets (B), “undetected” resonances . . .;
- Electroweak properties: electroweak form factors; radiative decays/transitions; semi-leptonic weak decays . . .;
- Strong (two-body) decays and interactions.

Tools:	Ingredients:	Achievements:
Field theoretical approaches (relativistically covariant)		
Lattice gauge theory	QCD	ground states $\longrightarrow$ excited states
Dyson-Schwinger / Bethe-Salpeter Eq.	Infrared Gluon prop.	meson ground states baryon g.s. (diquark-quark)
- inst. approx. **Salpeter Equation**	Confinement Instanton effects	mesons and baryons
Quantum mechanical approaches (“relativised” quark kinematics/dynamics;) currents: parameterised or (covariantly) from Dirac’s front-, instant-, point form		
Constituent Quark Model	Confinement OGE $\longrightarrow$ Fermi-Breit	mesons and baryons
Constituent Quark Model	Confinement GBE	baryons (M < 1.8 GeV)
Constituent Quark Model	Hypercentric interactions + FB	baryons (M < 1.8 GeV)

Constituent Quark Models

and many other approaches (algebraic treatment with collective variables, (chiral) soliton models etc.)

Here the focus is on: Constituent Quark Models

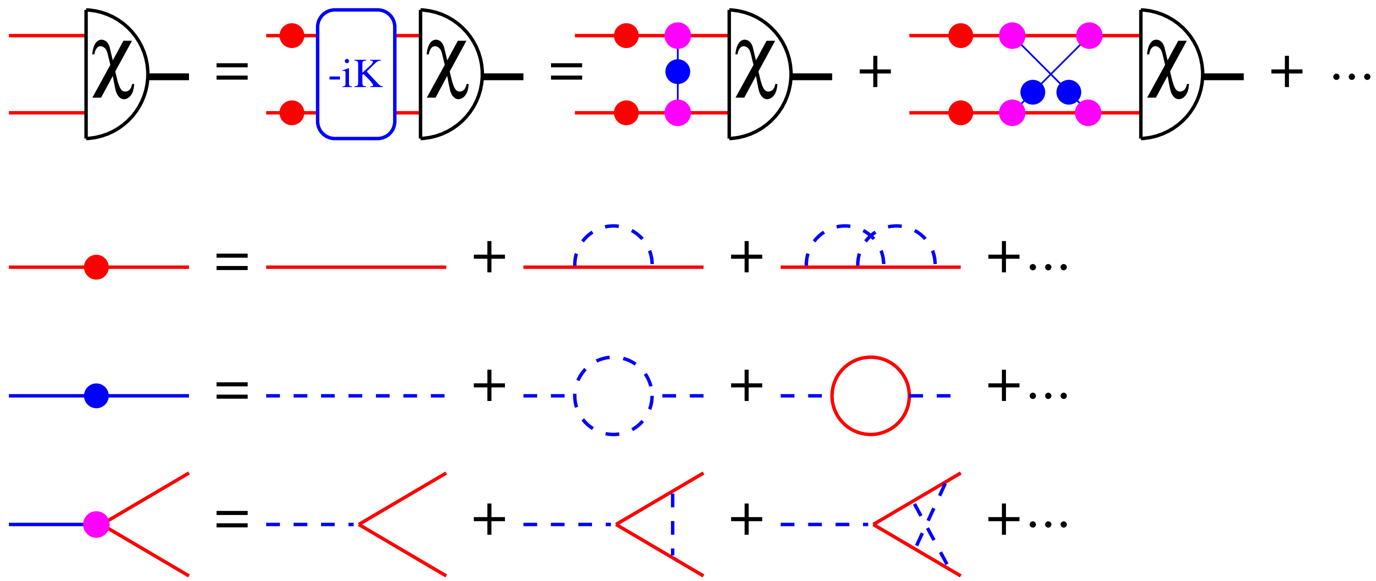
- Basic Assumption:
  (the majority of) meson and baryon excitations can be described by $q\bar{q}$– and q^3– bound states of (constituent) quarks, respectively; the coupling to strong decay channels can be treated perturbatively ...
- constitutes a framework to judge what is exotic (glueballs, hybrids, multiquark-states) ...
- Light flavoured (u, d, s) systems:
  Even with constituent quark masses, quarks moving in a hadron are not really slow; in general the total mass differs appreciably from the sum of the constituent masses $\Rightarrow$ relativistically covariant description $\Leftarrow$ large momentum transfers:
  - Relativistic bound state equations (Bethe-Salpeter, Dyson-Schwinger)
  - (Dirac's (instant-, front-) point form of Relativistic Quantum Mechanics)
- Extension to heavy flavoured systems

Relativistic bound state equations ($q\bar{q}$)

Bound states of 4-momentum $\bar{P}$ ($\bar{P}^2 = M^2$) described by BETHE-SALPETER-amplitude

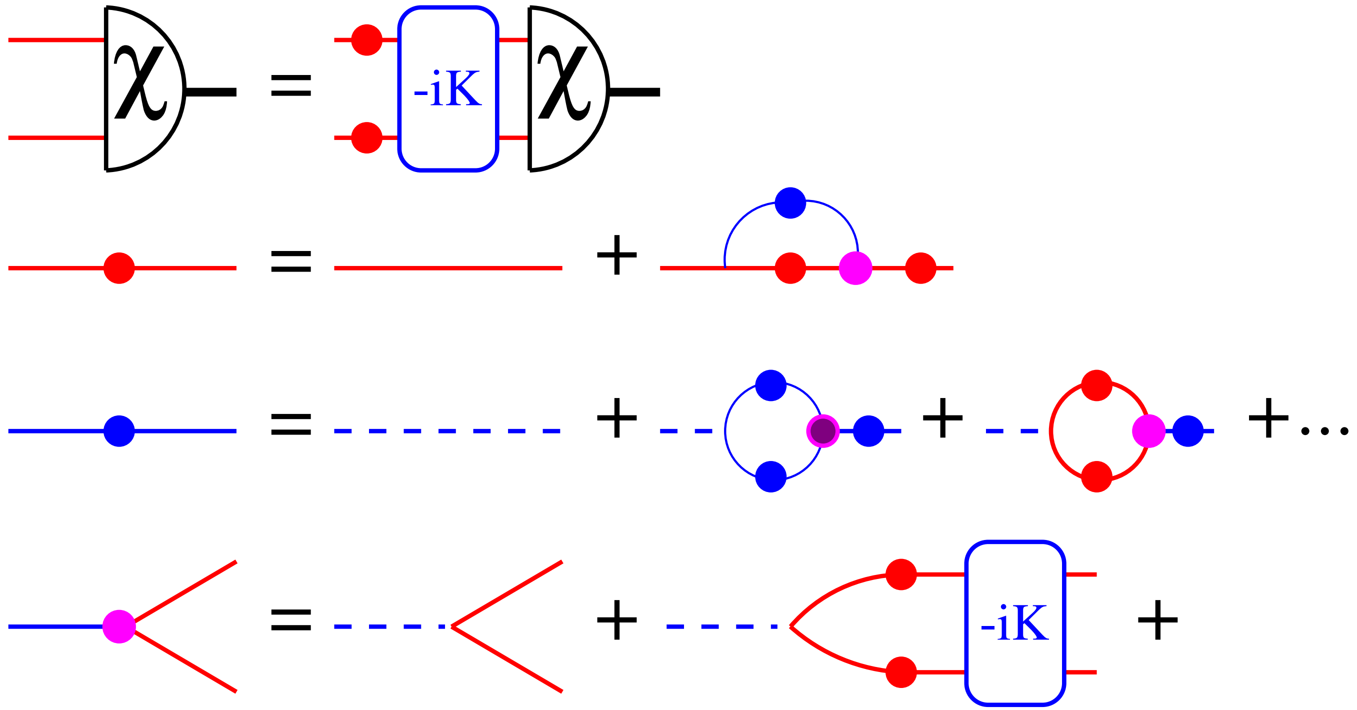
$$\chi_{\alpha\beta}(x_1, x_2) := \langle 0 | \, T \left[\psi^1_\alpha(x_1) \bar{\psi}^2_\beta(x_2) \right] | \bar{P} \rangle$$

fulfil the homogeneous BETHE-SALPETER equation:



and involve full (dressed) propagators for fermions, exchange bosons and full (dressed) vertex functions: This leads to the skeleton-expansion: *i.e.* an infinite set of coupled DYSON-SCHWINGER- and BETHE-SALPETER-equations:

Skeleton-expansion, approximations



In order to solve this in practise one truncates this expansion, makes an *Ansatz* for some n-point function and solves the equations (BETHE-SALPETER-equation for two particles or the DYSON-SCHWINGER-equation for the self-energy) of lower order.

⇒ renormalisation-group-improved rainbow-ladder approach (DSE) based on an effective gluon propagator with a specific infrared behaviour

P. Maris, C.D. Roberts: "Dyson-Schwinger Equations: A tool for hadron physics", Int. J. Mod. Phys. E**12** (2003) 297; nucl-th/0301049, (2003)

Further approximations ...

A simplified ANSATZ is to assume that the fermion propagator has the free form

$$S(p) \approx i\,\left[\gamma^\mu p_\mu - m + i\varepsilon\right]^{-1}$$

and to account for the self-energy contributions by introducing a constituent mass m. One might approximate the irreducible interaction kernel by a single gluon exchange in COULOMB-gauge, perhaps with a running coupling $\alpha_S(k^2)$:

$$K(P;p,p+k) = 4\pi\,\alpha_S(-k^2)\,\frac{1}{(2\pi)^4}$$
$$\left[\frac{\gamma^0(1)\gamma^0(2)}{|\vec{k}|^2} + \frac{1}{k^2+i\varepsilon}\left(\vec{\gamma}(1)\vec{\gamma}(2) - \frac{1}{|\vec{k}|^2}(\vec{\gamma}(1)\cdot\vec{k})(\vec{\gamma}(2)\cdot\vec{k})\right)\right],$$

where the first term describes the instantaneous COULOMB-potential, since

$$\frac{4\pi}{(2\pi)^4}\int \mathrm{d}^3k\,\mathrm{e}^{i(\vec{x}\cdot\vec{k})}\,\frac{1}{|\vec{k}|^2}\int \mathrm{d}k^0\,\mathrm{e}^{-ik^0t} = \frac{1}{r}\delta(t)\,,$$

if we neglect the k^2 dependence of α_S and where $r = |\vec{x}| = |\vec{x}_1 - \vec{x}_2|$. If in addition we make the no-retardation limit, $k^2 \to -|\vec{k}|^2$ we obtain an instantaneous OGE-potential.

Instantaneous approximation

In the following we shall consider such instantaneous kernels

$$K(P, p, p') = V(p_\perp, {p_\perp}'), \text{ with } p_\perp := p - p_\|,\ p_\| := \frac{(P\cdot p)}{P^2} P\,,$$

or (in the restframe of the particle)

$$K(P = (M, \vec{0}), p, p') = V(\vec{p}, \vec{p}\,')$$

in general.

- motivated by the success of the (non-relativistic) Constituent Quark Model
- implementation of confinement by a string-like potential

Defining the SALPETER-amplitude

$$\Phi(\vec{p}) = \int \frac{\mathrm{d}p^0}{2\pi}\, \chi(p^0, \vec{p})\Big|_{P=(M,\vec{0})}\,,$$

introducing projectors on positive and negative energy solutions $\Lambda_i^\pm(\vec{p}) := \frac{\omega_i(\vec{p}) \pm H_i(\vec{p})}{2\omega_i(\vec{p})}$, with $H_i(\vec{p}) = \gamma_0\left((\vec{\gamma}\cdot\vec{p}) + m_i\right)$ the DIRAC-one-particle hamiltonian and $\omega_i(\vec{p}) = \sqrt{m_i^2 + |\vec{p}|^2}$, and integrating the l.h.s. and the r.h.s of the BETHE-SALPETER-equation over p^0 we obtain, for instantaneous interaction kernels and free-form propagators, in the rest frame of the particle-antiparticle system the SALPETER-equation:

SALPETER-equation

$$\Phi(\vec{p}) = \Lambda_1^-(\vec{p})\gamma_0 \frac{\left[\int \frac{d^3p'}{(2\pi)^3} V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')\right]}{M+\omega_1(\vec{p})+\omega_2(\vec{p})}\gamma_0\Lambda_2^+(-\vec{p}) - \Lambda_1^+(\vec{p})\gamma_0 \frac{\left[\int \frac{d^3p'}{(2\pi)^3} V(\vec{p},\vec{p}\,')\Phi(\vec{p}\,')\right]}{M-\omega_1(\vec{p})-\omega_2(\vec{p})}\gamma_0\Lambda_2^-(-\vec{p})$$

Normalisation

$$\int \frac{d^3p}{(2\pi)^3} \mathrm{tr}\left[\Phi^\dagger(\vec{p})\Lambda_1^+(\vec{p})\Phi(\vec{p})\Lambda_2^-(-\vec{p}) - \Phi^\dagger(\vec{p})\Lambda_1^-(\vec{p})\Phi(\vec{p})\Lambda_2^+(-\vec{p})\right] = 2M\,.$$

The SALPETER-equation constitutes the basis of virtually all constituent quark models:

⇒ **full** SALPETER-equation (instantaneous BSE)

⇒ **reduced** SALPETER-equation (“relativised” SCHRÖDINGER-equation: relativistic kinetic energy, relativistic corrections to the potential (in: $\Lambda^\pm$) (“R”CQM))

St. Godfrey, N. Isgur, Phys. Rev. **32** (1985) 189; S. Capstick, W. Roberts, Prog. Part. Nucl. Phys., **45**, (2000) 241

Light Mesons with the SALPETER-equation

The instantaneous interaction kernel (potential) V contains a

- confinement potential:

$$\int \frac{d^3p'}{(2\pi)^3} V(\vec{p},\vec{p}')\Phi(\vec{p}') = \int \frac{d^3p'}{(2\pi)^3} \mathcal{V}_C(|\vec{p}-\vec{p}'|^2)\Gamma\Phi(\vec{p}')\Gamma\,,$$

  where $\mathcal{V}_C(|\vec{p}-\vec{p}'|^2)$ is the FOURIER-transform of a linearly rising potential $\mathcal{V}_C(|\vec{x}_q-\vec{x}_{\bar{q}}|) = a_C + b_C \cdot |\vec{x}_q - \vec{x}_{\bar{q}}|$, with a "suitable" spin-dependence, given by the DIRAC-structure Γ, chosen to minimise spin-orbit effects.

- spin-flavour dependent interaction from instanton effects:

$$\Delta\mathcal{L}(2) = \frac{3}{16}\sum_i \sum_{\substack{k,l\\m,n}} \sum_{\substack{c_k,c_l\\c_m,c_n}} g_{\text{eff}}(i)\epsilon_{ikl}\epsilon_{imn}\left(\frac{3}{2}\delta_{c_kc_n}\delta_{c_lc_m} - \frac{1}{2}\delta_{c_kc_m}\delta_{c_nc_l}\right)$$
$$\left[\left(\bar{\Psi}_{k,c_k}\ \mathbb{1}\ \Psi_{n,c_n}\right)\ \left(\bar{\Psi}_{l,c_l}\ \mathbb{1}\ \Psi_{m,c_m}\right) + \left(\bar{\Psi}_{k,c_k}\ \gamma^5\ \Psi_{n,c_n}\right)\ \left(\bar{\Psi}_{l,c_l}\ \gamma^5\ \Psi_{m,c_m}\right)\right]$$

  where $i,k,l,m,n \in \{u,d,s\}$ are flavour and $c_k,c_l,c_m,c_n \in \{r,g,b\}$ colour indices.

  - flavour antisymmetric; $U_A(1)$ symmetry breaking; acts on $J=0$ only.

SALPETER-model parameters

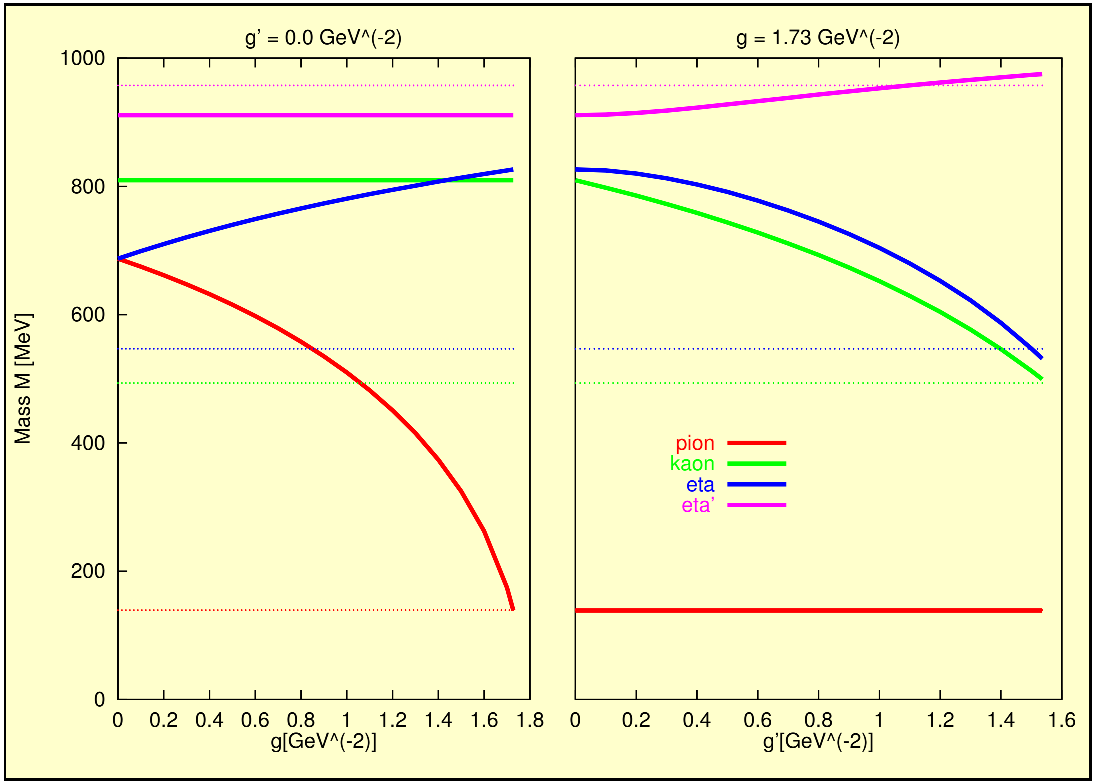
$$\int \frac{d^3p'}{(2\pi)^3} V_{\text{III}}(\vec{p},\vec{p}')\Phi(\vec{p}') = 4\, G(g,g') \int \frac{d^3p'}{(2\pi)^3} \mathcal{R}_\Lambda(\vec{p},\vec{p}') \left(\mathbb{1}\text{tr}\left[\Phi(\vec{p}\,')\right] + \gamma^5\text{tr}\left[\Phi(\vec{p}\,')\gamma^5\right]\right) ,$$

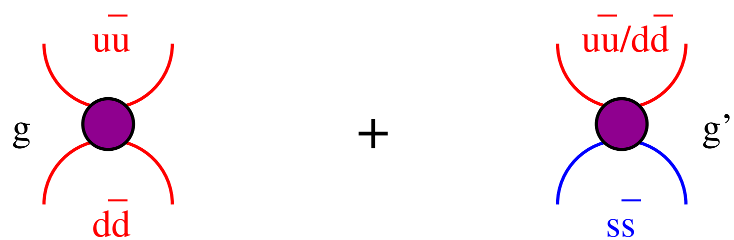
where $\mathcal{R}_\lambda$ represents a regularisation function ($\Rightarrow$ finite range (0.3–0.4 fm)) and $G(g,g')$ is a flavour matrix.

