Study of Fragmentation Functions in e^+e^- Annihilation

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Abstract

This White Paper is focused on studies of single and dihadron multidimensional fragmentation functions in e^+e^- annihilation, and their role in studies of the 3D structure of nucleons and nuclei. This document is a result of a community-wide effort in nuclear science following a series of workshops on physics of the 3D structure of hadrons over the past few years. It emphasizes the relevance of activities on hadronization studies at Belle and BaBar for user communities of BNL and JLab.

We summarize the crucial ingredients needed to accomplish the QCD science program in the U.S., established with the CEBAF accelerator at JLab and the future EIC, as well as BNL, CERN and JLab experiments. We give examples of the importance of recent measurements at Belle and BaBar for our understanding of the nucleon structure and outline what a program at present and future facilities can contribute to our knowledge of fragmentation functions.

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I. INTRODUCTION

Fragmentation functions describe the phenomenon of hadronization, i.e., how quarks or gluons transform into hadrons before ever being detected as free particles. They are essential manifestations of Quantum Chromodynamics (QCD) and confinement. The present White Paper outlines the current and future opportunities on measurements of fragmentation functions, emphasizing their relevance for the study of the structure of the nucleon.

Fragmentation functions appear in electron-positron annihilation, but also in semi-inclusive DIS (SIDIS) and proton-proton collisions to hadrons, where they are combined with parton distribution functions (PDFs). Electron-positron annihilation data at the intensity frontier allow precision measurements of the spin and transverse momentum dependence of the fragmentation process into single and di-hadrons. The planned improvement in luminosity by a factor of 40 at Belle II, coupled with improved particle identification and vertexing capabilities will allow for a multi-dimensional analysis of fragmentation functions.

The ensuing dramatic improvement in the knowledge of fragmentation functions will be an important ingredient in the global analysis of the nucleon structure from semi-inclusive Deep-Inelastic-Scattering and *pp* collision data, thus contributing in a unique way to the challenge of understanding how partons build up all intrinsic properties of the nucleon, in particular how they share the nucleon's spin. This is one of the main goals of the upgraded 12 GeV CEBAF at JLab and the future Electron Ion Collider (EIC).

Many fundamental aspects of the structure of the nucleon are precluded to inclusive DIS experiments and can be accessed only in SIDIS, where fragmentation functions are involved. For instance, SIDIS makes it possible to pin down the flavor dependence of PDFs, or to access chiral-odd PDFs, such as the transversity distribution. More broadly, SIDIS allows us to extend the frontiers of our knowledge of parton dynamics and reconstruct multidimensional maps of the distributions of partons in the nucleon, encoded in Transverse Momentum Distributions (TMDs). This is one of the major goals of the JLab upgrade and the construction of the EIC. For the success of this program, precision measurements of the full kinematic dependence of the corresponding fragmentation functions are crucial. An example for the importance of e^+e^- data for this program is given by recent Belle and and BaBar analysis accessing transverse spin dependent fragmentation functions. Together with SIDIS data, they enabled the extraction of transversity in a global analysis and made important contributions to the formulation of the science case of JLab and the EIC.

II. STUDIES OF SPIN AND FLAVOR DEPENDENCE OF TMDS

Precision measurements of spin and azimuthal asymmetries in semi-inclusive pion and kaon production off unpolarized, longitudinally polarized and transversely polarized targets provide access to the spin and flavor dependences of transverse momentum distributions (TMDs) of quarks in the valence region [1–10].

The proposed measurements in the wide range of Q^2 ($1 < Q^2 < 6$) at upcoming CLAS12 experiment in JLab, would allow studies of Q^2-dependences of TMDs, and in particular the Sivers TMD [10] predicted to have very non-trivial evolution properties [11–17]. SIDIS experiments will measure variety of observables including cross sections, multiplicities and spin asymmetries for all combinations of target and beam spins.

Following the TMD factorization theorem one can write the SIDIS differential cross section in the leading order approximations and for the hadrons produced in the current fragmentation region as a sum involving structure functions, which are convolutions of various TMD PDFs and TMD FFs. These structure functions encode all the possible polarization correlations that occur in this process and can be accessed experimentally through specific angular modulations. The unpolarized structure function, for example, is given by the following expression [11, 18–20]

$$F_{UU,T}(x,z,P_T^2,Q^2) = \sum_a \mathcal{H}^a(Q^2) \iint d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_\perp\, f_1^a\big(x,k_T^2;Q^2\big) \\ \times D_1^{a\to h}\big(z,p_\perp^2;Q^2\big)\ \delta\big(z\boldsymbol{k}_T + \boldsymbol{p}_\perp - \boldsymbol{P}_T\big) + Y\big(Q^2,P_T^2\big) + \mathcal{O}\big(M/Q\big), \tag{1}$$

where $\mathcal{H}^a(Q^2)$ is a perturbatively calculable factor, $f_1^a(x,k_T^2;Q^2)$ is the unpolarized TMD PDF describing an unpolarized parton of flavor a carrying longitudinal momentum fraction x and transverse momentum $\boldsymbol{k_T}$ at the renormalization scale chosen to be equal to Q. $D_1^{a\to h}(z,p_T^2;Q^2)$ is the TMD fragmentation function for an unpolarized parton of flavor a fragmenting into an unpolarized hadron h which carries the longitudinal momentum fraction z and acquires the transverse momentum $\boldsymbol{p}_\perp$ (with respect to the fragmenting quark) during the fragmentation process. The term Y is introduced to ensure a matching to the perturbative calculations at high transverse momentum. The expressions for the polarized structure functions are analogous to the unpolarized one, except they involve spin-dependent PDFs and FFs [21–23].

As can be seen clearly from Eq. 1, extraction of underlying TMD from measured structure functions for different polarizations of leptons and nucleons will require detailed measurements of corresponding unpolarized and polarized fragmentation functions.

The large acceptance of the CLAS12 detector at Hall-B and the SoLID detector at Hall-A of JLab, as well as detectors at EIC would allow studies of multihadron events. Of particular interest are recently proposed studies of Sivers TMD using dihadron production off the transversely polarized nucleons [24, 25]. Extraction of Sivers TMD with completely different systematics from single-hadron case will require new measurements at Belle and BaBar of underlying TMD dihadron fragmentation functions.

III. STUDIES OF FRAGMENTATION FUNCTIONS AT BELLE AND BABAR

The cleanest, and often the only practical, way to access fragmentation functions is in the process $e^+ + e^- \to \gamma^*/Z_0 \to q + \bar{q}$. Here, a $q\bar{q}$ pair is created in e^+e^- annihilation that then subsequently fragments into hadrons. For a given quark flavor to be produced in the annihilation, the center of mass energy of the reaction should be larger than the quark mass threshold. The relative fractions of different produced flavors are determined by their couplings to the intermediate neutral bosons, which for the case of photon-exchange are proportional to the squares of the charges of the quarks. The differential cross section expression for inclusive one hadron production then can be written as a convolution of process-dependent perturbative factor $\mathcal{C}$ and the universal fragmentation function

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dzdQ^2} = \sum_{i=q,\bar{q},g} \mathcal{C}_i(z,Q^2) \otimes D_1^{i\to h}(z,Q^2), \tag{2}$$

where σ_0 is the total hadronic cross section. Then the fragmentation functions can be extracted from such cross section measurements via phenomenological parametrizations [26–28]. Nevertheless, compared to our current knowledge of PDFs, the fragmentation functions are still not well determined.

