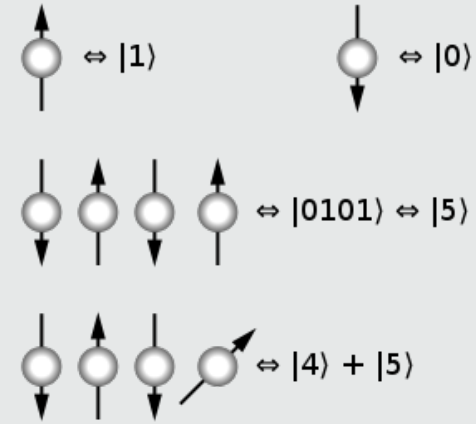
THE QCD ABACUS

circa 2400 b.c Abacus

?

circa 20xx a.d.



qubits can be in a superposition of all the clasically allowed states

Rich Brower, Boston University

March 28, 2018

NP_CI Argonne

QUANTUM LINKS (AKA D-THEORY)

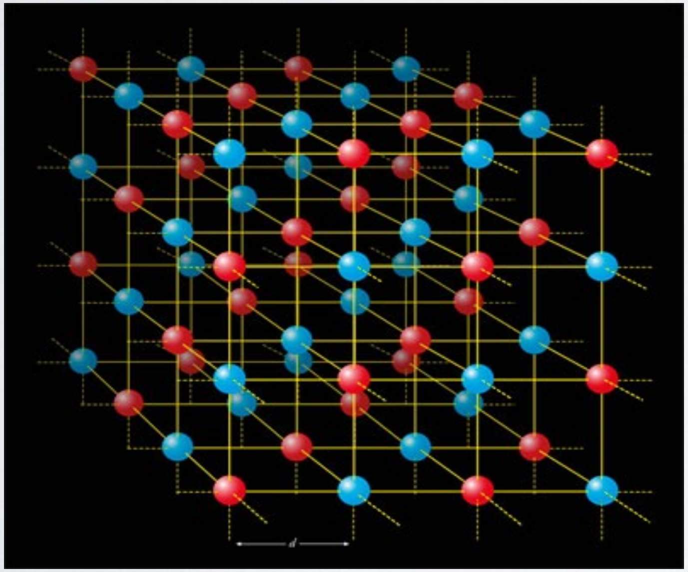
Early effort to developed "efficient classical computing in Lattice Gauge Theory"!

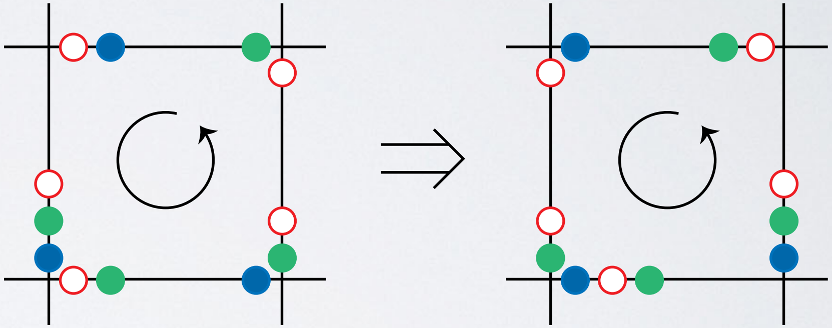
- *D. Horn, Finite Matrix Model with Continuous Local Gauge Inv. Phys. Lett. B100 (1981)*
- *P. Orland, D. Rohrlich, Lattice Gauge Magnets: Local Isospin From Spin Nucl. Phys. B338 (1990) 647*
- *S. Chandrasekharan, U-J Wiese Quantum links models: A discrete approach to gauge theories Nucl. Phys. B492 (1997)*
- *R. C. Brower, S. Chandrasekharan, U-J Wise , QCD as quantum link model, Phys. Rev D 60 (1999).*
- *R. C. Brower, The QCD Abacus: APCTP-ICPT Conference, Seoul, Korea, May (1997)*
- *R. C. Brower, S. Chandrasekharan, U-J Wiese, D-theory: Field quantization … discrete variable Nucl. Phys. B (2004).*
- *U-J Wiese, See recent talk at FNAL QC workshop Dec 6-7, 2017 & lots of published work and many others.*

