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## First observation of excited states in $^{114}\mathrm{Cs}$ : Spectroscopy of the yrast $v(h_{11/2})\otimes\pi(h_{11/2})$ band

J. F. Smith, 1,\* A. M. Fletcher, 1 C. J. Chiara, 2,3 M. P. Carpenter, 4 H. J. Chantler, 5 C. N. Davids, 4 J. L. Durell, 1 D. B. Fossan, 2 S. J. Freeman, 1 R. V. F. Janssens, 4 T. Koike, 2 F. G. Kondev, 4 D. R. LaFosse, 2 J. C. Lisle, 1 D. Patel, 1 E. S. Paul, 5 W. Reviol, 3 D. G. Sarantites, 3 D. Seweryniak, 4 K. Starosta, 2,† R. Wadsworth, 6 and A. N. Wilson 6,‡ 1 Schuster Laboratory, University of Manchester, Manchester M13 9PL, United Kingdom

2 Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, New York 11794-3800, USA

3 Department of Chemistry, Washington University, St. Louis, Missouri 63130, USA

4 Argonne National Laboratory, Argonne, Illinois 60439, USA

5 Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 7ZE, United Kingdom

6 Department of Physics, University of York, Heslington, York YO10 5DD, United Kingdom

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Excited states have been observed for the first time in the very neutron-deficient  ${}^{114}_{55}\text{Cs}_{59}$  nucleus. The assignment to  ${}^{114}\text{Cs}$  was made by detecting  $\gamma$  rays in coincidence with evaporated charged particles and with recoiling evaporation residues. A rotational band has been observed up to spin  $21\hbar$  (tentatively  $25\hbar$ ). Excitation-energy systematics and a study of quasiparticle alignments suggest that the band is based on the  $\nu(h_{11/2}) \otimes \pi(h_{11/2})$  configuration. Unlike bands based on this configuration in heavier cesium isotopes, only one signature partner is observed, and there is no evidence for signature inversion. These observations agree with theoretical calculations which predict that Coriolis-induced signature splitting should dominate at N=59.

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Odd-odd nuclei with  $N \simeq Z$  are particularly interesting due to the fact that the unpaired neutron and proton can occupy similar orbitals with a large spatial overlap in their wave functions. Such nuclei are ideal cases in which to study the effects of residual neutron-proton (np) interactions. The very neutron-deficient Z = 55 cesium isotopes are interesting in this respect. These isotopes are well deformed with quadrupole deformations of  $\beta_2 \simeq 0.3$  [1–3]. The properties of rotational excitations in these nuclei-such as quasiparticle alignments—can be used to characterize intrinsic states and infer structural information. The cesium isotopes with  $N \lesssim 65$  ( $A \lesssim 120$ ) have both their neutron and proton Fermi levels lying within the  $h_{11/2}$  subshell. With Z = 55 and  $\beta_2 \simeq 0.3$ , the proton Fermi level lies close to the [550]1/2<sup>-</sup> orbital, and as N approaches 55, the neutron Fermi level moves towards the low- $\Omega h_{11/2}$  orbitals. As a consequence, the spatial overlap of the unpaired neutron and proton increases, resulting in an increased np interaction. This effect has been seen to perturb rotational alignments in the neutron-deficient cesium isotopes [2]. Furthermore, the phenomenon known as signature inversion [1,3-5], which is observed in the yrast  $\nu(h_{11/2}) \otimes \pi(h_{11/2})$  bands in the neutron-deficient cesium isotopes, has, in some theoretical approaches, been attributed to the effects of the np interaction [6-8]. It is thus of great interest to study  $^{114}_{55}$ Cs<sub>59</sub>, which lies close to the N=Z line and in which, therefore, effects arising from the np interaction should be apparent.