The high precision SIDIS cross section measurements at upcoming experiments at JLab12 and the planned EIC will require a well determined FF to make full use of the anticipated datasets to explore unknown aspects of the nucleon structure, such as it transverse momentum and flavor dependence of both unpolarized and polarized PDFs.

Furthermore, fragmentation functions describe the formation of QCD bound states (hadrons) from primordial quarks. They are therefore intrinsically linked to confinement in QCD and provide access to fundamental soft QCD quantities. Due to the dual nature of fragmentation functions and parton distribution functions, these quantities are important beyond the fragmentation process. Examples are TMD evolution soft factors which are universal between PDFs and FFs. Compared to the nucleon structure, which can be evaluated on the lattice, fragmentation functions cannot be computed using lattice QCD techniques due to the non-inclusive final state.

This section will concentrate on Belle, BaBar and future Belle II measurements for three reasons: The center of mass energy of these experiments is closest to the SIDIS experiments, they are the only collaborations that are still actively doing data analysis and most importantly, the Belle and BaBar datasets are orders of magnitude larger than previous e^+e^- experiments e.g. at LEP, SLAC and HERA.

A. Belle and BaBar Dataset and Setup

Arguably, the advent of the B-factories at KEK and SLAC for the first time enabled high precision FF measurements. They collected very large datasets of e^+e^- annihilation events around $\sqrt{s} = 10.58$ GeV, the mass of the $\Upsilon(4S)$ resonance. Belle at KEK collected an integrated luminosity of about 1 ab^{-1} whereas BaBar collected about half of that. Both experiments recorded about 90% of the data on resonance and 10% off resonance. However, since resonant production of Υ mesons that decay into $B\bar{B}$ pairs is only a fraction of the cross-section that can easily be excluded using event shape variables, the complete dataset can be used to study the fragmentation of light and charm quarks into hadrons. The complete dataset of Belle corresponds to about $2 \cdot 10^9$ events.

Both Belle and BaBar are general purpose detectors with vertex detectors, a central drift chamber, particle identification and electromagnetic calorimetry in the central region [29]. For the purpose of fragmentation function measurement of identified hadrons, one of the important differences between the detectors lies in their central PID detectors. BaBar is using a DIRC detector, whereas Belle relies on an aerogel cherenkov counter in conjunction with a time of flight detector. Both BaBar and Belle use dE/dx information from the central drift chamber. This means that the π/K separation at BaBar performs better than at Belle, which is important for measurements with multi kaon final states. However, Belle II, the successor of the Belle experiment which is described below, add state-of-the-art particle identification as well.

B. Recent Results

Recent results from Belle and BaBar played a significant role in the global analysis of SIDIS and $p+p$ data. This section highlights some of them. This gives an indication of the precision that can be reached using existing data.

1. *Hadron Multiplicities*

Multiplicities of charged and neutral hadrons have been measured over a wide range of energies. They are an important input for the extraction of unpolarized fragmentation functions [30, 31]. For the purpose of this white paper, we concentrate on the measurement of light meson multiplicities, which are most relevant for SIDIS measurements of the nucleon structure and on fractional energies $z > 0.1$ where hadron mass corrections do not play a significant role anymore in the extraction of fragmentation functions. Figure 1 shows the world data for identified charged pion production. They are dominated by recent measurements by Belle [32] on 68 fb^{-1} from which high precision kaon multiplicities were extracted as well. BaBar showed measurement of charged particle multiplicities [33] on a smaller dataset of 0.91 fb^{-1}, which are also shown in fig. 1 on the right. However, with systematic errors, the precision of the measurements are comparable. The Belle and BaBar data fill in a region at high z where previously no data was available. A new global fit of pion production data, that includes the new Belle and BaBar data [28] also shows that the high precision data from the two experiments agree well within their errors. In addition to being an important ingredient to extract fragmentation functions that are needed e.g. in SIDIS, the precision of the data also allows to investigate pertubative QCD techniques that are relevant for PDFs, such as threshold resummation and hadron mass corrections [34].

2. *Transverse Spin Dependent Fragmentation Functions*

The chiral-odd distribution functions, that describe the correlations between the transverse spin and the transverse momenta in nucleons such as the transversity or

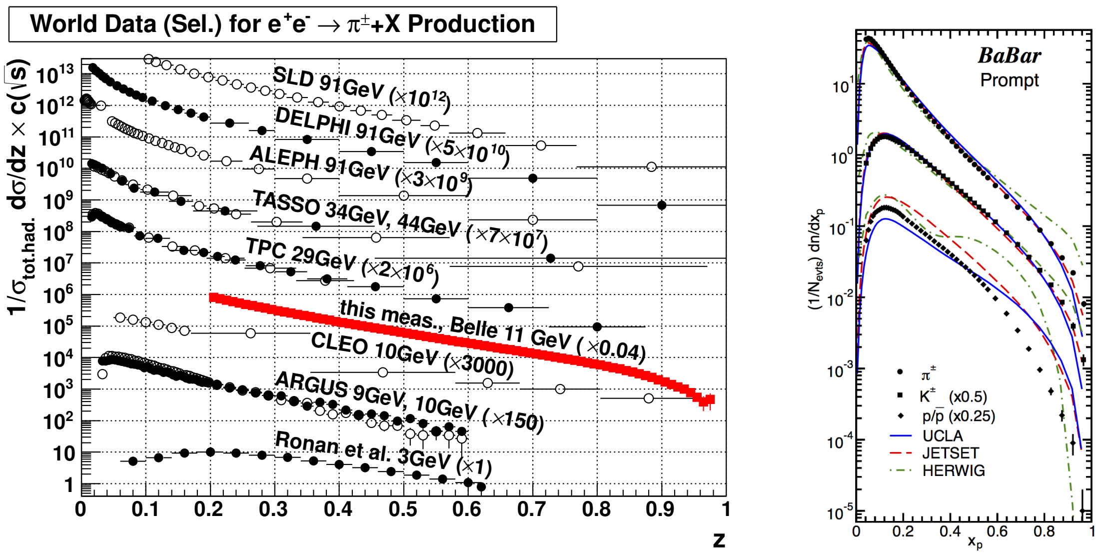

FIG. 1: Selected World data on pion multiplicities. The recent Belle measurement is shown in red (left). The right figure shows pion, kaon and proton multiplicities measured at BaBar. Shown here are the results for prompt production which excludes particles produced in weak decays.

the Boer-Mulders PDFs. These PDFs can only be measured in conjunction with other chiral-odd functions, since the cross sections of the processes described by these functions are physical observables and have to be chiral-even. In SIDIS, this has to be a chiral-odd fragmentation function, which in the case of an unpolarized hadron production is the Collins fragmentation function [35]. Here, one way to think of the Collins fragmentation functions into unpolarized mesons is as quark polarimeters leading to a left-right asymmetry of the produced hadrons. This effect describes the correlation between the transverse spin of the quark and the transverse momentum of the produced hadron.

