The quark intrinsic motion in a covariant approach

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(based on collaboration and discussions
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Outline

- A few comments on quark kinematics and effects of Lorentz invariance
- TMDs: numerical predictions based on covariant QPM
- Summary

Intrinsic motion

... is required by QM, a few examples:

electrons in atom *non-relativistic motion, OAM & spin are decoupled*

$$d \approx 10^{-10} m, \quad p \approx 10^{-3} MeV, \quad m_e \approx 0.5 MeV, \quad \beta \approx 0.002$$

nucleons in nucleus

$$d \approx 10^{-15} m, \quad p \approx 10^{2} MeV, \quad m_N \approx 940 MeV, \quad \beta \approx 0.1$$

quarks in nucleon *relativistic motion, OAM & spin cannot be decoupled*

$$d \approx 10^{-15} m, \quad p \approx 10^{2} MeV, \quad m_e \approx 5 MeV, \quad \beta \approx 1$$

Kinematic variables

... intrinsic motion generates the quark momenta ***p*, *p_L*, *p_T*, *OAM***...

Instead of ***$p_L \equiv p_1$*** the light cone variable is commonly used.

$$x = \frac{p_0 - p_1}{P_0 - P_1}$$

Advantages:

- Lorentz invariance (along collision axis)
- Simple interpretation in the infinite momentum frame
- Relation to Bjorken variable which appears in the DIS data

$$x_B = \frac{Q^2}{2Pq}$$

Kinematics of DIS

Bjorken variable

$$x_B = \frac{Q^2}{2Pq}$$

Light cone ratio

$$x = \frac{p_0 - p_1}{P_0 - P_1}$$

depends on kinematics of:

probing lepton **quark (parton)**

enters:

Structure functions **Distribution functions**

and is important for:

Experimentalists. **Theorists.**

- Despite their different origin both the variables can be identified at sufficiently large Q^2:

$$\boxed{x_B \simeq x \equiv \frac{p_0 - p_1}{P_0 - P_1}}$$

- Constraint:

$$\boxed{0 \leq x_B \leq 1} \Rightarrow \boxed{0 \leq \frac{p_0 - p_1}{P_0 - P_1} \leq 1}$$

Kinematics - further conditions

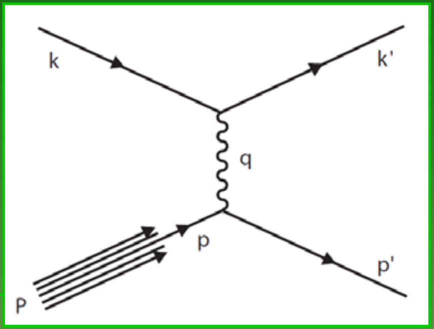


FIG. 1.

- *Lorentz invariance:* It means that the theoretical description in terms of the standard kinematical variables P, p, q, x_B, x (see Fig. 1) can be boosted also to the nucleon rest frame.

-in an opposite case the description is apparently incomplete...

- *Rotational-symmetry:* The kinematical region $\mathcal{R}^3$ of the quark intrinsic momenta $\mathbf{p} = (p_1, p_2, p_3)$ in the nucleon rest frame has rotational-symmetry (i.e. $\mathbf{p}\in\mathcal{R}^3 \Rightarrow \mathbf{p}' = \mathbf{R}\mathbf{p} \in \mathcal{R}^3$, where $\mathbf{R}$ is any rotation in $\mathcal{R}^3$). For example, in terms of the covariant QPM means that probabilistic distribution of the quark momenta is controlled by some function $G(pP/M, Q^2)$

*P.Z., Phys.Rev.D **85**, 037501(2012)*

-to simplify discussion, only leading order is considered...

Rest frame:

$$x = \frac{p_0 - p_1}{M}$$

AND

$$0 \le \frac{p_0 - p_1}{M} \le 1$$

rot. sym. $\Rightarrow$

$$0 \le \frac{p_0 + p_1}{M} \le 1$$

Combinations (+,-) of both imply:

$$0 \le |p_1| \le p_0 \le M, \qquad |p_1| \le \frac{M}{2}$$

rot. sym. $\Rightarrow$

$$0 \le p_T \le p_0 \le M, \qquad p_T \le \frac{M}{2}$$

$$p_T = \sqrt{p_2^2 + p_3^2}$$

$$0 \le |p| \le p_0 \le M, \qquad |p| \le \frac{M}{2}$$

$$|p| = \sqrt{p_1^2 + p_2^2 + p_3^2}$$

Shortly:

If we assume Lorentz invariance and rotational symmetry in the rest frame, then:

$$\boxed{0 \leq x \leq 1} \quad \Rightarrow \quad \boxed{p_T < M/2}$$

OR in other words the conditions:

A. Lorentz invariance
B. Rotational symmetry
C. $\boxed{p_T > M/2}$
D. $\boxed{0 \leq x \leq 1}$

are contradictory!

For the on-mass-shell approach the more strict relations are obtained, e.g.

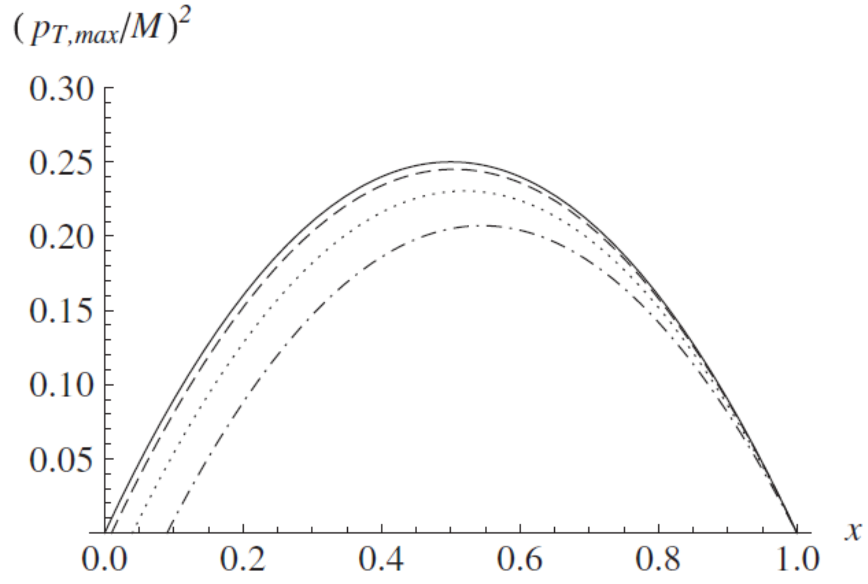


FIG. 2. Upper limit of the quark transversal momentum as a function of x for $\mu = 0$ (solid line), 0.1 (dashed line), 0.2 (dotted line) and 0.3 (dash-dotted line).

