

# TMDs AND THE 3-D STRUCTURE OF HADRONS IN PP-COLLISIONS AT CERN

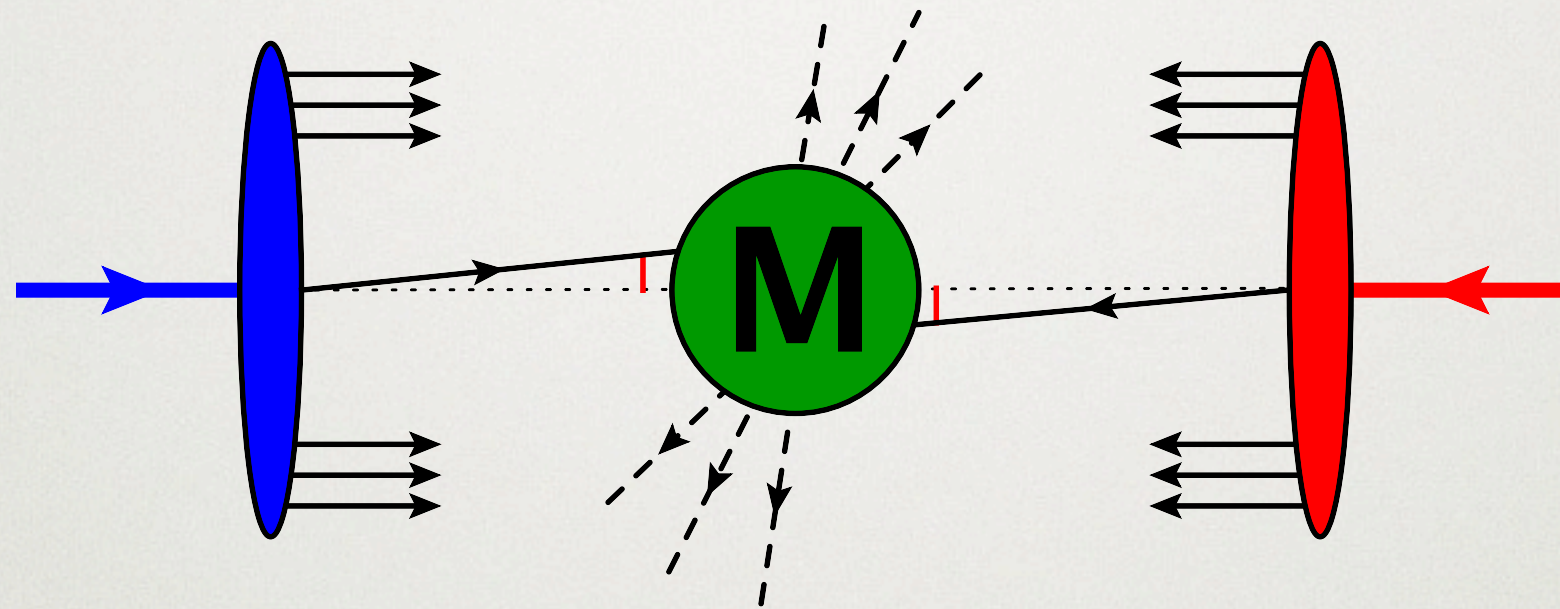
MARC SCHLEGEL  
UNIVERSITY OF TUEBINGEN

“DRELL-YAN SCATTERING AND STRUCTURE OF HADRONS”,  
ECT\*, TRENTO, MAY 24, 2012



# PROTON-PROTON COLLISIONS

physical (factorized) picture



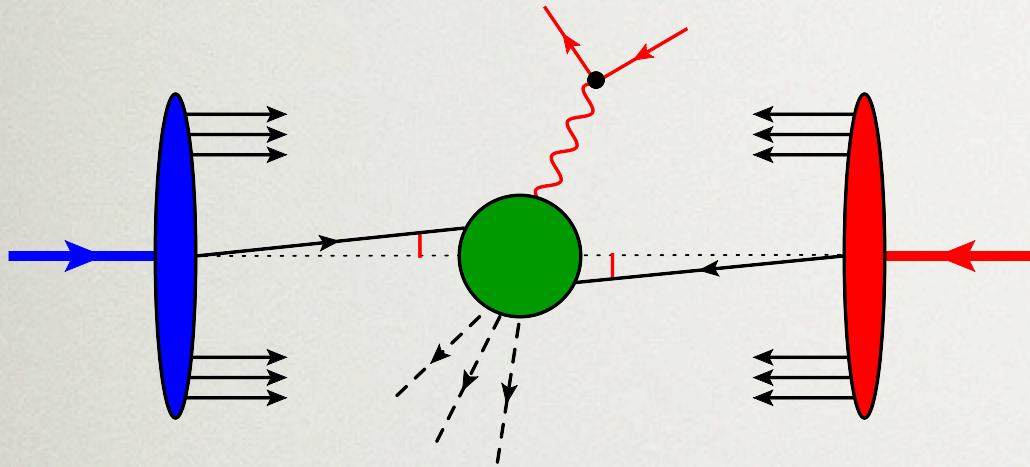
Total momentum of all *detected* final state events:

$$q = q_1 + q_2 + q_3 + \dots \rightarrow y, Q^2, q_T$$

- collinear factorization: **all final states!**  $\frac{d\sigma}{dQ^2} \frac{d\sigma}{dQ^2 d^2q_T} (\Lambda_{QCD} \ll q_T)$
- TMD factorization: **color-singlet** final states only:  $\frac{d\sigma}{dQ^2 d^2q_T} (\Lambda_{QCD} \sim q_T \ll Q)$

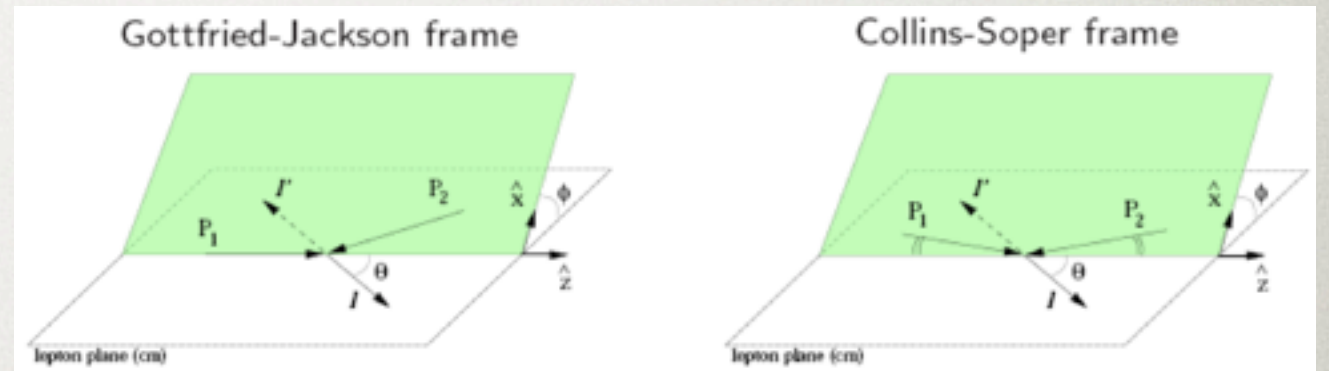


# DRELL-YAN PROCESS



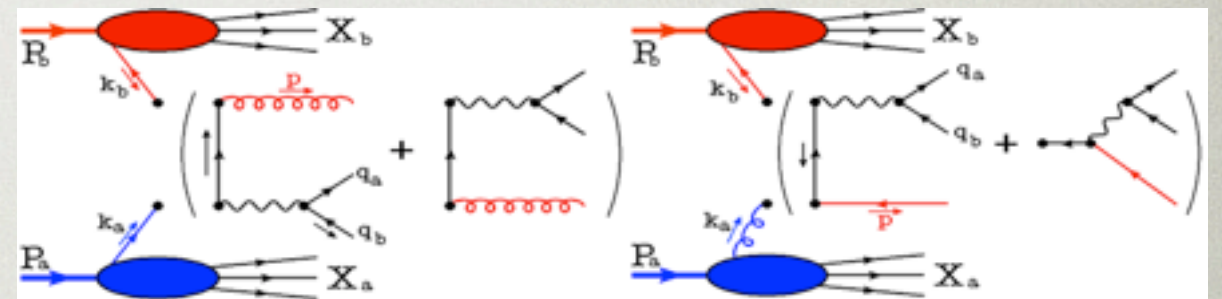
$$\frac{d^6\sigma}{d^4q d\Omega} = 2 \frac{d^6\sigma}{dy dQ^2 d^2\vec{q}_T d\Omega}$$

Kinematics easy in dilepton rest frame



## Collinear factorization

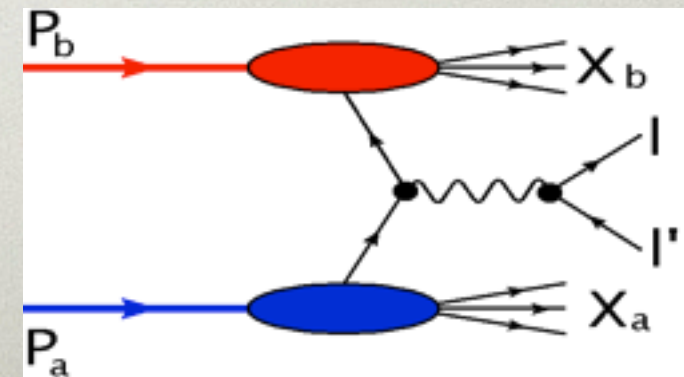
→ large  $q_T$  from undetected parton



## TMD factorization

→ small  $q_T$  from intrinsic parton momenta

→ *only* quark - antiquark interactions!





# TMD FACTORIZATION (DY)

DY: Separation into *Leptonic* + *Hadronic* Tensor

$$\frac{d\sigma}{d^4q d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

All-order TMD factorization theorem

$$W^{\mu\nu} \sim \int d^2k_{aT} d^2k_{bT} \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T) \text{Tr}[\hat{M}^\mu \Phi(x_a, \vec{k}_{aT}) (\hat{M}^\nu)^\dagger \bar{\Phi}(x_b, \vec{k}_{bT})] + Y^{\mu\nu}$$

$$q_T \ll Q$$

$$q_T \simeq Q$$

Unpolarized proton  $\rightarrow$

$$\Phi_U^q(x, \vec{k}_T) = \frac{1}{2} \gamma^- f_1^q(x, \vec{k}_T^2) + \gamma^- \gamma^i \gamma_5 \frac{\epsilon_T^{ij} k_T^j}{2M} h_1^\perp(x, \vec{k}_T^2)$$

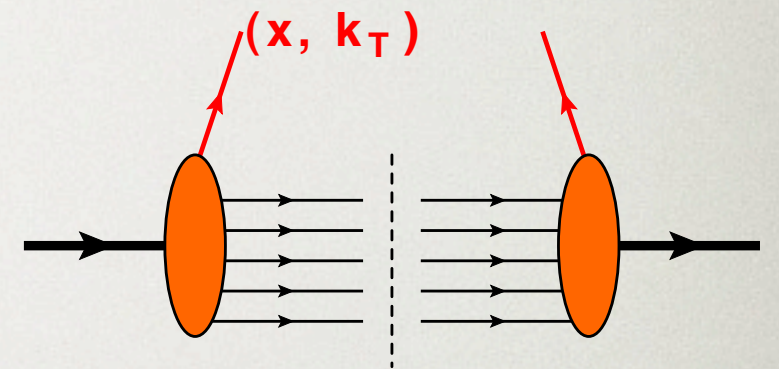
Boer-Mulders effect  $\rightarrow$

$$\frac{d\sigma_{UU}}{d^4q d\Omega} \propto \mathcal{C}[f_1^q f_1^{\bar{q}}] + \cos(2\phi) \mathcal{C}[h_1^{\perp,q} h_1^{\perp,\bar{q}}] + \mathcal{O}(\Lambda/Q)$$



# (NAIVE) TMD DEFINITION

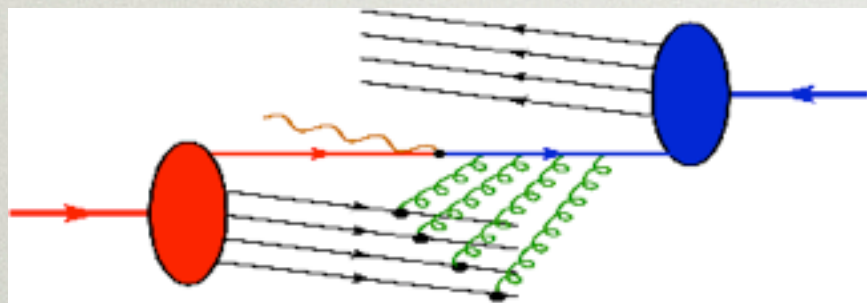
Implement “intrinsic” transverse parton momentum  $k_T$   
 → opportunity to study different aspects of hadron spin structure (e.g. 3-d momentum structure, spin-orbit correlations, etc.)



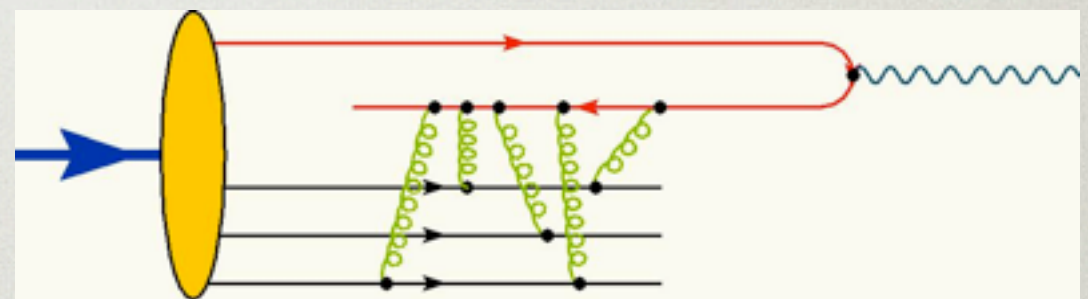
$$\Phi_{ij}(x, \vec{k}_T; S) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ik \cdot z} \langle P, S | \bar{\psi}_j(0) \mathcal{W}_{\text{SIDIS/DY}}[0, z] \psi_i(z) | P, S \rangle \Big|_{z^+=0}$$

→ Wilson line: Initial/Final State Interactions, process dependence

Initial State Interactions: Drell-Yan



Final State Interactions: SIDIS



→ sign switch of Sivers and Boer-Mulder function “T-odd”

$$f_{1T}^\perp|_{DIS} = -f_{1T}^\perp|_{DY}$$

$$h_1^\perp|_{DIS} = -h_1^\perp|_{DY}$$

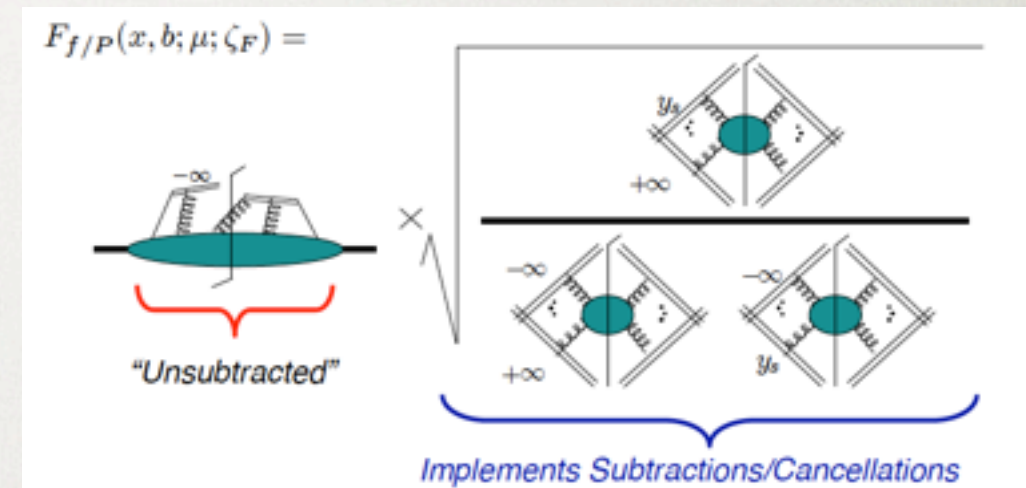


# TMDs AND EVOLUTION

[Aybat, Rogers, PRD83, 114042; Collins' "Foundations of pQCD"]

Exact TMD definition beyond tree-level:

- 1) Wilson lines are off the light cone
  - $\xi$  regulates light cone divergences
  - "unsubtracted" TMD
- 2) "Soft factors" implemented



$$f_1^q(x, \vec{b}_T^2; \mu; \xi) = Z_F Z_2 \lim_{y \rightarrow -\infty} \left( f_1^{q, \text{unsub}}(x, \vec{b}_T^2; \mu; y_P - y) \times \sqrt{\frac{S(\vec{b}_T^2; -y, y_s)}{S(\vec{b}_T^2; -y, y) S(\vec{b}_T^2; y_s, y)}} \right)$$

Evolution equations for  $\xi$  (Collins-Soper evolution)

$$\frac{\partial \ln f_1^q(x, \vec{b}_T^2; \mu; \xi)}{\partial \ln \sqrt{\xi}} = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left( \frac{S(\vec{b}_T^2; y_s, -\infty)}{S(\vec{b}_T^2; \infty, y_s)} \right)$$

anomalous dimensions

$$\frac{d \ln f_1^q(x, \vec{b}_T^2; \mu; \xi)}{d \ln \mu} = \gamma_F(g(\mu); \xi/\mu^2)$$

$$\frac{d}{d\mu} \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left( \frac{S(\vec{b}_T^2; y_s, -\infty)}{S(\vec{b}_T^2; \infty, y_s)} \right) = -\gamma_K(g(\mu))$$



# TRANSVERSE MOMENTUM DEPENDENCE

[Aybat, Rogers, Qiu, Collins; Anselmino, Boglione, ...]