For bosons I will use the "Fermionic pairs" (or what Uwe-Jens calls the "rishon")

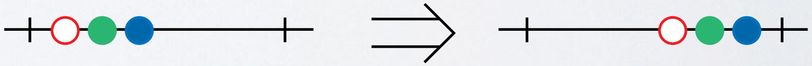
4D QUANTUM LINKS

$$\hat{H} = \beta \sum_{x,\mu\neq\nu} \mathrm{Tr}[\hat{U}_{x,\mu}\hat{U}_{x+\hat{\mu},\nu}\hat{U}^{\dagger}_{x+\hat{\nu},\mu}\hat{U}^{\dagger}_{x,\nu}] + \sum_{x,\mu} [\det\hat{U}_{x,\mu} + \det\hat{U}^{\dagger}_{x,\mu}]$$



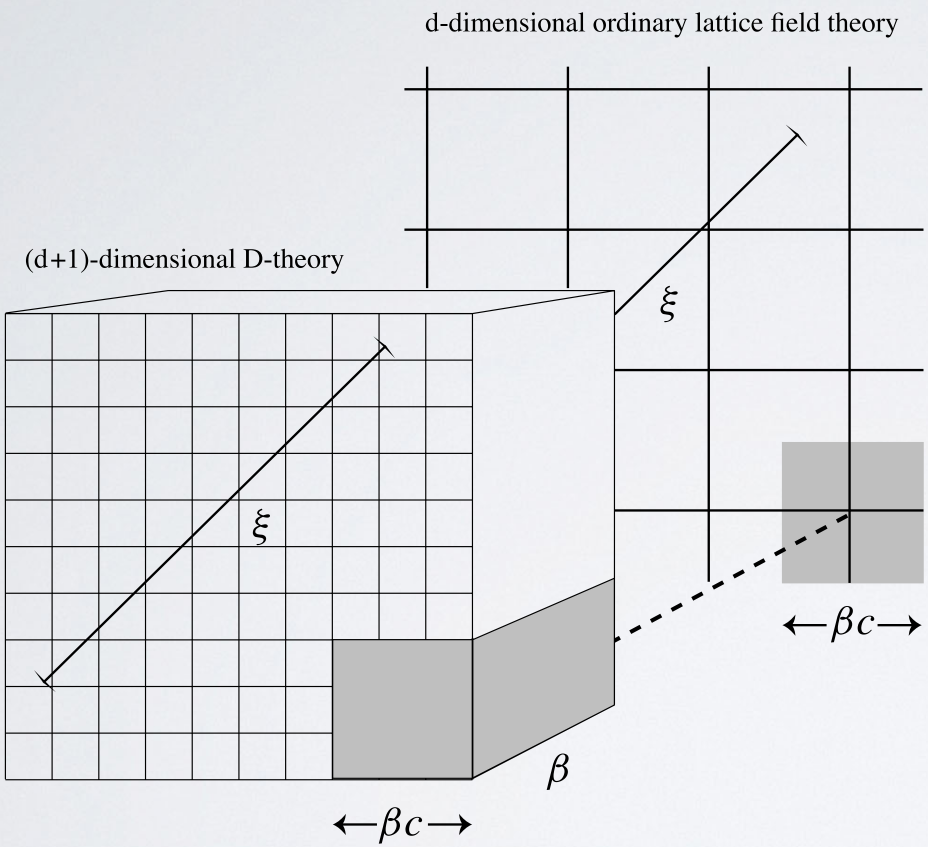


Tr Up



det U $_{x,\mu}$

D-THEORY: FERMIONIC GAUGE, SCALAR AND DIRAC LATTICES



$$Z = \mathrm{Tr}\exp(-\beta \hat{H}).$$

Original Motivation: MAKE
Easy Bosonic into Hard Fermionic
Theories with maybe smart cluster
(aka worm) algorithms?

$$\xi = 1/am \sim \exp[c\beta/g^2]$$

Asymptotic Freedom of 2d spin and 4d gauge implies exponential dimensional reduction as $\beta \to \infty$

QUANTUM SPINS: O(3)