$$x \geq \frac{m^2}{M^2}$$

$$p_T^2 \leq M^2\left(x - \frac{m^2}{M^2}\right)(1-x)$$

… and particularly for massless quarks:

$$\langle p_T^2(x)\rangle \leq M^2 x(1-x)$$

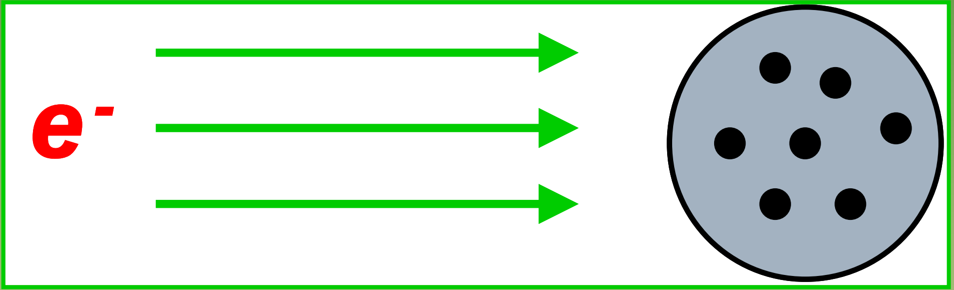
J. Sheiman Nucl. Phys., **B171**, 445 (1980)

The results of kinematical analysis can be illustrated by the covariant QPM which is based on the same inputs:

LORENTZ INV. & ROT. SYMMETRY & $x=x_B$

3D covariant parton model

- General framework



$$\Delta\sigma(x, Q^2) \sim |A|^2$$

$$|A|^2 = L_{\alpha\beta} W^{\alpha\beta}$$

The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0)d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2},$$

which are expected to depend effectively on Q^2. These distributions measure the probability to find a quark in the state

$$u(p, \lambda\mathbf{n}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

$W^{\alpha\beta} \Rightarrow$

$$F_1(x, Q^2)$$
$$F_2(x, Q^2)$$
$$g_1(x, Q^2)$$
$$g_2(x, Q^2)$$

Structure functions

Input: 3D distribution functions in the proton rest frame

The distributions allow to define the generic functions G and ΔG:

$$G(p_0) = \sum_q e_q^2 G_q(p_0), \quad G_q(p_0) \equiv G_q^+(p_0) + G_q^-(p_0),$$

$$\Delta G(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0)$$

from which the structure functions can be obtained.

F_1, F_2 – exact and manifestly covariant form:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

... similarly for g_1, g_2 :

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right]$$
$$\times\,\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right]$$
$$\times\,\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

Comment:

In the limit of usual collinear approach assuming $p = xP$, (i.e. intrinsic motion is suppressed!) one gets known relations between the structure and distribution functions:

$$F_2(x) = x\sum_q e_q^2 q(x)$$

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 (q^+(x) - q^-(x))$$

3D covariant parton model

Model implies relations and rules:

- between 3D distributions and structure functions
- **LI** & **RS** generate relations between distributions: WW relation, sum rules WW, BC, ELT; helicity↔transversity, transversity↔pretzelosity; relations between different TMDs, recently also TMDs↔PDFs

*…see A.Efremov, P.Schweitzer, O.Teryaev and P.Z., Phys.Rev.D **83**, 054025(2011) and citations therein.*

TMDs

$\phi(x, \mathbf{p}_T)_{ij}$ light-front correlators

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\,\phi(x,\mathbf{p}_T)] = f_1(x,\mathbf{p}_T) - \frac{\varepsilon^{jk}p_T^j S_T^k}{M} f_{1T}^{\perp}(x,\mathbf{p}_T)$$

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\gamma_5\,\phi(x,\mathbf{p}_T)] = S_L\, g_1(x,\mathbf{p}_T) + \frac{\mathbf{p}_T\cdot\mathbf{S}}{M} g_{1T}^{\perp}(x,\mathbf{p}_T)$$

$$\frac{1}{2}\,\mathrm{tr}[i\sigma^{j+}\gamma_5\,\phi(x,\mathbf{p}_T)] = S_T^j h_1(x,\mathbf{p}_T) + S_L\frac{p_T^j}{M} h_{1L}^{\perp}(x,\mathbf{p}_T)$$

$$+ \frac{(p_T^j p_T^k - \frac{1}{2}\mathbf{p}_T^2\delta^{jk})S_T^k}{M^2} h_{1T}^{\perp}(x,\mathbf{p}_T) + \frac{\varepsilon^{jk}p_T^k}{M} h_1^{\perp}(x,\mathbf{p}_T)$$

LI & **RS** generate relations also between some TMDs !

*A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **80**, 014021(2009)*

PDF-TMD relations

1. UNPOLARIZED

$$f_1^a(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\,\frac{d}{dy}\left[\frac{f_1^a(y)}{y}\right]_{y=\xi(x,\mathbf{p}_T^2)}$$

$$\xi(x,\mathbf{p}_T^2) = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

For details see:
P.Z. Phys.Rev.D **83**, 014022 (2011), **arXiv:0908.2316 [hep-ph]**
A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **83**, 054025(2011)
arXiv:0912.3380 [hep-ph], arXiv:1012.5296 [hep-ph]

The same relation was shortly afterwards obtained independently:
U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D **81**, 036010 (2010),
arXiv:0909.5650 [hep-ph]