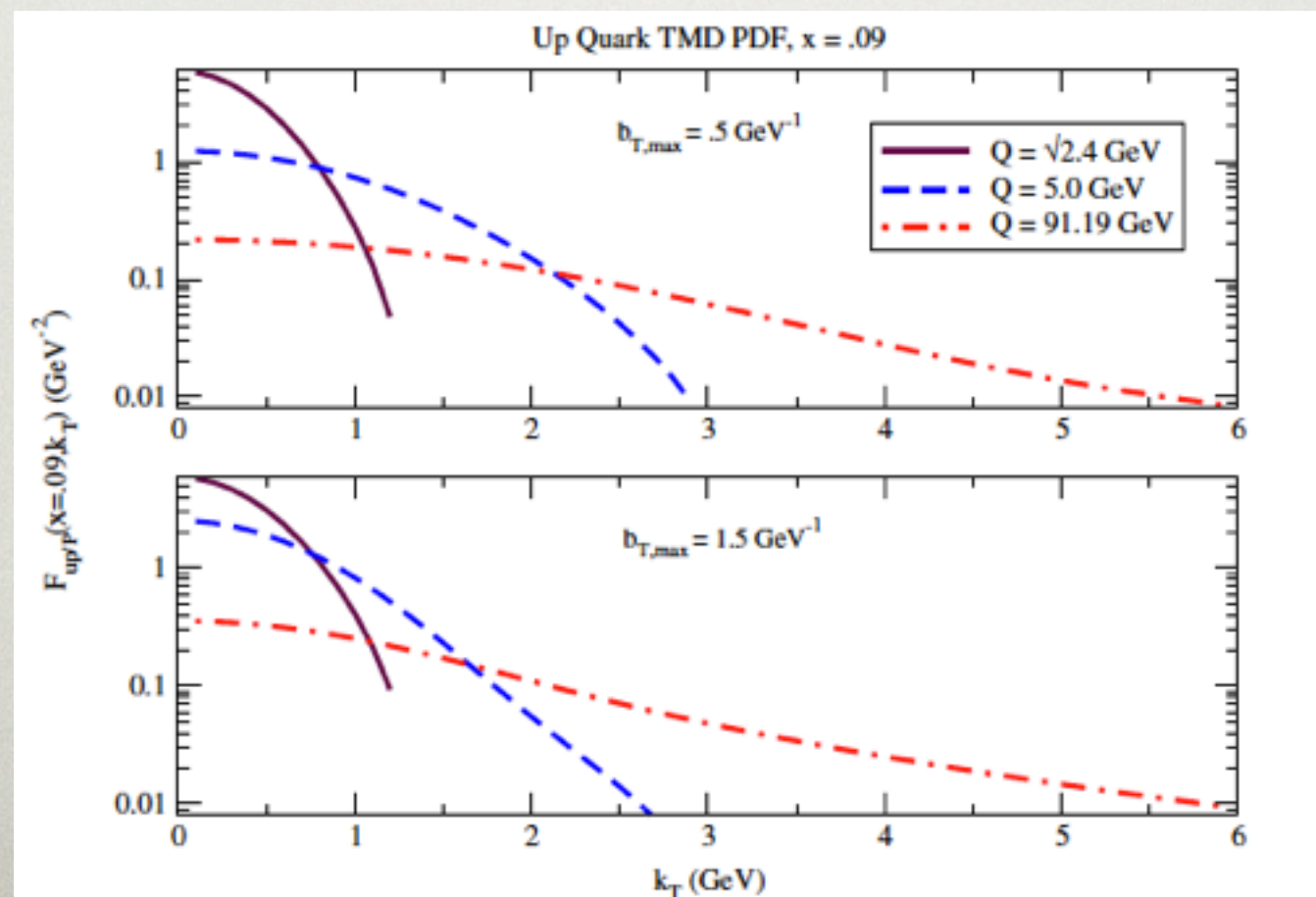
## Solution of evolution equation

$$f_1^q(x, \vec{b}_T^2; \mu; \xi) = \sum_{q'} \left( \tilde{C}_{qq'} \otimes q(x) \right) \Big|_{\mu \propto 1/b_*} e^{S_{\text{pert}}(b_*)} \Big|_{\mu \propto 1/b_*} e^{g_q(x, b_T) + \frac{1}{2} g_K(b_T) \ln \frac{\xi}{\xi_0}}$$

TMD at large  $k_T$

perturbative Sudakov factor

non-perturbative input



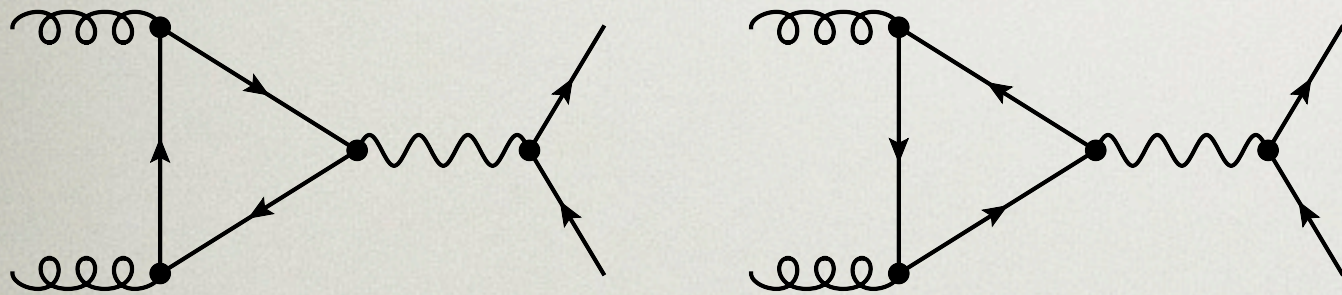


# PROTON-PROTON COLLISIONS BEYOND DY

Further leptonic final states available for TMD factorization

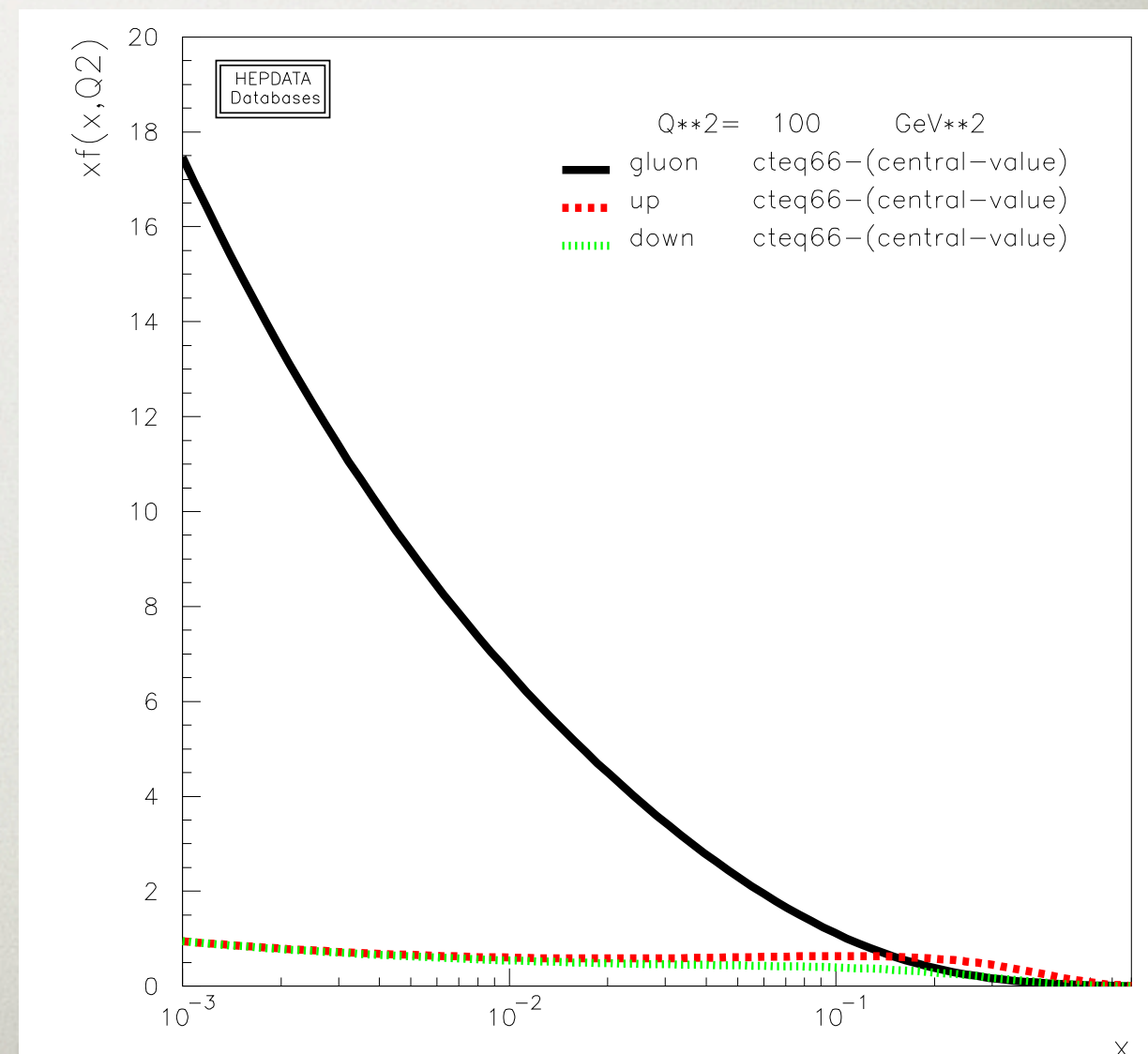
$$p + p \rightarrow \left( (l\bar{l}), (\gamma\gamma), (\gamma l\bar{l}), (l'\bar{l}')(l\bar{l}), \dots \right) + X$$

- Gluon-Gluon fusion at NNLO through loops  
→ known to contribute in coll. factorization
- No gluon fusion in Drell-Yan (Furry's Theorem)



- Chance to study gluon TMDs at low x

$$x_a x_b = \frac{Q^2}{S(1 + q_T^2/Q^2)} \sim \left( \frac{10 - 100 \text{ GeV}}{7000 - 14000 \text{ GeV}} \right)^2$$



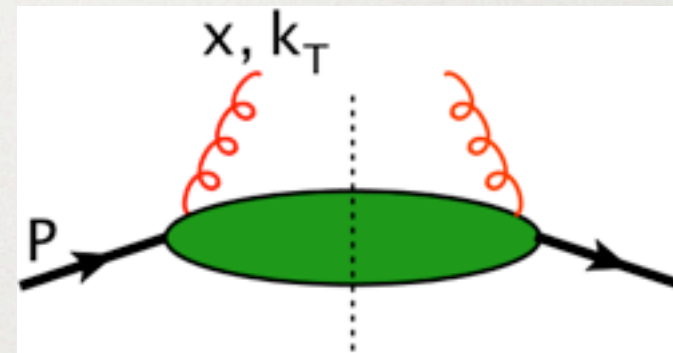


# Gluon TMDs

$$\Gamma^{ij}(x, \vec{k}_T) = \frac{1}{xP^+} \int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ik \cdot z} \langle P, S | F^{+i}(0) \mathcal{W}[0; z] F^{+j}(z) | P, S \rangle \Big|_{z^+=0}$$

$\Gamma^{[T-even]}(x, \vec{k}_T)$		$\Gamma^{[T-odd]}(x, \vec{k}_T)$	
	flip		flip
U	$f_1^g$ $h_1^{\perp g}$		
L	$g_{1L}^{\perp g}$		$h_{1L}^{\perp g}$
T	$g_{1T}^{\perp g}$	$f_{1T}^{\perp g}$	$h_1^g$ $h_{1T}^{\perp g}$

[Mulders, Rodriues, PRD 63,094021]



- \* gluonic correspondence to “Boer-Mulders”:  
T-even
- \* unpolarized gluons in transversely pol. proton: gluon Sivers function
- \* gluonic transversity / pretzelosity / wormgears: T-odd
- \* no chirality
- \* two collinear PDFs



# Processes discussed w.r.t. gluon TMDs

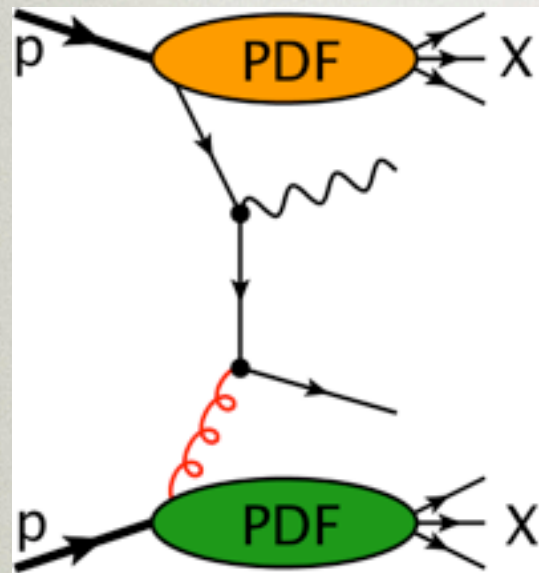
Gluon TMDs do not appear in Drell-Yan or SIDIS...

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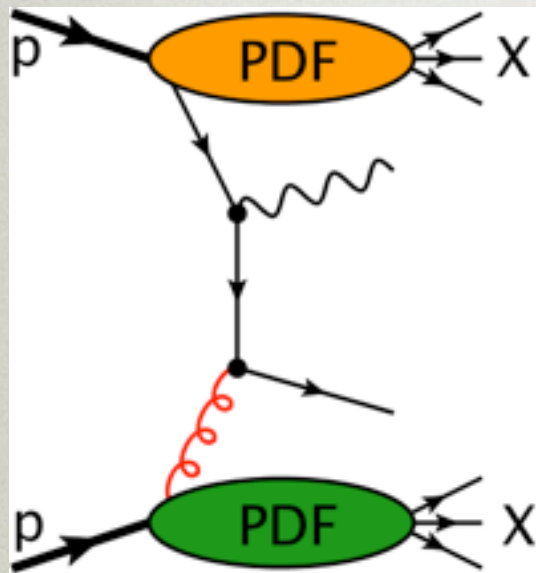
## Jet / Hadron production in pp - collisions

Spin dependent processes feasible at RHIC  
colored final states: problems with TMD factorization



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Gluon TMDs do not appear in Drell-Yan or SIDIS...

