Formulating nonabelian gauge theories for a quantum computer
Practical implementation of Wilson’s formulation of the Feynman path integral on a classical computer:

$32^3 \times 64$ lattice size: millions of degrees of freedom
Hilbert space size $\sim e^{\text{millions}}$. Lattice QFT: sample it!
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... $e^{\text{“millions” d}}$ wave function
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1d wave function  2d wave function  … $e^{\text{"millions"}}$ $d$ wave function

Amazingly, the ground state wave function of glue + quark/antiquark pairs is possible to sample effectively.
at zero baryon number
at zero baryon number
...
Problem at nonzero baryon number!
One needs an exponentially large number of samples to approximate the wave function...
Problem at nonzero baryon number: how hard is the sign problem?

CPU effort to study matter at nuclear density in a box of given size
Give or take a few powers of 10...

![Graph showing CPU effort vs box size for different temperatures.]

- CPU effort grows exponentially with $L^3/T\nucl$. 
- $\nucl = 10^{-6}$.
- 10 sec on 1 GF laptop for 2 lattice, $a=0$.
- $\exp(2VNUC/(m_B^3/2))$.
- $\times f(m_B)$.
- 1 Exaflop x year = $10^9$ GFlops x yr.

![Radius of 238U as a reference point.]

neutron stars will take a little while....
Apparently at significant baryon number density the wave function explores a large Hilbert space.

Hilbert space is VERY BIG, growing exponentially with the number of particles.

Classical computers are ill-equipped for searching this space when you don’t know where to start.

Can a quantum computer help?
What do these cartoons mean?
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Just a problem with a bad basis for expanding our wave functions?
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A typical contribution to the QCD vacuum...complicated, but simply generated from local Wilson Yang-Mills action
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A typical contribution to the QCD vacuum...complicated, but simply generated from local Wilson Yang-Mills action

Yes! The “good” basis will exhibit complex entanglement in space, color, spin relative to “theory basis” which is based on principles of locality, gauge invariance, statistics
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E.g. sign problems in QCD at finite density closely related to chiral symmetry breaking and the existence of a light pion.
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QCD @ finite density behaves like a frustrated system...destroys simple long-range pion correlations in vacuum - presumably through entanglement.
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QCD @ finite density behaves like a frustrated system...destroys simple long-range pion correlations in vacuum - presumably through entanglement.

Quantum circuits are an efficient way to create highly entangled nonlocal states from few-qubit interactions...maybe they can help!
Many potential applications for a quantum computer to study the Standard Model and beyond:

- Finite baryon density
- Real time dynamics
- Nontrivial topology
- N=4 SUSY, matrix models for quantum gravity
- Chiral gauge theory...
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All projects require a formulation of nonabelian gauge theories for a quantum computer...

...and that path begins with restricting the Hilbert space to something finite and then understanding how to fix that damage to the theory.
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...and that path begins with restricting the Hilbert space to something finite and then understanding how to fix that damage to the theory.

This talk: a couple comments on Yang-Mills theory

- Truncation in Hamiltonian formulation
- Gauge invariance
Starting point: the Hamiltonian derived from Wilson’s lattice gauge theory (Kogut & Susskind, 1975)
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Gluons allow quark phases to rotate independently...

...like the differential in a car
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Gluons allow quark phases to rotate independently...

...like the differential in a car

- On every link:
  - $U \in G$ (matrix in gauge group)
  - $U \rightarrow L U R^\dagger$
  - $L \in G, R \in G$. 

The continuum Yang-Mills Hamiltonian (no quarks):

- Fix $A_0 = 0$ gauge

\[ H = \frac{1}{2} \left( g^2 \mathbf{E}_a \mathbf{E}_a + \frac{1}{g^2} \mathbf{B}_a \mathbf{B}_a \right), \quad \left[ A^i_a, E^j_b \right] = i\hbar \delta^{ij} \delta_{ab} \]

- Impose Gauss law constraint on physical states:

\[ D_i E_i |\psi\rangle = 0 \]
The continuum Yang-Mills Hamiltonian (no quarks):

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Kogut-Susskind (lattice) Yang-Mills Hamiltonian:

- Fix $U=1$ gauge on temporal links, $U$ on spatial links \( \triangleright \) operators

- $\vec{B}_a \vec{B}_a \rightarrow -\text{Re Tr } \hat{U} \quad \text{(product of U's around plaquette)}$

- $\vec{E}_a \vec{E}_a \rightarrow \hat{\ell}_a^2 = \hat{r}_a^2 \quad \text{(Casimir operator)}$