Cesium isotopes with  $A \lesssim 117$  are difficult to study experimentally. The most effective method of populating states with high angular momenta in these nuclei is to use fusion-evaporation reactions. However, when the compound nuclei themselves are very neutron deficient, the evaporation of  $\alpha$  particles and protons is favored compared to that of neutrons, and the cross sections for the production of the most neutron-deficient nuclei are small. Recently, about ten  $\gamma$ -ray transitions have been assigned to <sup>113</sup>Cs using the method of recoil-decay tagging [9]; that work aside, the most neutron-deficient cesium isotopes in which excited states have been observed are  $^{117}$ Cs (odd  $\hat{A}$ ) [2] and  $^{116}$ Cs (odd-odd) [3]. The isotope  $^{114}$ Cs is known to decay by  $\beta$ -delayed proton and  $\alpha$ -particle emission, but with small branching ratios of 0.07(2) and  $1.6(6) \times 10^{-3}$ , respectively [10,11], rendering recoil-decay tagging very difficult. In the present work, excited states have been observed for the first time in  $^{114}$ Cs, by detecting  $\gamma$ -rays in coincidence with prompt evaporated particles and with evaporation residues.

Excited states in <sup>114</sup>Cs were populated using the <sup>58</sup>Ni(<sup>58</sup>Ni, pn) reaction. The <sup>58</sup>Ni beam at 230 MeV was provided by the ATLAS accelerator system at Argonne National Laboratory. The beam was incident upon a selfsupporting 500- $\mu$ g/cm<sup>2</sup> <sup>58</sup>Ni foil. Emitted  $\gamma$  rays were detected using the Gammasphere spectrometer [12] which consisted of 101, 75%-efficient, escape-suppressed, germanium detectors arranged in 16 rings of constant polar angle,  $\theta$ . The Microball [13], consisting of 95 CsI(Tl) scintillators, was used to detect evaporated protons and  $\alpha$  particles. The recoiling evaporation residues were dispersed according to their mass (M) to charge (q) ratio by the Argonne Fragment Mass Analyzer (FMA) [14] and were detected in a parallel-grid avalanche counter (PGAC) at the focal plane. The FMA effectively enabled the mass number, A, of the evaporation residues to be determined. The residues were implanted into a double-sided silicon strip

<sup>\*</sup>Corresponding address: John. F. Smith@Manchester.ac.uk

<sup>&</sup>lt;sup>†</sup>Present address: NSCL, Michigan State University, East Lansing, Michigan 48824, USA.

<sup>&</sup>lt;sup>‡</sup>Present address: Department of Nuclear Physics, RSPhysSE, Australian National University, Canberra ACT 0200, Australia.

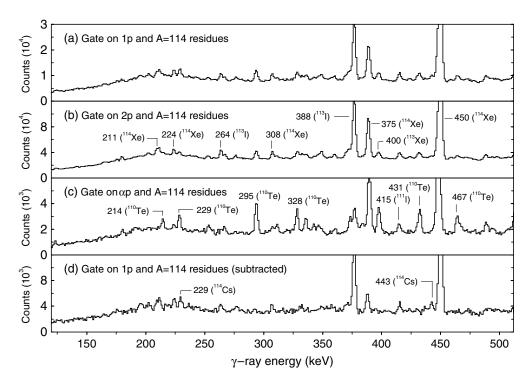


FIG. 1. Gamma-ray spectra gated on various combinations of charged particles and A=114 evaporation residues. The  $\gamma$ -ray peaks in the spectra of panels (b) and (c) will be present the spectrum on panel (a) due to either a proton or an  $\alpha$  particle, respectively, going undetected. The spectrum shown in panel (d) is that of panel (a) with a fractions of the spectra of panels (b) [0.11] and (c) [0.91] subtracted. On panels (b) and (c), the most prominent peaks are marked with transition energies in keV, and the isotopic origin of the  $\gamma$  ray in parentheses. On panel (d) the 229- and 443-keV  $\gamma$  rays, assigned to  $^{114}$ Cs, are marked.

detector (DSSD) behind the focal plane of the FMA. The horizontal position at the focal plane was measured by the PGAC; the DSSD was not used in work presented here. The experimental details have been thoroughly described in Ref. [15].