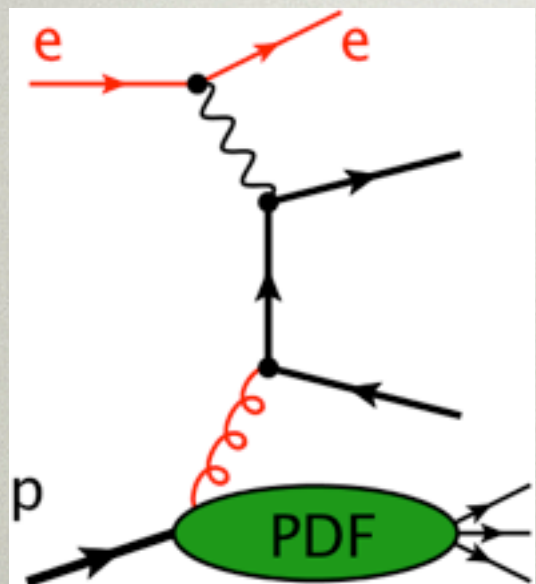


## Jet / Hadron production in pp - collisions

Spin dependent processes feasible at RHIC  
colored final states: problems with TMD factorization

## Heavy Quark production in ep - collisions

[Boer, Brodsky, Mulders, Pisano, PRL 106, 132001]



TMD factorization ok!

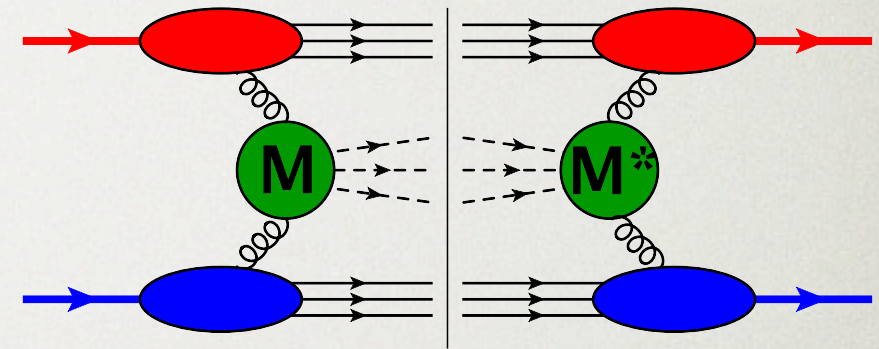
Spin dependent gluon TMDs: EIC  
(Nucleon) spin independent gluon TMDs: EIC / HERA(?)



# GLUON - GLUON INTERACTIONS

General gluonic TMD expression:

$$\frac{d\sigma}{d^4q d\Omega \dots} (q_T \ll Q) \propto \mathcal{C}[\Gamma^{ij}(x_a, \vec{k}_{aT}) \Gamma^{kl}(x_b, \vec{k}_{bT})] \sum_I (\mathcal{M}^{ik;I} (\mathcal{M}^{jl,I})^*)$$



unpolarized proton:

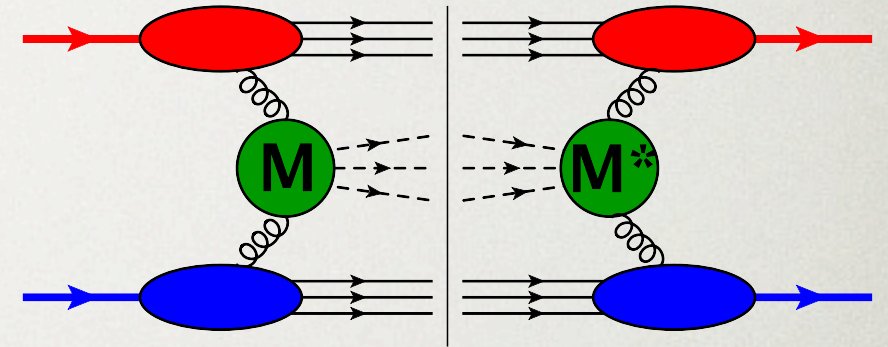
$$\Gamma_U^{ij}(x, \vec{k}_T) = \frac{\delta^{ij}}{2} f_1^g(x, \vec{k}_T^2) + \frac{k_T^i k_T^j - \frac{1}{2} \vec{k}_T^2 \delta^{ij}}{2M^2} h_1^{\perp g}(x, \vec{k}_T^2)$$



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unpolarized proton:

$$\Gamma_U^{ij}(x, \vec{k}_T) = \frac{\delta^{ij}}{2} f_1^g(x, \vec{k}_T^2) + \frac{k_T^i k_T^j - \frac{1}{2} \vec{k}_T^2 \delta^{ij}}{2M^2} h_1^{\perp g}(x, \vec{k}_T^2)$$

Calculation of partonic amplitudes

$$\varepsilon_\lambda^\mu(k_a) = (0, 1, i\lambda, 0) / \sqrt{2}$$

→ convenient to use gluon polarization vectors

→ helicity amplitudes

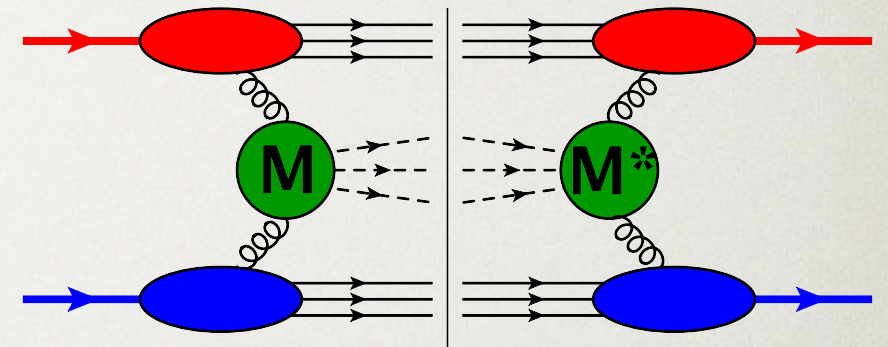
$$\delta_T^{\mu\nu} = \sum_\lambda \varepsilon_\lambda^\mu(k_{a/b}) (\varepsilon_\lambda^\nu)^*(k_{a/b})$$



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unpolarized proton:

$$\Gamma_U^{ij}(x, \vec{k}_T) = \frac{\delta^{ij}}{2} f_1^g(x, \vec{k}_T^2) + \frac{k_T^i k_T^j - \frac{1}{2} \vec{k}_T^2 \delta^{ij}}{2M^2} h_1^{\perp g}(x, \vec{k}_T^2)$$

Calculation of partonic amplitudes

$$\varepsilon_\lambda^\mu(k_a) = (0, 1, i\lambda, 0) / \sqrt{2}$$

→ convenient to use gluon polarization vectors

→ helicity amplitudes

$$\delta_T^{\mu\nu} = \sum_\lambda \varepsilon_\lambda^\mu(k_{a/b}) (\varepsilon_\lambda^\nu)^*(k_{a/b})$$

Helicity correlator:

$$\Gamma_{\lambda_1 \lambda_2}(x, \vec{k}_T) \equiv \Gamma^{ij}(x, \vec{k}_T) (\varepsilon_{\lambda_1}^i(k))^* (\varepsilon_{\lambda_2}^j(k))^* = \frac{1}{2} \left( \delta_{\lambda_1, \lambda_2} f_1^g(x, \vec{k}_T^2) + \delta_{\lambda_1, -\lambda_2} \frac{(k_x - i\lambda_1 k_y)^2}{2M^2} h_1^{\perp g}(x, \vec{k}_T^2) \right)$$

helicity non-flip

helicity flip



# GENERAL GG - STRUCTURES

---

General TMD cross section in the helicity formalism

$$\frac{d\sigma}{d^4q d\Omega \dots} (q_T \ll Q) \propto \mathcal{C}[\Gamma_{\lambda_a \lambda_{a'}}(x_a, \vec{k}_{aT}) \Gamma_{\lambda_b \lambda_{b'}}(x_b, \vec{k}_{bT})] \sum_I \left( \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_{a'} \lambda_{b'}}^I)^* \right)$$

Decomposition into four structures



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Decomposition into four structures

$$\mathcal{C}[f_1^g f_1^g] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_a \lambda_b}^I)^* \right) \rightarrow F_1(Q, \Omega, \dots) \rightarrow \text{helicity non-flip, } \phi \text{- independent, survives } q_T\text{-integration}$$



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$$\mathcal{C} \left[ \frac{(k_{ax}^2 - k_{ay}^2)(k_{bx}^2 - k_{by}^2) + 4k_{ax}k_{bx}k_{ay}k_{by}}{16M^4} h_1^{\perp g} h_1^{\perp g} \right] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_a}^I (\mathcal{M}_{-\lambda_a - \lambda_a}^I)^* \right) \rightarrow F_2(Q, \Omega, \dots) \rightarrow \text{double helicity flip, } \phi \text{- independent, vanishes upon } q_T\text{-integration}$$



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General TMD cross section in the helicity formalism

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## Decomposition into four structures

$$\mathcal{C}[f_1^g f_1^g] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_a \lambda_b}^I)^* \right) \rightarrow F_1(Q, \Omega, \dots) \rightarrow \text{helicity non-flip, } \phi \text{- independent, survives } q_T\text{-integration}$$

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$$\mathcal{C} \left[ \frac{(k_{ax}^2 - k_{ay}^2)}{4M^2} h_1^{\perp g} f_1^g \right] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{-\lambda_a \lambda_b}^I)^* \right) \rightarrow F_{3,a}(Q, \Omega, \dots) \rightarrow \text{single helicity flip, } \cos(2\phi) \text{- mode, weighted } q_T\text{-integration}$$

$$\mathcal{C} \left[ \frac{(k_{bx}^2 - k_{by}^2)}{4M^2} f_1^g h_1^{\perp g} \right] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_a - \lambda_b}^I)^* \right) \rightarrow F_{3,b}(Q, \Omega, \dots) \rightarrow \text{single helicity flip, } \cos(2\phi) \text{- mode, weighted } q_T\text{-integration}$$



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## Decomposition into four structures

$$\mathcal{C}[f_1^g f_1^g] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_a \lambda_b}^I)^* \right) \rightarrow F_1(Q, \Omega, \dots) \rightarrow \text{helicity non-flip, } \phi \text{- independent, survives } q_T\text{-integration}$$

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$$\mathcal{C} \left[ \frac{(k_{ax}^2 - k_{ay}^2)}{4M^2} h_1^{\perp g} f_1^g \right] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{-\lambda_a \lambda_b}^I)^* \right) \rightarrow F_{3,a}(Q, \Omega, \dots) \rightarrow \text{single helicity flip, } \cos(2\phi) \text{- mode, weighted } q_T\text{-integration}$$

$$\mathcal{C} \left[ \frac{(k_{bx}^2 - k_{by}^2)}{4M^2} f_1^g h_1^{\perp g} \right] \left( \sum_I \mathcal{M}_{\lambda_a \lambda_b}^I (\mathcal{M}_{\lambda_a - \lambda_b}^I)^* \right) \rightarrow F_{3,b}(Q, \Omega, \dots) \rightarrow \text{single helicity flip, } \cos(2\phi) \text{- mode, weighted } q_T\text{-integration}$$

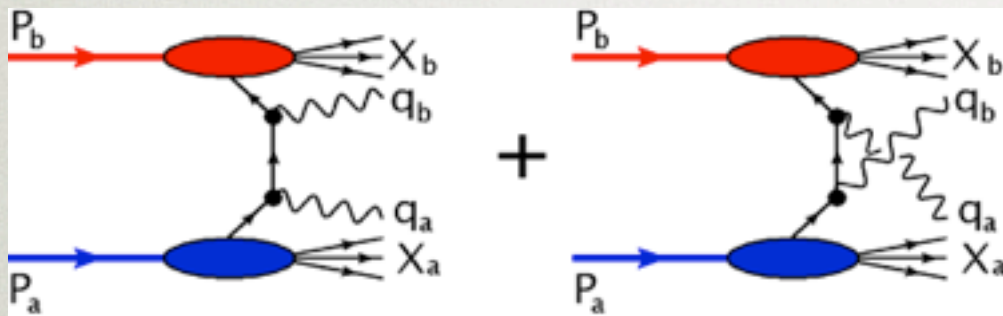
$$\mathcal{C} \left[ \frac{(k_{ax}^2 - k_{ay}^2)(k_{bx}^2 - k_{by}^2) - 4k_{ax}k_{bx}k_{ay}k_{by}}{16M^4} h_1^{\perp g} h_1^{\perp g} \right] \left( \sum_I \mathcal{M}_{\lambda_a - \lambda_a}^I (\mathcal{M}_{-\lambda_a \lambda_a}^I)^* \right) \rightarrow F_4(Q, \Omega, \dots) \rightarrow \text{double helicity flip, } \cos(4\phi) \text{- mode, weighted } q_T\text{-integration}$$



# Box diagrams

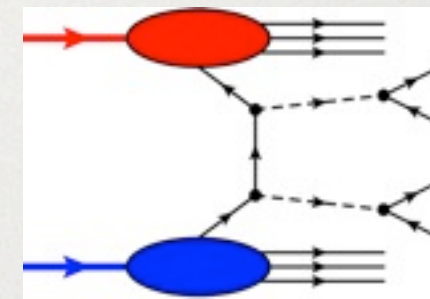
## Diphotons

quark - antiquark interactions

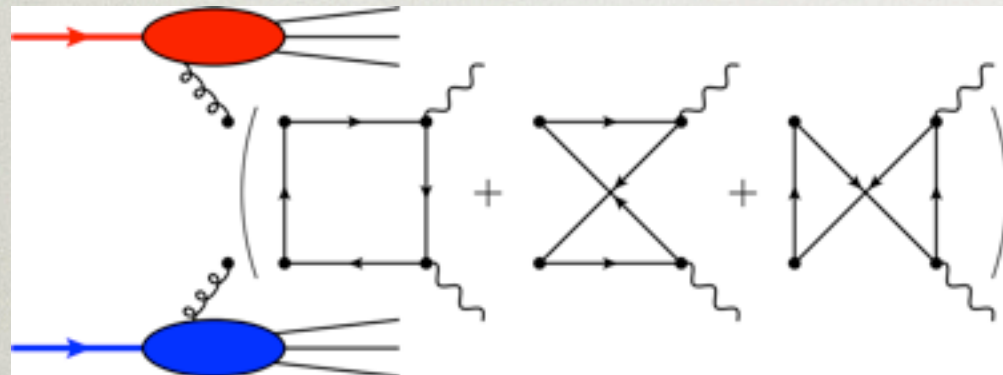


## Other leptonic final states:

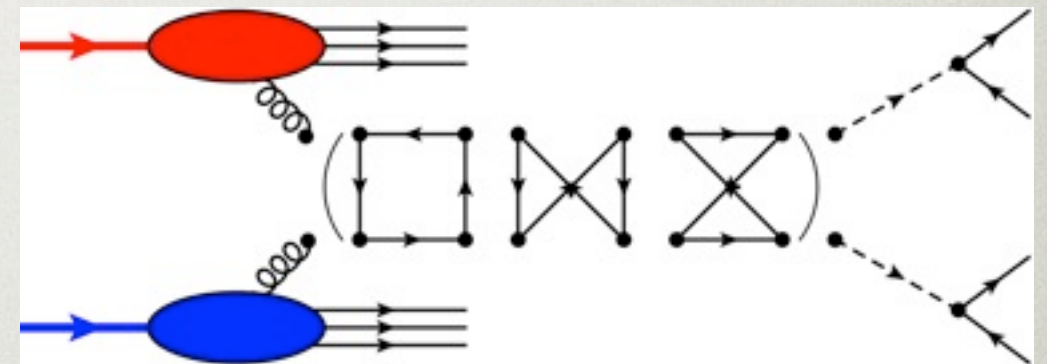
→ ZZ, WW, 4l, Zγ, 2lγ



gluon TMDs at  $O(\alpha_s^2)$



[Qiu, M.S., Vogelsang, PRL 107, 062001 (2011)]



[Boer, den Dunnen, Pisano, MS, Vogelsang, in prep.]

