Higher twist E-loss, in medium evolution and the EIC

or

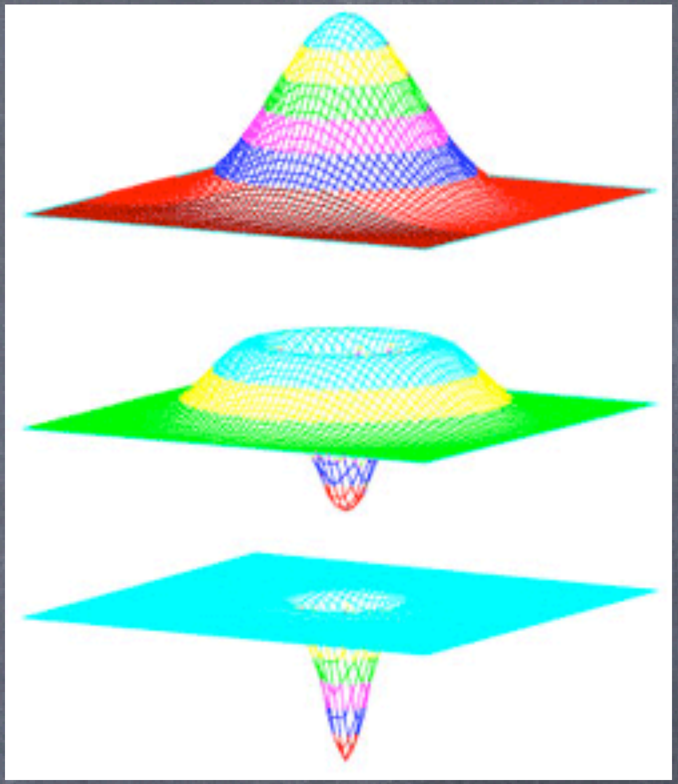
What do nuclei look like to a microscopic colored probe

Abhijit Majumder

The Ohio-State University

THE QUESTION:

Electron scattering gives the charge density or quark density inside a nucleon



J. Arrington, G. Miller

What does the gluon density look like ?

How confined are low-x gluons (fluctuations)?

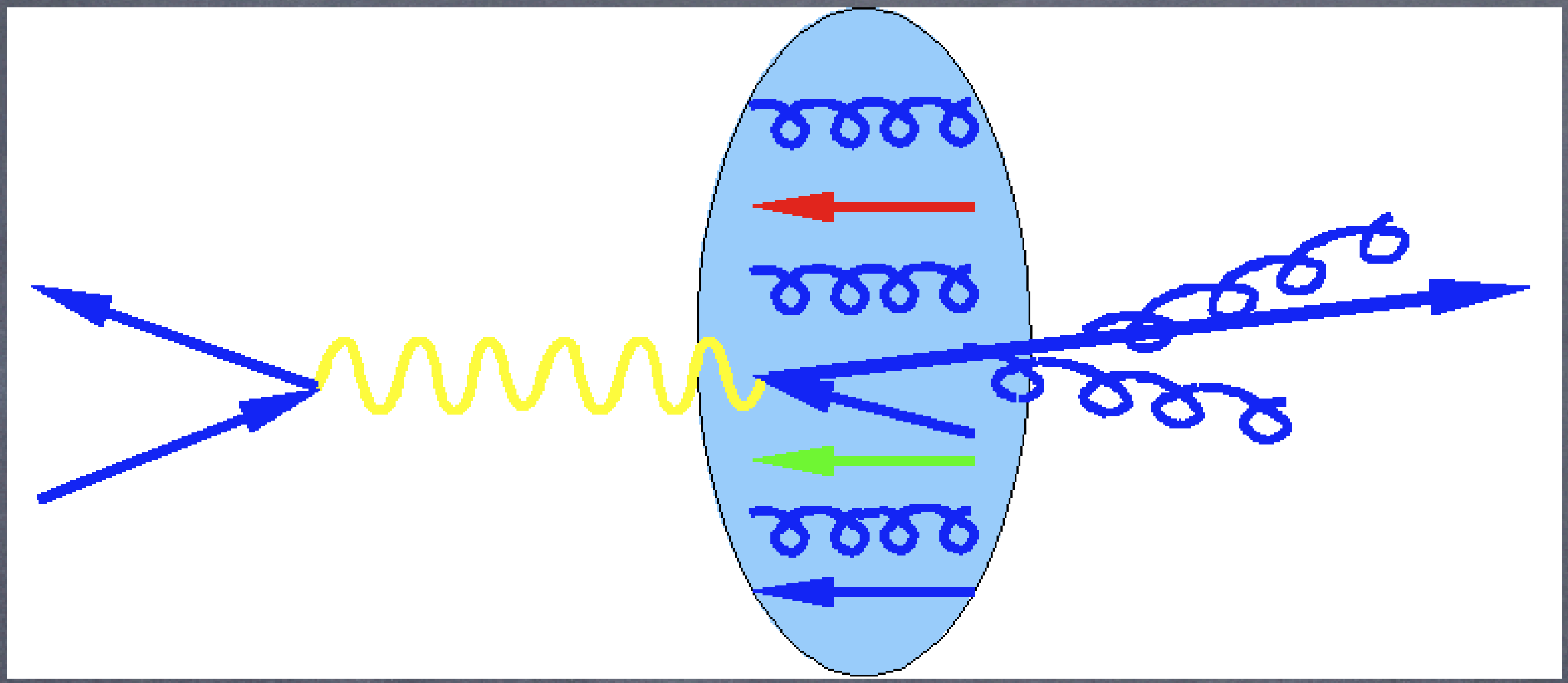
Are there other long range correlations in nuclei ?

How does this change with resolution ?

Need a colored probe !

A second order probe!

DIS with a hard jet in the final state



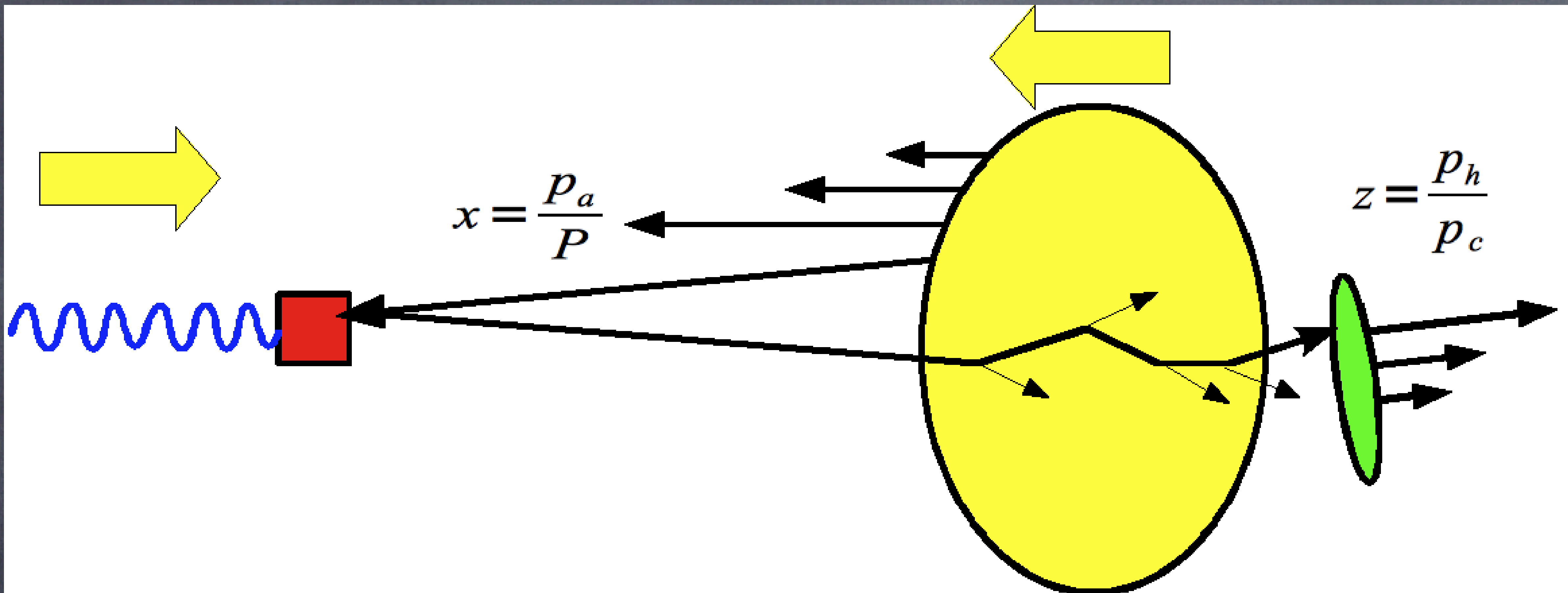
Note: the produced jet (not the photon) is the probe

Lack of precise control on parameters as in regular DIS!

