

**Quasi- $\gamma$  band and odd-even staggering effect in  $^{102}\text{Ru}$** S. Lalkovski,<sup>1</sup> G. Rainovski,<sup>2,3,\*</sup> K. Starosta,<sup>4</sup> M. P. Carpenter,<sup>5</sup> D. B. Fossan,<sup>2</sup> S. Finnigan,<sup>3</sup> S. Ilieva,<sup>1</sup> P. Joshi,<sup>6</sup>  
T. Koike,<sup>2</sup> E. S. Paul,<sup>3</sup> N. Pietralla,<sup>2</sup> C. Vaman,<sup>2</sup> and R. Wadsworth<sup>6</sup><sup>1</sup>*Faculty of Physics, St. Kliment Ohridski University of Sofia, 1164 Sofia, Bulgaria*<sup>2</sup>*Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, New York 11794-3800, USA*<sup>3</sup>*Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 7ZE, United Kingdom*<sup>4</sup>*National Superconducting Cyclotron Laboratory, Michigan State University, 164 S. Shaw Lane, East Lansing, Michigan 48824, USA*<sup>5</sup>*Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439, USA*<sup>6</sup>*Department of Physics, University of York, Heslington, York YO1 5DD, United Kingdom*

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Excited states of the nucleus  $^{102}\text{Ru}$  have been studied using the  $^{96}\text{Zr}(^{10}\text{B}, p3n)$  reaction at a beam energy of 42 MeV. The emitted  $\gamma$  rays were detected with the Gammasphere spectrometer. The known positive-parity structure built on the second  $2^+$  state was extended up to the  $10^+$  and  $11^+$  states. This structure has been interpreted as a quasi- $\gamma$  band. Its energy staggering as a function of spin has been analyzed in order to derive information on the type of triaxiality present in  $^{102}\text{Ru}$ , a nucleus that serves as one of the cores for chiral structures in the mass  $A \approx 100$  region.

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In the past few years, the observation of chiral twin bands [1] in the mass  $A \approx 130$  [2–7] and  $A \approx 100$  [8,9] regions has attracted significant theoretical and experimental efforts. These two nearly degenerate,  $\Delta I = 1$  bands of the same parity result from a perpendicular coupling of three vectors: the angular momentum of the valence high- $j$  particle-like protons (neutrons), the angular momentum of the valence high- $j$  hole-like neutrons (protons), and the rotational spin of the rigid triaxial core. When the rotation of the triaxial core is sufficiently high (comparable to the spins of the valence particles  $\approx 4-5\hbar$ ), this coupling causes a spontaneous breaking of the chiral symmetry in the intrinsic body-fixed frame. The observation of chiral bands provides experimental evidence for existence of stable triaxial shapes in nuclei.

One of the questions related to the conditions for breaking the chiral symmetry is the origin of rigidity of the triaxially deformed odd-odd nucleus. The simplest suggestion is that the rigid triaxiality is a property of the core itself. However, the even-even nuclei in the mass  $A \approx 130$  and 100 region, which can be considered as cores for the odd-odd chiral rotors, show spectra typical of transitional  $\gamma$ -soft nuclei in these mass regions of interest [10]. Thus, other scenarios leading to stable triaxial shapes for odd-odd nuclei must be considered. The interaction between the valence particles in odd-even and odd-odd nuclei and the soft even-even core can stabilize the core toward a certain deformed shape. For instance, the study of even-odd nuclei shows that the presence of a high- $j$ , odd-valence nucleon could drive the  $\gamma$ -soft core of the odd-mass nuclei toward an axially symmetric shape [11]. Triaxiality can come about by combining deformation-driving effects of valence particles and valence holes in odd-odd nuclei. It is expected that the presence of

two opposite shape-driving unpaired nucleons from high- $j$  orbitals could stabilize the  $\gamma$ -soft core into a rigid triaxial shape.