- \* no colored final state ⇒ TMD factorization ok
- \* only Initial State Interactions, past-pointing Wilson lines
- \* gauge invariance ⇒ box finite ⇒ effectively tree-level



# Unpolarized $pp \rightarrow \gamma\gamma X$ Cross-Section at $q_T \ll Q$

$$\frac{d\sigma_{UU}}{d^4q d\Omega} \sim \left( \frac{2}{\sin^2 \theta} \right) \left( (1 + \cos^2 \theta) [f_1^q \otimes f_1^{\bar{q}}] + \cos(2\phi) \sin(2\theta) [h_1^{\perp q} \otimes h_1^{\perp \bar{q}}] \right)$$

quark contributions  $\rightarrow$  almost identical to DY



# Unpolarized $pp \rightarrow \gamma\gamma X$ Cross-Section at $q_T \ll Q$

$$\frac{d\sigma_{UU}}{d^4q d\Omega} \sim \left( \frac{2}{\sin^2 \theta} \right) \left( (1 + \cos^2 \theta) [f_1^q \otimes f_1^{\bar{q}}] + \cos(2\phi) \sin(2\theta) [h_1^{\perp q} \otimes h_1^{\perp \bar{q}}] \right)$$

quark contributions  $\rightarrow$  almost identical to DY

$$+ \left( \frac{\alpha_s}{2\pi} \right)^2 \left( \mathcal{F}_1 [f_1^g \otimes f_1^g] + \mathcal{F}_2 [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi) \mathcal{F}_3 [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi) \mathcal{F}_4 [h_1^{\perp g} \otimes h_1^{\perp g}] \right)$$

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$\mathcal{F}_i(\theta) \rightarrow$  non-trivial functions of  $\cos(\theta)$  and  $\sin(\theta)$  (Logarithms from quark loop)



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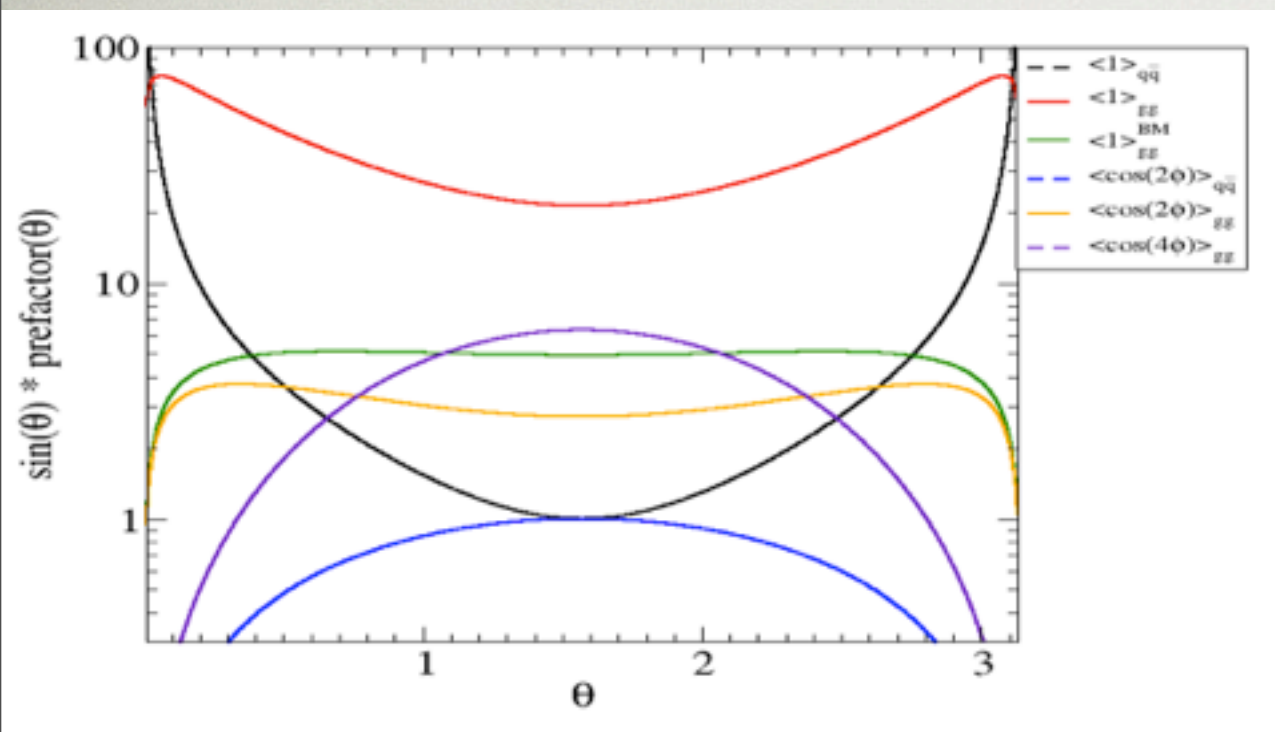
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- $\cos(4\phi)$  modulation a pure gluonic effect
- $\cos(2\phi) \rightarrow$  sign of gluon  $h_1^\perp$
- requires  $p_T$  & isolation cuts for the photons
- powerful in combination with DY  
 $\rightarrow$  map out quark TMDs in DY  $\rightarrow$  gluon TMDs in  $\gamma\gamma$



# NUMERICAL ESTIMATE

RHIC energy:  $\sqrt{S} = 500 \text{ GeV}$

Positivity bounds

$$|h_1^{\perp, g}| \leq \frac{2M^2}{k_T^2} f_1^g \quad |h_1^{\perp, q}| \leq \frac{M}{k_T} f_1^q$$

Gaussian ansatz:

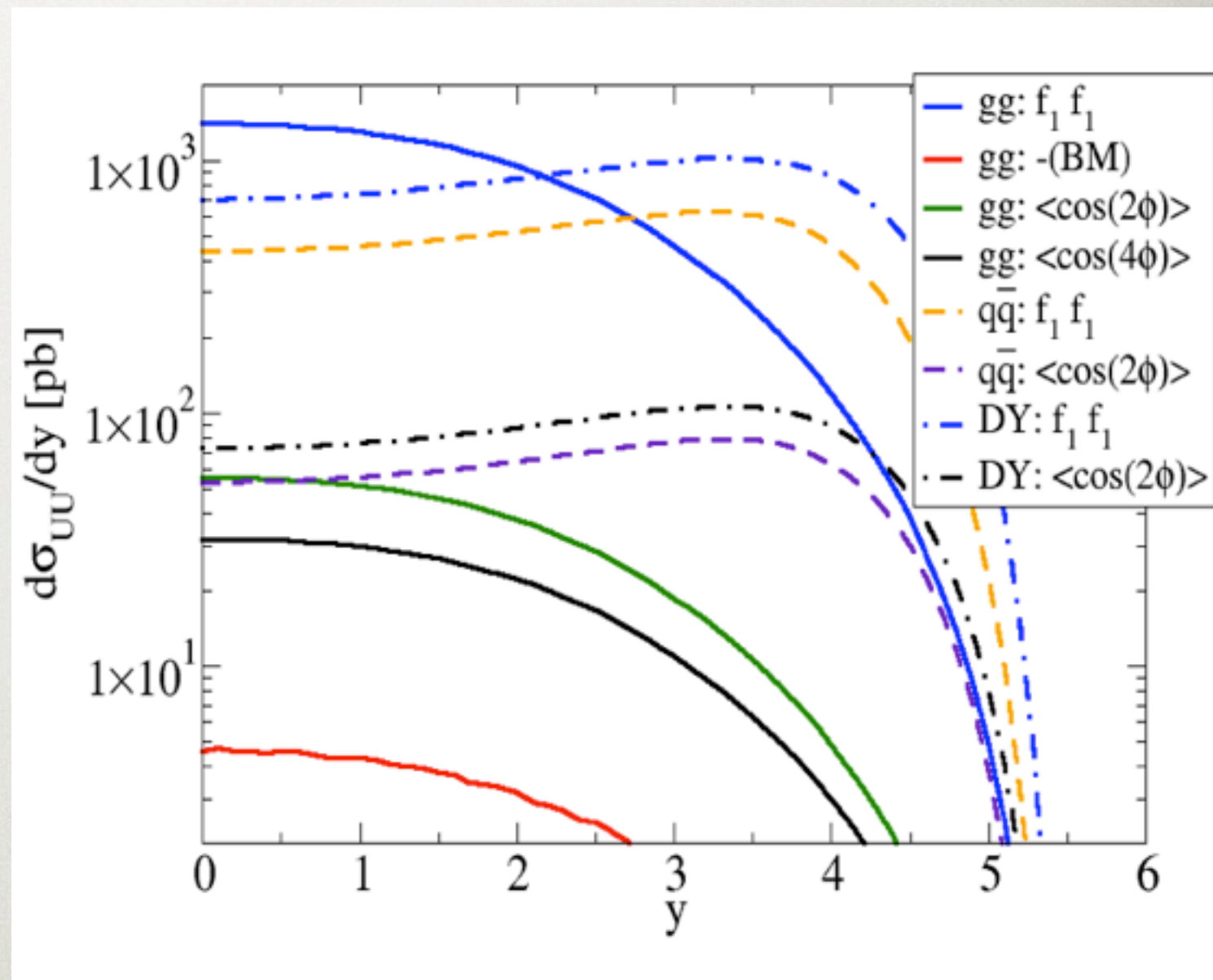
$$f_1^{q/g}(x, k_T^2) = f_1^{q/g}(x) e^{-k_T^2 / \langle k_{T, q/g}^2 \rangle}$$

Gaussian widths:

$$\langle k_{T, q}^2 \rangle = \langle k_{T, g}^2 \rangle = 0.5 \text{ GeV}^2$$

$p_T$ -cuts for each photon:

$$p_T^\gamma > 1 \text{ GeV}$$



→ Gluon TMDs feasible at RHIC at mid-rapidity!



# Gluon Sivers Effect

(Transverse) Spin dependent photon pair cross section:

$$\frac{d\sigma_{TU}}{d^4q d\Omega} \sim S_T \sin \phi_S \left[ \frac{2}{\sin^2 \theta} (1 + \cos^2 \theta) [f_{1T}^{\perp,g} \otimes f_1^{\bar{q}}] \right. \\ \left. + \left( \frac{\alpha_s}{2\pi} \right)^2 \left( \mathcal{F}_1 [f_{1T}^{\perp,g} \otimes f_1^g] + \mathcal{F}_2 [h_1^g \otimes h_1^{\perp,g}] + \mathcal{F}_2 [h_{1T}^{\perp,g} \otimes h_1^{\perp,g}] \right) \right] + \dots$$

