Linear response on a Quantum Computer

Alessandro Roggero & Joseph Carlson (LANL)

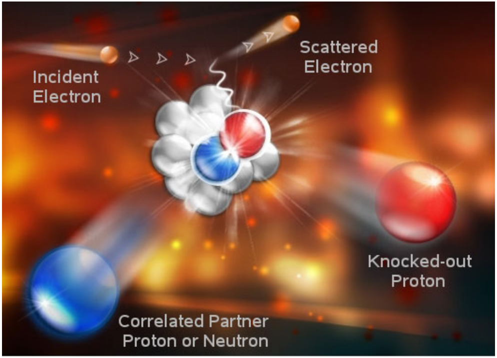
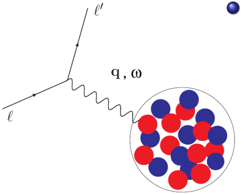


figure from JLAB collab.

NPQI - ANL - 28 Mar, 2018

Inclusive cross section and the response function



- xsection completely determined by response function

$$R(q,\omega) = \sum_f \left| \langle f | \hat{O}(q) | 0 \rangle \right|^2 \delta\left(\omega - E_f + E_0\right)$$

  - excitation operator $\hat{O}(q)$ specifies the vertex

Same structure not only in NP but also condensed matter, cold atoms,. . .

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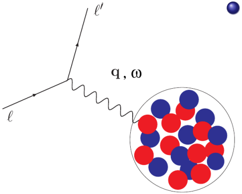

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Extremely challenging classically for strongly correlated quantum systems

- limited to small systems
- reliant on approximations that are difficult to control (efficiently)

Response functions on a Quantum Computer

- use time correlation functions (Terhal&DiVincenzo(2000), Ortiz et al. (2001))

Ingredients for response calculation in frequency space

- an oracle that prepares the ground state (QAA, VQE, Spectral Combing, ...)
- an oracle for time evolution (Berry et al. (2015),Hao Low et al. (2016))

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By performing quantum phase estimation (Kitaev(1996), Abrams&Lloyd(1999)) with M ancilla qubits we will measure frequency ν with probability:

$$P(\nu) = \sum_f |\langle f|E\rangle|^2 \, \delta_M\left(\nu - E_f + E_0\right)$$

- finite width approximation of $R(q,\omega)$
- need only $M \sim \log_2(1/\Delta\omega)$ ancillae
- evolution time $t \sim Poly(\text{sys.size})/\Delta\omega$

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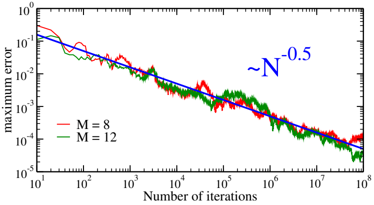
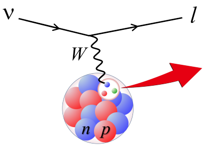


Exclusive response for neutrino oscillation experiments



Goals for ν oscillation exp.

- neutrino masses
- accurate mixing angles
- CP violating phase

$$P(\nu_\alpha \to \nu_\alpha) = 1 - sin^2(2\theta) sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

- need to use measured reaction products to constrain E_ν of the event

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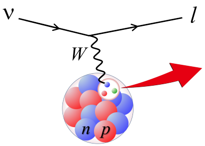

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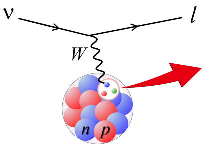

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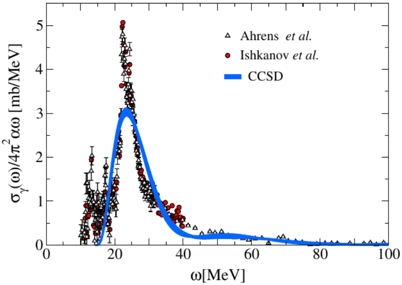
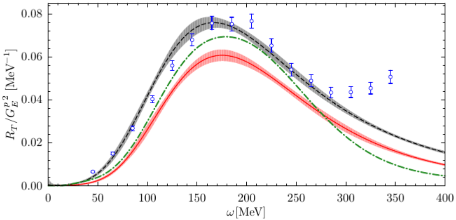
STAY TUNED more details coming out soon: Roggero & Carlson (in prep.)

Response functions on classical computers

Bacca et al. (2013) LIT+CC



Lovato et al. (2016) GFMC



Quantum Phase Estimation

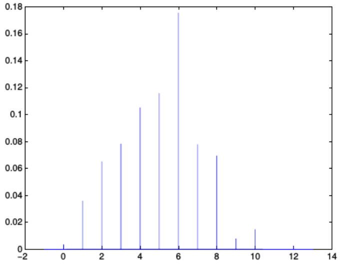
Kitaev (1996), Brassard et al. (2002), Svore et. al (2013), Weibe & Granade (2016)

QPE is a general algorithm to estimate eigenvalues of a unitary operator

$$U|\xi_k\rangle = \lambda_k|\xi_k\rangle \, , \lambda_k = e^{2\pi i\phi_k} \quad \Leftarrow \quad U = e^{-itH}$$

