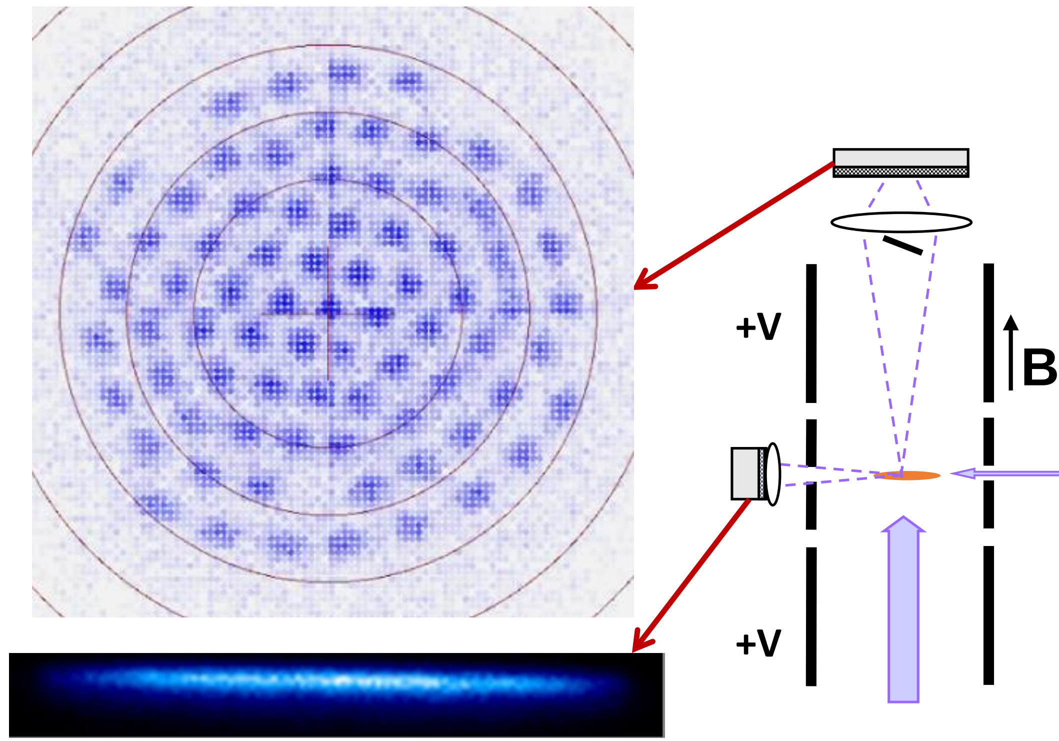
Quantum sensing and simulation with single plane crystals of trapped ions

John Bollinger
NIST-Boulder
Ion storage group

Justin Bohnet (Honeywell), Kevin Gilmore, Elena Jordan, Brian Sawyer (GTRI) , Joe Britton (ARL)

theory –Rey goup (JILA/NIST)
Freericks group (Georgetown)
Dan Dubin (UCSD)

- motional amplitude sensing
- quantum simulation – measure quantum dynamics with OTOC

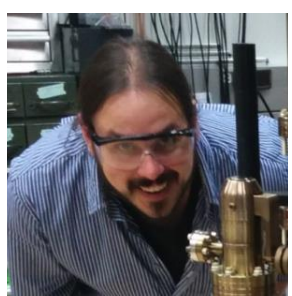


NIST ion storage group

Justin Bohnet

Kevin Gilmore

Elena Jordan



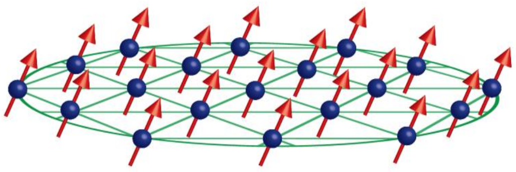
Brian Sawyer
GTRI

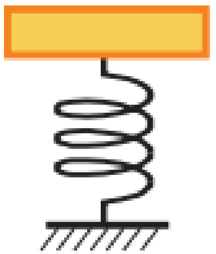
Joe Britton
ARL

Outline:

- Penning trap features
  - high field qubit, modes

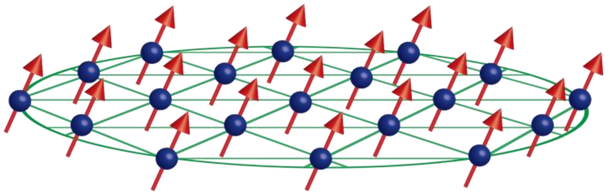
- sensing small COM (center-of-mass) motion
  - spin-dependent forces





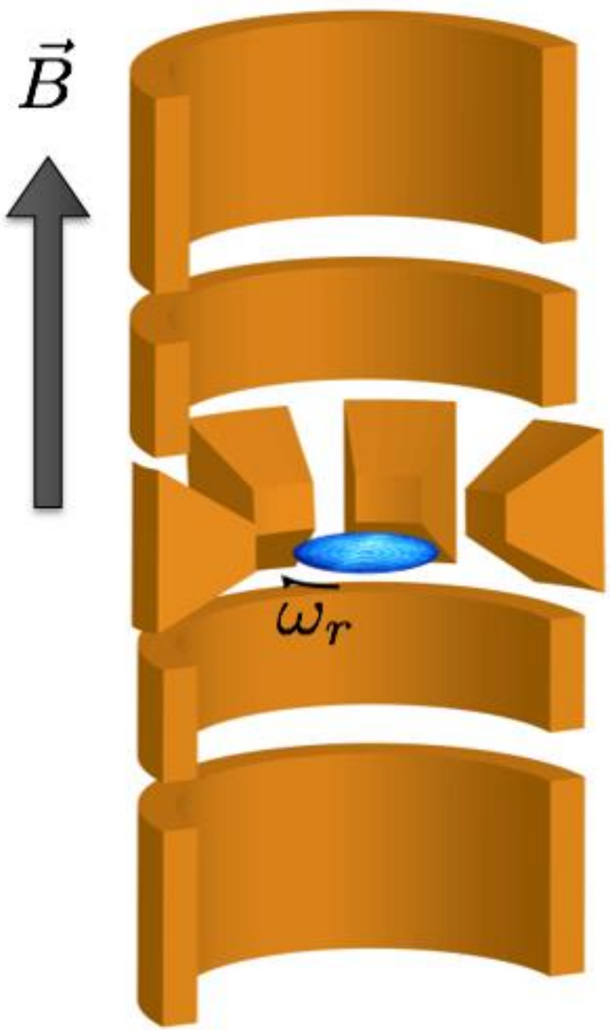
- Quantum simulation with ion crystals in a Penning trap
  - engineering Ising interactions with spin-dependent forces

$$H_{\text{Ising}} = \frac{1}{N}\sum_{i<j} J_{i,j}\sigma_i^z\sigma_j^z$$



  - Loschmidt echo and out-of-time order correlation functions

Penning trap: many particle confinement with static fields

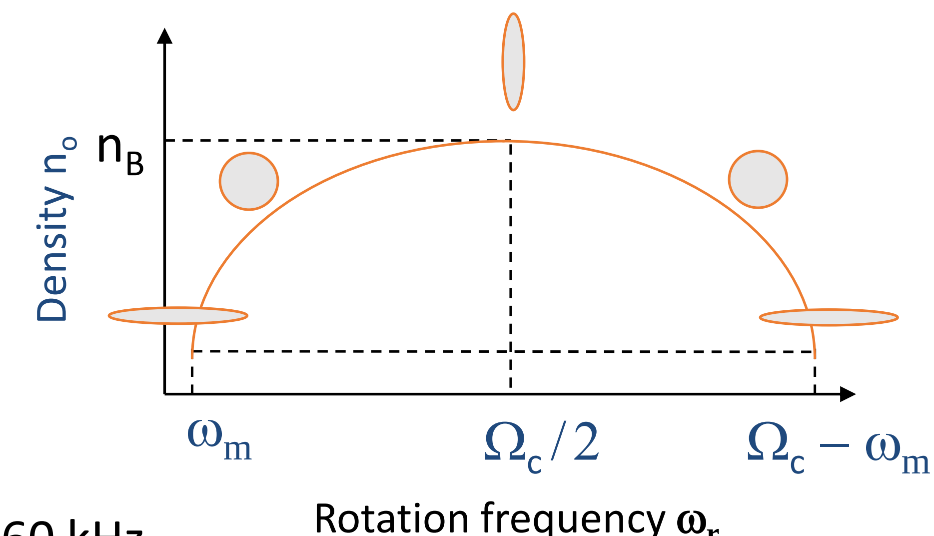


- radial confinement due to rotation – ion plasma rotates $v_\theta = \omega_r\, r$ due to ***ExB*** fields

in rotating frame, Lorentz force is directed radially inward

$$\varphi_{trap}(r,z) \approx \frac{1}{2} m\omega_z^2\left(z^2 - \frac{r^2}{2}\right) \quad \overset{\text{rotating frame}}{\Longrightarrow}$$

$$\varphi_{rot}(r,z) = \frac{1}{2} m\omega_z^2\left(z^2 + \left(\frac{\omega_r(\Omega_c-\omega_r)}{\omega_z^2} - \frac{1}{2}\right) r^2\right)$$



$^9Be^+$, B_0 = 4.5 T

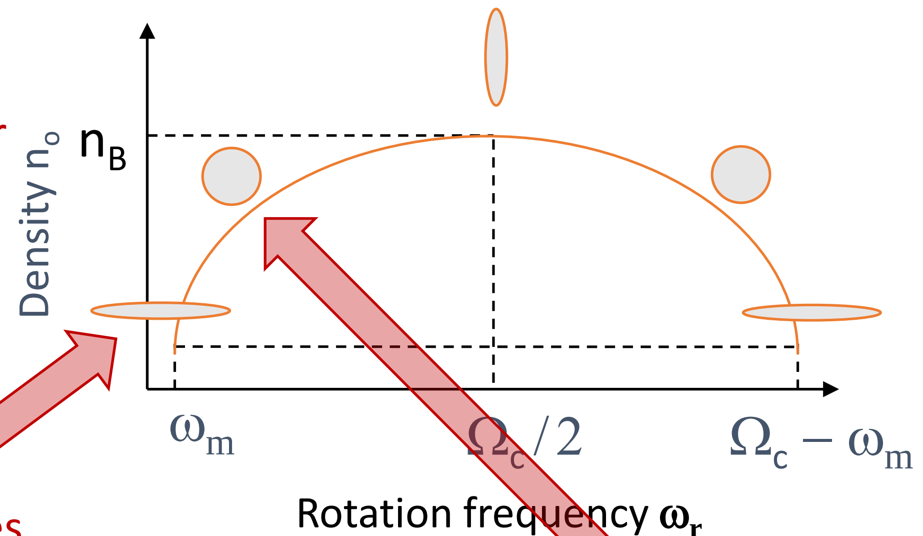
$\frac{\Omega_c}{2\pi} \sim$ 7.6 MHz, $\frac{\omega_z}{2\pi} \sim$ 1.6 MHz, $\frac{\omega_m}{2\pi} \sim$ 160 kHz

Ion crystals form as a result of minimizing Coulomb potential energy

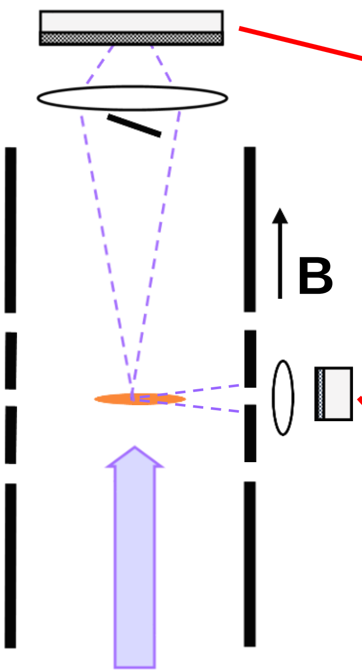
T→ 0.4 mK (Doppler laser cooling) $\Rightarrow$ ${q^2}/{a_{WS}} \gg k_B T$, $2a_{WS}\sim$ ion spacing

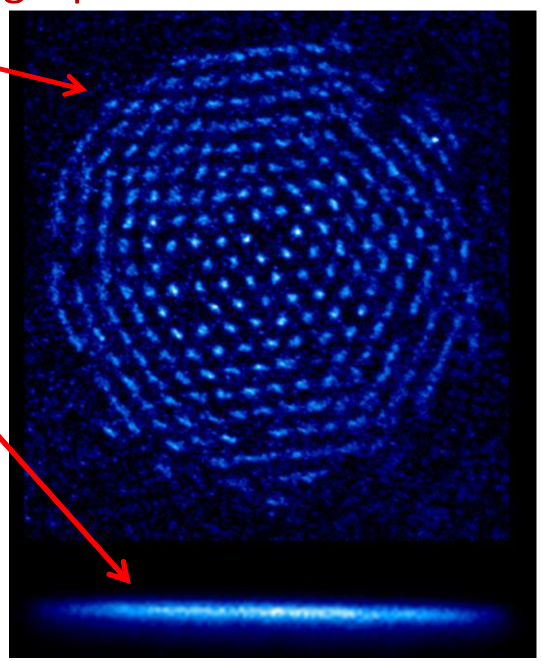
type of crystal, nearest neighbor ion spacing depend on ω_r

Mitchell et.al., Science (1998)



single planes





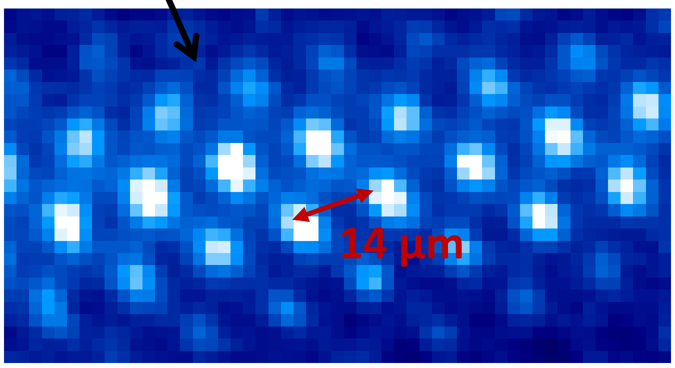
0.5 mm

bcc crystals with N>100 k

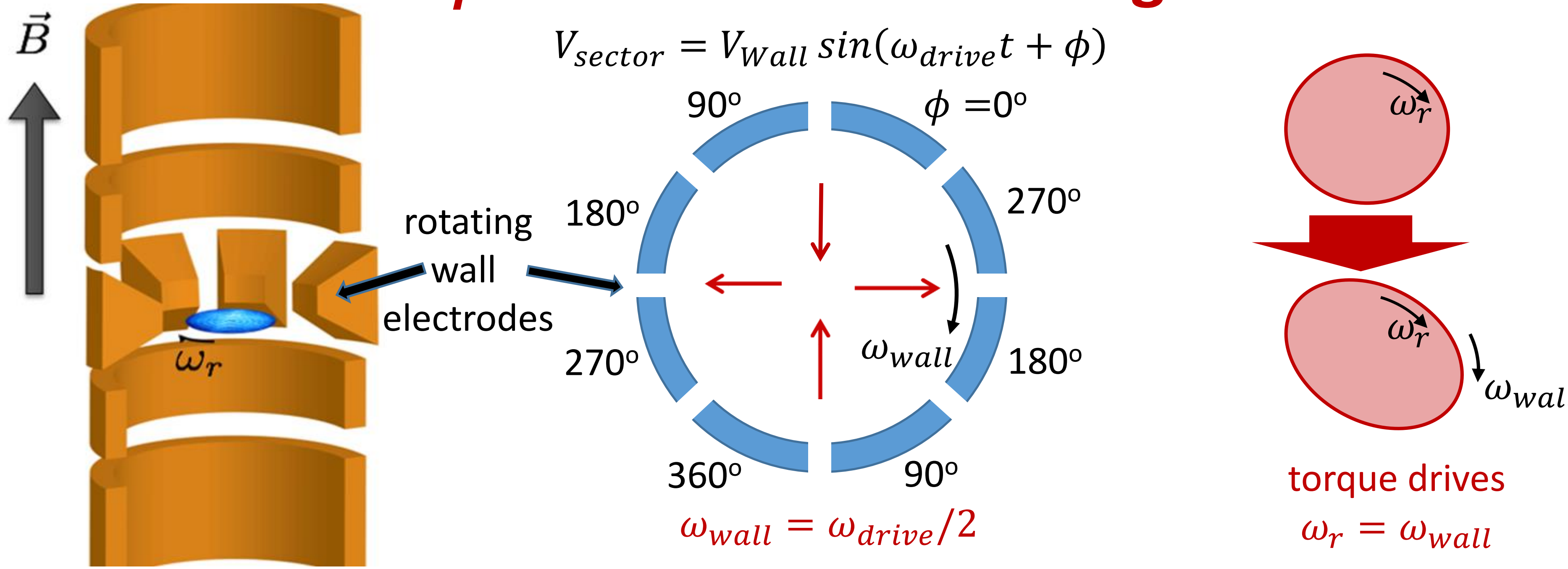
observed with:

Bragg scattering

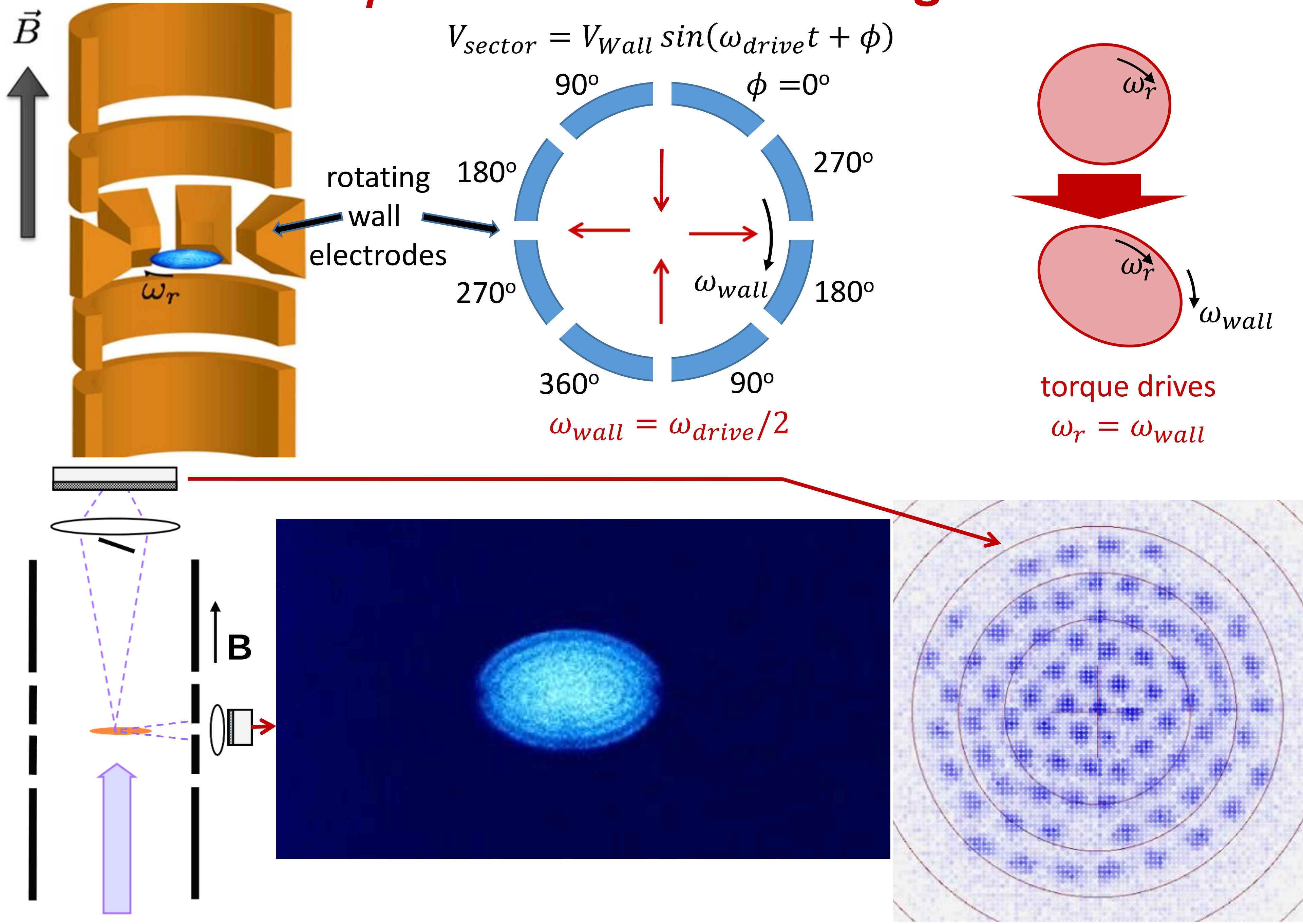
ion fluorescence imaging



Precise ω_r control with a rotating electric field



Precise ω_r control with a rotating electric field

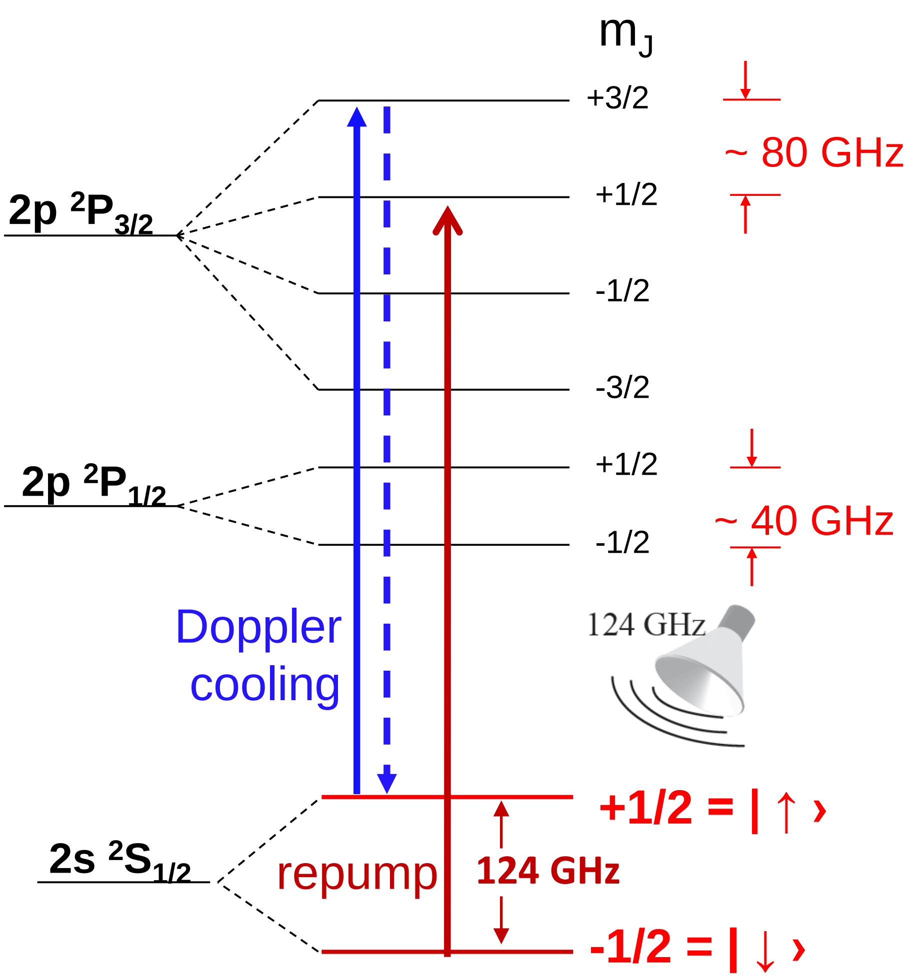


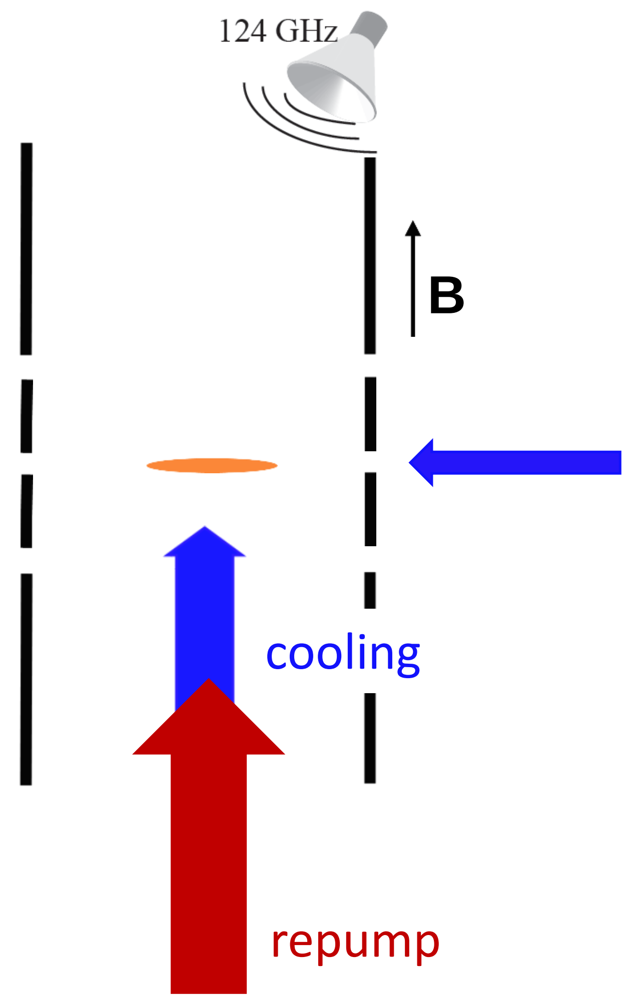
Be$^+$ high magnetic field qubit

^{9}Be$^+$, B ~ 4.5 T, $\omega_o/2\pi$ ~124.1 GHz

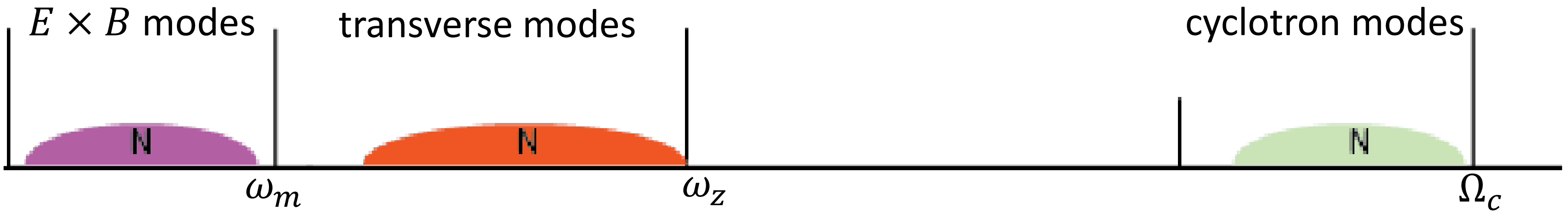
$$H_{\mu W} = \sum_i B_\perp \hat{\sigma}_i^x ,$$