Boer, Jakob and Mulders [36] proposed a measurement of the Collins fragmentation in unpolarized e^+e^- annihilation that makes use of the fact that the polarization of the quark and anti-quark produced in the annihilation are correlated. Both Belle and BaBar measured the Collins fragmentation function for pions in back-to-back correlations of pions [37][38] using the so-called double ratio method. Here the ratio of unlike- over like-sign combinations is used to cancel acceptance effects and

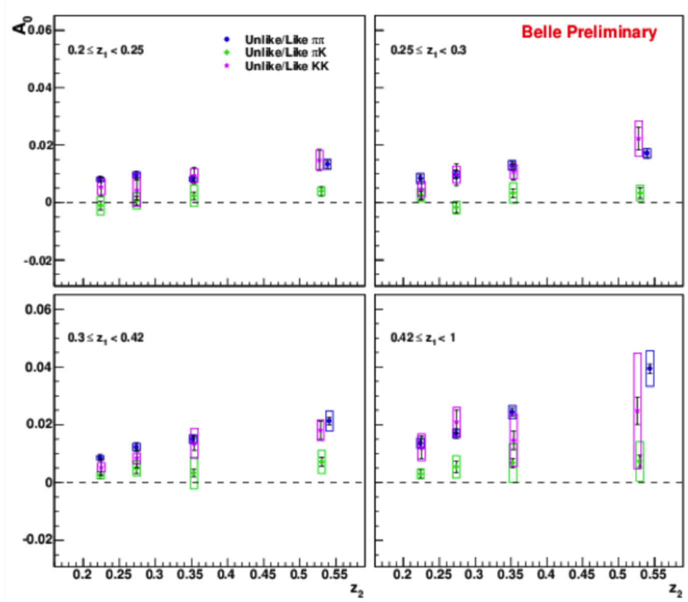

FIG. 2: Collins asymmetry in e^+e^- production for identified kaons. Systematic errors, which are in large part due to particle identification corrections, are significant.

asymmetries due to QCD radiation, which is charge independent. This is necessary, since one does not measure a spin asymmetry in which acceptance effects and the like cancel. Figure 3 shows the BaBar measurement of the Collins asymmetry, which provided an important cross-check of the Belle measurement and also measured the p_T dependence of the effect. Belle recently also showed preliminary results for the Collins effect in kaon production [39], which is shown in Fig. 2. The measurement errors are still dominated by systematic effects from the particle identification, which require further studies.

The Collins fragmentation function $H_1^\perp(z, p_\perp)$, described the modulation of the transversely polarized quark's fragmentation function to an unpolarized hadron with respect to the azimuthal angle between the transverse momentum of the hadron $\boldsymbol{p}_\perp$ and the transverse spin of the quark. Naturally, the contribution of the Collins term to the polarized fragmentation function vanishes when integrating over $\boldsymbol{p}_\perp$ (see, e.g., [40]).The evolution of such TMD quantities is a very active field of research (see e.g. [41, 42] and references therein). The evolution of fragmentation functions and parton-distribution function uses the same soft kernels, which have to be measured. Therefore measuring the Collins effect at a significantly lower energy (the evolution is

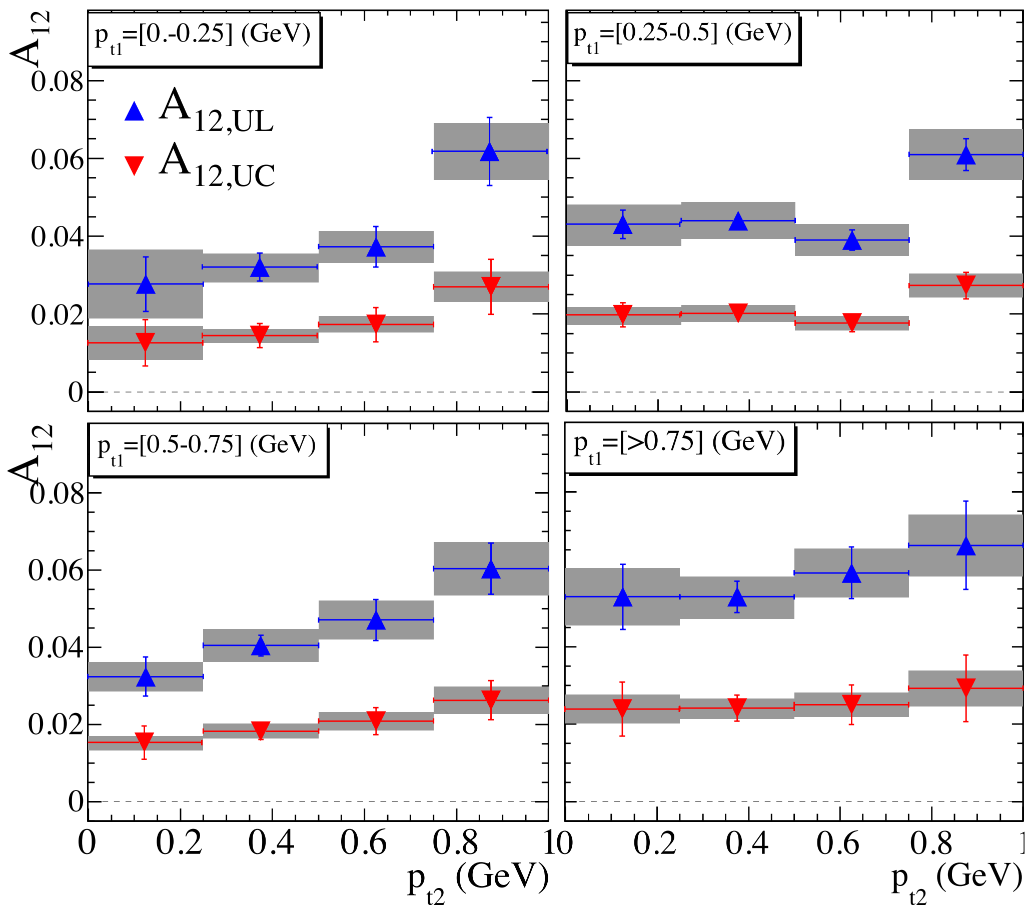

FIG. 3: Collins asymmetry for identified pions in e^+e^- annihilation from BaBar vs P_t.

dependent on $\log(Q^2)$) can help us understand TMD evolution. Such a measurement has recently been done by the BESIII collaboration at a center of mass energy of about 3.6 GeV, showing significantly enhanced asymmetries compared to the ones measured at Belle.

