Pion-Nucleon Sigma Term in Global Color Symmetry Model

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Outline

I. Introduction

II. Brief Description of the GCM

III. The $\sigma_{\pi N}$ in Free Space and in Nuclear Matter

IV. Application to Chiral Quark Condensate

V. Summary and Remarks
I. Introduction

Pion-nucleon sigma term is significant in

\[ M_N = M_{N,b} + \sigma_{\pi N} \]

Nucleon mass decomposition

\[ 2\langle \bar{s}s \rangle_\rho = 1 - \frac{\sigma_0}{\sigma_{\pi N}} \]

s-quark content in nucleon

Chiral symmetry breaking effect in nucleon

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_{\pi N}}{2m_q|\langle \bar{q}q \rangle_0|} \rho \]

Chiral symmetry restoration in nuclear matter

may probably be helpful in searching

Higgs boson and supersymmetric particles, Dark matter

Can not be measured directly !!

\[ \sigma_{\pi N} = \sigma(0) \neq \sigma(2m_\pi^2) \]
Pion-nucleon sigma term has been studied in
Chiral perturbation theory
Chiral quark models,
Lattice QCD,
……

**Different approach gives quite different result**

- as small as $(18 \pm 5)\text{ MeV}$
- as large as $(80 \sim 90)\text{ MeV}$
- generally $(45 \pm 7)\text{ MeV}$
- recently $(50 \sim 80)\text{ MeV}$

**References**

- PRD59, 054504 (1999)
- EPJ A22, 89 (2004);
- PLB253, 252, 260; PRD63, 054026
- PRD69, 034003; PRC71, 065201
Two Puzzles in Strong Interaction at low energy:

- Chiral Symmetry and its Spontaneous Breaking
- Color Confinement

Characteristics Identifying Quark Deconfinement and Chiral Symmetry Restoration:

- Hadron Properties
- Vacuum Structure

Confinement Mechanism:
- Flux Tube, 
- Intuitive view?

Lattice QCD, pQCD (Factorization Re-summation), 

NJL model, QMC, QMF, Truncated DSE, Instanton, GCM, …
How the Chiral Symmetry is Restored?

Quark condensates are usually taken as characteristics of Vacuum Structure Order Parameters.

Theoretical approaches:
- Composite-operator,
- Sum rules, QMC, NJL,
- Walecka model,
- Dirac-Brueckner,
- S-D Equation,
- Instanton dilute liquid model, …

Different results have been obtained!!

ANL Workshop, Aug. 29-Sept.2
How to understand the difference is a puzzle!!
II. Brief Description of the GCM

1. The Main Point of Global Color Symmetry Model

- The global color symmetry model (GCM) is based upon an effective quark-quark interaction defined through a truncation of QCD.

\[
S = \int d^4x \left\{ \bar{q} \left[ \gamma_\mu \cdot \partial_\mu + m \right] \delta(x-y) q(y) - \frac{g^2}{2} j_\mu^a(x) D_{\mu\nu}(x-y) j_\nu^a(y) \right\},
\]

where \( j_\mu^a = \bar{q}(x) \lambda^a_\mu / 2 \gamma_\mu q(x) \) is the quark current,
\( D_{\mu\nu}(x-y) \) is the effective two-point gluon propagator.

Taking $D_{\mu\nu}(x - y) = \delta_{\mu\nu} D(x - y)$

and applying the Fierz reordering

$$\Lambda^\theta = \frac{1}{2} K^a \otimes C^b \otimes F^c$$

with

$$\{K^a\} = \{I, i\gamma_5, \frac{i}{\sqrt{2}} \gamma_\mu, \frac{i}{\sqrt{2}} \gamma_\mu \gamma_5\}$$