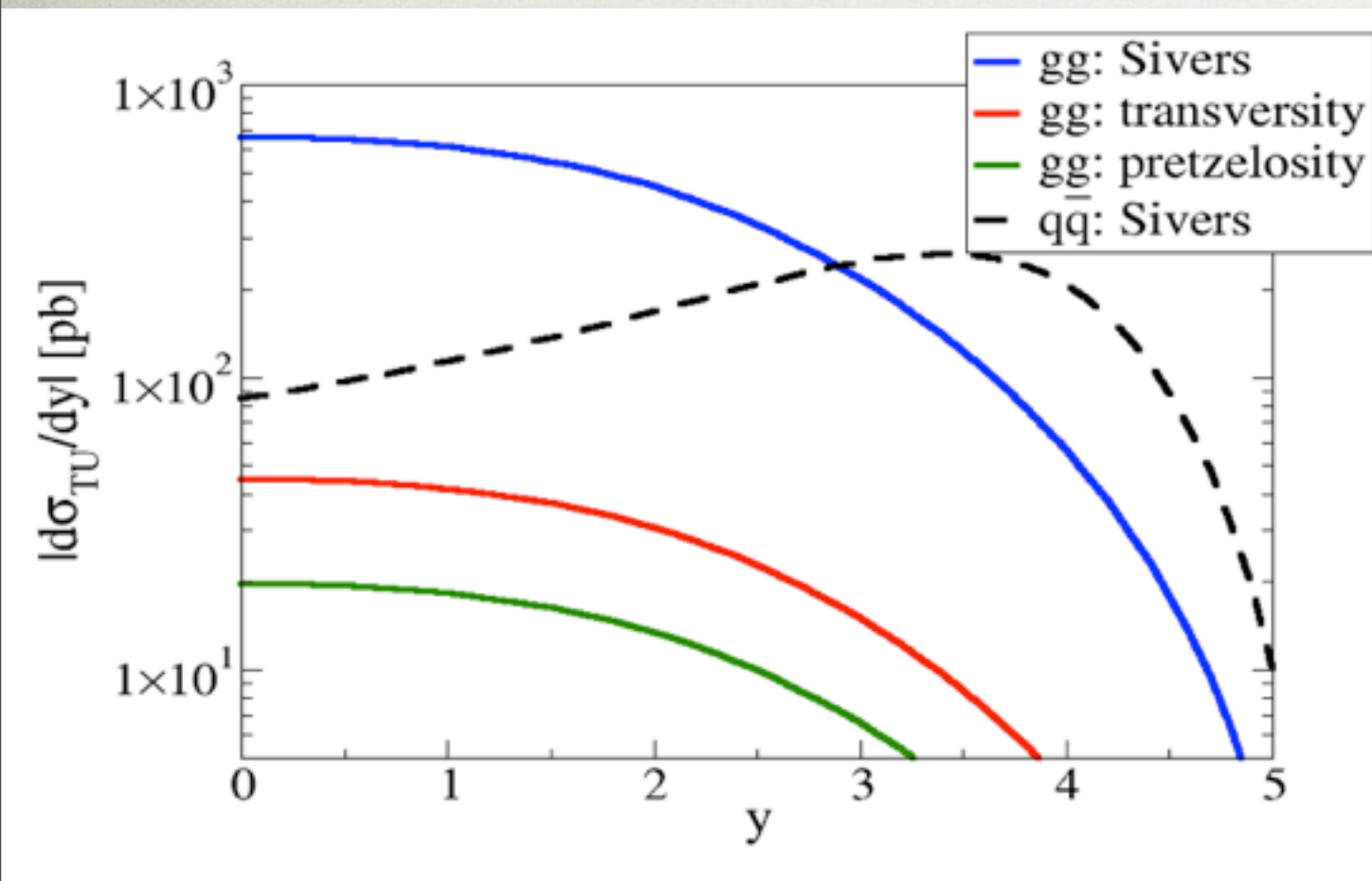


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Estimates for RHIC 500 GeV



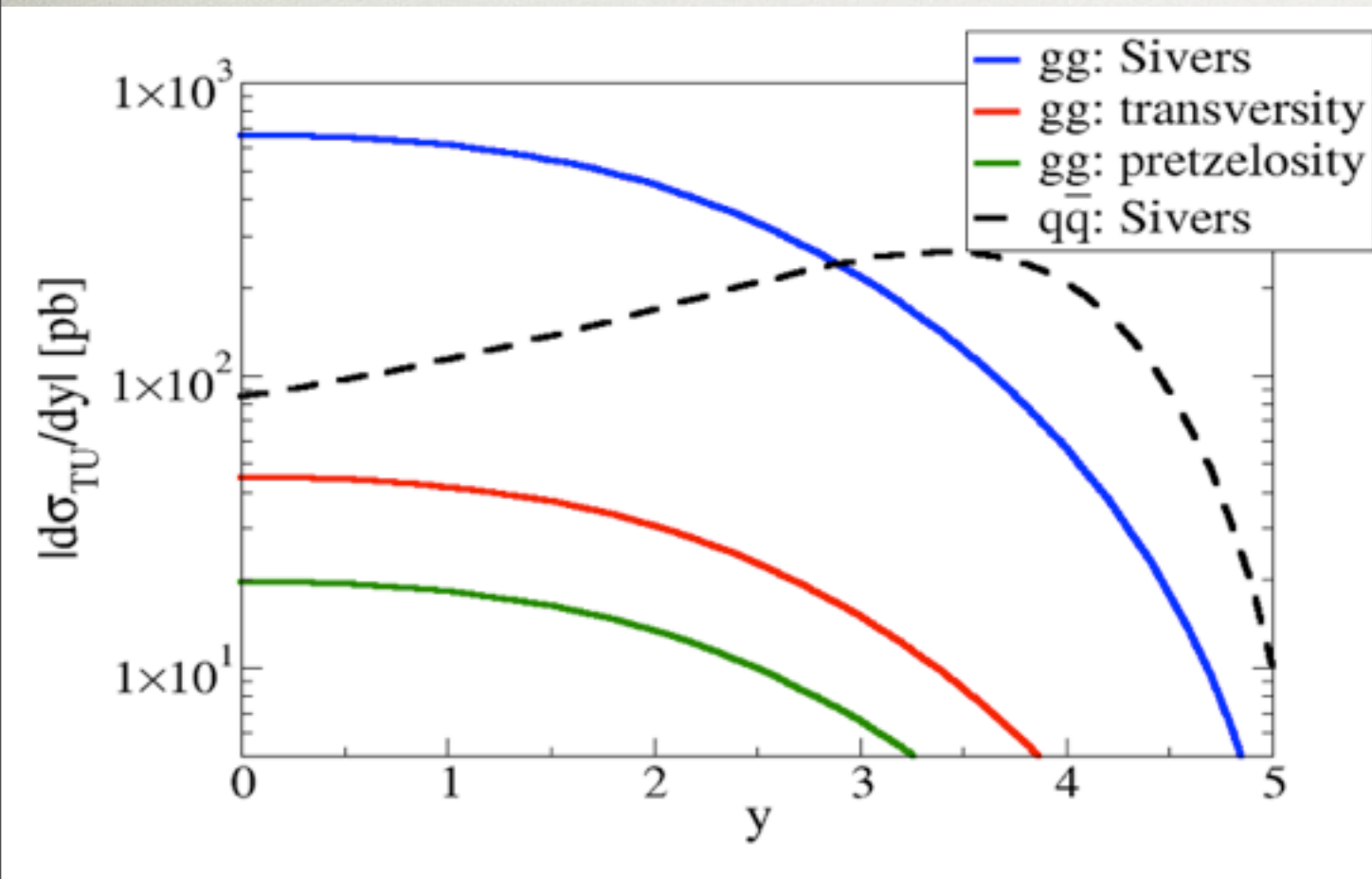


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- Flavor cancellation for quark Sivers func.  

$$f_{1T}^{\perp,u} \simeq -f_{1T}^{\perp,d}$$
 → bound only for u-quarks

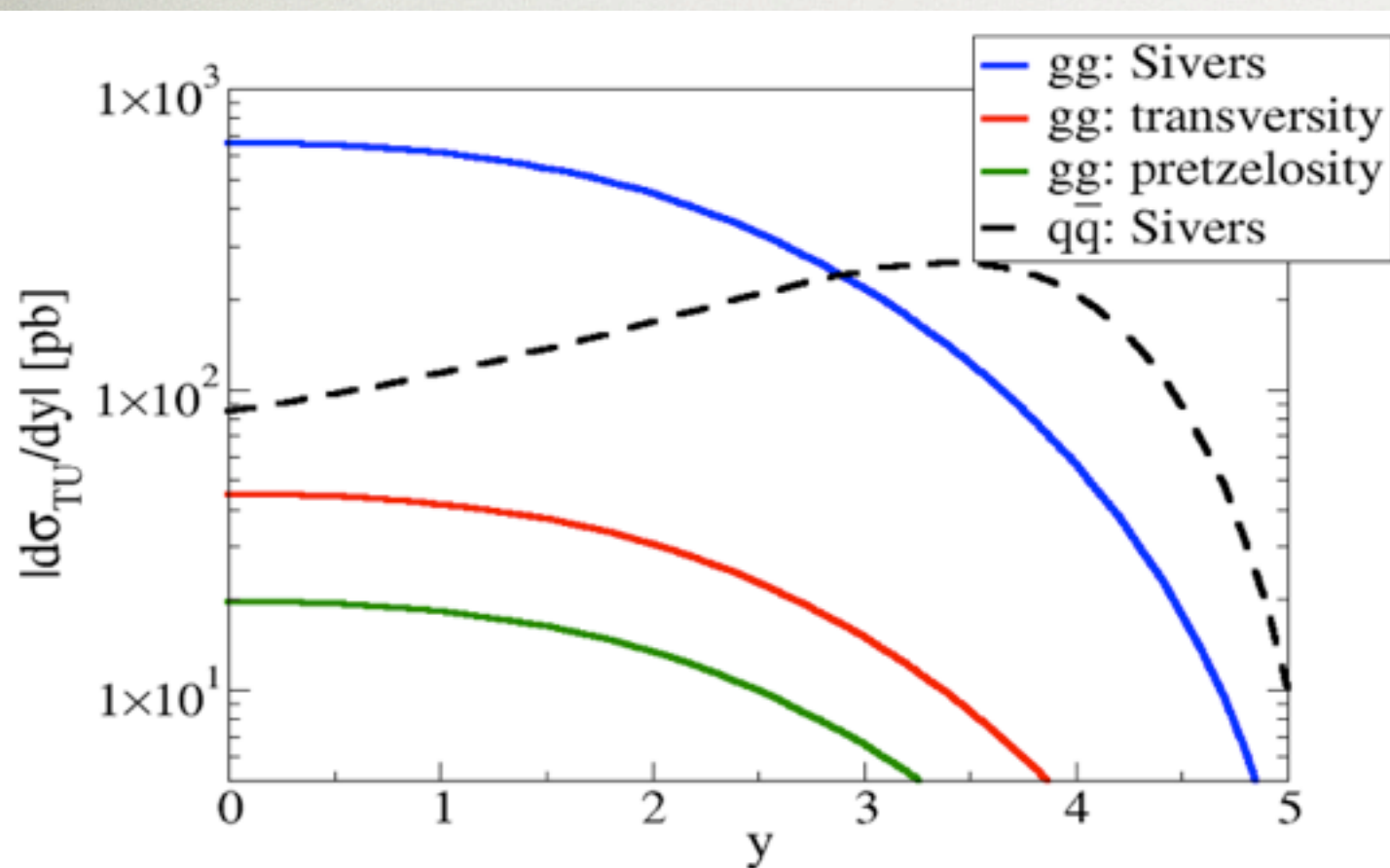


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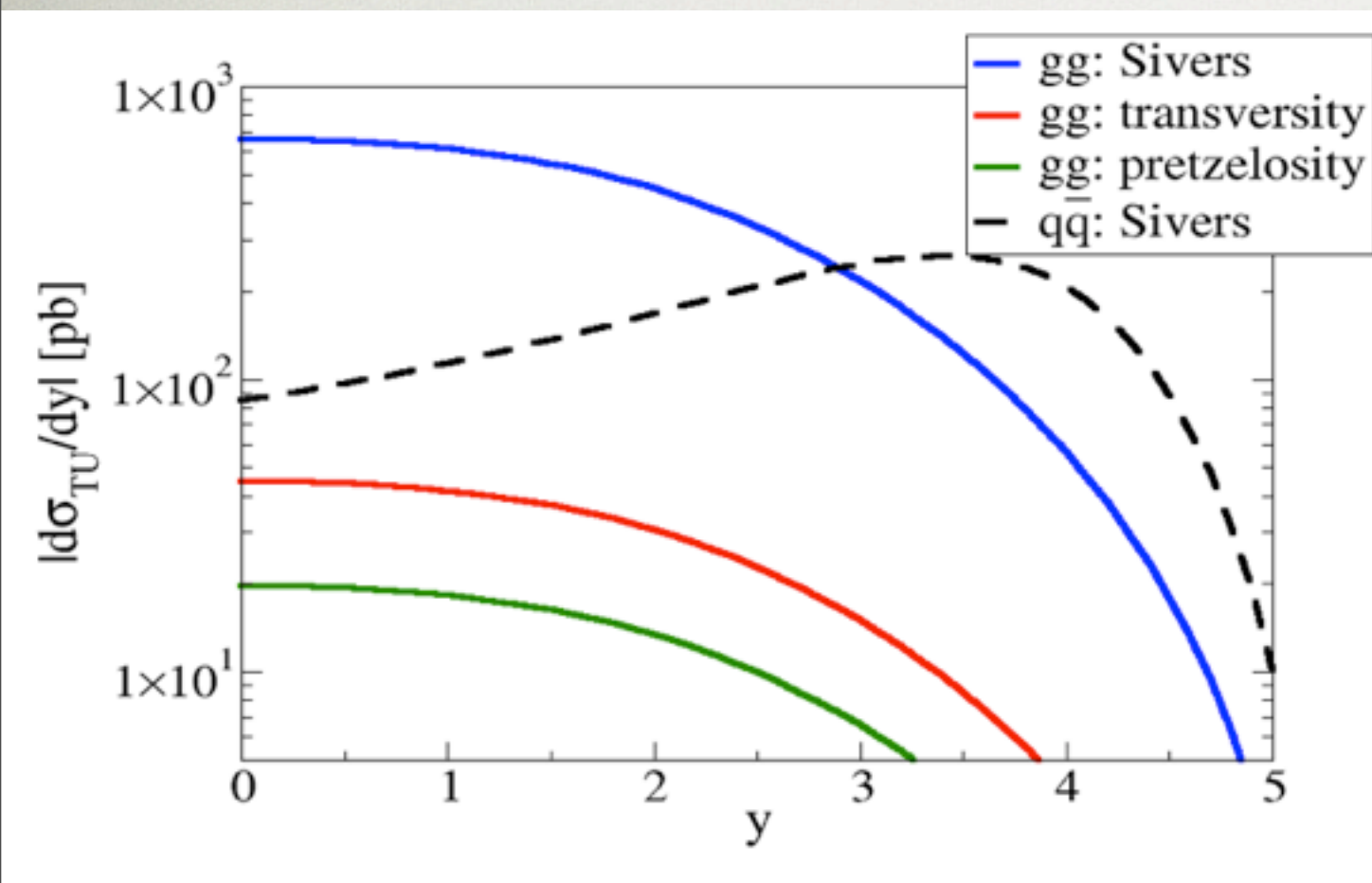


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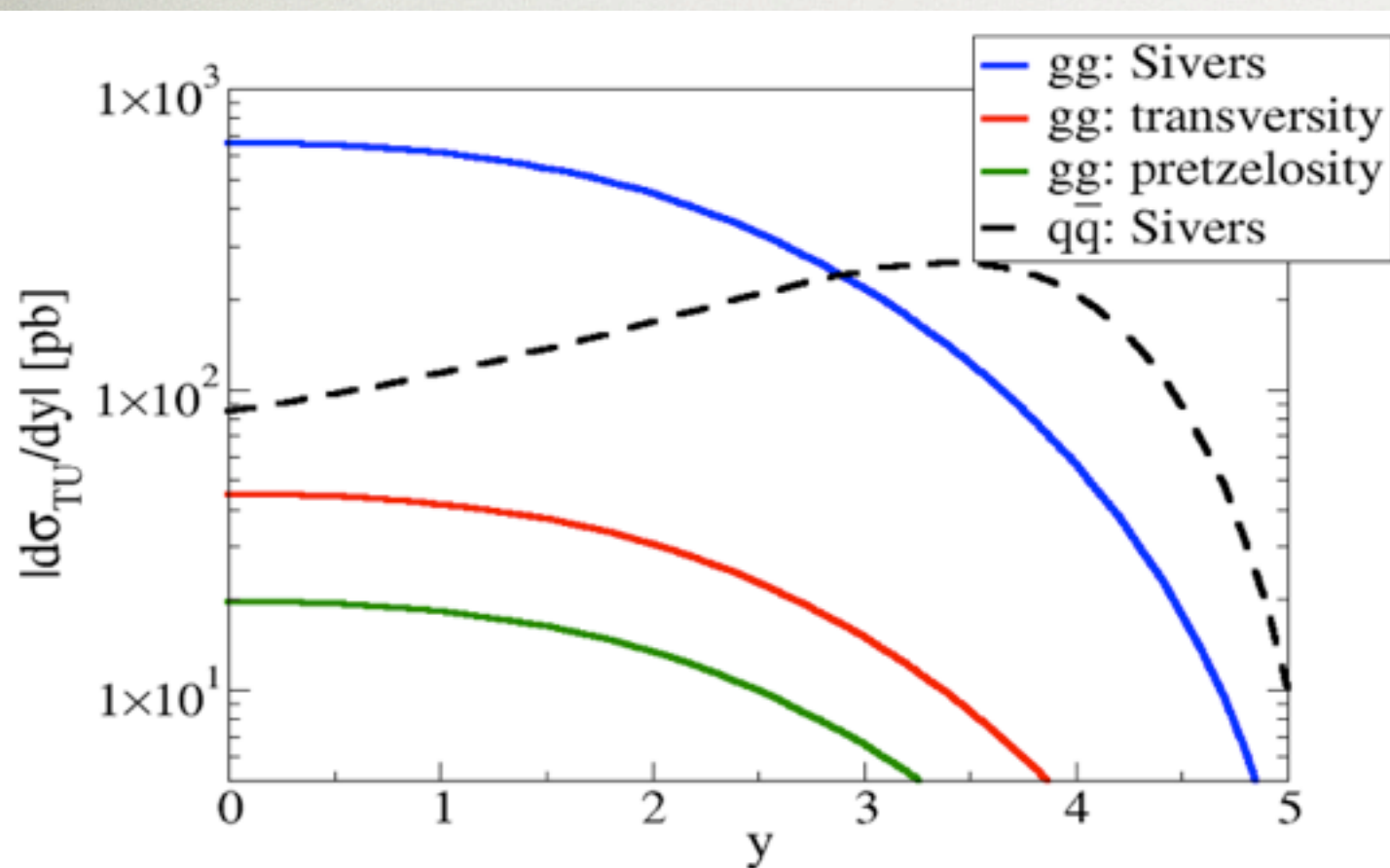


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- Gluons dominate at mid-rapidity, quarks at large rapidity
- Effects by gluon “transv. / pretzel.” small



# DIPHOTONS IN COLLINEAR FACTORIZATION

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[Nadolsky, Balazs, Berger, Yuan; Catani, Grazzini, de Florian]

Procedure for CSS-resummation:

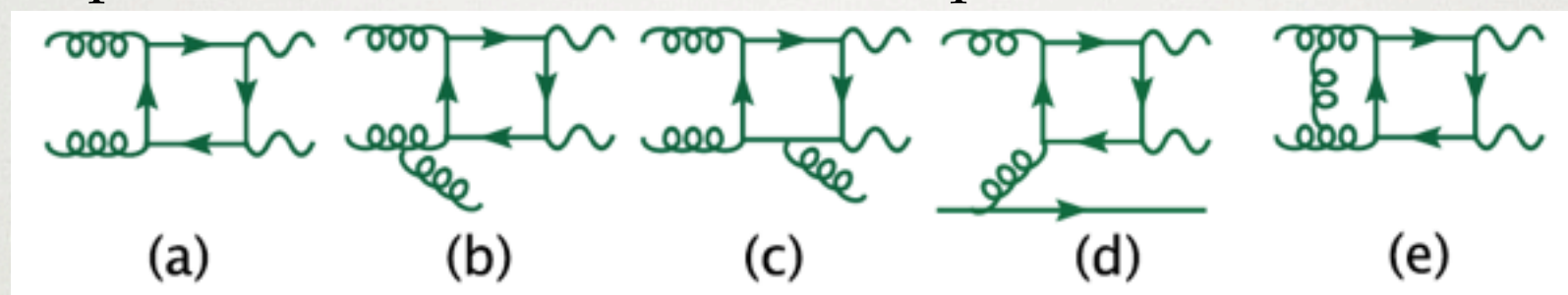


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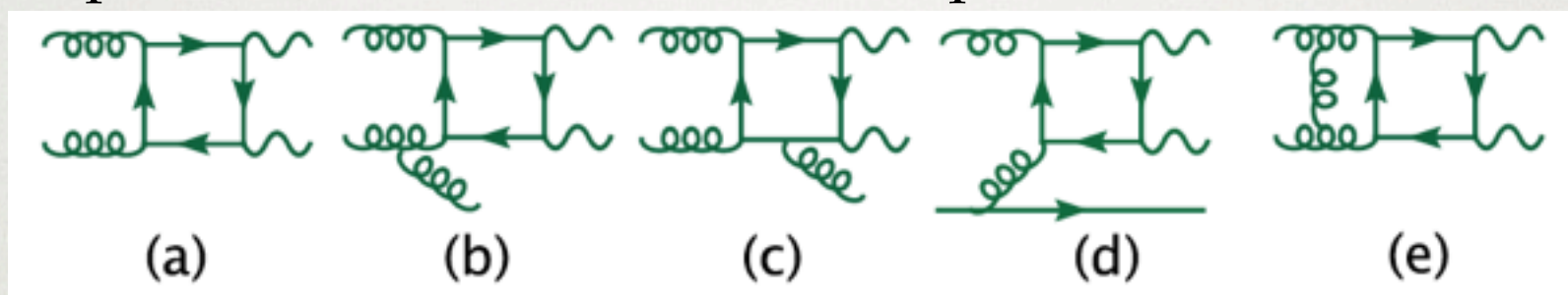


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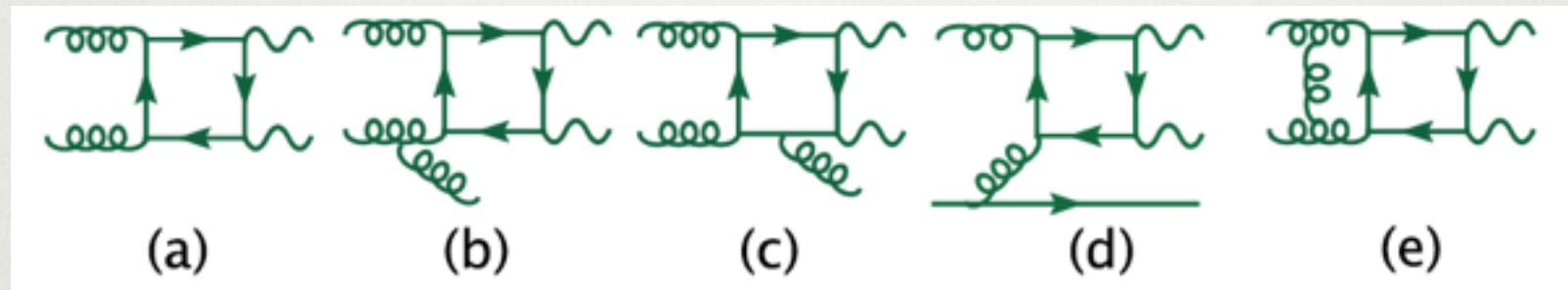


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3) Fourier transform into  $b_T$ -space

$$W(Q, Q_T, y, \Omega_*) = \int \frac{d\vec{b}}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \tilde{W}(Q, b, y, \Omega_*).$$

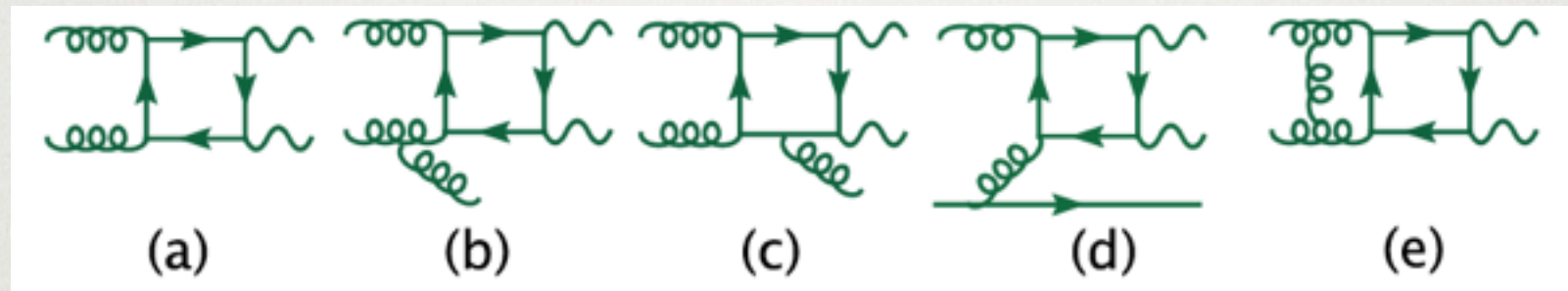


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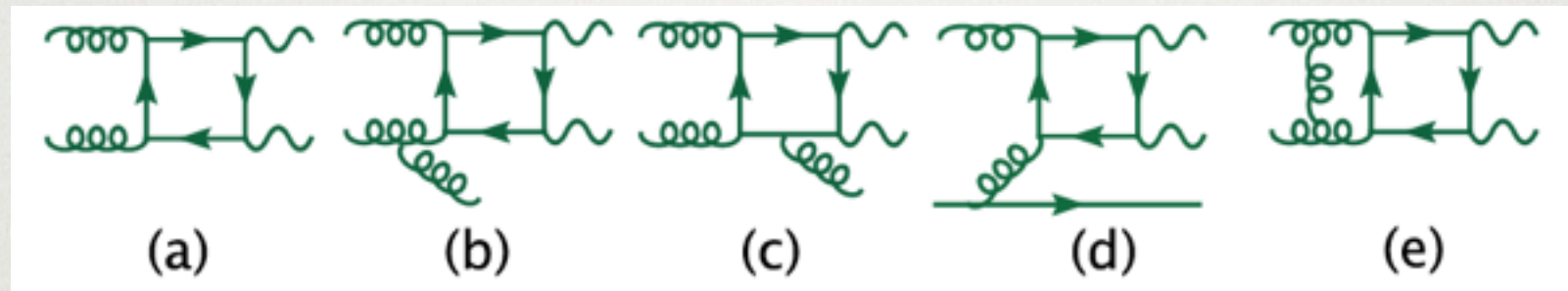


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5) Result

$$\tilde{W}(y, Q, b_T, \Omega) = e^{S_{\text{Sud}}(Q, b_T)} \left( F_1(\Omega) (C_q \otimes q)^2 + F_3(\Omega) ((C_q \otimes q)(\tilde{C}_q \otimes q) + (\tilde{C}_q \otimes q)(C_q \otimes q)) + (F_2(\Omega) + F_4(\Omega)) (\tilde{C}_q \otimes q)^2 \right) (x_a, x_b, 1/b_T)$$

→ Structure similar to TMD result! → Matching of coll. and TMD formalism



# GLUON TMDs IN HEAVY IONS

[Metz, Zhou; Dominguez, Qiu, Xiao, Yuan]

Definitions for large nuclei A:

Weizsäcker-Williams distribution:

$$M_{\text{WW}}^{ij} = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - i\vec{k}_\perp \cdot \vec{\xi}_\perp} \langle A | F^{+i}(\xi^- + y^-, \xi_\perp + y_\perp) L_{\xi+y}^\dagger L_y F^{+j}(y^-, y_\perp) | A \rangle$$
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Matrix elements → Gaussian weighting over color sources

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→ no saturation of pos. bound

$$x f_{1,\text{WW}}^g(x, k_\perp) \simeq S_\perp \frac{N_c^2 - 1}{4\pi^3} \frac{\mu_A}{k_\perp^2} \quad (k_\perp \gg Q_s)$$

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→ no saturation of pos. bound

$$x h_{1,\text{DP}}^{\perp g}(x, k_\perp) = 2x f_{1,\text{DP}}^g(x, k_\perp)$$

$$= \frac{k_\perp^2 N_c}{\pi^2 \alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp \cdot \vec{\xi}_\perp} e^{-(Q_{sq}^2 \xi_\perp^2 / 4)}$$

$$x f_{1,\text{WW}}^g(x, k_\perp) \approx S_\perp \frac{N_c^2 - 1}{4\pi^3} \frac{\mu_A}{k_\perp^2} \quad (k_\perp \gg Q_s)$$

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→ saturation at large  $k_T$

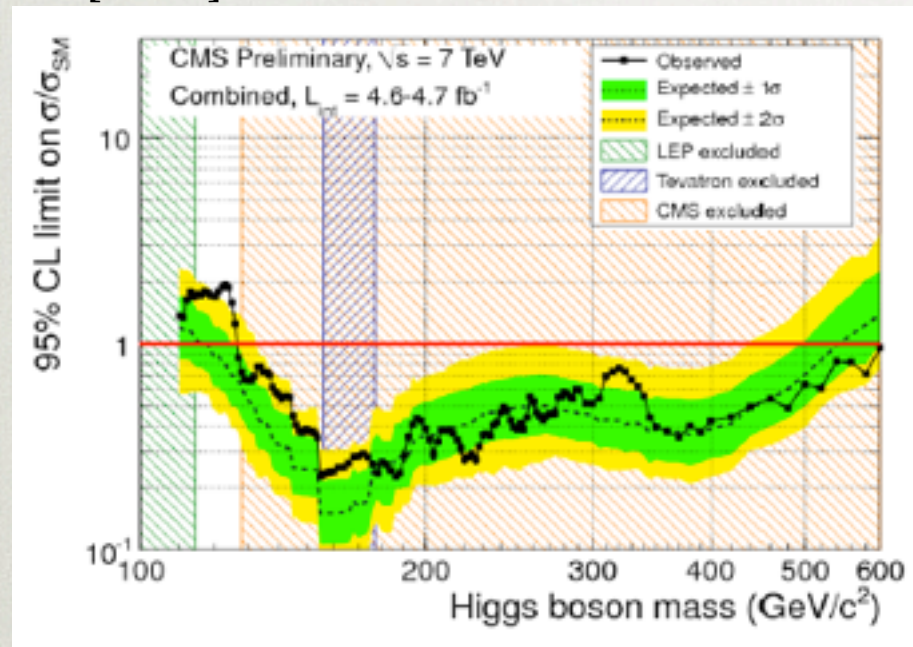
→ Saturation for all  $x, k_T$ !



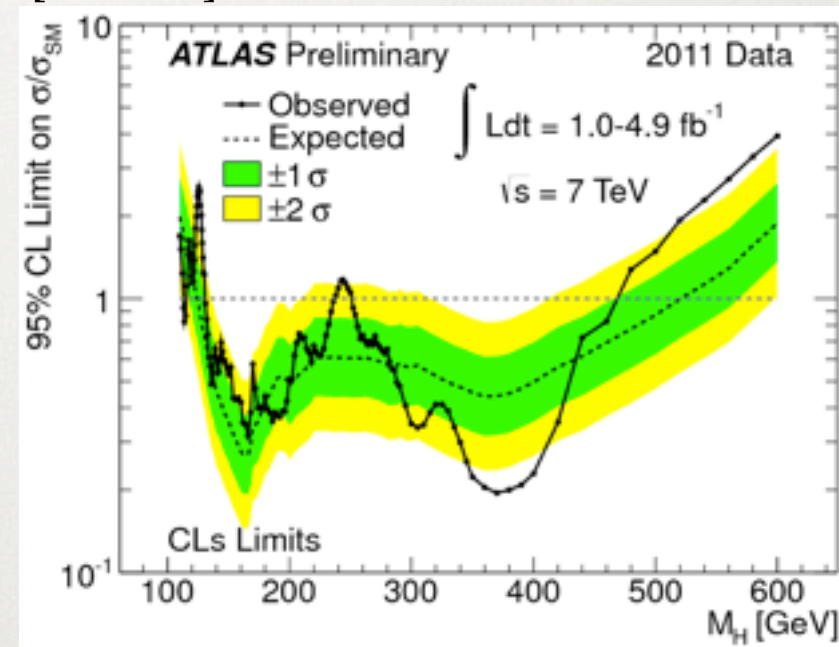
# RELATION TO LHC PHYSICS

Search for the Higgs boson: → mass  $m_H$ , total decay width  $\Gamma_H$

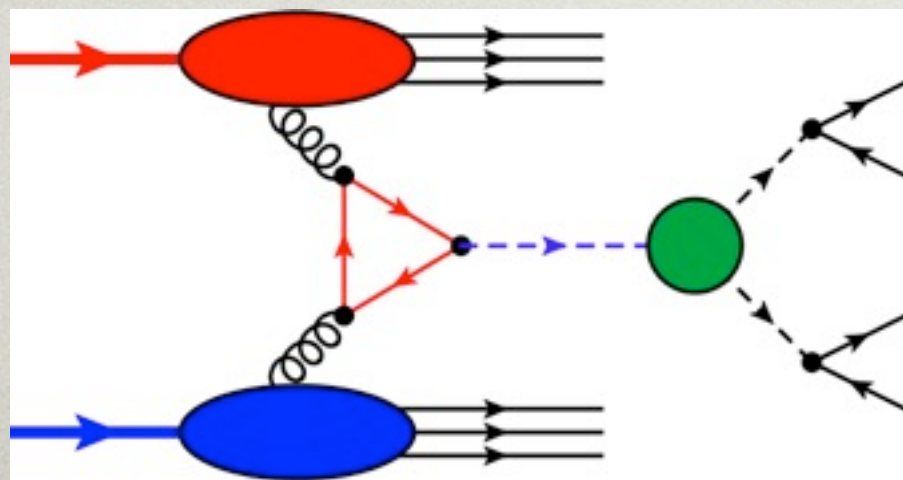
[CMS]



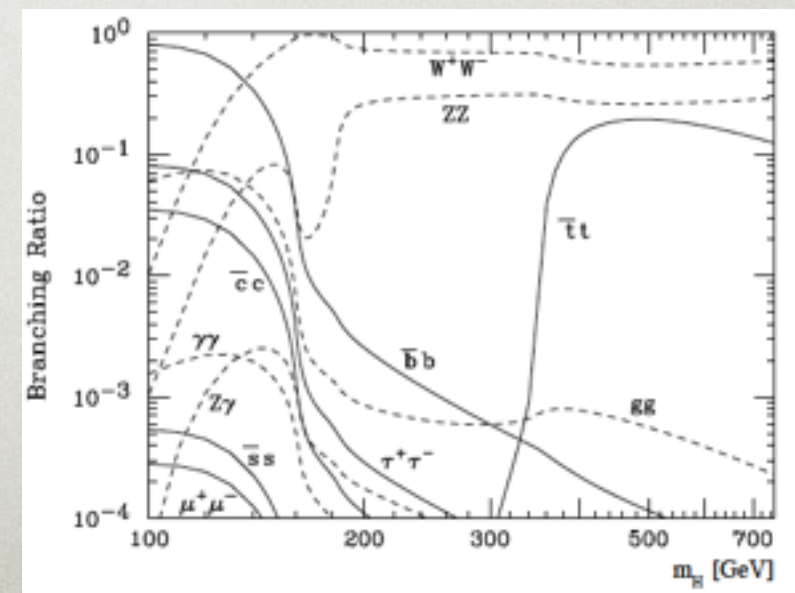
[ATLAS]



main production mechanism: gluon fusion



Higgs decay branching ratio





# LINEARLY POLARIZED GLUONS AND HIGGS PRODUCTION

[BOER, DEN DUNNEN, PISANO, M.S., VOGELSANG, PRL 108, 032002 (2012)]

*Can gluonic TMDs be useful for the LHC?*

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Once a scalar particle (Higgs!?) is found..... want to determine its parity.



