Quantum sensing and simulation with single plane crystals of trapped ions

John Bollinger
NIST-Boulder
Ion storage group

Justin Bohnet (Honeywell), Kevin Gilmore,
Elena Jordan, Brian Sawyer (GTRI),
Joe Britton (ARL)

theory – Rey group (JILA/NIST)
Freericks group (Georgetown)
Dan Dubin (UCSD)

• motional amplitude sensing
• quantum simulation – measure quantum dynamics with OTOC
Outline:

- Penning trap features
  - high field qubit, modes

- sensing small COM (center-of-mass) motion
  - spin-dependent forces

- Quantum simulation with ion crystals in a Penning trap
  - engineering Ising interactions with spin-dependent forces

\[ H_{\text{Ising}} = \frac{1}{N} \sum_{i<j} J_{i,j} \sigma_i^z \sigma_j^z \]

- Loschmidt echo and out-of-time order correlation functions
Penning trap: many particle confinement with static fields

- radial confinement due to rotation – ion plasma rotates $\nu_\theta = \omega_r \ r$ due to $E \times B$ fields in rotating frame, Lorentz force is directed radially inward

$$
\varphi_{\text{trap}} (r, z) \approx \frac{1}{2} m \omega_z^2 \left(z^2 - \frac{r^2}{2}\right) \quad \Rightarrow
$$

$$
\varphi_{\text{rot}} (r, z) = \frac{1}{2} m \omega_z^2 \left(z^2 + \left(\frac{\omega_r (\Omega_c - \omega_r)}{\omega_z^2} - \frac{1}{2}\right) r^2\right)
$$

$^9\text{Be}^+, B_0 = 4.5 \ T$

$$
\frac{\Omega_c}{2\pi} \sim 7.6 \ MHz, \ \frac{\omega_z}{2\pi} \sim 1.6 \ MHz, \ \frac{\omega_m}{2\pi} \sim 160 \ kHz
$$
Ion crystals form as a result of minimizing Coulomb potential energy

\[ T \rightarrow 0.4 \text{ mK (Doppler laser cooling)} \Rightarrow \frac{q^2}{a_{WS}} \gg k_B T, 2a_{WS} \sim \text{ion spacing} \]

Type of crystal, nearest neighbor ion spacing depend on \( \omega_r \).


Bcc crystals with \( N > 100 \text{ k} \) observed with:
- Bragg scattering
- Ion fluorescence imaging

14 \( \mu \text{m} \)
0.5 mm
Precise $\omega_r$ control with a rotating electric field

$$V_{\text{sector}} = V_{\text{Wall}} \sin(\omega_{\text{drive}} t + \phi)$$

$\phi = 0^\circ$

$$\omega_{\text{wall}} = \omega_{\text{drive}} / 2$$

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torque drives

$\omega_r = \omega_{\text{wall}}$
Precise $\omega_r$ control with a rotating electric field

\[ V_{\text{sector}} = V_{\text{Wall}} \sin(\omega_{\text{drive}}t + \phi) \]

\[ \phi = 0^\circ \]

\[ \omega_{\text{wall}} = \omega_{\text{drive}} / 2 \]

\[ \omega_r = \omega_{\text{wall}} \]

Rotating wall electrodes

Torque drives $\omega_r$ = $\omega_{\text{wall}}$
Be\(^+\) high magnetic field qubit

\[ ^9\text{Be}^+ , \ B \sim 4.5 \ T, \ \omega_0 / 2\pi \sim 124.1 \text{ GHz} \]

\[ H_{\mu W} = \sum_i B_\perp \hat{\sigma}_i^x , \]

\[ B_\perp > 10 - 15 \text{ kHz} \]
Transverse (drumhead) modes

\[ E \times B \text{ modes} \quad \text{transverse modes} \quad \text{cyclotron modes} \]

\[ \omega_m \quad \omega_z \quad \Omega_c \]
Transverse (drumhead) modes

Modes characterized by eigenfrequency $\omega_m$ and eigenvector $b_{i,m}$

Freericks group, PRA (2013)
Baltrush, Negretti, Taylor, Calarco, PRA (2011)
Dubin, UCSD
Transverse (drumhead) modes

$E \times B$ modes
transverse modes
cyclotron modes

$\omega_m$
$\omega_z$
$\Omega_c$

Spin precession

Measure mode spectrum with spin-dependent force

Spin-dependent force frequency $\mu$ (kHz)

$\omega_z$
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- Loschmidt echo and out-of-time order correlation functions
Motional amplitude sensing or
Trapped ions as sensitive $\vec{E}$-field and force detectors

Maiwald, et al., Nature Physics 2009 – 1 yN Hz$^{-1/2}$
Hempel et al., Nature Photonics 2013 – detect single photon recoil
Shaniv, Ozeri, Nature Communications, 2017 – high sensitivity ($\sim$28 zN Hz$^{-1/2}$) at low frequencies