After 75 h of beamtime, with a trigger requirement of either (i) four or more suppressed germanium-detector signals, (ii) three or more suppressed germanium-detector signals plus a PGAC signal, or (iii) a DSSD signal,  $7.6 \times 10^8$  events were written to magnetic tape. Much of the offline analysis proceeded using two- and three-fold  $\gamma$ -ray coincidences. Initially, all of the data were used to increment a two- and a three-dimensional histogram, known as a *matrix* and a *cube*, respectively. One-dimensional spectra were projected from these histograms using the RADWARE data-analysis programs [16]. Analysis revealed that about 15 evaporation residues were produced; the most intense products were  $^{113}I$  (3p evaporation), <sup>112</sup>Te (4p), and <sup>110</sup>Te ( $\alpha$ 2p), which constituted 46%, 36%, and 12% of the data, respectively. The isotopes  $^{114}$ Xe (2p) and  $^{113}$ Xe (2pn) were also populated, each with a fraction of about 2% of the data; these isotopes are of particular relevance here as their presence indicates that both two-particle evaporation and neutron evaporation take place following the reaction. In the data from the Microball, discrimination between protons and  $\alpha$  particles was made using the methods described in Ref. [13]. The particle-detection efficiencies were measured to be  $\sim$ 85% for protons and  $\sim$ 65% for  $\alpha$  particles.

The FMA was found to have a transport efficiency of  $\sim$ 10%, and an M/q resolution of 1 in 350.

In order to identify  $\gamma$ -ray transitions in <sup>114</sup>Cs,  $\gamma$ -ray spectra were incremented with gates on all combinations of charged particles likely to be evaporated in the reaction, and on different M/q values (effectively, A). Spectra gated on A=114 evaporation residues and one proton (1p), two protons (2p), and one proton plus an  $\alpha$  particle  $(\alpha p)$  are shown in Figs. 1(a), 1(b), and 1(c), respectively. The spectrum gated on (A=114 and (a) shown in Fig. 1(a) is mostly composed of (a) rays from (a) rays are present due to events where one or more evaporated particles has gone undetected, or where (a) values for (a) rays in all of the spectra shown in Fig. 1 is due to overlap of the tails of the (a) peaks (a) and (a) with those for (a) and (a) and (a) respectively.

The peaks corresponding to  $\gamma$  rays from 1p channels in the 1p-gated spectrum [Fig. 1(a)] were accentuated by subtracting fractions of spectra gated on higher charged-particle multiplicities, such as 2p [Fig. 1(b)], and  $\alpha p$  [Fig. 1(c)]. In the resulting spectrum [Fig. 1(d)],  $\gamma$  rays at 229 and 443 keV, which were not present in the spectra gated on 2p and  $\alpha p$ , are more pronounced. The PGAC spectrum gated on these  $\gamma$  rays revealed peaks at the positions of (M=114)/q; the positions were verified by projecting PGAC spectra gated on known  $\gamma$  rays in  $^{114}$ Xe, and other known products, as

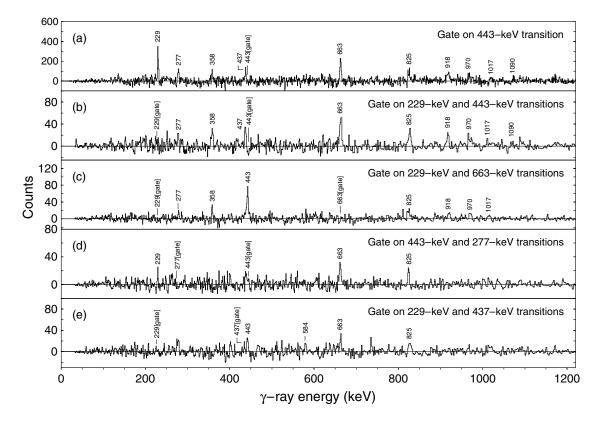


FIG. 2. Representative  $\gamma$ -ray spectra. Panel (a), projected from the 1p-gated matrix, shows the spectrum in coincidence with the 443-keV  $\gamma$  ray. The spectra shown in panels (b) to (e) are projected by double-gating in the 1p-gated coincidence cube, as indicated. The peaks which are labeled, with  $\gamma$ -ray energies in keV, correspond to  $\gamma$  rays assigned to  $^{114}$ Cs. The 663- and 825-keV  $\gamma$  rays on panel (e) arise from coincidences with the low-energy tail of the more intense 443-keV  $\gamma$  ray.