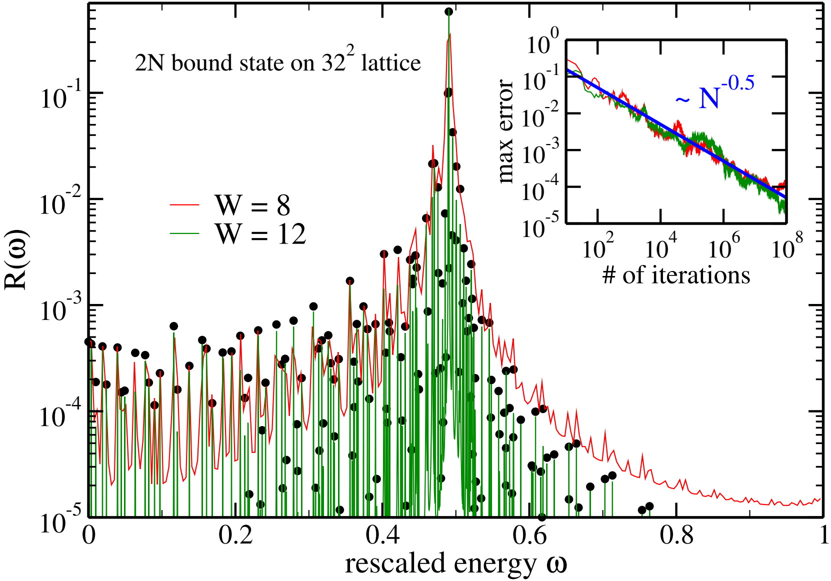
- starting vector $|\psi\rangle = \sum_k c_k|\xi_k\rangle$
- store time evolution $|\psi(t)\rangle$ in auxiliary register of m qubits
- perform (Quantum) Fourier transform on the auxiliary register
- measures will return λ_k with probability $P(\lambda_k) \approx |c_k|^2$

Ovrum&Hjorth-Jensen (2007)



to get $|GS\rangle$ a good $|\psi\rangle$ is critical

Test on classical computer



Final state properties from a Quantum Computer

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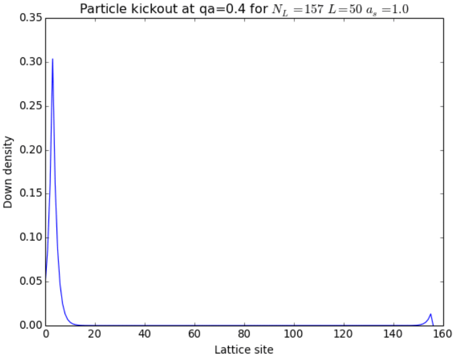
$$|out\rangle_\nu \sim \sum_f \langle f|\hat{O}(q)|0\rangle|f\rangle \quad \text{with } E_f - E_0 = \nu \pm \Delta\omega$$

- we can then measure eg. 1- and 2-particle momentum distributions



Caveat

- need to further time-evolve to extract information on asymptotic states in the detectors

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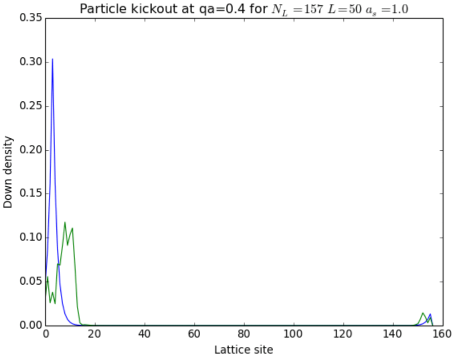

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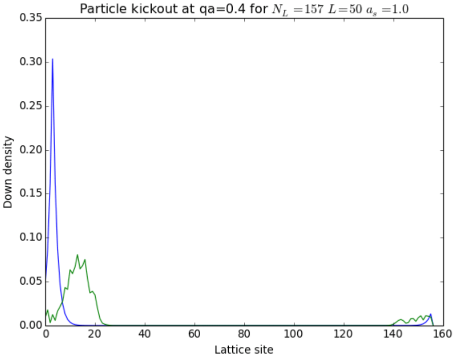

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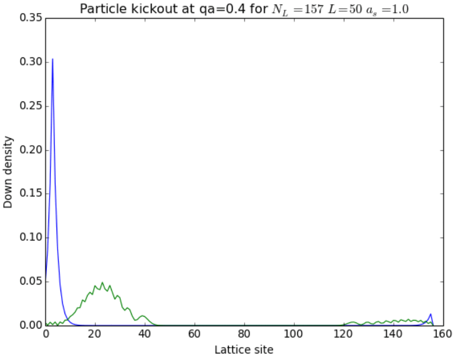

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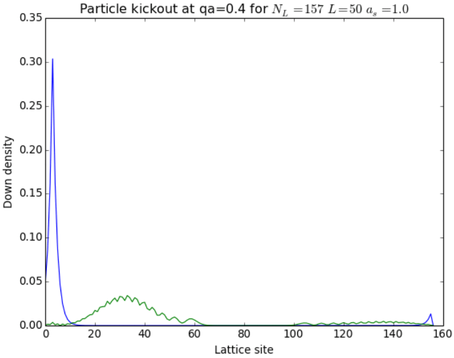

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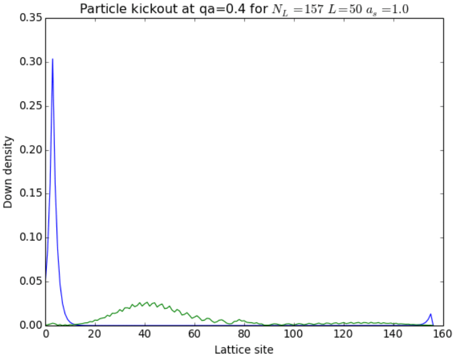

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