Polarized Parton Distributions and their uncertainties

Workshop on Drell-Yan Scattering and the Structure of Hadrons

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Outline

1. Motivation
   - Issues in Standard PDF determination
2. The NNDPF procedure
   - A general overview
   - Monte Carlo sampling, Neural Networks and minimization
3. Towards `NNPDFpol1.0`
   - Experimental dataset, PDF parametrization, theoretical constraints
   - Preliminary results: the `NNPDFpol1.0` parton set
4. Conclusions
   - Summary and outlook

1. Motivation

Issues in standard PDF determination

- Extraction of a set of functions with error bands from a set of data points.
- We need an error band, i.e. a probability density $\mathcal{P}[\Delta q(x)]$ in the space of PDFs:

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\Delta q \, \mathcal{P}[\Delta q] \mathcal{O}[\Delta q]$$

$$\sigma^2_{\mathcal{O}} = \int \mathcal{D}\Delta q \, \mathcal{P}[\Delta q] (\mathcal{O}[\Delta q] - \langle \mathcal{O} \rangle)^2$$

Standard approach

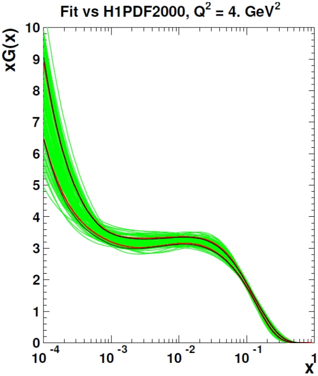
1. Choose a fixed functional form like
$$\Delta q_i(x, Q_0^2) = A_i x^{b_i} (1-x)^{c_i} (1+\dots)$$
2. Determine best-fit parameters
3. Errors determined via Gaussian linear error propagation

But...

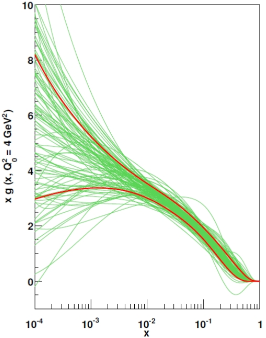
1. Is the parametrization flexible enough?
2. What is the error associated to any particular choice?
3. Need to rely on linear error propagation

Simple functional forms vs Neural Networks

HERA-LHC 2009 PDF benchmarks



simple functional forms

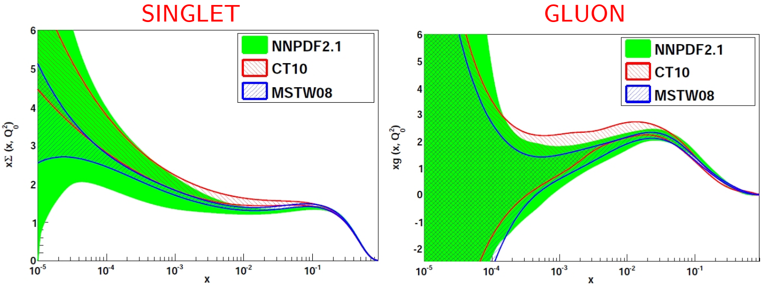


Neural Networks

- Simple functional forms $\Delta q(x) = Ax^b(1-x)^c P(x)$
  $\longrightarrow$ systematic underestimation of uncertainties $\Rightarrow$ tolerance
- Artificial Neural Networks as universal interpolants
  $\longrightarrow$ reduce theoretical bias from choice of PDF functional form

NNPDF: a new approach to PDF fitting based on
Monte Carlo sampling and Neural Networks

NNPDF2.1 at NLO

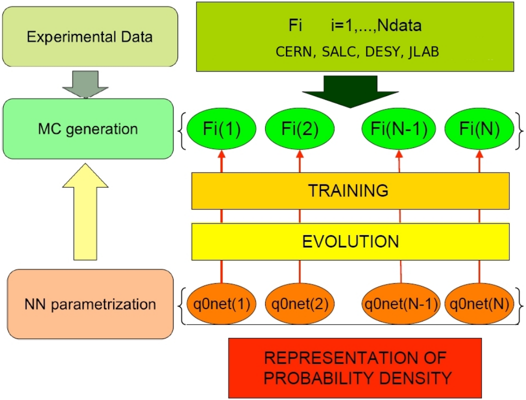


The NNPDF Collaboration, Nucl.Phys. B849 (2011) 296, 1101.1300

Successfully applied in the unpolarized case
Routinely used in LHC data analysis and theory prediction
Most recent global fit: `NNPDF2.1` available at LO, NLO and NNLO