The observation of chiral twin bands in  $^{104,106}\text{Rh}$  [8,9] motivates studies of triaxiality in neighboring even-even nuclei,  $^{102}\text{Ru}$  and  $^{106}\text{Pd}$ . The known structures of these nuclei suggest that they are rather  $\gamma$  soft in their ground state. It is therefore of significant interest to study the softness of the  $\gamma$  deformation in these nuclei at intermediate spin values,  $I = 6 - 12\hbar$ , because this is the spin regime of core rotation where the chiral symmetry breaking is observed in the odd-odd nucleus  $^{104}\text{Rh}$  [8]. One approach to the problem is to study the structure of  $\gamma$  bands in these nuclei in the indicated spin range. The sequence of excitation energies in the  $\gamma$  band is sensitive to the type of triaxiality present: the rigid triaxial rotor generates a spectrum with the clusterization of the spin states  $(2_2^+, 3_1^+)$ ,  $(4_2^+, 5_1^+)$ ,  $\dots$ , etc. [12], while the  $\gamma$ -soft triaxiality gives rise to a sequence of states  $2_2^+$ ,  $(3_1^+, 4_2^+)$ ,  $(5_1^+, 6_2^+)$ ,  $\dots$ , etc. [11], where the states in parentheses are degenerate in the case of full  $\gamma$  independence. However, the experimental information about the structure of the  $\gamma$  bands in the mass  $A \approx 100$  region in the spin range of interest is scarce. The purpose of the present study is to extend our knowledge of the  $\gamma$  band in  $^{102}\text{Ru}$  up to spins that are relevant for the formation of chiral structures in neighboring odd-odd nuclei.

Excited states of  $^{102}\text{Ru}$  were populated in the  $^{96}\text{Zr}(^{10}\text{B}, 3n1p)$  reaction at a beam energy of 42 MeV. The target consisted of two stacked, isotopically enriched  $^{96}\text{Zr}$  foils with a thickness of  $520 \mu\text{g}/\text{cm}^2$  each. The  $^{10}\text{B}$  beam was provided by the Argonne tandem linear accelerator system (ATLAS) accelerator at the Argonne National Laboratory.  $\gamma$  rays were detected with the Gammasphere spectrometer [13]. Events were recorded when at least four Ge detectors fired in prompt coincidence. A total of  $5.8 \times 10^8$  events of  $\gamma$ -ray fold 4 or higher were collected. The events were sorted offline into an

\*On leave from Faculty of Physics, St. Kliment Ohridski University of Sofia, 1164 Sofia, Bulgaria.