- $[\hat{\ell}_a, \hat{U}] = -T_a \hat{U}$, \quad $[\hat{r}_a, \hat{U}] = \hat{U} T_a$

- Impose at each site: $\sum (\hat{\ell}_a + \hat{r}_a) |\psi\rangle = 0$
The Hilbert space: the link operators are coordinates in the gauge group, the $\ell_a, r_a$ operators are their conjugates.
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"coordinate" basis:

$$\langle g|g'\rangle = \delta(g - g'), \quad \int dg |g\rangle\langle g| = 1$$
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“coordinate” basis:

\[
\langle g|g' \rangle = \delta(g - g') , \quad \int dg |g\rangle\langle g| = 1
\]

“momentum” basis:

\[
\langle Rab|R' a' b' \rangle = \delta_{RR'}\delta_{aa'}\delta_{bb'} , \quad \sum_{Rab} |Rab\rangle\langle Rab| = 1
\]

\[
\langle Rab|g \rangle \equiv \sqrt{\frac{d_R}{|G|}} D_{ab}^{(R)}(g)
\]

Irreducible representations of \( G \)
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Irreducible representations of \( G \)

A Formulation of Lattice Gauge Theories for Quantum Simulations

*Erez Zohar and Michele Burrello, Phys. Rev. D 91, 054506*
E.g. U(1): particle on a circle

$|g⟩ \rightarrow |φ⟩$, \quad φ ∈ [0, 2π)

$|Rab⟩ \rightarrow |L⟩$, \quad L ∈ Z, \quad D^R_{ab}(g) \rightarrow e^{iLφ}$
E.g. $U(1)$: particle on a circle

$|g\rangle \rightarrow |\phi\rangle$, \hspace{0.5cm} \phi \in [0, 2\pi)$

$|Rab\rangle \rightarrow |L\rangle$, \hspace{0.5cm} L \in \mathbb{Z}, \hspace{0.5cm} D_{ab}^R(g) \rightarrow e^{iL\phi}$

E.g. $SU(2)$: particle on a 3-sphere

$|g\rangle \rightarrow |\tilde{\theta}\rangle$

$|Rab\rangle \rightarrow |jmm'\rangle$, \hspace{0.5cm} D_{ab}^R(g) \rightarrow D_{mm'}^{(j)}(\tilde{\theta})$ \hspace{1cm} (Wigner d-matrices)
E.g. U(1): particle on a circle
\[ |g\rangle \rightarrow |\phi\rangle \ , \ \ \phi \in [0, 2\pi) \]

\[ |R_{ab}\rangle \rightarrow |L\rangle \ , \ \ L \in \mathbb{Z} \ , \ \ D^{R}_{ab}(g) \rightarrow e^{iL\phi} \]

E.g. SU(2): particle on a 3-sphere
\[ |g\rangle \rightarrow |\tilde{\theta}\rangle \]

\[ |R_{ab}\rangle \rightarrow |j m m'\rangle \ , \ \ \ D^{R}_{ab}(g) \rightarrow D^{(j)}_{m m'}(\tilde{\theta}) \]  \hspace{1cm} \text{(Wigner d-matrices)}

Even with spatial lattice, we have an infinite-dimension Hilbert space:

- The |g⟩ states take continuous values
- The |Rab⟩ states are discrete, but there are \( \infty \) of them
“Latticize” G?
“Latticize” $G$?

Nice graphics algorithms, but not lattices (e.g. generally no useful families of finite subgroups of $G$, so no gauge symmetry)...
“Latticize” G?

Nice graphics algorithms, but not lattices (e.g. generally no useful families of finite subgroups of G, so no gauge symmetry)...

.... except for \( Z_N \in U(1) \)
Cutoff on $|\text{Rab}\rangle$ states (canonical momentum cutoff)?

E.g. $U(1)$, cutoff on $L$

- $-L_0$
- $0$
- $L_0$

$L$
Cutoff on $|R_{ab}\rangle$ states (canonical momentum cutoff)?

E.g. $U(1)$, cutoff on $L$

E.g. $SU(2)$, cutoff on $j$: 

\[ L_0 \quad 0 \quad L_0 \]

\[ m \]

\[ j \]

\[ m' \]
Cutoff on $|\text{Rab}\rangle$ states (canonical momentum cutoff)?

E.g. $U(1)$, cutoff on $L$.

E.g. $SU(2)$, cutoff on $j$.