Similarly to the measurements of the Collins fragmentation function of single hadrons in back-to-back correlation of hadron pairs, one can measure the di-hadron interference fragmentation function (IFF) $H_1^{\sphericalangle}$ in the correlation of back-to-back di-hadron pair. The IFF $H_1^{\sphericalangle}$ is sensitive to the transverse polarization of the outgoing quark similarly as the Collins fragmentation function. However, the correlation survives an integration over the intrinsic transverse momentum in the fragmentation process. Therefore $H_1^{\sphericalangle}$ can be used in a collinear framework e.g. for the extraction of transversity from SIDIS. This is an important cross-check with the extractions using

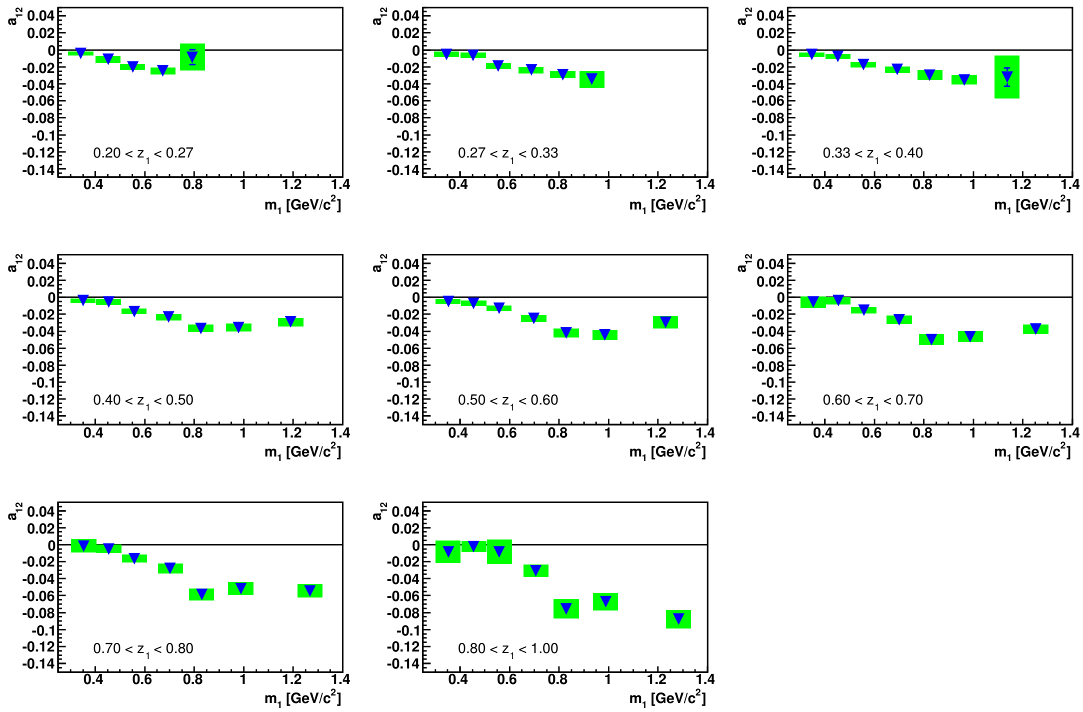

FIG. 4: Asymmetries proportional to the Interference fragmentation function vs. invariant mass m of the hadron pair for different bins in the fractional energy z of the parent quark carried by the hadron pair.

Collins asymmetries, but it is also free from complications that appear in the TMD factorization frameworks, such as strong evolution effects which are not entirely perturbatively calculable and potential breaking of factorization in p+p collisions. The Belle measurement [43] does not use the double ratio method as the Collins analysis, since acceptance and radiation effects can be controlled in the di-hadron measurement. Therefore the result shown in Figure 4 is directly proportional to $H_1^{\sphericalangle}\bar{H}_1^{\sphericalangle}$. The second term describes the fragmentation of the anti-quark.

C. Potential Future measurements at Belle/BaBar data and Belle II

Important measurements of fragmentation functions can be made with existing data taken at Belle and BaBar. Analyzing BESIII data to extract fragmentation functions at lower scales is also possible in principle, but only little data is available off-resonance. In addition the center-of-mass energy at which BESIII is measuring makes it difficult to clearly define a thrust axis that is a proxy for the quark axis or allows to separate hemispheres of quark and anti-quark. Therefore we concentrate here on measurements at Belle and BaBar and future measurements at Belle II. In the following, we will highlight some of the measurements that can be done on existing data and which will important for a successful SIDIS program:

1. Transverse Momentum Dependence of Spin Averaged Fragmentation Functions and of the Collins Fragmentation Function

The transverse momentum dependence of the TMD fragmentation functions is needed, since all measurements that involve them, contain two chiral-odd objects and a convolution over the transverse momentum. In addition, measurements like Collins asymmetries in SIDIS are inherently dependent on the relative transverse momentum of the hadron with respect to the quark axis. For this reason the transverse momentum dependence has to be known. Similarly, the transverse momentum dependence of the unpolarized functions has to be known, because they appear in the denominator of measured asymmetries. More importantly though, the transverse momentum dependence of the fragmentation process has to be known to extract non-collinear aspects of the nucleon structure that would get convoluted with the fragmentation function. The transverse momentum dependence can be measured with respect to the jet axis, a proxy for the outgoing quark axis, or theoretically cleaner, in back-to-back hadron pairs.

2. Unpolarized Di-Hadron Fragmentation Functions and Transverse Momentum Dependence of Di-Hadron Fragmentation Functions

Di-hadron fragmentation functions can provide a cleaner access to the polarized spin structure of the proton at twist-2 and twist-3. Belle has provided data on $\frac{H_1^{\sphericalangle}}{D_1}$, the ratio of transverse polarization dependent fragmentation function over its polarization independent counterpart (note that D_1 in this context is the di-hadron FF). In order to extract $H_1^{\sphericalangle}$, D_1 has to be known, which is currently taken from Pythia simulations. Therefore a measurement of D_1 is important. Recently [25], it was proposed that two-hadron SIDIS measurements provide a new way to access the Sivers function in the nucleon, with larger basis than in the single hadron productions case that will allow for flavor separation of Sivers PDF. Such extraction though is crucially hinged on the knowledge of the fully unintegrated dihadron FF, which makes the measurement of these functions very interesting as well.

3. Twist-3 Fragmentation Functions

In recent years, a growing interest is being devoted to higher-twist effects providing access to largely unknown quark-gluon correlations. They are expected to play a significant role in experiments operating at a low-to-medium Q^2 regime. Higher-twist FFs are essential in disentangling the different contributions that appear in many SIDIS observables. For example, the longitudinal polarized beam or target single-spin asymmetries in single pion semi-inclusive electroproduction were identified as pure twist-3 effects, being proportional to the coupling of leading and higher-twist TMDs and FFs.