2. The NNPDF approach

A general overview on the methodology



Ingredients:
Monte Carlo sampling and Neural Networks

Ingredient 1: Monte Carlo sampling of experimental data

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming multi-Gaussian data probability distribution

$$g_{1,p}^{(\mathrm{art}),(k)}(x,Q^2) = \left[1 + r_{c,p}^{(k)}\sigma_{c,p} + r_{s,p}^{(k)}\sigma_{s,p}\right] g_{1,p}^{(\mathrm{exp})}(x,Q^2)$$

$\sigma_{c,p}$: correlated systematics $\sigma_{s,p}$: statistical errors (also uncorrelated systematics)

$r_{c,p}^{(k)}$, $r_{s,p}^{(k)}$: Gaussian random numbers

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

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- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

- Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{O}[\Delta q_k]$$

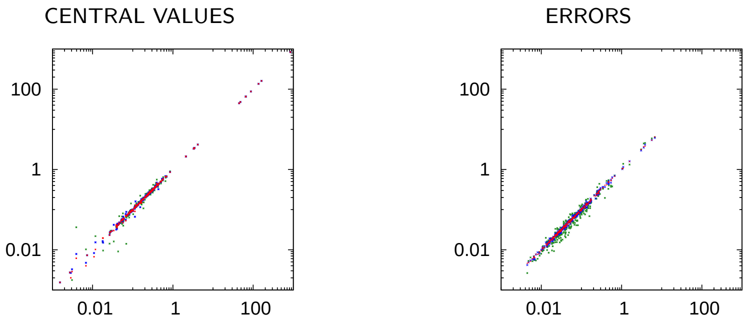
...and the same is true for errors, correlations etc.

- No need to rely on linear propagation of errors
- Possibility to test for non-Gaussian behaviour in fitted PDFs

DETERMINING SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy
- Accuracy of few % requires $\sim$ 250 replicas

Scatter plots: MC replicas vs experimental data [$N_{\rm rep}$ 10 100 250]



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DETERMINING SAMPLE SIZE

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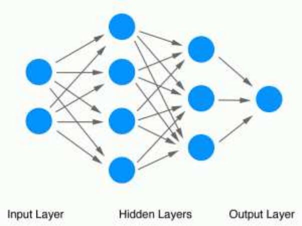
$r\,[g_1]$	1.00
$\left\langle \delta g_1^{(\text{exp})} \right\rangle_{\text{dat}}$	0.177
$\left\langle \delta g_1^{(\text{gen})} \right\rangle_{\text{dat}}$	0.177
$r\left[\delta g_1^{(\text{gen})}\right]$	1.000
$\left\langle \text{cov}^{(\text{exp})} \right\rangle_{\text{dat}}$	0.025
$\left\langle \text{cov}^{(\text{gen})} \right\rangle_{\text{dat}}$	0.025
$r\left[\text{cov}^{(\text{gen})}\right]$	1.000

- We have defined the covariance matrix

$$\text{cov}_{pq} = \left(\sigma_{c,p}\sigma_{c,q} + \delta_{pq}\sigma_{s,p}\sigma_{s,q}\right) g_{1,p} g_{1,q}$$

Ingredient 2: Neural Networks

A convenient functional form
providing redundant and flexible parametrization
used as a generator of random functions in the PDF space

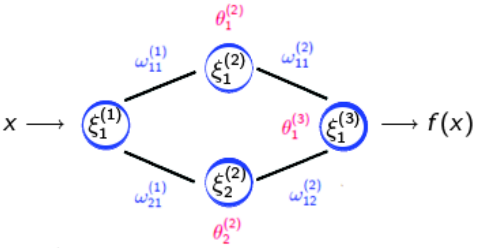


$$\xi_i^{(l)} = g\left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (weights and thresholds)
- activation determined according to a non-linear function

EXAMPLE: THE SIMPLEST 1-2-1 NN



$$f(x) \equiv \xi_1^{(3)} = \left\{ 1 + \exp\left[\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}} \right] \right\}^{-1}$$

Recall: $$\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right) \quad ; \quad g(x) = \frac{1}{1 + e^{-x}}$$

Ingredient 2: Neural Networks

NEURAL NETWORKS

- Parametrize each polarized PDF replica with flexible Neural Network

DSSV, AAC, LSS, BB	NNPDFpol
$\mathcal{O}(10-20)$ parameters	$\mathcal{O}(200)$ parameters

- Train NN to determine the best fit for each replica
- Compute an ensemble of observables and compare to experimental data

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- Train NN to determine the best fit for each replica
- Compute an ensemble of observables and compare to experimental data

MAIN FEATURES

- Only require smoothness of the fitted function
- Do not require any other prejudice on *a priori* functional form
- Reduce the bias associated to the choice of some functional form