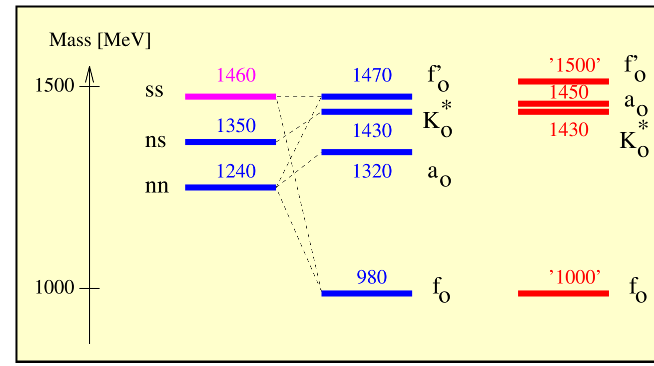
Parameters of the SALPETER model:

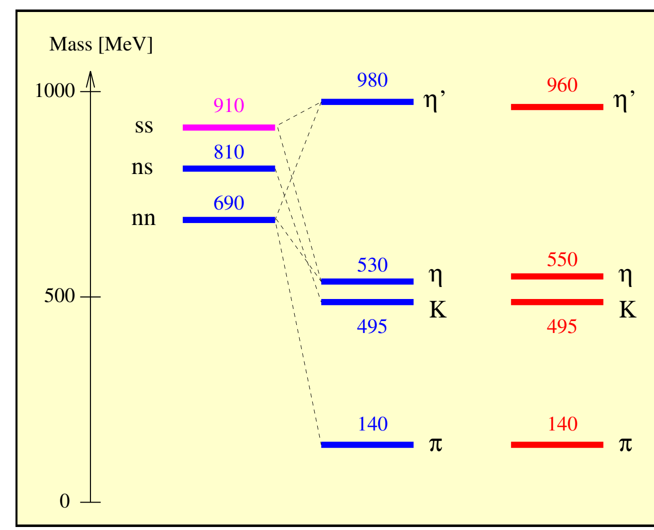
		Model $\mathcal{A}$		Model $\mathcal{B}$	
masses	m_n	306	MeV	419	MeV
	m_s	503	MeV	550	MeV
confinement	a_C	-1751	MeV	-1135	MeV
	b_C	2076	MeV/fm	1300	MeV/fm
	$\Gamma \cdot \Gamma$	$\frac{1}{2}(\mathbb{1} \cdot \mathbb{1} - \gamma_0 \cdot \gamma_0)$		$\frac{1}{2}(\mathbb{1} \cdot \mathbb{1} - \gamma_5 \cdot \gamma_5 - \gamma^\mu \cdot \gamma_\mu)$	
instanton	g	1.73	GeV^{-2}	1.63	GeV^{-2}
induced	g'	1.54	GeV^{-2}	1.35	GeV^{-2}
interaction	λ	0.30	fm	0.42	fm

(pseudo)scalar mesons





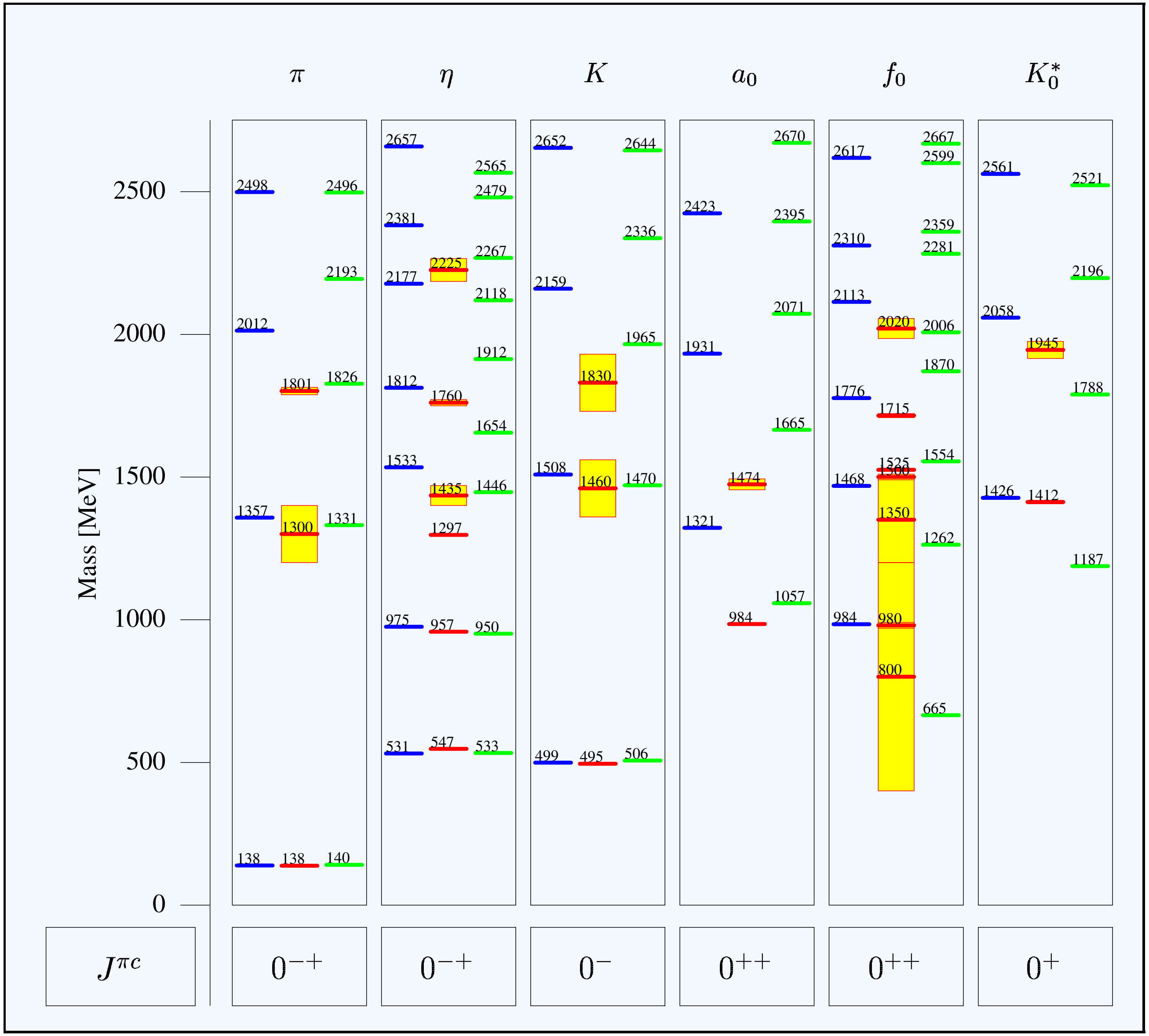




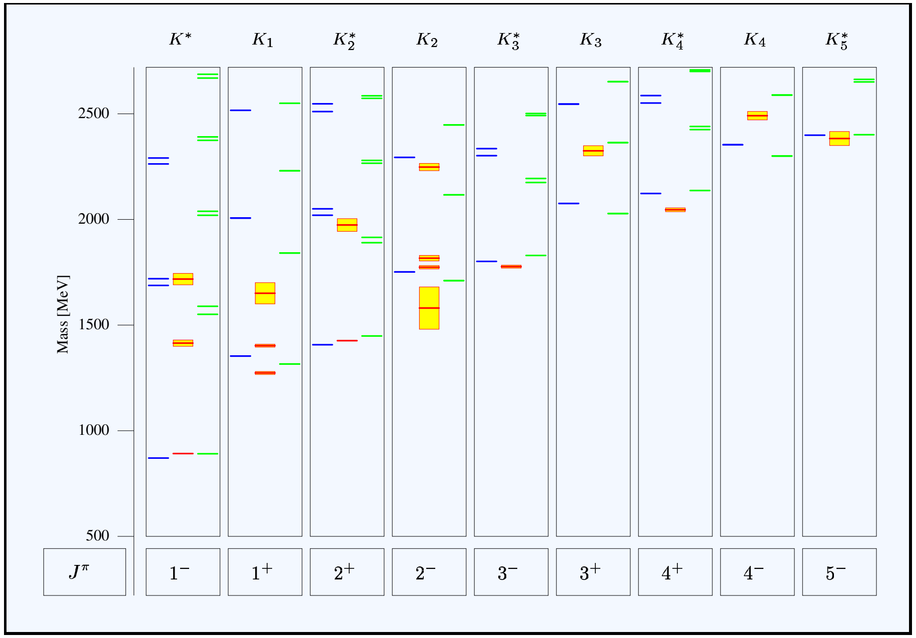
$V_C + V_{III}$ Exp.

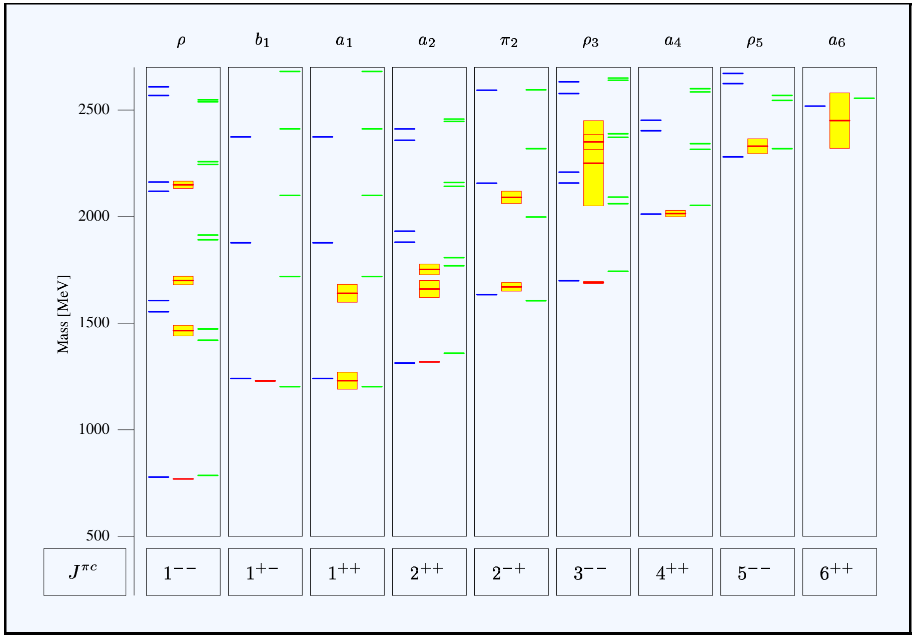
V_{III} mixes $u\bar{u}, d\bar{d}, s\bar{s}$ or $8_F, 1_F$ for (pseudo)scalars

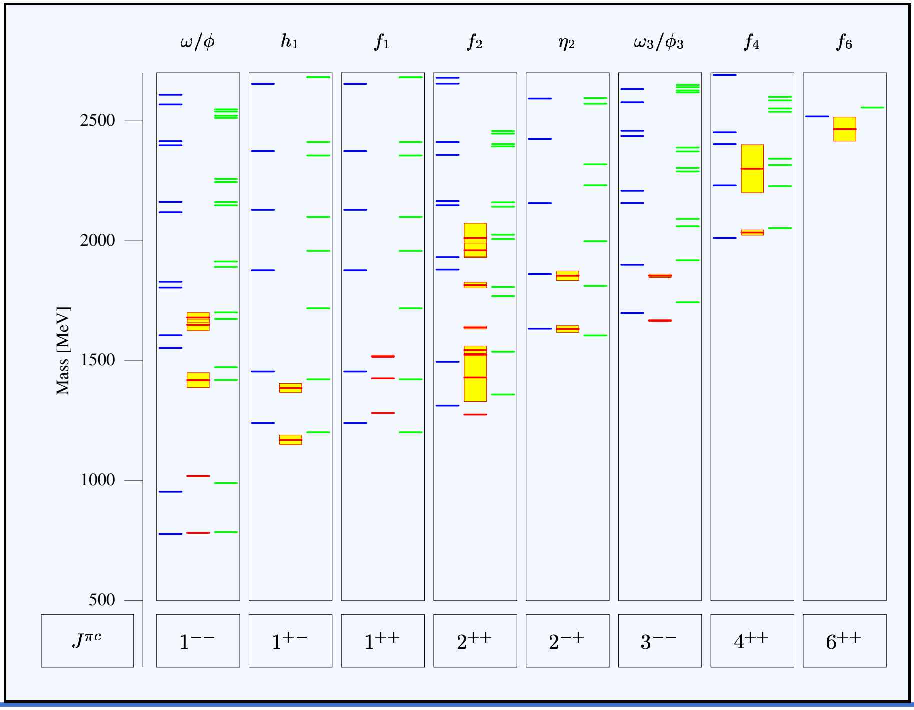
(pseudo)scalar excitation spectrum



Meson Spectra







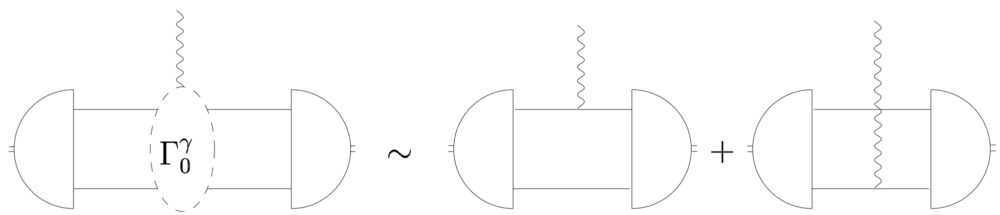
Meson Form Factors

The meson form factors for the transitions $\mathcal{M}(P) \to \mathcal{M}(P')\gamma^*(q)$ with a photon virtuality $q^2 = (P - P')^2 =: -Q^2$ are defined via the current matrix elements by:

$$J^\mu := \langle \mathcal{M}(P') \,| j^\mu(0) |\, \mathcal{M}(P) \rangle = \mathcal{Q} \cdot f_{\mathcal{M}}(Q^2) \left(P + P'\right)^\mu \ .$$

The lowest order contribution to the current m.e. is:

$$\begin{aligned} J_0^\mu \quad = \quad & -e_1 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left[\bar{\Gamma}(p - \frac{q}{2}) S_1(\frac{P}{2} + p - q)\gamma^\mu S_1(\frac{P}{2} + p)\Gamma(p) S_2(-\frac{P}{2} + p)\right] \\ & + e_2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left[\bar{\Gamma}(p + \frac{q}{2}) S_1(\frac{P}{2} + p)\Gamma(p) S_2(-\frac{P}{2} + p)\gamma^\mu S_2(-\frac{P}{2} + p + q)\right] . \end{aligned}$$



The vertex function in the rest frame of the meson $P = (m, \vec{0})$ follows from

$$\boxed{\Gamma(\vec{p})_{(M,\vec{0})} = -i \int \frac{d^3p'}{(2\pi)^3} \left[V(\vec{p}, \vec{p}\,')\Phi(\vec{p}\,')\right] \ ,}$$

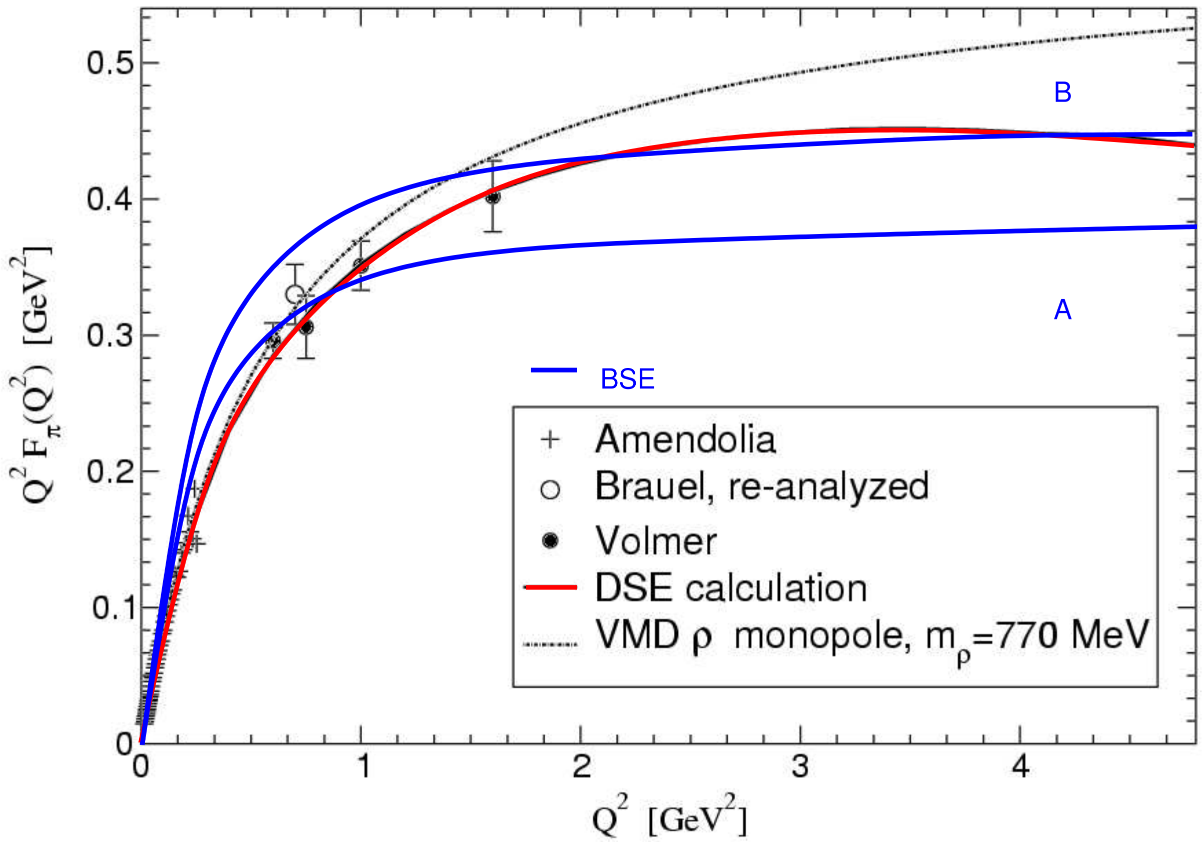
where $\Gamma(p)_P := S_1^{-1}\left(\frac{P}{2} + p\right) \chi_P(p) S_2^{-1}\left(-\frac{P}{2} + p\right)$

Charged Pion Form Factor

and the BETHE-SALPETER-amplitude for any on-shell momentum P with $P^2 = M^2$ is then given by

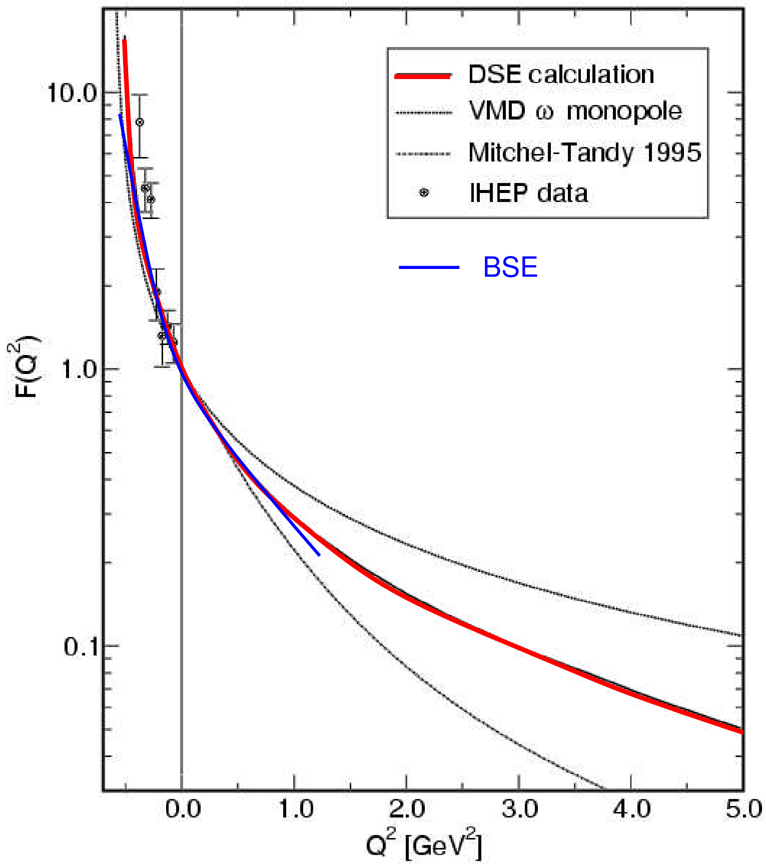
$$\chi_P(p) = S_{\Lambda_P} \chi_{(M,\vec{0})} S^{-1}_{\Lambda_P} \,,$$

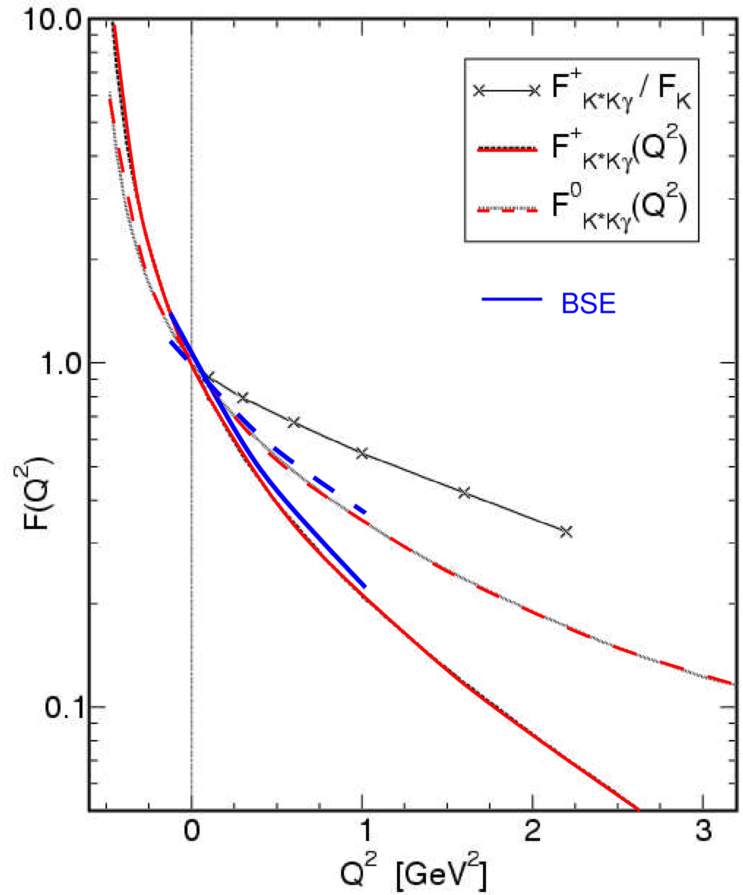
where S_Λ denotes the transformation of DIRAC-spinors .