In order to study this with pQCD:

the jet scale (virtuality) has to be hard on entry and exit

Why an EIC, or why a Large scale ? allows for a factorized approach



$$d\sigma = \int dx \; G(x) \; d\hat{\sigma} \; \tilde{J}$$

$$\tilde{J} = J_{vac} + \int dL \, f(\hat{q} \ldots, L, Q^2) \times J_{vac}$$

Philosophy of calculations

1) Calculate the effect of gluon interactions on basic, yet unobservable processes

2) Use these to construct more complicated observable processes

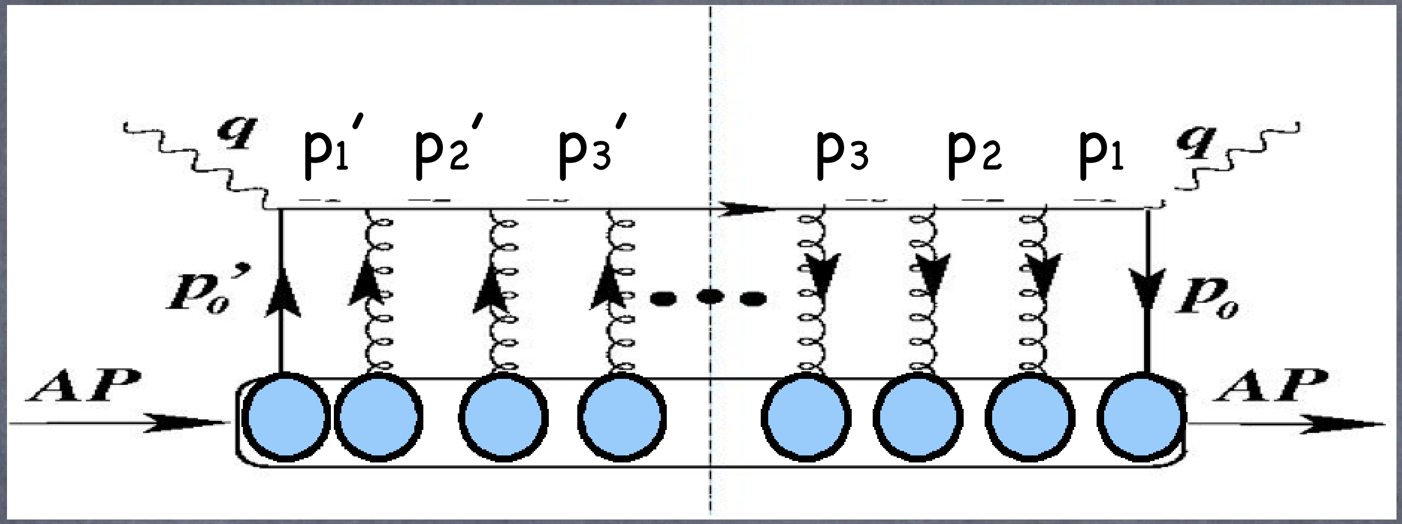
3) Quantify the effect in a handful of transport coeffs.

4) Relax assumptions on gluon distribution and repeat

1) single parton without radiation

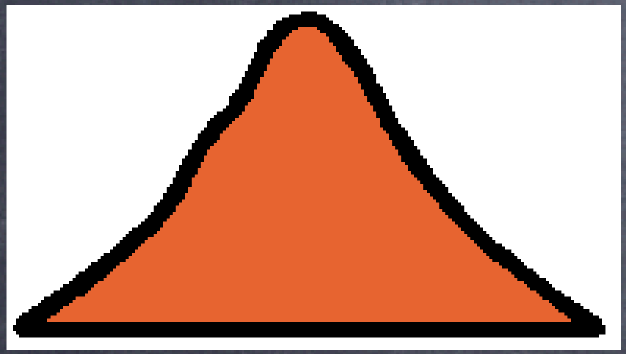
a) transverse broadening

$$p^+ = \frac{p^0 + p_z}{\sqrt{2}}$$



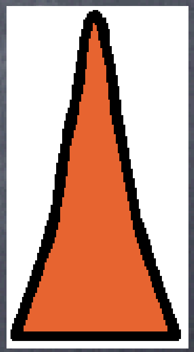
$$p^- = \frac{p^0 - p_z}{\sqrt{2}}$$

Assuming independent scattering off nucleons gives a diff. equation



$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$

$$\langle p_\perp^2 \rangle = 4Dt$$



$$\hat{q} = \frac{p_\perp^2}{L^-} = \frac{4\pi^2 \alpha_S C_R}{N_c^2 - 1} \int \frac{dy^-}{2\pi} \frac{d^2 y_\perp d^2 k_\perp}{(2\pi)^2} e^{-i\left(\frac{k_\perp^2}{2q^-} y^- - k_\perp \cdot y_\perp\right)}$$

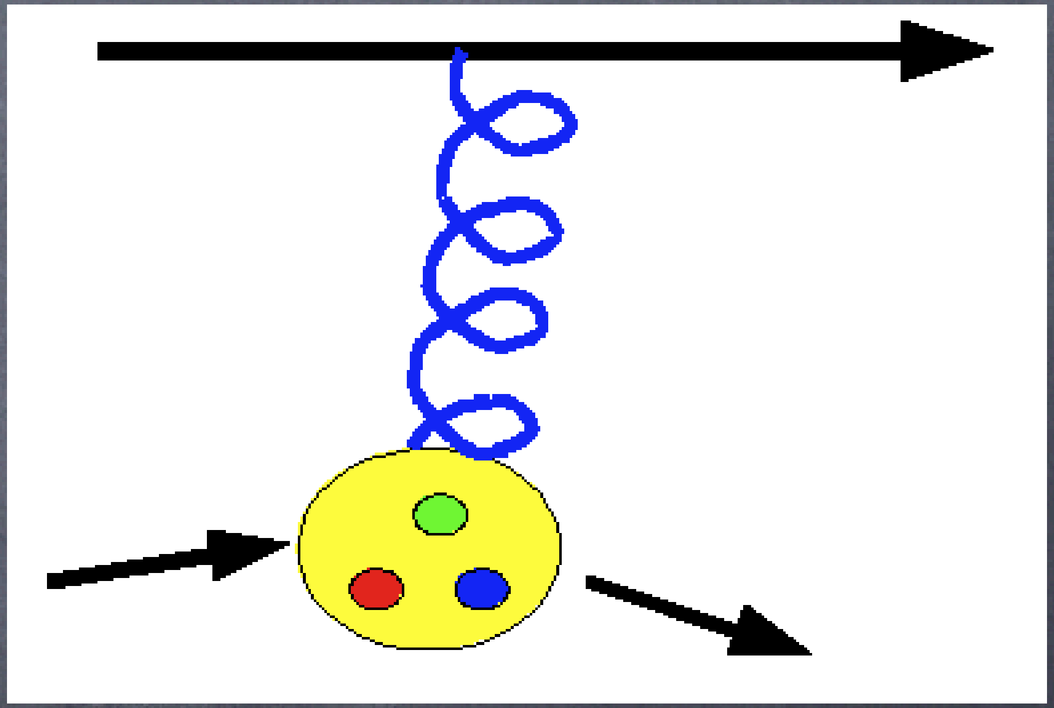
$$\times \quad \langle F^{\mu\alpha} v_\alpha (y^-, y_\perp) F_\mu^\beta (0) v_\beta \rangle$$

b) Longitudinal drag and diffusion

A close to on shell parton has a 3-D distribution

$$p^+ = \frac{p_\perp^2}{2p^-}$$

$$f(\vec{p}) \equiv \delta^2(p_\perp^2)\delta(p^- - q^- + k^-)$$

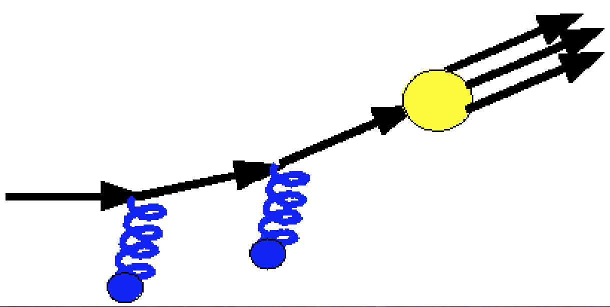


Using the same analysis, we get a drag. and diff. term

$$\frac{\partial f(p^-, L^-)}{\partial L^-} = c_1 \frac{\partial f}{\partial p^-} + c_2 \frac{\partial^2 f}{\partial^2 l^-}$$

c_1 is dE/dL, calculate in a deconfined quasi-particle medium.

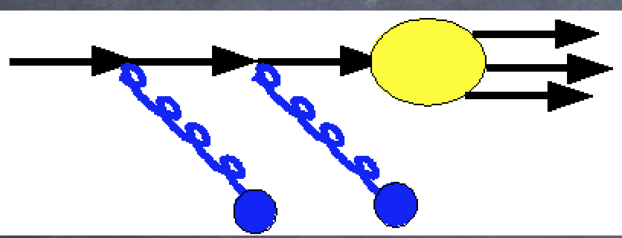
There are a 4 medium properties which modify the parton $\hat{q}$, $\hat{e}$ = dE/dL and $\hat{f}$ = dN/dL



$$D\left(\frac{\vec{p}_h}{|\vec{p}+\vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_T^2 \rangle_L}{L}$$

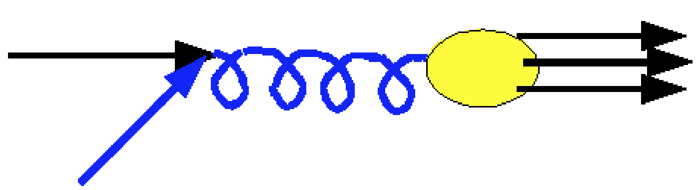
Transverse momemtum diffusion rate



$$D\left(\frac{p_h}{p-k}, m_J^2\right)$$

$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

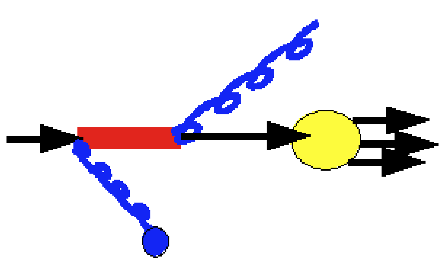
Elastic energy loss rate also diffusion rate e_2