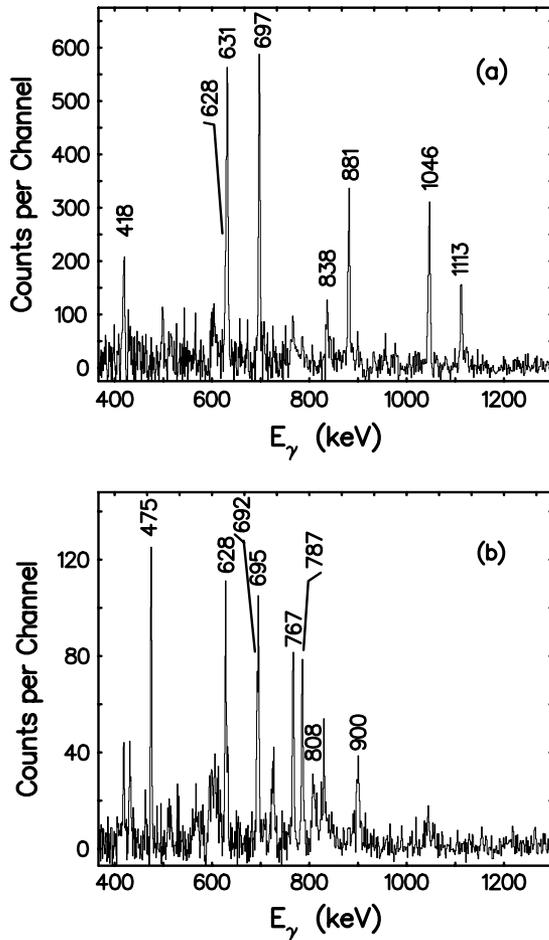


FIG. 1. Sample coincidence spectra showing the transitions from the quasi- $\gamma$  band in  $^{102}\text{Ru}$ . (a) Double gated on the 475- and 815-keV transitions. (b) Sum of all possible triple gates in the even-spin part of the quasi- $\gamma$  band. Peaks labeled with their energy in keV are assigned to  $^{102}\text{Ru}$ . Unlabeled peaks are contaminants from other reaction channels or from stronger populated structures in  $^{102}\text{Ru}$ .

$E_\gamma$ - $E_\gamma$ - $E_\gamma$ - $E_\gamma$  hypercube for subsequent analysis with the RADWARE programs [14]. The calculated relative cross section [15] of the  $3n1p$  channel leading to  $^{102}\text{Ru}$  is about 3% of the strongest reaction channels  $^{96}\text{Zr}(^{10}\text{B},xn)^{101,102,103}\text{Rh}$ , which account for about 92% of the total cross section (900 mb). In spite of the fact that  $^{102}\text{Ru}$  was populated in such a weak channel, the resolving power of Gammasphere and the use of high-fold coincidence data allowed us not only to extend the high-spin part of the level scheme of  $^{102}\text{Ru}$ , but also to reveal some nonyrast structures, which are weakly populated in fusion-evaporation reactions. In particular, we revealed the structure built on the second  $2^+$  state, the quasi- $\gamma$  band. Sample coincidence  $\gamma$ -ray spectra showing the transitions from the quasi- $\gamma$  band are shown in Fig. 1.

To deduce the multipole order of  $\gamma$  rays, we analyzed angular-correlation ratios, based on the directional correlation formalism [16]. For this purpose,  $\gamma$ - $\gamma$  coincidences between detectors placed at forward and backward angles (two rings at  $50.1^\circ$  and  $58.3^\circ$  with  $\Theta_{\text{av}} = 53^\circ$  and two rings at  $129.9^\circ$

TABLE I. Measured properties of the  $\gamma$ -ray transitions in the quasi- $\gamma$  band in  $^{102}\text{Ru}$ .

$E_\gamma$ (keV) <sup>a</sup>	$I_\gamma$ <sup>b</sup>	$R$ <sup>c</sup>	$I_i \rightarrow I_f$
417.8(4)	2.9(4)	0.60(15)	$3_1^+ \rightarrow 2_2^+$
474.7(1)	100(3)	1.03(1)	$2_1^+ \rightarrow 0_1^+$
628.4(3)	5.1(7)	1.11(16) <sup>d</sup>	$2_2^+ \rightarrow 2_1^+$
631.0(2)	89(3)	1.05(1)	$4_1^+ \rightarrow 2_1^+$
692.4(4)	1.6(2)	0.94(11) <sup>d</sup>	$4_2^+ \rightarrow 4_1^+$
694.8(4)	3.0(6)		$4_2^+ \rightarrow 2_2^+$
697.2(3)	5.4(6)	1.07(12)	$5_1^+ \rightarrow 3_1^+$
712.4(5)	<1		$6_2^+ \rightarrow 6_1^+$
727.1(2)	28(1)	1.07(3)	$10_1^+ \rightarrow 8_1^+$
766.7(2)	69(2)	1.03(1)	$6_1^+ \rightarrow 4_1^+$
786.8(4)	1.8(2)	1.15(10)	$6_2^+ \rightarrow 4_2^+$
808.4(5)	<1		$(8_2^+) \rightarrow 6_2^+$
815.1(4)	2.5(3)	1.04(10)	$7_1^+ \rightarrow 5_1^+$
830.7(2)	59(2)	1.04(2)	$8_1^+ \rightarrow 6_1^+$
837.9(5)	<1		$(11_1^+) \rightarrow 9_1^+$
881.3(4)	1.5(2)	0.99(9)	$9_1^+ \rightarrow 7_1^+$
899.8(5)	<1		$(10_2^+) \rightarrow (8_2^+)$
1046.3(3)	7.7(9)	0.71(12)	$3_1^+ \rightarrow 2_1^+$
1103.4(3)	3.3(2)		$2_2^+ \rightarrow 0_1^+$
1113.2(4)	2.2(3)	0.73(6)	$5_1^+ \rightarrow 4_1^+$
1161.5(5)	<1		$7_1^+ \rightarrow 6_1^+$