CLASSICAL O(3) MODEL

$$Z = \int \mathcal{D}\vec{s}\ \exp(-\frac{1}{g}S[\vec{s}]) \qquad S[\vec{s}] = -\sum_{\langle x,y\rangle} \vec{s}_x \cdot \vec{s}_y \quad , \quad \vec{s}_x \cdot s_x = 0$$

$$\vec{s}_x \to a_i^\dagger(x)\vec{\sigma}^{ij}a_j(x) \quad \text{or} \quad \hat{S}_{ij} = a_i^\dagger(x)a_j(x) \quad \text{with} \quad Tr[\hat{S}_x] = 0;$$

$$\{a_i(x), a_j^\dagger(x)\} = \delta_{ij}\delta_{xy} \quad , \quad \{a_i^\dagger(x)a_j^\dagger(x)\} = 0 \quad , \quad \{a_i(x), a_j(x)\} = 0$$

QUANTUM O(3)

$$Z = \mathrm{Tr}\exp(-\beta\hat{H}). \qquad \hat{H} = \sum_{\langle x,y\rangle} Tr[\hat{S}_x\hat{S}_y] \quad , \quad Tr[\hat{S}_x] = 0$$

Global Rotation: $\vec{J} = \sum_x Tr[\vec{\sigma}\hat{S}_x] \implies [\vec{J}, \hat{H}] = 0$

Local Fermion No: $\hat{F}_x = Tr[\hat{S}_x] \implies [\hat{F}_x, \hat{H}] = 0$

OPERATOR QUANTUM GAUGE LINK

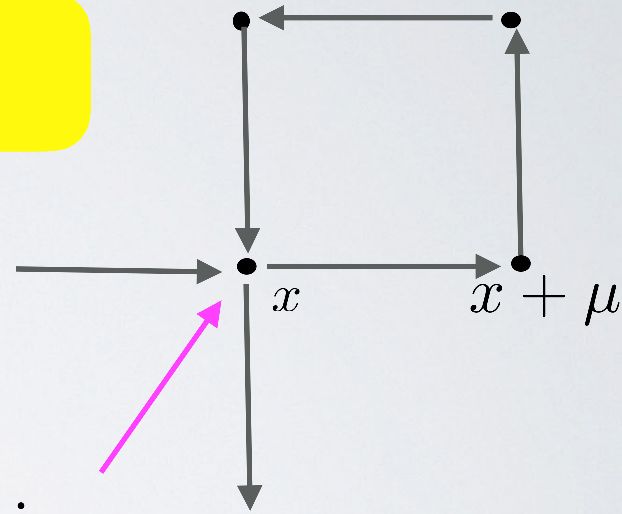
On each link (x,x $+\ \mu$) introduce $2N_c$complex fermion

$a_, a_i^\dagger$ right(+) moving and $b_j, b_j^\dagger$ left(-) moving fluxon

LINK:

$$a_i \longrightarrow b_j^\dagger$$

$$U_{ij}(x, x+\mu) \quad \to \quad \hat{U}_{ij} = a_i(x)\, b_j^\dagger(x+\mu)$$



$$\{a_i, a_j^\dagger\} = \delta_{ij} \qquad \{b_i, b_j^\dagger\} = \delta_{ij}$$

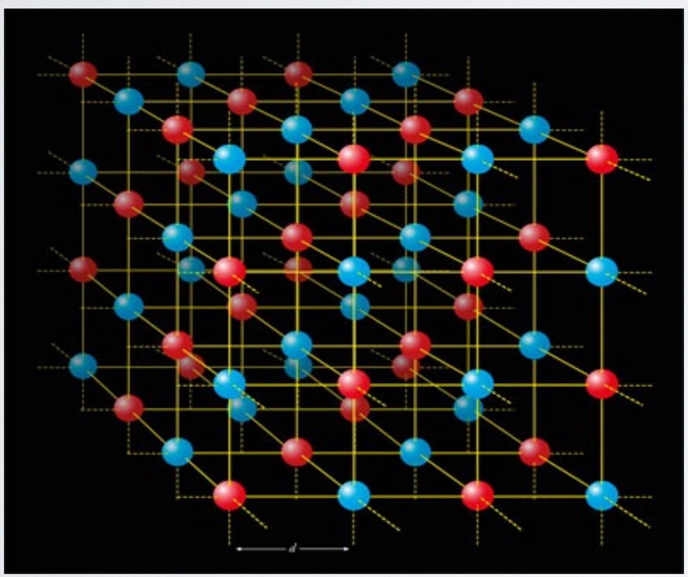
Local Gaue Operators $\quad \Omega_{ij}(x) = a_i^\dagger(x) a_j(x) + \cdots$