P. Maris, C.D. Roberts, nucl-th/0301049

$\omega\pi\gamma$ - $K^*K\gamma$–transition form factors





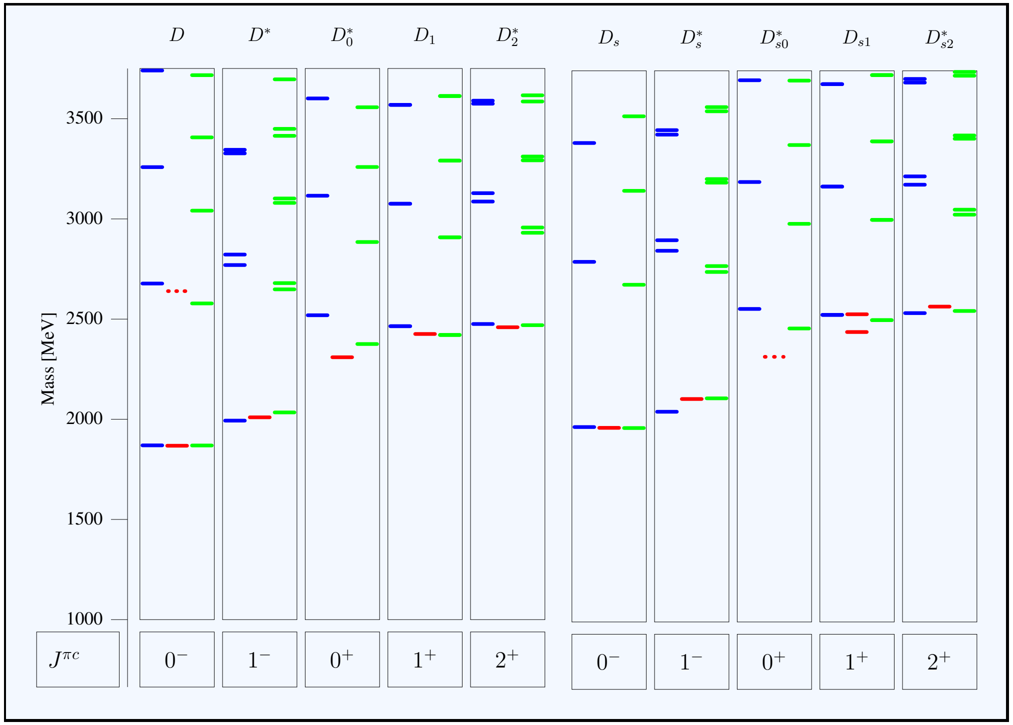
P. Maris, C.D. Roberts, nucl-th/0301049

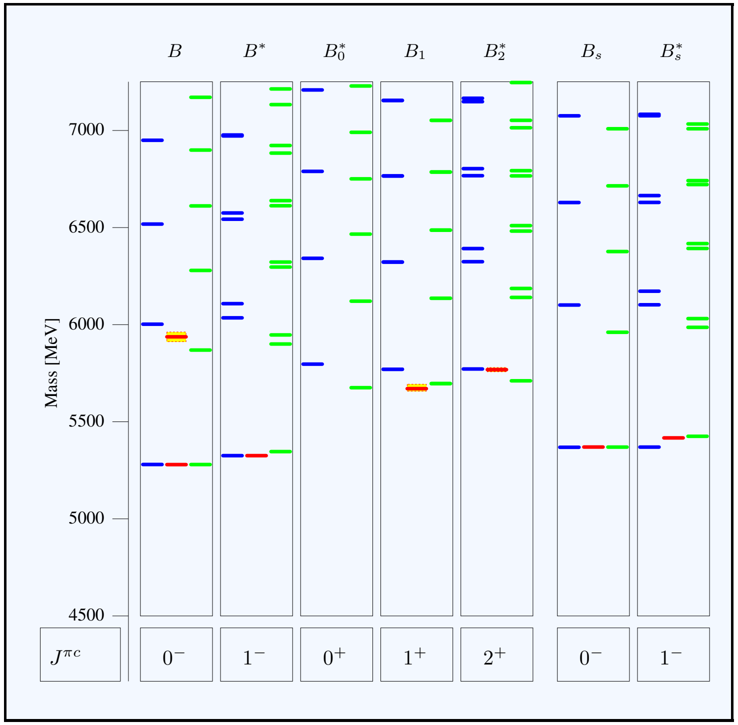
calc. (BSE)

D- and B-mesons

(formal extension to $f_i \in \{c, b\}$)

- Mass spectra:



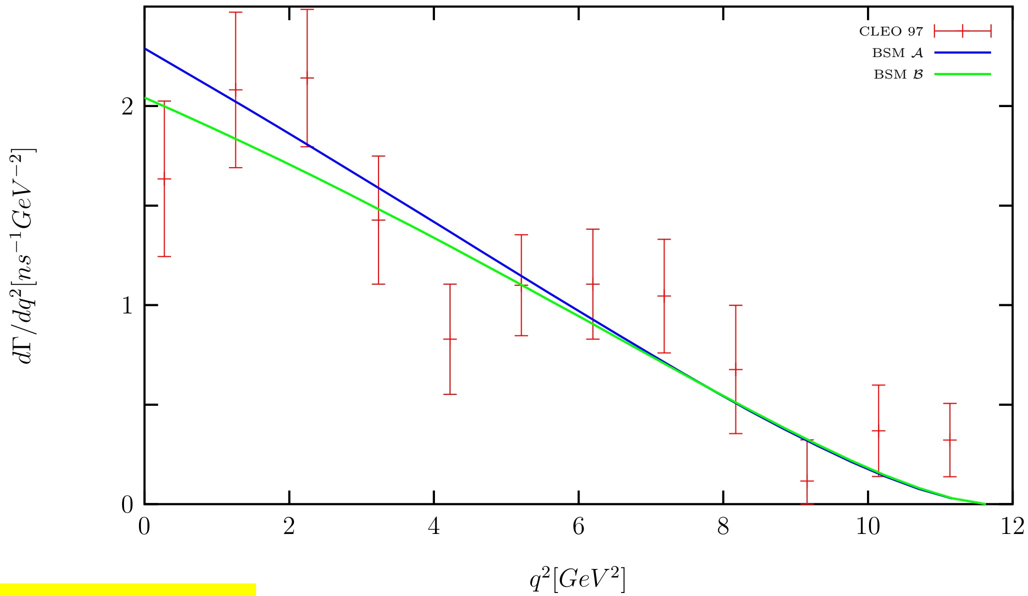


- semileptonic decays
- hadronic weak decays (with factorisation)

D. Merten *et al.*, Eur. Phys. J. A **13** (2002) 477

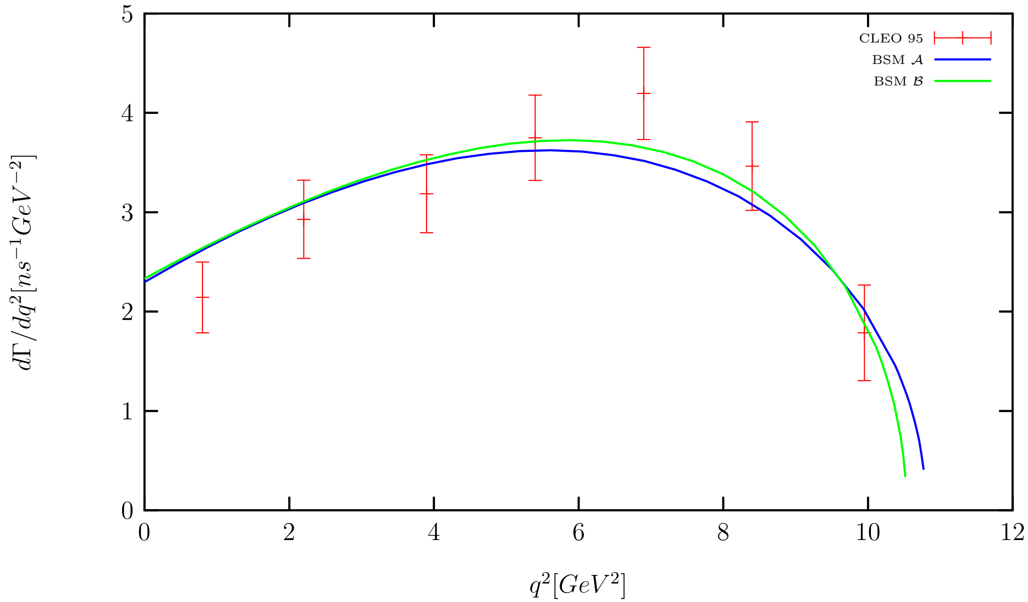
Semileptonic decays $B \to D^{(*)}\ell\bar{\nu}$

$B \to D\ell\bar{\nu}$:



$|V_{cb}^{PDG}| = 0.037 - 0.043$

$B \to D^*\ell\bar{\nu}$:



$$|V_{cb}^{FIT}| = \begin{array}{l} \mathcal{A}: 0.034 \pm 0.001 \\ \mathcal{B}: 0.035 \pm 0.001 \end{array}$$

B- and D-semi-leptonic decay observables

$B \to D^{(*)}$ decay observables ($\Gamma[10^{13}|V_{cb}|^2 s^{-1}]$)

	exp	mod $\mathcal{A}$	mod $\mathcal{B}$	ISGW2
$\Gamma(B \to D)$		1.05	0.93	1.19
$\Gamma(B \to D^*)$		2.78	2.64	2.48
Γ_L/Γ_T	1.24 ± 0.16	1.14	1.20	1.04
Γ_+/Γ_-		0.23	0.27	
R_1	$1.18 \pm 0.30 \pm 0.12$	1.18	1.10	1.27
R_2	$0.71 \pm 0.22 \pm 0.07$	0.94	0.87	1.02

$D_s \to \eta/\eta'/\phi$ decay observables ($\Gamma[10^{10} s^{-1}]$)

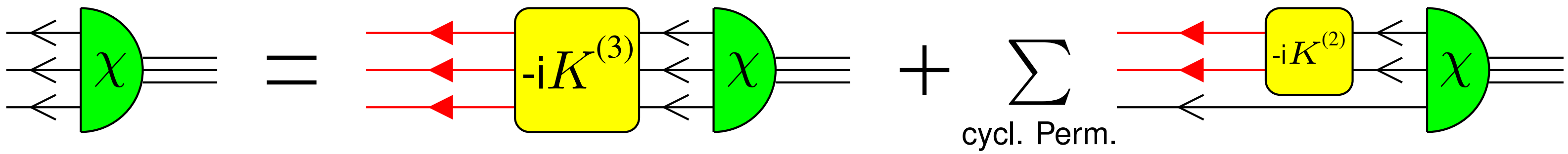
	exp	mod $\mathcal{A}$	mod $\mathcal{B}$	ISGW2
$\Gamma(D_s \to \eta)$	5.24 ± 1.41	4.05	3.11	3.5
$\Gamma(D_s \to \eta')$	1.80 ± 0.69	1.27	1.75	3.0
$\Gamma(D_s \to \phi)$	4.03 ± 1.01	7.89	9.67	4.6
Γ_L/Γ_T	0.72 ± 0.18	1.20	1.42	0.96
Γ_+/Γ_-		0.20	0.33	
$A_1(0)$		0.66	0.79	
$V(0)/A_1(0)$	1.92 ± 0.32	1.77	1.30	2.1
$A_2(0)/A_1(0)$	1.60 ± 0.24	0.85	0.63	1.3

B- and D-semi-leptonic decay observables (II)

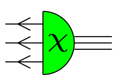
$D \to K^{(*)}$ decay observables ($\Gamma[10^{10}s^{-1}]$)

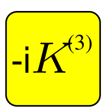
	exp	mod $\mathcal{A}$	mod $\mathcal{B}$	ISGW2
$\Gamma(D \to K)$	7.97 ± 0.36	7.51	7.26	10.0
$\Gamma(D \to K^*)$	4.55 ± 0.34	7.64	10.08	5.4
Γ_L/Γ_T	1.14 ± 0.08	1.29	1.48	0.94
Γ_+/Γ_-	0.21 ± 0.04	0.23	0.34	
$A_1(0)$	0.56 ± 0.04	0.69	0.81	
$V(0)/A_1(0)$	1.82 ± 0.09	1.54	1.18	2.0
$A_2(0)/A_1(0)$	0.78 ± 0.07	0.81	0.62	1.3

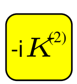
Baryons: q^3-Bethe-Salpeter-Equation



describes bound states of mass $M^2 = \bar{P}^2$ and total momentum $\bar{P} = p_1 + p_2 + p_3$, where:

- 

$:= \langle 0 | T \psi(x_1)\, \psi(x_2)\, \psi(x_3) | \bar{P} \rangle$, Bethe-Salpeter-Amplitude
-  $= \langle 0 | T \psi(x)\, \bar{\psi}(x') | 0 \rangle = S_F(x - x')$, full quark propagator
- 

irreducible **three**-particle kernel
- 

irreducible **two**-particle kernel

Salpeter-Equation

Free constituent quark propagators and instantaneous interaction kernels $\Rightarrow$

$$\mathcal{H}\Phi_M^\Lambda = M\Phi_M^\Lambda$$

Eigenvalue equation for baryon mass M with:

- Salpeter-Amplitude: $\Phi_M(\vec{p}_\xi, \vec{p}_\eta) := \int \frac{\mathrm{d}p_\xi^0}{2\pi}\frac{\mathrm{d}p_\eta^0}{2\pi}\,\chi_M(p_\xi, p_\eta)$

  Projection: $\Phi_M^\Lambda := \left[\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ + \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right]\Phi_M$
- Φ_M^Λ in baryon rest frame, $\overline{M} = (M, \vec{0})$
- Salpeter-Hamilton-Operator: $\mathcal{H} = \mathcal{H}\left(\boxed{V^{(3)}}, \boxed{V^{(2)}}\right)$

Norm: $\langle\Phi_M^\Lambda|\Phi_M^\Lambda\rangle = \int \frac{\mathrm{d}p_\xi^3}{2\pi}\frac{\mathrm{d}p_\eta^3}{2\pi}\,\Phi_M^{\Lambda\,\dagger}(p_\xi, p_\eta)\,\Phi_M^\Lambda(p_\xi, p_\eta) = 2M$

$\Rightarrow$ induces a (positive definite) scalar product $\langle\Phi_1|\Phi_2\rangle$

Salpeter Hamiltonian

... approximate treatment of $V^{(2)}$ ...:

$$
\begin{aligned}
(\mathcal{H}\Phi_M)(\vec p_\xi,\vec p_\eta) = & \sum_{i=1}^{3} H_i \; \Phi_M(\vec p_\xi,\vec p_\eta) \\
+ \quad & \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ + \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right) \\
& \quad \gamma^0 \otimes \gamma^0 \otimes \gamma^0 \int \frac{d^3p'_\xi}{(2\pi)^3}\,\frac{d^3p'_\eta}{(2\pi)^3}\; V^{(3)}(\vec p_\xi,\vec p_\eta,\vec p\,'_\xi,\vec p\,'_\eta)\;\Phi_M(\vec p\,'_\xi,\vec p\,'_\eta) \\
+ \quad & \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ - \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right) \\
& \quad \gamma^0 \otimes \gamma^0 \otimes \mathbb{1} \int \frac{d^3p'_\xi}{(2\pi)^3} \left[V^{(2)}(\vec p_\xi,\vec p\,'_\xi) \otimes \mathbb{1}\right] \Phi_M(\vec p\,'_\xi,\vec p_\eta) \\
& + \text{ cycl. perm. (123)}
\end{aligned}
$$

- $\Lambda_i^{\pm}(\vec p_i) := \frac{\omega_i \pm H_i}{2\omega_i}$ Energy projectors
- $H_i(\vec p_i) := \gamma^0\,(\boldsymbol{\gamma}\cdot\vec p_i + m_i)$ Dirac Hamiltonian

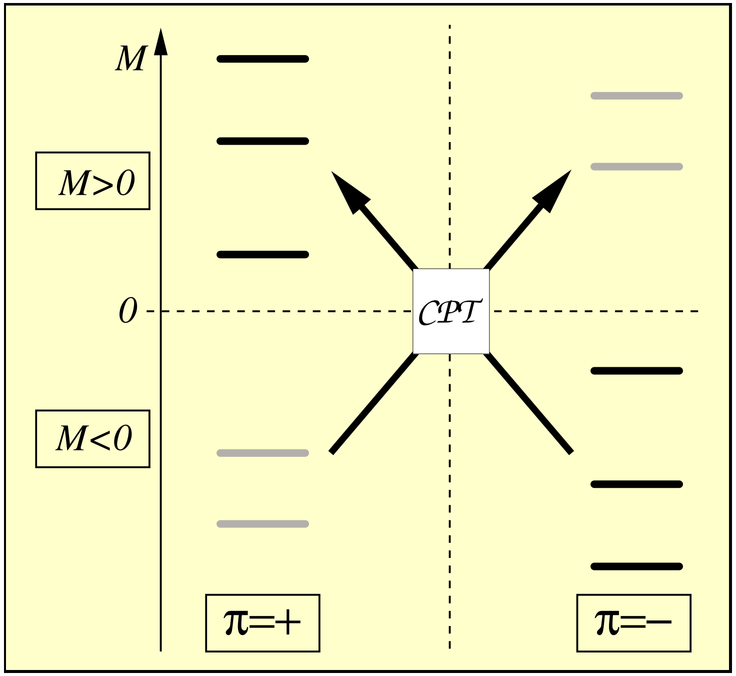
... solved by diagonalisation in a large finite basis...

$\mathcal{CPT}$-symmetry of the Salpeter equation

$\mathcal{H}$ is not positive definite with respect to the norm $\langle . | . \rangle$!

$\rightarrow$ There are positive and negative mass eigenvalues M !

Spectrum (schematically):



$\mathcal{CPT}$ transforms solutions Φ^{π}_{-M} with parity π and negative energy $-M$ into a solution with parity $-\pi$ and positive energy M:

$$\Phi^{-\pi}_{M} = \bigotimes_{i=1}^{3} \gamma^0 \gamma^5 \; \Phi^{\pi}_{-M}$$

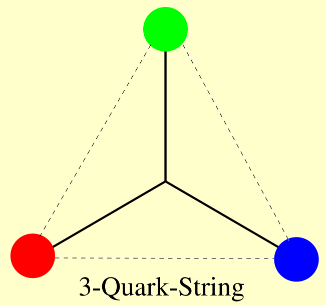
$\Rightarrow$ 1-1-correspondence with states of NRCQM appears.

But: Baryon states with positive and negative parity are coupled !

Confinement and instanton induced interaction

- Quark confinement realized by a phenomenological **string potential** for 3 quarks: (*Ansatz* similar to NRCQM)

$$V^{(3)}_{\text{Conf}}(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}) = \mathsf{A_3} + \mathsf{B_3}\sum_{\mathbf{i<j}} |\mathbf{x_i}-\mathbf{x_j}|$$



with Dirac structure:

$$\mathsf{A}_3 = a\ \tfrac{3}{4}\Big[\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\gamma^0\otimes\mathbb{1}\otimes\gamma^0+\mathbb{1}\otimes\gamma^0\otimes\gamma^0\Big]$$

$$\mathsf{B}_3 = b\ \tfrac{1}{2}\Big[-\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\gamma^0\otimes\mathbb{1}\otimes\gamma^0+\mathbb{1}\otimes\gamma^0\otimes\gamma^0\Big]$$

Spin-orbit effects are small and Regge trajectories are quantitatively correct.

- Spin dependent mass splittings form **'t Hooft's interaction** (induced by instantons):

$$V^{(2)}_{\text{'t Hooft}}(\mathbf{x_1}-\mathbf{x_2}) = \frac{\mathbf{1}}{\lambda^{\mathbf{3}}\pi^{\frac{\mathbf{3}}{\mathbf{2}}}}\ \exp\left(-\frac{|\mathbf{x_1}-\mathbf{x_2}|^{\mathbf{2}}}{\lambda^{\mathbf{2}}}\right)\cdot$$

$$-4\underbrace{\left(g_{nn}\ \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nn)+g_{ns}\ \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(ns)\right)}_{\text{flavour-dependent coupling}}\left[\mathbb{1}\otimes\mathbb{1}+\gamma^5\otimes\gamma^5\right]\ \mathcal{P}^{\mathcal{D}}_{S_{12}=0}$$

⇒ spin/flavour-antisymmetric quark pairs;

⇒ does not act on: flavour-decuplet, spin-symmetric states;

⇒ no $\vec{L}\cdot\vec{S}$, no tensor forces.

Model parameters

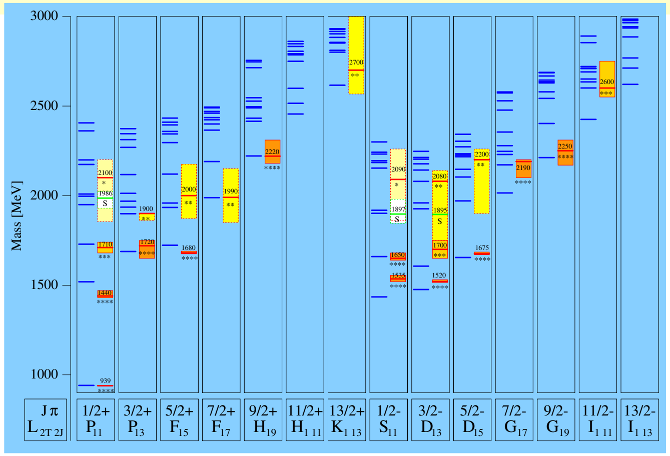
		parameter	value
quark-	'nonstrange'	m_n	330 Mev
masses	'strange'	m_s	670 Mev
confinement	offset	a	-744 MeV
	slope	b	470 MeV fm^{-1}
't Hooft's	nn-coupling	g_{nn}	136.0 MeV fm^3
force	ns-coupling	g_{ns}	94.0 MeV fm^3
	effective range	λ	0.4 fm

Parameters are fixed by

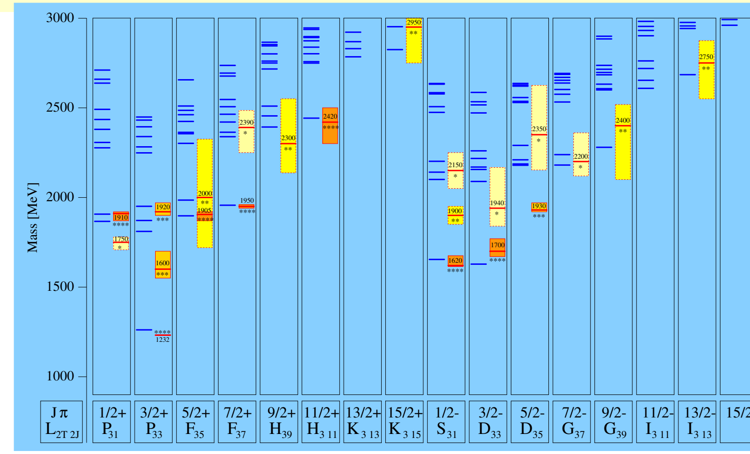
- the Δ-Regge trajectory
  $\longrightarrow$ Confinement parameters a, b and m_n
- baryon ground-states (octet und decuplet)
  $\longrightarrow$ g_{nn}, g_{ns}, λ and m_s

Light-flavoured Baryons

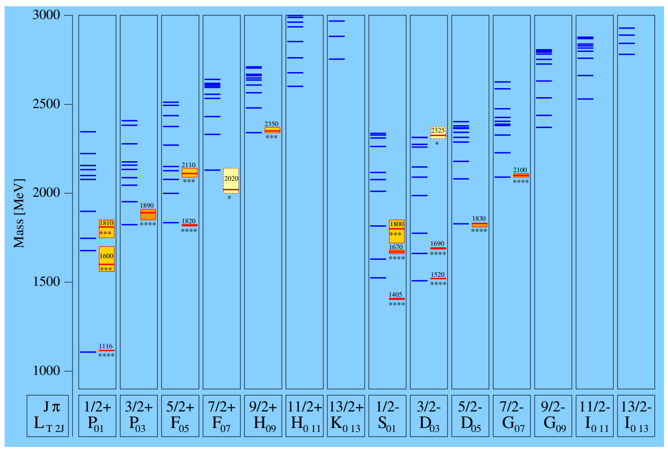
N :