Biercuk et al., Nature Nanotechnology, 2010 – 100-ion crystal (400 yN Hz$^{-1/2}$)

Basic idea: map motional amplitude onto spin precession

<table>
<thead>
<tr>
<th>Single ion</th>
<th>N ion crystal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$\leftrightarrow$</td>
<td>$\leftrightarrow$</td>
</tr>
</tbody>
</table>

$\leftrightarrow$ for N=100

$N$ ion crystal
- Less projection noise
- Smaller zero-point motion, $z_{zpt} \approx 2$ nm

$\sim \frac{1}{\sqrt{N}}$

N+1 levels

$N=61$

$N=124$
Sensing small center-of-mass motion

\[ F_{\uparrow}(t) = -F_{\downarrow}(t) = F_0 \cos(\mu t) \]

\[ H_I = \sum_i F_0 \cos(\mu t) \hat{z}_i \hat{\sigma}_i^z \]

Implement classical COM oscillation:
\[ \hat{z}_i \rightarrow \hat{z}_i + Z_c \cos(\omega t + \phi) \]

\[ H_I \approx F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \sum_i \frac{\hat{\sigma}_i^z}{2} \]
\[ = F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \hat{S}_z \]

For \( \mu = \omega \), produces spin precession with rate \( \propto F_0 \cdot Z_c \cos(\phi) \)
Measuring spin precession

Precession $\theta$,
\[
\theta = \frac{F_0}{\hbar} Z_c \tau \cos(\phi)
\]
\[-\frac{F_0}{\hbar} Z_c \tau < \theta < \frac{F_0}{\hbar} Z_c \tau \]

Probability of measuring spin up:
\[
\langle P_\uparrow \rangle = \frac{1}{2} \left( 1 - e^{-\Gamma \tau} \langle \cos \theta \rangle \right)
\]
\[= \frac{1}{2} \left( 1 - e^{-\Gamma \tau} J_0 \left( \frac{F_0}{\hbar} Z_c \tau \right) \right)\]
Measuring spin precession

\[ \langle P_\uparrow \rangle (\%) \]

ODF Difference Frequency \(\mu\) (kHz)

\[ \hat{x}, \hat{y}, \hat{z} \]

COM mode

tilt mode

\[ \omega \]

\[ \omega_z \]
Sensitivity limits/ signal-to-noise

\[ \frac{Z_c^2}{\delta Z_c^2} \approx \frac{\langle P_{\uparrow} \rangle - \langle P_{\uparrow} \rangle_{bck}}{\delta (\langle P_{\uparrow} \rangle - \langle P_{\uparrow} \rangle_{bck})} \]

SNR limited due to noise from fluctuations in \( \phi \)

50 pm – smallest detected amplitude

Small signal limits due to:
projection noise
spontaneous emission

\[ \left. \frac{Z_c^2}{\delta Z_c^2} \right|_{\text{limiting}} = \left[ \frac{Z_c}{0.2 \text{ nm}} \right]^2 \]

Gilmore et al.,
PRL 2017
Sensing small center-of-mass motion

Future:

- Fixed phase sensing off-resonance (i.e. fixed $\phi$ in $Z_c \cos(\omega t + \phi)$)
  - 74 pm in single experimental trial
  - $18 \text{ pm}/\sqrt{\text{Hz}}$
  - Exploit spins: squeezed states

- On-resonance with COM mode
  - Enhance force and electric field sensitivities by $Q \sim 10^6$
  - Protocols for evading zero-point fluctuations, backaction ??
  - 20 pm amplitude from a resonant 100 ms coherent drive
    - force/ion of $5 \times 10^{-5}$ yN
    - electric field of 0.35 nV/m
Potential for dark matter search  
(axions and hidden photons)

20 pm amplitude from a resonant 100 ms coherent drive

- force/ion of $5 \times 10^{-5}$ yN
- electric field of 0.35 nV/m

$\epsilon = \frac{E}{3.3 \frac{nV}{m}} \times 10^{-12}$

Technical improvement: EIT cooling

Morigi PRA 67 (2003); exp results with smaller ion numbers: Innsbruck, NIST

\[ ^9\text{Be}^+ \]

\[ B = 4.45 \text{T} \]

\[ \text{level splitting} \sim 80 \text{ GHz} \]

\[ 2s^2p^2P_{3/2} \]

\[ 2s^2p^2P_{1/2} \]

\[ 2s^2s^2S_{1/2} \]