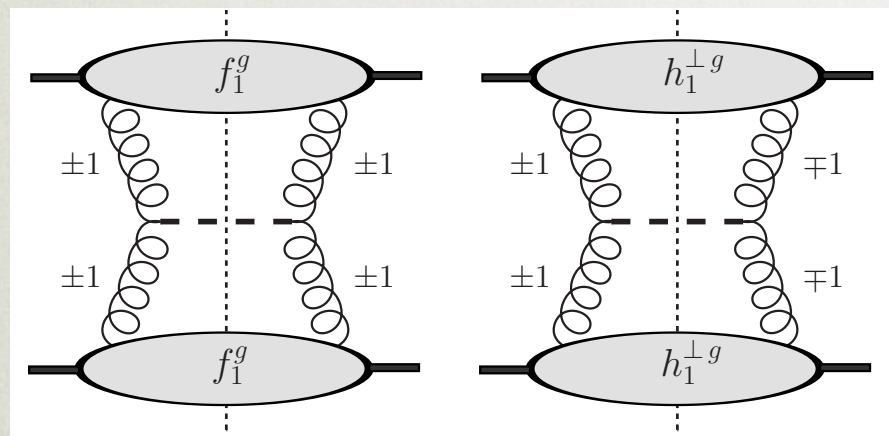
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linearly polarized gluons sensitive to Higgs parity

$$[f_1^g \otimes f_1^g] \pm [h_1^{\perp g} \otimes h_1^{\perp g}]$$

+: scalar Higgs    -: pseudoscalar Higgs



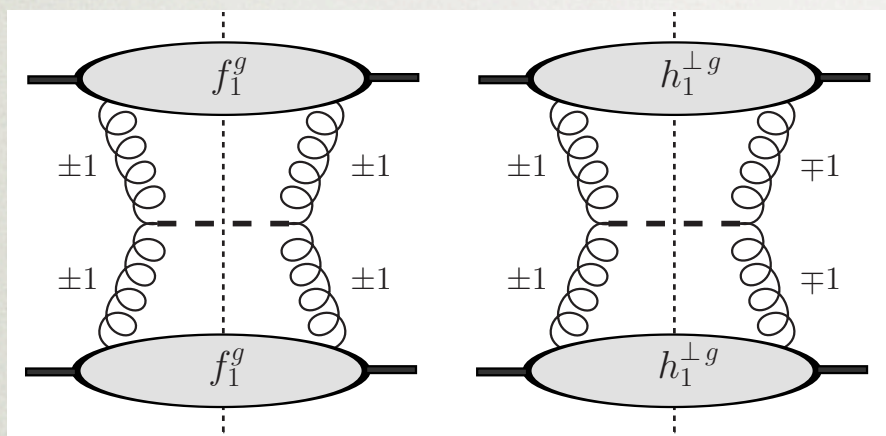
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$$R = \frac{[h_1^{\perp g} \otimes h_1^{\perp g}]}{[f_1^g \otimes f_1^g]}$$

→ precise q<sub>T</sub> measurement may offer a way to determine Higgs parity



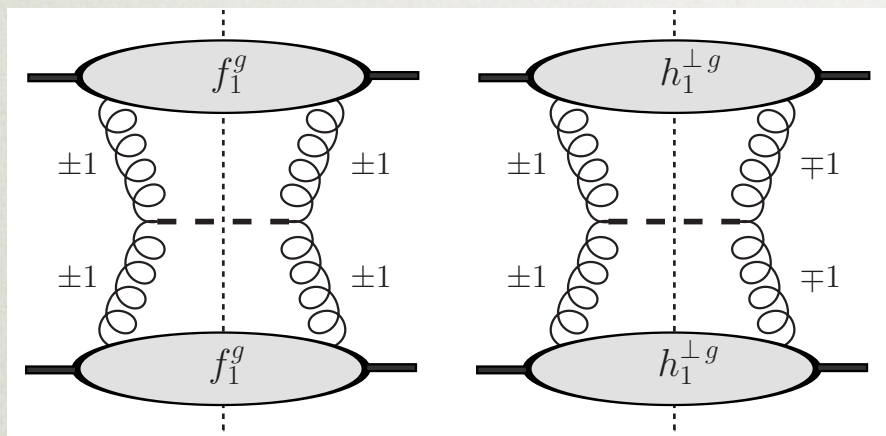
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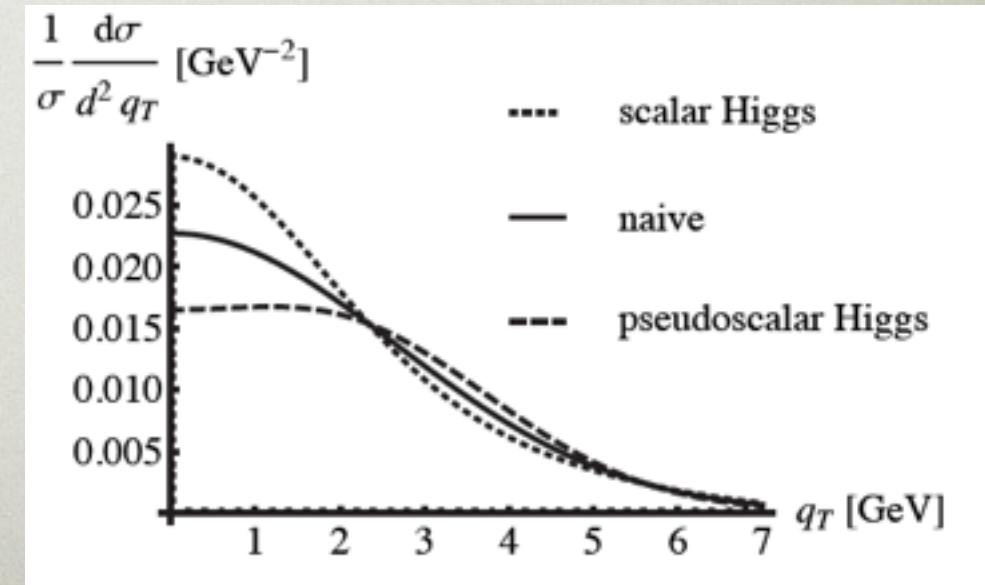
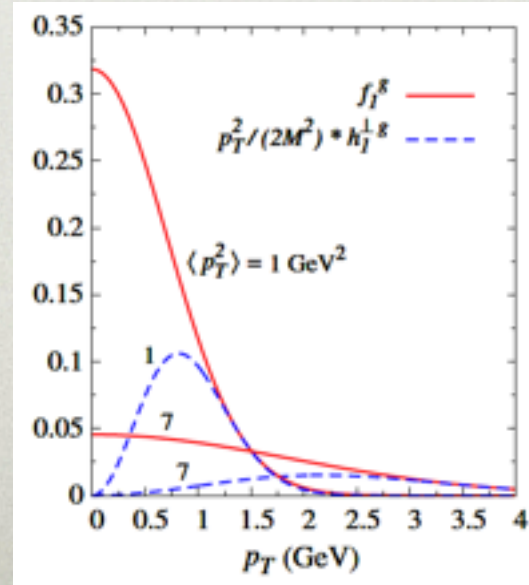
Numerical estimate:

Gaussian ansatz +

saturation

$$\langle p_T^2 \rangle = 7 \text{ GeV}^2$$

$$\frac{1}{\sigma} \frac{d\sigma}{d^2\vec{q}_T} = [1 \pm R(q_T)] \frac{1}{2\pi \langle p_T^2 \rangle} e^{-q_T^2/2\langle p_T^2 \rangle}$$





# RESUMMED GLUON TMDs IN HIGGS PRODUCTION

---

[Sun, Xiao, Yuan, PRD 84,094005]

Use of “old-fashioned” TMD approach to derive CSS-resummation

for pure Higgs production:  $P_a + P_b \rightarrow H + X$



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$$S(\vec{l}_T, \mu, \rho) \left( f_{1,a}^{g,\text{unsub}} f_{1,b}^{g,\text{unsub}} + w h_{1,a}^{\perp g,\text{unsub}} h_{1,b}^{\perp g,\text{unsub}} \right)$$



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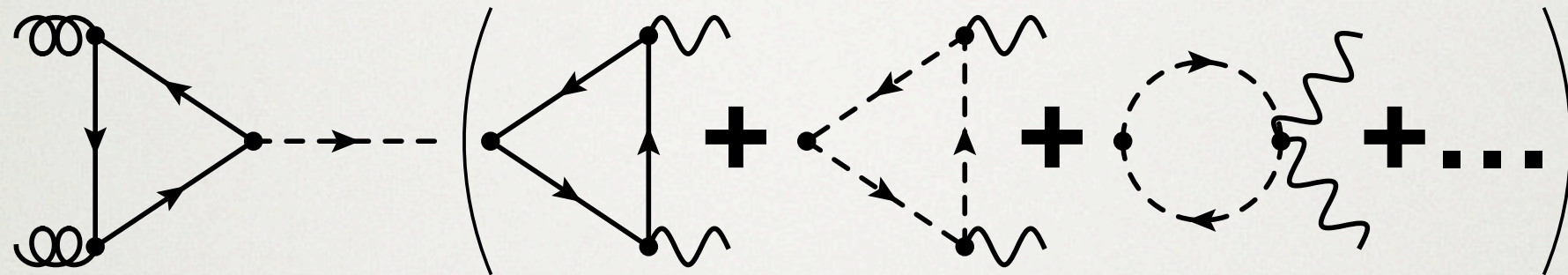
4) Perturbative tail of the gluon TMDs:

$$f_1^g \otimes f_1^g + h_1^{\perp g} \otimes h_1^{\perp g} = \int d^2 b_T e^{-iq_T \cdot b_T} \left[ e^{-S_{\text{Sud}}(M^2, 1/b_T^2)} \left( (C_q \otimes q)^2 + (\tilde{C}_q \otimes q)^2 \right) \right](x_a, x_b, 1/b_T)$$

$\rightarrow$  agrees with conventional collinear resummation approach [Catani, Grazzini]



# Including Higgs decay: $gg \rightarrow H/A \rightarrow \gamma\gamma$

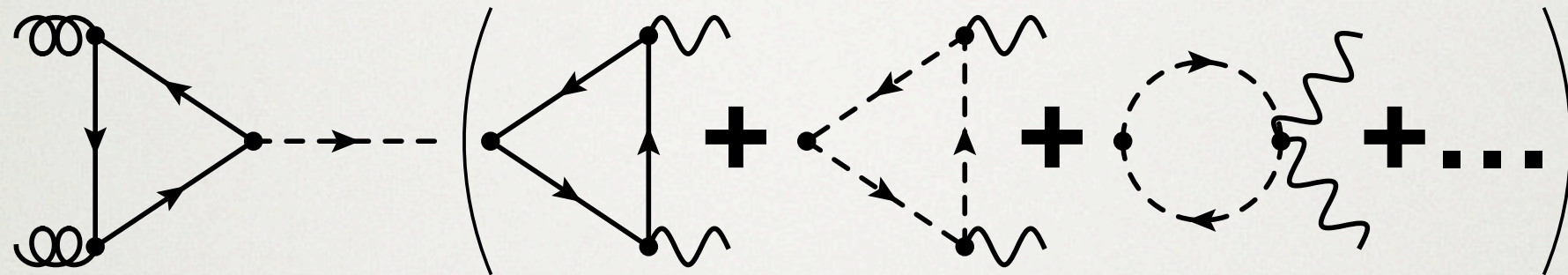


$\phi$  - integrated cross section of Higgs + box:

$$\int d\phi \frac{d\sigma^{gg}}{d^4q d\Omega} \propto \bar{\mathcal{F}}_1 [f_1^g \otimes f_1^g] + \bar{\mathcal{F}}_2 [h_1^{\perp g} \otimes h_1^{\perp g}]$$

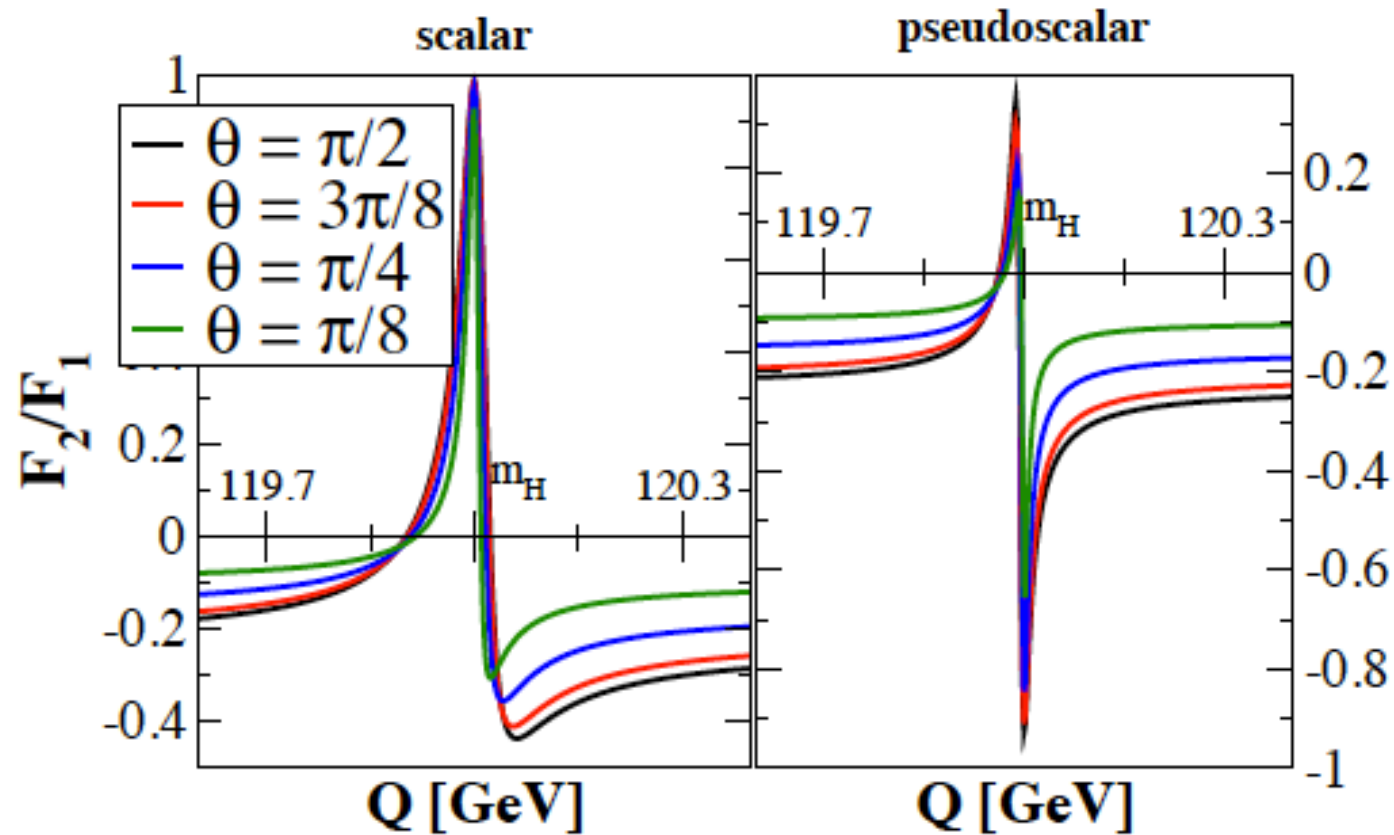


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$Q \neq m_H$ :  $\bar{\mathcal{F}}_1 \gg \bar{\mathcal{F}}_2$

box dominant

$Q \sim m_H$ :  $\bar{\mathcal{F}}_1 \simeq \bar{\mathcal{F}}_2$

Higgs dominant (pole of the propagator)

Sign signature preserved at the pole!  
small total Higgs width  $\rightarrow$  good  $Q$  resolution

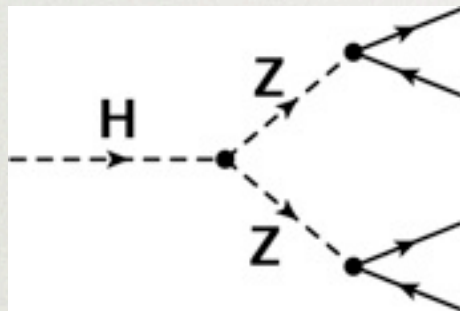


# 4 LEPTON PRODUCTION

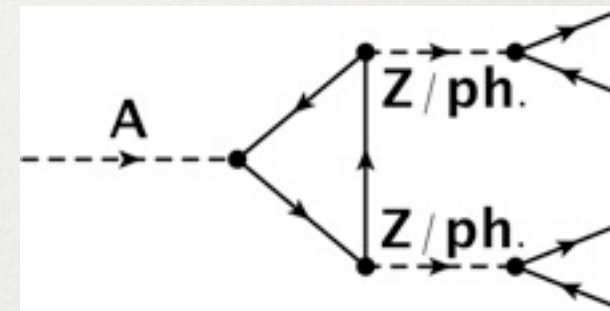
[Boer, den Dunnen, Pisano, MS, Vogelsang, in prep.]