Measurements of higher-twist di-hadron FFs would also help to understand the collinear picture of the proton up to the twist-3 level, using longitudinal single-spin asymmetries in Di-hadron SIDIS, where this FFs couple to elusive twist-3 PDFs, $e(x)$ in the beam SSA case and $h_L(x)$ for the target SSA case. Due to presence of the unknown higher-twist DiFF $\tilde{G}_1^{\sphericalangle}$, analysis of the data could be done only combining sets of data with polarized and unpolarized targets, introducing additional systematic errors. The knowledge of the higher twist DiFF $\tilde{G}_1^{\sphericalangle}$, will constitute an essential step toward the final extraction of collinear higher twist PDFs $e(x)$ and $h_L(x)$[44]. A measurement of functions like $\tilde{G}_1^{\sphericalangle}$ in e^+e^- will be important for studies of quark-gluon correlations in hard processes in general, and measurements of twist-3 distributions $e(x)$, and $h_L(x)$ at JLab, in particular. However, theoretical work is still needed to extract this FF from e^+e^- annihilation data.

4. Flavor Dependence and Polarizing Fragmentation Functions

In addition to the fragmentation into charged pions, detecting kaons or neutral π^0 and η allows to access to the flavor structure of the fragmentation functions. This information is important to understand the relative differences of asymmetries observed for pion and kaons in SIDIS or between π^0 and η transverse single spin asymmetries in p+p and use them to access the flavor dependence of the nucleon spin structure. The polarizing fragmentation function into Λ baryons can be seen as the equivalent of the Sivers PDFs for FFs, and its measurement will therefore allow the study of related QCD effects [45].

5. Measurement Prospects Using Belle/BaBar Data and at Belle II

Existing Belle and BaBar analysis, which are performed on most of the existing dataset can give a guidance of the precision that can be achieved. Multidimensional binning, in particular if transverse momentum dependence is to be taken into account needs larger datasets, if the expected precision of JLab12 or the EIC should be reached. Belle II [46] is an upgrade of the Belle detector and is situated at SuperKEKB which is scheduled to start up in early 2016. SuperKEKB provides 40$\times$ the instantaneous luminosity than was available at KEKB and the Belle II collaboration plans to take about 50^{-1} ab in the first seven years of running. In addition to provide orders of magnitude more data than Belle, Belle II is also a state of the art detector with significantly improved vertex detection and particle identification. Figure 5 shows a direct comparison of the Belle and Belle II detectors. The kaon efficiency will be improved by the imaging Time-of-Propagation detector alone to 95% over most of the kinematic range while fake rates are on the percent level. This makes the measurement of multi-kaon final states feasible. The improved vertex detection will allow unbiased correction for charm contributions. Figure 6 shows the relative fraction of charm production in the measurement of the Collins fragmentation function in πK pairs at Belle. On average, charm quarks are produced almost 40% of the time. Final states with kaons have a higher charm fraction and the charm contribution is highest at lower z since pions and kaons come for D-meson decays. In SIDIS

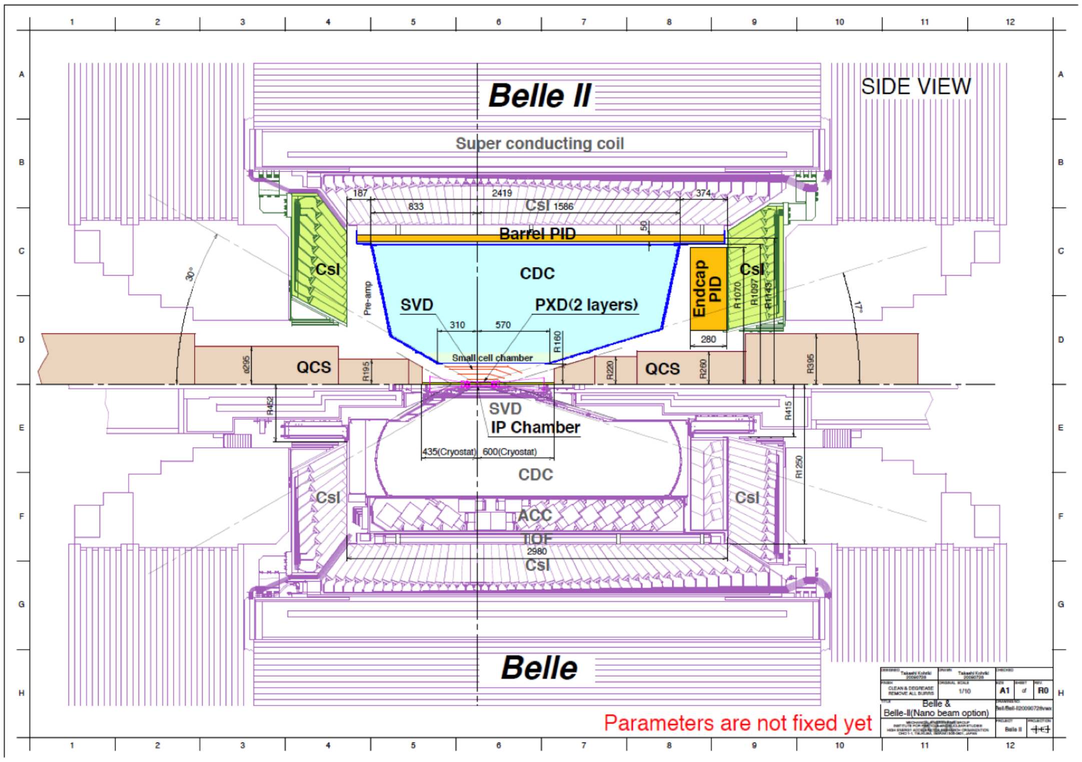

FIG. 5: Comparison of the Belle detector with the Belle II detector. Notable differences are the new vertex detectors and the iToP Barrel PID. In addition, the other subsystem received electronics upgrades.

charm production is negligible, so the contribution to the fragmentation process has to be corrected for. This is done either with models for the charm fragmentation or using charm enhanced samples. Currently charm-enhanced samples are produced by selecting $D^{(*)}$-mesons via a cut on their invariant mass. This method suffers from low statistics and a bias of the phase space. Therefore the vertex detection capabilities of Belle II, together with the amount of available data, will help to significantly reduce the systematic errors on our current measurements.