$$B_\perp > 10-15\ \text{kHz}$$

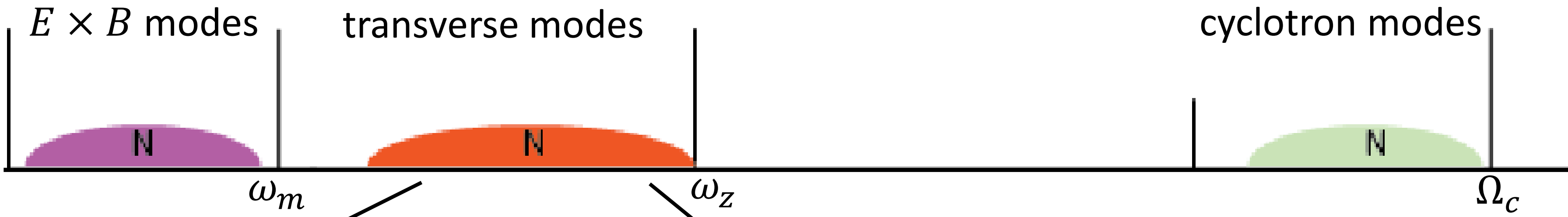


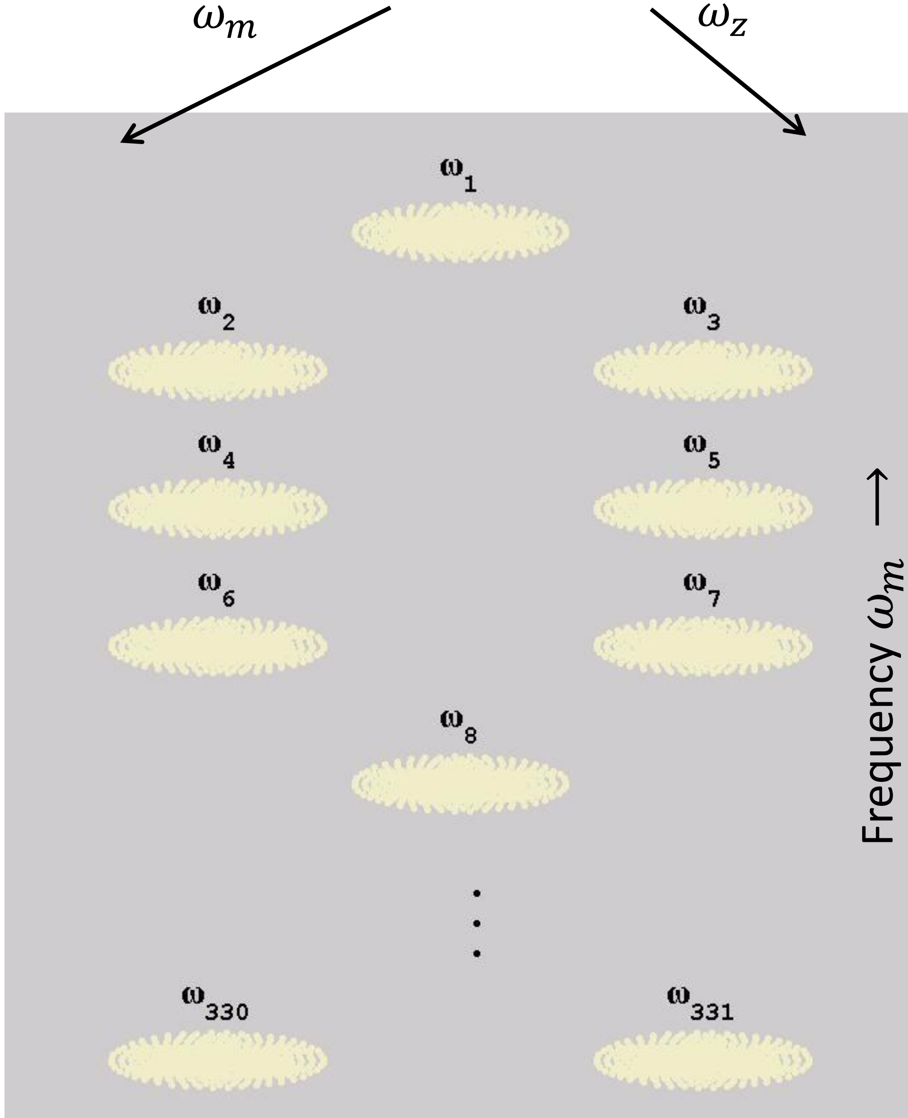


Transverse (drumhead) modes



Transverse (drumhead) modes

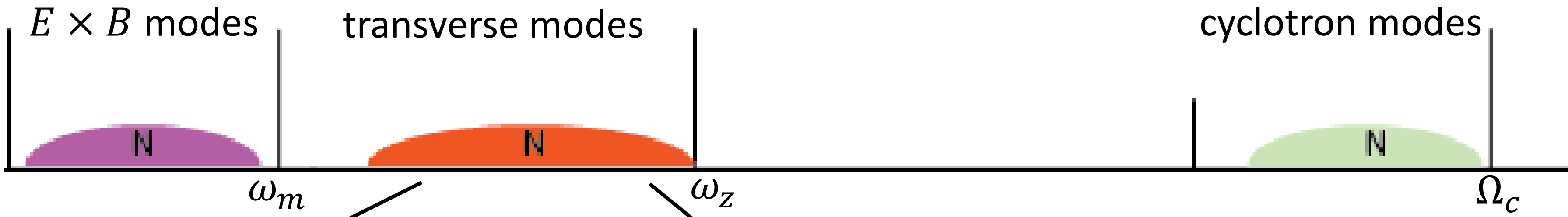


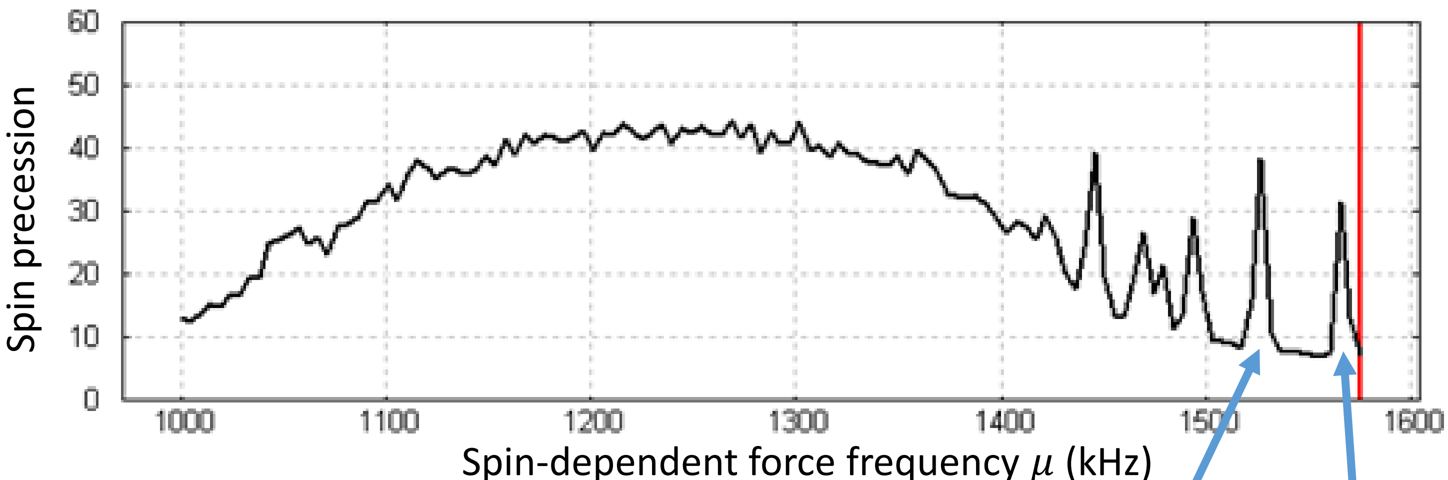


Freericks group, PRA (2013)
Baltrush, Negretti, Taylor, Calarco, PRA (2011)
Dubin, UCSD

Modes characterized by eigenfrequency ω_m and eigenvector $b_{i,m}$

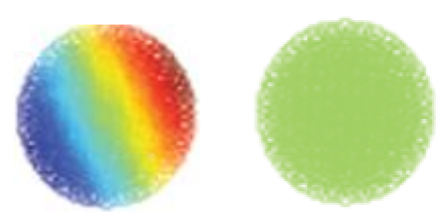
Transverse (drumhead) modes





Measure mode spectrum with spin-dependent force

tilt mode COM mode

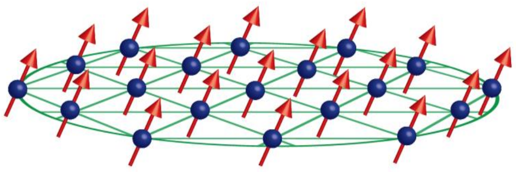


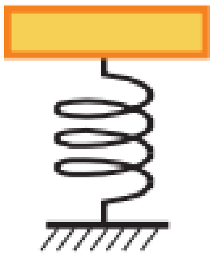
ω_z

Outline:

- Penning trap features
  - high field qubit, modes

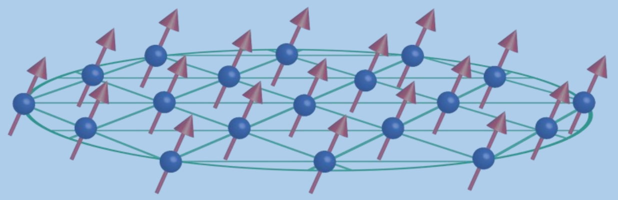
- sensing small COM (center-of-mass) motion
  - spin-dependent forces





- Quantum simulation with ion crystals in a Penning trap
  - engineering Ising interactions with spin-dependent forces

$$H_{\text{Ising}} = \frac{1}{N}\sum_{i<j} J_{i,j}\sigma_i^z\sigma_j^z$$



  - Loschmidt echo and out-of-time order correlation functions

Motional amplitude sensing or Trapped ions as sensitive $\vec{E}$-field and force detectors

Maiwald, *et al.*, Nature Physics 2009 – 1 yN Hz$^{-1/2}$

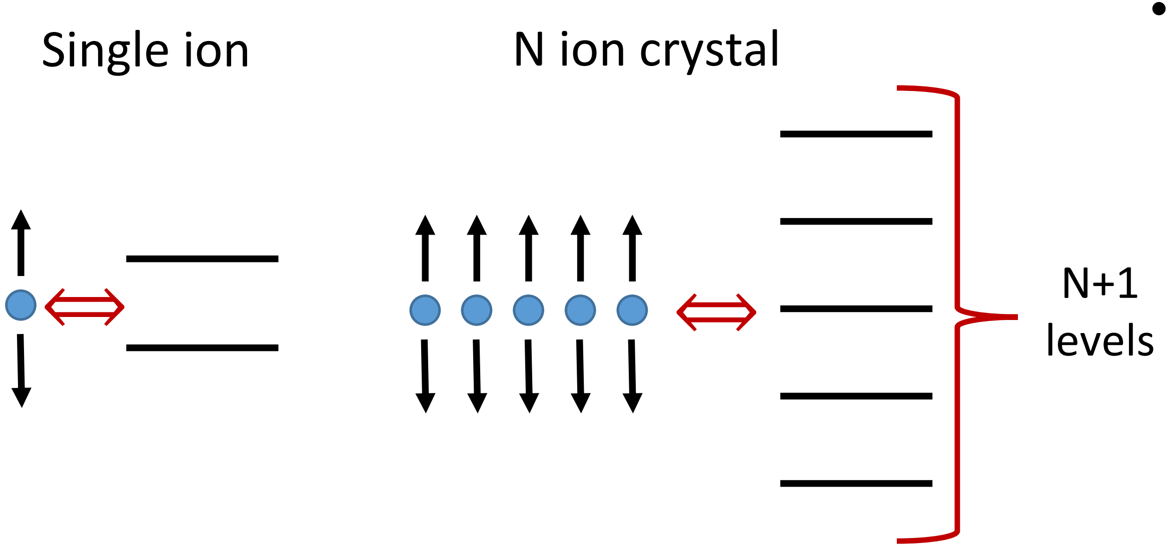
Hempel *et al.*, Nature Photonics 2013 – detect single photon recoil

Shaniv, Ozeri, Nature Communications, 2017 – high sensitivity (~28 zN Hz$^{-1/2}$) at low frequencies

⋮

Biercuk *et al.*, Nature Nanotechnology, 2010 – 100-ion crystal (400 yN Hz$^{-1/2}$)

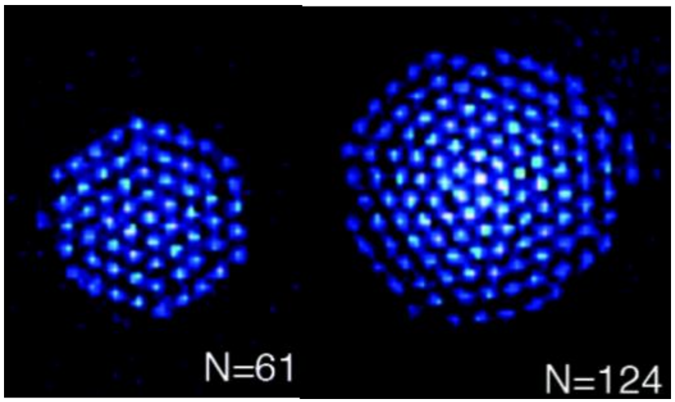
Basic idea: map motional amplitude onto spin precession



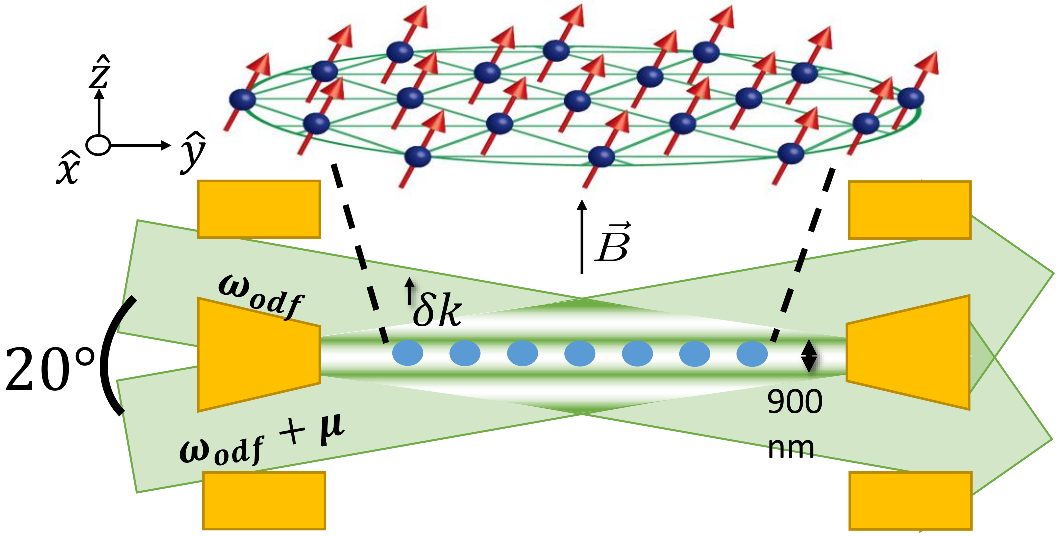
<u>N ion crystal</u>

- Less projection noise
- Smaller zero-point motion, $z_{zpt} \approx 2\ \text{nm}$ for N=100

$\sim \dfrac{1}{\sqrt{N}}$



Sensing small center-of-mass motion



$$F_\uparrow(t) = -F_\downarrow(t) = F_0 \cos(\mu t)$$

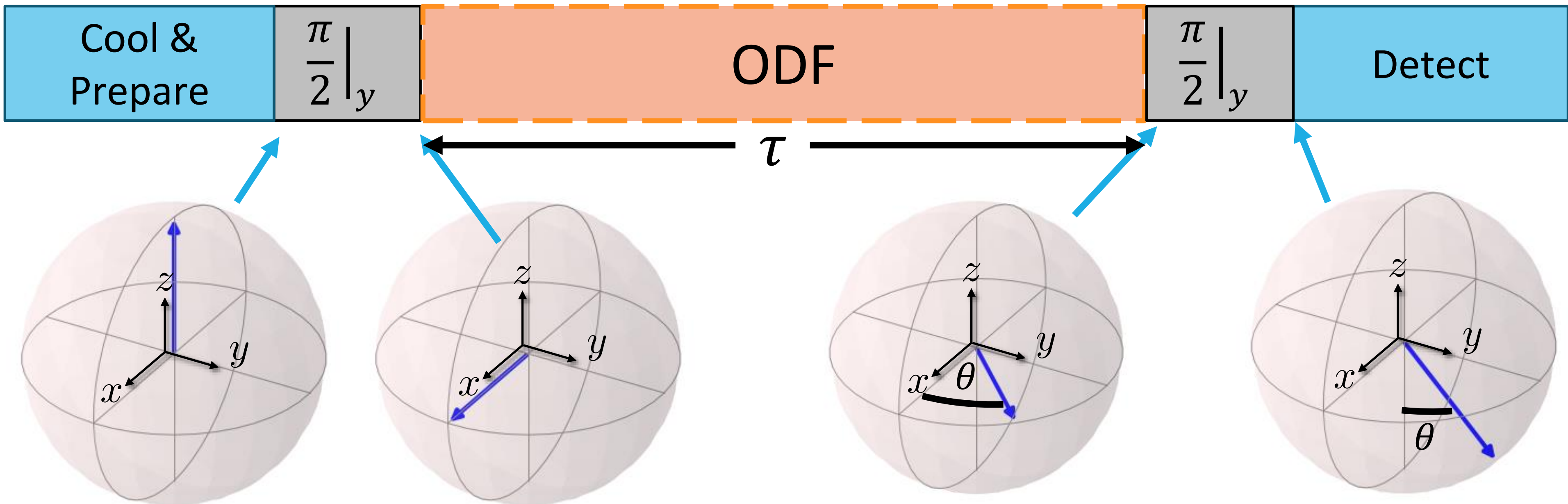
$$H_I = \sum_i F_0 \cos(\mu t)\, \hat{z}_i \hat{\sigma}_i^z$$

Implement classical COM oscillation: $\hat{z}_i \rightarrow \hat{z}_i + Z_c\, cos(\omega t + \phi)$

$$H_I \cong F_0 \cdot Z_c\, cos[(\omega - \mu)t + \phi] \sum_i \frac{\hat{\sigma}_i^z}{2}$$
$$= F_0 \cdot Z_c\, cos[(\omega - \mu)t + \phi]\ \hat{S}_z$$

For $\mu = \omega$, produces spin precession with rate $\propto F_0 \cdot Z_c \cos(\phi)$