$$\{C^b\} = \{\frac{4}{3} I, \frac{i}{\sqrt{3}} \lambda\}$$

$$\{F^c\} = \{\frac{1}{\sqrt{2}} I, \frac{\vec{r}}{\sqrt{2}}\}$$

to the quark field, one has the current term

$$\frac{g^2}{2} \int d^4 x d^4 y j_\mu^a(x) D_{\mu\nu}(x - y) j_\nu^a(y)$$

$$= -\frac{g^2}{2} \int d^4 x d^4 y j^\theta(x, y) D(x - y) j^\theta(y, x)$$

- The $j^\theta(x, y) = \bar{q}(x) \Lambda q(y)$ is a bilocal current, and can be transferred into an auxiliary Bose-field $B^\theta(x, y)$. 
- The GCM action can be rewritten as

\[
S[\bar{q}, q, B^\theta(x, y)] = \iint d^4x d^4y \left[ \bar{q}(x) G^{-1}(x, y; B^\theta) q(y) + \frac{B^\theta(x, y) B^\theta(y, x)}{2g^2 D(x - y)} \right],
\]

with

\[
G^{-1}(x, y; B^\theta) = \gamma \cdot \partial \delta(x - y) + \Lambda^\theta B^\theta(x, y).
\]

- The bilocal field \( B^\theta(x, y) \) can be generally given as

\[
B^\theta(x, y) = B^\theta_v(x - y) + \sum_{i, \theta} \phi^\theta_i \left( \frac{x + y}{2} \right) \Gamma^\theta_i (x - y),
\]

- The vacuum configuration \( B^\theta_v(x, y) \) can be obtained by solving the equation

\[
\frac{\partial S[B^\theta(x, y)]}{\partial B^\theta(x, y)} \bigg|_{B^\theta(x, y) = B^\theta_v(x - y)} = 0.
\]
In practical calculation, one can fix the $B^\theta_v$ in two ways.

One is by solving the rainbow Dyson-Schwinger equation

$$G^{-1}(x, y) = i\gamma \cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D(p - q) \frac{t^a}{2} \gamma_\mu G(x, y) \gamma_\mu \frac{t^a}{2}$$

with $G^{-1} = iA(p^2)\gamma \cdot p + \Lambda^\theta B^\theta_v(p^2)$.

Another is by modeling dressed quark propagator directly.

For example

$$A = 1, \quad B^\theta_v(q) = m(q^2).$$

where $m(q)$ is the dynamical quark mass.

The relative excitation can be fixed as

$$\Gamma_\sigma = B^\theta_v, \quad \Gamma_\pi = i\gamma_5 \tau^7 B^\theta_v.$$

It has been successful in describing hadron properties, chiral parameters, and so on.
2. GCM in Strongly Interacting Matter

(1) General Method

- The quark field and the bilocal field in nuclear matter with chemical potential $\mu$

\[
q(x) \implies q' = e^{\mu x_4} q(x),
\]

\[
B^\theta(x,y) \implies B^\theta(x,y; \mu) = e^{\mu x_4} B^\theta(x,y)e^{-\mu y_4}.
\]

- The action of the GCM at finite chemical potential is

\[
S = \int d^4x d^4y \bar{q}'(x) \left[ \gamma \cdot \partial \delta(x-y) - \mu \gamma_4 + \Lambda^\theta B^\theta(x,y; \mu) \right] q'(y)
+ \int d^4x d^4y \frac{B^\theta(x,y) B^\theta(y,x)}{2g^2 D(x-y)},
\]

\[\text{(8a)}\]

i.e.,

\[
S = -\text{Tr} \ln \left[ \gamma \cdot \partial \delta(x-y) - \mu \gamma_4 + \Lambda^\theta B^\theta \right]
+ \int d^4x d^4y \frac{B^\theta(x,y) B^\theta(y,x)}{2g^2 D(x-y)}.
\]

\[\text{(8b)}\]
Generally, the bilocal field $B^\theta(x, y; \mu)$ can be written as

$$B^\theta(x, y; \mu) = B^\theta_0(x, y; \mu) + \sum_i \Gamma_i^\theta \phi_i^\theta \left( \frac{x + y}{2} \right),$$

(9)

