1. Beta Decay is Important, Hard

2. What’s Been Done So Far

3. Future Work
Beta-Decay Far From Stability is Important

- Half-lives are particularly important at the r-process “ladders” \((N=50, 82, 126)\) where abundances peak. These half-lives determine the r-process time scale.

- But others are also important; affect abundances between peaks.

Yes, uncertainties in astrophysics dwarf those in nuclear physics, but we won’t really understand the nucleosynthesis until both are reduced.
Calculating Beta Decay is Hard...

(though it gets easier in neutron-rich nuclei).

To calculate beta decay between two states, you need:

- an accurate value for the decay energy $\Delta E$ (since $T_{1/2} \propto \Delta E^{-5}$)
- matrix elements of the GT operator $\vec{\sigma} \tau_-$ and (sometimes) “forbidden” operators $\vec{r} \vec{\sigma} \tau_-$ between the two states. [Most of the strength of is above threshold, btw].

So your nuclear structure model must do a good job with nuclear masses, spectra, and wave functions, and to simulate the r process it must do the job in almost all isotopes.
Usefulness of RIA-Lite

RIA – $1^+$ ion yields for reacceleration

Reacceleration covers most of the path except near $N = 126$. Theory — no matter how phenomenological — should take it from there.
Approaches Tried So Far

1. Macroscopic/Microscopic Mass Model + Schematic QRPA
2. Self-consistent Skyrme Mean-Field Theory (HFB) + QRPA*
3. Relativistic QRPA
4. Shell Model
5. Finite Fermi Systems Theory*

Wave functions in all these models lack correlations, something that is compensated for by renormalizing $g_A$. 
The only calculation applied globally.

- Masses through, e.g., finite-range droplet model with shell corrections.
- QRPA: $V \approx \kappa \vec{\sigma} \tau_- \cdot \vec{\sigma} \tau_+$ with $\kappa \propto 1/A$.
- First-forbidden (in “gross theory”) added to GT a few years ago. Shortens half-lives some at $N = 82$, more at $N = 126$. 

![Graph showing $\beta$-decay half-lives comparison](image)
Self-Consistent Skyrme HFB + Canonical-Basis QRPA

Just one calculation, in ladder nuclei, with no forbidden decay.

- Same energy-density functional in HFB and QRPA, so that most unknown physics is in functional. Skyrme functional SkO’ chosen because did best with GT distributions.

- One free parameter: strength of $T = 0$ pairing (it’s zero in schematic QRPA.) Adjusted in each of the three regions to reproduce measured lifetimes.
Half-lives at ladder nuclei are shorter than in global approach at $N = 50$ and 82 (even with forbidden contribution in global calculations), longer at $N = 126$. 

![Graph showing half-lives at different neutron numbers](image)
Further Development of HFB+QRPA

So far: Improving the isovector “T-odd” Energy Functional

\[ \mathcal{E} = \int (\mathcal{H}_{\text{even}} + \mathcal{H}_{\text{odd}}) \, d^3r \]

\[ \mathcal{H}_{\text{odd}} = C_s \vec{s}(\vec{r}) \cdot \vec{s}(\vec{r}) \rightarrow g_0' \]
\[ + C_\Delta \vec{s}(\vec{r}) \cdot \nabla^2 \vec{s}(\vec{r}) \]
\[ + C_T \vec{s}(\vec{r}) \cdot \vec{T}(\vec{r}) \rightarrow g_0', g_1' \]

with \( \vec{s}(\vec{r}) \) the spin density and \( \vec{T}(\vec{r}) \) the “kinetic” spin density. [The \( g' \) are the nuclear-matter spin-isospin “Landau parameters”.] Only enough data to determine one combination of parameters \( g_0' \).
The graph shows the relationship between $B_{res}/B_{tot}$ and $\Delta E$ (MeV) as a function of $g_0$. Different isotopes are represented with distinct symbols: $^{208}$Pb (filled square), $^{124}$Sn (diamond), and $^{112}$Sn (circle). The $B_{res}/B_{tot}$ values increase with $g_0$, while $\Delta E$ decreases with increasing $g_0$. The graph suggests a linear trend for each isotope, indicating a consistent pattern across different isotopes.
The best available approach, at present, *if* you have large model space, good effective interactions and transition operators, etc. Two calculations reported for \( N = 82 \), with no forbidden contributions.

- Martinez-Pinedo and Langanke
- B.A. Brown et al

Half-lives in two calculations are similar, the first group’s even shorter than in HFB+QRPA.

But extension to deformed nuclei still a long way off.
A Green’s-function-based approximation to the self consistent HFB+ QRPA

- Like HFB+QRPA, FFST applied only in spherical nuclei, though approach is applicable generally.
- Currently includes forbidden transitions, which have moderate effect at $N = 82$, large effect at $N = 126$. Half-lives at $N = 82$ are between those of HFB+QRPA and shell model.
The experimental β-decay half-lives for the $N=82$ chain taken from the NUBASE compilation [57] compared to DF3.
The Near Future (I hope)

- Extend HFB+QRPA to **deformed nuclei** to make HFB+QRPA approach “global”. HFB part already done.
  1. Makes Skyrme HFB approach global (HFB part already done).
  2. Opens up more data for tuning GT properties of Skyrme functionals
- Improve T-even parts of functionals (e.g. through Dobaczeweski’s program).
In this section, we present the results of calculations performed for all particle-bound even-even nuclei. In the pairing channel, we used a pure volume contact interaction scheme that was not focused on weakly bound nuclei. In the lower panel, showing two-neutron separation energies, results are shown for those nuclei with both number projections. The Skyrme SLy4 interaction and volume contact pairing were used.

Note that on the proton-rich side the lighter of them may be obtained as this was adjusted with special emphasis on the properties of neutron matter. At present, the smaller of $N - Z$ was positive. For this figure, calculations for a given mass number were performed by building THO basis states from spherical wave functions corresponding to oblate, spherical, and prolate deformations, respectively. Depending on properties of a given nucleus, we could therefore obtain one, two, or three solutions with different shapes. For each obtained solution we performed a PNP calculation of the total energy. The lowest energy was obtained using the prescription developed in the preceding section. The value used in the procedure was obtained in the following way. From the starting HFB+HO calculation, we determined the $\delta$-value (Fig. 2) for which the nuclides with both $N$ and $Z$ even deformations were calculated within the HFB+THO method with Lipkin-Nogami correction followed by exact particle-number projection. The Skyrme SLy4 interaction and volume contact pairing were used.

Deformed HFB Already Done

(Stoitsov et al, 2003)
Deformed Skyrme-QRPA

Need this to go away from ladder nuclei. But basis can contain $10^5$-$10^6$ states. Can use Lanczos-RPA algorithm developed by Johnson and Bertsch.

Tested in schematic model
Extended FFTS: includes more complex configurations

- Improves agreement with low-lying excitation spectrum in like-particle channels.
- No problem in principle with extending to beta decay.
- Could make strong and variable $T=0$ pairing less necessary.

![Graph of PDR in Ca Isotopes]

Preliminaries  So Far  Future
The further you have to extrapolate, the worse off you are. RIA (or sRIA or ssRIA...) will reduce extrapolation to zero for some parts of $r$-process path, and reduce it to manageable levels for others.