Measuring spin precession



Precession θ,

$$\theta = \frac{F_0}{\hbar} Z_c \tau \cos(\phi)$$

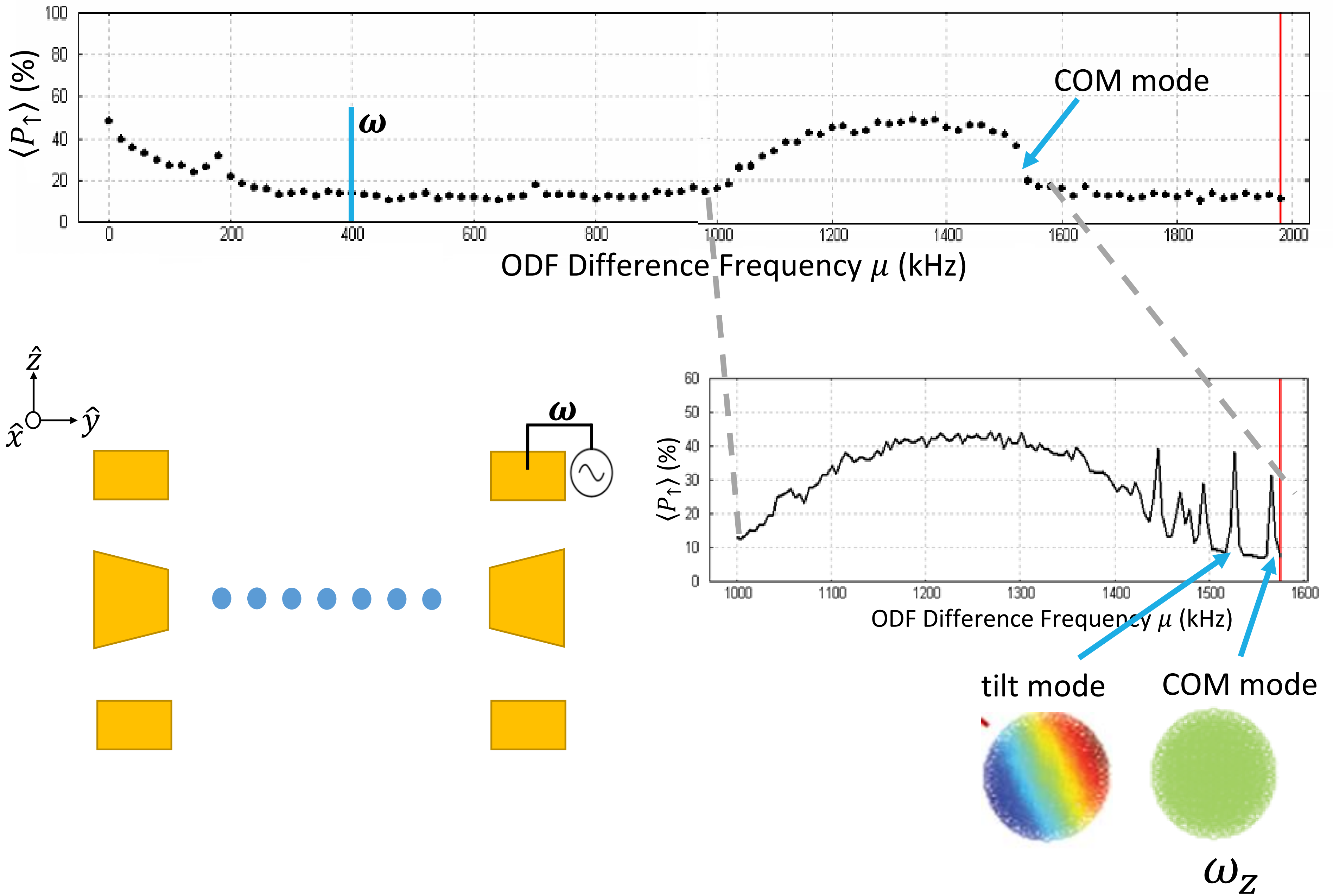
$$-\frac{F_0}{\hbar} Z_c \tau < \theta < \frac{F_0}{\hbar} Z_c \tau$$

Probability of measuring spin up:

$$\langle P_\uparrow \rangle = \frac{1}{2}\left(1 - e^{-\Gamma\tau} \langle \cos\theta \rangle\right)$$

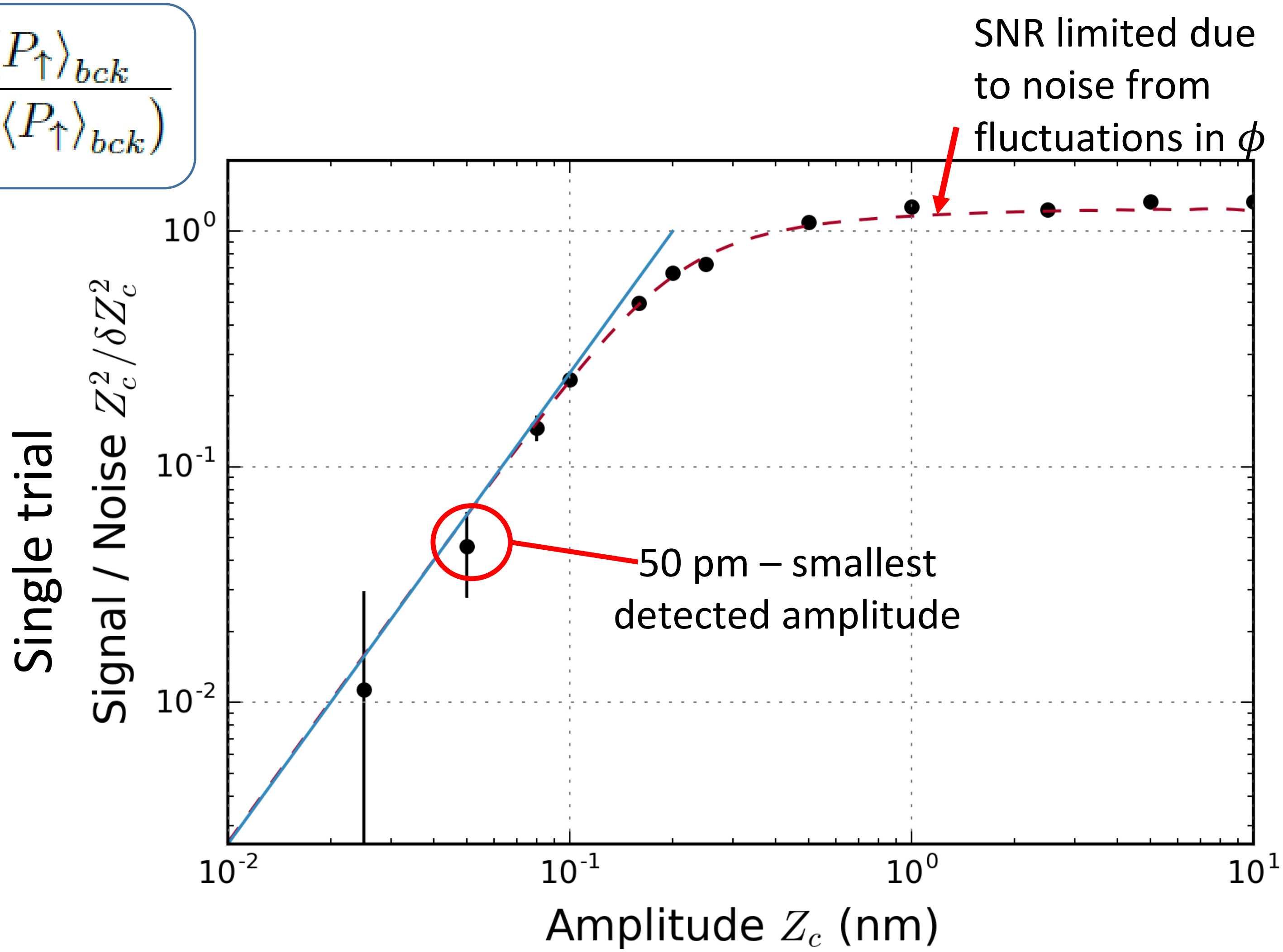
$$= \frac{1}{2}\left(1 - e^{-\Gamma\tau} J_0\left(\frac{F_0}{\hbar} Z_c \tau\right)\right)$$

Measuring spin precession



Sensitivity limits/ signal-to-noise

$$\frac{Z_c^2}{\delta Z_c^2} \approx \frac{\langle P_\uparrow \rangle - \langle P_\uparrow \rangle_{bck}}{\delta\left(\langle P_\uparrow \rangle - \langle P_\uparrow \rangle_{bck}\right)}$$



Small signal limits due to:
projection noise
spontaneous emission

$$\left.\frac{Z_c^2}{\delta Z_c^2}\right|_{\text{limiting}} = \left[\frac{Z_c}{0.2\ \text{nm}}\right]^2$$

Gilmore et al., PRL 2017

Sensing small center-of-mass motion

Future:

- Fixed phase sensing off-resonance (i.e. fixed ϕ in $Z_c cos(\omega t + \phi)$)
  - 74 pm in single experimental trial
  - 18 pm/$\sqrt{Hz}$
  - Exploit spins: squeezed states

- On-resonance with COM mode
  - Enhance force and electric field sensitivities by $Q{\sim}10^6$
  - Protocols for evading zero-point fluctuations, backaction ??
  - 20 pm amplitude from a resonant 100 ms coherent drive
    - force/ion of 5×10^{-5} yN
    - electric field of 0.35 nV/m

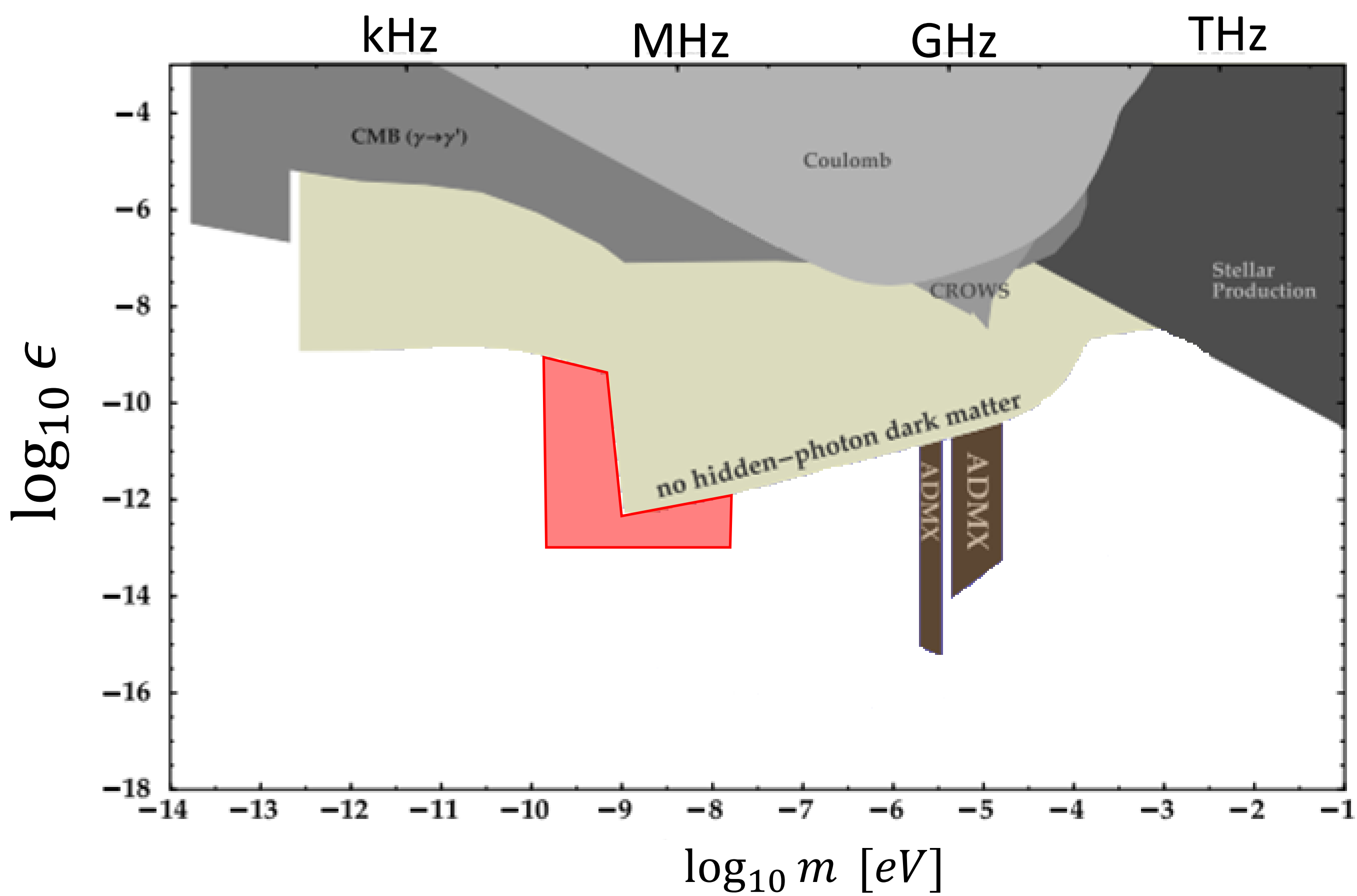
Potential for dark matter search
(axions and hidden photons)

20 pm amplitude from a resonant 100 ms coherent drive

- force/ion of 5×10^{-5} yN
- electric field of 0.35 nV/m

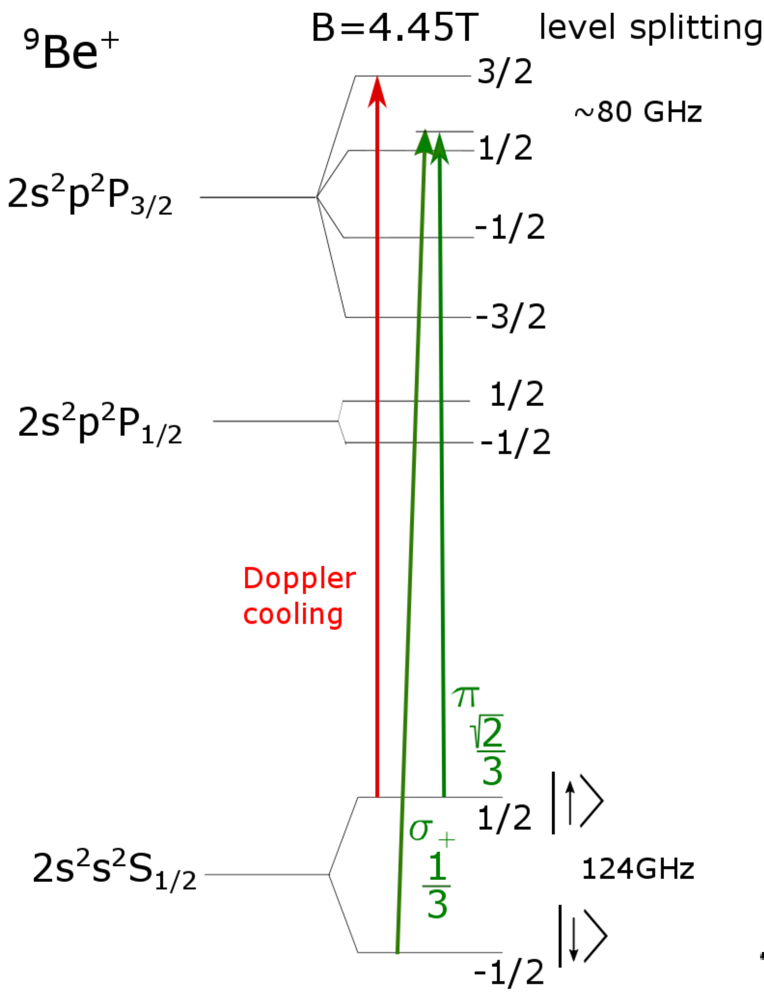
S. Chaudhuri, *et al.*, Phys. Rev. D (2015).