In this talk we assume m→0

PDF-TMD relations

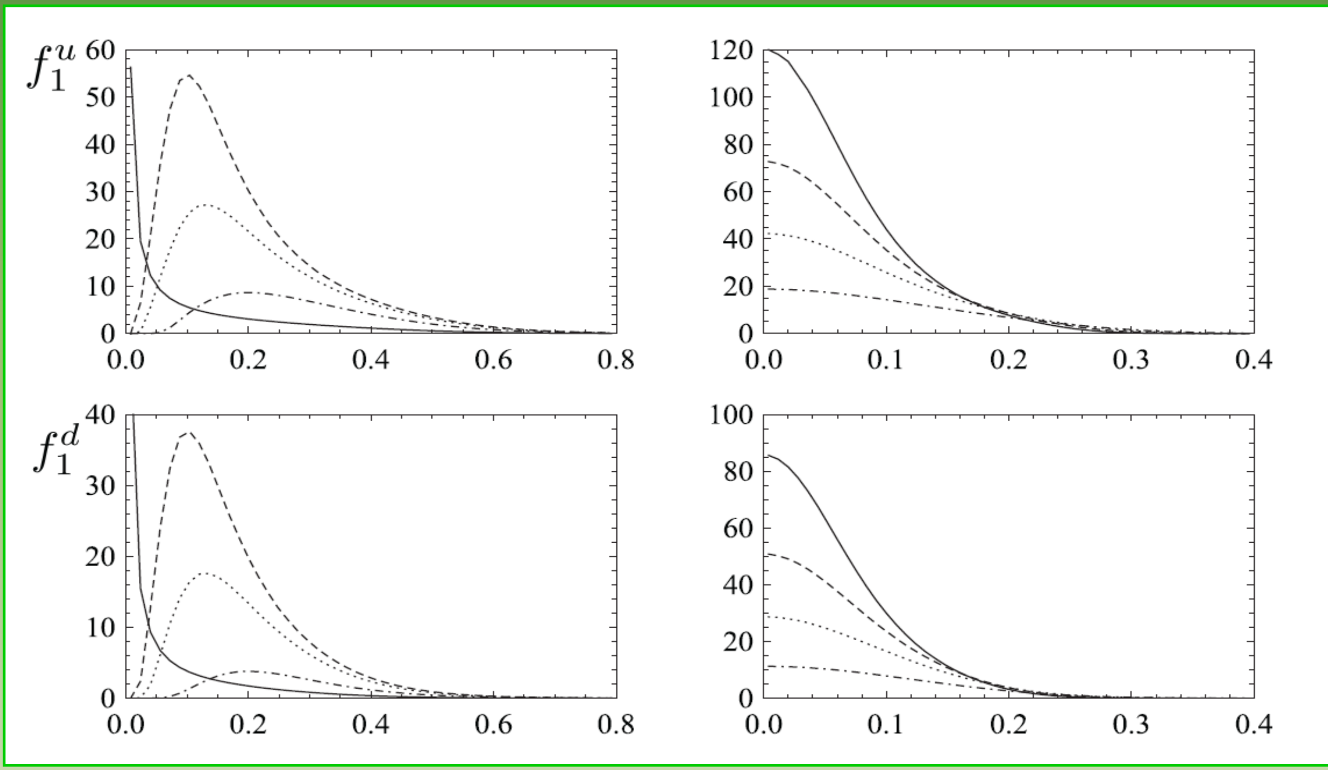
2. POLARIZED

$$
\begin{aligned}
g_1^a(x,\mathbf{p}_T) &= \frac{2x-\xi}{2} K^a(x,\mathbf{p}_T) \ , \\
h_1^a(x,\mathbf{p}_T) &= \frac{x}{2} K^a(x,\mathbf{p}_T) \ , \\
g_{1T}^{\perp a}(x,\mathbf{p}_T) &= K^a(x,\mathbf{p}_T) \ , \\
h_{1L}^{\perp a}(x,\mathbf{p}_T) &= -K^a(x,\mathbf{p}_T) \ , \\
h_{1T}^{\perp a}(x,\mathbf{p}_T) &= -\frac{1}{x} K^a(x,\mathbf{p}_T) \ .
\end{aligned}
$$

Known $f_1(x)$, $g_1(x)$ allow us to predict some unknown TMDs

$$
K^a(x,\mathbf{p}_T) = \frac{2}{\pi \xi^3 M^2} \left(2 \int_\xi^1 \frac{dy}{y}\, g_1^a(y) + 3 g_1^a(\xi) - x\, \frac{dg_1^a(\xi)}{d\xi} \right)
$$

Numerical results:



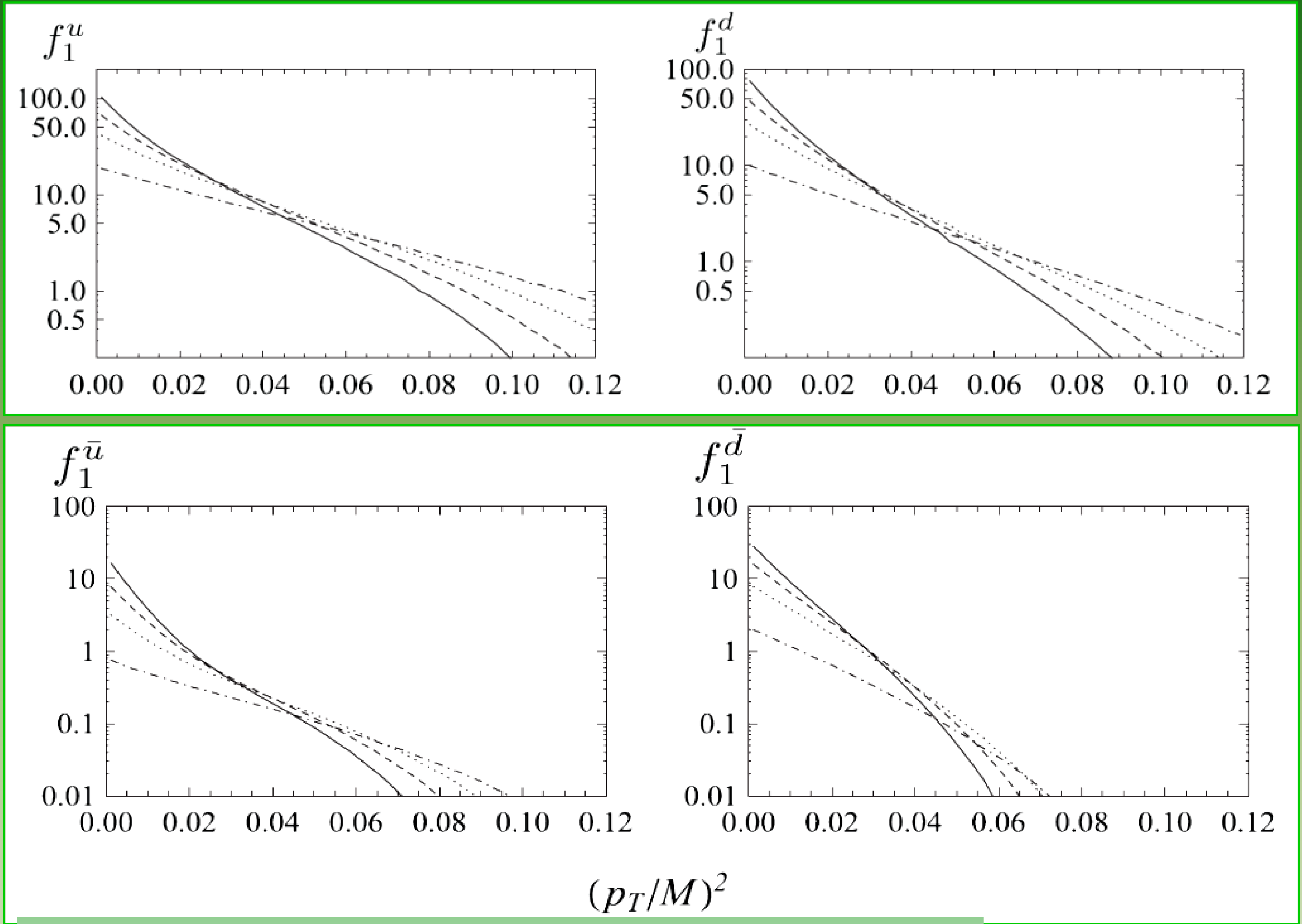
x **p_T / M**

p_T / M		***x***
q(x)	———	*0.15*
0.10	- - - - - - - -	*0.18*
0.13	···················	*0.22*
0.20	—·—·—·—	*0.30*

Input for $f_1(x)$
MRST LO at *4 GeV*^{2}

Another model approaches to TMDs give compatible results:

1. U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D 81, 036010 (2010)
2. C.Bourrely, F.Buccellla, J.Soffer, Phys.Rev. D 83, 074008 (2011)



x
0.15
0.18
0.22
0.30

- Gaussian shape – is supported by phenomenology
- $<p_T^2>$ depends on x , is smaller for sea quarks
- $<p_T>$ < 0.1GeV, p_T/M < 0.5

…corresponds to our former results on momentum distributions in the rest frame, see
PZ, Eur.Phys.J. C52, 121 (2007)

$$f_1^q(x) \rightarrow P_q(p_T)$$

Input for $f_1(x)$
MRST LO at *4 GeV*^{2}

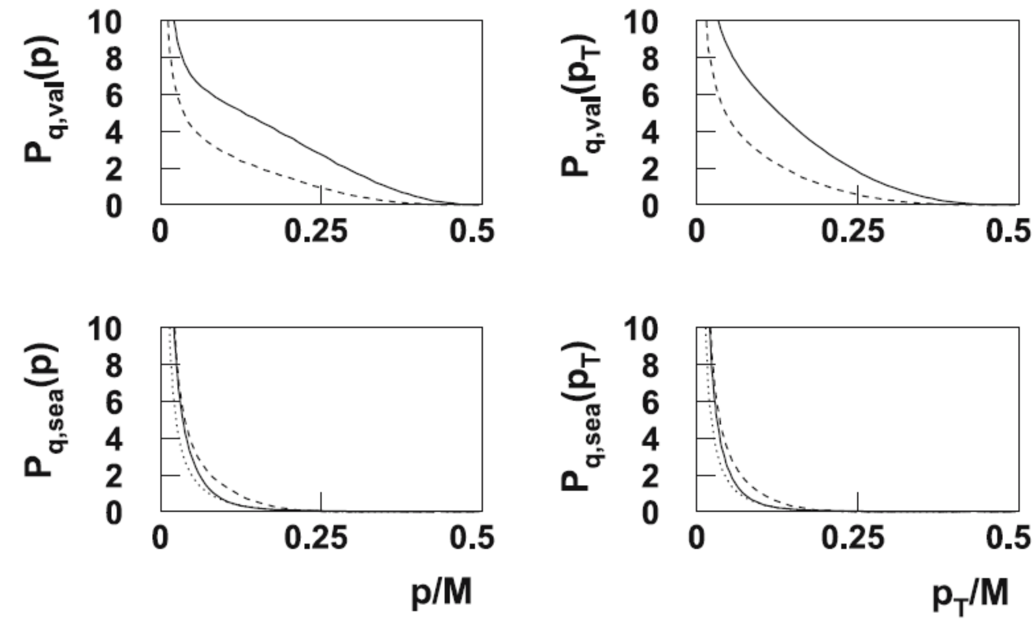
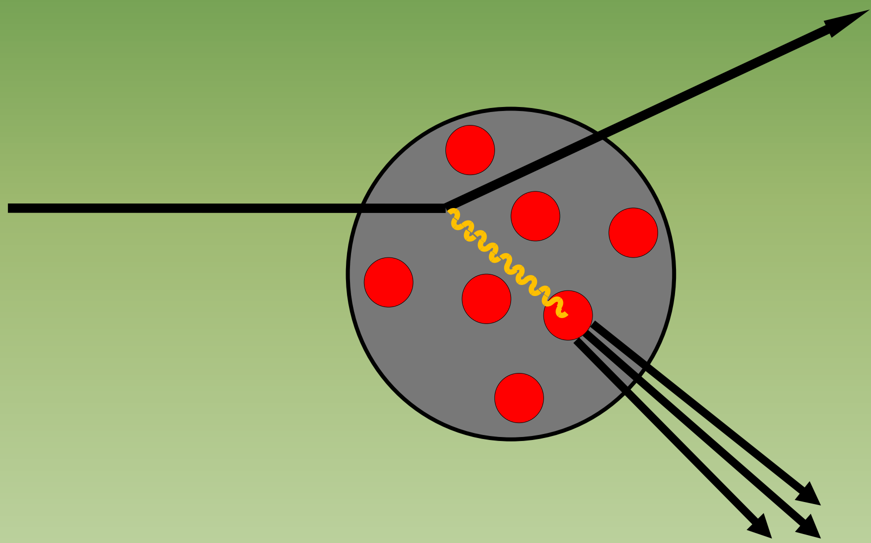


Fig. 1. The quark momentum distributions in the rest frame of the proton: the p and p_{T} distributions for valence quarks $P_{q,\mathrm{val}} = P_q - P_{\bar{q}}$ and sea quarks $P_{\bar{q}}$ at $Q^2 = 4\,\mathrm{GeV}^2$. Notation: $u, \bar{u}$ is indicated by a *solid line*, $d, \bar{d}$ by a *dashed line* and $\bar{s}$ by a *dotted line*

Calculation of $\langle p \rangle_{q,\mathrm{val}}$ gives roughly 0.11 GeV/c for u and 0.083 GeV/c for d quarks. Since $G_q(p)$ has rotational symmetry, the average transversal momentum can be calculated to be $\langle p_{\mathrm{T}} \rangle = \pi/4 \cdot \langle p \rangle$.

Two sets of DIS data and methods of obtaining $\langle p_T \rangle$:



I. Leptonic data:

Structure functions $F_2(x_B,Q^2)$

$\langle p_T \rangle \approx 0.1$ GeV/c

II. Hadronic data:

Azimuthal asymmetry

SIDIS, Cahn effect

$\langle p_T \rangle \approx 0.6$ GeV/c

I. Leptonic data

Available methods are based on approaches in which bounds of *x* imply bounds of p_T :

- **Statistical models:**

R. S. Bhalerao, N. G. Kelkar, and B. Ram, Phys. Lett. B **476**, 285 (2000).
J. Cleymans and R. L. Thews, Z. Phys. C **37**, 315 (1988).
C. Bourrely, J. Soffer, and F. Buccella, Eur. Phys. J. C **23**, 487 (2002); Mod. Phys. Lett. A **18**, 771 (2003); Eur. Phys. J. C **41**, 327 (2005); Mod. Phys. Lett. A **21**, 143 (2006); Phys. Lett. B **648**, 39 (2007).

- **Covariant models:**

J. D. Jackson, G. G. Ross, and R. G. Roberts, Phys. Lett. B **226**, 159 (1989).
P. Zavada, Phys. Rev. D **83**, 014022 (2011).