IV. MODELING OF FRAGMENTATION FUNCTIONS

The models of the fragmentation functions are challenging to build, as they describe the transition probabilities of the fast moving partons from perturbative regime into non-perturbative hadronic bound states. One of the earliest models for the un-

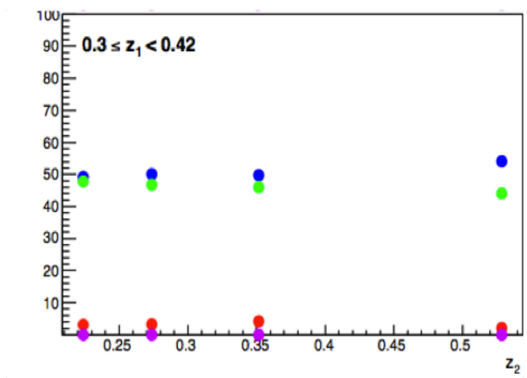

FIG. 6: Fractional charm (green), uds(blue), bottom (red) and τ (purple) contributions to the di-hadron Collins measurement for one z_1 bin using πK pairs at Belle. The charm contribution reaches 50% in some bins.

polarized fragmentation functions was the so-called quark-jet model built by Field and Feynman [47, 48], where the quark hadronization was described as a sequential emission of infinite number of hadrons and the probability for each hadron emission was parameterized using an empirical form.

Currently there are two main classes of models for the fragmentation functions. The first type describes the complete hadronization of the colored partons involved in the reaction, such as the Lund string model [49–51] and the Cluster model [52–54]. These models were implemented in modern Monte-Carlo event generators PYTHIA8 [55] and HERWIG++ [56] respectively, with vast array of improvements and flexible parameterizations of the relevant physical phenomena to provide a good description of the experimental data. On the other hand, the connection of these models to the low energy QCD is weak, and due to their extensive ad-hoc parameterizations their predictive power is limited. Finally, these models currently lack the description of the polarized quark hadronization, thus cannot describe phenomena involving the Collins single hadron and Interference dihadron fragmentation functions. The model studies of the Collins function within the Lund string model framework were pioneered in [57] for pions using the Lund string mechanism, which was later applied to vector mesons [58], with a recursive non-relativistic model detailed in [59], though there are no current implementations of these developments.

A second class of fragmentation models are those employing the spectator-approximation, where a quark emits a single hadron leaving behind an on-shell spectator (usually an other quark). This approximation allows to derive analytical forms for the corresponding fragmentation functions. A range of such models have been built for both unpolarized and Collins fragmentation function [60–66], as well as dihadron fragmentation functions [67, 68], employing various QCD-inspired phenomenological models. These models on the other hand can only directly access the favored fragmentation functions, and typically also need to introduce additional phenomenological parameters to describe the hadron emission with small light-cone momentum fraction where the effects of the multiple hadron emissions by the quark are strong. Moreover, the recent measurements by the COMPASS collaboration [69] hint at the common underlying mechanism for both Collins fragmentation and Interference dihadron fragmentation functions, that currently cannot be simultaneously described by the spectator type models.

A yet another approach is the recently proposed Nambu–Jona-Lasinio–jet (NJL-jet) model [70–72] that employ the quark-jet hadronization framework of Field and Feynman, but instead of phenomenological parametrization use spectator type calculations performed within QCD-inspired NJL quark model for the single hadron emission probability input. Further developments of these model involved the calculations of the TMD fragmentation functions, Collins effect, as well as dihadron fragmentation function employing a Monte Carlo implementation of this model [24, 40, 73, 74]. In a recent toy-model study [25], the NJL-jet demonstrated that the mechanism that can explain the correlations between the Collins FFs and IFF observed by COMPASS in [69].

A recent review of the fragmentation functions (including the collinear interference fragmentation functions not discussed here) can be found in [75]. Precision measurements of transverse momentum dependent fragmentation functions will be important for understanding of the dynamics of the hadronization in general and the role of spin-orbit correlations in hadronization process in particular.

V. SUMMARY

In summary precision measurements of the transverse momentum dependent fragmentation functions describing the single and double hadron production are crucial for accomplishing the physics program of the upgraded JLab and motivation of construction of the EIC.

Significant progress in our knowledge of transverse momentum dependent fragmentation functions into single pions, both polarization independent and transverse polarization dependent, can be achieved with the existing Belle and BaBar data. However, data from the Belle II experiment, which will be commissioned in 2016 at KEK will be crucial for the multi-dimensional analysis of fragmentation function with a precision that matches the expected statistical errors from JLab12 and EIC experiments. Belle II will be necessary to measure multi-kaon final states and reduce systematic errors from particle identification and charm contributions to the level of the expected statistical precision.

[1] H. Avakian *et. al.*, *Probing the proton's quark dynamics in sidis at 12 gev*, *JLab Experiment E12-06-112* (2006).

[2] H. Avakian *et. al.*, *Studies of spin-orbit correlations in pion electroproduction in dis with polarized hydrogen and deuterium targets*, *JLab Experiment E12-07-107* (2007).

[3] H. Avakian *et. al.*, *Studies of the boer-mulders asymmetry in kaon electroproduction with hydrogen and deuterium targets*, *JLab Experiment E12-09-008* (2009).

[4] H. Avakian *et. al.*, *Studies of spin-orbit correlations in kaon electroproduction in dis with polarized hydrogen and deuterium targets*, *JLab Experiment E12-09-009* (2009).

[5] R. Ent, P. Bosted, and H. Mkrtchyan, *Transverse momentum dependence of semi-inclusive pion production*, *JLab Experiment E12-09-017* (2009).

[6] G. Cates, E. Cisbani, G. Franklin, A. Puckett, and W. B., *Measurement of semi - inclusive pion and kaon electroproduction in the dis regime on a transversely polarized 3he target using the super bigbite and bigbite spectrometers in hall a*, *JLab Experiment E12-09-018* (2009).

[7] H. Gao, J.-P. Chen, X. Jiang, and J.-C. Peng, *E12-10-006 an update to pr12-09-014: Target single spin asymmetry in semi-inclusive deep-inelastic electro pion production on a trasversely polarized 3he target at 8.8 and 11 gev*, *JLab Experiment E12-10-006* (2010).

[8] J.-P. Chen, J. Huang, Y. Qiang, and Y. Wenbiao, *E12-11-007 asymmetries in semi - inclusive deep - inelastic electro - production of charged pion on a longitudinally polarized he - 3 target at 8.8 and 11 gev*, *JLab Experiment E12-11-007* (2008).

[9] H. Avakian *et. al.*, *Measurement of transversity with dihadron production in sidis with transversely polarized target*, *JLab Experiment C12-12-009* (2012).

[10] H. Avakian *et. al.*, *Transverse spin effects in sidis at 11 gev with a transversely polarized target using the clas12 detector*, *JLab Experiment C12-11-111* (2011).

[11] S. M. Aybat and T. C. Rogers, *TMD Parton Distribution and Fragmentation Functions with QCD Evolution*, *Phys.Rev.* **D83** (2011) 114042, [arXiv:1101.5057].

[12] P. Sun and F. Yuan, *Energy Evolution for the Sivers Asymmetries in Hard Processes*,

Phys.Rev. **D88** (2013) 034016, [arXiv:1304.5037].

