Reaction Models of Electromagnetic Meson Production in the Nucleon Resonance Region

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Data of $\gamma p$ reaction cross sections
• Challenge:

**Extensive data** of electromagnetic production of
π, η, K, ω, φ, and ππN (ρN, πΔ)

↓↓

Understand the structure of nucleon resonances (N* )

↓↓

Understand non-perturbative QCD:
– Confinement of constituent quarks
– Chiral dynamics of meson cloud of baryons
- Traditional practice:
  Amplitude Analyses of data
  \[ \rightarrow \]
  Extract \( N^* \) parameters
  No interpretation of \( N^* \) parameters

- Theoretical task:
  Develop Dynamical Reaction Models
  \[ \rightarrow \]
  Extract \( N^* \) parameters
  Also attempt to interpret \( N^* \) parameters in terms of QCD:

  - Hadron Models (now)
  - Lattice QCD (near future)
Question:
Why do we need reaction models ??
(difficult and complicated)

Answer:
Many-year’s experiences in nuclear physics

Herman Feshbach:
”Much of the present-day understanding of nuclear structure has been gained from the study of nuclear reactions. For this purpose it is necessary to understand the dynamics of nuclear reactions, while at the same time methods must be developed that permit the extraction of nuclear structure information.”
Develop reaction models
to separate *reaction mechanisms* from *nuclear structure*

(Optical potentials, DWIA, Coupled-channel, multiple-scattering etc.)

→

Replace *nuclear* → *hadron*
Similar situation in the study of $N^*$ structure

→

Develop *reaction models*
to separate *reaction mechanisms* from *hadron structure*
Question:

What could be wrong in using the information from amplitude analyses (Particle Data Group) to test theoretical models of $N^*$ structure?

- Analyses are often guided by numerical simplicity in many-parameters fits

→

Answer:

Results from the study of $N$-$\Delta$ transitions in the past 10 years
In a dynamical reaction model of $\gamma N \rightarrow \pi N$:

$$\tilde{\Gamma}_{\gamma N \rightarrow \Delta} = \Gamma_{\gamma N \rightarrow \Delta} + \tilde{\Gamma}_{\pi N \rightarrow \Delta} G_{\pi N}(E) v_{\gamma \pi}$$

Quark Model Prediction

pion cloud effect
\( \gamma N \rightarrow \Delta \) Magnetic Dipole \( G_M(Q^2) \)

- Pion cloud has a very large effect on \( G_M \); about 40% at \( Q^2 = 0 \)
- At \( Q^2 = 0 \), \( G_M^{\text{bare}} \sim G_M^{SU(6)} \sim 2. \)

\[ \rightarrow \]

Resolve a long-standing puzzle!!
Possible Plan

- QCD
- LQCD
- Data
- Models
- Reaction Theory
- Amplitude Analyses
- N - N^*
This talk:

- Review the current reaction models of meson production reactions
- Report the status of a reaction model being developed by Argonne-Osaka-Schizuoka collaboration
  (A. Matsuyama, T. Sato, T.-S. H. Lee)
General Consideration

Question:

What is the structure of $N^*$?

Theoretical guidance:

Chiral symmetry in QCD is broken spontaneously

$\rightarrow$

$N^*$ can be described in terms of constituent quarks and meson clouds

$\rightarrow$

$|N^*\rangle = |N^*_0\rangle + |N^*_0\pi\rangle + |N^*_0\pi\pi\rangle + \cdots$

$|N^*_0\rangle = |qqq\rangle$

$q = \text{constituent quarks}$
A tractable reaction model is based on:

- each quark core ($N_0^*$) is treated as an elementary field $\psi_B$
- $N_0^*$ structure is defined by form factors

Data $\rightarrow$ [form factors] $\leftarrow$ hadron models/Lattice QCD

Starting Interaction Lagrangian:

$$L_I = \sum_{B,B'} \sum_M \bar{\psi}_{B'} [\Gamma^0 \phi_M] \psi_B$$

- $\psi_B =$ bare $N, \Delta, N_1^*, N_2^* \cdots$
- $\phi_M = \gamma, \pi, \eta, K, \omega, \phi \cdots$
- $\Gamma^0$ must be consistent with chiral symmetry:

$$L_I \sim \bar{\psi}_N [\gamma_5 \gamma^\mu \partial_\mu \phi_\pi] \psi_N + \cdots$$
Approximations:

- **Ladder** Bethe-Salpeter equations
  - I. Afnan and collaborators
  - N. Kaiser, E. Oset, M. Lutz et al

- **Three-dimensional ladder** Bethe-Salpeter equations
  - Julich coupled-channel model
  - F. Gross and Y. Suyra
  - Dubna-Mainz-Taipei (MAID-DMT) model (S.N. Yang)
  - V. Pascalutsa, J. Tjon, G. L. Caia, L. Wright
  - Many earlier $\pi N$ models

- **Unitary** transformation method
  - T. Sato, A. Matsuyama, and T.-S. H. Lee
  - M. Fuda
Focus on:

Formulation based on *unitary transformation method*

→

- Derive most of the current *reaction models*
Method of Unitary Transformation

- Start with a Lagrangian $L(x)$ of relativistic quantum field theory
- Apply the canonical quantization to define Hamiltonian density

$$h(x) = \sum_B \pi_B(x) \partial_0 \psi_B(x) + \sum_M \pi_M(x) \partial_0 \phi_M(x) - L(x)$$

- Define Hamiltonian in Fock-space

$$H = \int d\vec{x} h(\vec{x}, t = 0)$$

- Apply unitary transformation to derive $H_{eff} = U^\dagger HU$ which leads to

Soluble few-body scattering equations
Example: \( L_I = \bar{\psi}_{B'} \Gamma^0 \phi_M \psi_B \) \( (B = N, \Delta, M = \pi, \gamma) \)

→ Hamiltonian:

\[
H = H_0 + H_I + H_{em},
\]

with

\[
H_I = \sum_{B, B'} \Gamma^0_{\pi B', B}
\]

\[
H_{em} = \int d\mathbf{x} A \cdot J
\]

\[
J^\mu = J^\mu_\pi + J^\mu_{B', B} + J^\mu_{B', B, \pi},
\]
• Decompose interaction term:

\[ H_I = H_1^P + H_1^Q, \]

Physical process: \( \Delta \leftrightarrow \pi N \)

\[ H_1^P = \Gamma_{\pi N,\Delta}^0 \]

Unphysical process: \( N \leftrightarrow \pi N, N \leftrightarrow \pi \Delta, \Delta \leftrightarrow \pi \Delta \)

\[ H_1^Q = \Gamma_{\pi N,N}^0 + \Gamma_{\pi \Delta,N}^0 + \Gamma_{\pi \Delta,\Delta}^0 \]
- Introduce unitary transformations

\[ U_n = \exp(iS_n) \]
\[ S_n \propto (H_I)^n \]

\[ H^{(n)} = U_n^\dagger U_{n-1}^\dagger \cdots H \cdots U_{n-1} U_n \]
\[ = H_{eff}(g^1, \cdots, g^n) + \sum_{m>n} [H^P(g^m) + H^Q(g^m)] \]

\( H_{eff}(g^1, \cdots, g^n) : \text{no unphysical processes (} H_I \propto g \)
Consider $n=2$:

\[ H = H_{\text{eff}}(g^1, g^2) = H_0 + V \]
\[ V = v^{bg} + \sum_{N_i^*}[\Gamma_i + \Gamma_i^\dagger] \]

$v^{bg}$: Non-resonant $MB \rightarrow M'B'$

$\Gamma_i$: $N^* \rightarrow MB$

\[ T(E) = V + V \frac{1}{E - H + i\epsilon} V \]

Main Feature: $V$ is energy-independent and hermitian

\[ \rightarrow \]

Unitarity condition is trivially satisfied.
$n=2$ interactions:

$\nu^{bg} : \text{Non-resonant}$

$\nu^R = \frac{\Gamma_i^\dagger \Gamma_i}{E - M_{N^*_i}} : \text{Resonant}$
Can derive

- Unitary Isobar Models:
  - MAID
  - Jlab/Yerevan UIM
- Multi-channel K-matrix models:
  - SAID
  - Giessen
  - Kent State University (KSU)
- Carnegie-Mellon Berkeley (CMB) Model
- Dynamical reaction models
Starting point:

- Relation between scattering $T$ and $K$ matrix:

\[
T(E) = V + V\left[\frac{P}{E - H_0} - i\pi\delta(E - H_0)\right]T(E)
\]

\[
K(E) = V + V\frac{P}{E - H_0}K(E)
\]

$P$ : the principal-value integration.