discussed in Ref. [15]. This analysis ruled out the possibility of the  $\gamma$  rays belonging to  $^{115}$ Cs, which would be populated in the p evaporation channel. The M/q ambiguity discussed above introduced the possibility of the  $\gamma$  rays belong to  $^{110}$ Cs; however, this would require p5n evaporation from the compound nucleus which is expected to have a very small cross section. The 229- and 443-keV  $\gamma$  rays were thus shown to be in coincidence with one evaporated proton and with A=114 evaporation residues, and have therefore been assigned to  $^{114}$ Cs. The intensities of these  $\gamma$  rays suggest a production cross section of about 50  $\mu$ b.

Once the assignment of  $\gamma$  rays to <sup>114</sup>Cs had been made, the gating conditions were relaxed in order to increase the number of counts in the spectra. To this end, a matrix and a cube were constructed, each of which were gated on 1p only. Gating on the 229- and 443-keV  $\gamma$  rays in the matrix established that they are in coincidence with each other, and also with several other  $\gamma$  rays. Representative spectra from the 1p-gated matrix and cube are shown in Fig. 2. Gamma-ray intensities, energies, and coincidence relationships derived from such spectra enabled the level structure shown in Fig. 3 to be deduced. A total of 12 transitions (four of which are tentative) have been assigned to <sup>114</sup>Cs. The main feature of the level scheme is a rotational band of seven transitions. It should be noted that there is a second, more intense 229-keV transition in the data, which is a known transition in <sup>110</sup>Te [17] [Fig. 1(c)].

The relative spins of the <sup>114</sup>Cs states were investigated by carrying out angular-distribution measurements. The method used was the same as that described in earlier publications, such as Refs. [2,3]. The measured angular distribution for the 229-keV transition is consistent with stretched-dipole character. The 443- and 663-keV transitions have distributions consistent with stretched-quadrupole character. Of the three transitions feeding into the main band, the 437- and 358-keV transitions were found to have distributions consistent with stretched-dipole character.

The yrast bands in the neighboring odd-odd cesium isotopes are based upon the  $\nu(h_{11/2}) \otimes \pi(h_{11/2})$  configuration. It is reasonable to assume that the band observed in <sup>114</sup>Cs is yrast, and that it is likely to be based upon the  $\nu(h_{11/2}) \otimes \pi(h_{11/2})$ configuration. The spin assignments of states within these bands were discussed in Ref. [3]. The excitation energies of states in these bands are observed to vary smoothly as a function of neutron number. The spin assignments of states in the <sup>114</sup>Cs band have therefore been tentatively made by comparing the excitation energies of states in the band, with those in neighboring cesium nuclei. Such a comparison is complicated by the observation of just one sequence of E2 transitions in <sup>114</sup>Cs, while two sequences (both signatures) are observed in the heavier isotopes. The <sup>114</sup>Cs band has therefore been compared to each of the two sequences in the neighboring nuclei in order to find the best agreement.

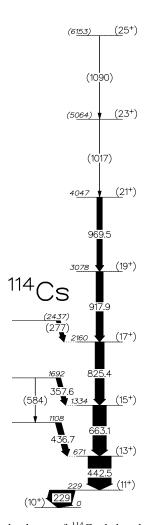


FIG. 3. The level scheme of  $^{114}$ Cs deduced in this work, with transition energies given in keV. The uncertainties on the transition energies range from 0.1 to 0.4 keV. The spin assignments are tentative and are discussed in the text. Transitions with energies in parentheses are placed in the level scheme tentatively.