[13] Z.-B. Kang, B.-W. Xiao, and F. Yuan, *QCD Resummation for Single Spin Asymmetries*, *Phys.Rev.Lett.* **107** (2011) 152002, [arXiv:1106.0266].

[14] S. M. Aybat, J. C. Collins, J.-W. Qiu, and T. C. Rogers, *The QCD Evolution of the Sivers Function*, *Phys.Rev.* **D85** (2012) 034043, [arXiv:1110.6428].

[15] M. G. Echevarri, A. Idilbi, A. Schafer, and I. Scimemi, *Model-Independent Evolution of Transverse Momentum Dependent Distribution Functions (TMDs) at NNLL*, *Eur.Phys.J.* **C73** (2013) 2636, [arXiv:1208.1281].

[16] M. Anselmino, M. Boglione, and S. Melis, *A Strategy towards the extraction of the Sivers function with TMD evolution*, *Phys.Rev.* **D86** (2012) 014028, [arXiv:1204.1239].

[17] M. G. Echevarria, A. Idilbi, Z.-B. Kang, and I. Vitev, *QCD Evolution of the Sivers Asymmetry*, *Phys.Rev.* **D89** (2014) 074013, [arXiv:1401.5078].

[18] J. C. Collins and D. E. Soper, *Back-to-back jets in qcd*, *Nucl. Phys.* **B193** (1981) 381.

[19] X. Ji, J. Ma, and F. Yuan, *Qcd factorization for semi-inclusive deep-inelastic scattering at low transverse momentum*, *Phys. Rev.* **D71** (2005) 034005, [hep-ph/0404183].

[20] J. Collins, *Foundations of perturbative QCD.* Cambridge University Press, 2011.

[21] P. J. Mulders and R. D. Tangerman, *The complete tree-level result up to order 1/q for polarized deep-inelastic leptoproduction*, *Nucl. Phys.* **B461** (1996) 197–237, [hep-ph/9510301].

[22] A. Bacchetta *et. al.*, *Semi-inclusive deep inelastic scattering at small transverse momentum*, *JHEP* **02** (2007) 093, [hep-ph/0611265].

[23] D. Boer, L. Gamberg, B. Musch, and A. Prokudin, *Bessel-Weighted Asymmetries in Semi Inclusive Deep Inelastic Scattering*, *JHEP* **1110** (2011) 021, [arXiv:1107.5294].

[24] H. H. Matevosyan, A. W. Thomas, and W. Bentz, *Dihadron Fragmentation Functions within the NJL-jet Model*, *Phys.Rev.* **D88** (2013) 094022, [arXiv:1310.1917].

[25] H. H. Matevosyan, A. Kotzinian, and A. W. Thomas, *Studies of Azimuthal Modulations in Two Hadron Fragmentation of a Transversely Polarised Quark*, *Phys.Lett.* **B731** (2014) 208–216, [arXiv:1312.4556].

[26] D. de Florian, R. Sassot, and M. Stratmann, *Fragmentation functions for pions, kaons, protons and charged hadrons*, *J. Phys. Conf. Ser.* **110** (2008) 022045, [hep-ph/0708.0769].

[27] M. Hirai, S. Kumano, T.-H. Nagai, and K. Sudoh, *Determination of fragmentation functions and their uncertainties*, *Phys.Rev.* **D75** (2007) 094009, [hep-ph/0702250].

[28] D. de Florian, R. Sassot, M. Epele, R. J. Hernandez-Pinto, and M. Stratmann, *Parton-to-Pion Fragmentation Reloaded*, arXiv:1410.6027.

[29] **BaBar Collaboration, Belle Collaboration** Collaboration, A. Bevan *et. al.*, *The Physics of the B Factories*, *Eur.Phys.J.* **C74** (2014), no. 11 3026, [arXiv:1406.6311].

[30] D. de Florian, R. Sassot, and M. Stratmann, *Global analysis of fragmentation functions for pions and kaons and their uncertainties*, *Phys.Rev.* **D75** (2007) 114010, [hep-ph/0703242].

[31] M. Epele, R. Llubaroff, R. Sassot, and M. Stratmann, *Uncertainties in pion and kaon fragmentation functions*, *Phys.Rev.* **D86** (2012) 074028, [arXiv:1209.3240].

[32] **Belle Collaboration** Collaboration, M. Leitgab *et. al.*, *Precision Measurement of Charged Pion and Kaon Differential Cross Sections in e+e- Annihilation at s=10.52GeV*, *Phys.Rev.Lett.* **111** (2013) 062002, [arXiv:1301.6183].

[33] **BaBar Collaboration** Collaboration, J. Lees *et. al.*, *Production of charged pions, kaons, and protons in e^+e^- annihilations into hadrons at $\sqrt{s}$=10.54GeV*, *Phys.Rev.* **D88** (2013) 032011, [arXiv:1306.2895].

[34] A. Accardi, D. P. Anderle, and F. Ringer, *Interplay of Threshold Resummation and Hadron Mass Corrections in Deep Inelastic Processes*, arXiv:1411.3649.

[35] J. C. Collins, *Fragmentation of transversely polarized quarks probed in transverse momentum distributions*, *Nucl. Phys.* **B396** (1993) 161–182, [http://arXiv.org/abs/hep-ph/9208213].

[36] D. Boer, R. Jakob, and P. J. Mulders, *Asymmetries in polarized hadron production in e^+e^- annihilation up to order 1/q*, *Nucl. Phys.* **B504** (1997) 345–380, [http://arXiv.org/abs/hep-ph/9702281].

[37] **BaBar Collaboration** Collaboration, J. Lees *et. al.*, *Measurement of Collins asymmetries in inclusive production of charged pion pairs in e^+e^- annihilation at*

BABAR, *Phys.Rev.* **D90** (2014) 052003, [arXiv:1309.5278].

[38] **Belle** Collaboration, R. Seidl *et. al.*, *Measurement of Azimuthal Asymmetries in Inclusive Production of Hadron Pairs in e+e- Annihilation at $\sqrt{s}$ = 10.58 GeV*, *Phys. Rev.* **D78** (2008) 032011, [arXiv:0805.2975].

[39] **BELLE** Collaboration, F. Giordano, *Kaon Collins Fragmentation Function Access at Belle*, *PoS* **DIS2014** (2014) 212.

[40] H. H. Matevosyan, A. W. Thomas, and W. Bentz, *Collins Fragmentation Function within NJL-jet Model*, *Phys.Rev.* **D86** (2012) 034025, [arXiv:1205.5813].

[41] J. Collins and T. Rogers, *Understanding the large-distance behavior of TMD parton densities and the Collins-Soper evolution kernel*, arXiv:1412.3820.

[42] Z.-B. Kang, A. Prokudin, P. Sun, and F. Yuan, *Nucleon Tensor Charge from Collins Azimuthal Asymmetry Measurements*, arXiv:1410.4877.