\[
\rightarrow
\]

\[
T(E) = K(E) - T(E)[i\pi\delta(E - H_0)]K(E)
\]

\[
\rightarrow
\]

Lead to on-shell relations between $T$ and $K$
Approaches:

- Start with $V = \nu^b + \nu^R$:

$$T_{a,b}(k_a, k_b, E) = V_{a,b}(k_a, k_b)$$

$$+ \sum_c \int \frac{d^k}{E - E_{M_c}(k) - E_{B_c}(k)} \frac{V_{a,c}(k_a, k)T_{c,b}(k, k_b)}{E + i\epsilon}$$

$a, b = \pi N, \gamma N, \eta N, \omega N, KY, \rho N, \pi \Delta, \sigma N$ (represent $\pi \pi N$)

- Need off-shell information
- Equations for Dynamical Models
• Start with $K$ matrix:

A matrix relation in partial-wave representation:

$$T_{a,b}(E) = \sum_c [(1 + iK(E))^{-1}]_{a,c}K_{c,b}(E)$$

$a, b = \pi N, \gamma N, \eta N, \omega N, KY, \rho N, \pi \Delta, \sigma N$ (represent $\pi \pi N$)

– Need only on-shell information
– Equations for K-matrix Models
Derivations

- Unitary Isobar Model (UIM):
  - start with $K$ matrix
  - channels: $\gamma N, \pi N$ (or $\eta N$)
  $$
  \gamma N \rightarrow \pi N \text{ amplitude:}
  $$
  \begin{align*}
  T_{\pi N, \gamma N} &= [1 + iK_{\pi N, \pi N}(E)]^{-1}K_{\pi N, \gamma N}(E) \\
  &= e^{i\delta_{\pi N}} \cos\delta_{\pi N} K_{\pi N, \gamma N}(E) \\
  \sim e^{i\delta_{\pi N}} \cos\delta_{\pi N} V_{\pi N, \gamma N}
  \end{align*}
  $$

$V_{\pi N, \gamma N} = \text{Tree-diagrams}$

$\delta_{\pi N} : \pi N$ phase shifts

→

Satisfy Watson Theorem in $W < 1.3$ GeV
– MAID and Jlab/Yerevan UIM:

1. Include of $N^*$ by using Walker’s parameterization
2. Unitarize the total amplitude

\[ T_{\pi N, \gamma N}(UIM) = e^\delta cos\delta[v_{\pi N, \gamma N}^{bg}] + \sum_{N_i^*} T_{\pi N, \gamma N}^{N_i^*}(W) \]

\[ T_{\pi N, \gamma N}^{N_i^*}(E) = f_{\pi N}(W) \frac{\Gamma^{tot} M_i \epsilon^{i \Phi_i}}{M_i^2 - W^2 - i M_i \Gamma^{tot}} A_{\gamma N}(W) \]

$\Phi_i$ : Unitarization Phase
Results from MAID and JLab/Yerevan UIM:
1. Successful in extracting $\Delta$ parameters
2. Can fit pion production data up to $W=2$ GeV
3. More will be discussed in I. Aznauryan’s talk

Comments:
Coupled-channel effects are not treated explicitly

\[
\gamma N \rightarrow (\pi \Delta, \rho N \cdot \cdot) \rightarrow \pi N \text{ is neglected }
\]

The extracted $N^*$ parameters in the second and third resonance regions need to be verified
- Multi-channel K-matrix models

- **SAID**:

Consider $\gamma N, \pi N, \pi \Delta$ (all inelastic channels)

\[ T_{\gamma N, \pi N}(SAID) = A_I (1 + iT_{\pi N, \pi N}) + A_R T_{\pi N, \pi N} \]

\[ A_I = K_{\gamma N, \pi N} - \frac{K_{\gamma N, \pi \Delta} K_{\pi N, \pi N}}{K_{\pi N, \pi \Delta}} \]

\[ A_R = \frac{K_{\gamma N, \pi \Delta}}{K_{\pi N, \pi \Delta}} \]
Actual analysis:

\[ A_I = \nu_{\gamma N, \pi N} + \sum_{n=0}^{M} \bar{p}_n z Q l_\alpha + n(z) \]

\[ A_R = \frac{m_\pi}{k_0} \left( \frac{q_0}{k_0} \right)^{t_\alpha} \sum_{n=0}^{N} p_n \left( \frac{E_\pi}{m_\pi} \right)^n \]

\( \bar{p}_n, p_n \): fitting parameters

\( N^* \) parameters are extracted by fitting the resulting amplitudes to a Briet-Wigner parameterization at \( W \rightarrow M^* \)
Results from SAID:

* determine $\pi N \to \pi N$ amplitudes
* determine $\gamma N \to \pi N$ multipole amplitudes
* extract $N^*$ parameters

Comments:

Its many-parameter parameterization of the non-resonant amplitudes need to be justified theoretically.