<sup>a</sup>Transition energy.

<sup>b</sup>Relative  $\gamma$ -ray intensity normalized to the intensity of the 474.7-keV transition ( $2_1^+ \rightarrow 0_1^+$ ).

<sup>c</sup>Angular-correlation ratios obtained by gating on stretched quadrupole transitions.

<sup>d</sup>Angular intensity ratios for  $\Delta I = 0$  dipole transitions are the same as for  $\Delta I = 2$  transitions.

and  $142.6^\circ$  with  $\Theta_{\text{av}} = 130^\circ$ )<sup>1</sup> and detectors placed close to  $90^\circ$  (three rings at  $80.7^\circ$ ,  $90.0^\circ$ , and  $99.3^\circ$  with  $\Theta_{\text{av}} = 90^\circ$ ) were sorted into an asymmetric  $E_\gamma$ - $E_\gamma$  matrix. Coincidence intensities of transitions were extracted from this matrix and used to calculate ratios  $R = I_\gamma(\Theta_{\text{av}}^{\text{gate}} = 90^\circ, \Theta_{\text{av}}^{\text{spectrum}} = 53^\circ, 130^\circ) / I_\gamma(\Theta_{\text{av}}^{\text{gate}} = 53^\circ, 130^\circ, \Theta_{\text{av}}^{\text{spectrum}} = 90^\circ)$ . An angular intensity ratio of 1.0 is expected if the gating and observed transitions are pure stretched transitions with the same multipole order. For the present detector geometry, a value of 0.65 is expected for a pure stretched dipole transition gated on a stretched quadrupole transition. The results from these measurements for the transitions in the quasi- $\gamma$  band in  $^{102}\text{Ru}$  are listed in Table I.

A partial level scheme of the quasi- $\gamma$  band in  $^{102}\text{Ru}$  deduced from the present experiment is presented in Fig. 2. The low-spin part of the structure built on the second  $2^+$  state of  $^{102}\text{Ru}$  had been previously identified in  $\gamma$ -ray spectroscopy studies following the  $\beta$  decay of  $^{102}\text{Rh}$  [17,18]. The structure had been further extended up to spin  $5^+$  [19,20] as spins and parities of the levels were assigned from directional correlation measurements [21,22]. In the present work, we have confirmed the ordering and spin assignments of these

<sup>1</sup>The average angle is calculated with respect to the number of detectors per ring.