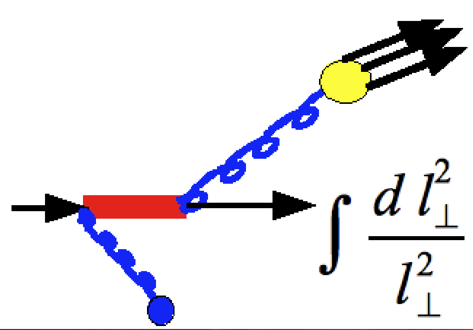


$$D_g\left(\frac{p_h}{p+k}, m_J^2\right)$$

$$\hat{f} = \frac{\langle \Delta N \rangle_L}{L}$$

Flavor (q <-> g) diffusion rate

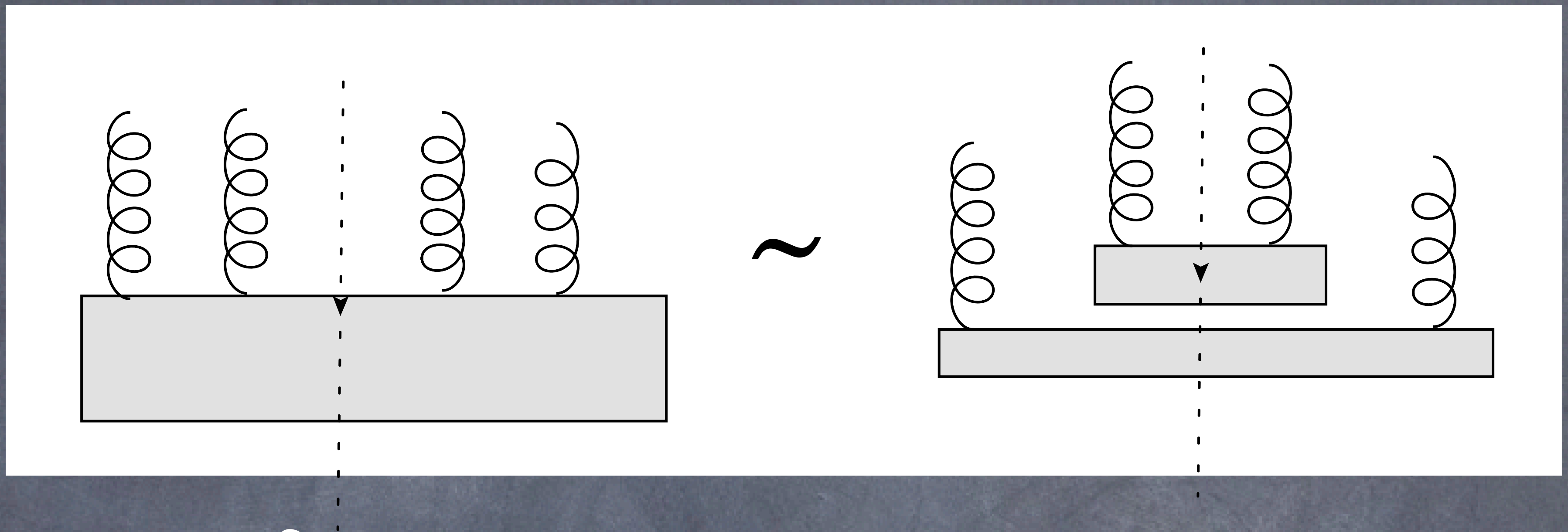




$$\int \frac{d\,l_\perp^2}{l_\perp^2} \int_{\frac{p_h}{p}}^{1} \frac{d\,y}{y} P(y)\, M(\vec{r}, y, l_\perp)\, D\left(\frac{p_h}{p\,y}\right)$$

couple the effect of these transport coeffs. with parton splitting

The number and form of the coeffs. is based on the short distance correlation approximation



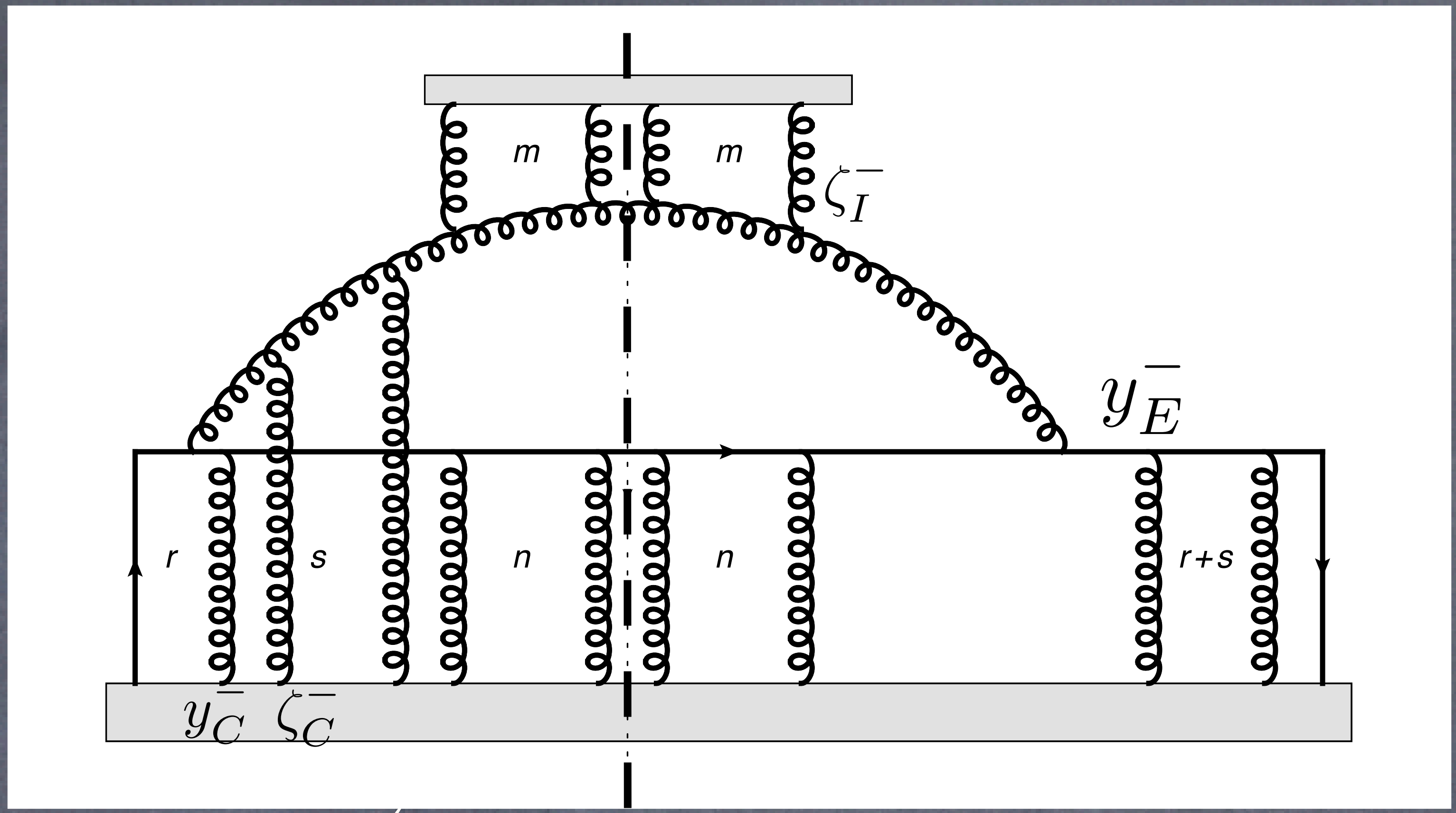
All four gluons from one nucleon: prop. to L

Two in one nucleon, two in another: prop. to L^2

2 n gluon expectation ---> n X 2 gluon expectation

What happens if low x gluons fluctuate over nucleons

Combine basic processes to calculate single gluon emission



$$\int dl_\perp dl_{q\perp} dy \text{ C.F. } \delta^2\left(q_\perp + l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{j=1}^{r} p_\perp^j - \sum_{l=1}^{m} k_\perp^l - \sum_{k=1}^{n} p_\perp^k\right)$$

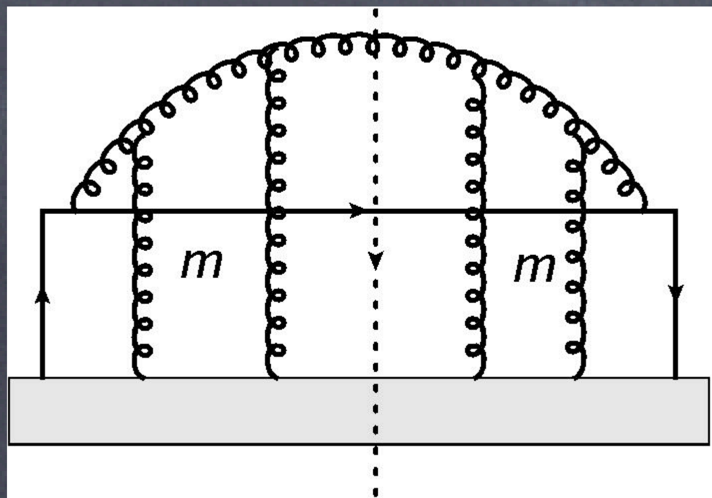
$$\frac{l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - \sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2} \cdot \frac{l_\perp - y\sum_{i=1}^{r+s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l}{\left(l_\perp - y\sum_{i=1}^{s} k_\perp^i - \sum_{l=1}^{m} k_\perp^l\right)^2}$$

$$\prod_{i=1}^{N}\int dy_i^- \frac{\int d^3\delta y_i \rho \langle p|A^+(y_i^- + \delta y_i^-, 0)A^+(y_i^-, -\delta y_\perp^i)|p\rangle}{2p^+(N_c^2-1)} e^{ik_\perp^i \delta y_\perp^i}$$

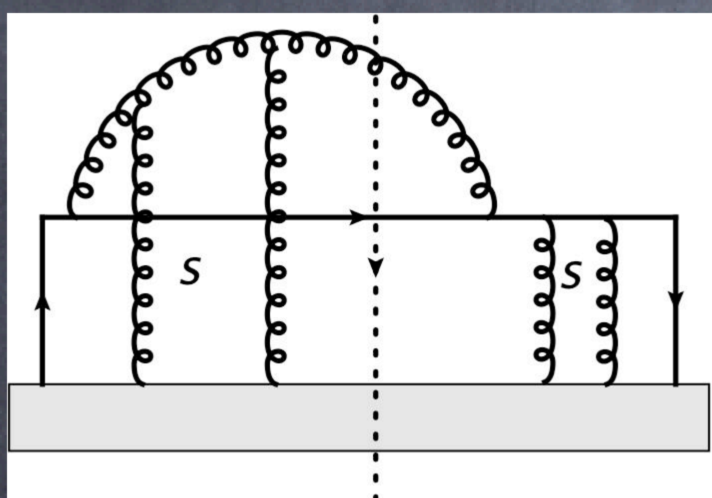
$$\left[\theta(\zeta_I^- - y_E^-)\left\{e^{-ip^+x_L y_E^-} - e^{-ip^+x_L\zeta_I^-}\right\} - \theta(\zeta_I^- - y_I^-)e^{-ip^+x_L y_I^-} - \theta(y_I^- - \zeta_I^-)e^{-ip^+x_L\zeta_I^-}\right.$$

$$\left.\left[\theta(\zeta_C^- - y_0^-)\left\{e^{ip^+x_L y_0^-} - e^{ip^+x_L\zeta_C^-}\right\} - \theta(\zeta_C^- - y_C^-)e^{ip^+x_L y_C^-} - \theta(y_C^- - \zeta_C^-)e^{ip^+x_L\zeta_C^-}\right]\right.$$