Coupled-channel effects are not treated explicitly.

The extracted $N^*$ parameters in the second and third resonance regions need to be verified.
– **Giessen Model**:  
Approximation: \( K = V = \text{Tree-diagrams} \)  
\[ T_{a,b}(\text{Giessen}) = \sum_c [(1 + iV(E))^{-1}]_{a,c} V_{c,b}(E) \]

**Results:**
* Fit both \( \pi N \) and \( \gamma N \) reaction data with channels: \( \gamma N, \pi N, \sigma N, \eta N, K\Lambda, K\Sigma \) and \( \omega N \).  
* Identify \( N^* : P_{31}(1750), P_{13}(1900), P_{33}(1920), D_{13}(1950) \)

**Comments:**  
**Multiple-scattering** effects in \( K \) matrix is neglected  
\[ \rightarrow \]
The extracted \( N^* \) parameters in the second and third resonance regions need to be **verified**
For deriving:

- Carnegie-Mellon Berkeley (CMB) Model
- Kent State University (KSU) model
- Dynamical models

Apply two-potential scattering formulation

for \( V = v^{bg} + \frac{\Gamma_{N*}^\dagger \Gamma_{N*}}{E - M_{N*}^0} \)

\[ T(E) = t^{bg}(E) + \frac{\tilde{\Gamma}_{N*}^\dagger(E)\tilde{\Gamma}_{N*}(E)}{E - M_{N*}^0 - \Sigma_{N*}(E)} \]

\[ t^{bg} = v^{bg} + v^{bg}G(E)t^{bg}(E) \]
Resonances are determined:

\[ \tilde{\Gamma}_{N^*} = \Gamma_{N^*} + \Gamma_{N^*} G(E)t^{bg}(E) \]

\[ \Sigma_{N^*}(E) = \Gamma_{N^*}^\dagger G(E)\tilde{\Gamma}_{N^*} \]

Main feature:

- Non-resonant effects on resonance parameters are identified
For multi-channel multi-resonances case:

\[
T_{a,b}(E) = t^{bg}_{a,b}(E) + \sum_{N_i^*,N_j^*} \Gamma_{N_i^*,a}^\dagger(E)[\hat{G}(E)]_{i,j} \Gamma_{N_j^*,b}(E)
\]

\[
t^{bg}_{a,b} = v^{bg}_{a,b} + \sum_c v^{bg}_{a,c} G_c(E) t^{bg}_{c,b}(E)
\]

\[
\Gamma_{N^*,a} = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t^{bg}_{b,a}
\]

\[
[\hat{G}(E)^{-1}]_{i,j}(E) = (E - M^0_{N_i^*}) \delta_{i,j} - \Sigma_{i,j}(E)
\]

\[
\Sigma_{i,j}(E) = \sum_a \Gamma_{N^*,a}^\dagger G_a(E) \Gamma_{N_j^*,a}
\]

\(a, b = \gamma N, \pi N, \eta N, \omega N, KY, \pi \Delta, \rho N, \sigma N \) (for \(\pi \pi N\))
Carnegie-Mellon Berkeley (CMB) Model

Set: \( v_{a,b}^{bg}(E) = \frac{\Gamma^\dagger_{L,a} \Gamma_{L,b}}{E - M_L} + \frac{\Gamma^\dagger_{H,a} \Gamma_{H,b}}{E - M_H} \) (separable form)

\[ V = v^{bg} + v^R = \sum_{i=N^*_i, L, H} \frac{\Gamma^\dagger_{i,a} \Gamma_{i,b}}{E - M_i} = \text{Separable} \]

\[ T_{a,b}(E) = \sum_{i,j} \Gamma^\dagger_{i,a} G_{i,j}(E) \Gamma_{j,b} \]

\[ G(E)_{i,j}^{-1} = (E - M^0_i) \delta_{i,j} - \sum_{i,j} (E) \]