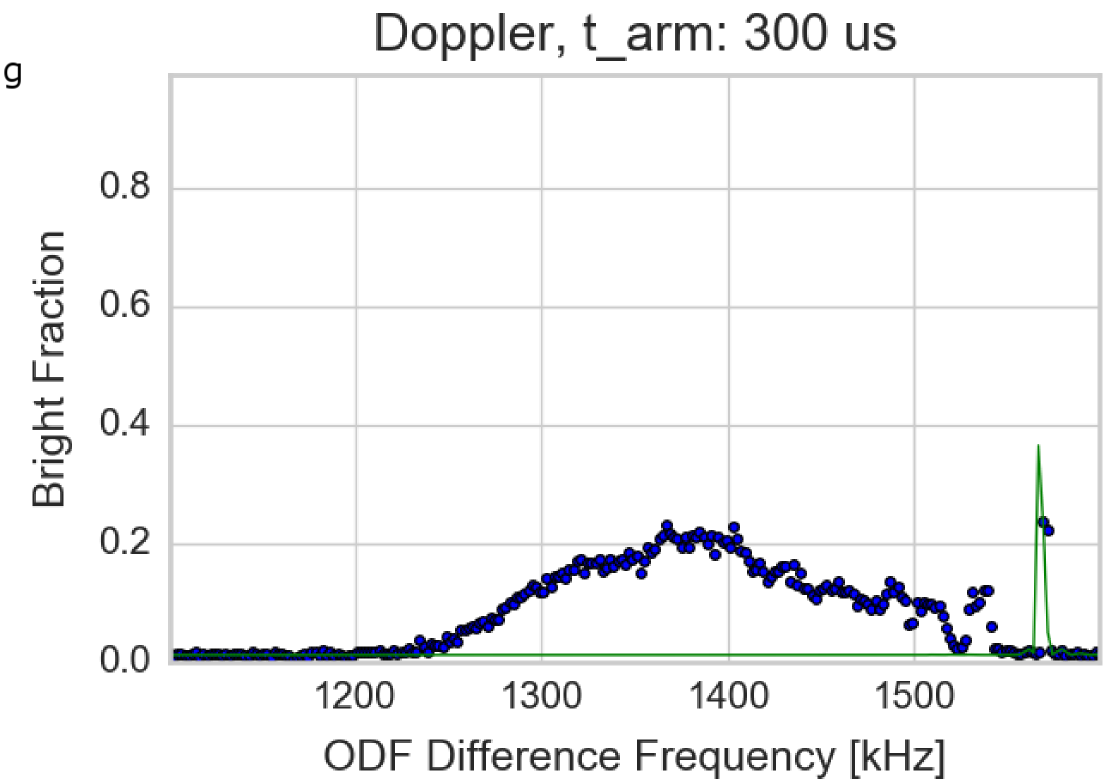
$$\epsilon = \frac{E}{3.3\,\frac{\mathrm{nV}}{\mathrm{m}}} * 10^{-12}$$

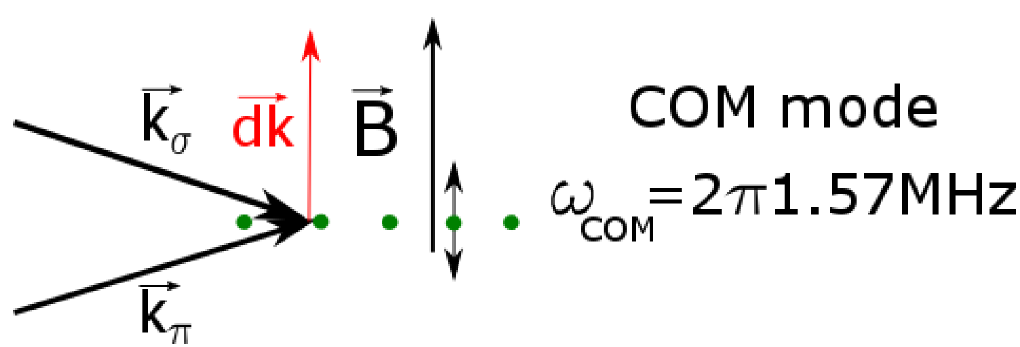


Technical improvement: EIT cooling

Morigi PRA 67 (2003); exp results with smaller ion numbers: Innsbruck, NIST

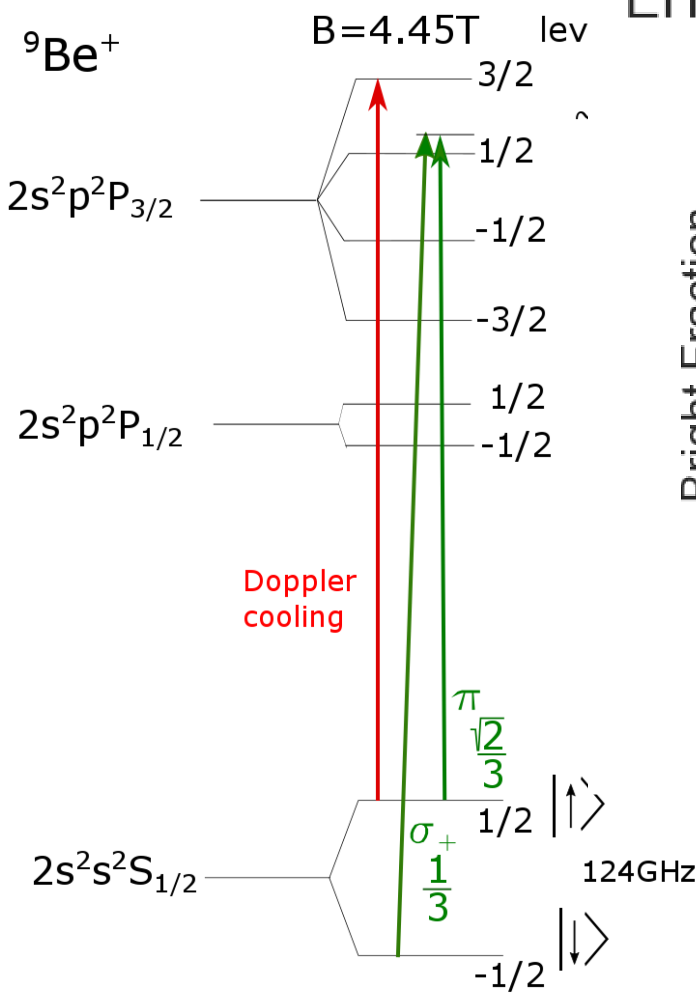


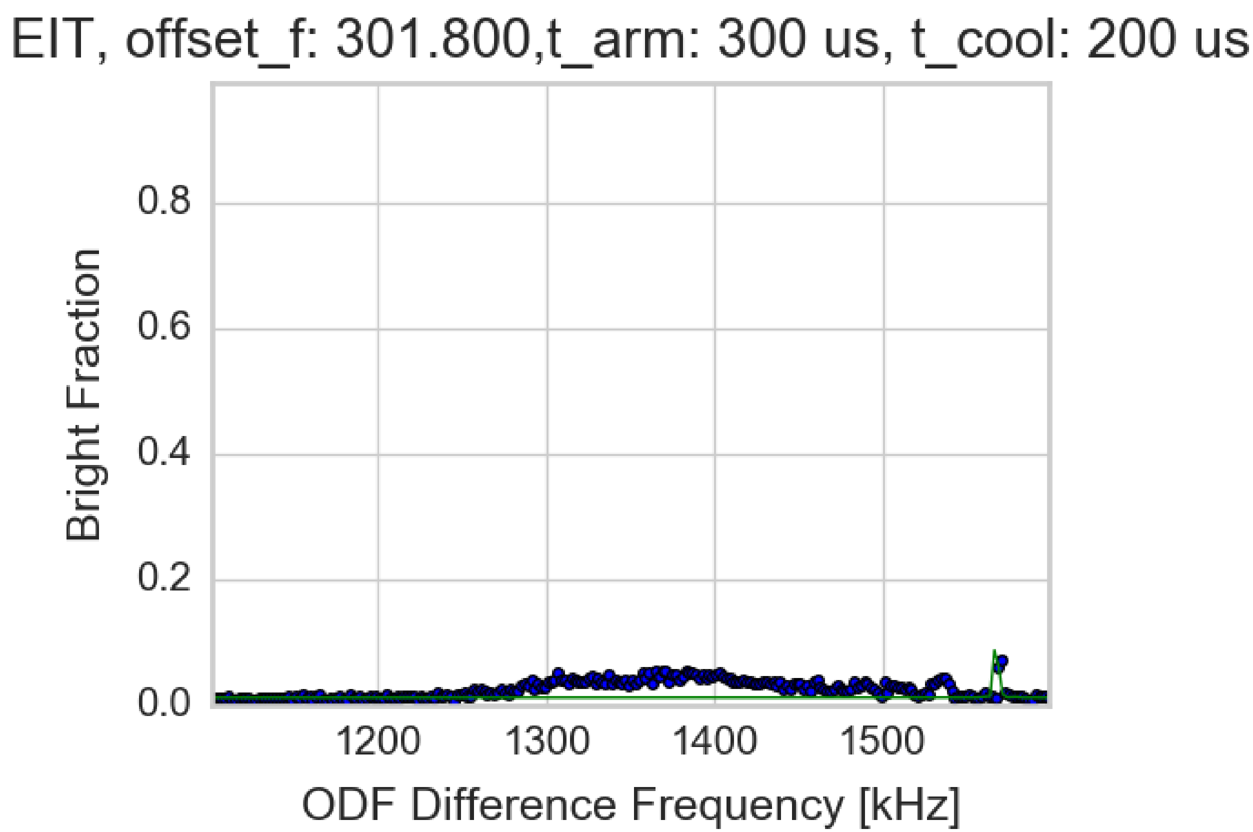


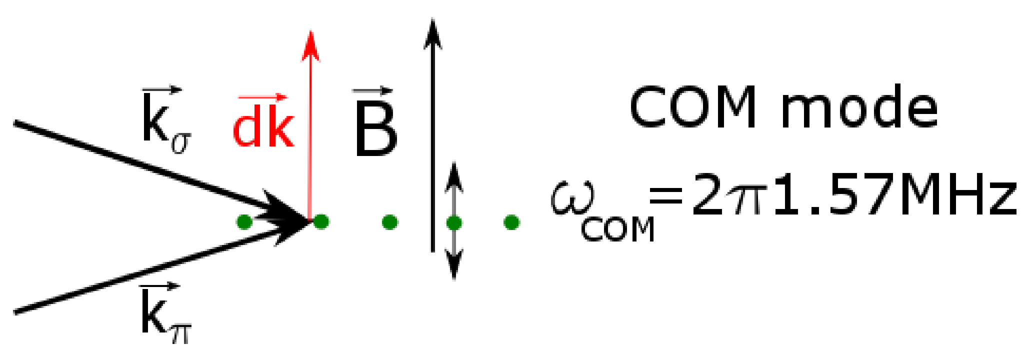


Technical improvement: EIT cooling

Morigi PRA 67 (2003); exp results with smaller ion numbers: Innsbruck, NIST



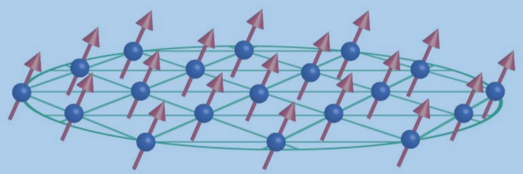


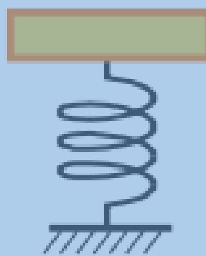


Outline:

- Penning trap features
  - high field qubit, modes

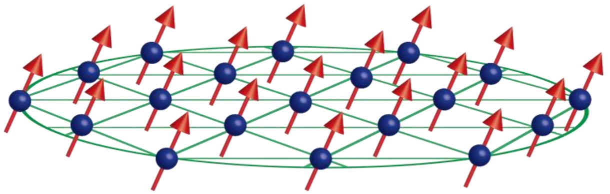
- sensing small COM (center-of-mass) motion
  - spin-dependent forces





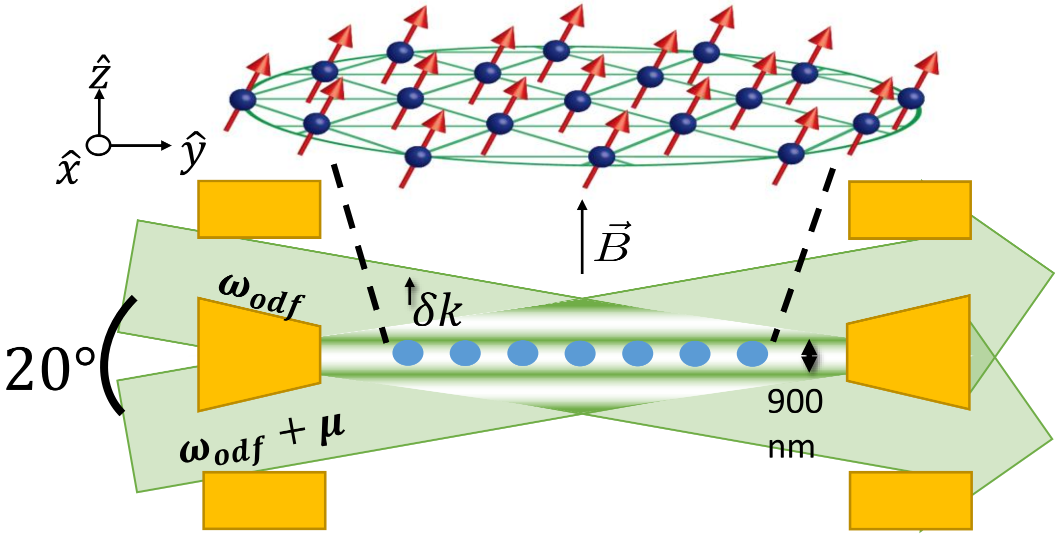
- Quantum simulation with ion crystals in a Penning trap
  - engineering Ising interactions with spin-dependent forces

$$H_{\text{Ising}} = \frac{1}{N}\sum_{i<j} J_{i,j}\sigma_i^z\sigma_j^z$$



  - Loschmidt echo and out-of-time order correlation functions

Sensing small center-of-mass motion



$$F_{\uparrow}(t) = -F_{\downarrow}(t) = F_0 \cos(\mu t)$$

$$H_I = \sum_i F_0 \cos(\mu t)\, \hat{z}_i \hat{\sigma}_i^z$$

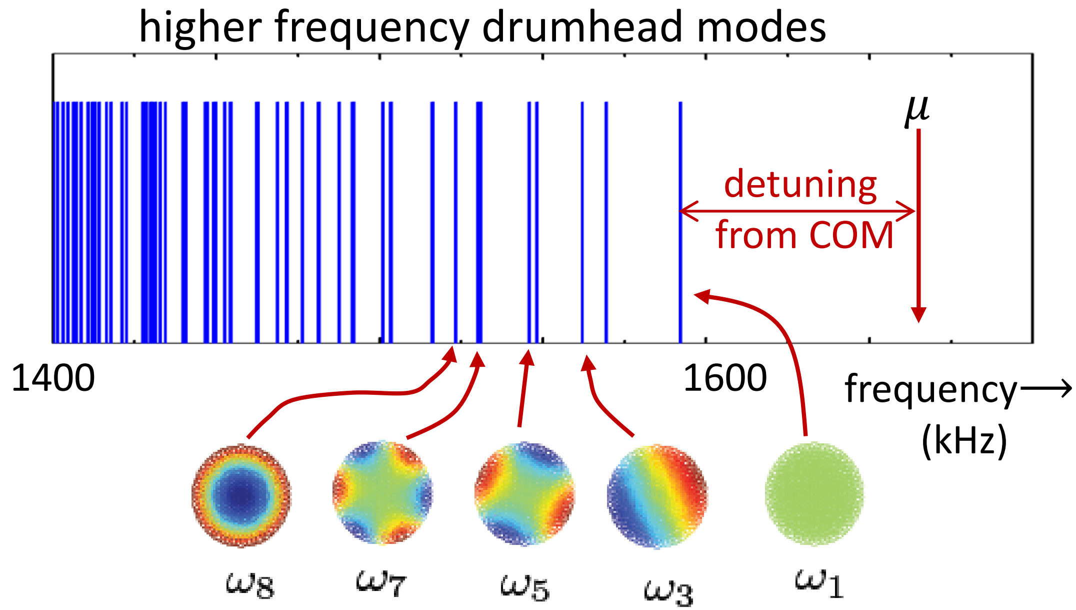
Implement classical COM oscillation: $\hat{z}_i \rightarrow \hat{z}_i + Z_c\, cos(\omega t + \phi)$

$$H_I \cong F_0 \cdot Z_c\, cos[(\omega - \mu)t + \phi] \sum_i \frac{\hat{\sigma}_i^z}{2}$$
$$= F_0 \cdot Z_c\, cos[(\omega - \mu)t + \phi]\ \hat{S}_z$$