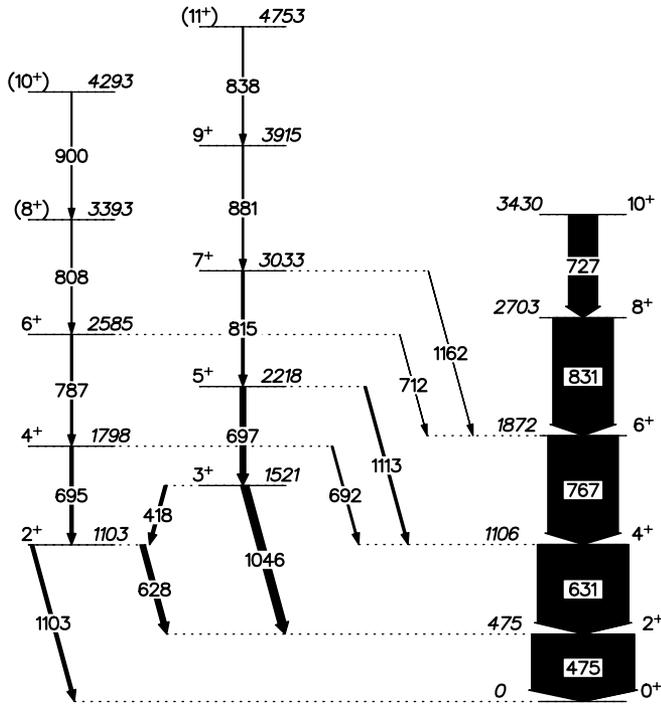


FIG. 2. Partial level scheme of  $^{102}\text{Ru}$  showing the decay of the quasi- $\gamma$  band. Energies of the  $\gamma$  transitions and levels are in keV. Arrow thickness is proportional to the  $\gamma$ -ray intensity.

states as proposed in [22]. Moreover, on top of the  $5^+$  state at 2218 keV, we have observed a cascade of three transitions of 815.1, 881.3, and 837.9 keV. These transitions are in coincidence with those of 697.2 and 1113.2 keV, which places them on top of the  $5^+$  state. Additionally, we observed a weak 1161.5-keV transition, which is in coincidence with the 881.3- and the 837.9-keV transitions but not with the 815.1-keV transition. This unambiguously shows that the 815.1- and 1161.5-keV transitions depopulate the 3033-keV state. The order of the transitions with energies 881.3 and 837.9 keV has been concluded from their intensities ratio (see Table I and Fig. 1). For the 815.1- and 881.3-keV transitions, we were able to extract angular correlation ratios which indicate their quadrupole character (see Table I), suggesting  $I^\pi = 7^+$  and  $I^\pi = 9^+$  for the states at 3033 and 3915 keV, respectively. Due to the low statistics, we cannot extract an angular correlation ratio for the 837.9-keV transition. However, this transition is in coincidence only with the transitions from the odd-spin part of the  $\gamma$  band and with transitions in the ground-state band below the  $6^+$  state, indicating that the 837.9-keV transition most likely represents the continuation of the odd-spin part of the  $\gamma$  band. Therefore, we have tentatively assigned  $I^\pi = (11^+)$  for the 4753-keV state. The even-spin part of the  $\gamma$  band is populated less intensely than the odd-spin part. We were also able to extend this part by three more transitions. Based on the angular correlation ratio for the 786.8-keV transition and the same consideration as for the 837.9-keV transition, we have assigned  $6^+$  for the 2585-keV state and tentatively propose ( $8^+$ ) and ( $10^+$ ) for the 3393- and 4293-keV states, respectively.