\[ \Sigma_{i,j}(E) = \sum_a \int k^2 dk \frac{\Gamma^\dagger_{i,a}(k) \Gamma_{j,a}(k)}{E - E_{Ma}(k) - E_{Ba}(k) + i\epsilon} \]
With appropriate variable changes: \( s = E^2 \)

\[ \Sigma_{i,j}(s) = \sum_c \gamma_{i,c} \Phi_c(s) \gamma_{j,c} \]

\[ \text{Re}[\Phi_c(s)] = \text{Re}[\Phi_c(s_0)] + \frac{s - s_{th,c}}{\pi} \int_{s_{th}}^{\infty} \frac{\text{Im}[\Phi_c(s')]}{(s' - s)(s' - s_0)} ds' \]

\[ \rightarrow \]

CMB model is analytic
Recent applications/extensions of CMB model:

- **Zagreb**: M. Batinic, A. Svarc and collaborators

  Consider **three channels**: $\pi N$, $\eta N$, $\sigma(\pi\pi)N$

- **PITT-ANL**: T. Varana, S. Dytman, T.-S. H. Lee

  Consider up to **eight channels**:

  $\pi N$, $\eta N$, $\pi\Delta$, $\rho N$, $\sigma(\pi\pi)N$, $\pi N^*(1440)$, $K\Lambda$, $\gamma N$
Results:

- $N^*$ in $S_{11}$ channel is better understood
- The interplay between channel coupling and $N^*$ excitation has been better understood
- Some extracted $N^*$ parameters are very different from PDG values
  - be careful in using PDG's values to test hadron models

Comments:

Its separable non-resonant amplitudes need to be justified theoretically

The extracted $N^*$ parameters in the second and third resonance regions need to be verified
• Kent State University (KSU) model

Start with

\[ T(E) = t^{bg}(E) + \frac{\bar{\Gamma}^{\dagger}_{N^*} \bar{\Gamma}_{N^*}}{E - M^{0}_{N^*} - \Sigma_{N^*}(E)} \]

One can derive exactly the **distorted-wave** form

\[ S(E) = 1 + 2iT(E) \]

\[ = \omega^{(+)}T[1 + 2i \frac{\Gamma^{\dagger}_{N^*}(E)\Gamma_{N^*}}{E - M^{0}_{N^*} - \Sigma_{N^*}(E)}]\omega^{(+)\dagger} \]

where

\[ \omega^{(+)} = 1 + G(E)t^{bg}(E) \]

(1)
\[ S(E) = \omega^{(+)} T^R R(E) \omega^{(+)} \]

\[ R(E) = 1 + 2i T^R(E) \]

\[ T^R(E) = \frac{\Gamma^\dagger_{N^*}(E) \Gamma_{N^*}(E)}{E - M^0_{N^*} - \Sigma_{N^*}(E)} \]

KSU separable parameterization:

\[ T^R(E) = \frac{K}{1 + iK} \sim x \frac{\Gamma/2}{E - M - i\Gamma/2} \]

\[ \omega^{(+)} = B_1 B_2 \cdots B_n \]

\[ B_i \sim e^{iX \Delta_i} \]

\( \Gamma, x \) and \( X \) are parameters in the fit
Results from KSU:

- fits to $E_{0+}$ of $\gamma N \rightarrow \pi N$
- being applied to study kaon production

Comments:

Its separable parameterization of non-resonant amplitude need to be justified theoretically

→

The extracted $N^*$ parameters in the second and third resonance regions need to be verified
Dynamical Models

Two equivalent approaches:

- Solve dynamical equations with \( V = v^{bg} + v^R \) directly:

\[
T_{a,b}(E) = V_{a,b} + \sum_c V_{a,c} G_c(E) T_{c,b}(E)
\]

\( a, b, c = \pi N, \gamma N, \eta N, \pi \Delta \cdots \)

Recent works:

- Julich Model : \( \pi N \)
- Fuda et al. : \( \pi N, \gamma N \)
- DMT Model : \( \pi N, \gamma N, \eta N \) (S.N. Yang’s talk)
- Ohio-Utrecht Model : \( \pi N, \gamma N \) (V. Pascalutsa’s talk)
- Chiral SU(3) models : \( KY, \omega N, \gamma N, \pi N \) (M. Lutz’s talk)
Use two-potential formulation to identify resonant mechanism

\[
T_{a,b}(E) = t_{a,b}^{bg}(E) + \sum_{N_i^*,N_j^*} \tilde{\Gamma}_{N_i^*,a}^{\dagger}[D^{-1}(E)]_{i,j} \tilde{\Gamma}_{N_j^*,b}
\]

\[
t_{a,b}^{bg}(E) = \nu_{a,b}^{bg} + \sum_c \nu_{a,c}^{bg} G_{c}(E) t_{c,b}^{bg}(E)
\]

\[
\tilde{\Gamma}_{N^*,a} = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_{b}(E) t_{b,a}^{bg}(E)
\]