U. D'Alesio, E. Leader, and F. Murgia, Phys. Rev. D **81**, 036010 (2010).

- **And others, e.g. Barbara Pasquini…**

$$\langle p_T \rangle \approx 0.1 \text{ GeV}/c$$

II. Hadronic data

Analysis is based on the Gaussian fit:

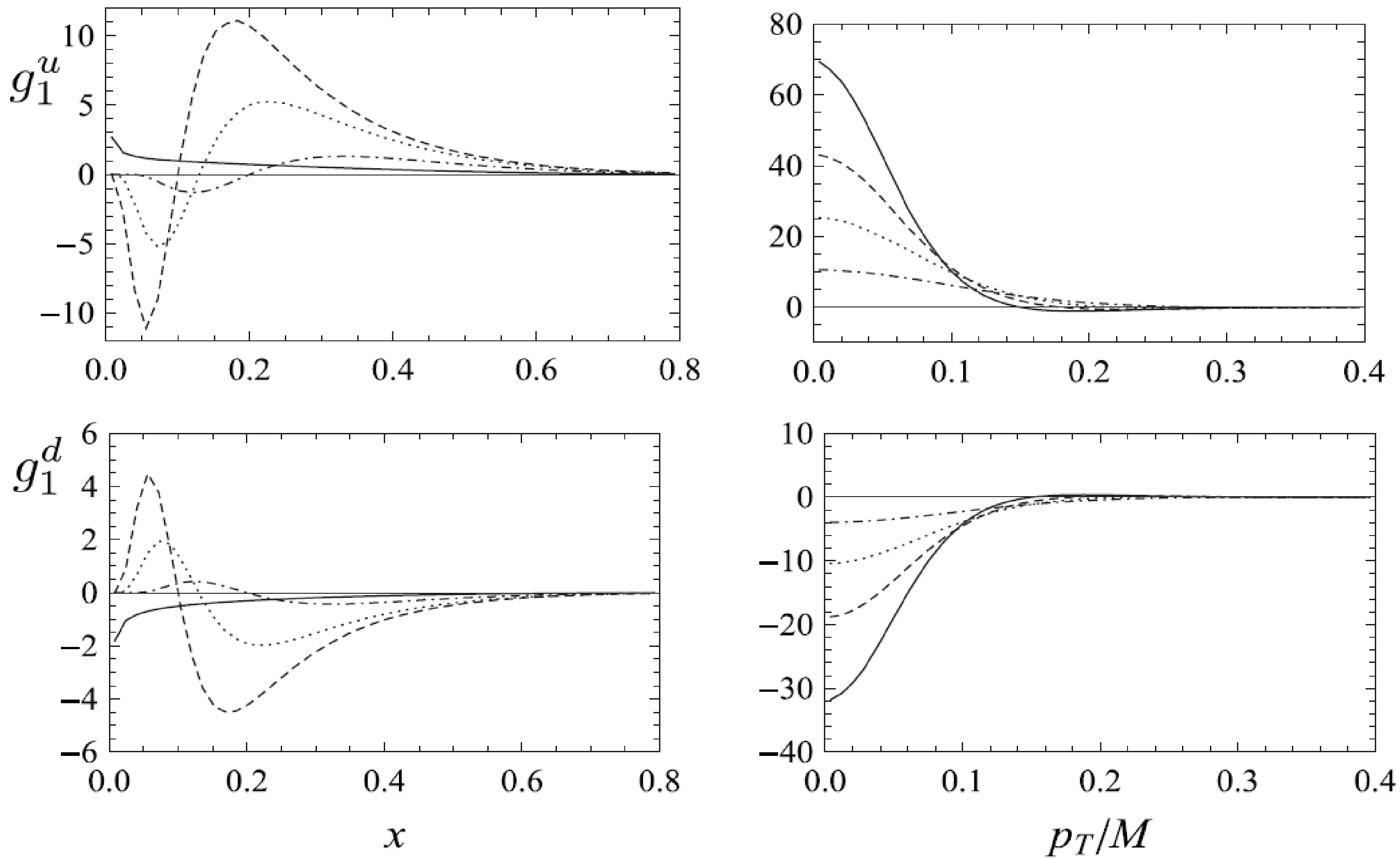
$$F_{f/P}(x,p_T) = f_{f/P}(x)\frac{\exp[-p_T^2/\langle p_T^2\rangle]}{\pi\langle p_T^2\rangle}$$

x, p_T are completely uncorrelated, no p_T bounds, strong p_L-p_T asymmetry…

P. Schweitzer, T. Teckentrup, and A. Metz, Phys. Rev. D **81**, 094019 (2010).

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, Phys. Rev. D **71**, 074006 (2005).
J. C. Collins, A. V. Efremov, K. Goeke, S. Menzel, A. Metz, and P. Schweitzer, Phys. Rev. D **73**, 014021 (2006).

$\langle p_T\rangle \approx 0.6$ GeV/c

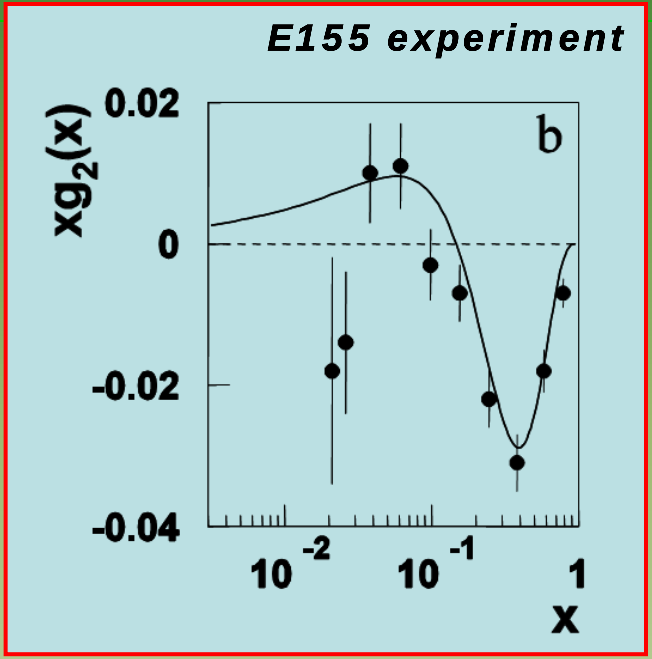


p_T/M		X
$g_{1q}(x)$	———	0.15
0.10	- - - - -	0.18
0.13	········	0.22
0.20	–·–·–·	0.30