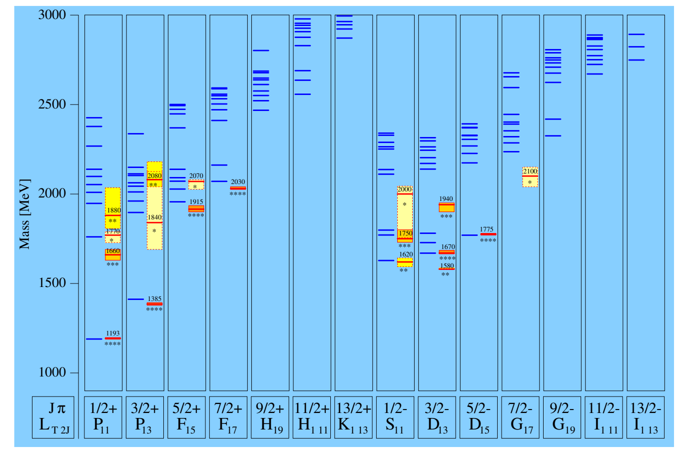
Δ :



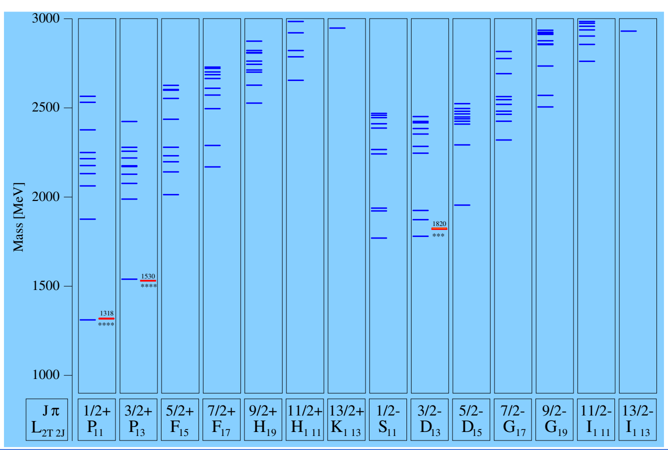
Λ :



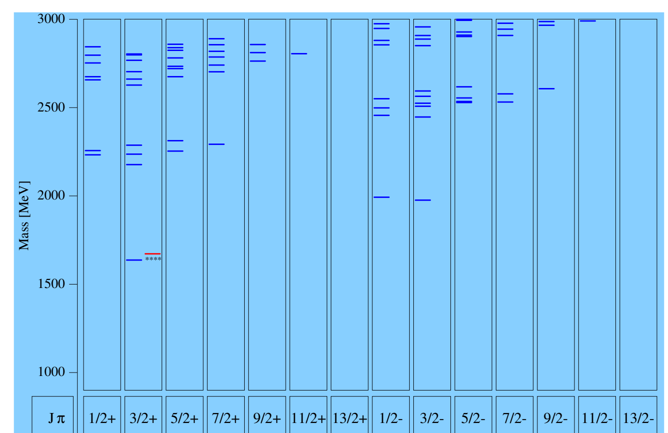
Σ :



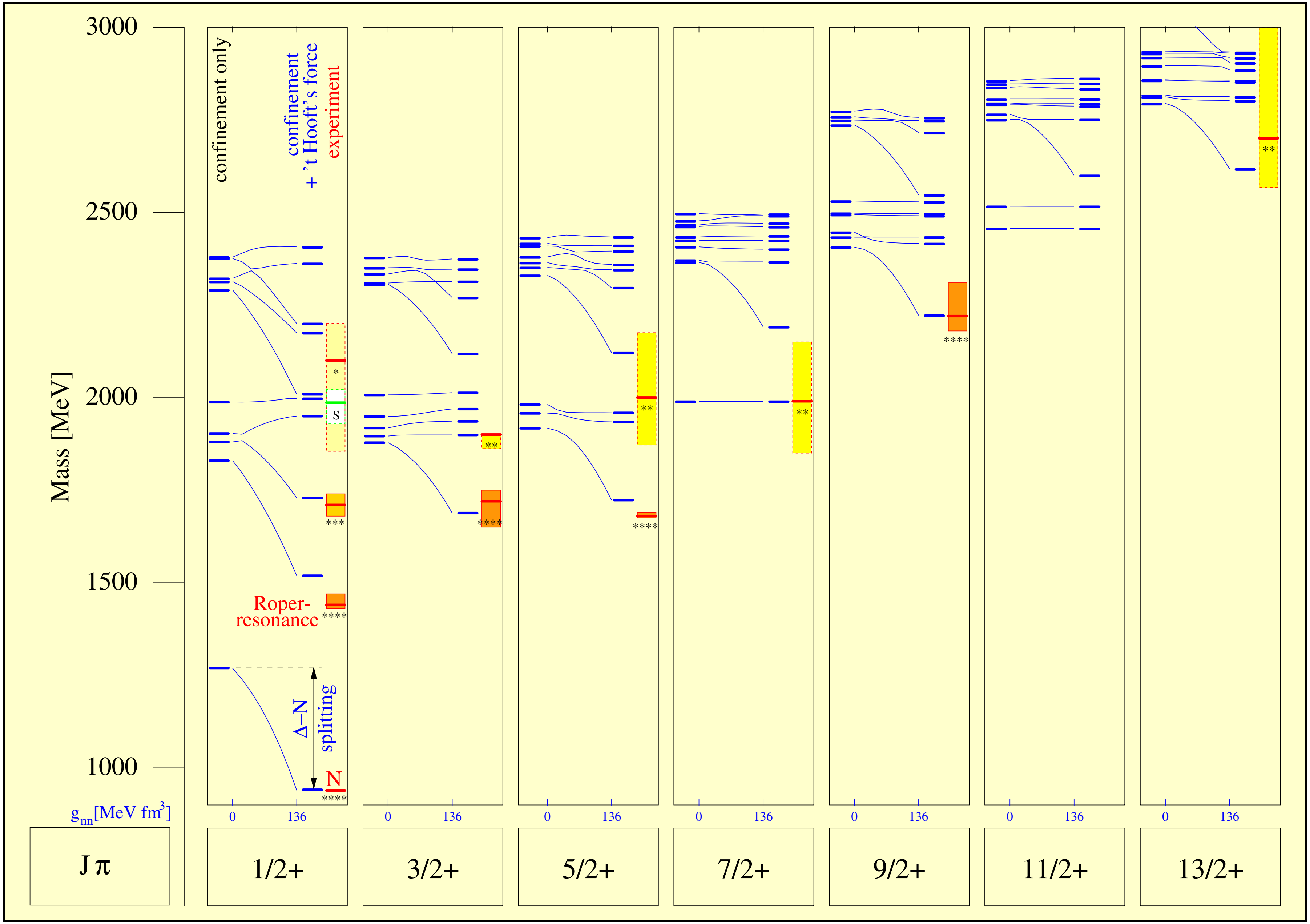
Ξ :



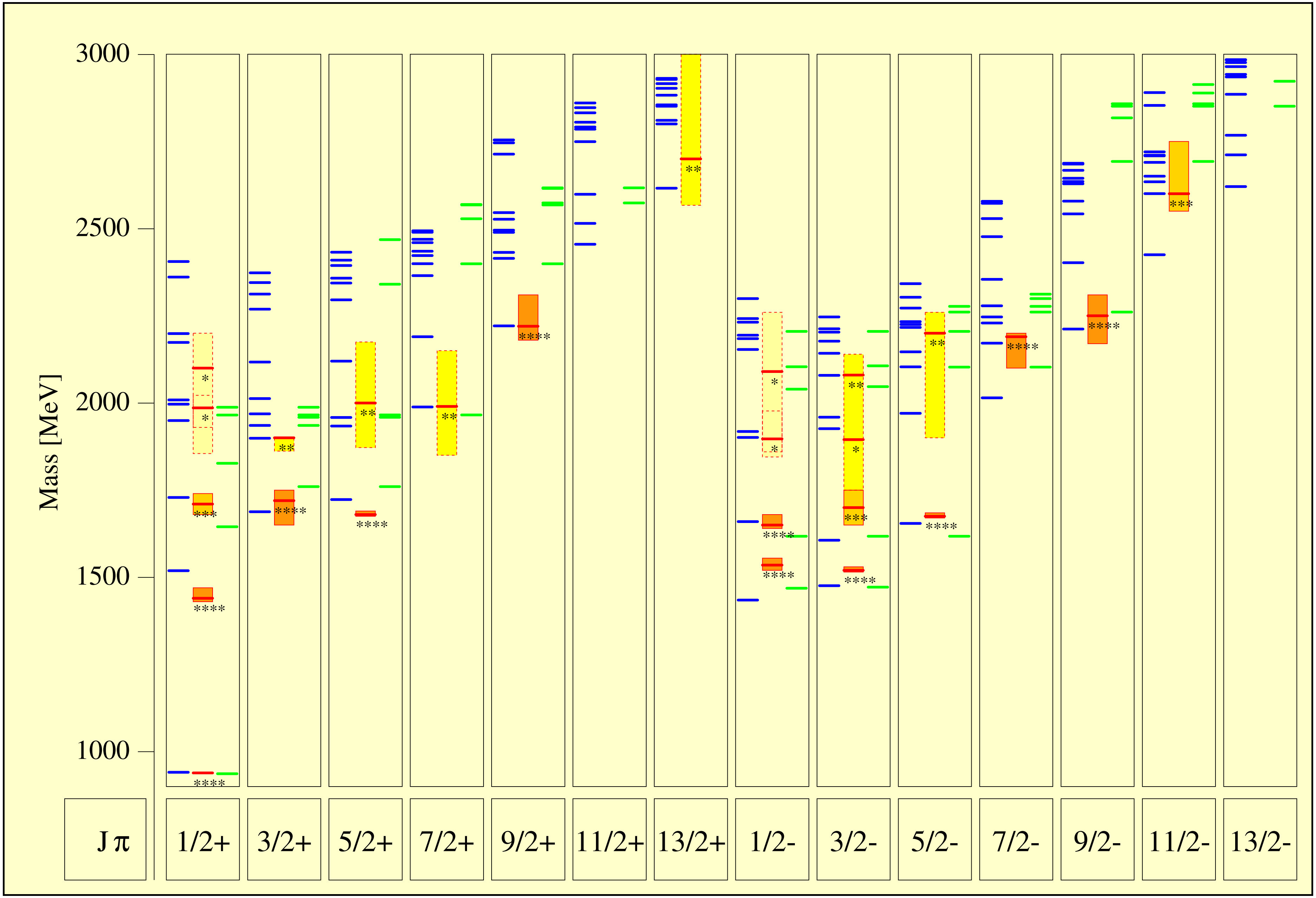
Ω :



Instanton-induced effects in the N^{*+}-spectrum



BSE ↔ NRCQM$(V_{\text{conf.}} + V_{III})$ N



other interactions with the Salpeter equation

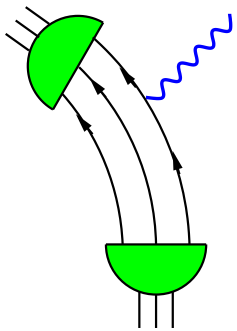
alternatively one could substitute the interaction:

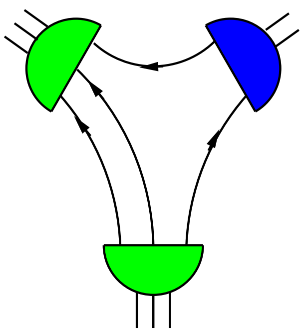
- instantaneous OGE and scalar confinement $\Rightarrow$ too large LS-effects, too large α_S, ...
- instantaneous GBE and scalar confinement $\Rightarrow$ too large LS-, and Tensor-effects ...

... a naive implementation of these interactions in the Salpeter-approach does not lead to a satisfactory mass spectrum ...

Electroweak properties

Electroweak currents and strong two-body decay amplitudes (as for mesons) calculated in the Mandelstam formalism, in lowest order parameterfree ...





Magnetic moment matrix element

$$\mu = \frac{\langle \Phi^\Lambda_M | \hat{\mu} | \Phi^\Lambda_M \rangle}{2M}$$

relativistic weight — single particle-ang. momenta — single particle-spins

$$\hat{\mu} := \frac{\omega_1 + \omega_2 + \omega_3}{M} \left\{ \sum_{\alpha=1}^{3} \frac{\hat{e}_\alpha}{2\omega_\alpha} \hat{l}^3_\alpha + \mathbb{1} \otimes \mathbb{1} \otimes \frac{\hat{e}_3}{2\omega_3} \Sigma^3 + \text{zykl. Perm.} \right\}$$

$$- \delta_{3i}\, \epsilon_{ijk} \frac{1}{M} \sum_{\alpha=1}^{3} \frac{\hat{e}_\alpha}{2\omega_\alpha} p^k_\alpha \sum_{\beta=1}^{3} \omega_\beta \frac{\partial}{\partial p^j_\beta}$$

— center-of-charge ang. momentum

Salpeter-Amplitude normalisation: $\langle \Phi^\Lambda_M | \Phi^\Lambda_M \rangle = 2M$

magnetic moments $[\mu_N]$

Baryon	BSE	Exp.	GBE
p	2.77	2.793	2.70
n	-1.71	-1.913	-1.70
Λ	-0.61	-0.613	-0.65
Σ^+	2.51	2.458	2.35
Σ^0	0.75	–	0.72
Σ^-	-1.02	-1.160	-0.92
Ξ^0	-1.33	-1.250	-1.24
Ξ^-	-0.56	-0.6507	-0.68
Δ^+	2.07	$2.7 \pm 1.5 \pm 1.3$	2.08
Δ^{++}	4.14	3.7 – 7.5	4.17
Ω^-	-1.66	-2.0200	-1.59

from: K. Berger, R.F. Wagenbrunn, W. Plessas, nucl-th/0407009

Tim van Cauteren, *et al.*: Eur. Phys. J. A**20** (2004) 283

Charge radius

Charge radius for a state with Salpeter-ampltude Φ_M:

$$\langle r^2 \rangle = \frac{\langle \Phi_M | \hat{r}^2 | \Phi_M \rangle}{2M}$$

where

$$\hat{r}^2 = \sum_{\alpha=1}^{3} \left\{ \frac{1}{2} \left[\frac{\Omega}{M} \left(\mathrm{i} \boldsymbol{\nabla}_{\boldsymbol{p_\alpha}} - \hat{\boldsymbol{R}} \right) + \mathrm{h.\,c.} \right] \right\}^2 \hat{q}_\alpha .$$

with $\hat{\boldsymbol{R}}$ the relativistic centre-of-mass:

$$\hat{\boldsymbol{R}} = \frac{1}{\Omega} \sum_{\alpha=1}^{3} \omega_\alpha \mathrm{i} \boldsymbol{\nabla}_{\boldsymbol{p_\alpha}} .$$

and Ω:

$$\Omega := \sum_{\alpha=1}^{3} \sqrt{m_\alpha^2 + \boldsymbol{p}_\alpha^2}$$

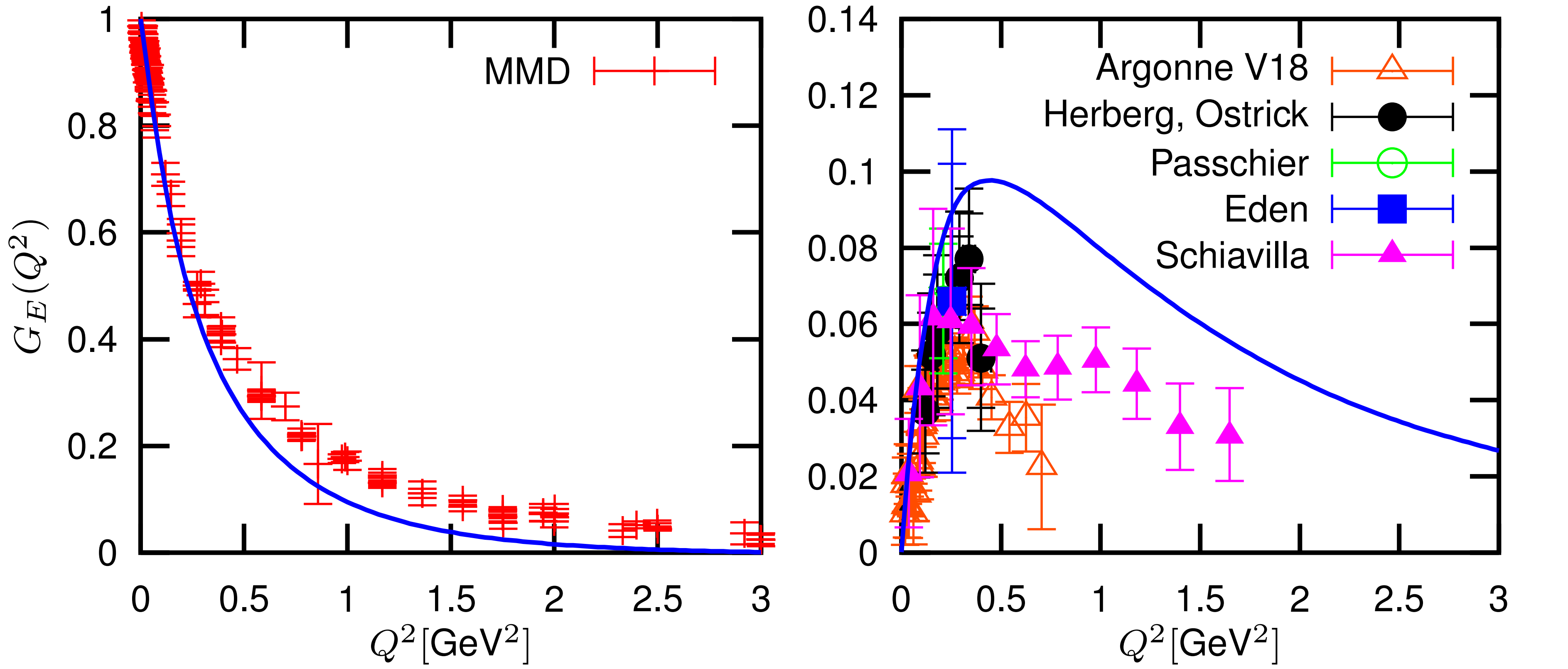
with $\hat{q}$ the quark charge operator.

Squared charge radii [fm]^2 – results

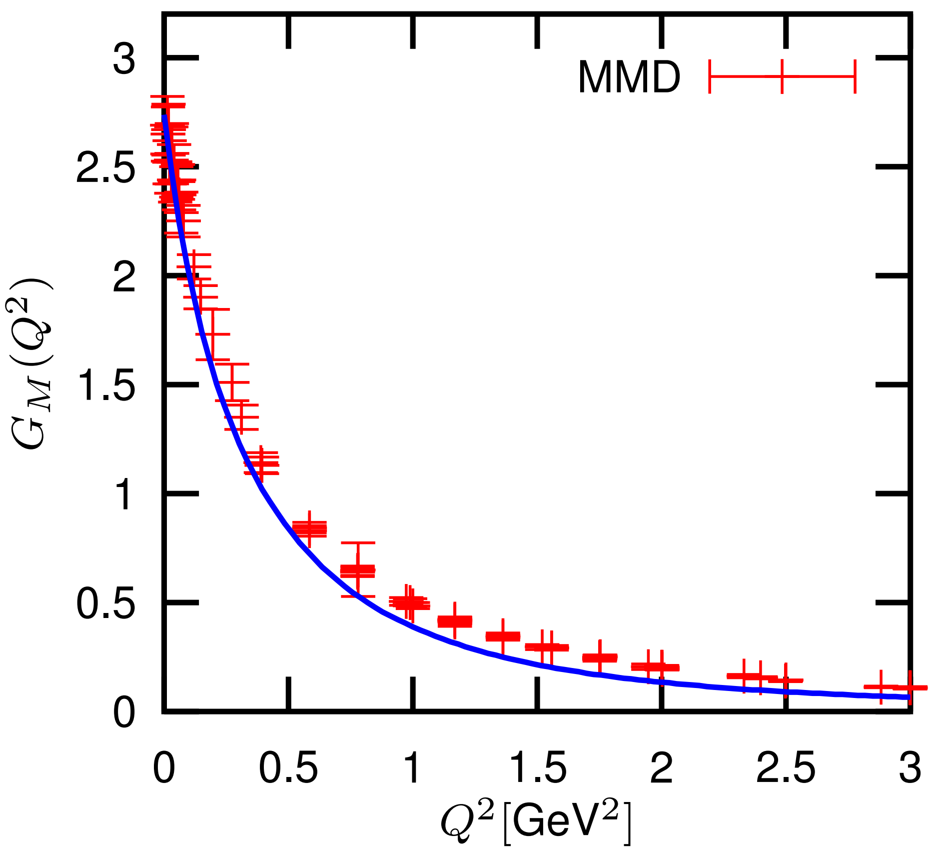
Baryon	$\chi PT^{(4)}_{IR/HB}$	exp	BSE	Baryon	BSE
p	$0.717\,/\,0.717$	0.757 ± 0.014	0.74	Δ^-	0.27
n	$-0.113\,/\,-0.113$	-0.1161 ± 0.0022	-0.187	Δ^0	0
				Δ^+	0.27
$\Lambda\Sigma^0$	$0.03 \pm 0.01\,/\,-0.09$	–	-0.120	Δ^{++}	0.55
Σ^+	$0.60 \pm 0.02\,/\,0.72$	–	0.66	Σ^{*+}	0.38
Σ^0	$-0.03 \pm 0.01\,/\,-0.08$	–	0.1	Σ^{*0}	0.05
Σ^-	$0.67 \pm 0.03\,/\,0.88$	$0.61 \pm 0.12 \pm 0.09$	0.45	Σ^{*-}	0.28
Ξ^0	$0.13 \pm 0.03\,/\,0.08$	–	0.068	Ξ^{*0}	0.12
Ξ^-	$0.49 \pm 0.05\,/\,0.75$	–	0.43	Ξ^{*-}	0.29
Λ	$0.11 \pm 0.02\,/\,0.00$	–	0.005	Ω^-	0.28