Recent Works

- Sato-Lee Model: $\pi N$, $\gamma N$
- Yoshimoto et al.: $\pi N$, $\eta N$, $\pi \Delta$
- Oh et al.: $\gamma N$, $\omega N$
- Julia-Diaz et al.: $\gamma N$, $K Y$, $\pi N$ (B. Julia-Diaz’s talk)
- Lee, Matsuyama, Sato (2004): $\pi N$, $\eta N$, $\gamma N$
DMT model (S.N. Yang’s talk)

Ohio-Utrecht Model (V. Pascalutsa’s talk)

Kaon production (B. Julia-Diaz’s talk)

Chiral SU(3) models (M. Lutz’s and E. Kolomeitsev’s talk)
Julich’s Coupled-channel Model


- **Channels**: $\pi N, \eta N, \sigma N, \pi \Delta, \rho N$.
- **$V$**: meson-exchange, s-channel $N^*$
- **fit**: $\pi N$ amplitudes up to 1.9 GeV
- **Main result**: $P_{11}$ is due to meson-baryon coupled-channel effects

Comments:

1. It does not satisfy $\pi \pi N$ unitarity condition
2. Need to check its predictions of $\pi N \rightarrow \pi \pi N$ and $\gamma N$ cross sections

Question on the nature of $P_{11}$ is still open
Coupled-channel study of $N^*$ in $S_{11}$

- **Channels**: $\pi N, \eta N, \gamma N$
- **Non-resonant int.**: tree-diagram of chiral Lagrangian
  \[ \mathcal{L}_{\pi N,\pi N}, \mathcal{L}_{\pi N,\eta N}, \mathcal{L}_{\eta N,\eta N} \]
  \[ \mathcal{L}_{\gamma N,\pi N}, \mathcal{L}_{\gamma N,\eta N} \]
- **2 $N^*$**: Related to constituent quark model

- **Finding**:
  1. Can describe the data only up to $W = 1.7$ GeV
  2. **Meson cloud** effect on $\gamma N \rightarrow N^*$ is about 20%
  3. Bare helicity amplitude is close to quark model

→

Consistent with the finding in the study of $\Delta$ excitation
\( \pi N \rightarrow \pi N \) amplitude

**T (S11)**
Two N*, Channels: pi N, eta N
\( \gamma N \rightarrow \pi N \) amplitude

Data from SAID

Re (p E0+)
Two N*, Channels: \( \pi N \), \( \eta N \)

Im (p E0+)
Two N*, Channels: \( \pi N \), \( \eta N \)
$\pi N \rightarrow \eta N$ amplitude

$\text{Re } T(\pi N, \eta N)$
Two $N^*$, Channels: $\pi N, \eta N$

$\text{Im } T(\pi N, \eta N)$
Two $N^*$, Channels: $\pi N, \eta N$
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Comments:

To explore $N^*$ from all $\pi N$ and $\gamma N$ data up to $W = 2.5$ GeV, we need to include coupling with $\pi\pi N$ channel

Develop coupled-channel model with $\pi\pi N$ channels

Argonne-Osaka-Shizuoka collaboration

A. Matsuyama, T. Sato, T.-S. H. Lee (in progress)
Coupled-channel model with $\pi\pi N$

\[ \rightarrow \]

Apply second-order unitary transformation method to derive $H_{eff}(g^2, g^3)$

$g$ : strong coupling constant of $H_I$

\[ H'' = U_2^\dagger [H'] U_2 \]

\[ H' = U_1^\dagger H U_1 \]
\[ = H_{eff}(g^2) + H'^Q_I(g^2) + H'^P_I(g^3) + H'^Q_I(g^3) + \cdots \]

Note:

SL model : $H_{eff}(g^2)$ in $\Delta \oplus \gamma N \oplus \pi N$
Evaluate $H_{eff}(g^2, g^3)$ in $MB \oplus \pi\pi N$