Input for g_1 :
LSS LO at $4\ GeV^2$

Comment

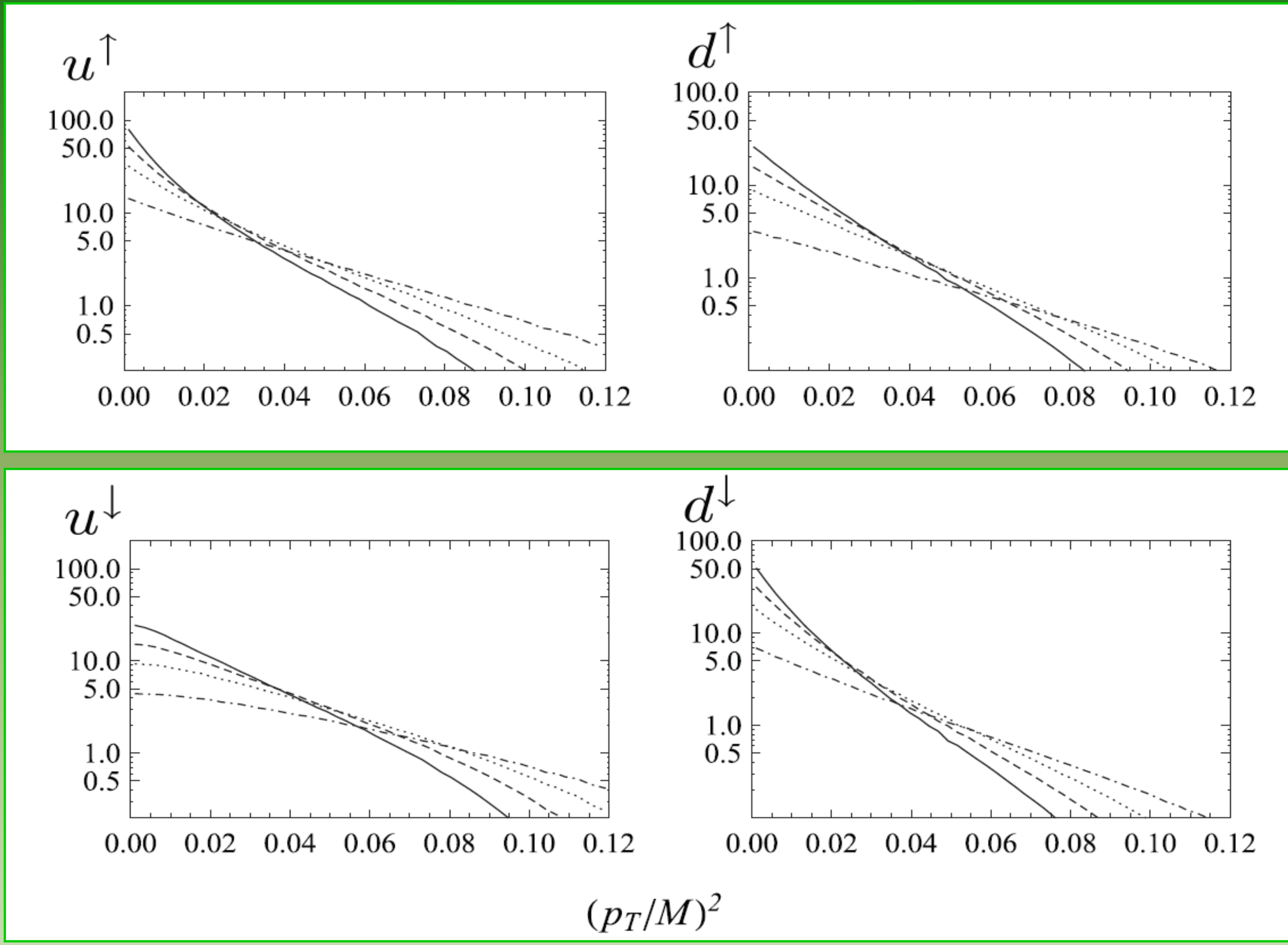
The situation is similar to $g_2(x)$:



P.Z. Phys.Rev.D **67**, 014019 (2003)

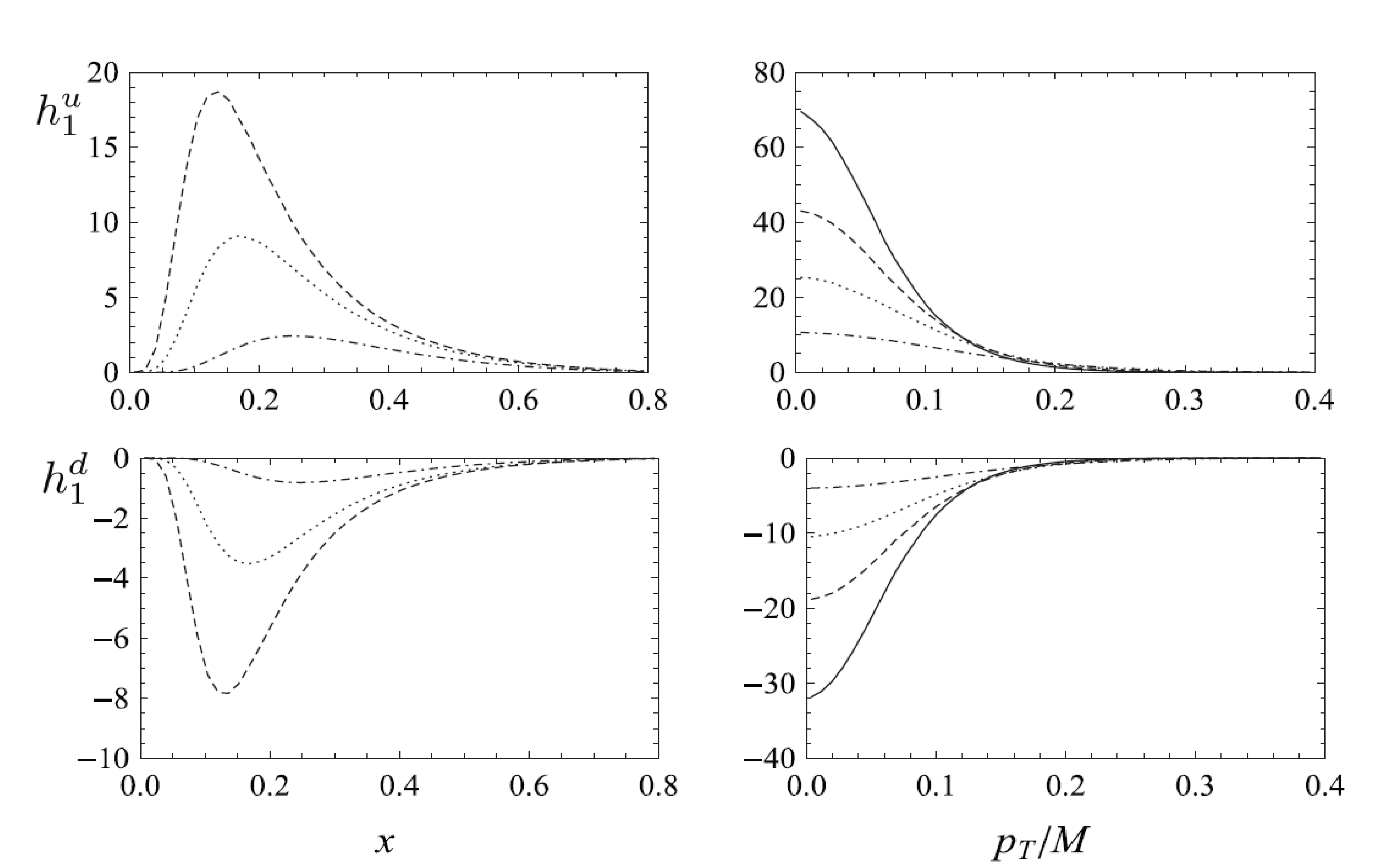
- In both cases the sign is correlated with the sign of p_L in the rest frame (in our approach)