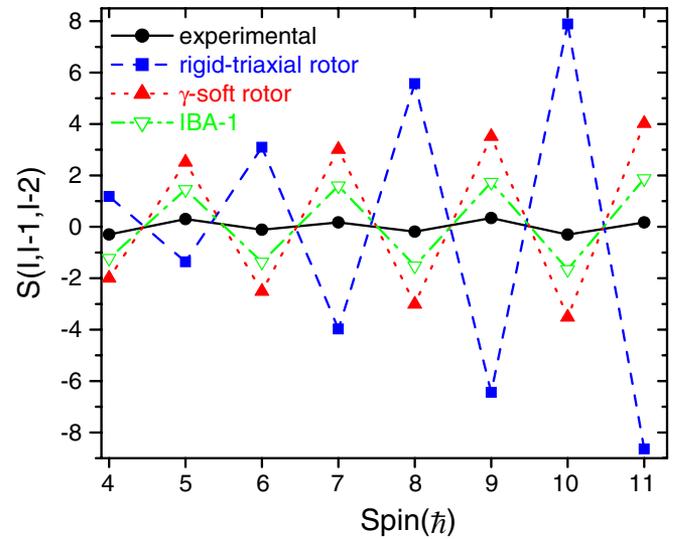


FIG. 3. (Color online) Experimentally observed staggering signatures in the quasi- $\gamma$  band in the  $^{102}\text{Ru}$  (filled circles, solid line), compared to the predictions of the rigid-triaxial model (filled squares, dashed line), the  $\gamma$ -soft model (filled triangles, dotted line), and the IBA-1 model (open triangles, dashed-dotted line). The lines between are drawn to guide the eye.

The existence of low-lying structures built on the second  $2^+$  state is a common feature for the Pd and Ru isotopes in the mass  $A \approx 100$  region [23]. These structures are built upon nonaxial vibrational modes and are usually called quasi- $\gamma$  bands. The term “quasi” is used to distinguish them from  $\gamma$  bands with  $K = 2$  occurring in axially deformed nuclei.

The energy sequence in quasi- $\gamma$  bands is well known to be sensitive to the triaxiality of a nucleus. The type of triaxiality can be qualitatively established by inspecting the relative positions of the even-spin part versus the odd-spin part of a quasi- $\gamma$  band [24]. The staggering signature is defined as [25]

$$S(I, I-1, I-2) = \frac{E(I) + E(I-2) - 2E(I-1)}{E(2_1^+)},$$

which has a constant value of  $1/3$  for a sequence with an  $I(I+1)$  spin dependence [24]. It has been shown [25] that  $S(I, I-1, I-2)$  for even  $I$  can clearly distinguish between  $\gamma$ -soft and  $\gamma$ -rigid potentials. For instance,  $S(4, 3, 2) = +1.67$  for triaxial rigid rotor ( $\gamma = 30^\circ$ ) and  $-2$  for  $\gamma$ -soft rotor [24].

The experimentally observed staggering signature for  $^{102}\text{Ru}$  is shown in Fig. 3. In the spin region below the first band crossing in the ground band, i.e.  $I \leq 10 \hbar$ , the staggering signatures are quite small with positive values for odd spins [ $0 < S(I, I-1, I-2) < 0.34$ ] and slightly negative values for even spins [ $0 > S(I, I-1, I-2) > -0.30$ ]. It is evident that the rigid triaxial scenario can be ruled out immediately, because the staggering for the rigid triaxial rotor model is of the opposite phase to that observed. To be quantitative, we calculated the expected staggering in the rigid triaxial rotor model [12]. For these calculations, we extracted the triaxiality parameter  $\gamma = 25.6^\circ$  ( $0 \leq \gamma \leq \pi/3$ ) from the relative excitation energies of the  $2_1^+$  and  $2_2^+$  states and the energy scaling parameter  $\hbar^2/4B\beta^2 = 83$  keV from the excitation energy of

the  $2_1^+$  state [12]. The resulting staggering is plotted in Fig. 3. Rigid triaxiality can be obviously discarded for  $^{102}\text{Ru}$  even in the spin range for chiral structures in neighboring odd-odd nuclei.

The other extreme of triaxial deformation is benchmarked by the limit of the  $\gamma$ -soft rotor model of Wilets and Jean [11]. This model is based on a potential that is independent of the  $\gamma$  variable but rigid in  $\beta$ . This model is considered as a structural benchmark, the geometrical analog of the SO(6) dynamical symmetry in the interacting boson model. In this limit, the model contains only one relevant parameter, namely, the energy scale, which was determined for  $^{102}\text{Ru}$  from the energy of the first  $2^+$  state. The calculations for  $^{102}\text{Ru}$  (see Fig. 3) correctly reproduce the phase of the staggering, but the amplitude is highly overestimated. This might indicate, on the one hand, that fluctuations of the triaxiality parameter  $\gamma$  are important for the structure of  $^{102}\text{Ru}$ , but, on the other hand, that not all possible degrees of triaxiality in  $^{102}\text{Ru}$  are allowed. Hence, the potential for the triaxial degree of freedom in  $^{102}\text{Ru}$  could be considered to be shallow and broad but still confined to a certain region.