One more ingredient: minimization and stopping

GENETIC ALGORITHM

> Standard minimization unefficient owing to the large parameter space
> and non-local x-dependence of the observables
> Genetic algorithm provides better exploration of the whole parameter space

- Set Neural Network parameters randomly
- Make clones of the parameter vector and mutate them
- Define a figure of merit or error function for the k-th replica

$$E^{(k)} = \frac{1}{N_{\text{rep}}} \sum_{i,j=1}^{N_{\text{rep}}} \left(g_{1,i}^{(\text{art})(k)} - g_{1,i}^{(\text{net})(k)} \right) \left((\text{cov})^{-1} \right)_{ij} \left(g_{1,j}^{(\text{art})(k)} - g_{1,j}^{(\text{net})(k)} \right)$$

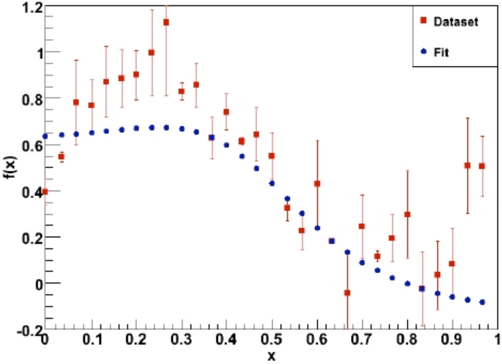
$g_{1,i}^{(\text{art})(k)}$: generated from Monte Carlo sampling
$g_{1,i}^{(\text{net})(k)}$: computed from Neural Network PDFs

- Select the best set of parameters and perform other manipulations (crossing, mutating, ...) until stability is reached.

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLEARNING

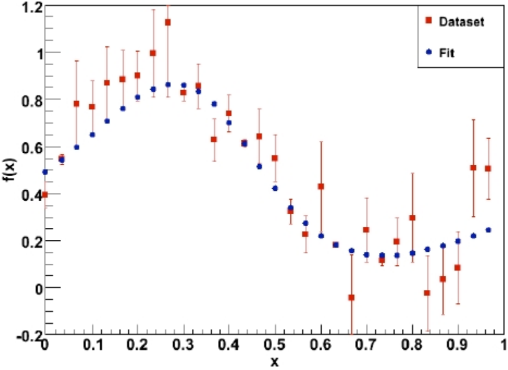


One more ingredient: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

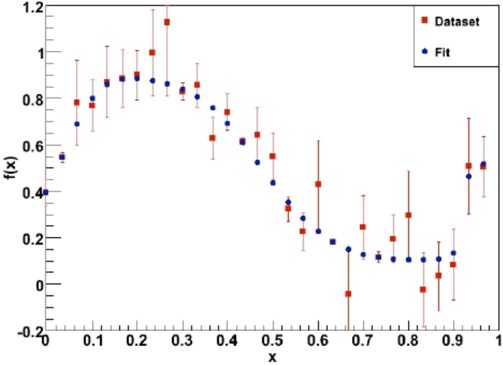
PROPER LEARNING



DRAWBACK

- NN can learn fluctuations owing to their flexibility

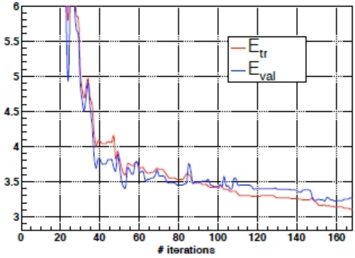
OVERLEARNING

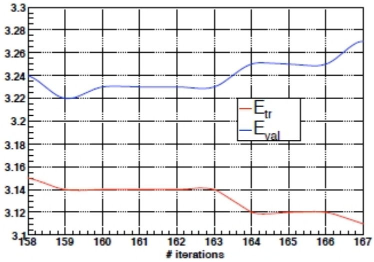


One more ingredient: minimization and stopping

CROSS-VALIDATION METHOD

- divide data into two subsets (training & validation)
- train the NN on training subset and compute χ^2 for each subset
- stop when χ^2 of validation subset no longer decreases (NN are learning fluctuations!)