$$q^{\uparrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q + g_1^q)$$

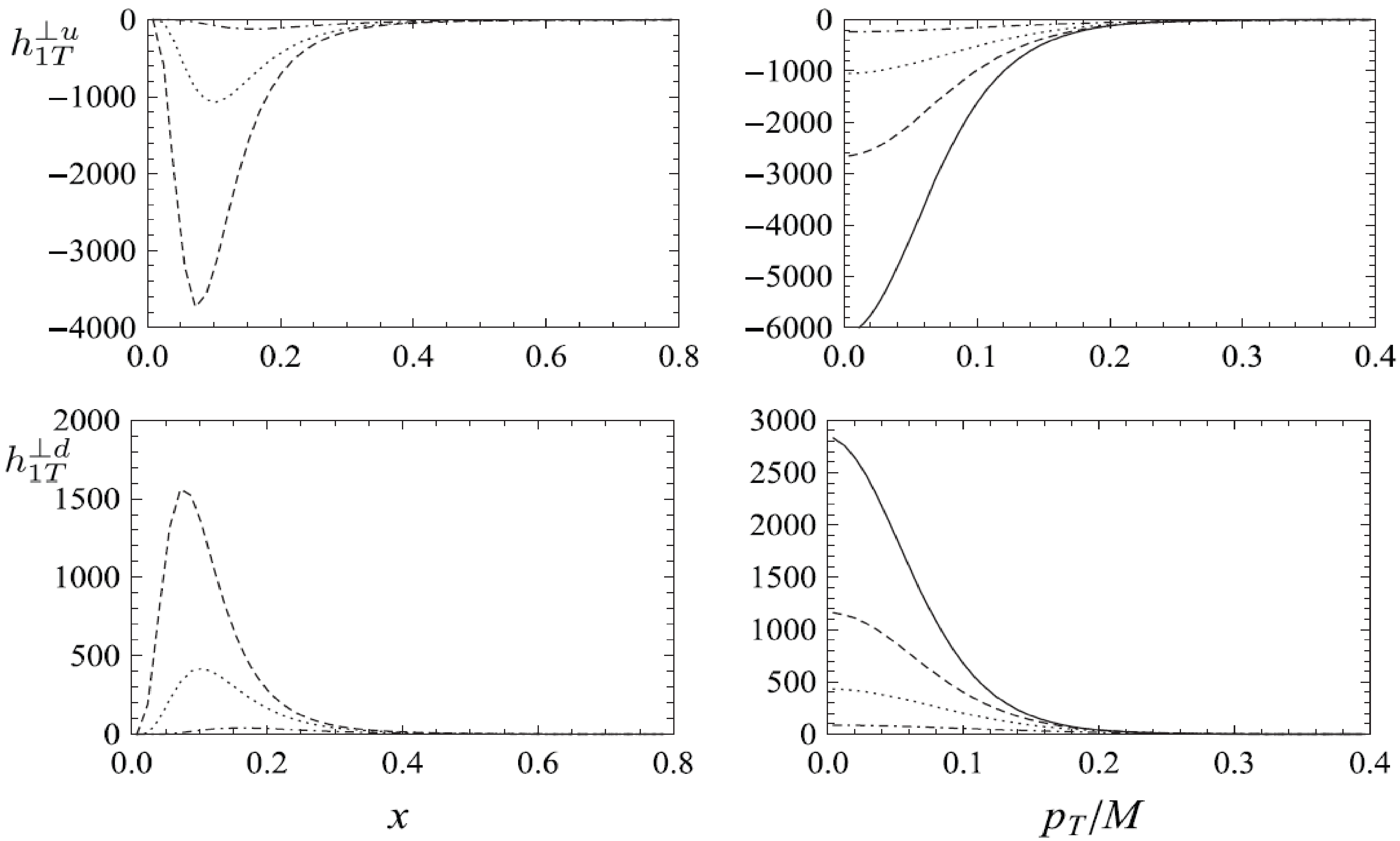


$$q^{\downarrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q - g_1^q)$$

x
0.15
0.18
0.22
0.30



p_T/M		x
		0.15
0.10		0.18
0.13		0.22
0.20		0.30



$h_{1T}^{\perp u}$
$h_{1T}^{\perp d}$
x
p_T/M

Remark on the covariant approach

Drawback of the covariant QPM: only leading order. Is the calculation of evolution feasible in a covariant approach?

1. Standard evolution:

$$\boxed{F_2^{p,n}, g_1^{p,n} \to q(x,Q_0^2), \Delta q(x,Q_0^2) \to q(x,Q^2), \Delta q(x,Q^2), g(x,Q^2)}$$

Modification: $\boxed{x = \frac{p_0 - p_L}{P_0 - P_L} \to \xi = \frac{pP}{M^2}}$ Rest frame: $\boxed{\xi = \frac{p_0}{M}}$

2. Covariant evolution:

$$\boxed{F_2^{p,n}, g_1^{p,n} \to G_q(\xi,Q_0^2), \Delta G_q(\xi,Q_0^2) \to G_q(\xi,Q^2), \Delta G_q(\xi,Q^2), G_g(\xi,Q^2)}$$

$$G_q = G_q^+ + G_q^-$$

$$\Delta G_q = G_q^+ - G_q^-$$

Last step:

$$\boxed{G_q(\xi,Q^2), \Delta G_q(\xi,Q^2) \to \mathbf{PDFs, TMDs, \ldots}}$$

Potential advantages

- Due to rot. sym. the number of variables does not change, but the new description is full 3D

- Covariant approach could provide an effective common framework for calculation with

(polarized + unpolarized) (PDFs + TMDs)

Summary

1. *We discussed kinematic constraints due to Lorentz invariance and rotational symmetry.*

2. *As an illustration we have presented some TMD predictions based on the covariant QPM.*

3. *We discussed significant differences in available estimates of the intrinsic $\langle p_T \rangle$.*

Thank you !

Backup slides

ROLE OF QUARKS IN PROTON SPIN

Angular momentum

- Total angular momentum consists of ***j=l+s***.
- In relativistic case ***l,s*** are not conserved separately, only ***j*** is conserved. So, we can have pure states of ***j*** *(j^2,j_z)* only, which are represented by the bispinor spherical waves:

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\boldsymbol{\omega}) \\ i^{-\lambda}\sqrt{p_0-m}\,\Omega_{j\lambda j_z}(\boldsymbol{\omega}) \end{pmatrix},$$

where $\boldsymbol{\omega} = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ (l defines the parity) and

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\, Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j-j_z}{2j}}\, Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j - \frac{1}{2},$$

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\, Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j+j_z+1}{2j+2}}\, Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j + \frac{1}{2}.$$

[P.Z. Eur.Phys.J. C52, 121 (2007)]

j=1/2

For $j = j_z = 1/2$ and $l = 0$:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}}\begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta\exp(i\varphi) \end{pmatrix} \end{pmatrix}.$$