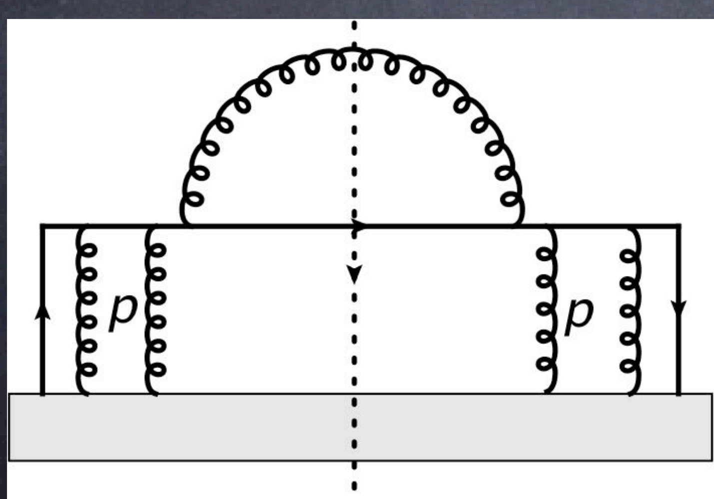
For D(z), integrate over broadening, expand in $1/l_\perp^2$



$$\sim C_A^m \int dy \frac{dl_\perp^2}{l_\perp^2} P(y) \int d\zeta^- \frac{2\hat{q}(\zeta^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-}\zeta^-\right)\right]$$

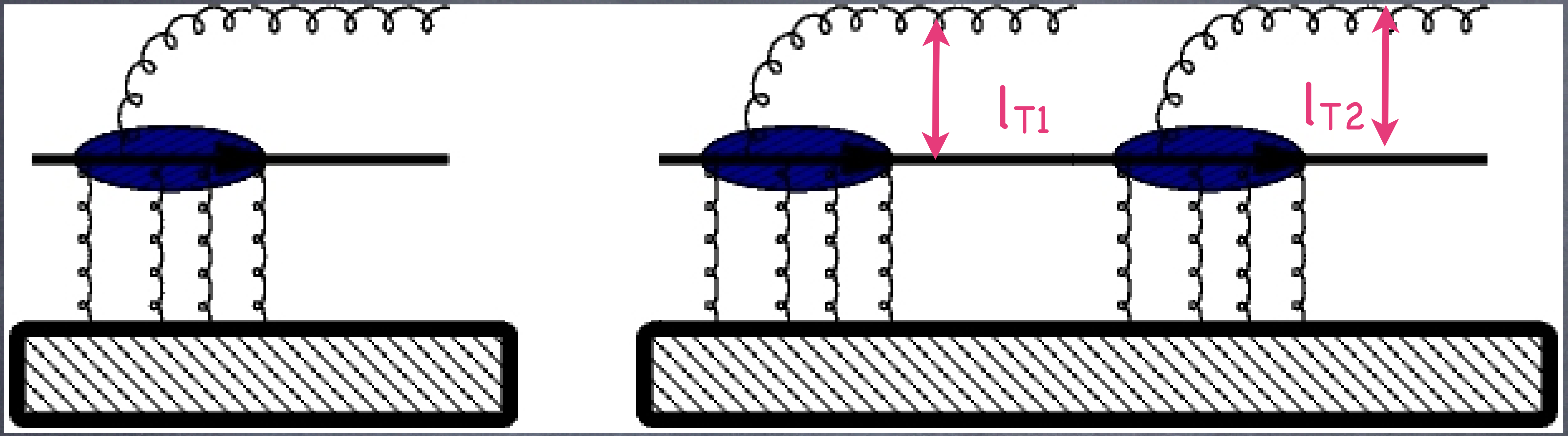


$$\sim -\left(\frac{C_A}{2}\right)^s \int dy \frac{dl_\perp^2}{l_\perp^2} y P(y) \int d\zeta^- \frac{\hat{q}(\zeta^-)}{2l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-}\zeta^-\right)\right]$$



$$\sim -(C_F)^p \int dy \frac{dl_\perp^2}{l_\perp^2} y^2 P(y) \int d\zeta^- \frac{\hat{q}_Q(\zeta^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-}\zeta^-\right)\right]$$

Need to repeat the kernel



What is the relation between subsequent radiations ?

To have unquestionable pQCD control need large Q^2

In the large Q^2 we can argue that there should be ordering of l_T. $l_\perp^1 \gg l_\perp^2$

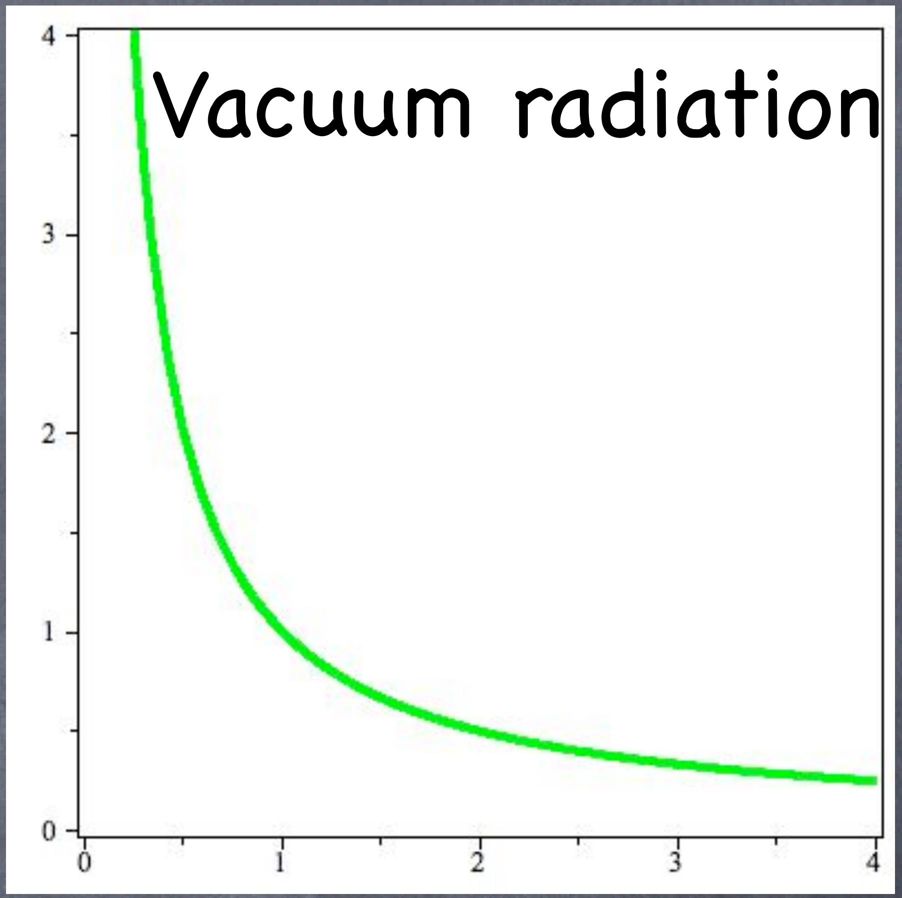
The same statement in a plot

Virtuality is like $l_\perp^2$, At leading log, CS goes as $dl_\perp^2/l_\perp^2$

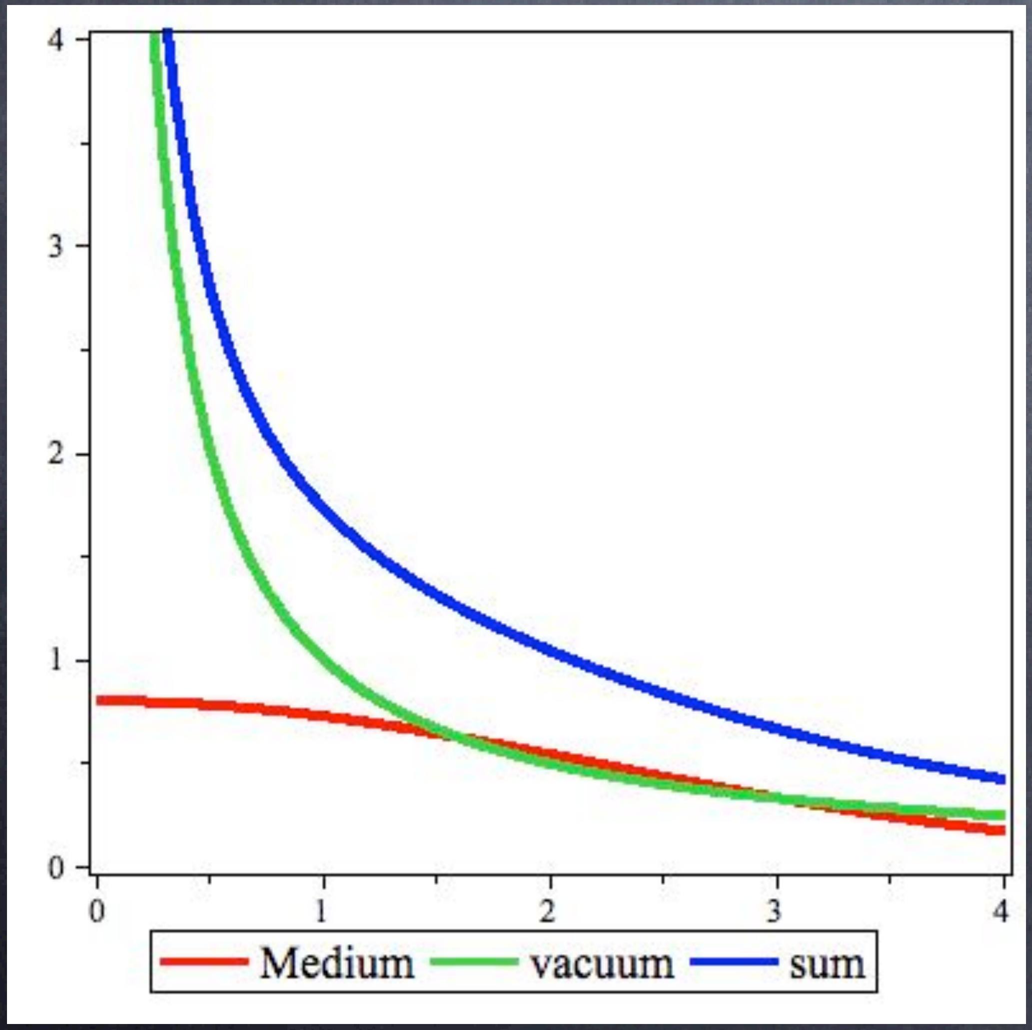
Integrating over this yields a $\log(\mu_1^2/\mu_2^2)$

Multiple emissions will yield large logs if strongly ordered

$$\frac{1}{l_\perp^2}$$



$$l_\perp^2$$



This CS is slightly modified in the medium

Include the largest correction from the medium

dσ = Log + # L,

If form is not too different, then sum with DGLAP

Testing all this for the single frag. func.