- The fluctuation amplitude

$$\Gamma^\theta_{\sigma} = B^\theta_0(x, y; \mu),$$

(10a)

$$\Gamma^\theta_{\pi} = i\gamma_\mu \vec{r} B^\theta_0(x, y; \mu).$$

(10b)

- The vacuum configuration can be determined by the saddle-point condition $\frac{\partial S}{\partial B^\theta_0} = 0$. Consequently, one gets the quark self-energy

$$\Sigma(p, \mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D(x - y) \frac{t^a}{2} \gamma_\nu \frac{1}{i\gamma_\nu \vec{q} + \Sigma(q, \mu) \gamma_\nu \frac{t^a}{2}}.$$

(11)

where $\vec{q} = \{\vec{q}, q_4 + i\mu\}$, $q_4 = i\omega_n = i(2n + 1)\pi T$.
Taking the conventional decomposition for the quark self-energy

\[ G^{-1}(p, \mu) = iA(p, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(p_4 + i\mu) + VB(\vec{p}), \]  

\[ \Sigma(p, \mu) = i[A(p, \mu) - 1]\vec{\gamma} \cdot \vec{p} + i[C(p, \mu) - 1]\gamma_4(p_4 + i\mu) + VB(\vec{p}), \]  

where \( V = \sigma + i\vec{\pi} \cdot \vec{\tau} \gamma_5 \) with restriction \( \sigma^2 + \vec{\pi}^2 = 1 \).

The \( A, B \) and \( C \) can be fixed by solving the rainbow D-S Eqs

\[ [A(\vec{p})-1]\vec{p}^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{A(\vec{q})\vec{q} \cdot \vec{p}}{A^2(\vec{q})\vec{q}^2 + C^2(\vec{q})\vec{q}_4^2 + B^2(\vec{q})}, \]  

\[ [C(\vec{p})-1]p_4^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{C(\vec{q})\vec{q}_4 \vec{p}_4}{A^2(\vec{q})\vec{q}^2 + C^2(\vec{q})\vec{q}_4^2 + B^2(\vec{q})}, \]  

\[ B(\vec{p}) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{B(\vec{q})}{A^2(\vec{q})\vec{q}^2 + C^2(\vec{q})\vec{q}_4^2 + B^2(\vec{q})}. \]
(2) Simple Approximation

Expanding the Eq. (11) in power of \( \mu \), we have

\[
G[\mu] = G - \mu G \Gamma^{(1)} G - \frac{\mu^2}{2} [G \Gamma^{(2)} G - 2G \Gamma^{(1)} G \Gamma^{(1)} G] + \cdots, \tag{15}
\]

with

\[
\Gamma^{(1)}(p) = \frac{\partial G^{-1}[\mu](p)}{\partial \mu} \bigg|_{\mu=0}, \quad \Gamma^{(2)}(p) = \frac{\partial^2 G^{-1}[\mu](p)}{\partial \mu^2} \bigg|_{\mu=0}.
\]

To fix the expansion coefficients \( \Gamma^{(1)} \) and \( \Gamma^{(2)} \), we define other two functions as

\[
\Lambda^{(1)}(p) = \Gamma^{(1)}(p) - \frac{\partial G^{-1}(p)}{\partial (-ip_4)}; \quad \Lambda^{(2)}(p) = \Gamma^{(2)}(p) - \frac{\partial^2 G^{-1}(p)}{\partial(-ip_4)^2}.
\]

Applying operations $\partial/\partial\mu$ and $\partial/\partial(-ip_4)$ on both sides of Eqs. (4) and (11), respectively, we obtain

$$\Lambda^{(1)}(p) = -\frac{4}{3} \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu G(q)\Lambda^{(1)}(q)G(q)\gamma_\nu, \quad (16)$$

and

$$\Lambda^{(2)}(p) = -\frac{4}{3} \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu \{G(q)\Lambda^{(2)}(q)G(q)$$

$$-2[G(q)\Gamma^{(1)}(q)G(q)\Gamma^{(1)}(q)G(q)$$

$$-G(q)\frac{\partial G^{-1}(q)}{\partial (-iq_4)}G(q)\frac{\partial G^{-1}(q)}{\partial (-iq_4)}G(q)]\} \gamma_\nu. \quad (17)$$