Different decay channels for scalar and pseudoscalar Higgs:

SM Higgs: tree-level vertex



BSM pseudoscalar Higgs: top-loop



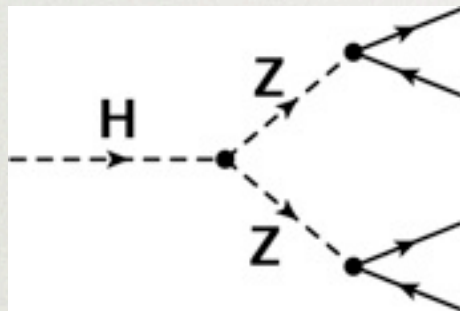


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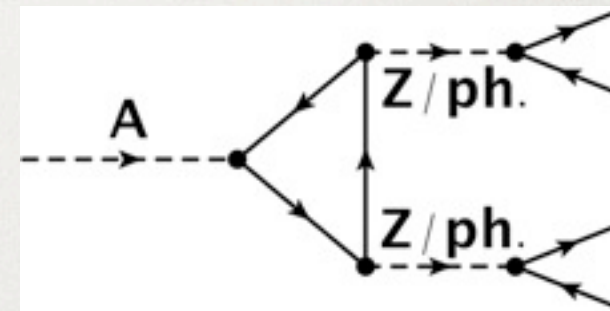
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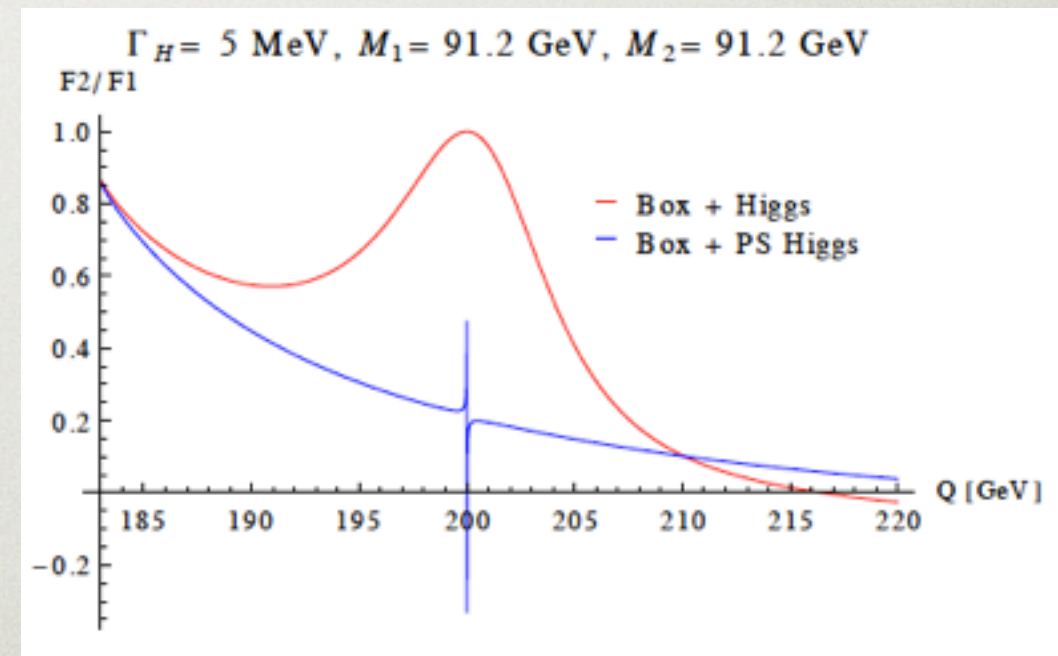
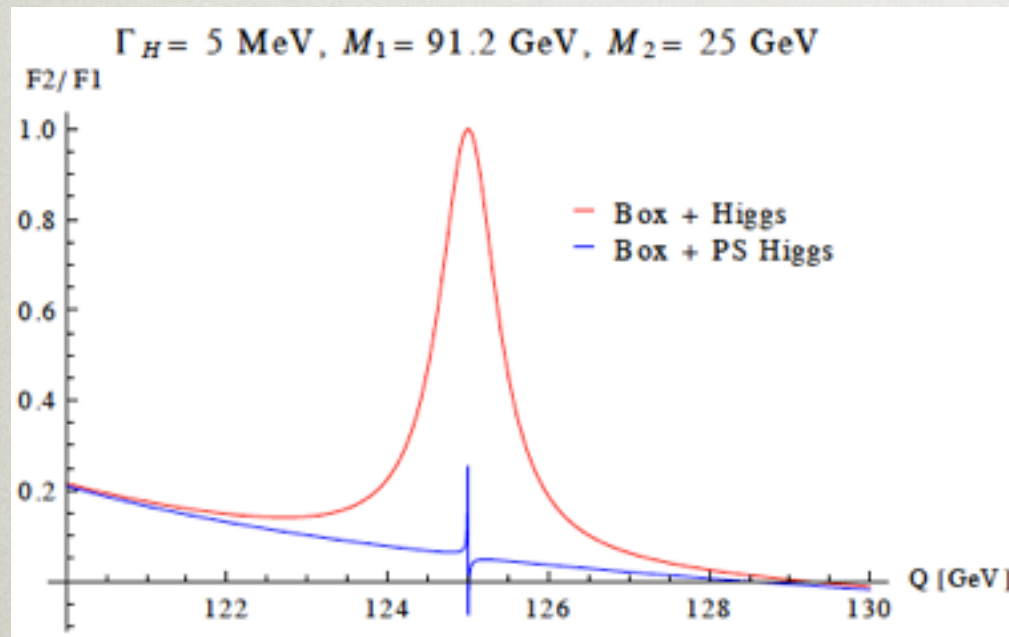


one on-shell Z:  $gg \rightarrow ZZ^*$

BSM pseudoscalar Higgs: top-loop



two on-shell Z:  $gg \rightarrow ZZ$



→ more difficult w.r.t. parity distinction, clean process experimentally

*Warning: multi-parton scattering!*

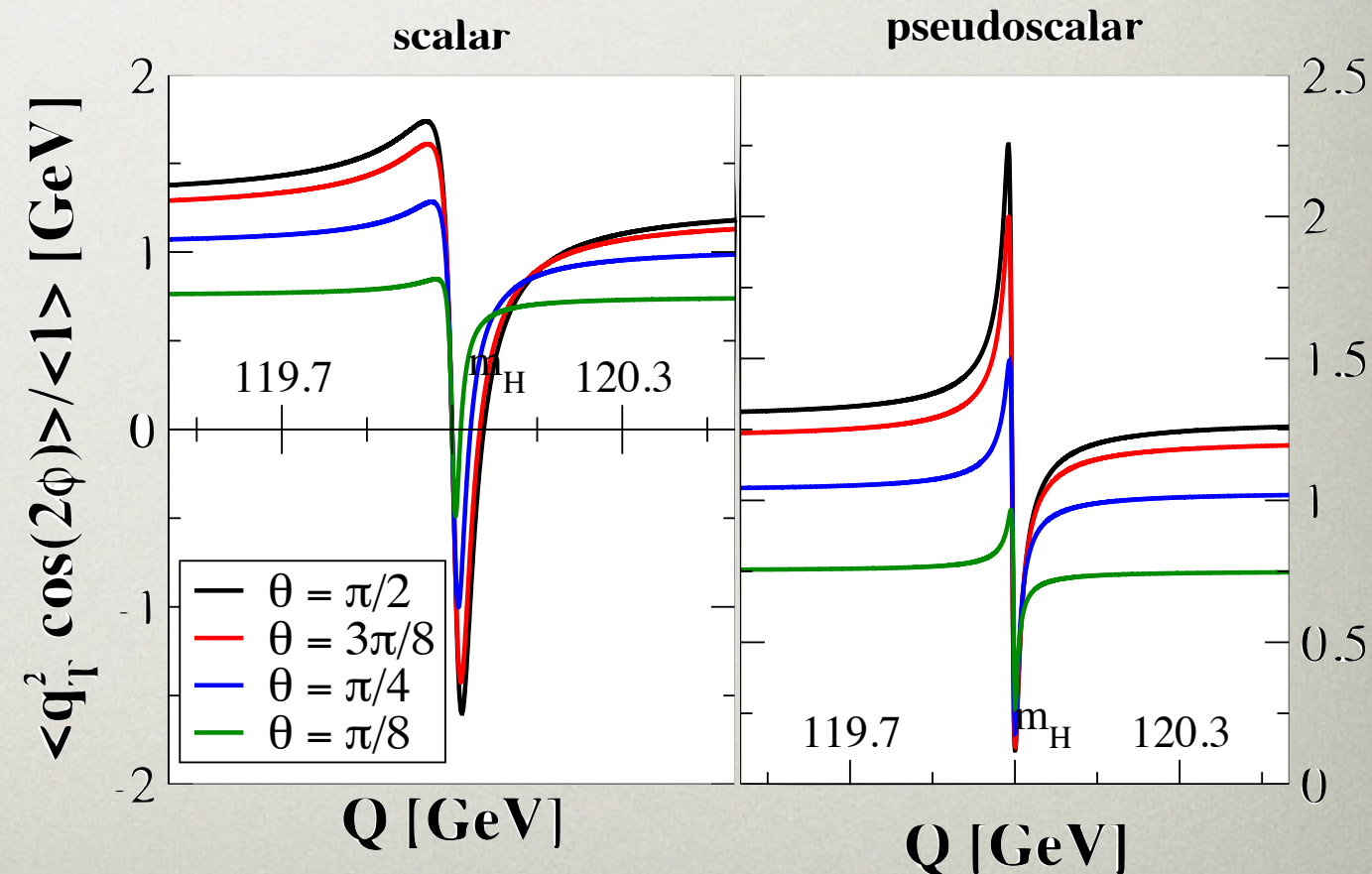
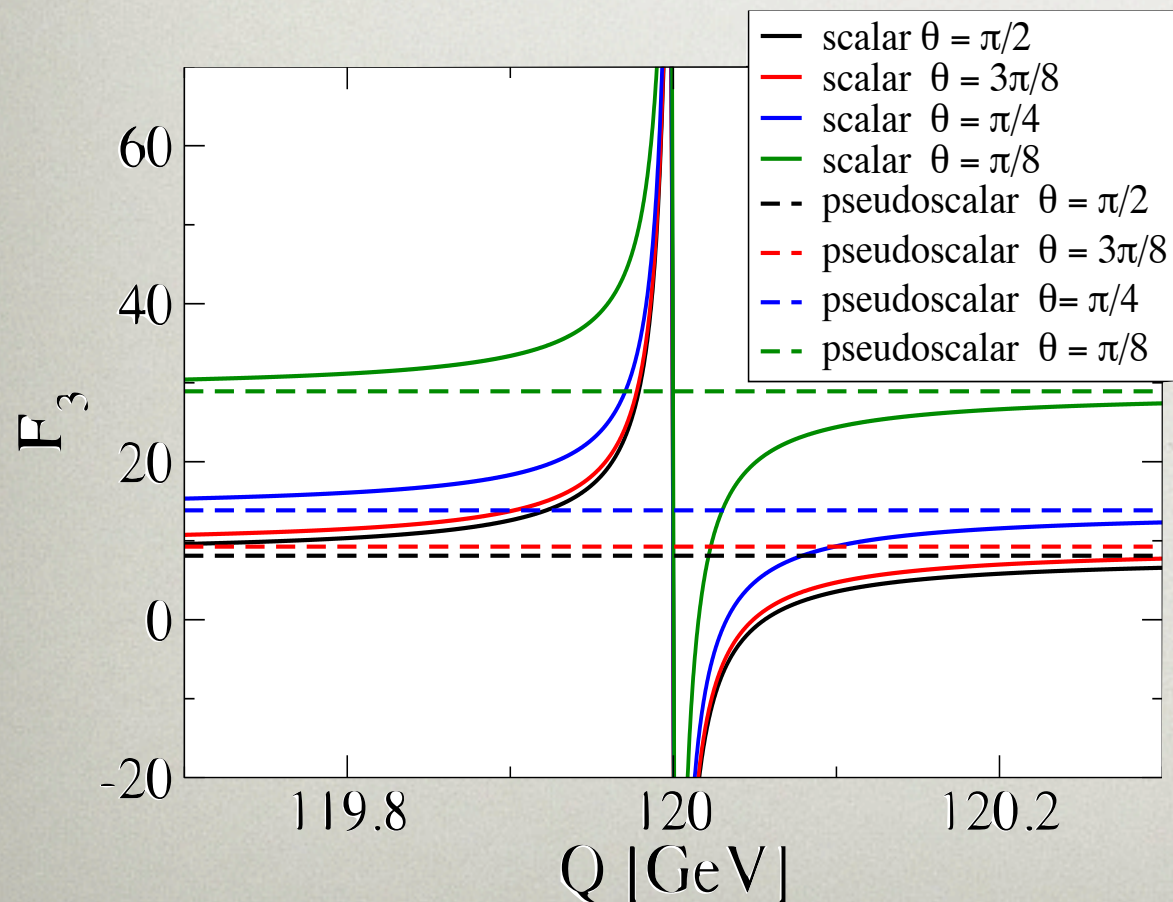


# May use also azimuthal $\cos(2\phi)$ modulation...

[Boer, den Dunnen, Pisano, MS, Vogelsang, in prep.]

$$\langle q_T^2 \cos(2\phi) \rangle = \int d^2 q_T d\phi q_T^2 \cos(2\phi) \frac{d\sigma}{d^4 q d\Omega} \sim \bar{\mathcal{F}}_3(\theta, Q^2) \left[ f_1^g \otimes h_1^{\perp g} + h_1^{\perp g} \otimes f_1^g \right]$$

scalar Higgs contributes to  $\bar{\mathcal{F}}_3$ , pseudoscalar doesn't  
 → offers alternative determination of Higgs parity  
 → theoretically cleaner (yes/no decision), experimentally harder












# Summary

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- Several alternative processes to DY at the LHC
- Gluon TMDs accessible at the LHC and RHIC
- Distribution of linearly polarized gluons from  $\cos(4\phi)$  - mode
- Linearly polarized gluons may be useful to pin down the parity of Higgs bosons
- Gluon Sivers- and Boer-Mulders effect may be feasible at RHIC (if  $\neq 0$ )
- To Do: Evolution of gluon TMDs



# SPIN-DEPENDENT TMDs

N \ q	U	L	T
U			
L			
T			

time-reversal odd

Plot courtesy of B. Musch

well-studied :

[experimentally & theoretically]

Sivers function

Boer-Mulders function

(naive) collinear limits:

unpolarized, helicity, transversity

“wormgear” functions

“pretzelosity”

quadrupole structure