Assuming nuclear p. d. f. = A X nucleon p. d. f.

we can construct the ratio of the frag. funcs.

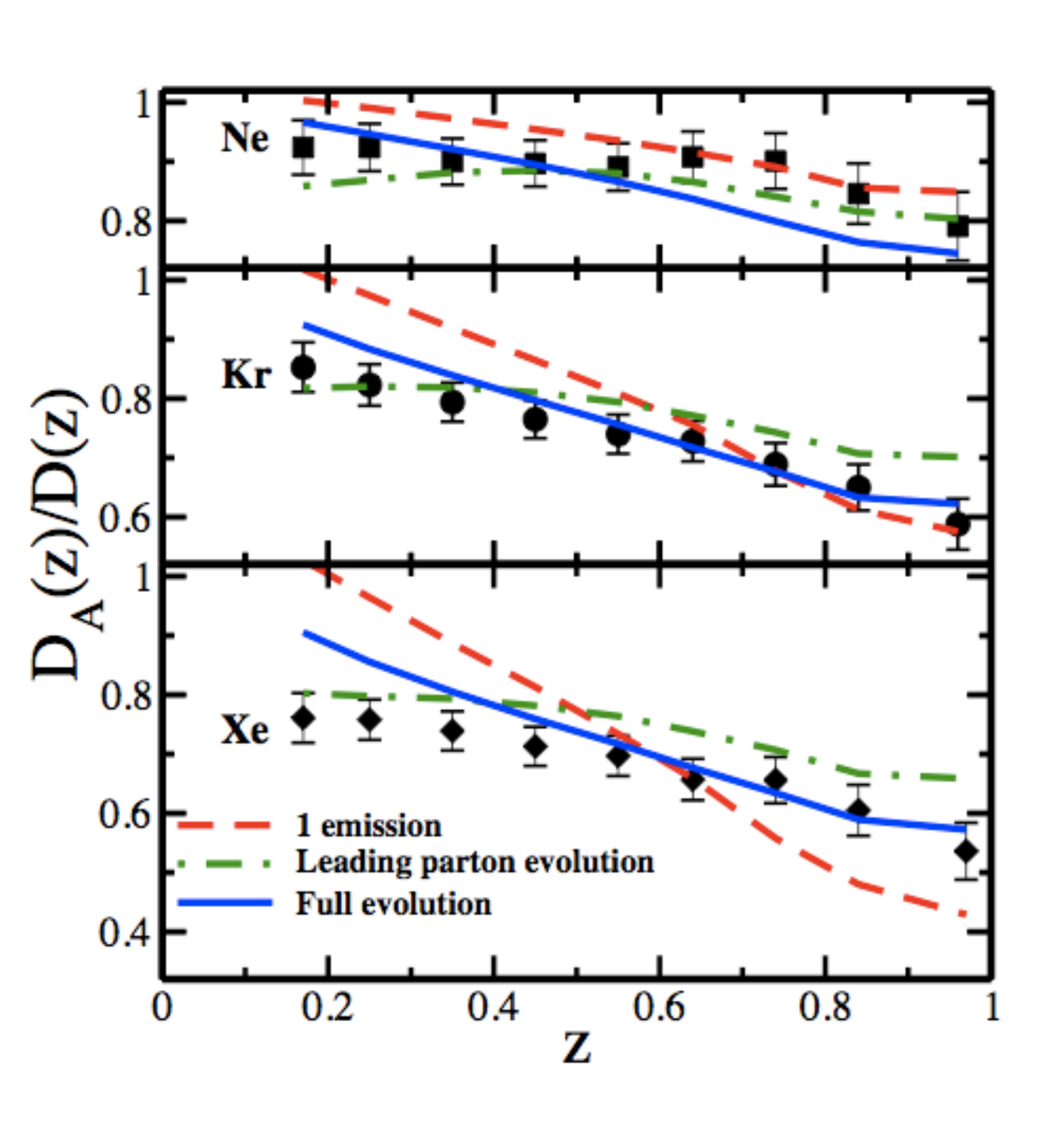
Data from HERMES at DESY

Three different nuclei

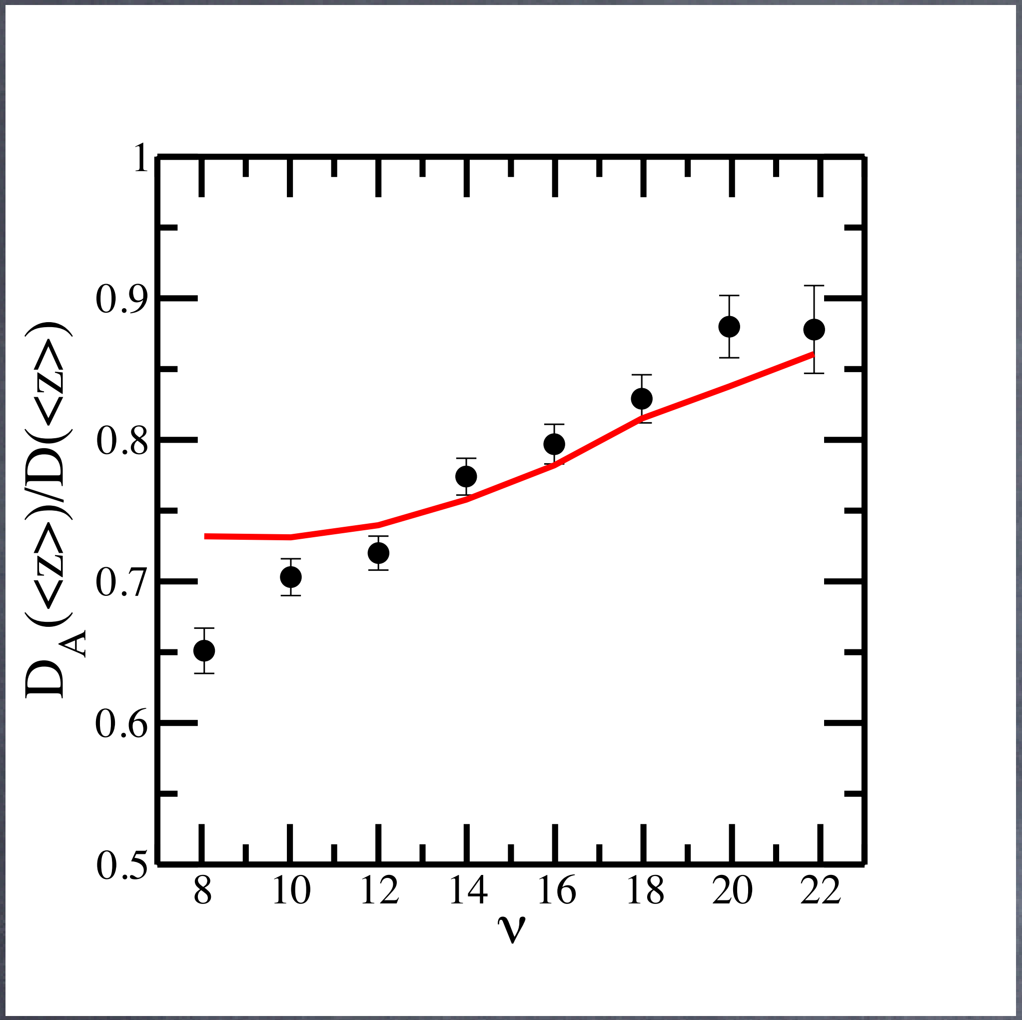
one $\hat{q}$ = 0.08GeV^2/fm

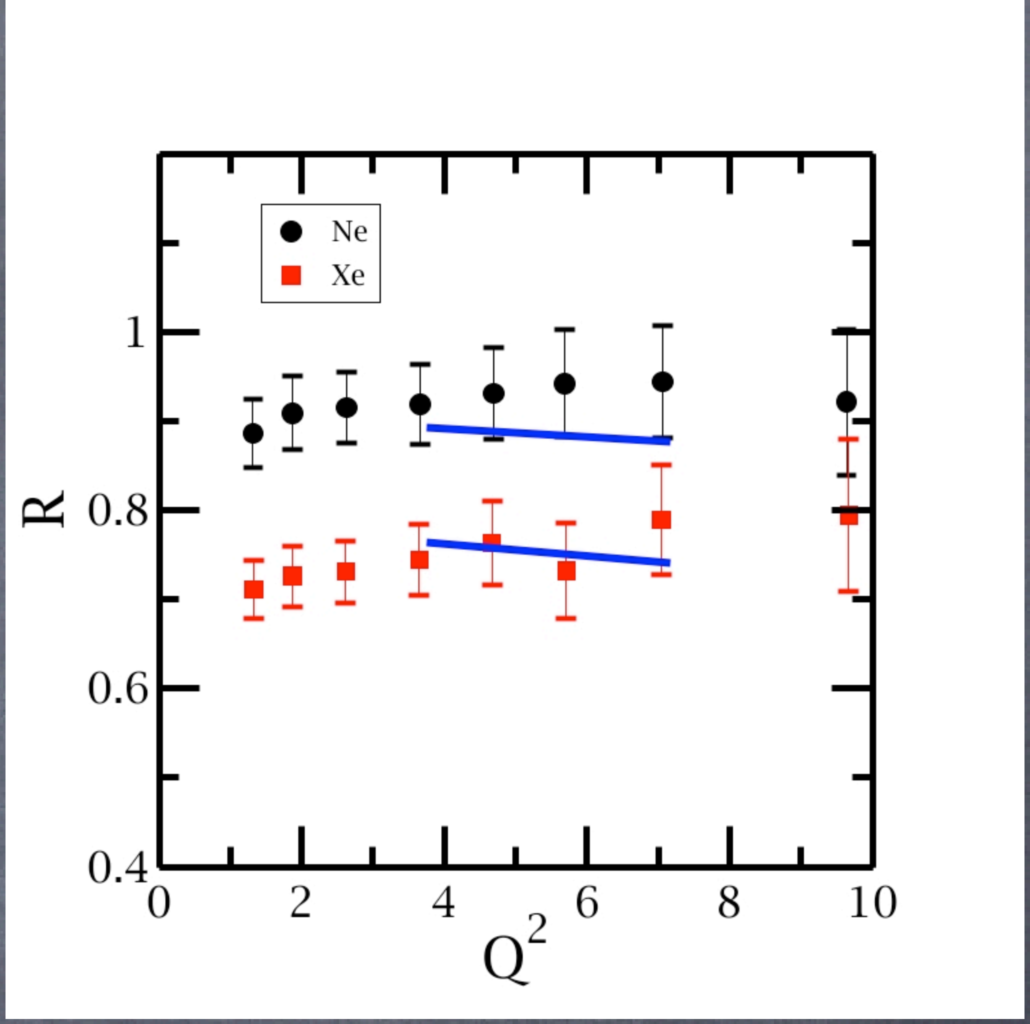
Fit one data point in Ne
everything else is prediction