Eq. (16) is just the meson Bethe-Salpeter Equation with ladder approximation.
Solutions of Eq.(16)

\[ <i> \quad \Lambda^{(1)}(p) = 0 \quad , \quad <ii> \quad \Lambda^{(1)}(p) = i \gamma_5 \frac{B(p^2)}{f_\pi}. \]

Our models

**model-1:**

\[ \Gamma^{(1)}(p) = \frac{\partial G^{-1}(p)}{\partial (-i p_4)}; \quad \Gamma^{(2)}(p) = \frac{\partial^2 G^{-1}(p)}{\partial (-i p_4)^2}. \]

**model 2:**

\[ \Gamma^{(1)}(p) = \frac{\partial G^{-1}(p)}{\partial (-i p_4)} + i \gamma_5 \frac{B}{f_\pi}; \quad \Gamma^{(2)}(p) = \frac{\partial^2 G^{-1}(p)}{\partial (-i p_4)^2} + i \gamma_5 E. \]
3. Relation Between the Chemical Potential and the Nuclear Matter Density

The pressure density

\[ P[\mu] = Tr \ln[G^{-1}] - \frac{1}{2} Tr[\Sigma G] \]

The nuclear matter density

\[ \rho = \frac{\partial P}{\partial \mu} = \frac{2}{3\pi^2} \mu^3 + 4 \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} [A(\vec{q}^2) - 1] \]
GCM, Global Color Symmetry Model: an effective field theory model of QCD

With the GCM, one can explore
the QCD foundation of bag models,
the chiral symmetry breaking and restoration,
the quark confinement and deconfinement, ••••••••••.
III. The $\sigma_{\pi N}$ in Free Space and in Nuclear Matter

1. In Free Space

According to Hellmann-Feynman theorem

$$\sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q}$$

Assuming $M_N = 3M_q$

we have

$$\sigma_{\pi N} = 3m_q \frac{\partial M_q}{\partial m_q}$$

The $M_q$ is the constituent quark mass and can be determined by the solution of the D-S equation

$$M_q = \frac{B}{A}$$
Taking
\[ \Gamma(p^2) = \frac{\partial G^{-1}(p)}{\partial m_q} \]
we have
\[ \Gamma(p, \mu) = 1 - \frac{4}{3} \int \frac{d^4 q}{(2\pi)^2} D_{\mu \nu} \gamma_\mu G(q, \mu) \Gamma(q, \mu) G(q, \mu) \gamma_\nu \]
It can be decomposed as
\[ \Gamma(p, \mu) = i \gamma \cdot p C(p^2) + D(p^2) \]
Then the chiral susceptibility can be given as
\[ \chi(p^2) = \frac{\partial M_q(p^2)}{\partial m_q} = \frac{A(p^2)D(p^2) - B(p^2)C(p^2)}{A^2(p^2)} \]
Taking \[ p^2 = M_q^2 \], we have
\[ \sigma_{\pi N} = 3m_q \left[ \chi(p^2 = M_q^2) \right]_{m_q \to 0} \]
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Taking the Munczek model of the effective gluon propagator

\[ D_{\mu\nu}(k) = 4\pi^2 \eta^2 \delta(k) \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \]

we have the solution of D-S equation at chiral limit

\[
\begin{align*}
A &= 2, \\
B &= \sqrt{\eta^2 - 4p^2}, \\
C &= -\frac{2}{\sqrt{\eta^2 - 4p^2}}, \\
D &= \frac{1}{2} \frac{\eta^2 + 8p^2}{\eta^2 - 8p^2},
\end{align*}
\]

\( \text{for } p^2 < \frac{\eta^2}{4} \)

\[
\begin{align*}
A &= \frac{1}{2} \left( 1 + \sqrt{1 + \frac{2\eta^2}{p^2}} \right), \\
B &= 0, \\
C &= 0, \\
D &= \frac{p^2 + \eta^2 + p^2 \sqrt{1 + \frac{2\eta^2}{p^2}}}{p^2 - \eta^2 + p^2 \sqrt{1 + \frac{2\eta^2}{p^2}}},
\end{align*}
\]