For the superposition

$$\Psi(\mathbf{p}) = \int a_k \psi_{kjlj_z}(\mathbf{p})dk; \qquad \int a_k^{\star} a_k dk = 1$$

the average spin contribution to the total angular momentum is calculated as

$$\langle s \rangle = \int \Psi^{\dagger}(\mathbf{p})\Sigma_z \Psi(\mathbf{p}) d^3p; \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_z & \cdot \\ \cdot & \sigma_z \end{pmatrix}.$$

Spin & orbital motion

$$\langle s_z \rangle = \int a_p^\star a_p \frac{(p_0 + m) + (p_0 - m)(\cos^2\theta - \sin^2\theta)}{16\pi p^2 p_0} d^3 p$$
$$= \frac{1}{2} \int a_p^\star a_p \left(\frac{1}{3} + \frac{2m}{3p_0} \right) dp.$$

$$\langle l_z \rangle = \frac{1}{3} \int a_p^\star a_p \left(1 - \frac{m}{p_0} \right) dp.$$

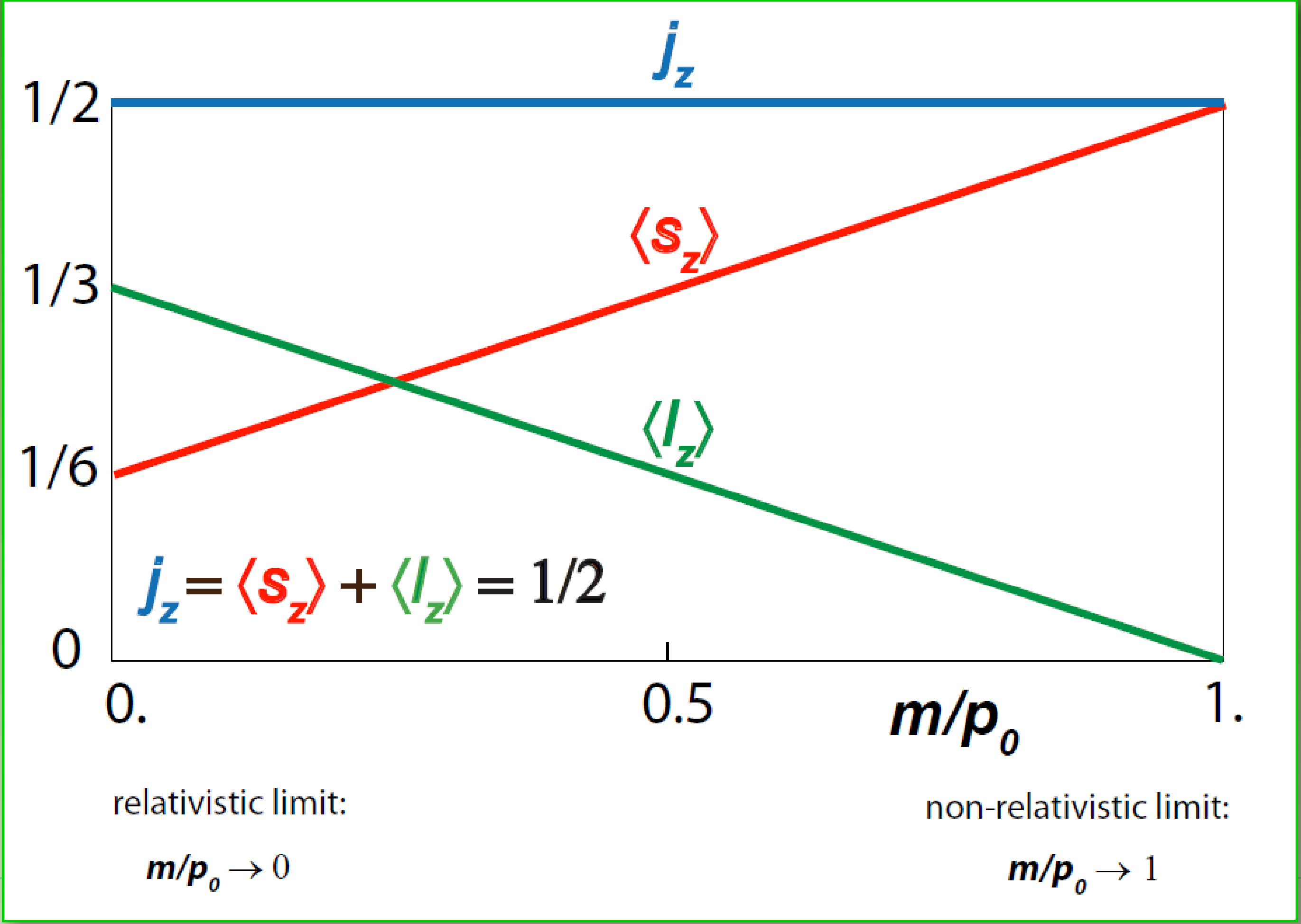
In relativistic limit:

$$m \ll p_0 \qquad \Rightarrow \qquad \langle s_z \rangle \to 1/6, \qquad \langle l_z \rangle \to 1/3.$$

... in general: $\boxed{\langle l_z \rangle = 2\langle s_z \rangle.}$

$\Longrightarrow$ **only 1 / 3 of j contributes to Σ**

Interplay of spin and orbital motion



Spin and orbital motion from PDF's

$$\langle s^q \rangle = \int g_1^q(x) dx.$$

$$\langle l^q \rangle = -\int h_{1T}^{\perp(1)q}(x) dx.$$

H. Avakian, A. V. Efremov, P. Schweitzer and F. Yuan
Phys.Rev.D81:074035(2010).

J. She, J. Zhu and B. Q. Ma
Phys.Rev.D79 054008(2009).

Our model:

$$\int g_1^q(x) dx = \frac{1}{2} \int \Delta G_q(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0} \right) d^3p.$$

$$-\int h_{1T}^{\perp(1)q}(x) dx = \frac{1}{3} \int \Delta G(p_0) \left(1 - \frac{m}{p_0} \right) d^3p.$$

Two pictures:

1. wavefunctions (bispinor spherical waves) & operators

$\langle s^q \rangle$	$\langle l^q \rangle$
$\frac{1}{2}\int a_p^* a_p \left(\frac{1}{3} + \frac{2m}{3p_0}\right) dp$	$\frac{1}{3}\int a_p^* a_p (1 - \frac{m}{p_0}) dp$

2. probabilistic distributions & structure functions (in our model)

$\int g_1^q(x) dx$	$-\int h_{1T}^{\perp(1)q}(x) dx$
$\frac{1}{2}\int \Delta G_q(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0}\right) d^3p$	$\frac{1}{3}\int \Delta G_q(p_0)(1 - \frac{m}{p_0}) d^3p$

$$a_p^* a_p dp \Leftrightarrow \Delta G_q(p_0) d^3p; \quad \Delta G_q(p_0) = G_q^+(p_0) - G_q^-(p_0)$$

$\Longrightarrow$ ***Also in our model OAM can be identified with pretzelosity!***