Can quantum computers aid light cone Hamiltonian calculations of partonic structure?

Adam Freese

Argonne National Laboratory

March 28, 2018

Why Hamiltonians?

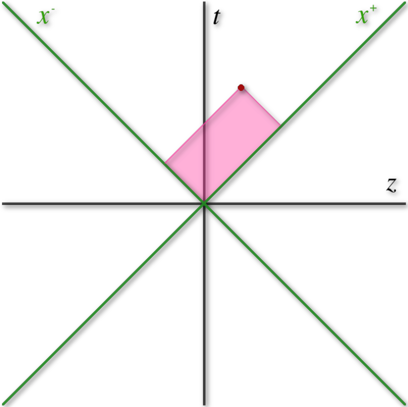
- Hamiltonian picture is more natural than Lagrangian in the AMO/condensed matter settings where quantum simulation is done.
- Jordan-Wigner transform allows interaction Hamiltonian for a lattice gauge theory to be mapped onto a Hamiltonian for spin interactions.
- The Hamiltonian picture allows dynamical, real-time descriptions of physical processes.
- **The challenge**: How can the Hamiltonian of QCD be mapped onto a real quantum simulator?
  - How can this be used to compute *the quark and gluon content of hadrons*?
- **The proposal**: *Light cone quantization* offers a simpler Hamiltonian formulation of QCD and other gauge theories.
  - Though it is geared towards momentum space rather than configuration space.

What is light cone quantization?

- Define **light cone coordinates**:

$$x^+ = t + z$$
$$x^- = t - z$$
$$\mathbf{x}_\perp = (x, y)$$

- Off-diagonal metric: $s^2 = x^+ x^- - \mathbf{x}_\perp^2$.
- The "energy" P^- generates translations in the "time" x^+.



Why light cone quantization?

- The Poincare group generators look like this:

Translations	**Boosts**	**Rotations**
$P^+ = P^0 + P^z$	$B_{\perp 1} = (K_1 + J_2)$	$S_{\perp 1} = (K_1 - J_2)$
$P^- = P^0 - P^z$	$B_{\perp 2} = (K_2 - J_1)$	$S_{\perp 2} = (K_2 + J_1)$
$\mathbf{P}_\perp = (P_x, P_y)$	K_3	J_3

- The operators J_3, $\mathbf{P}_\perp$, and $\mathbf{B}_\perp$ generate a **Galilean subgroup** in the transverse plane.
- The longitudinal boost K_3 merely rescales all these operators.
- The theory is invariant under both K_3 and the Galilean subgroup.
- This produces **non-relativistic group structure** on the light cone.
    - Promising if we want to use a non-relativistic Hamiltonian to simulate QCD!

Why light cone quantization?

- The dispersion relation includes **no square roots**:

$$i\partial_+|\Psi\rangle = p^-|\Psi\rangle = \left(\frac{m^2 + \mathbf{p}_\perp^2}{p^+}\right)|\Psi\rangle$$

- A remarkably similar structure to the non-relativistic dispersion relation in two dimensions.
- The true Hamiltonian of a field theory is more complicated—it involves integrals of fields.
- But in light cone quantization, the full Hamiltonian is **the sum of free and interaction parts**:

$$H = T + U = T + (V + F + S + C)$$

See Brodsky *et al.*, Phys. Rep. 301 (1998) for full description of the terms.

Fock state decomposition

- Fock space decomposition gives us a particle content picture.
- This picture falls naturally out of light cone quantization, and is **invariant under boosts and the Galilean subgroup**.
- Fock space is the basis we want to use if we're aiming to compute the quark/gluon content of a hadron.
- In light cone quantization, the Schwinger model bound state has a very simple particle content: it's just an electron and a positron!

On a computer: discretized light cone quantization

- Traditionally, computations are done in *momentum space*.
- Periodic boundary conditions are used, too.
  - This ensures conservation of charges (energy, color, *etc.*).
- P^+ has a discrete spectrum; choosing an eigenvalue for P^+ sets the **harmonic resolution** and truncates the Fock space.
- One constructs all color-singlet states for some P^+ and then diagonalizes the Hamiltonian P^-.
- $p^+ \to \infty$ returns the continuum limit.
- **The difficulty**: a harmonic resolution as low as 5 can produce a Hilbert space as big as 10^{20} dimensions.
  - This is not a tractable problem on a classical computer.
  - *Can a quantum computer make this more tractable?*

Challenges

- DLCQ computations are traditionally done in momentum space.
  - Can quantum simulations be done in momentum space?
    - With periodic boundary conditions?
  - Can the state space of an AMO or condensed matter system simulate a truncated Fock space?
  - Is this a problem better posed for a universal quantum computer? (*e.g.*, running a fast matrix diagonalization algorithm?)