Engineering quantum magnetic couplings

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \underbrace{\hat{z}_j}_{\sum_{m=1}^{N} b_{jm} \sqrt{\frac{\hbar}{2M\omega_m}} \left(\hat{a}_m^{\dagger} e^{i\omega_m t} + \hat{a}_m e^{-i\omega_m t} \right)} \cdot \hat{\sigma}_j^z =$$

N drumhead eigenvalues ω_m and eigenvector $\vec{b}_m$



$\hat{U}$ $\hat{U}$ (t) $\hat{U}$ (t)

Produces spi

- useful metrolo
- source of deco

Infinite range ⟹Single axis twisting

$$H_{Ising} = \frac{J}{N} \sum_{i<j} \sigma_i^z \sigma_j^z = \frac{2J}{N} S_z^2$$

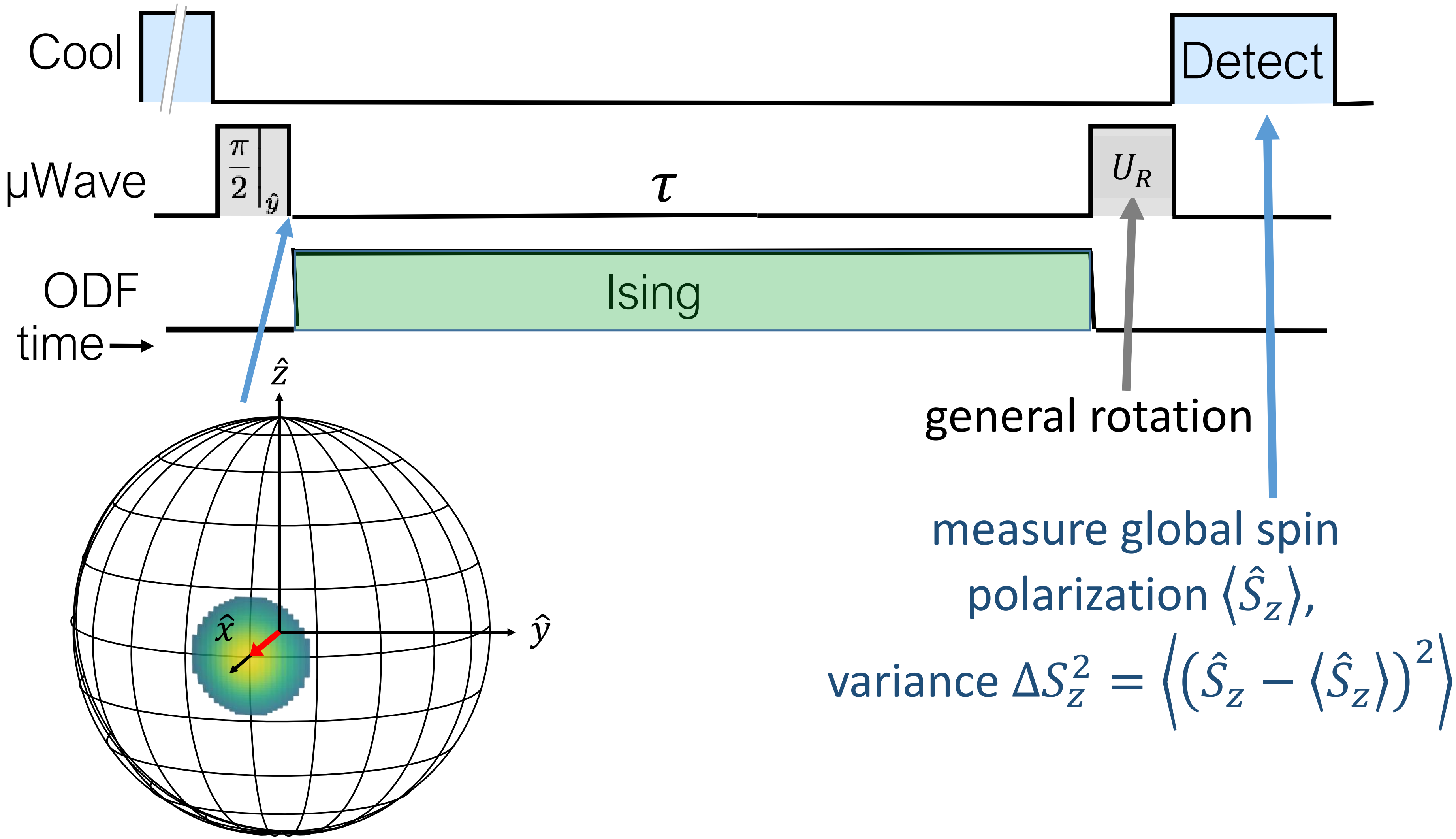
where $S_z = \sum_i \frac{\sigma_i^z}{2}$

generates a “cat state” $\frac{1}{\sqrt{2}} \{ |\uparrow\uparrow\uparrow\cdots\uparrow\rangle_x + |\downarrow\downarrow\downarrow\cdots\downarrow\rangle_x \}$

at long times τ, such that $\frac{2J}{N}\tau = \frac{\pi}{2}$

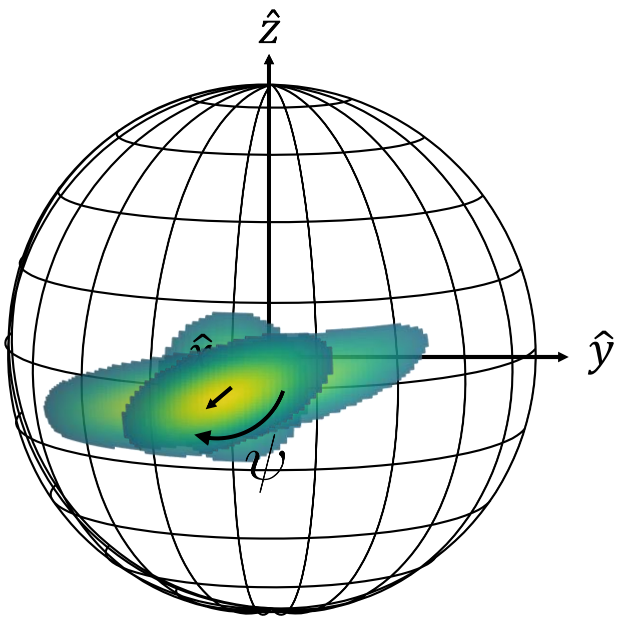
Benchmarking quantum dynamics

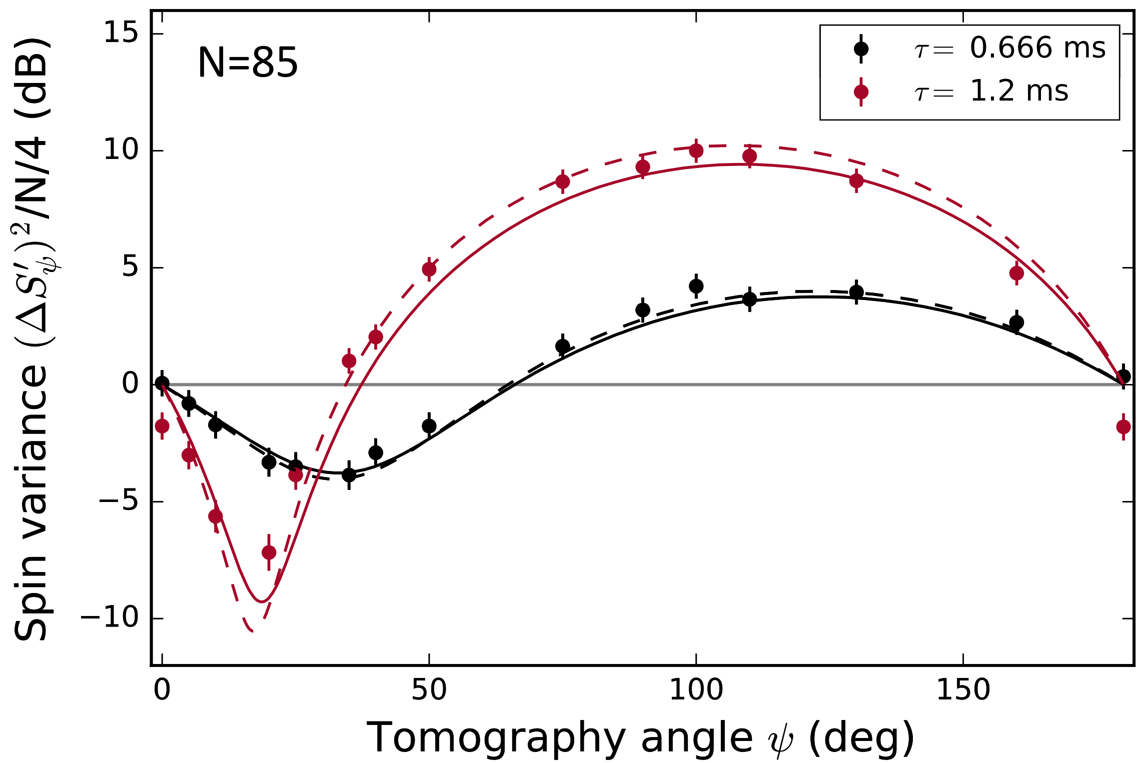
- employ infinite range interactions $H_{Ising} \approx \frac{2J}{N} S_z^2, S_z \equiv \sum_i \sigma_i^z /2$
- prepare eigenstate of $H_\perp = \sum_i B_\perp \hat{\sigma}_i^x$, turn on H_{Ising}



Benchmarking quantum dynamics

Bohnet *et al.*, *Science* 352, 1297 (2016)

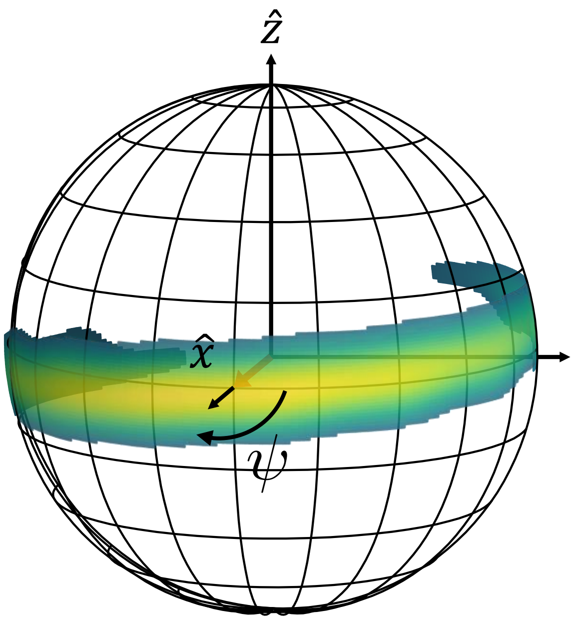


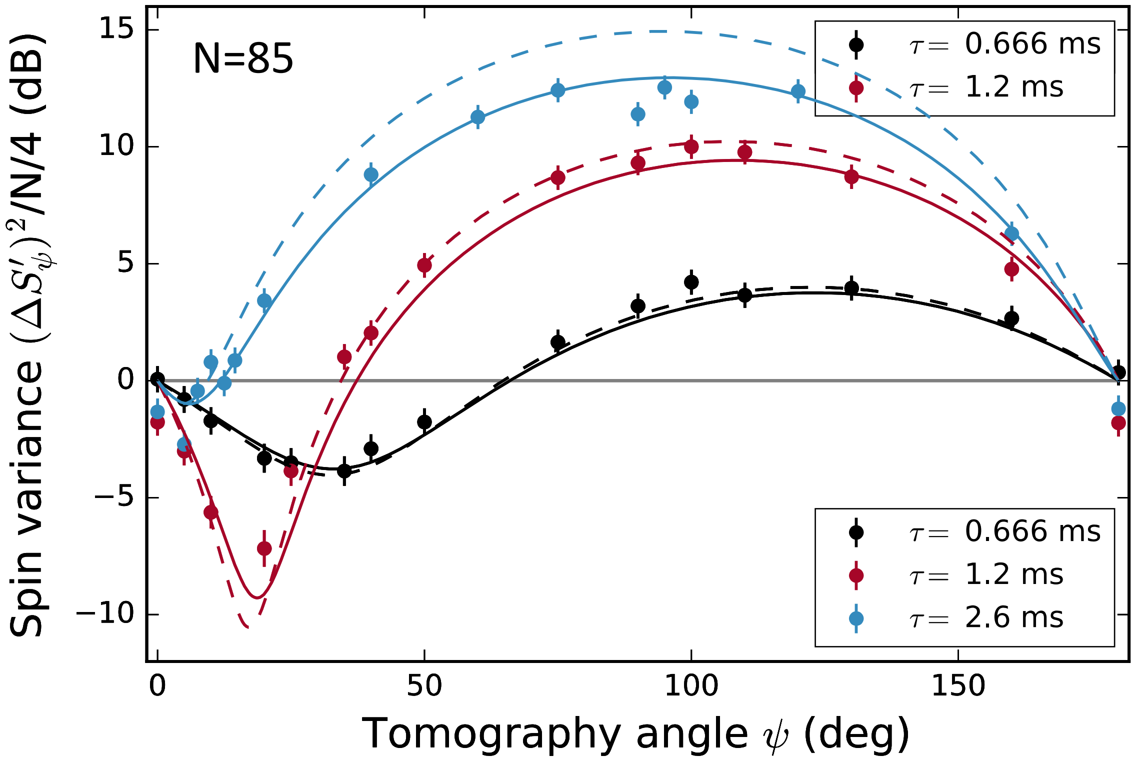


- Measurements of Ramsey squeezing parameter $\Rightarrow$ prove entanglement for $25 < N < 220$
- Largest inferred squeezing: -6.0 dB

Benchmarking quantum dynamics

Bohnet *et al.*, *Science* 352, 1297 (2016)





Out-of-time-order correlation functions

$$F(t) \equiv \langle\psi|W(t)^{\dagger}V^{\dagger}W(t)V|\psi\rangle \text{ where } \mathrm{W(t)} = \mathrm{e}^{iHt}W(0)e^{-iHt},$$
$$[V, W(0)] = 0$$

$$Re[F(t)] = 1 - \langle|[W(t), V]|^2\rangle/2$$

$\Rightarrow$ **measures failure of initially commuting operators to commute at later times**

$\Rightarrow$ **quantifies spread or scrambling of quantum information across a system's degrees of freedom**

Swingle et al., arXiv:1602.06271; Shenker et al., arXiv:1306.0622; Kitaev (2014)

Difficult to measure $\Leftrightarrow$ requires time-reversal of dynamics

time reversal is possible in many quantum simulators!