\( \text{for } p^2 > \frac{\eta^2}{4} \)

Then

\[
\chi(p^2) = \frac{3}{4} \frac{\eta^2}{\eta^2 - p^2} \quad \text{and} \quad \sigma = \frac{9}{2} m_q
\]
Beyond the chiral limit, the D-S equation should be solved numerically.
The deduced pion-nucleon sigma term reads
2. In Nuclear Matter

In the Munczek model, the solution of the D-S equation with finite chemical potential $\mu$ can be expressed in the same form as that in free space with replacement $p_4 \rightarrow p_4 + i\mu$. However, the boundary condition cannot be fixed directly, we take

$$\text{Re}[\tilde{p}^2 - \frac{\eta^2}{4}] < a\mu^2$$

Then

$$\sigma_{\pi N}(\mu) = \frac{9}{2} m_q \frac{\eta^2}{\eta^2 + 4\mu^2}$$
IV. Application to Quark Condensate

1. Direct Application

Model independent relation

\[
\frac{\langle \bar{q}q \rangle_{\rho}}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{2 \langle \bar{q}q \rangle_0} \frac{d \varepsilon}{dm_q}
\]

with approximation \( \varepsilon = \rho M_N + \delta \varepsilon \)

we have

\[
\frac{\langle \bar{q}q \rangle_{\rho}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_{\pi N}}{2m_q \langle \bar{q}q \rangle_0} \rho
\]

Numerical result:

Chang, Liu, Guo, hep-ph/0508192
2. Understanding the variation behavior of $\langle \bar{q}q \rangle_\mu$

\[ |\langle \bar{q}q \rangle_\mu| = Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} G(q). \]

With the $g[\mu]$ determined above, we have

\[ |\langle \bar{q}q \rangle| = a + c\mu + b\mu^2 \]

with

\[ a = Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} G(q), \]

\[ c = -Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} G(q)\Gamma^{(1)}(q)G(q), \equiv 0 \]

\[ b = -Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} \left\{ \frac{1}{2}G(q)\Gamma^{(2)}(q)G(q) \right. \]

\[ -G(q)\Gamma^{(1)}(q)G(q)\Gamma^{(1)}(q)G(q) \} , \]

Chang, Liu, et al., hep-ph/0508094
With effective gluon propagator

\[ D_{\mu\nu}(k) = t_{\mu\nu} 4\pi^2 d \frac{\chi^2}{k^4 + \Delta}, \]

where \( d = \frac{12}{27} \) and \( t_{\mu\nu} = \delta_{\mu\nu} \).

🔥 Calculated chiral quark condensate in vacuum and the response to the chemical potential

<table>
<thead>
<tr>
<th>( \Delta [\text{GeV}^4] )</th>
<th>( \chi [\text{GeV}] )</th>
<th>( a^{1/3} )</th>
<th>( b/a ) in model-1</th>
<th>( b/a ) in model-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-1} )</td>
<td>1.77</td>
<td>290</td>
<td>0.177</td>
<td>-4.073</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>1.33</td>
<td>250</td>
<td>0.325</td>
<td>-6.309</td>
</tr>
<tr>
<td>( 10^{-4} )</td>
<td>0.95</td>
<td>217</td>
<td>0.594</td>
<td>-10.913</td>
</tr>
<tr>
<td>( 10^{-6} )</td>
<td>0.77</td>
<td>205</td>
<td>0.782</td>
<td>-15.450</td>
</tr>
</tbody>
</table>

Parameters are taken from Phys. Lett. B 405, 8 (1997), with \( f_\pi \) being fixed as \( f_\pi = 87 \text{MeV} \).
The classical potential corresponding to the effective gluon propagator

\[ V(r) = -\frac{d\pi \chi^2}{r \sqrt{\Delta}} e^{-r \frac{4 \sqrt{\Delta}}{4}} \sin[r \frac{\sqrt{\Delta}}{4}]. \]
Variation of the condensate in vacuum: increase with the increasing of confinement strength