$\rightarrow$

Coupled-channel model with $\pi\pi N$

\[
H_{eff} = H_0 + H_I
\]

\[
H_I = \Gamma_V + v_{2\leftrightarrow 2} + v_{2\leftrightarrow 3} + v_{3\leftrightarrow 3}
\]

with

\[
\Gamma_V = \sum_{N^*} \sum_{MB} [\Gamma_{N^*\leftrightarrow MB}] + h_{\rho\leftrightarrow\pi\pi} + h_{\sigma\leftrightarrow\pi\pi}
\]

- $MB$: $\gamma N, \pi N, \eta N, \pi\Delta, \rho N, \sigma N$

- $\Gamma_V$: bare vertex interactions
Non-resonant interactions

\[ v_{2 \leftrightarrow 2} = \sum_{M B, M'B'} v_{M B, M'B'} + v_{\pi\pi} \]  

\[ (2) \]

\[ v_{M B, M'B'} : \text{meson-baryon interactions} \]

\[ v_{\pi\pi} : \pi\pi \text{ interaction.} \]

\[ v_{2 \leftrightarrow 3} : \pi N \leftrightarrow \pi\pi N \]

\[ v_{3 \leftrightarrow 3} : \pi\pi N \leftrightarrow \pi\pi N \]

\[ v_{\pi\pi}, h_{\rho \leftrightarrow \pi\pi}, h_{\sigma \leftrightarrow \pi\pi} : \text{from } \pi\pi \text{ scattering} \]
$v_{2\leftrightarrow 3}$:

$iS H_1^Q iS$

$H_1^Q iS iS$

$H_1^Q iS iS$

$v_{3\leftrightarrow 3}$:

$H_1^Q S'$

$S' H_1^Q$

$S' H_1^Q$
where

\[ T_{\gamma N, \pi \pi N}(E) = T_{\gamma N, \pi \Delta}(E) G_{\pi \Delta} \Gamma_{\Delta, \pi N} \]
\[ + T_{\gamma N, \rho N}(E) G_{\rho N} h_{\rho, \pi \pi} \]
\[ + T_{\gamma N, \sigma N}(E) G_{\sigma N} h_{\sigma, \pi \pi} \]

and

\[ T_{\gamma N, MN}(E) = t_{\gamma N, MN}(E) + \sum_{N^*} \frac{\bar{\Gamma}_{\gamma N \rightarrow N^*}(E) \bar{\Gamma}_{N^* \rightarrow MN}(E)}{E - M_{N^*}^0 - \sum_{N^*}(E)} \]

\[ G_{MB}(E) = \frac{1}{E - E_B - E_M - \Sigma(E)} \]

\[ \Sigma(E) = \text{Self-energy of the unstable } \Delta, \rho, \text{ and } \sigma \]
The dressed vertices are

\[ \bar{\Gamma}_{N^* \rightarrow MB} = \Gamma_{N^* \rightarrow MB} + \sum_{M'B'} \Gamma_{N^* \rightarrow M'B'} G_{M'B'}(E) X_{M'B', MB} \]

\[ \uparrow \]

\[ \text{Bare} \]

Non-resonant amplitudes are:

\[ X_{MB, M', B'}(E) = Z_{MB, M'B'}(E) \]

\[ + \sum_{M''B''} Z_{MB, M''B''}(E) G_{M''B''}(E) X_{M''B'', M'B'}(E) \]
The channel-coupling interactions are:

\[ Z_{MB,MB'} = \nu_{MB,MB'}(E) + Z^{(\pi\pi N)}_{MB,MB'}(E) \]  \hspace{1cm} (3)

Three-body $\pi\pi N$ cut is in

\[ Z^{(\pi\pi N)}_{MB,MB'}(E) = Z^{(E)}_{MB,MB'}(E) + Z^{(I)}_{MB,MB'}(E) \]
Example:

\[ Z_{MB,M'B'}^{(E)}(E) = < MB | \Gamma_V \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} \Gamma_V | B'M' > \]

\[ Z_{MB,M'B'}^{(I)}(E) = < MB | \Gamma_V \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} t_{\pi\pi N,\pi\pi N}(E) \frac{P_{\pi\pi N}}{E - H_0 + i\epsilon} \Gamma_V | M'B' > \]

→

Apply the Spline-function expansion method to solve the $\pi\pi N$ coupled-channel equations on real-axis