Doppler cooling

Doppler, t_arm: 300 \text{ us}
Technical improvement: EIT cooling

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$^9\text{Be}^+$

B = 4.45T

$2s^2p^2 P_{3/2}$

$2s^2p^2 P_{1/2}$

Doppler cooling

$2s^2s^2 S_{1/2}$

EIT, offset_f: 301.800, t_arm: 300 us, t_cool: 200 us

Bright Fraction

ODF Difference Frequency [kHz]

COM mode

$\omega_{\text{COM}} = 2\pi \times 1.57\text{MHz}$
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Implement classical COM oscillation: \( \hat{z}_i \rightarrow \hat{z}_i + Z_c \cos(\omega t + \phi) \)

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\[ = F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \hat{S}_z \]
Engineering quantum magnetic couplings

\[
\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \hat{z}_j \cdot \hat{\sigma}_j^z =
\]

\[
\sum_{m=1}^{N} b_{jm} \sqrt{\frac{\hbar}{2M\omega_m}} (\hat{a}_m^* e^{i\omega_m t} + \hat{a}_m e^{-i\omega_m t})
\]

N drumhead eigenvalues \(\omega_m\) and eigenvector \(\vec{b}_m\)

Infinite range \(\implies\) Single axis twisting

\[
H_{Ising} = \frac{J}{N} \sum_{i<j} \sigma_i^z \sigma_j^z = \frac{2J}{N} S_Z^2
\]

where \(S_Z = \sum_i \frac{\sigma_i^z}{2}\)

generates a “cat state” \(\frac{1}{\sqrt{2}} \{|\uparrow\uparrow\uparrow\cdots\uparrow\rangle_x + |\downarrow\downarrow\downarrow\cdots\downarrow\rangle_x\}\)

at long times \(\tau\), such that \(\frac{2J}{N} \tau = \frac{\pi}{2}\)

 Produces spin
• useful metrology
• source of decoherence
Benchmarking quantum dynamics

- employ infinite range interactions $H_{Ising} \approx \frac{2J}{N} S_z^2$, $S_z \equiv \sum_i \sigma_i^z / 2$
- prepare eigenstate of $H_\perp = \sum_i B_\perp \hat{\sigma}_i^x$, turn on $H_{Ising}$

Cool  

$\mu$Wave

ODF time

$\hat{Z}$

$\hat{X}$

$\hat{Y}$

general rotation

measure global spin polarization $\langle \hat{S}_z \rangle$, variance $\Delta S_z^2 = \left\langle (\hat{S}_z - \langle \hat{S}_z \rangle)^2 \right\rangle$

$\sum_i \sigma_i^z / 2$
• Measurements of Ramsey squeezing parameter ⇒
  prove entanglement for $25 < N < 220$
• Largest inferred squeezing: -6.0 dB
Benchmarking quantum dynamics

Bohnet et al., *Science* 352, 1297 (2016)

Spin variance $(\Delta S_\psi^2/N)/4$ (dB)

Tomography angle $\psi$ (deg)

- $\tau = 0.666$ ms
- $\tau = 1.2$ ms
- $\tau = 2.6$ ms

$N=85$
Out-of-time-order correlation functions

\[ F(t) \equiv \langle \psi | W(t)^\dagger V^\dagger W(t) V | \psi \rangle \] where \[ W(t) = e^{iHt} W(0) e^{-iHt} \], \[ [V, W(0)] = 0 \]

\[ \text{Re}[F(t)] = 1 - \langle ||[W(t), V]|^2 \rangle / 2 \]

⇒ measures failure of initially commuting operators to commute at later times
⇒ quantifies spread or scrambling of quantum information across a system’s degrees of freedom


Difficult to measure ⇔ requires time-reversal of dynamics

time reversal is possible in many quantum simulators!
Time reversal of the Ising dynamics

\[ H_{\text{Ising}} = \frac{J}{N} \sum_{i<j} \hat{\sigma}_i^z \hat{\sigma}_j^z, \quad \frac{J}{N} \approx \frac{F_0^2}{\hbar 4m \omega_z} \cdot \frac{1}{\mu - \omega_z} \]

Change \( \mu = \omega_z + \delta \) (antiferromagnetic) to \( \mu = \omega_z - \delta \) (ferromagnetic)
Multiple quantum coherence protocol

- Probe higher-order coherences and correlations (Pines group, 1985)

\[ |\psi_0\rangle, \langle \psi_0 | \psi_f \rangle^2 \]
Multiple quantum coherence protocol

\[ |\psi_0\rangle \]

\[ \begin{array}{c}
\text{Cool} \quad R_y \quad H_{Ising} \quad R_x(\phi) \quad -H_{Ising} \quad R_y \quad \text{Detection}
\end{array} \]

\[ \tau \quad \tau \quad \text{time} \]

\[ \langle S_x \rangle = \langle \psi_0 | e^{iH_{Ising} \tau} e^{i\phi S_x} e^{-iH_{Ising} \tau} S_x e^{iH_{Ising} \tau} e^{-i\phi S_x} e^{-iH_{Ising} \tau} |\psi_0\rangle \]

\[ = \frac{2}{N} \langle \psi_0 | e^{iH_{Ising} \tau} W^\dagger e^{-iH_{Ising} \tau} V^\dagger e^{iH_{Ising} \tau} W e^{-iH_{Ising} \tau} V |\psi_0\rangle \]

\[ W^\dagger(t) \quad V^\dagger(0) \quad W(t) \quad V(0) \]