This maintains gauge symmetry, gives finite Hilbert space, but what does it do to the physics?
A cutoff on E ("p") gives dispersion in B ("x")

Photons or gluons have minimum B.B energy contribution...will give a mass gap depending inversely on the cutoff on E.

• Can this be quantified?
• Is there a "Symanzik action", RG group for the effects of this cutoff?
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U(1) example:
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U(1) example:

\[ \hat{U}_{LL'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]
A cutoff on \( E \) ("p") gives dispersion in \( B \) ("x")

Photons or gluons have minimum \( B.B \) energy contribution...will give a mass gap depending inversely on the cutoff on \( E \).

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\( U(1) \) example:

With \( Z_N \) discretization of \( G \), very similar:

\[
\hat{U}_{LL'} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

With cutoff on \( L \):

\[
\hat{U}_{LL'} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
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\end{pmatrix}
\]
A cutoff on $E$ ("p") gives dispersion in $B$ ("x")

Photons or gluons have minimum $B.B$ energy contribution...will give a mass gap depending inversely on the cutoff on $E$.

- Can this be quantified?
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\textbf{U(1) example:}

With cutoff on $L$:

\[ \hat{U}_{LL'} = \begin{pmatrix}
  0 & 1 & 0 & 0 & 0 & 0 \\
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  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix} \]

With $\mathbb{Z}_N$ discretization of $G$,

very similar:

\[ \hat{U}_{LL'} = \begin{pmatrix}
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix} \]

- Confining, Coulomb & Higgs phases
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U(1) example:

With cutoff on L: \[ \hat{U}_{LL'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]

With \( Z_N \) discretization of \( G \), very similar:

\[ \hat{U}_{LL'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]

► Confining, Coulomb & Higgs phases
Nevertheless, toy models on small lattices with low cutoffs can be interesting in their own right, and perhaps feasible in near-term
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Example: “glueballs” in SU(2), 2+1 dimensions, four lattice sites.
Nevertheless, toy models on small lattices with low cutoffs can be interesting in their own right, and perhaps feasible in near-term

Example: “glueballs” in SU(2), 2+1 dimensions, four lattice sites.

minimal:
- no glueballs in 1+1 dimensions
- no bluebells in 2+1 with less than 1 plaquette
\[ |j_{01}, m_{01}, m'_{01}\rangle \quad |j_{02}, m_{02}, m'_{02}\rangle \quad |j_{13}, m_{13}, m'_{13}\rangle \quad |j_{23}, m_{23}, m'_{23}\rangle \]
\[ |j_{01}, m_{01}, m'_{01}\rangle \rightarrow |j_{02}, m_{02}, m'_{02}\rangle \rightarrow |j_{13}, m_{13}, m'_{13}\rangle \rightarrow |j_{23}, m_{23}, m'_{23}\rangle \rightarrow |j_{01}, m_{01}, m'_{01}\rangle \]

\( \ell^\alpha, r^\alpha \in \mathfrak{su}(2) \)
Gauge invariance constraint at each vertex

\[ \ell^\alpha, r^\alpha \in \mathfrak{su}(2) \]
Gauge invariance constraint at each vertex \( \ell^\alpha, r^\alpha \in \mathfrak{su}(2) \)

general state:
\[
|\psi\rangle = |j_0, m_0, m'_0\rangle |j_{13}, m_{13}, m'_{13}\rangle |j_{23}, m_{23}, m'_{23}\rangle |j_{02}, m_{02}, m'_{02}\rangle
\]
The matrix element of the space of gauge invariant states has dimension 2 of terms of the following form with the appropriate weights:

where the subscripts indicate which link, and I have lumped all the links where

For the four link model there are four sites, and each site has a link entering and leaving it. Consider the action of

Putting these results together I find (numerically...need to do analytically)

There are two operators in

For SU(2) this leads to the state

Using the fact that

we have

and

The gauge invariant constraint at each vertex is:

\[ \ell^\alpha, r^\alpha \in \mathfrak{su}(2) \]

The general state is:

\[ |\psi\rangle = |j_{01}, m_{01}, m'_{01}\rangle |j_{13}, m_{13}, m'_{13}\rangle |j_{23}, m_{23}, m'_{23}\rangle |j_{02}, m_{02}, m'_{02}\rangle \]

gauge invariant state:

\[ |\mathcal{I}\rangle = \frac{1}{(2j + 1)^2} \sum_{m_i = -j}^j (-1)^{m_0 + m_3} |j, m_0, m_1\rangle_{[01]} |j, -m_0, m_2\rangle_{[02]} |j, m_1, m_3\rangle_{[13]} |j, m_2, -m_3\rangle_{[23]} \]
SU(2) Hilbert space for one link, cut off at $j=3$