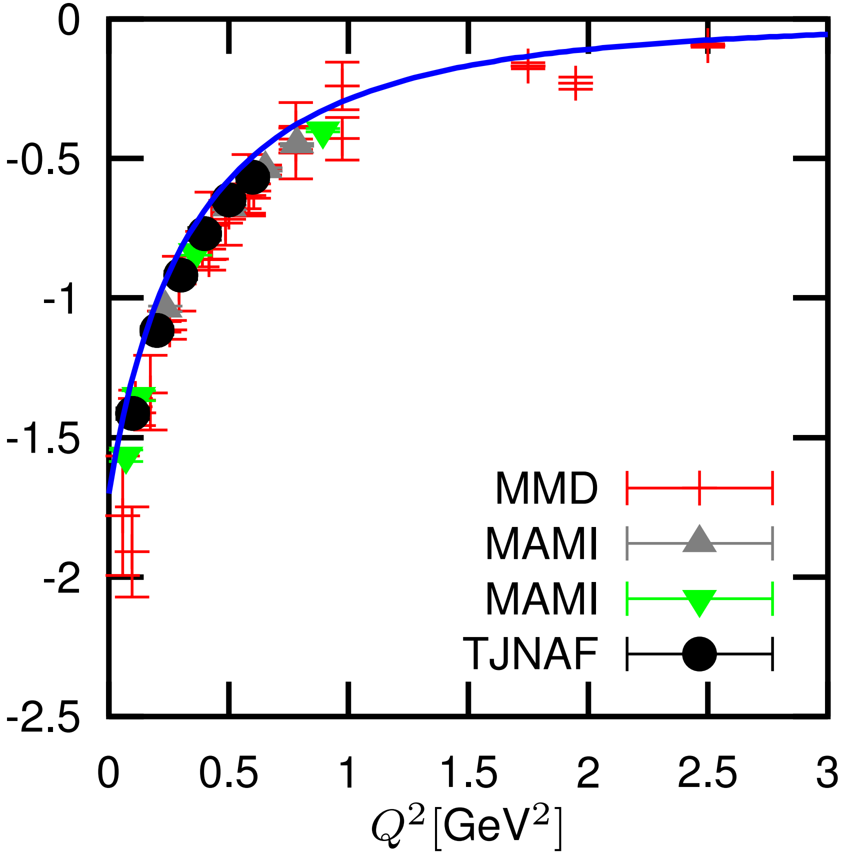
B. Kubis, U.-G. Meißner, Eur. Phys. J. **C 18** (2001) 747

RCQM electric nucleon form factors

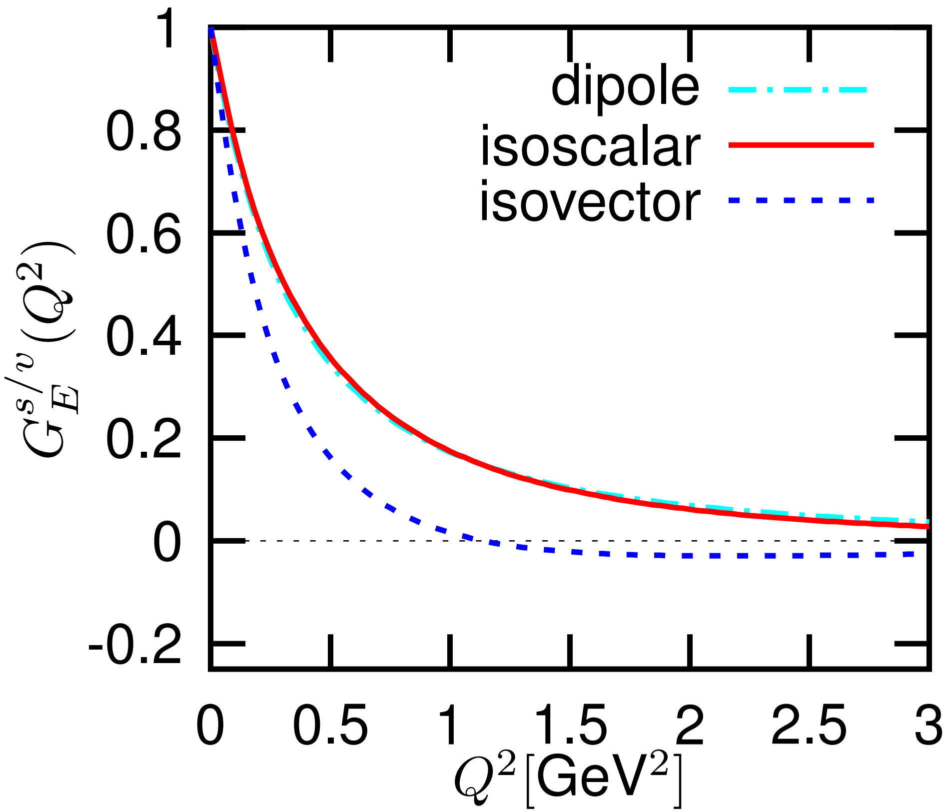


RCQM magnetic nucleon form factors



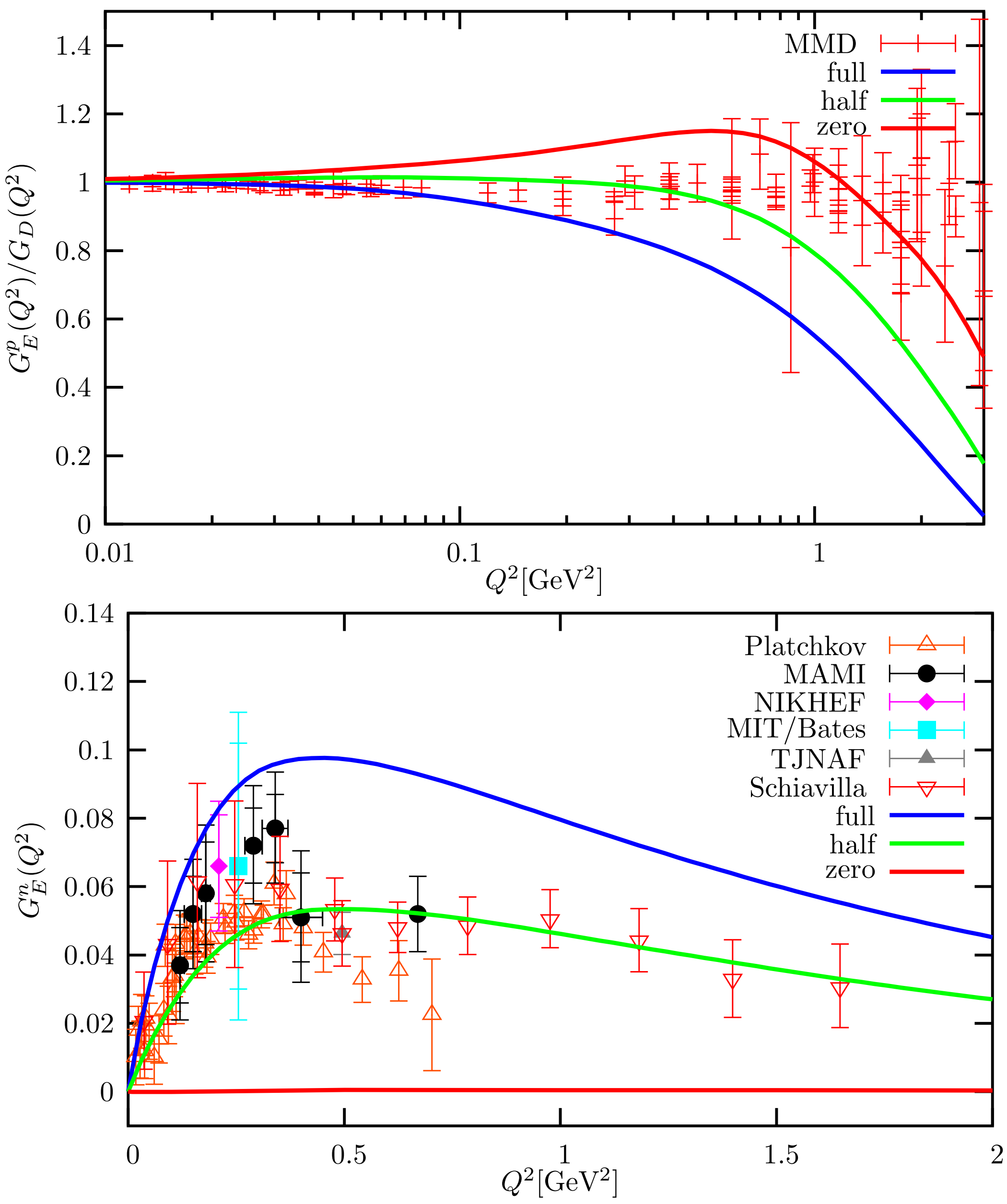


RCQM: isovector ↔ isoscalar



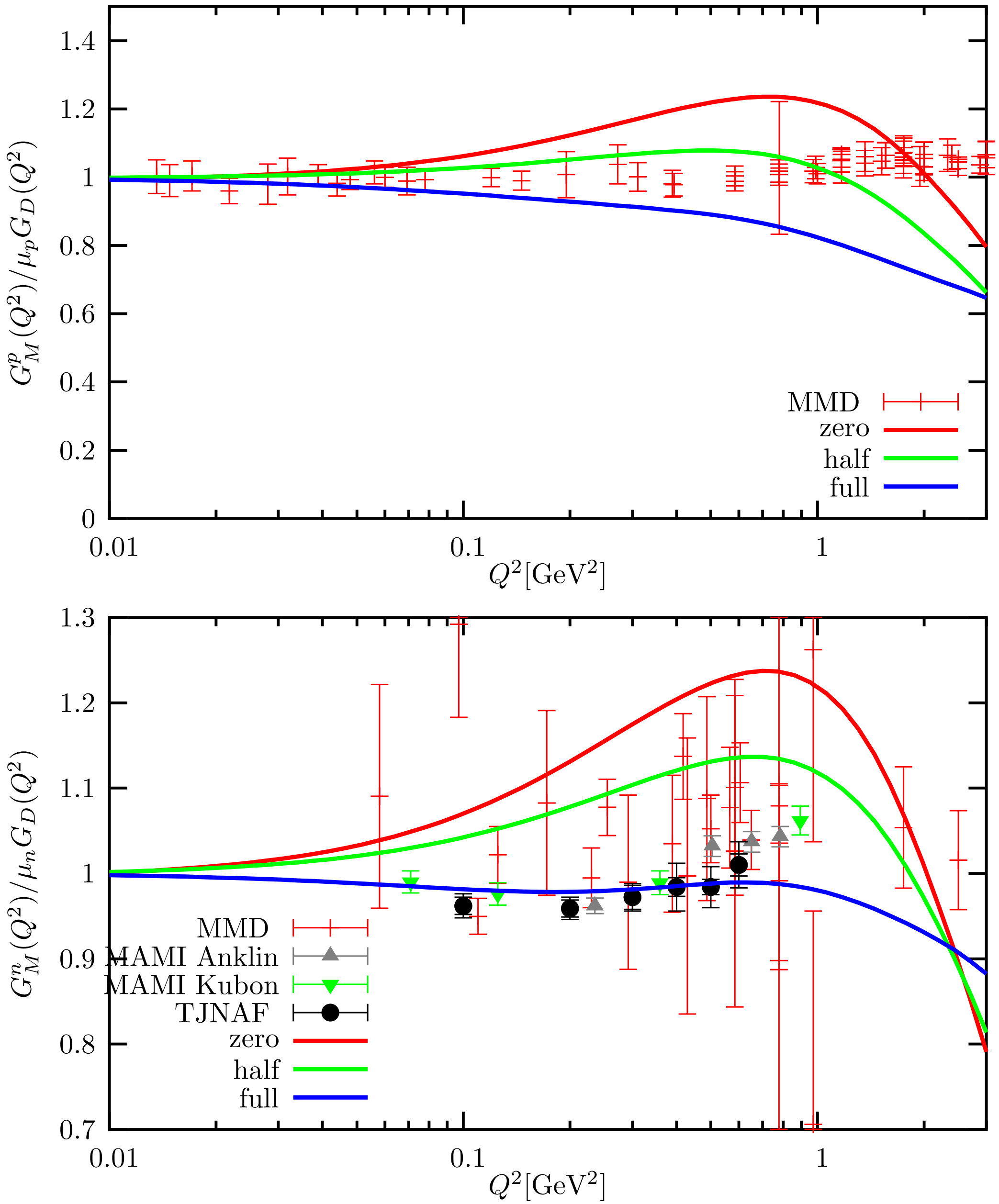
isoscalar electric form factor: dipole shape

RCQM nucleon electric form factors



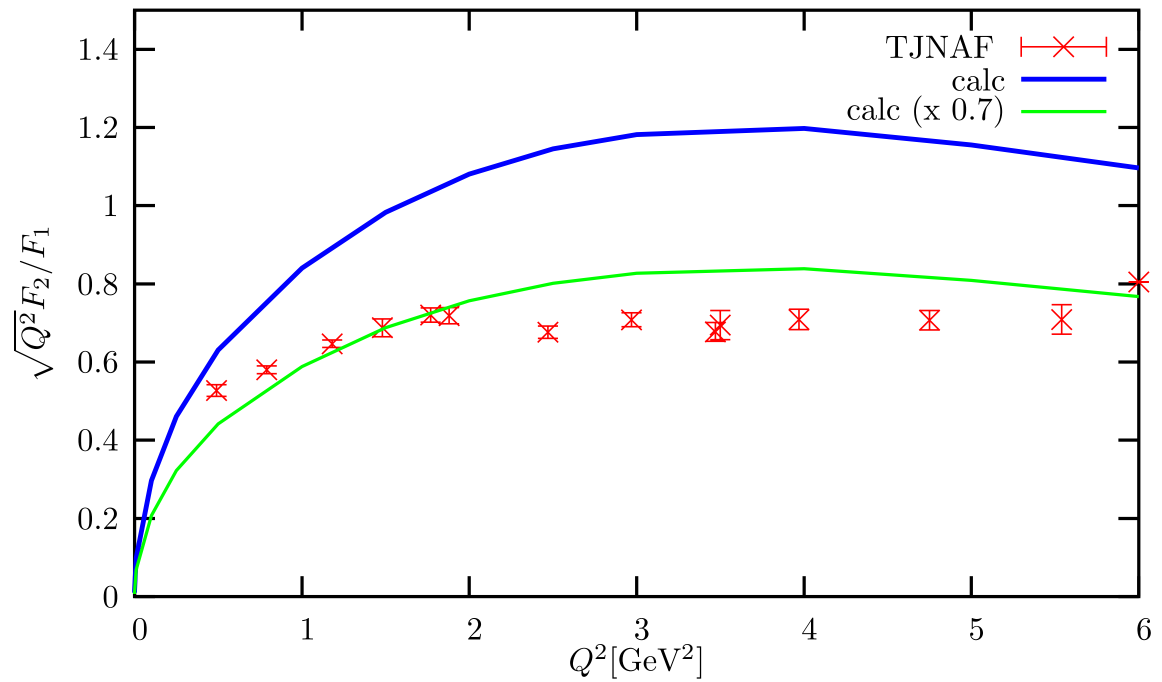
varying the strength of the instanton induced spin-flavour dependent interaction: 0.0, 0.5, 1.0 of the value determined by the spectrum

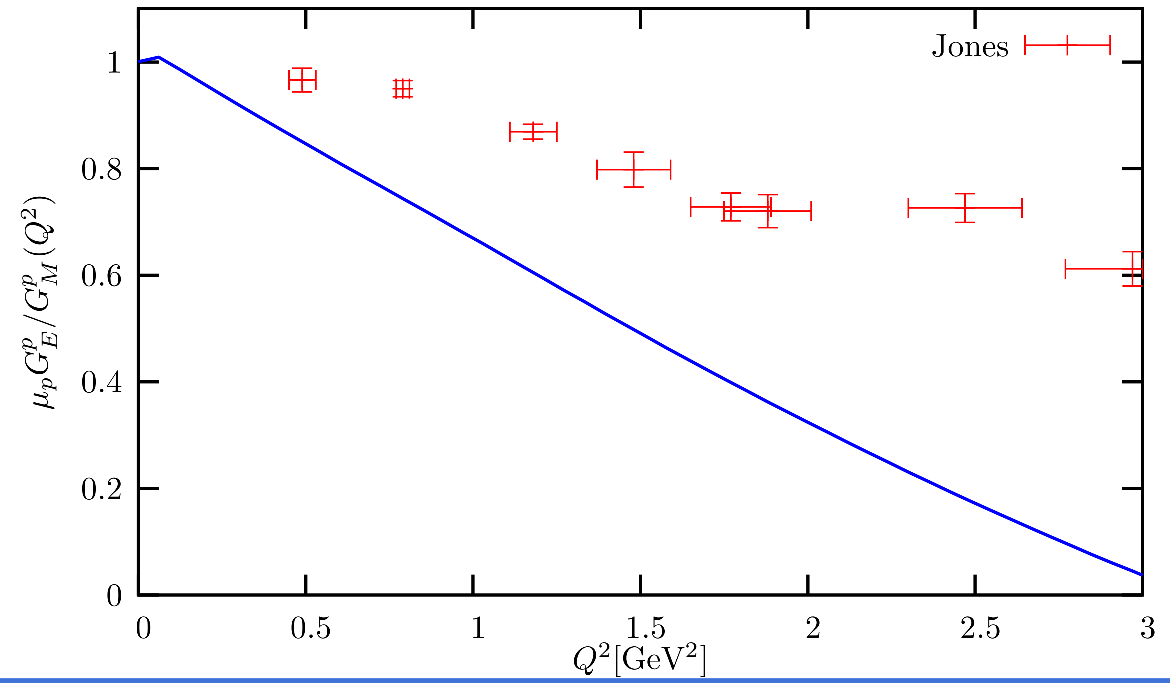
RCQM nucleon magnetic form factors



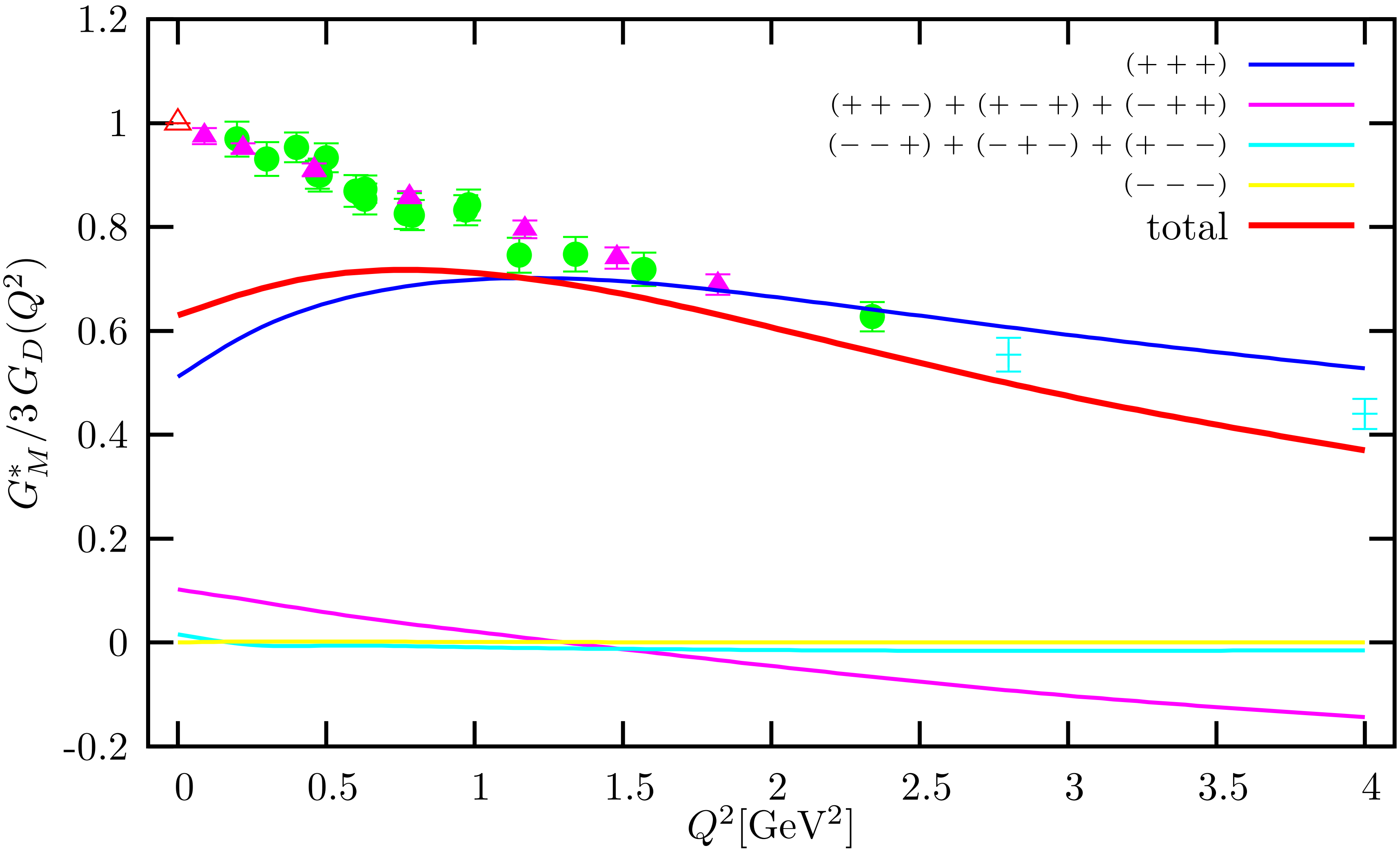
varying the strength of the instanton induced spin-flavour dependent interaction: 0.0, 0.5, 1.0 of the value determined by the spectrum