The excitation-energy systematics are shown in Fig. 4. The even- and odd-spin states are shown by filled and open circles, respectively, for each isotope. For  $^{114}$ Cs, the observed data are shown twice, by filled and open circles, relative to what would be the  $10^+$  state in both (odd- and even-spin) scenarios. In the odd-spin scenario (open circles), the  $10^+$  state would be the lowest state observed, whereas in the even-spin scenario (filled circles) the  $10^+$  state would be that at 229 keV.

Clearly, the assignment of either odd or even spins would result in reasonable systematic agreement of the excitation energies. In order to assign the most likely spins to the states, it is useful to consider other arguments. The yrast states of neighboring (even-odd)  $^{117}_{55}\mathrm{Cs}_{62}$  [2] and (odd-even)  $^{113}_{54}\mathrm{Xe}_{61}$  [23] form decoupled  $\pi(h_{11/2})$  and  $\nu(h_{11/2})$  bands, respectively, each with band-head spin of  $11/2\hbar$ . If the neutron and proton in  $^{114}\mathrm{Cs}$  are also decoupled, their parallel coupling will result in a band-head spin of  $11\hbar$ . The  $^{114}\mathrm{Cs}$  band has therefore been tentatively assigned to have odd spins, as shown on Fig. 4. Given these spin assignments, the level scheme of  $^{114}\mathrm{Cs}$  extends to the  $21^+$  (tentatively  $25^+$ ) state. Three presumed-

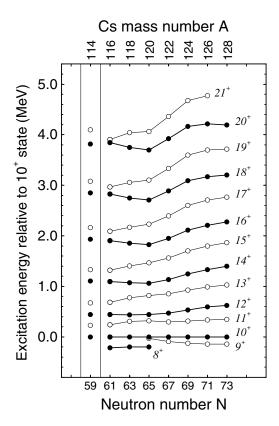


FIG. 4. Excitation-energy systematics of states in  $v(h_{11/2}) \otimes \pi(h_{11/2})$  bands. The data for  $^{116-128}$ Cs are taken from Refs. [1,3,18–22]. The spin assignments were discussed in Ref. [3]. The excitation energies are given relative to the  $10^+$  states in each of the bands. For  $^{114}$ Cs, the two sets of data, given by open and filled circles, each represent the observed data. The open circles assume that lowest state observed is the  $10^+$  state, and that the state at 229 keV is the  $11^+$  state giving the band odd spins. The filled circles assume that the state at 229 keV is the  $10^+$  state (with the lowest state observed having spin and parity  $9^+$ ) and that the band has even spins. The open circles are thus offset from the filled circles by 229 keV.

M1/E2 transitions are observed to feed the  $13^+$ ,  $15^+$ , and  $17^+$  states. It is assumed that the side-feeding states do not form the signature partner of the main band, primarily because of their weak population. Also, extended total Routhian surface (TRS) calculations [24], which reproduce signature splittings in neighboring cesium nuclei reasonably well, predict the signature splitting to be > 0.5 MeV for rotational frequencies above 0.3 MeV/ $\hbar$  in  $^{114}$ Cs [3].

The aligned angular momentum of the  $^{114}$ Cs band has been compared to theoretical predictions, and to analogous bands in neighboring nuclei. Initially, TRS [25] calculations were performed for all likely configurations involving an  $h_{11/2}$  proton. The deformations of the considered configurations were found to be very similar, with  $0.225 \leqslant \beta_2 \leqslant 0.238$ ,  $\beta_4 \approx 0.045$ , and  $5^\circ \leqslant \gamma \leqslant 10^\circ$ . For a deformation of  $\beta_2 = 0.229$ ,  $\beta_4 = 0.045$  and  $\gamma = 5^\circ$ , cranked shell model (CSM) [26] calculations predicted the alignments of pairs of  $h_{11/2}$  neutrons at rotational frequencies of 0.32 MeV/ $\hbar$  (EF in the usual nomenclature [2]), 0.47 MeV/ $\hbar$  (FG), and 0.50 MeV/ $\hbar$  (EH). The alignments of  $h_{11/2}$  protons were predicted to occur at 0.39 MeV/ $\hbar$  (ef),