Physics meaning of the chiral quark condensate:
Mean spectral density at zero energy of quark over the gluon configurations and the four-volume,

Quantitative expression: Banks-Casher relation

\[ -\langle \bar{q}q \rangle = \pi \nu_q(0) \]

Confinement \leftrightarrow\text{attractive potential}
Increase of confinement strength
\text{attractive potential stronger} \quad \nu(0) \text{ increases}
Response to the chemical potential: positive in model-1; negative in model-2.

Increase of chemical potential

\[ \Delta \nu(0) > 0 \]

confinement more strong

Model-1: \[ \Delta \nu_q(0) = \Delta \nu(0) > 0 \]

The response is positive!

Model-2: \[ \Delta \nu_q(0) + \Delta \nu_\pi(0) = \Delta \nu(0) \]

the pion decreases \[ \Delta \nu(0) \]

\[ \Delta \nu_\pi(0) > 0 \]

The response is negative!
With effective gluon propagator

\[ D_{\mu\nu}(k) = \tau_{\mu\nu} 4\pi^2 \xi \frac{k^2}{\omega^2} e^{-k^2/\omega^2}, \]

where \( \tau_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu / k^2 \)

♦ Calculated chiral quark condensate in vacuum and the response to the chemical potential

<table>
<thead>
<tr>
<th>( \xi ) [GeV]</th>
<th>( \omega ) [GeV]</th>
<th>( a^{1/3} )</th>
<th>( b/a ) in model-1</th>
<th>( b/a ) in model-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.0</td>
<td>0.5</td>
<td>251</td>
<td>(-1.441 \times 10^{-5})</td>
<td>(-5.954)</td>
</tr>
<tr>
<td>25.0</td>
<td>0.45</td>
<td>246</td>
<td>(-1.396 \times 10^{-5})</td>
<td>(-6.571)</td>
</tr>
<tr>
<td>45.0</td>
<td>0.4</td>
<td>246</td>
<td>(-1.281 \times 10^{-5})</td>
<td>(-7.237)</td>
</tr>
</tbody>
</table>

Parameters are taken from Phys. Rev. D 65, 094026 (1997), with \( f_\pi \) being fitted as \( f_\pi = 93\text{MeV} \)
The classical potential corresponding to the effective gluon propagator

\[ V(r) = \frac{\sqrt{\pi} \xi \omega^3}{8} (\omega^2 r^2 - 6) e^{-\frac{\omega^2 r^2}{4}}. \]

An interaction with confinement and screening at middle distance and “asymptotic free” property at small distance.
Variation characteristics of the quark condensate:

♦ The condensate in the vacuum increases with the ascend of the confinement strength.

♦ The response in model-2 is negative.

♦ The response in model-1 is negative, due to the “asymptotic free” property.
V. Summary and Remarks

- The pion-nucleon sigma term in free space and in nuclear matter are studied in the GCM (truncated DSE).
- It is found that \[ \sigma_{\pi N}(\mu) = \frac{9}{2} m_q \frac{\eta^2}{\eta^2 + 4\mu^2} \] in the Munczek model of effective gluon propagator.
- The chiral quark condensate decreases slowly than the linear behavior as the density dependence of the \( \sigma_{\pi N} \) is taken into account.
- The decreasing characteristic of the chiral quark condensate is attributed to the pion configuration.
Remark (2)

- The quark confinement strengthens the chiral quark condensates.
- The difference of the variation behavior of the condensate arises from the difference of the configuration of the vacuum.
- The vacuum may consist both quark condensate and Goldstone bosons.
Remark (3)

- The dependence of the effective gluon propagator on the chemical potential has not yet been taken into account.
- Quantitative description of the variation rate of the response against the confinement strength is expected.
- The chiral quark condensate may not be a good characteristic to identify the restoration process.
Thanks !!!