RCQM G_E^p/G_M^p and F_2/F_1 at large Q^2





RCQM $N - \Delta$ magnetic transition form factor



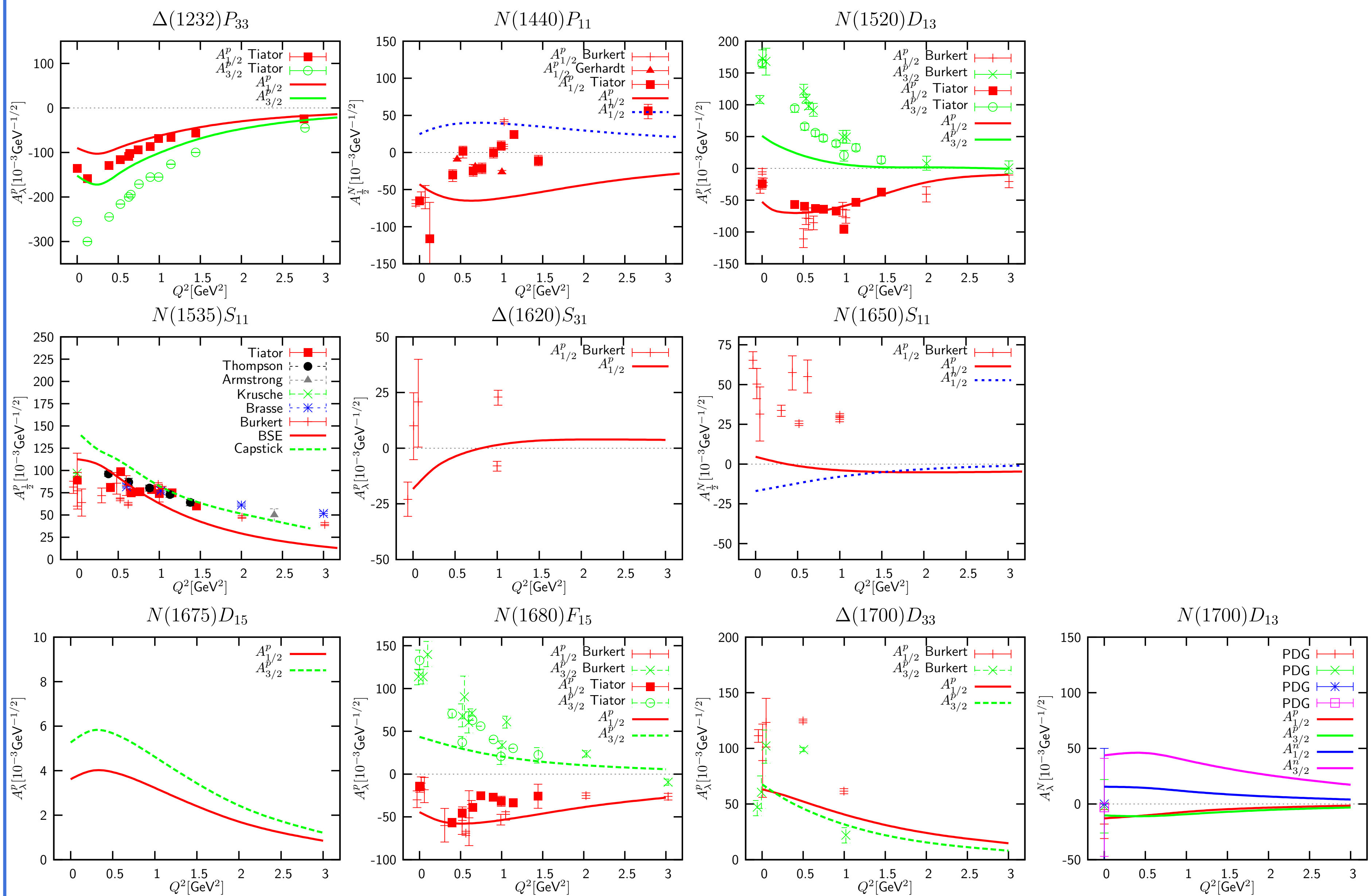
lower components of Dirac-spinors:
relevant for large Q^2 behaviour

help a bit at low $Q^2 \Rightarrow$ “pion cloud”?

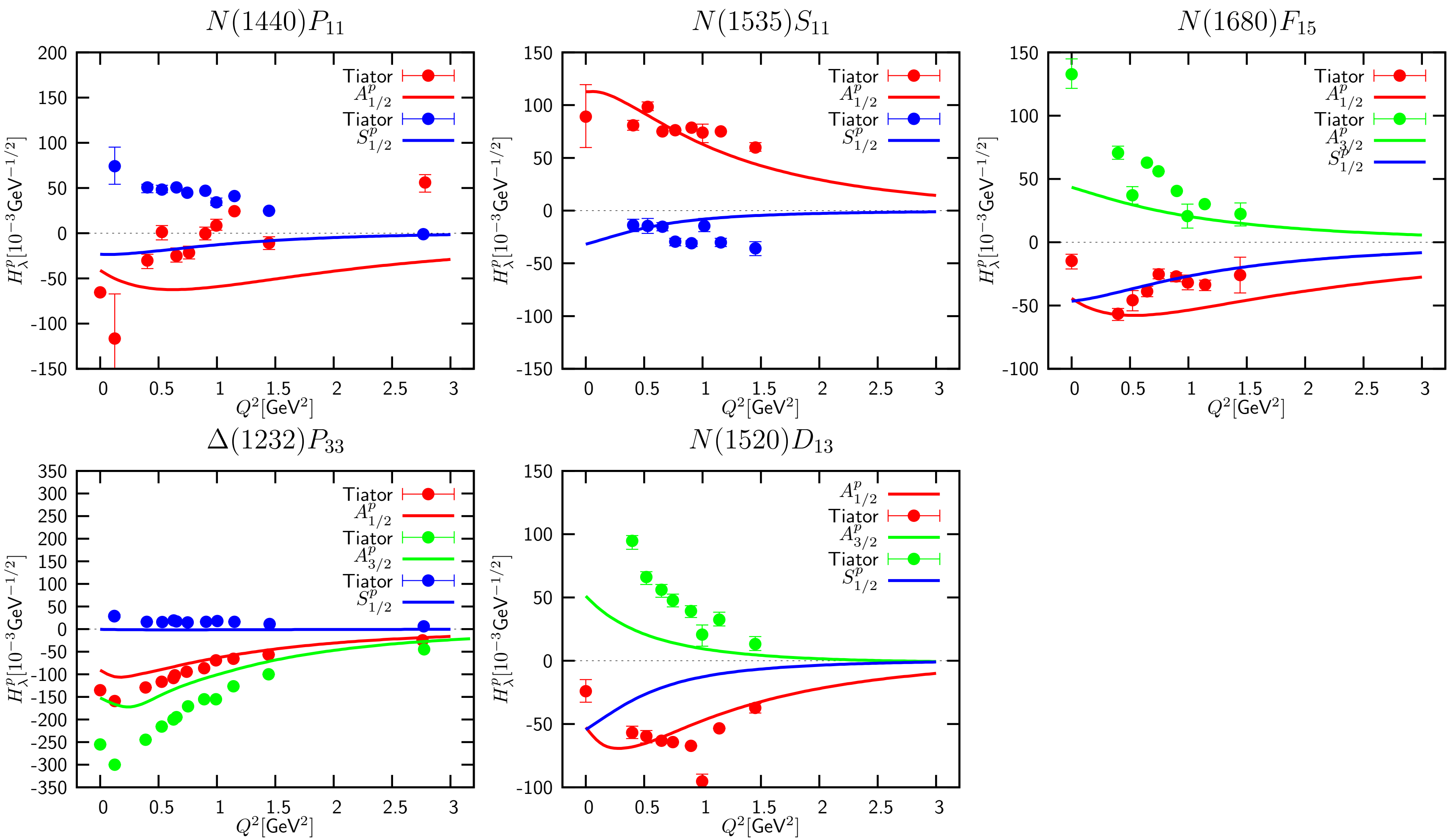
Photon couplings (helicity amplitudes) $[10^{-3}\text{GeV}^{-\frac{1}{2}}]$

state		Calc.	PDG		Calc.	PDG
$P_{33}(1232)$	$A^N_{1/2}$	-89	-135 ± 6			
	$A^N_{3/2}$	-152	-255 ± 8			
$S_{11}(1535)$	$A^p_{1/2}$	113	90 ± 30	$A^n_{1/2}$	-75	-46 ± 27
$S_{11}(1650)$	$A^p_{1/2}$	5	53 ± 16	$A^n_{1/2}$	-16	-15 ± 21
$D_{13}(1520)$	$A^p_{1/2}$	-53	-24 ± 9	$A^n_{1/2}$	1	-59 ± 9
	$A^p_{3/2}$	51	166 ± 5	$A^n_{3/2}$	-52	-139 ± 11
$D_{13}(1700)$	$A^p_{1/2}$	-13	-18 ± 13	$A^n_{1/2}$	16	0 ± 50
	$A^p_{3/2}$	-10	-2 ± 24	$A^n_{3/2}$	-42	-3 ± 44
$D_{15}(1675)$	$A^p_{1/2}$	4	19 ± 8	$A^n_{1/2}$	-25	-43 ± 12
	$A^p_{3/2}$	5	15 ± 9	$A^n_{3/2}$	-33	-58 ± 13
$P_{11}(1440)$	$A^p_{1/2}$	-48	-65 ± 4	$A^n_{1/2}$	27	40 ± 10
$P_{11}(1710)$	$A^p_{1/2}$	53	9 ± 22	$A^n_{1/2}$	-27	-2 ± 14
$S_{31}(1620)$	$A^N_{1/2}$	18	27 ± 11			
$D_{33}(1700)$	$A^N_{1/2}$	63	104 ± 15			
	$A^N_{3/2}$	68	85 ± 22			

Helicity amplitudes



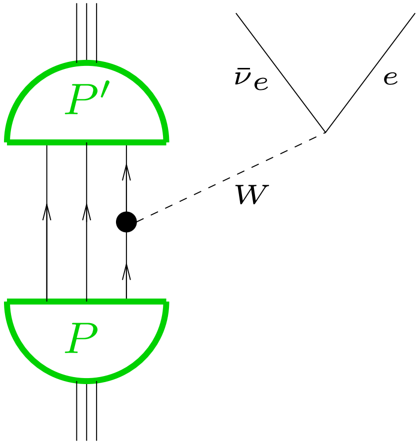
Helicity ampitudes $A^p_{1/2}$, $A^p_{3/2}$, $S^p_{1/2}$



L. Tiator, D. Drechsel, S. Kamalov, M. M. Giannini, E. Santopinto and A. Vassallo, Eur. Phys. J. A **19** (2004) 55 [arXiv:nucl-th/0310041].

(Simon Kreuezer)

Semi-leptonic decays



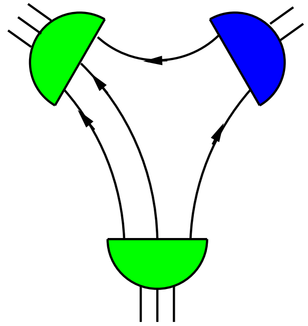
$\Gamma\ [10^6 s^{-1}]$	Exp.	Calc.
$\Lambda \to p\, e^- \bar{\nu}_e$	3.16 ± 0.06	3.10
$\Sigma^+ \to \Lambda\, e^+ \nu_e$	0.25 ± 0.06	0.20
$\Sigma^- \to \Lambda\, e^- \bar{\nu}_e$	0.38 ± 0.02	0.34
$\Sigma^- \to n\, e^- \bar{\nu}_e$	6.9 ± 0.2	4.91
$\Xi^0 \to \Sigma^+\, e^- \bar{\nu}_e$	0.93 ± 0.14	0.91
$\Xi^- \to \Sigma^0\, e^- \bar{\nu}_e$	0.5 ± 0.1	0.51
$\Xi^- \to \Lambda\, e^- \bar{\nu}_e$	3.3 ± 0.2	2.30
$\Lambda \to p\, \mu^- \bar{\nu}_\mu$	0.60 ± 0.13	0.47
$\Sigma^- \to n\, \mu^- \bar{\nu}_\mu$	3.04 ± 0.27	1.60
$\Xi^- \to \Lambda\, \mu^- \bar{\nu}_\mu$	2.1 ± 1.3	1.04
$\Omega^- \to \Xi^0\, e^- \bar{\nu}_e$	68 ± 34	46

g_A/g_V	Exp.	Calc.
$n \to p\, e^- \bar{\nu}_e$	1.2670 ± 0.0035	1.21
$\Lambda \to p\, e^- \bar{\nu}_e$	-0.718 ± 0.015	-0.82
$\Sigma^- \to n\, e^- \bar{\nu}_e$	0.340 ± 0.017	0.25
$\Xi^0 \to \Sigma^+\, e^- \bar{\nu}_e$	$1.32\,^{+0.21}_{-0.17} \pm 0.05$	1.38
$\Xi^- \to \Lambda\, e^- \bar{\nu}_e$	-0.25 ± 0.05	-0.27

Strong decay widths

$N\pi$ decay widths Γ[MeV] — $\Delta\pi$ decay widths Γ[MeV]

Decay	BSE	GBE	$^3P_0$	PDG	Decay	BSE	$^3P_0$	PDG
$S_{11}(1535) \to N\pi$	33	93	216	$(68 \pm 15)^{+45}_{-23}$	$\to \Delta\pi$	1	2	< 2
$S_{11}(1650) \to N\pi$	3	29	149	$(109 \pm 26)^{+29}_{-4}$	$\to \Delta\pi$	5	13	$(6 \pm 5)^{+2}_{0}$
$D_{13}(1520) \to N\pi$	38	17	74	$(66 \pm 6)^{+8}_{-5}$	$\to \Delta\pi$	35	35	$(24 \pm 6)^{+3}_{-2}$
$D_{13}(1700) \to N\pi$	0.1	1	34	$(10 \pm 5)^{+5}_{-5}$	$\to \Delta\pi$	88	778	seen
$D_{15}(1675) \to N\pi$	4	6	28	$(68 \pm 7)^{+14}_{-5}$	$\to \Delta\pi$	30	32	$(83 \pm 7)^{+17}_{-6}$
$P_{11}(1440) \to N\pi$	38	30	412	$(228 \pm 18)^{+65}_{-65}$	$\to \Delta\pi$	35	11	$(88 \pm 18)^{+25}_{-25}$
$P_{33}(1232) \to N\pi$	62	34	108	$(119 \pm 0)^{+5}_{-5}$				
$S_{31}(1620) \to N\pi$	4	10	26	$(38 \pm 7)^{+8}_{-8}$	$\to \Delta\pi$	72	18	$(68 \pm 23)^{+14}_{-14}$
$D_{33}(1700) \to N\pi$	2	3	24	$(45 \pm 15)^{+15}_{-15}$	$\to \Delta\pi$	52	262	$(135 \pm 45)^{+45}_{-45}$

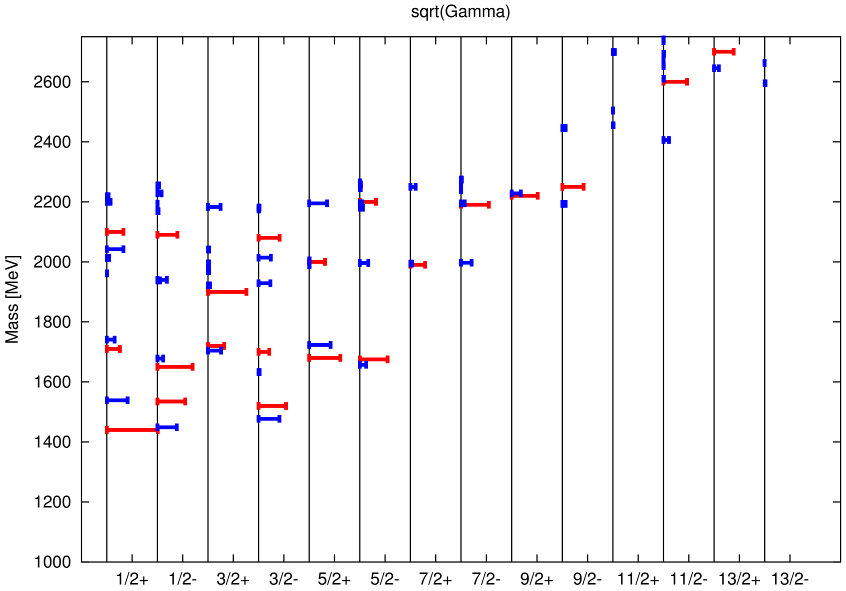


$^3P_0$: S. Capstick, W. Roberts, Phys.Rev. D49 (1994) 4570-4586

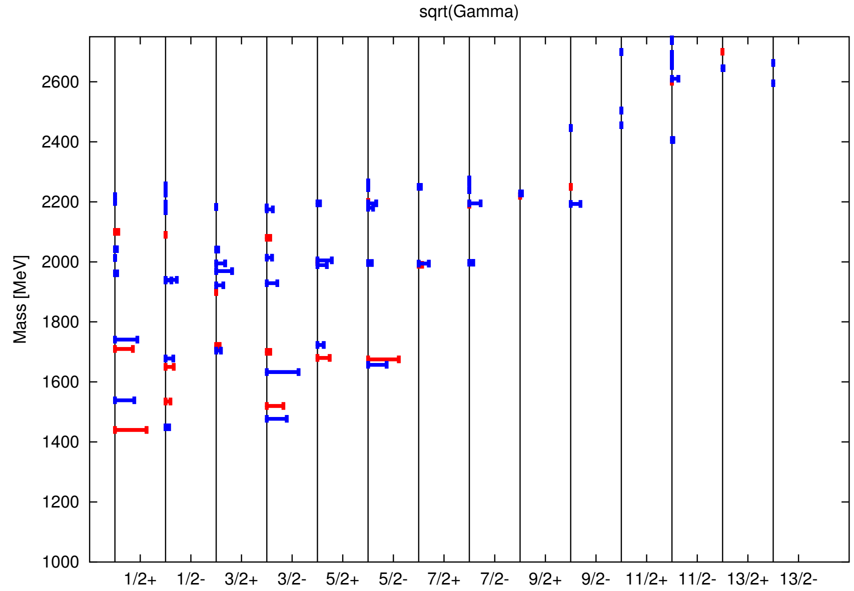
REME (GBE): W. Plessas, nucl-th/306021

Strong decay amplitudes exp. calc.