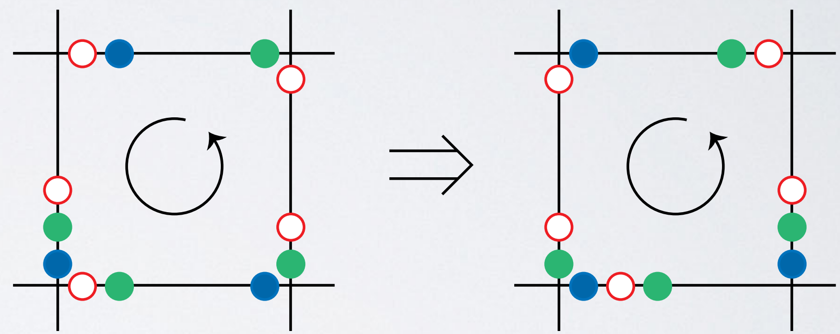
For SU(3) QCD have SU(6) per Link Lie Algebra $\quad \begin{bmatrix} a_i a_j^\dagger & a_i b_j^\dagger \\ b_i a_j^\dagger & b_i b_j^\dagger \end{bmatrix}$

Hilbert Space is a large qubit array of color vectors in 4d space-time

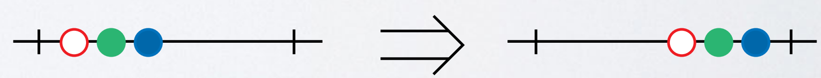
5TH AXIS FINITE

$$\hat{H} = \beta \sum_{x,\mu\neq\nu} \mathrm{Tr}[\hat{U}_{x,\mu}\hat{U}_{x+\hat{\mu},\nu}\hat{U}^{\dagger}_{x+\hat{\nu},\mu}\hat{U}^{\dagger}_{x,\nu}] + \sum_{x,\mu} [\mathrm{det}\hat{U}_{x,\mu} + \mathrm{det}\hat{U}^{\dagger}_{x,\mu}]$$





Tr Up



det U $_{x,\mu}$

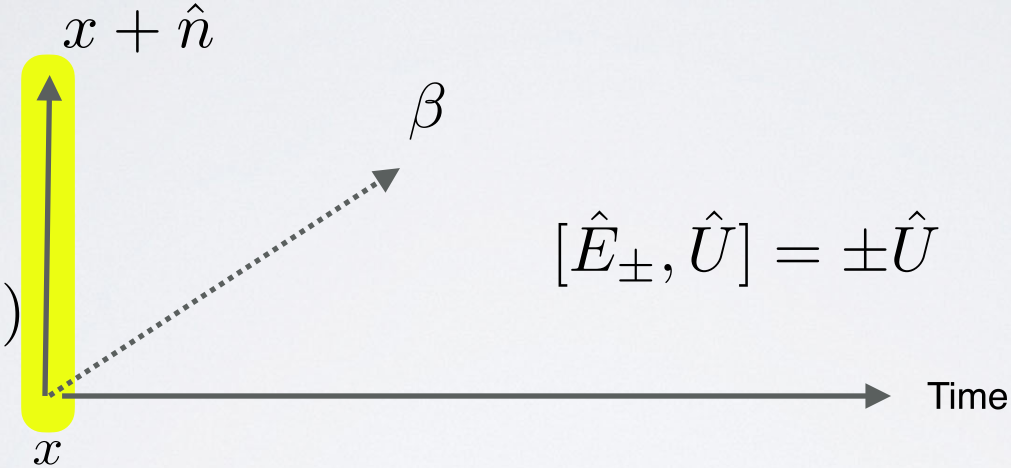
QC MINKOWSKI DYAMICS

- Discrete 5th Axis Ls Layers (with Domain Wall Fermions)
- Change mu = 4 axis to Time with "Domain Wall" Hamiltonian.