Hilbert space dimension for $L$ links, cutoff $J$:

$$\left[ \sum_{j=0}^{J} (2j + 1)^2 \right]^L = \left[ \frac{(1 + J)(1 + 2J)(3 + 4J)}{3} \right]^L$$

140 states
4-link SU(2) model:
4-link SU(2) model:

general state:

$$|\psi\rangle = |j_{01}, m_{01}, m'_{01}\rangle |j_{13}, m_{13}, m'_{13}\rangle |j_{23}, m_{23}, m'_{23}\rangle |j_{02}, m_{02}, m'_{02}\rangle$$

dimension of Hilbert space with cutoff $J$:

$$D = \left[ \frac{(1 + J)(1 + 2J)(3 + 4J)}{3} \right]^4$$
For the matrix element of

The matrix element of the space of gauge invariant states has dimension 2

We see that for this to transform as a singlet will require (i) where the subscripts indicate which link, and I have lumped all the links where acting at site

For the four link model there are four sites, and each site has a link entering and leaving it. Consider the action of

For SU(2) this leads to the state

\[ |\psi_i \rangle = j_{01}, m_{01}, m_{01}' \rangle j_{13}, m_{13}, m_{13}' \rangle j_{23}, m_{23}, m_{23}' \rangle j_{02}, m_{02}, m_{02}' \rangle \]

The dimension of the Hilbert space with cutoff \( J \):

\[ D = \left[ \frac{(1 + J)(1 + 2J)(3 + 4J)}{3} \right]^4 \]

gauge invariant state:

\[ |\psi_j \rangle = \frac{1}{(2j + 1)^2} \sum_{m_i = -j}^j (-1)^{(m_0 + m_3)} |j, m_0, m_1 \rangle_{[01]} |j, -m_0, m_2 \rangle_{[02]} |j, m_1, m_3 \rangle_{[13]} |j, m_2, -m_3 \rangle_{[23]} \]

Same \( j \) on all links; all \( m \)'s summed

dimension of gauge invariant subspace with cutoff \( J \):

\[ D_{\text{inv}} = 2J + 1 \]
4-link SU(2) model:

**general state:**

\[ |\psi\rangle = |j_{01}, m_{01}, m'_{01}\rangle |j_{13}, m_{13}, m'_{13}\rangle |j_{23}, m_{23}, m'_{23}\rangle |j_{02}, m_{02}, m'_{02}\rangle \]

**dimension of Hilbert space with cutoff \( J \):**

\[ D = \left[ \frac{(1 + J)(1 + 2J)(3 + 4J)}{3} \right]^4 \]

**gauge invariant state:**

\[ |\psi_j\rangle = \frac{1}{(2j + 1)^2} \sum_{m_i=-j}^{j} (-1)^{-(m_0+m_3)} |j, m_0, m_1\rangle_{[01]} |j, -m_0, m_2\rangle_{[02]} |j, m_1, m_3\rangle_{[13]} |j, m_2, -m_3\rangle_{[23]} \]

**dimension of gauge invariant subspace with cutoff \( J \):**

\[ D_{\text{inv}} = 2J + 1 \]

e.g.: \( J=3 \):

\[ D = 384, 160, 000 \]

\[ D_{\text{inv}} = 7 \]

\[ \geq 29 \text{ qubits} \]

\[ \geq 3 \text{ qubits} \]

D. B. Kaplan ~ Argonne Nat’l Lab ~ 3/29/18
If you aren’t shocked by gauge invariance, you haven’t understood it!
The SU(2) glue ball spectrum can be calculated quickly (Mathematica) for this simple system (because gauge invariance can be imposed analytically):

For low cutoff, can this be simulated on an existing quantum computer? Stay tuned.
Conclusions:

• Small scale nonabelian gauge theories, far from the continuum limit, can likely be simulated in on a digital quantum computer in the near term (like U(1) Schwinger model, M. Savage talk)

• There exists a straightforward formalism for representing gauge theories with a finite Hilbert space, suitable for computation

• Theorists need to better understand the physics of the cutoff on the Hilbert space for eventual large scale computations

• A vast majority of states simulated are unphysical unless the Hamiltonian can be projected onto the gauge invariant states... E.g. by using dual gauge fields? \[ \vec{B} = \vec{\nabla} \times \vec{a}, \quad \vec{E} = \vec{\nabla} \times \vec{b} \]
“Looks like a fair amount of overtime might be called for”