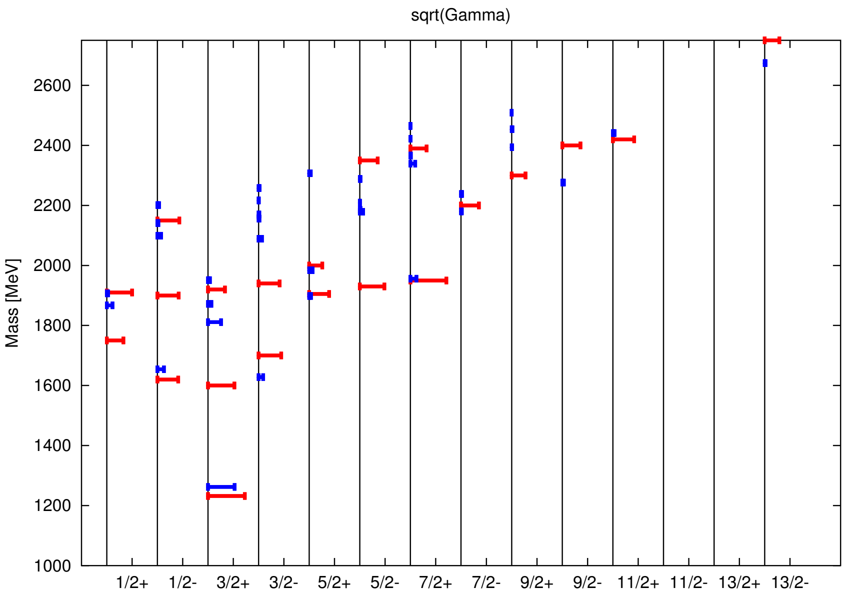
$N \longrightarrow N\pi$



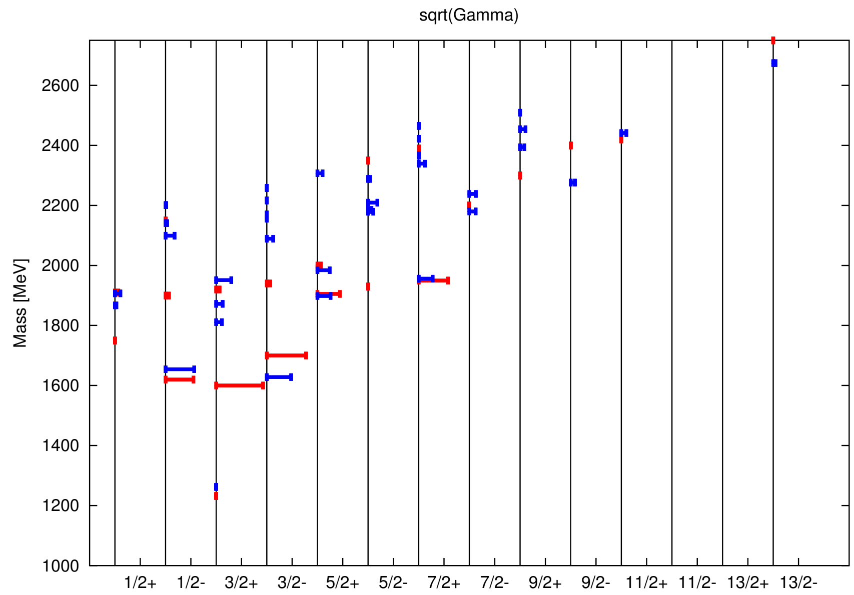
$N \longrightarrow \Delta\pi$



$\Delta \longrightarrow N\pi$

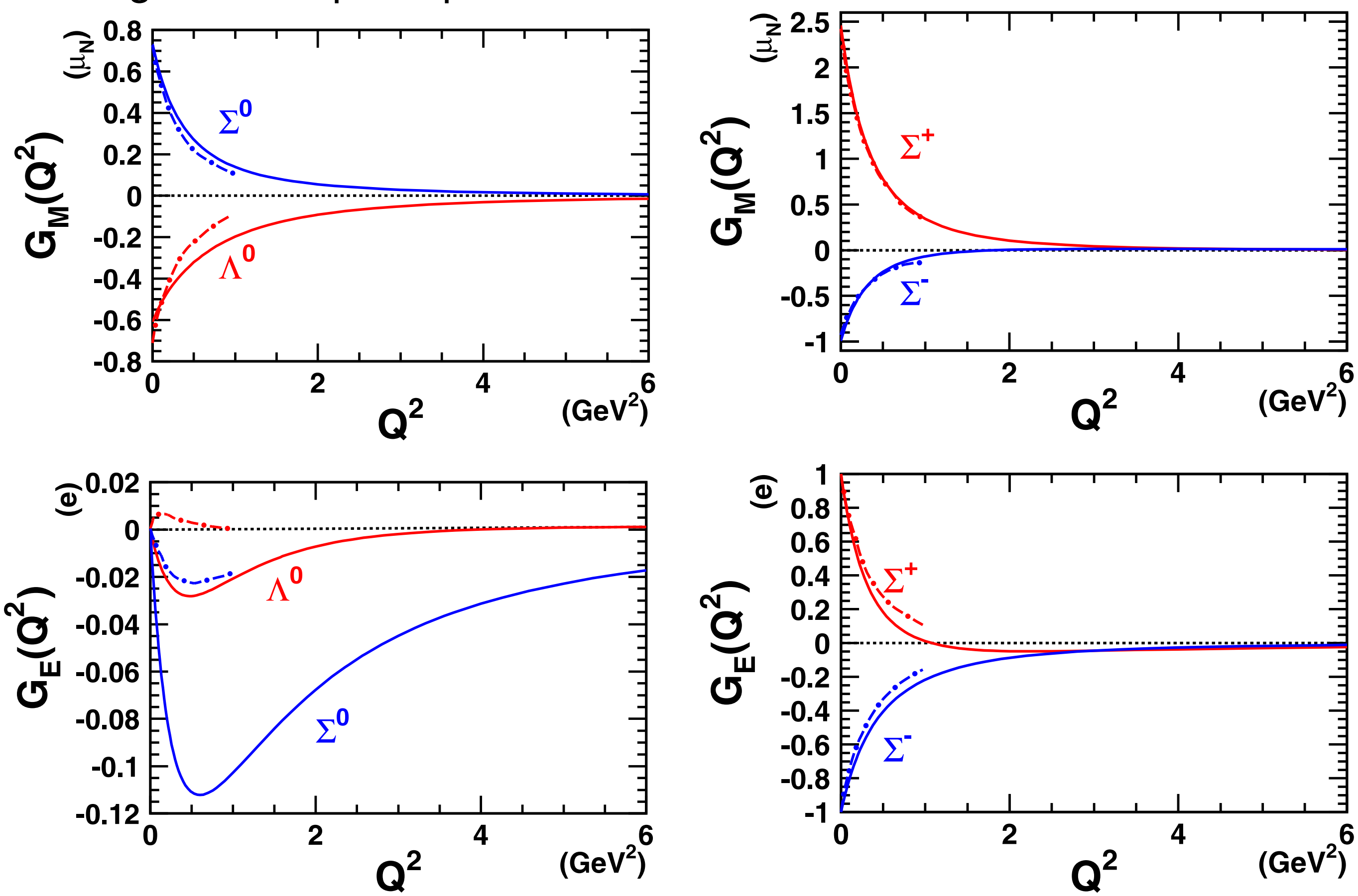


$\Delta \longrightarrow \Delta\pi$



RCQM hyperon form factors

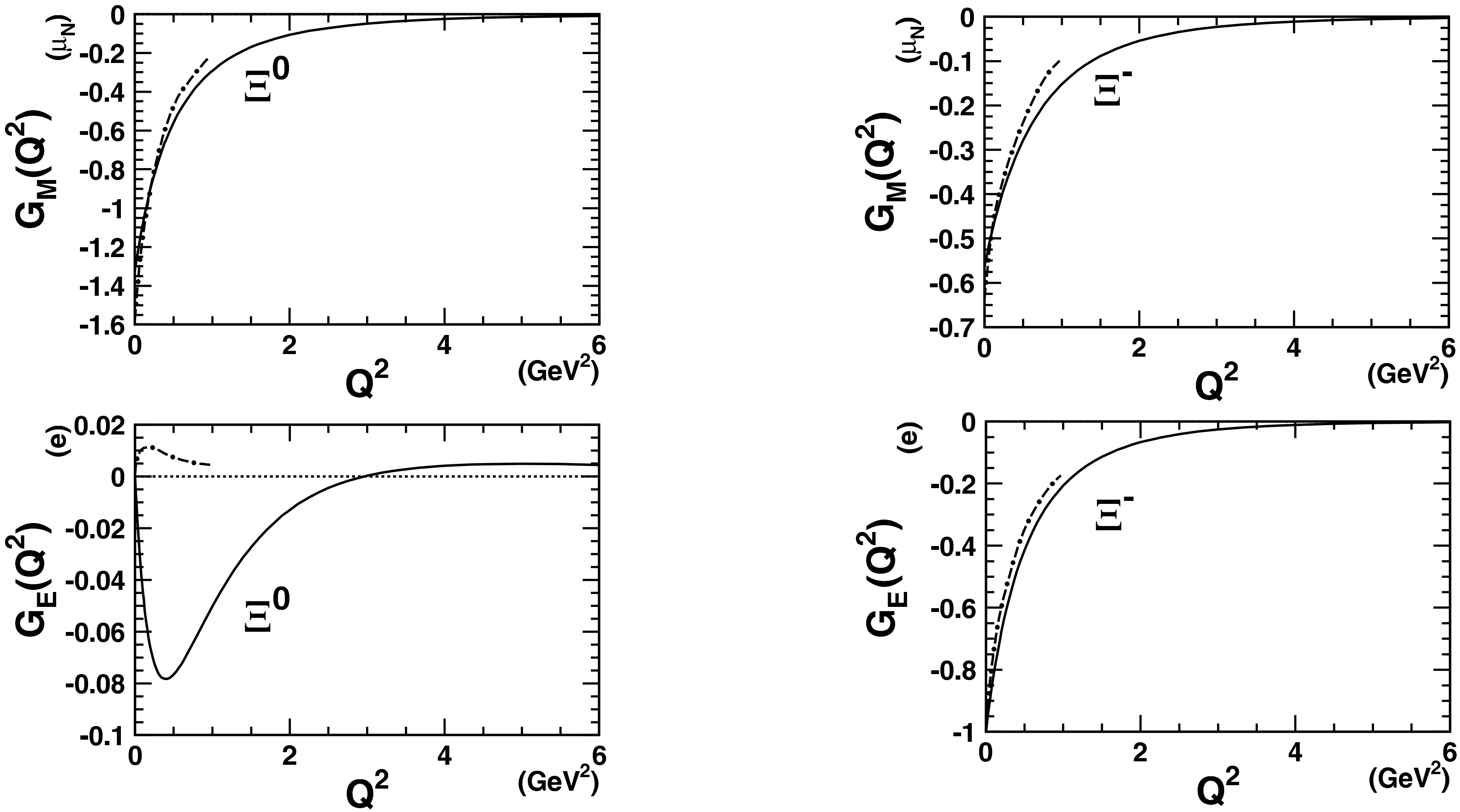
context: electromagnetic coupling to hyperons to be used as guidlines in hadronic models for strange meson photoproduction



BSE (solid): T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283

ChQSM (dashed-dotted): H. Ch. Kim *et al.*, Phys. Rev. **D53** (1996) 4013

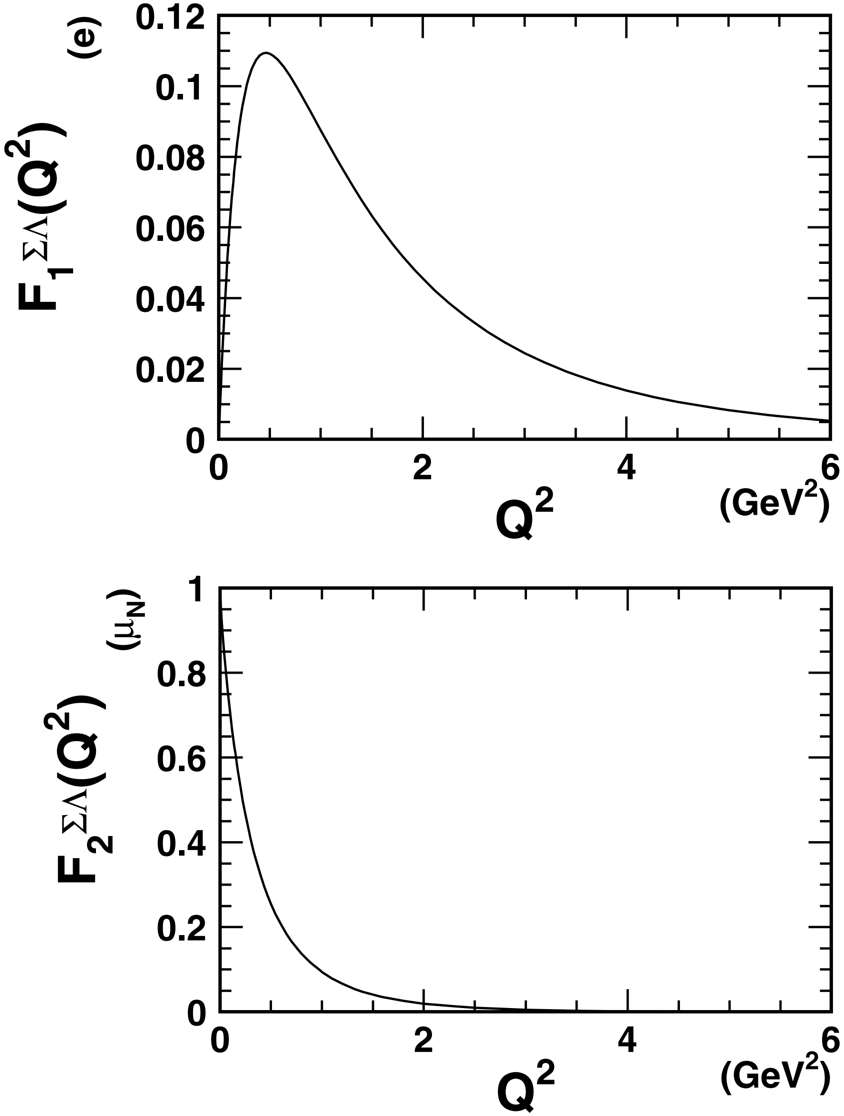
RCQM Ξ-hyperon form factors



BSE (solid): T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283

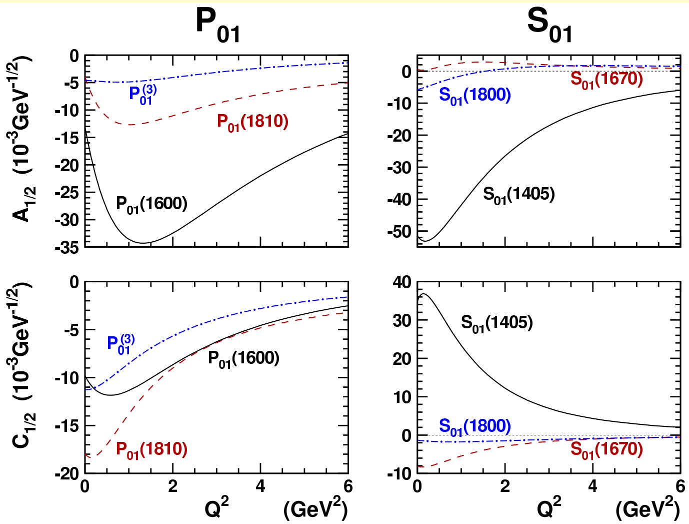
ChQSM (dashed-dotted): H. Ch. Kim *et al.*, Phys. Rev. **D53** (1996) 4013

RCQM hyperon transition form factors



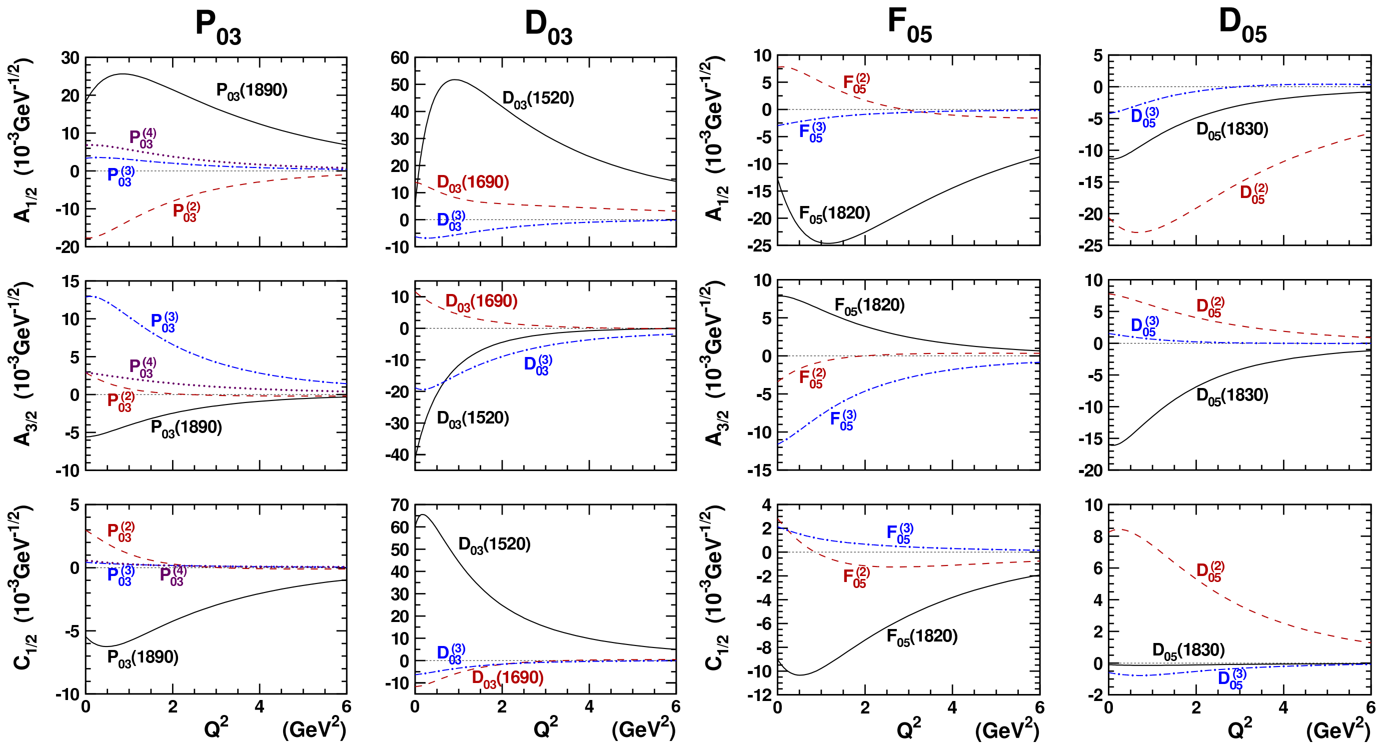
BSE: T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283;
T. van Cauteren *et al.*, nucl-th/0407017

RCQM hyperon $\Lambda \to \Lambda + \gamma$ helicity amplitudes

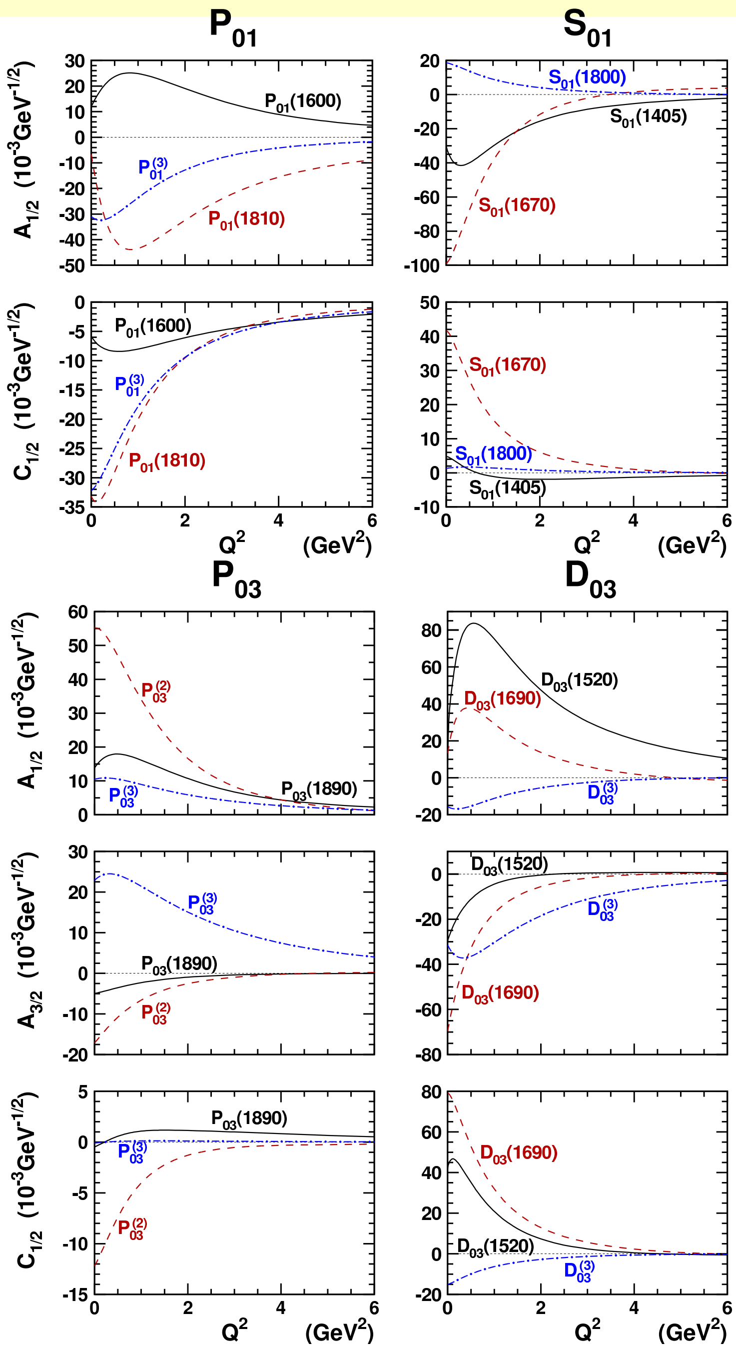


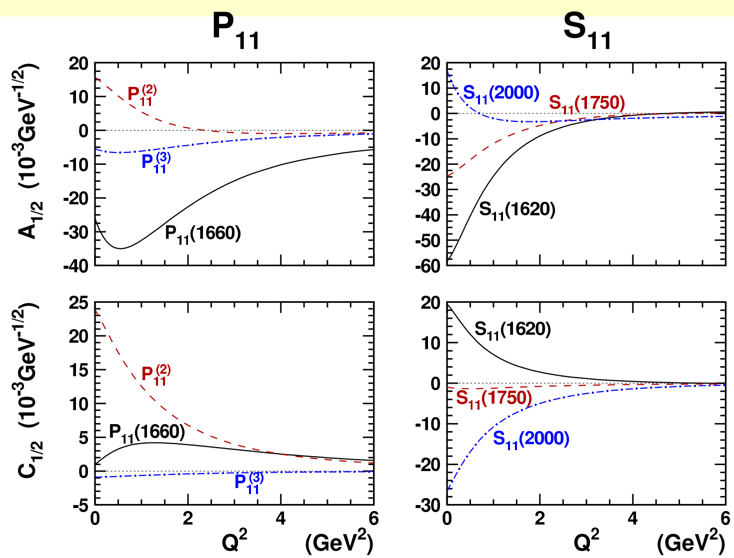
$\Lambda \to \Lambda\gamma\,;\, A[10^{-3}\text{GeV}^{-\frac{1}{2}}]\,;\, \Gamma[\text{MeV}]$

state	$\lvert A_{\frac{1}{2}} \rvert$	$\lvert A_{\frac{3}{2}} \rvert$	Γ(calc)	Γ(exp)
$S_{01}(1405)$	51.5	-	0.912	$0.027^{+0.008}_{-0.008}$
$D_{03}(1520)$	5.50	41.2	0.258	$0.125^{+0.042}_{-0.038}$



$\Lambda \to \Sigma + \gamma$ $\Sigma \to \Lambda + \gamma$

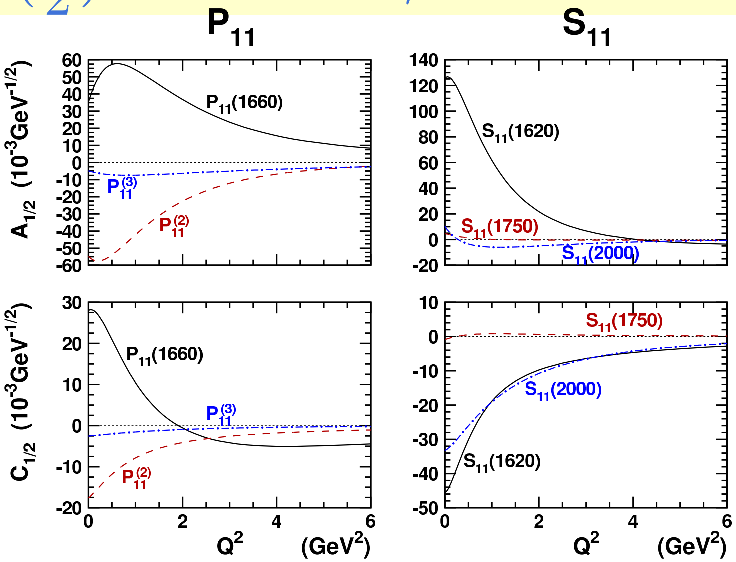




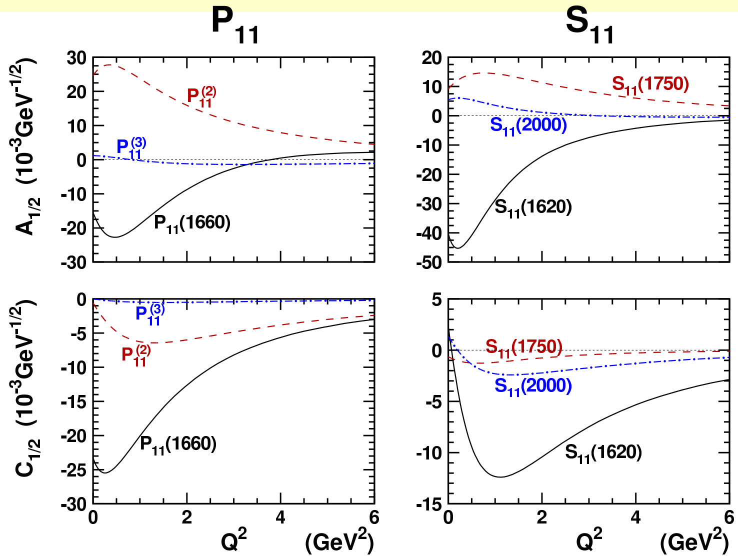
$\Lambda \to \Sigma\gamma$; $A[10^{-3}\text{GeV}^{-\frac{1}{2}}]$; Γ[MeV]

state	$\lvert A_{\frac{1}{2}}\rvert$	$\lvert A_{\frac{3}{2}}\rvert$	Γ(calc)	Γ(exp)
$S_{01}(1405)$	30.3	-	0.233	$0.010^{+0.004}_{-0.004}$
				$0.0123^{+0.007}_{-0.007}$
$D_{03}(1520)$	23.3	30.0	0.157	$0.304^{+0.076}_{-0.070}$