LINK $\hat{U}_{ij}$

$\hat{E}_{ij}(x, x+\hat{n})$

$\hat{n} = 1, 2, 3$



$$[\hat{E}_{\pm}, \hat{U}] = \pm\hat{U}$$

$$\hat{H}_0 = -\frac{1}{4g^2}\sum_{\square}\mathrm{Tr}[\hat{U}_{\square} + \hat{U}^{\dagger}_{\square}] + g^2\sum_{\langle x,y\rangle} Tr[\hat{E}^2_{+}(x,y) + \hat{E}^2_{-}(x,y] + \text{quarks}$$

SEE: Uwe-Jens Wiese, Ultracold Quantum Gases and Lattice Systems: Quantum Simulation of Lattice Gauge Theories Nucl. Phys. A931 (2014) 246-256

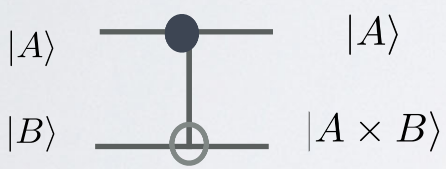
FERMIONIC QUBIT GATES

Each $a^\dagger a + a a^\dagger = 1$ $aa = a^\dagger a^\dagger = 0$

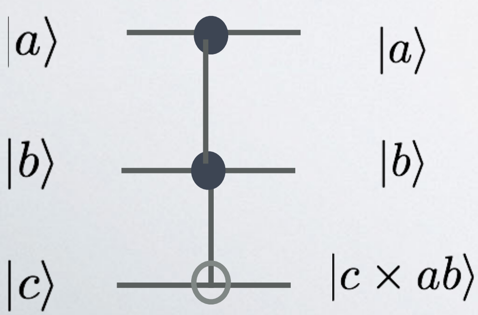
On Each qubit $a^\dagger(\alpha|1\rangle + \beta|0\rangle) = \beta|1\rangle$ $a(\alpha|1\rangle + \beta|0\rangle) = \alpha|0\rangle$

$$a^\dagger + a \implies X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (a + a^\dagger)^2 - 1 \implies H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$(a^\dagger + a)/i \implies Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad (2a^\dagger a - 1) \implies Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



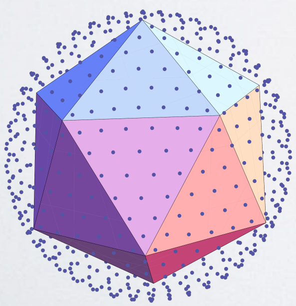
$$(1 - a^\dagger a) + a^\dagger a (b + b^\dagger) \implies \text{ContrNOT}$$

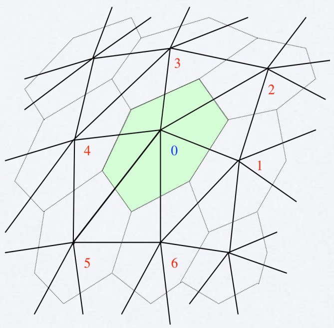


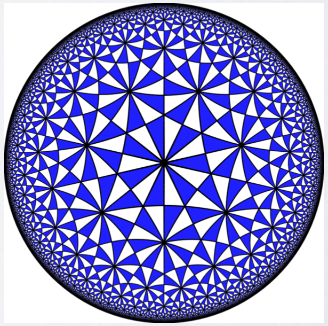
$$(1 - a^\dagger a b^\dagger b) + a^\dagger a b^\dagger b (c^\dagger + c) \implies \text{Toffoli}$$

QUANTUM "RISHONS" FOR GAUGE & SPIN & DIRAC ?

- HOW GENERAL IS TOTAL FERMIONIC APPROACH?
- WHAT CLASSICAL COMPLEXITY VS   QC COMPLEXITY?
- START WITH 1+1 and 2 + 1
- Set up by CFT wavefunction ?
- More General Simplicial Lattice for CFT & Gauge/AdS ?







See https://arxiv.org/abs/1803.08512   https://arxiv.org/abs/1610.08587