Q^2 = 3GeV^2 , ν = 16-20 GeV



The ν and Q^2 dependence





Many approximations made!

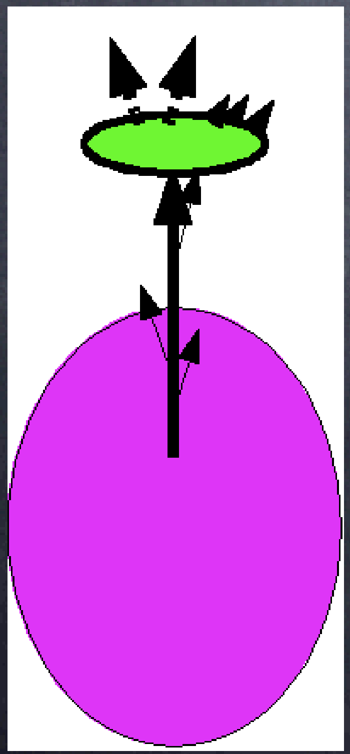
$$\tilde{D}(z, Q^2, \nu)\Big|_{\zeta}^{\zeta_f} \rightarrow \tilde{D}(z, Q^2, \nu)\Big|_{\zeta_i}^{\zeta_f}$$

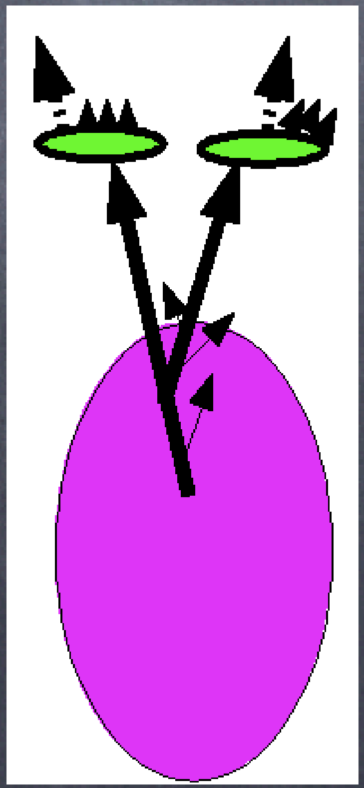
Dihadrons, yet another test of the formalism

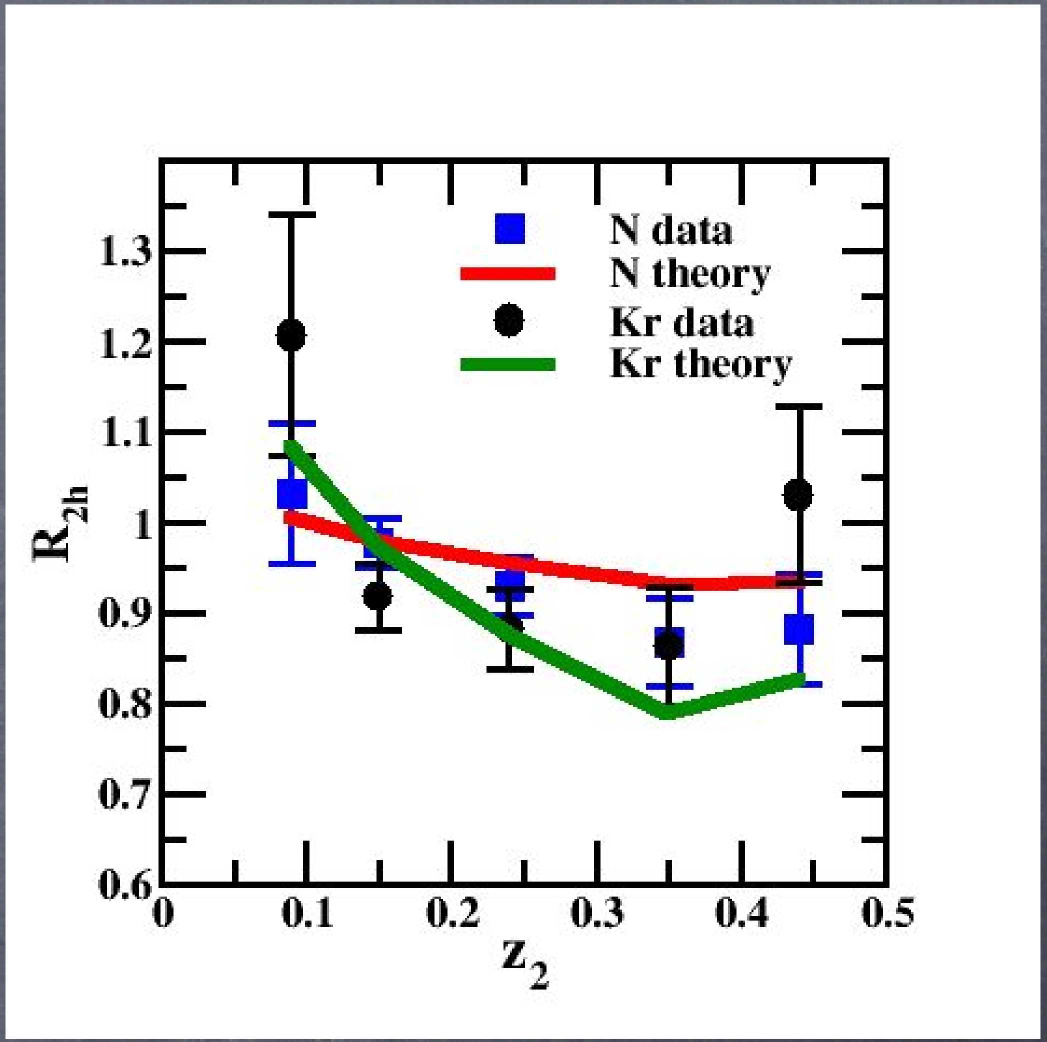
Works in DIS with no additional parameters

Works in HIC with no additional parameters

Requires the same non-pert. input a dihadron fragmentation func.

