Here we give the standard definition of multipole amplitudes of photoproduction of pseudo-scalar mesons ($M = \pi, \eta, K$) on the nucleon (N). In the center of mass system, the formula relating the un-polarized differential cross section of $\gamma(\vec{q})+N(-\vec{q}) \to M(\vec{k})+N(-\vec{k})$ is

$$\frac{d\sigma_0}{d\Omega} = \frac{1}{4}\sum_{m_{s_N}=\pm 1/2}\ \sum_{m'_{s_N}=\pm 1/2}\ \sum_{\lambda=\pm 1}\frac{k}{q}| < m'_{s_N}|F_{CGLN}|m_{s_N} > |^2 \tag{1}$$

where

$$F_{CGLN} = \sum_{i=1,4} O_i F_i(\theta, E) \tag{2}$$

Here we have defined $E = q + E_N(q) = E_M(k) + E_N(k)$, where $E_\alpha(p) = \sqrt{m_\alpha^2 + p^2}$ and m_α is the mass of particle α. The operators in the above equation are:

$$\begin{aligned}
O_1 &= -i\vec{\sigma}\cdot\vec{\epsilon}_\lambda \\
O_2 &= -[\vec{\sigma}\cdot\hat{k}][\vec{\sigma}\cdot(\hat{q}\times\vec{\epsilon}_\lambda] \\
O_3 &= -i[\vec{\sigma}\cdot\hat{q}][\hat{k}\cdot\vec{\epsilon}_\lambda] \\
O_4 &= -i[\vec{\sigma}\cdot\hat{k}][\hat{k}\cdot\vec{\epsilon}_\lambda]
\end{aligned}$$

where $\hat{k} = \vec{k}/|\vec{k}|$ and $\hat{k} = \vec{q}/|\vec{q}|$, and $\vec{\sigma}$ is the standard Pauli operator. The quantization direction is chosen to be in the $\hat{z}$-direction, and the photon polarization vectors are $\vec{\epsilon}_\pm = \frac{\mp}{\sqrt{2}}[\hat{x} \pm i\hat{y}]$.

The Chew-Goldberger-Low-Nambu(CGLM) amplitudes are related to multipole amplitudes by

$$\begin{aligned}
F_1(\theta, E) &= \sum_l [P'_{l+1}(x)E_{l+}(E) + P'_{l-1}(x)E_{l-}(E) + lP'_{l+1}(x)M_{l+}(E) + (l+1)P'_{l-1}(x)M_{l-}(E)] \\
F_2(\theta, E) &= \sum_l [(l+1)P'_l(x)M_{l+}(E) + lP'_l(x)M_{l-}(E)] \\
F_3(\theta, E) &= \sum_l [P''_{l+1}(x)E_{l+}(E) + P''_{l-1}(x)E_{l-}(E) - P''_{l+1}(x)M_{l+}(E) + P''_{l-1}(x)M_{l-}(E)] \\
F_4(\theta, E) &= \sum_l [-P''_l(x)E_{l+}(E) - P''_l(x)E_{l-}(E) + P''_l(x)M_{l+}(E) - P''_l(x)M_{l-}(E)]
\end{aligned}$$

where $x = \hat{k}\cdot\hat{q} = cos\theta$, l is the orbital angular momentum of the final πN system, $P'_l(x) = dP_l(x)/dx$ and $P''_l(x) = d^2P_l(x)/d^2x$ are the derivatives of the Legendre function $P_l(x)$ with the understanding that $P'_{-1} = P''_{-1} = 0$.

The formula for using the multipole ampltitudes to calculate the polarization observables can be found in "A. Sandorfi, S. Hoblit, H, Kamano, T.-S. H. Lee, J.Phys. G38, 053001 (2011)