[43] A. Vossen *et. al.*, *Observation of the interference fragmentation function for charged pion pairs in e^+e^- annihilation near $\sqrt{s} = 10.58$ GeV*, private communication.

[44] S. Pisano *et. al.*, *Higher-twist collinear structure of the nucleon through di-hadron sidis on unpolarized hydrogen and deuterium.*, *JLab Experiment E12-06-112B/E12-09-008B* (2014).

[45] D. Boer, Z.-B. Kang, W. Vogelsang, and F. Yuan, *Test of the Universality of Naive-time-reversal-odd Fragmentation Functions*, *Phys.Rev.Lett.* **105** (2010) 202001, [arXiv:1008.3543].

[46] **Belle-II Collaboration** Collaboration, T. Abe *et. al.*, *Belle II Technical Design Report*, arXiv:1011.0352.

[47] R. D. Field and R. P. Feynman, *Quark Elastic Scattering as a Source of High Transverse Momentum Mesons*, *Phys. Rev.* **D15** (1977) 2590–2616.

[48] R. D. Field and R. P. Feynman, *A parametrization of the properties of quark jets*, *Nucl. Phys.* **B136** (1978) 1.

[49] B. Andersson, G. Gustafson, and C. Peterson, *A Statistical Model for Quark Fragmentation Into Mesons with Emphasis on Vector Meson Contributions*, *Nucl. Phys.* **B135** (1978) 273.

[50] T. Sjostrand, *The Lund Monte Carlo for Jet Fragmentation*, *Comput. Phys.*

Commun. **27** (1982) 243.

[51] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand, *Parton Fragmentation and String Dynamics*, *Phys. Rept.* **97** (1983) 31–145.

[52] G. Marchesini and B. Webber, *Simulation of QCD Jets Including Soft Gluon Interference*, *Nucl.Phys.* **B238** (1984) 1.

[53] B. Webber, *A QCD Model for Jet Fragmentation Including Soft Gluon Interference*, *Nucl.Phys.* **B238** (1984) 492.

[54] G. Marchesini and B. Webber, *Monte Carlo Simulation of General Hard Processes with Coherent QCD Radiation*, *Nucl.Phys.* **B310** (1988) 461.

[55] T. Sjostrand, S. Mrenna, and P. Z. Skands, *A Brief Introduction to PYTHIA 8.1*, *Comput.Phys.Commun.* **178** (2008) 852–867, [arXiv:0710.3820].

[56] M. Bahr, S. Gieseke, M. Gigg, D. Grellscheid, K. Hamilton, *et. al.*, *Herwig++ Physics and Manual*, *Eur.Phys.J.* **C58** (2008) 639–707, [arXiv:0803.0883].

[57] X. Artru, J. Czyzewski, and H. Yabuki, *Single spin asymmetry in inclusive pion production, Collins effect and the string model*, *Z.Phys.* **C73** (1997) 527–534, [hep-ph/9508239].

[58] J. Czyzewski, *Single spin asymmetry of vector meson production in the string model*, *Acta Phys.Polon.* **27** (1996) 1759–1766, [hep-ph/9606390].

[59] X. Artru and Z. Belghobsi, *String model for spinning quark jets*, *AIP Conf.Proc.* **1444** (2011) 97–105.

[60] R. Jakob, P. J. Mulders, and J. Rodrigues, *Modelling quark distribution and fragmentation functions*, *Nucl. Phys.* **A626** (1997) 937–965, [hep-ph/9704335].

[61] A. Bacchetta, R. Kundu, A. Metz, and P. Mulders, *The Collins fragmentation function: A Simple model calculation*, *Phys.Lett.* **B506** (2001) 155–160, [hep-ph/0102278].

[62] A. Bacchetta, R. Kundu, A. Metz, and P. J. Mulders, *Estimate of the collins fragmentation function in a chiral invariant approach*, *Phys. Rev.* **D65** (2002) 094021, [hep-ph/0201091].

[63] L. P. Gamberg, G. R. Goldstein, and K. A. Oganessyan, *A Mechanism for the T odd pion fragmentation function*, *Phys.Rev.* **D68** (2003) 051501, [hep-ph/0307139].

[64] A. Bacchetta, A. Metz, and J.-J. Yang, *Collins fragmentation function from gluon rescattering*, *Phys.Lett.* **B574** (2003) 225–231, [hep-ph/0307282].

[65] D. Amrath, A. Bacchetta, and A. Metz, *Reviewing model calculations of the Collins fragmentation function*, *Phys. Rev.* **D 71** (2005) 114018, [hep-ph/0504124].

[66] A. Bacchetta, L. P. Gamberg, G. R. Goldstein, and A. Mukherjee, *Collins fragmentation function for pions and kaons in a spectator model*, *Phys. Lett.* **B659** (2008) 234–243, [hep-ph/0707.3372].

[67] A. Bacchetta and M. Radici, *Modeling dihadron fragmentation functions*, *Phys. Rev.* **D74** (2006) 114007, [hep-ph/0608037].

[68] A. Bacchetta, F. Ceccopieri, A. Mukherjee, and M. Radici, *Asymmetries involving dihadron fragmentation functions: from DIS to e+e- annihilation*, *Phys. Rev.* **D79** (2009) 034029, [arXiv:0812.0611].

[69] **COMPASS Collaboration** Collaboration, C. Adolph *et. al.*, *A high-statistics measurement of transverse spin effects in dihadron production from muon-proton semi-inclusive deep-inelastic scattering*, *Phys.Lett.* **B736** (2014) 124–131, [arXiv:1401.7873].

[70] T. Ito, W. Bentz, I. Cloet, A. Thomas, and K. Yazaki, *The NJL-jet model for quark fragmentation functions*, *Phys.Rev.* **D80** (2009) 074008, [arXiv:0906.5362].

[71] H. H. Matevosyan, A. W. Thomas, and W. Bentz, *Calculating Kaon Fragmentation Functions from NJL-Jet Model*, *Phys.Rev.* **D83** (2011) 074003, [arXiv:1011.1052].

[72] A. Casey, H. H. Matevosyan, and A. W. Thomas, *Calculating Dihadron Fragmentation Functions in the NJL- jet model*, arXiv:1202.4036.

[73] H. H. Matevosyan, A. W. Thomas, and W. Bentz, *Monte Carlo Simulations of Hadronic Fragmentation Functions using NJL-Jet Model*, *Phys.Rev.* **D83** (2011) 114010, [arXiv:1103.3085].

[74] H. H. Matevosyan, W. Bentz, I. C. Cloet, and A. W. Thomas, *Transverse Momentum Dependent Fragmentation and Quark Distribution Functions from the NJL-jet Model*, *Phys.Rev.* **D85** (2012) 014021, [arXiv:1111.1740].

[75] M. Radici, *Unpolarized and Polarized Fragmentation Functions*, *Nuovo Cim.* **C035N2** (2012) 69–77, [arXiv:1111.3383].