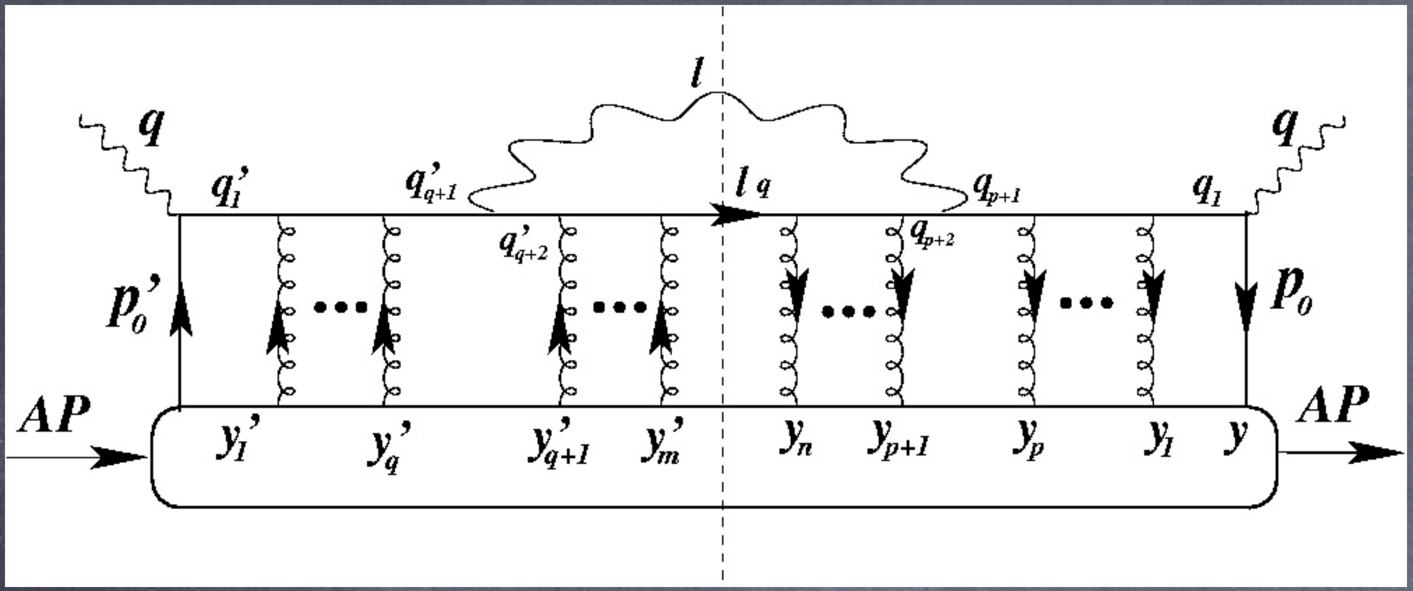


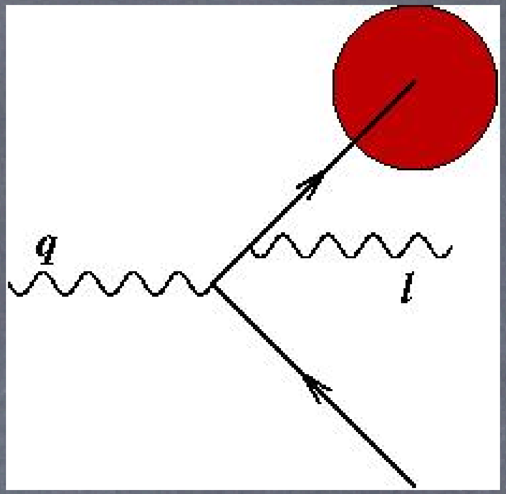


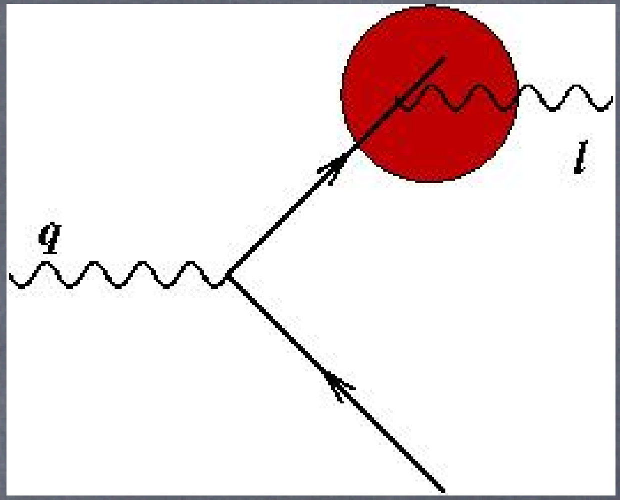


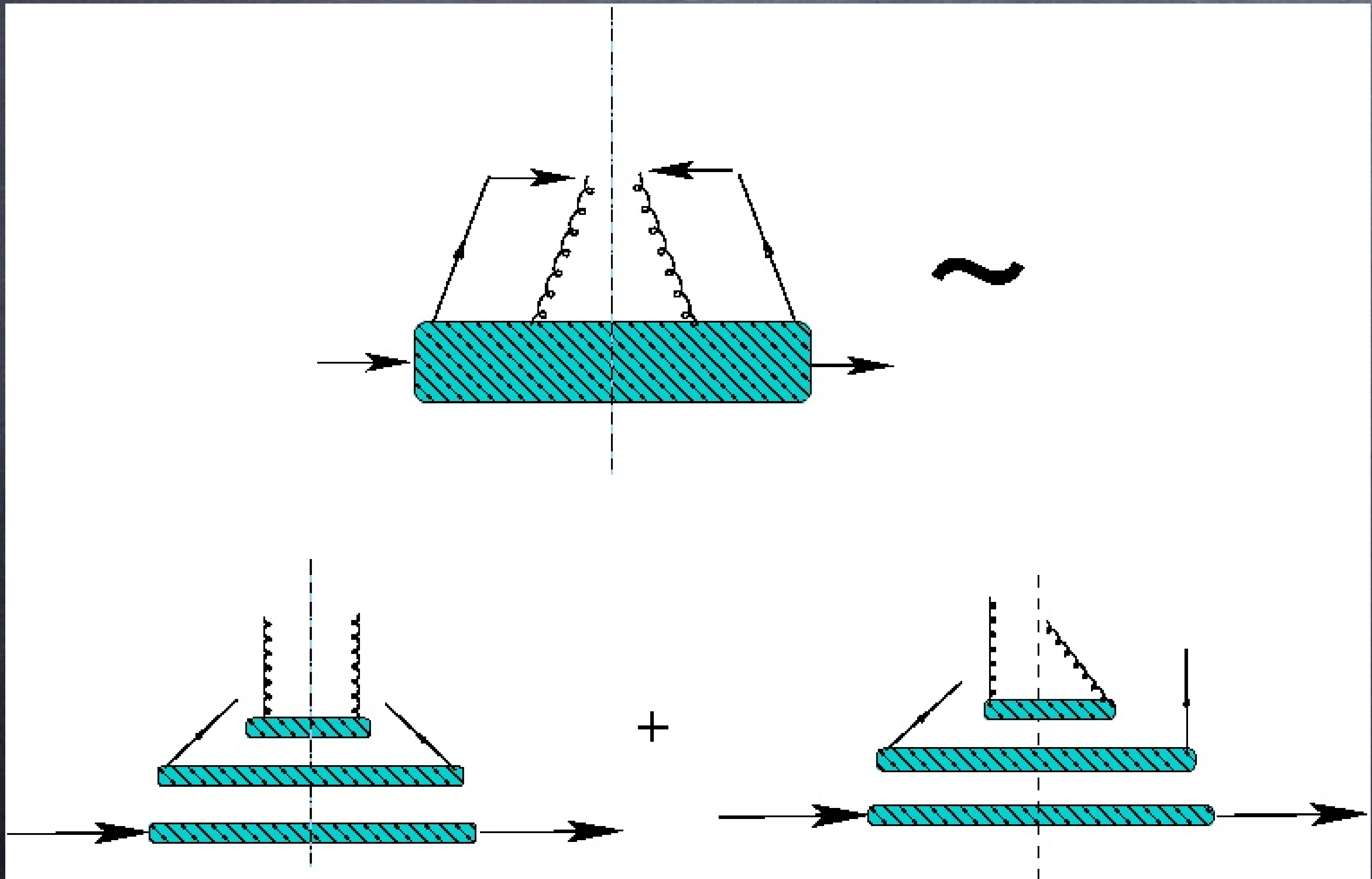
Relaxing the assumptions on the gluon correlation

consider photon Brem.









This is basically a gluon GPD

It does not yet involve color correlations over several nucleons

only momentum correlations

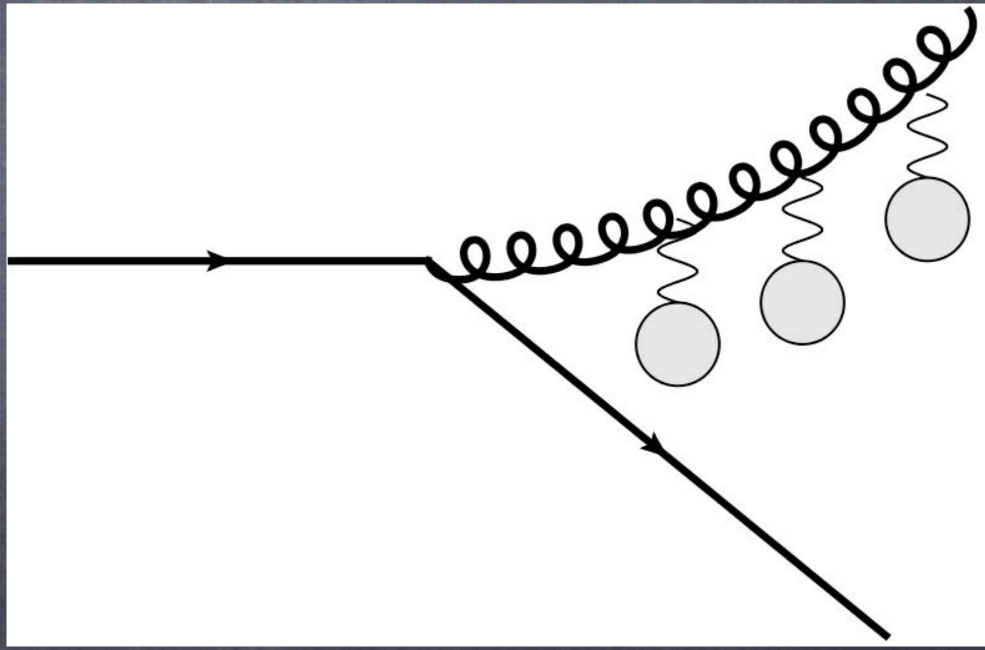
Turning all this into a Monte-Carlo not so trivial

A Monte-Carlo tracks the momenta of each of the partons

Need to use calculated double differential distribution

$$\frac{d\sigma}{dl_\perp^2\, dl_{q\perp}^2} \propto \int \frac{dy d^2 l_\perp d^2 l_{q\perp}}{2\pi^2} \frac{\alpha_s C_F P(y)}{l_\perp^2 y} \int_0^{L^-} d\zeta^- D(\zeta^-) \left\{ 2 - 2\cos\left(p^+ x_L \zeta^-\right)\right\} \left[\left(\frac{4 - 2\vec{l}_\perp \cdot \nabla_{l_{q\perp}}}{l_\perp^2} \right) \frac{e^{-\frac{l_{q\perp}^2}{4\int dy^- D(y^-)}}}{4\pi \int dy^- D(y^-)} + \nabla^2_{l_{q\perp}} \frac{e^{-\frac{l_{q\perp}^2}{4\int dy^- D(y^-)}}}{4\pi \int dy^- D(y^-)} - \left(\int_{\zeta^-}^{L^-} dy^- D(y^-) \right) \left\{ \frac{2\vec{l}_\perp \cdot \nabla_{l_{q\perp}} \nabla^2_{l_{q\perp}} - 4\nabla^2_{l_{q\perp}}}{l_\perp^2} \right\} \frac{e^{-\frac{l_{q\perp}^2}{4\int dy^- D(y^-)}}}{4\pi \int dy^- D(y^-)} \right]. \quad (95)$$

$l_{q\perp}$ is the off-set from the quark and gluon momenta being equal and opposite



Integrating out the $l_{q\perp}$

$$\frac{d\sigma}{dl_\perp^2} \sim C_A^m \int dy \frac{dl_\perp^2}{l_\perp^2} P(y) \int d\zeta^- \frac{2\hat{q}(\zeta^-)}{l_\perp^2} \left[2 - 2\cos\left(\frac{l_\perp^2}{2q^-} \zeta^- \right) \right]$$

Conclusions: what is missing ?

1) The scale evolution of the transport coefficients

2) Incorporation of elastic loss and diffusion (just done!)

3) A complete NLO calculation to estimate the error

4) Extension to Monte-Carlo simulations (underway!)

6) Going beyond the lowest order and diagonal coeffs.