The experimentally observed staggering in the quasi- $\gamma$  band in  $^{102}\text{Ru}$  is relatively small with respect to both extreme triaxial cases (see Fig. 3). Additionally, the staggering signatures  $S(4, 3, 2)$  and  $S(6, 5, 4)$  have small negative values of  $-0.297$  and  $-0.111$ , respectively. These values are significantly smaller than the values observed systematically for well-known  $\gamma$ -soft nuclei [Xe, Ba, and Pt isotopes with  $S(4, 3, 2) \leq -1$  [25]]. This suggests that the deviations from an axially symmetric shape are moderate in the  $^{102}\text{Ru}$ . Such a situation might be covered within the limits of the interacting boson approximation (IBA)-1 model with a two-body interaction [26]. We used the standard Hamiltonian in the extended consistent  $Q$  formalism with a centrifugal term [27]

$$H = \varepsilon_d n_d + \kappa Q^x \cdot Q^x + \kappa' L \cdot L.$$

In the framework of this model, we obtained a satisfactory fit of the level energies in the ground-state band up to the first band crossing with the following parameters:  $\varepsilon_d = 0.57$  MeV,  $\chi = -0.4$ ,  $\kappa = -20$  keV, and  $\kappa' = 8.5$  keV. We have to point out that the simultaneous description of the near-vibrational

ground band and the weakly staggered quasi- $\gamma$  band is a problem. The small value of  $\chi$  (for this moderate boson number  $N_B = 7$ ) is similar to though slightly larger than the values found in Ba and Ce isotopes [28], which indicates indeed that  $^{102}\text{Ru}$  is a  $\gamma$ -soft nucleus. The odd-even spin energy staggering in the quasi- $\gamma$  band, obtained with the above parametrization, is smaller than the staggering predicted by the  $\gamma$ -independent model, but it still overestimates the experimental data. This comparison suggests that the effective deviations from an axially symmetric shape in  $^{102}\text{Ru}$  are appreciable. More likely, the potential of  $^{102}\text{Ru}$  is quite soft in the triaxial degree of freedom but might have a minimum centered away from the axially symmetric value  $\gamma = 0^\circ$ . For a more accurate description of  $^{102}\text{Ru}$  with the interacting boson model (IBM), it is necessary to include three-body terms in the Hamiltonian [29]. Though such a procedure might allow quantitative determination of the  $\gamma$  dependence of the potential in  $^{102}\text{Ru}$  [25], it is beyond the scope of our present analysis.

In summary, we have extended the quasi- $\gamma$  band in  $^{102}\text{Ru}$  up to spin values which allow for a detailed study of the odd-even spin energy staggering in the spin regime relevant to chiral structures in the neighboring odd-odd nuclei. The theoretical analysis of the staggering in the framework of the  $\gamma$ -independent [11] and IBA-1 [26] models indicates that the  $^{102}\text{Ru}$  nucleus has considerable  $\gamma$  softness, but its triaxial deformation  $\gamma$  might be confined in a region away from axial symmetry. Therefore, it is likely that  $^{102}\text{Ru}$  attains a certain stiffness in the  $\gamma$  degree of freedom, which, with the addition of two valence nucleons, evolves into rigid triaxiality, forming the chiral rotor  $^{104}\text{Rh}$ . However, it has to be noted that the collectivity in  $^{102}\text{Ru}$ , having a vibrational ground band and a quasi- $\gamma$  band with near-rotational behavior, is difficult to understand using simple structural benchmarks and requires more sophisticated theoretical descriptions.

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