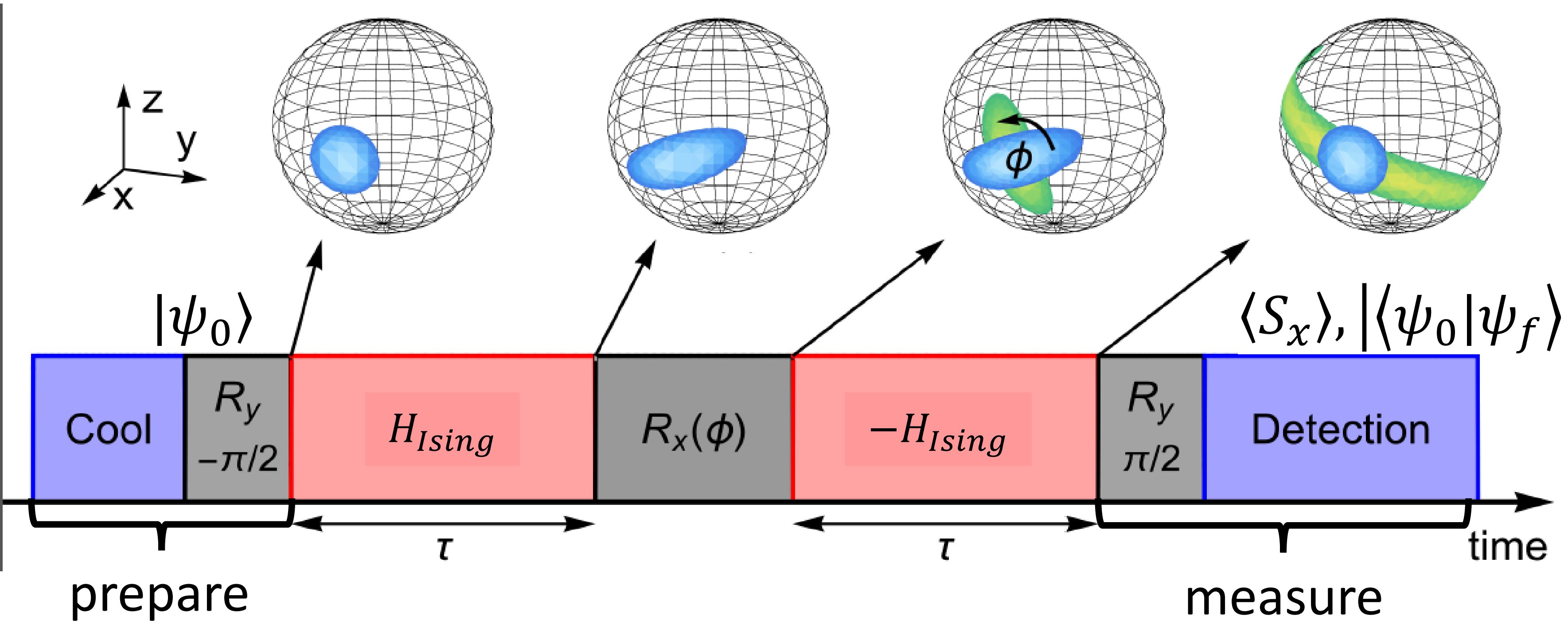
Time reversal of the Ising dynamics

$$H_{Ising} = \frac{J}{N} \sum_{i<j} \hat{\sigma}_i^z \hat{\sigma}_j^z, \quad \frac{J}{N} \cong \frac{F_0^2}{\hbar 4 m \omega_z} \cdot \frac{1}{\mu - \omega_z}$$

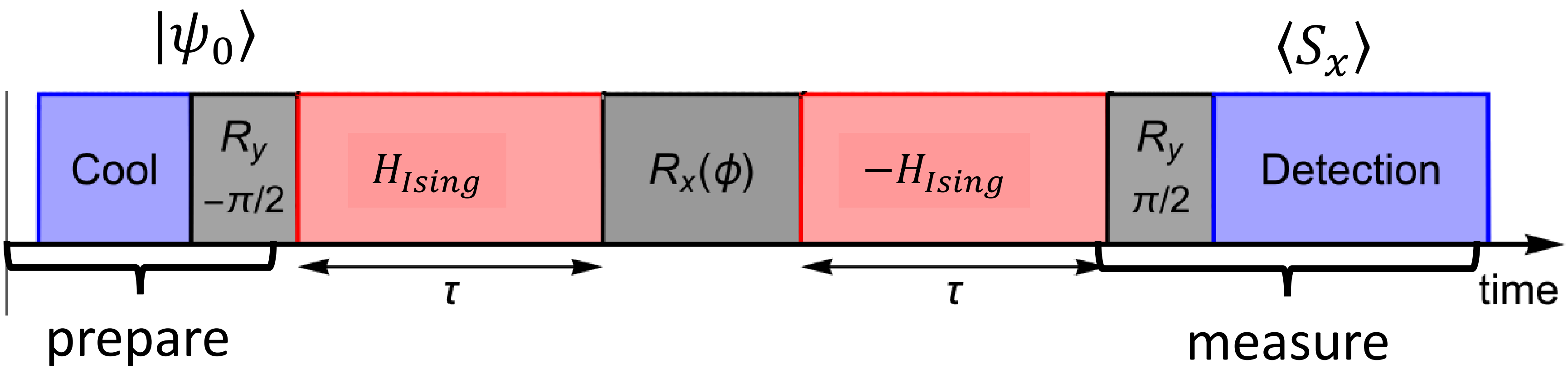
Change $\mu = \omega_z + \delta$ (antiferromagnetic)
to $\mu = \omega_z - \delta$ (ferromagnetic)

Multiple quantum coherence protocol

- Probe higher-order coherences and correlations (Pines group, 1985)



Multiple quantum coherence protocol



$$\langle S_x\rangle = \langle\Psi_0|\, e^{iH_{Ising}\tau}\, e^{i\phi S_x}\, e^{-iH_{Ising}\tau} S_x\, e^{iH_{Ising}\tau}\, e^{-i\phi S_x}\, e^{-iH_{Ising}\tau}|\Psi_0\rangle$$

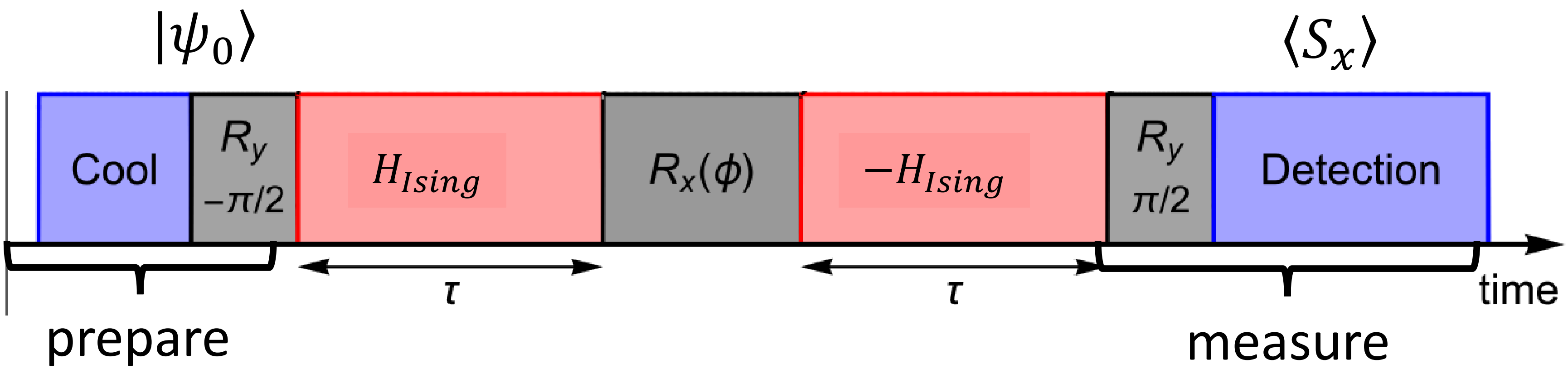
$$= \frac{2}{N}\langle\Psi_0|\, \underbrace{e^{iH_{Ising}\tau}\, W^\dagger\, e^{-iH_{Ising}\tau}}_{W^\dagger(t)} \underbrace{V^\dagger}_{V^\dagger(0)}\, \underbrace{e^{iH_{Ising}\tau}\, W\, e^{-iH_{Ising}\tau}}_{W(t)} \underbrace{V}_{V(0)}|\Psi_0\rangle$$

Out-of-time-order correlation (OTOC) function

$\Rightarrow$ **quantifies spread or scrambling of quantum information across a system's degrees of freedom**

Swingle et al., arXiv:1602.06271; Shenker et al., arXiv:1306.0622; Kitaev (2014)

Multiple quantum coherence protocol

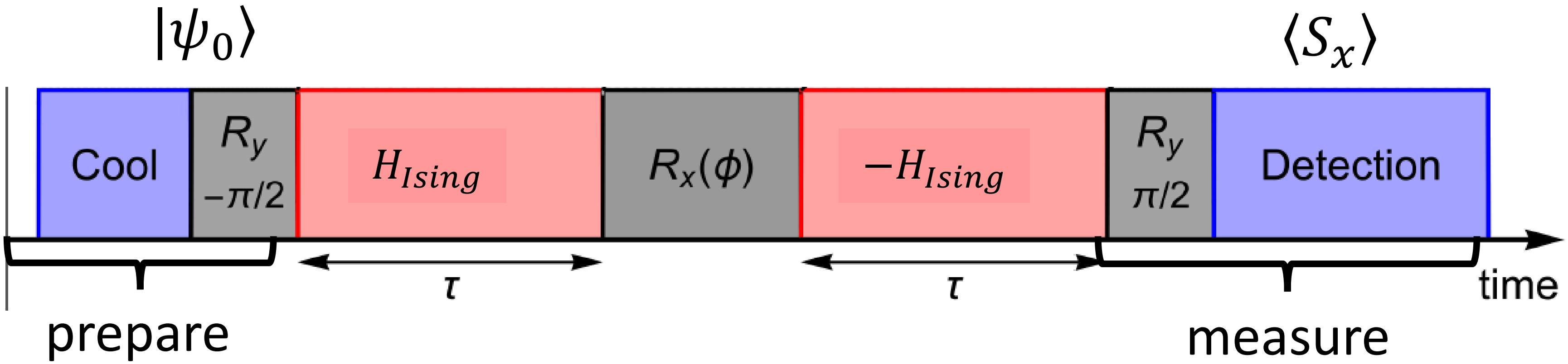


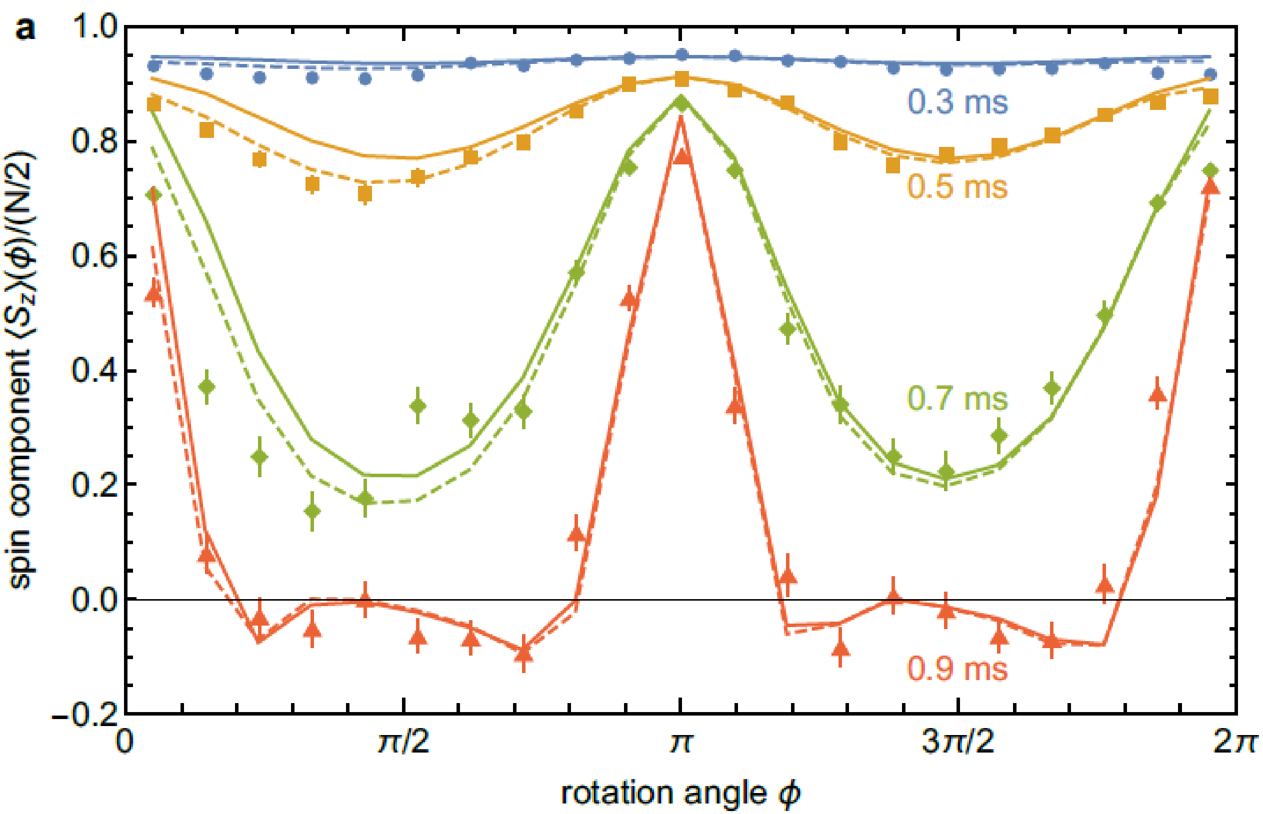
$$\langle S_x \rangle = \langle \Psi_0 | \, e^{iH_{Ising}\tau} \, e^{i\phi S_x} \, e^{-iH_{Ising}\tau} S_x \, e^{iH_{Ising}\tau} \, e^{-i\phi S_x} \, \underbrace{e^{-iH_{Ising}\tau} |\Psi_0\rangle}_{\equiv |\Psi\rangle}$$

$$= \sum_m \langle \Psi | C_m | \Psi \rangle \, e^{i\phi m} \qquad C_m = \sum \underbrace{\sigma_1^z \sigma_4^y \ldots \sigma_k^z}_{\text{At least m terms}}$$

m^{th} order Fourier coefficient $\langle \Psi | C_m | \Psi \rangle$ indicates $|\Psi\rangle$ has correlations of at least order m

MQC protocol – $\langle S_x \rangle$ measurement





$$H_{Ising} = J/N \sum_{i<j} \sigma_i^z \sigma_j^z$$

$J \lesssim 5kHz$

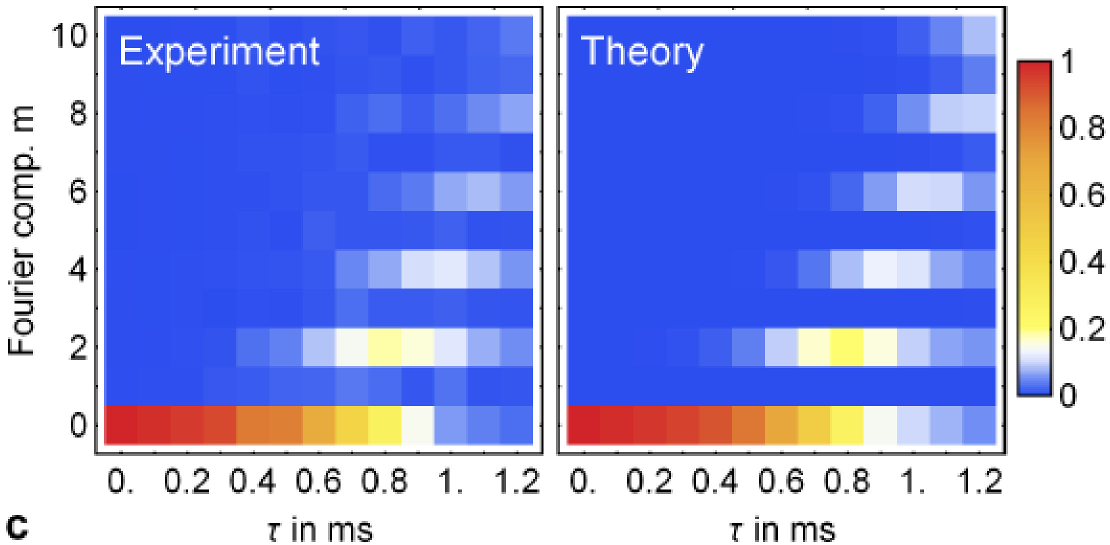
$N = 111$

$\Gamma = 93Hz$

[Gärttner, Bohnet et al. Nature Physics 2017]

Fourier transform of magnetization

[Gärttner, Bohnet et al. Nature Physics 2017]



- Measure build-up of 8-body correlations
- Only global spin measurement
- Illustrates how OTOCs measure spread of quantum information

Summary:

- trapped ion crystals – motional amplitude sensing below the zero-point fluctuations
- employed spin-squeezing, OTOCs to benchmarked quantum dynamics with long range Ising interactions