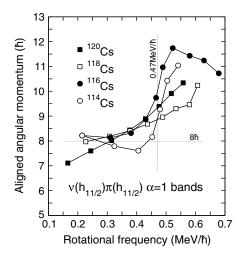


FIG. 5. Aligned angular momenta in the  $v(h_{11/2})\otimes\pi(h_{11/2})$  odd-spin bands in  $^{114,116,118,120}$ Cs. The data for  $^{116,118,120}$ Cs are taken from Refs. [1,3,19]. For all data points, a reference configuration with the Harris parameters [27] $\mathcal{J}_0=17.0~\mathrm{MeV^{-1}}\hbar^2$  and  $\mathcal{J}_1=25.8~\mathrm{MeV^{-3}}\hbar^4$  [28] has been subtracted. A value of K=3 has been assumed for all of the bands. The initial value of the aligned angular momentum ( $\sim 8\hbar$ ) and the alignment frequency (0.47 MeV/ $\hbar$ ) for  $^{114}$ Cs are marked.

0.63 MeV/ $\hbar$  (fg), and 0.61 MeV/ $\hbar$  (eh). The alignments of pairs of positive-parity neutrons and protons, (from the  $g_{7/2}$  and  $d_{5/2}$  subshells) were not predicted to occur below 0.6 MeV/ $\hbar$ . The aligned angular momentum of the <sup>114</sup>Cs band is shown in Fig. 5 in comparison with the odd-spin sequences of the  $\nu(h_{11/2}) \otimes \pi(h_{11/2})$  bands in <sup>116,118,120</sup>Cs [1,3,19]. The first  $h_{11/2}$  neutron (EF) and  $h_{11/2}$  proton (ef) alignments are blocked in the <sup>116,118,120</sup>Cs bands. Similarly, neither of these alignments are observed in the band in <sup>114</sup>Cs. The upbend at 0.47 MeV/ $\hbar$  in <sup>114</sup>Cs is tentatively assigned to be the  $h_{11/2}$ 

neutron (FG) alignment. These observations are all consistent with the proposed  $v(h_{11/2}) \otimes \pi(h_{11/2})$  configuration.

Bands based on the  $\nu(h_{11/2})\otimes\pi(h_{11/2})$  configuration in the neighboring odd-odd  $^{116-126}\mathrm{Cs}$  isotopes [1,3,18–22] exhibit signature inversion [1,3-5] whereby, at low rotational frequencies, states which are expected to be favored (in energy) are displaced upwards, above the unfavored states. The favored signature [3,29] of the  $v(h_{11/2}) \otimes \pi(h_{11/2})$  configuration is expected to be  $\alpha_f = 1$ , corresponding to odd spins. If the <sup>114</sup>Cs spin assignments are correct, the band has favored signature and signature inversion is not observed. For <sup>114</sup>Cs, it is expected that both the proton and neutron Fermi levels will be close to the  $\Omega = 1/2 h_{11/2}$  orbital, causing the Coriolisinduced signature splitting to be too large for signature inversion to occur. This simple expectation is backed up by theoretical predictions: extended TRS calculations [3,24] and triaxial particle-rotor model calculations [30] predict explicitly that signature inversion will not be observed below N = 61 in the cesium isotope chain.

In summary, excited states have been observed for the first time in the very neutron-deficient  $^{114}\mathrm{Cs}$  isotope. A rotational band based on the  $v(h_{11/2})\otimes\pi(h_{11/2})$  configuration has been observed up to the  $21^+$  (tentatively  $25^+$ ) state. Tentative spin assignments have been made by studying excitation-energy systematics. The alignment of the second pair of  $h_{11/2}$  neutrons is observed, in agreement with predictions of the cranked shell model. In the present data, there is no evidence for signature inversion, which is in agreement with extended-TRS and triaxial particle-rotor model calculations. From the present work, there is no evidence to suggest that the nuclear structure of  $^{114}\mathrm{Cs}$  is influenced by np interactions.

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