* The method was developed in $NN \rightarrow \pi NN$ studies by Matsuyama and Lee (1986)
First Results from Unitary $\pi\pi N$ calculations

**Objective:** explore the starting parameters

- $h_{\rho,\pi\pi}$ and $h_{\sigma,\pi\pi}$ from fitting $\pi\pi$ phase shifts (by Johnstone and Lee)
- $\nu_{2,2}$ are calculated using the coupling constants of Julich’s model

$v_{a,b}$

where $a, b = \gamma N, \pi N, \eta N, \rho N, \pi \Delta, \sigma N$

have been constructed

- $N^* \rightarrow \gamma N, \pi N, \eta N, \rho N, \pi \Delta$ are taken from the $^3P_0$ model of Capstick and Roberts and/or from PDG
$d\sigma/dM_{\pi N}$ of $\pi^+ p \rightarrow \pi\pi N$

$\pi^+ p \rightarrow \pi\pi N$

Unitary $\pi\pi N$ Calculation

$\pi^+ p \rightarrow \rho N \rightarrow \pi\pi N$

$\pi^+ p \rightarrow \pi \Delta \rightarrow \pi\pi N$
$d\sigma/dM_{\pi\pi}$ of $\pi^+p \rightarrow \pi\pi N$

Unitarity $\pi\pi N$ calculations

$\pi^+p \rightarrow \pi\pi N$

$\pi^+p \rightarrow \rho N \rightarrow \pi\pi N$

$\pi^+p \rightarrow \pi\Delta \rightarrow \pi\pi N$
Effect of coupled-channel and $\pi\pi N$ cut on amplitudes

$$\pi N(P_{11}) \rightarrow \pi\Delta (P_{11})$$

$W = 1750$ MeV

\[ X = Z + Z G X \]

$Z = v_{bg} + Z^{(\pi\pi N)}$

$Re (X)$

$p$ (MeV)

$Re (X)$
Effect of coupled-channel and $\pi \pi N$ cut on amplitudes

$$\pi N(P_{11}) \rightarrow \pi \Delta (P_{11})$$

$W = 1750$ MeV

$Z = v_{bg} + Z^{(\pi \pi N)}$

Breakup threshold
Effect of $\pi\pi N$ cut

$\pi^+ p \rightarrow \pi\pi N$

Effects of $\pi\pi N$ cut

Unitary $\pi\pi N$ cal.

No Z-diagram

$\pi\pi N$ cut
Effect of $\pi\pi N$ cut

$\pi^+ p \rightarrow \pi\pi N$

Unitary $\pi\pi N$ cal.

No Z - Diagram

$\rho$ $\pi$

$N$ $\Delta$
\[ \gamma p \rightarrow \pi N, \eta N, \pi\pi N(\pi\Delta, \rho N) \]
\[ \gamma p \rightarrow \pi N, \eta N, \pi \pi N(\pi \Delta, \rho N) \]
Fits to $\pi N$ scattering amplitudes

$S_{11}$:

Re $T$ (S11)\newline
2 N*, Channels: pi-N, eta-N, pi-Delta, rho-N

Im $T$ (S11)\newline
Two N*, Channels: pi-N, eta-N, pi-D, rho-N
$P_{11}$:

**Re $T (P_{11})$**

2 $N^*$, Channels: pi-N, eta-N, pi-Delta, rho-N

**Im $T (P_{11})$**

2 $N^*$, Channels: pi-N, eta-N, pi-Delta, rho-N
If coupled-channel multiple scattering and $\pi\pi N$ cut structure is neglected

- Moscow/Jlab Isobar Model of $\gamma N \rightarrow \pi\pi N$
  (V. Mokeev’s talk)
- Amplitude analyses of $\gamma N \rightarrow \pi\pi N$ of RPI/JLab group
Concluding Remarks

- We have developed a dynamical approach for investigating $N^*$ in $\pi N$ and $\gamma N$ reactions.

- It is tractable and systematic in getting $H_{eff}$ from relativistic quantum field theory.

- The model in the $\Delta$ region can describe most of the current data.

- Numerical methods for solving coupled-channel model including $\pi\pi N$ has been well developed with some preliminary results.

- $\pi\pi N$ unitarity cut is crucial in predicting $\pi\pi N$ production cross sections and identifying ”missing” resonances.

- Much more work is needed to carry out complete coupled-channel analyses of all meson production data.