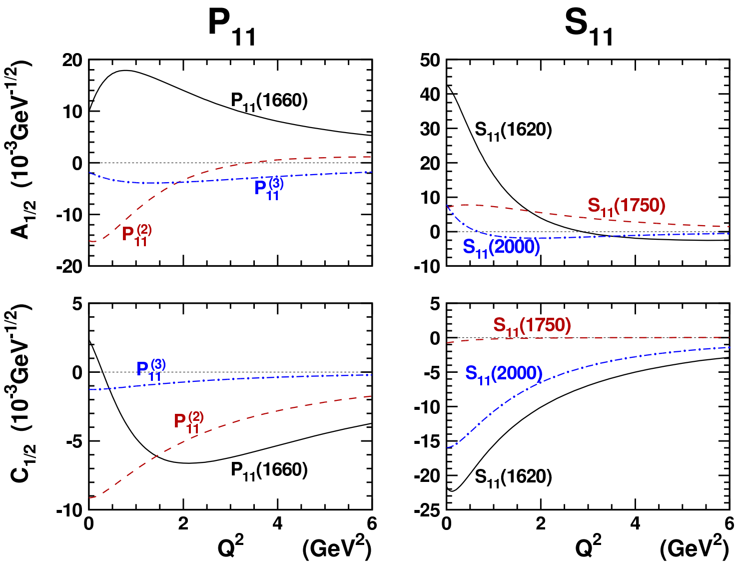
$\Sigma(\frac{1}{2}) \to \Sigma + \gamma$



Σ^+

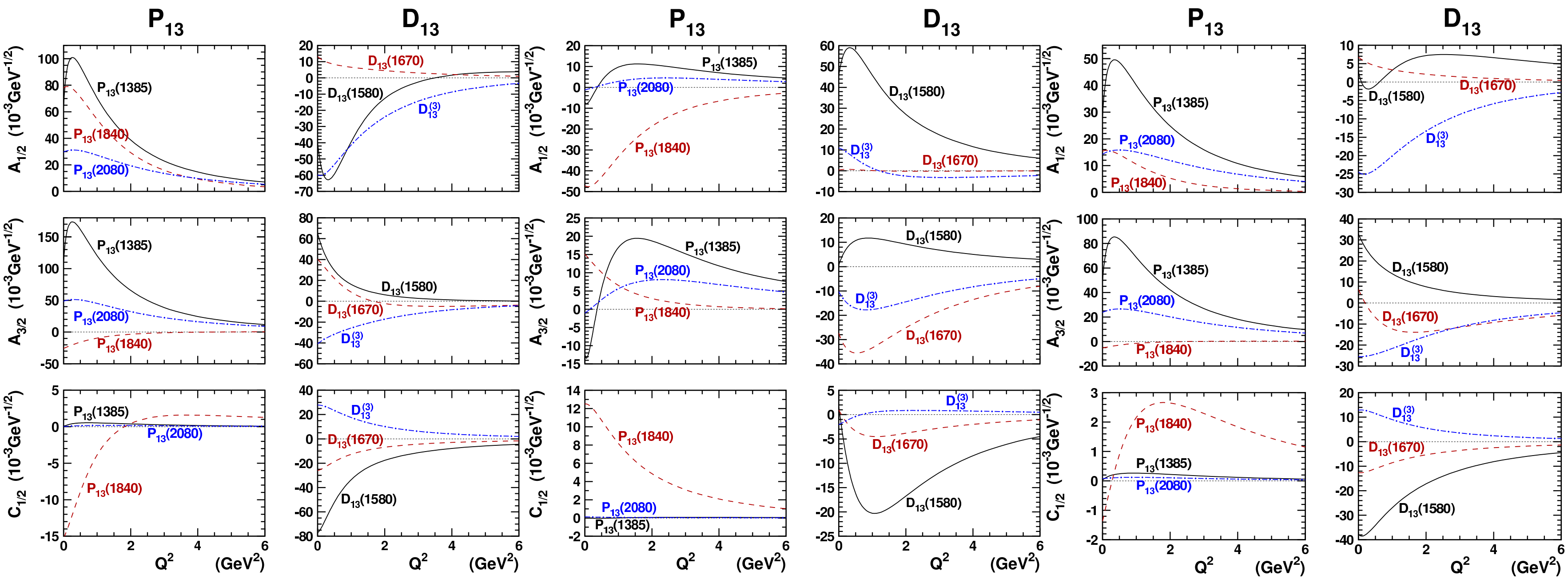


Σ^-



Σ^0

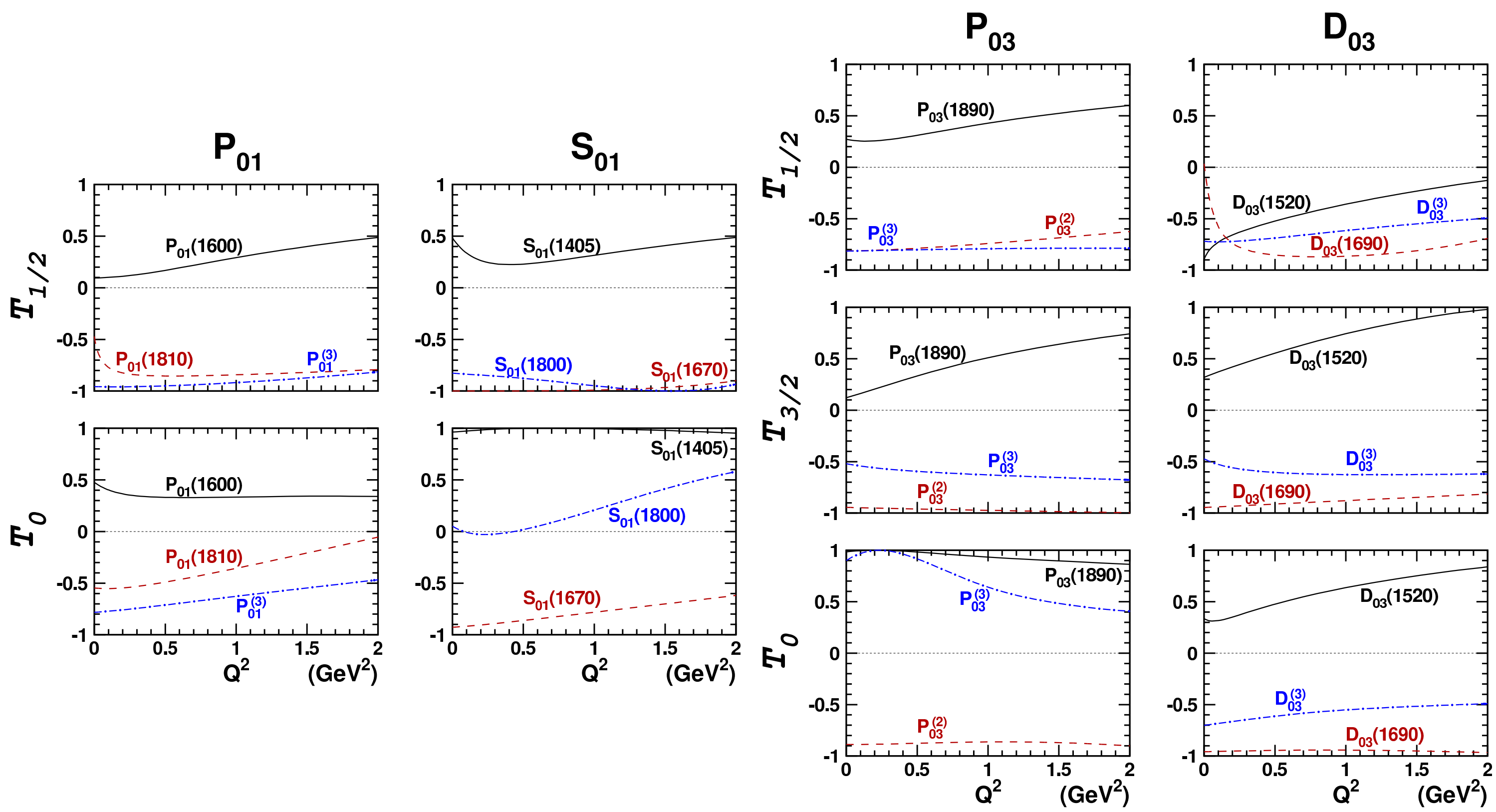
$\Sigma(\frac{3}{2}) \rightarrow \Sigma + \gamma$



Σ^+ Σ^- Σ^0

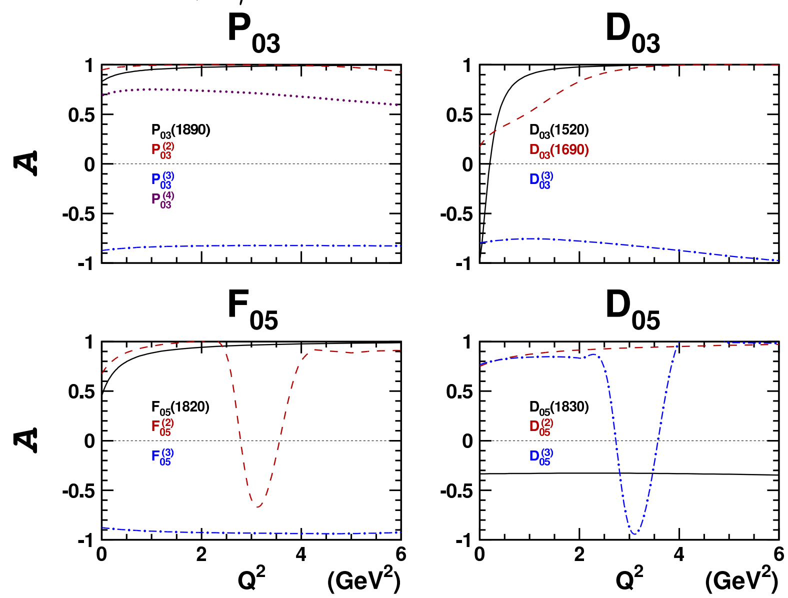
Isospin Asymmetries $\Lambda \to \Lambda/\Sigma + \gamma$

$$T_\lambda := \frac{\left|A^\Lambda_\lambda\right|^2 - \left|A^\Sigma_\lambda\right|^2}{\left|A^\Lambda_\lambda\right|^2 + \left|A^\Sigma_\lambda\right|^2}\,, \qquad T_0 := \frac{\left|C^\Lambda_{\frac{1}{2}}\right|^2 - \left|C^\Sigma_{\frac{1}{2}}\right|^2}{\left|C^\Lambda_{\frac{1}{2}}\right|^2 + \left|C^\Sigma_{\frac{1}{2}}\right|^2}$$

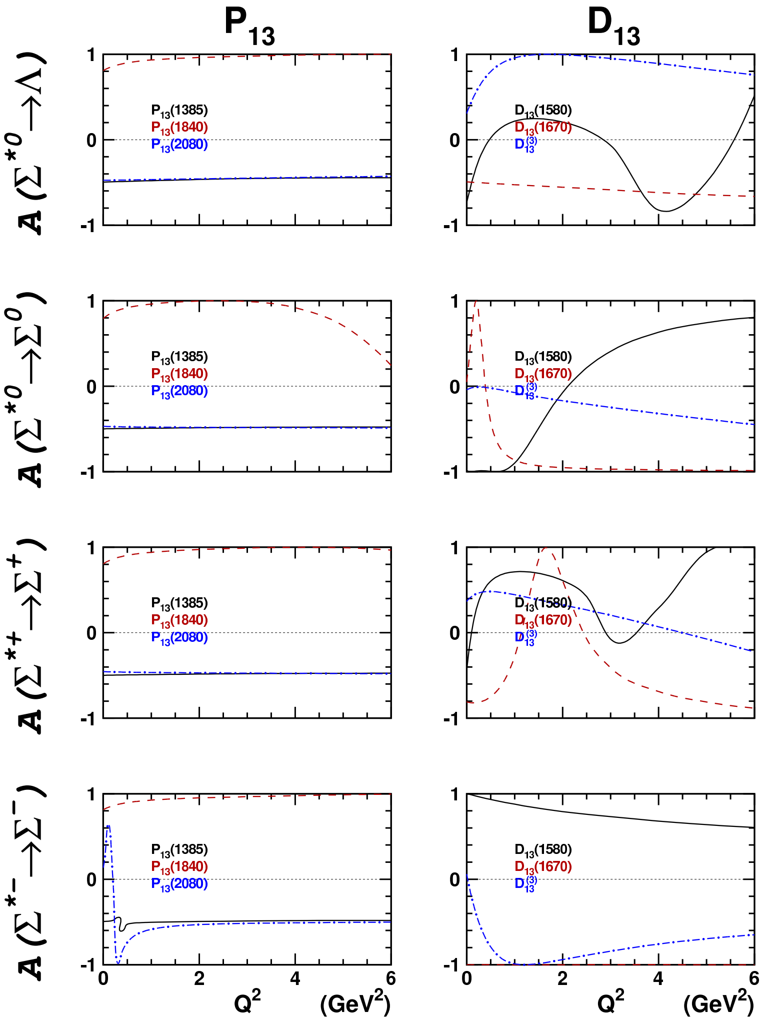


Helicity Asymmetries $Y \rightarrow \Lambda/\Sigma^0 + \gamma$

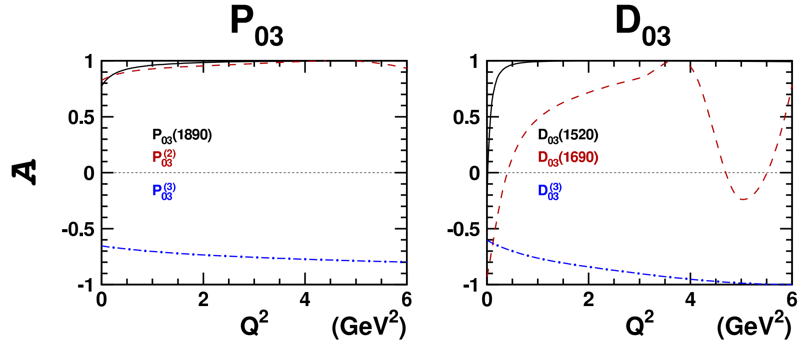
$\Lambda \rightarrow \Lambda + \gamma$



$$\mathcal{A} := \frac{\left|A_{\frac{1}{2}}\right|^2 - \left|A_{\frac{3}{2}}\right|^2}{\left|A_{\frac{1}{2}}\right|^2 + \left|A_{\frac{3}{2}}\right|^2}$$



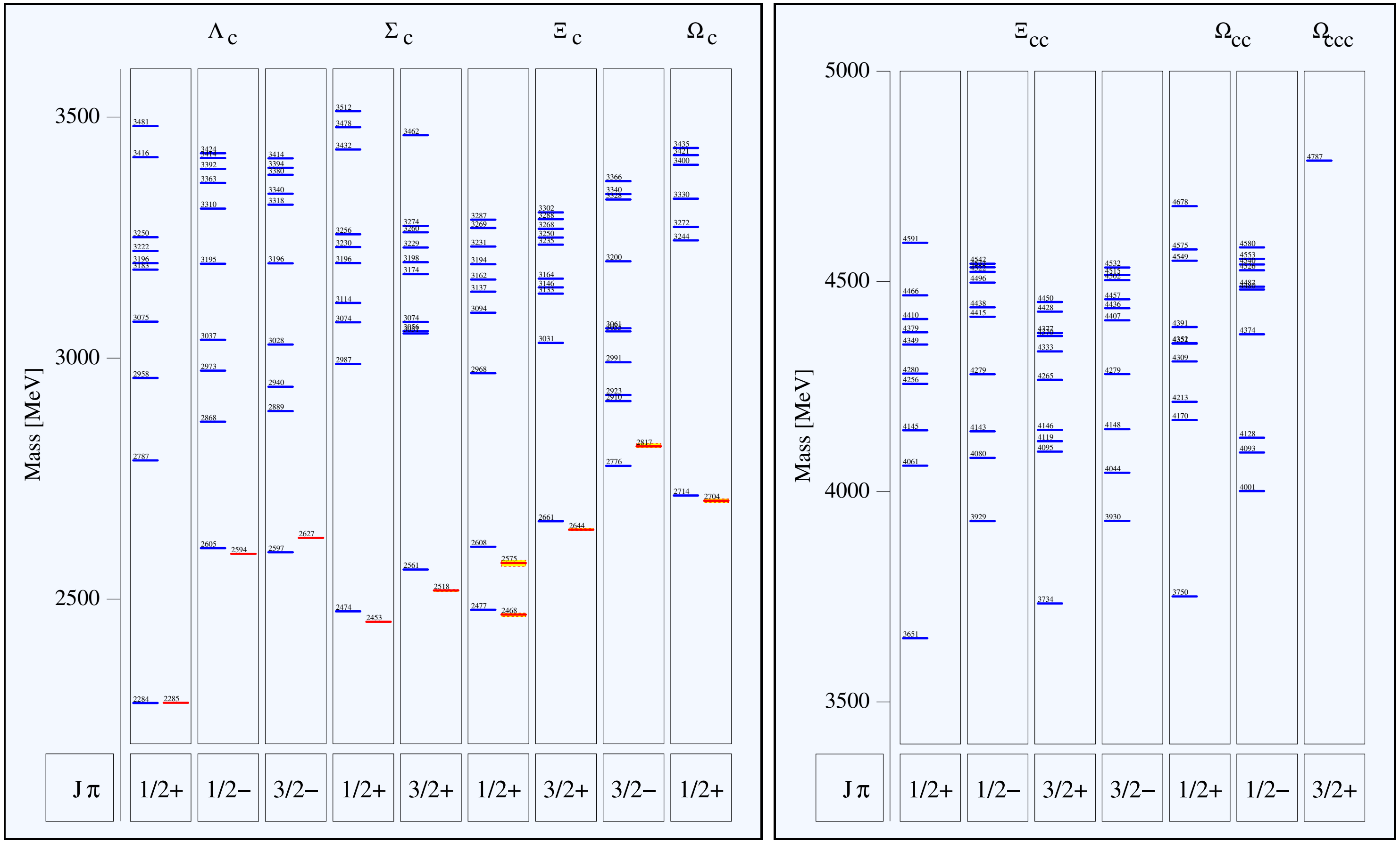
$\Lambda \rightarrow \Sigma^0 + \gamma$



charmed baryons

(formal extension to $f_i = c$)

- Mass spectra:



- semileptonic decays (prelim.):

$\Gamma[\Lambda_c^+ \to \Lambda\, e^+\, \nu_e]$: $1.58\ 10^{11}$ s^{-1}(calc) $\leftrightarrow$ $(1.02 \pm 0.30)\ 10^{11}$ s^{-1}(exp)

$\Gamma[\Lambda_c^+ \to \Lambda\, \mu^+\, \nu_\mu]$: $1.40\ 10^{11}$ s^{-1}(calc) $\leftrightarrow$ $(0.97 \pm 0.34)\ 10^{11}$ s^{-1}(exp)

Finale

- Constituent Quark Models provide a very useful tool for relating various hadron properties: mass spectra, electroweak form factors, decay amplitudes
- constitute a reference frame for discriminating exotics
- Frameworks:
  - Field Theory
    - Bethe-Salpeter/Dyson-Schwinger-equation
    - . . . with instantaneous potentials (full Salpeter Equation) (confinement + instanton induced interaction)

    $\Rightarrow$ parameter-free calculation of amplitudes (in lowest order)
  - Quantum Mechanics
    - Quark Dynamics from a “relativised” Schrödinger Equation on the basis of OGE or GBE + confinement.
    - Amplitudes in Dirac’s point-form formulation or parametrised
- a unified description of light-flavoured mesons and baryons up to high masses and spins has been achieved, implementing confinement by a string-like potential, in the “R”CQM with e.g. OGE-based or GBE-based quark dynamics and (rather efficiently) with instanton-induced interactions in the Salpeter framework; extension to heavy-flavoured hadrons in progress
- Implementation of relativistic covariance is extremely important for the quark dynamics and the description of amplitudes.

Publications

1. M. Koll, R. Ricken, D. Merten, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **9** (2000) 73
2. R. Ricken, M. Koll, D. Merten, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **9** (2000) 221
3. U. Löring, K. Kretzschmar, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **10** (2001) 309–346
4. U. Löring, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **10** (2001) 395–446
5. U. Löring, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **10** (2001) 447–486
6. D. Merten, R. Ricken, M. Koll, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **13** (2002) 477–491
7. D. Merten, U. Löring, K. Kretzschmar, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **14** (2002) 477–489
8. B.C. Metsch, U. Löring, D. Merten, H.R. Petry, Eur. Phys. J. A **18** (2003) 189–192
9. D. Merten, U. Löring, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **18** (2003) 193–195
10. R. Ricken, M. Koll, D. Merten, Eur. Phys. J. A **18** (2003) 667–689
11. T. van Cauteren, D. Merten, T. Corthals, S. Janssen, B.C. Metsch, H.R. Petry, J. Ryckebusch, Eur. Phys. J. A**20** (2004) 283