7) Making more general transport coefficients for multi-gluon correlations

Back up

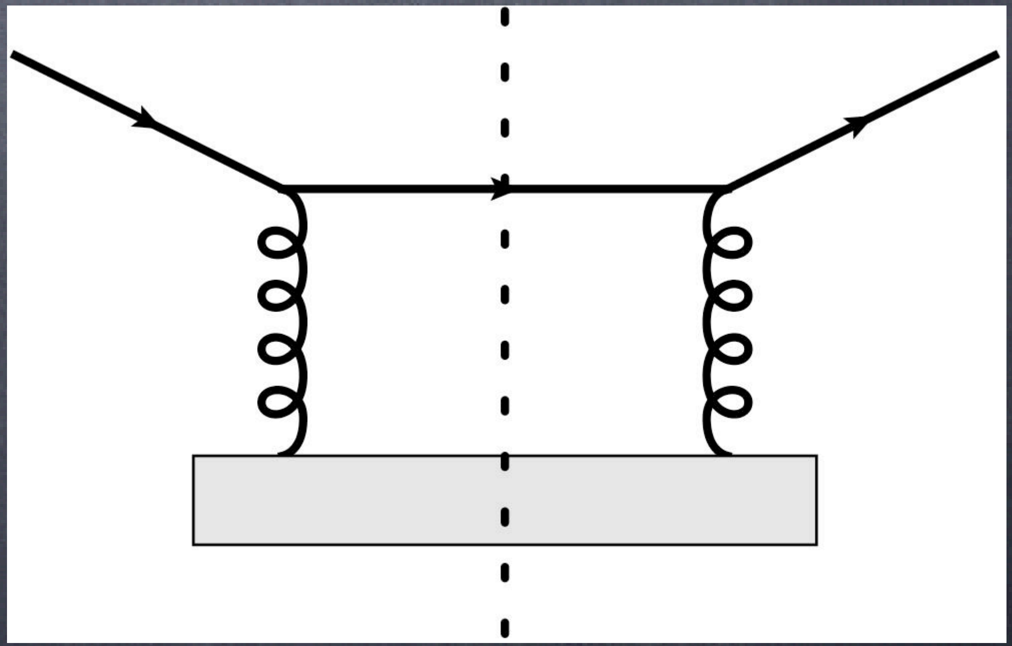
What is "higher twist" ?

A knitting technique ?

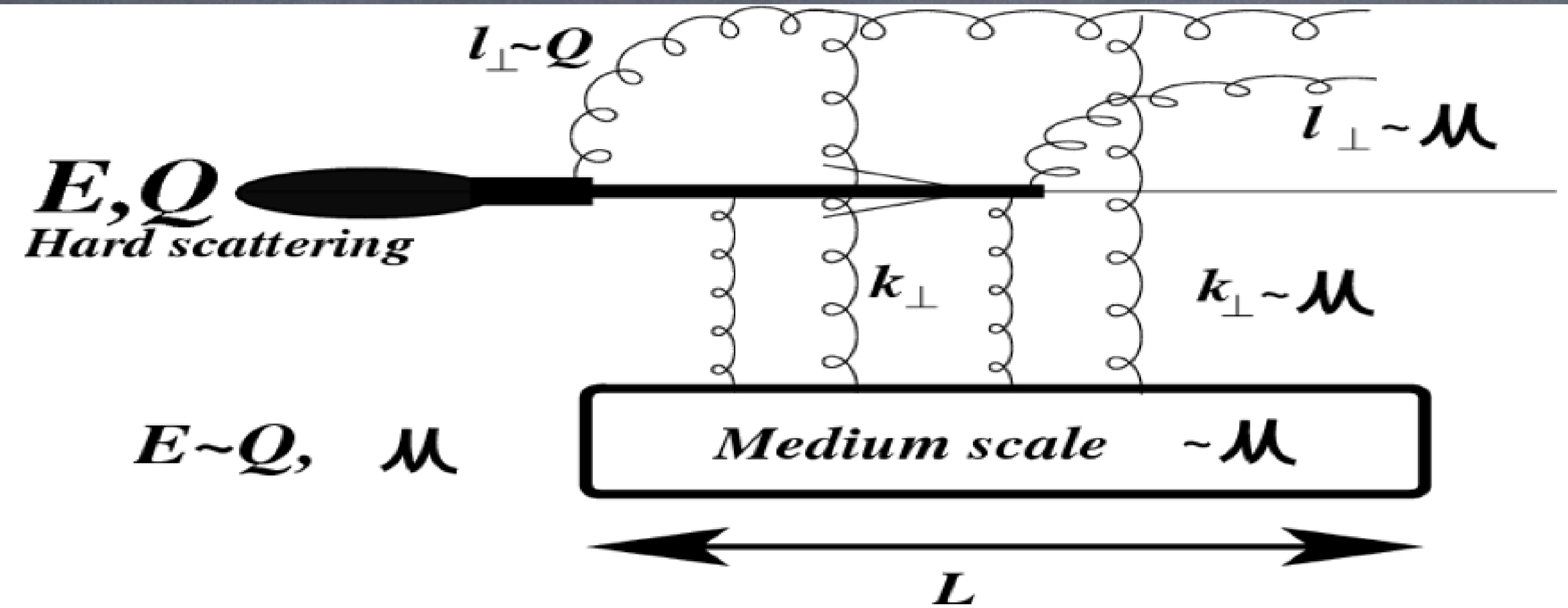
A technical term referring to the inclusion of multiple scattering effects on Hard processes

Involves the Lorentz force[2] $F^{\mu\nu} v_\nu F_\mu^\alpha v_\alpha$

Twist $t = d - s$ = 2



Possibility of setting up a rigorous theory at some large Q, compare directly with experiment no fudge!



Jet forward energy: $E, q^- \sim Q \gg m_J \gg \Lambda_{QCD}$ ***mass of proton,***

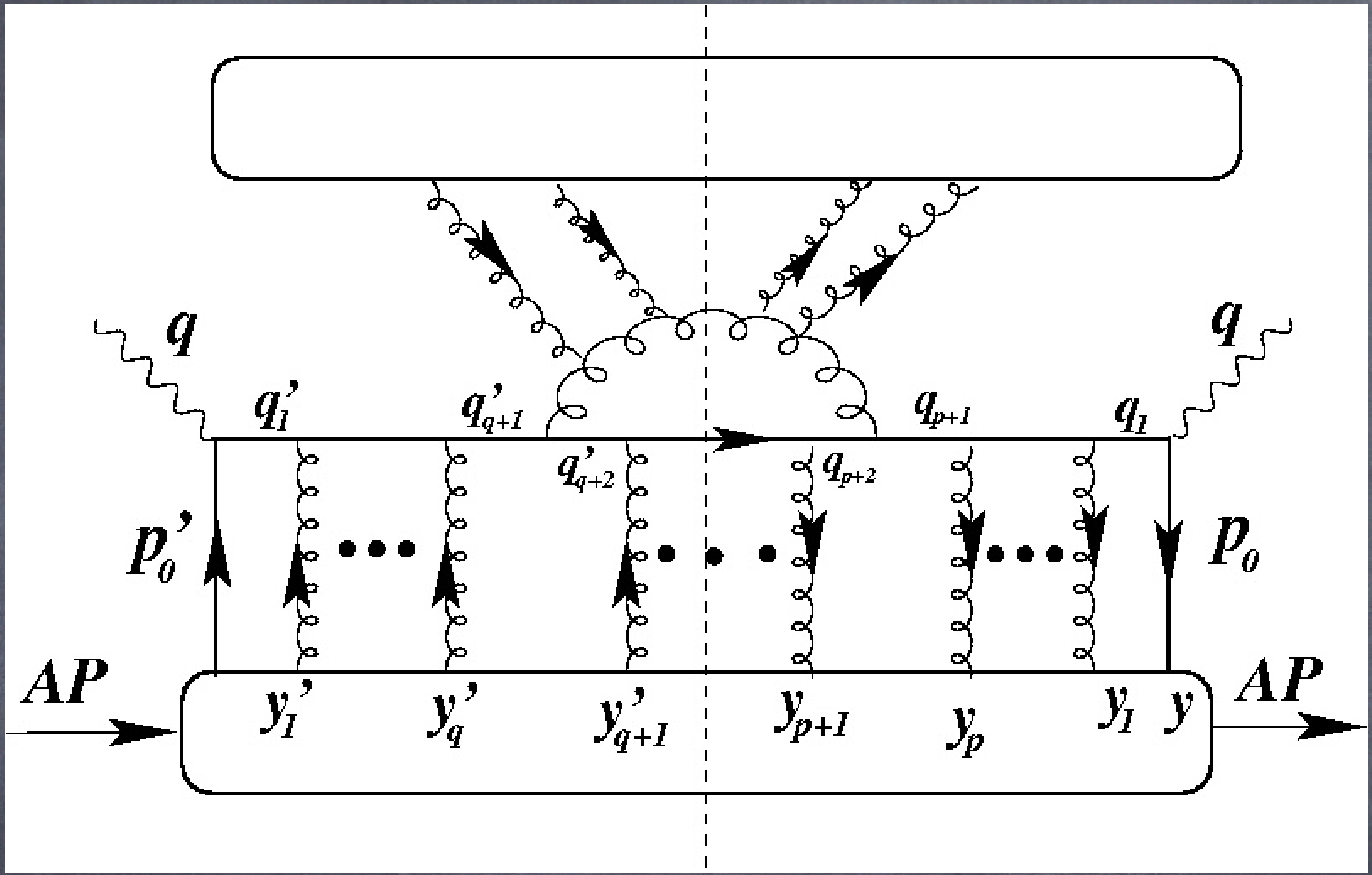
Virtuality of photon: $Q \gg l_\perp \leqslant m_J$ ***Virtuality of jet,***

Radiated gluon momentum: $\left[\dfrac{l_\perp^2}{2q^- y}, yq^-, l_\perp\right]$

Soft medium gluons $\lambda_{QCD} \ll k_\perp \ll l_\perp$ ***However!*** $A^{\frac{1}{3}} k_\perp \leqslant l_\perp$

$L \sim A^{\frac{1}{3}}$ ***A, atomic number of the nucleus,***

The single gluon emission kernel



Calculate 1 gluon emission with quark & gluon N-scattering
with transverse broadening and elastic loss built in
Finally solved analytically, in large Q^2 limit.