Future directions:

- transverse field, variable range interaction, longitudinal fields

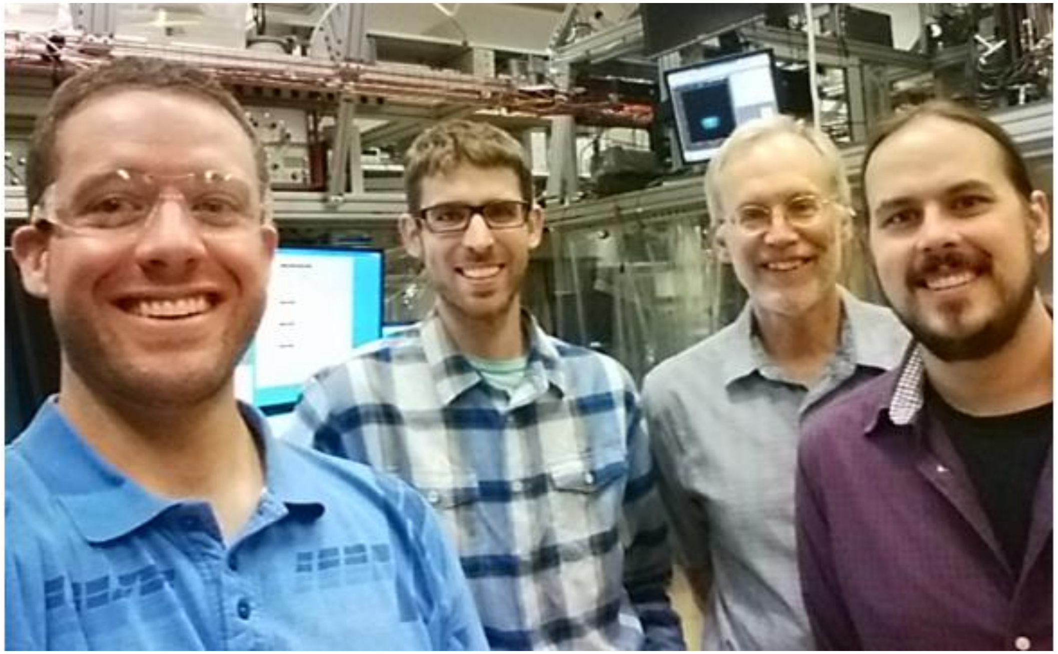
$$\sum_{i<j} J_{i,j}\sigma_i^z \sigma_j^z + B_\perp \sum_i \sigma_i^x + \sum_i h_i \sigma_i^z$$

- spin-phonon models (Dicke model)

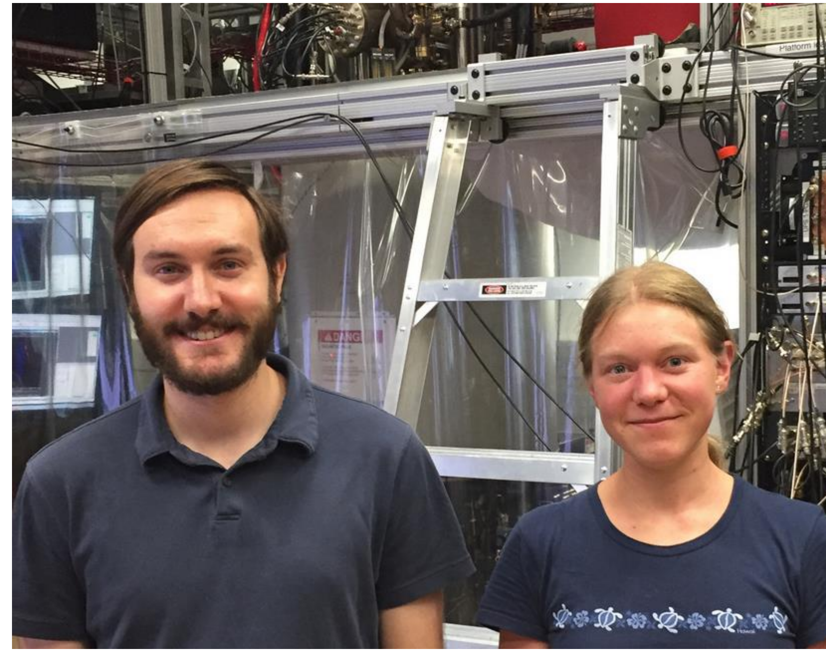
$$-\delta a^\dagger a - \frac{g_0}{\sqrt{N}}\left(a + a^\dagger\right)S_z + B_\perp S_x$$ arXiv:1711.07392

- mitigate decoherence, improve single ion readout
- 3-dimensional crystals with thousands of ions?

Lab selfie ~ 2014



2017



Joe Britton
ARL

Justin Bohnet
Honeywell

Brian Sawyer
GTRI

Kevin Gilmore
CU grad student

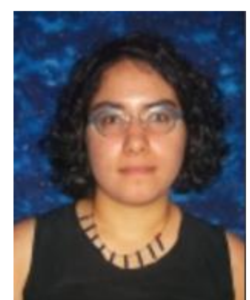
Elena Jordan
Leopoldina PD

Theory

Ana Maria Rey

Martin Gärttner

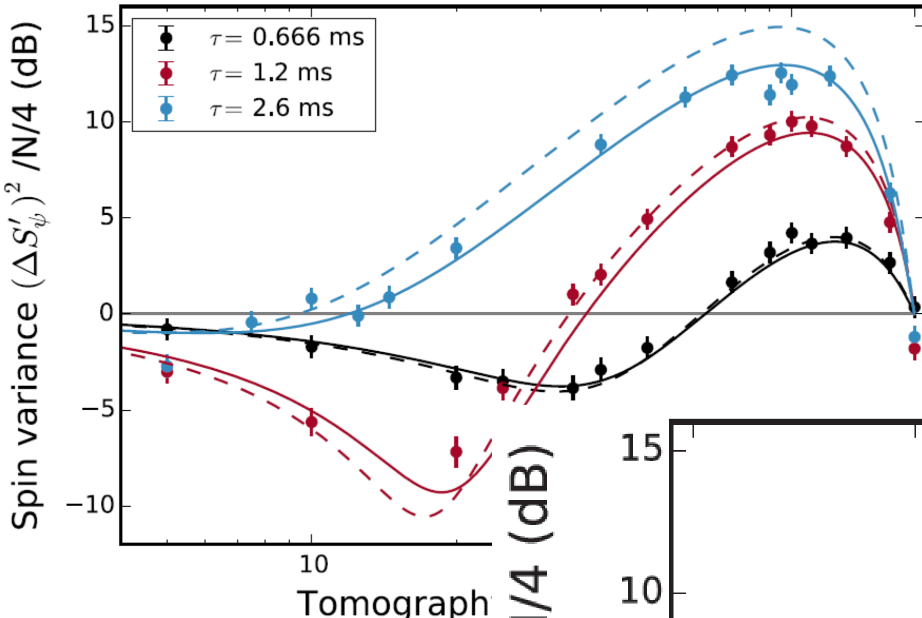
Michael Wall



Arghavan Safavi-Naini

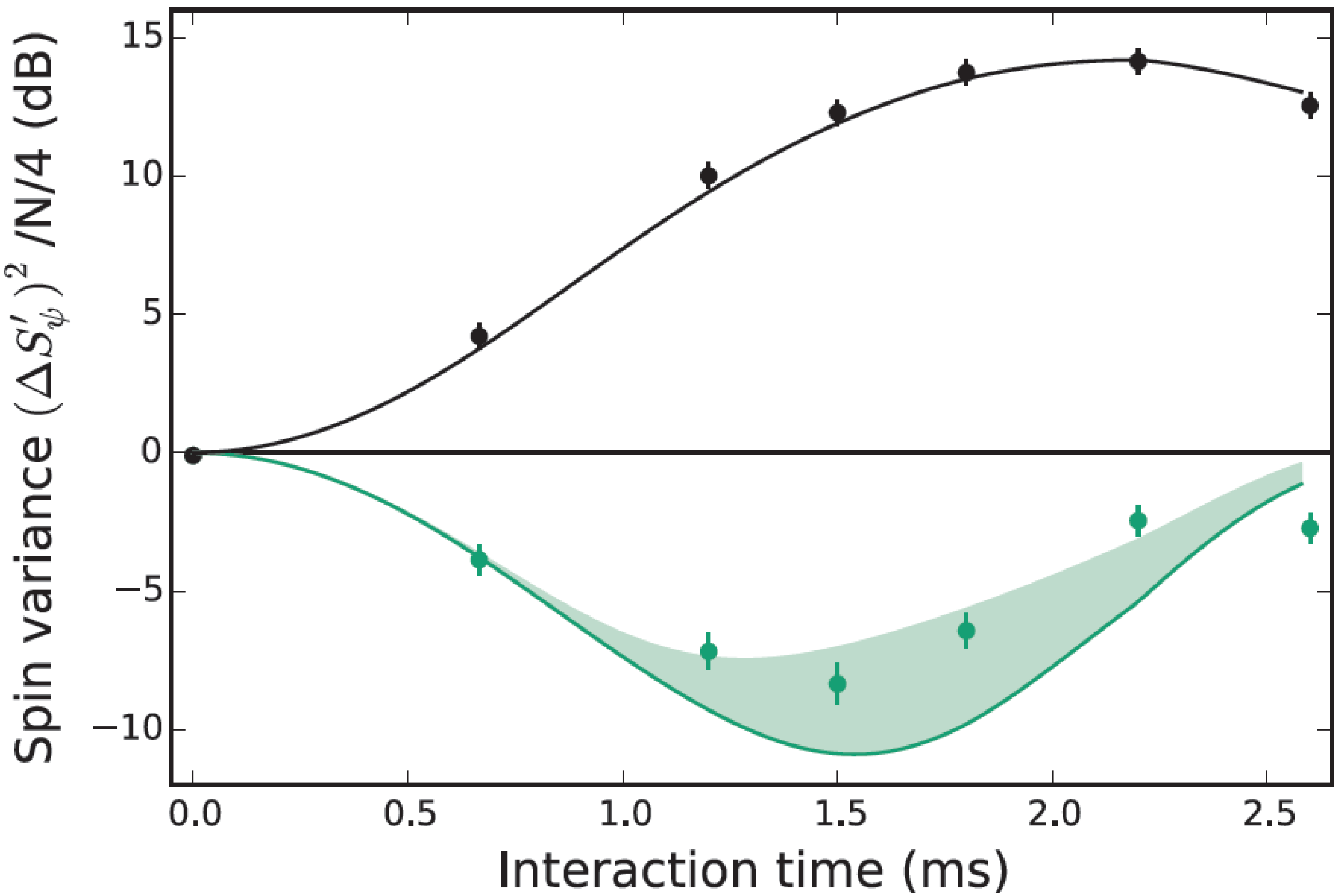
Michael Foss-Feig (ARL)

Benchmarking quantum dynamics and entanglement



N = 85

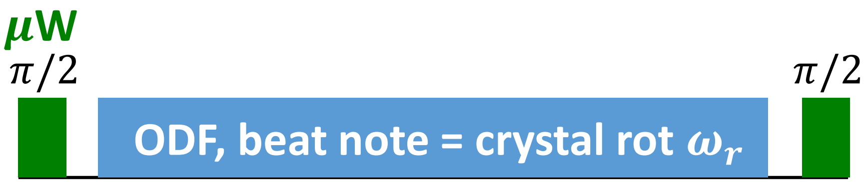
Time dependence of squeezed and anti-squeezed variance

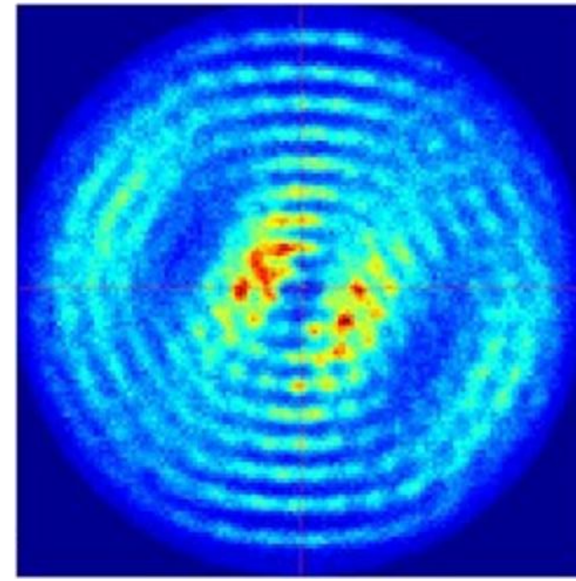


Bohnet *et al.*, Science352 (2016)

Writing a spin gradient

method: generate Stark shift gradient in the rotating frame

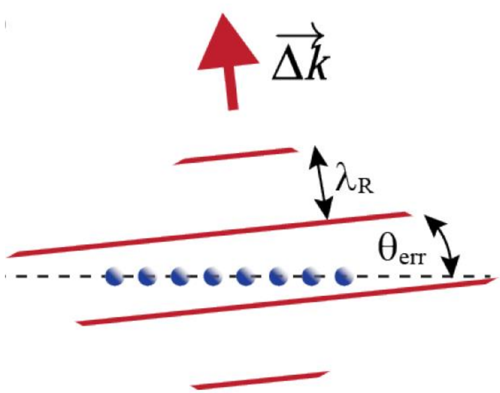


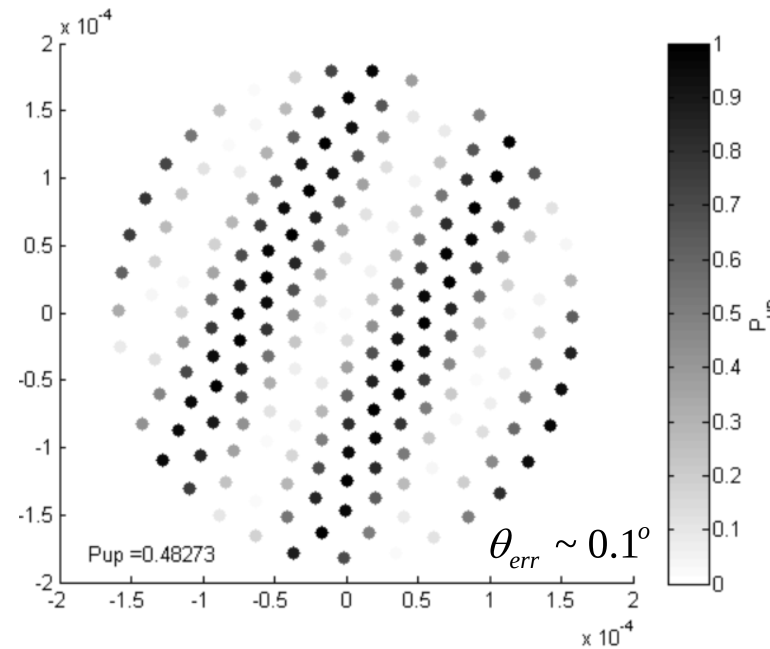


$$H_{ODF} = \sum_j \frac{F_o}{\Delta k} \cos\left[\Delta k \sin(\theta_{err}) \overbrace{R_j \cos(\omega_r t + \phi_j)}^{x_{lab,j}} - \mu t\right] \hat{\sigma}_j^z$$

$\mu = \omega_r$ produces static Stark shift in the rotating frame

$$\approx \sum_j \frac{F_o}{\Delta k} J_1\left(\Delta k \sin(\theta_{err}) R_j\right) \sin(\phi_j) \hat{\sigma}_j^z \equiv \sum_j h_j \hat{\sigma}_j^z$$





Random field Ising model $\quad \frac{1}{N} \sum_{i<j} J_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z + \sum_j h_j \hat{\sigma}_j^z$

In-plane modes

Freericks group, PRA 87 (2013)