Out-of-time-order correlation (OTOC) function

⇒ quantifies spread or scrambling of quantum information across a system’s degrees of freedom

Multiple quantum coherence protocol

\[ |\psi_0\rangle \]

\[ \langle S_x \rangle = \langle \psi_0 | e^{iH_{Ising} \tau} e^{i\phi S_x} e^{-iH_{Ising} \tau} S_x e^{iH_{Ising} \tau} e^{-i\phi S_x} e^{-iH_{Ising} \tau} |\psi_0\rangle \]

\[ = \sum_m \langle \psi | C_m |\psi \rangle e^{i\phi m} \quad C_m = \sum \sigma_1^z \sigma_4^y \ldots \sigma_k^z \]

At least m terms

\[ m^{th} \text{ order Fourier coefficient} \langle \psi | C_m |\psi \rangle \text{ indicates} \quad |\psi\rangle \text{ has correlations of at least order } m \]
MQC protocol – $\langle S_x \rangle$ measurement

$|\psi_0\rangle$

$H_{\text{Ising}}$

$R_x(\phi)$

$-H_{\text{Ising}}$

$R_y \pi/2$

Detection

$\langle S_x \rangle$

Cool

$R_y -\pi/2$

$\tau$

prepare

measure

time

$H_{\text{Ising}} = J/N \sum_{i<j} \sigma_i^z \sigma_j^z$

$J \lesssim 5kHz$

$N = 111$

$\Gamma = 93Hz$

Fourier transform of magnetization


- Measure build-up of 8-body correlations
- Only global spin measurement
- Illustrates how OTOCs measure spread of quantum information
Summary:

• trapped ion crystals – motional amplitude sensing below the zero-point fluctuations

• employed spin-squeezing, OTOCs to benchmarked quantum dynamics with long range Ising interactions

Future directions:

• transverse field, variable range interaction, longitudinal fields

\[ \sum_{i<j} J_{i,j} \sigma_i^z \sigma_j^z + B_\perp \sum_i \sigma_i^x + \sum_i h_i \sigma_i^z \]

• spin-phonon models (Dicke model)

\[ -\delta a^\dagger a - \frac{g_0}{\sqrt{N}} (a + a^\dagger) S_z + B_\perp S_x \quad \text{arXiv:1711.07392} \]

• mitigate decoherence, improve single ion readout

• 3-dimensional crystals with thousands of ions?
Lab selfie ~ 2014

Joe Britton  
ARL

Justin Bohnet  
Honeywell

Brian Sawyer  
GTRI

Kevin Gilmore  
CU grad student

Elena Jordan  
Leopoldina PD

Theory

Ana Maria Rey

Martin Gärttner

Michael Wall

Arghavan Safavi-Naini

Michael Foss-Feig (ARL)
Benchmarking quantum dynamics and entanglement

Time dependence of squeezed and anti-squeezed variance

Spin variance $(\Delta S'_{\phi})^2 / N/4$ (dB)

N = 85

Bohnet et al., Science 352 (2016)
Writing a spin gradient

method: generate Stark shift gradient in the rotating frame

\[
\mu \mathbf{W} \begin{array}{c} \pi/2 \\ \pi/2 \end{array}
\]
ODF, beat note = crystal rot \( \omega_r \)

\[
H_{ODF} = \sum_j \frac{F_o}{\Delta k} \cos[\Delta k \sin(\theta_{err}) R_j \cos(\omega_r t + \phi_j) - \mu t] \hat{\sigma}_j^z
\]

\( \mu = \omega_r \) produces static Stark shift in the rotating frame

\[
\approx \sum_j \frac{F_o}{\Delta k} J_1(\Delta k \sin(\theta_{err}) R_j) \sin(\phi_j) \hat{\sigma}_j^z \equiv \sum_j h_j \hat{\sigma}_j^z
\]

Random field Ising model

\[
\frac{1}{N} \sum_{i<j} J_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z + \sum_j h_j \hat{\sigma}_j^z
\]
In-plane modes

$E \times B$ modes

transverse modes

Lowest frequency $E \times B$ modes

Lowest frequency cyclotron modes

Freericks group, PRA 87 (2013)