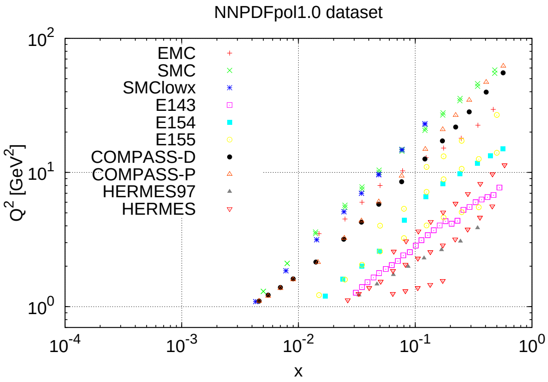




The best fit does not coincide with the χ^2 absolute minimum

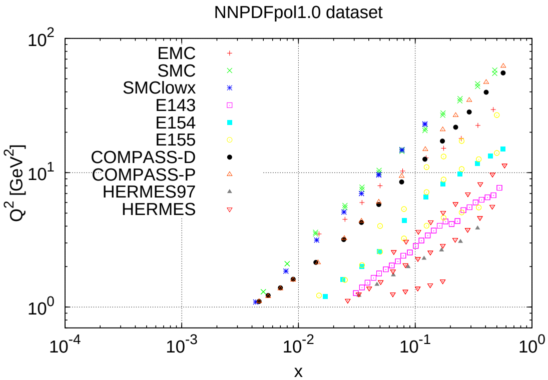
3. Towards NNPDFpol1.0

Experimental dataset



$$g_1(x,Q^2) = A_1(x,Q^2)\frac{F_2(x,Q^2)}{2x(1+R(x,Q^2))}(1+\gamma^2) \qquad \gamma^2 = \frac{4M_N^2x^2}{Q^2}$$

Experimental dataset



1. All relevant polarized DIS data on proton, neutron and deuteron targets
2. Kinematical cuts to remove the sensitivity to dynamical higher-twist
   - $Q^2 > 1 Gev^2$
   - $W^2 = Q^2(1-x)/x \geq 6.25 GeV^2$ (C. Simolo, Ph.D. Thesis. arXiv:0807.1501)

(higher twist terms added to observables and fitted to data become compatible with zero)

Input polarized PDF basis

1. Four polarized PDFs (gluon + linear combinations of light quarks)
   - singlet $\Delta\Sigma(x) \equiv \Sigma_{i=1}^{n_f}(\Delta q_i(x) + \Delta\bar{q}_i(x))$
   - gluon $\Delta g(x)$
   - triplet $\Delta T_3(x) \equiv (\Delta u(x) + \Delta\bar{u}(x)) - (\Delta d(x) + \Delta\bar{d}(x))$
   - octet $\Delta T_8(x) \equiv (\Delta u(x) + \Delta\bar{u}(x)) + (\Delta d(x) + \Delta\bar{d}(x) - 2(\Delta s(x) + \Delta\bar{s}(x)))$

   provided the definition for any flavor i and for gluon

$$\Delta q_i(x, Q^2) = q_i^{\uparrow\uparrow}(x, Q^2) + \bar{q}_i^{\uparrow\uparrow}(x, Q^2) - q_i^{\uparrow\downarrow}(x, Q^2) + \bar{q}_i^{\uparrow\downarrow}(x, Q^2)$$

$$\Delta g(x, Q^2) \equiv g^{\uparrow\uparrow}(x, Q^2) - g^{\uparrow\downarrow}(x, Q^2)$$

2. At initial scale $Q_0^2 = 1 GeV^2$
3. Assume all heavy quarks are generated radiatively
4. Adopt $\alpha_s(M_Z^2) = 0.119$, $m_c = 1.4$ GeV, $m_b = 4.75$ GeV
5. All PDFs refer to proton target (proton and neutron related by isospin)

Theoretical constraints

1. Positivity of physical cross sections bounds structure function

$$|g_1(x,Q^2)| \le F_1(x,Q^2)$$

Modify $E^{(k)}$ by a Lagrange multiplier $\lambda_{\rm pos} \sim 10^{10}$, $x \in [10^{-5}, 0.9]$, $Q^2 \in [1,100]$ GeV^2

$$E^{(k)} \longrightarrow E^{(k)} - \lambda_{\rm pos} \sum_{p=1}^{N_{dat,pos}} \theta\left(-|g_{1,p}^{(net)(k)}(x,Q^2)| - g_{1,p}^{(net)(k)}(x,Q^2)\right) g_{1,p}^{(net)(k)}(x,Q^2)$$

Replicas not fulfilling positivity bound are strongly penalized

2. Sum rules

$$[\Delta T_3(Q_0^2)] \equiv \int_0^1 dx \Delta T_3(x,Q_0^2) = a_3 \qquad [\Delta T_8(Q_0^2)] \equiv \int_0^1 dx \Delta T_8(x,Q_0^2) = a_8$$

$$a_3 = 1.2701 \pm 0.0025 \qquad a_8 = 0.585 \pm 0.025$$

Imposed through normalization of triplet and octet PDFs
a_3 and a_8 randomly selected for each replica within the allowed experimental uncertainty
uncertainty on a_8 up to 30% will be studied
a_3 will also be fitted in order to test Bjorken sum rule

PDF Parametrization

$$\Delta\Sigma(x,Q_0^2) = (1-x)^{m_{\Delta\Sigma}} x^{-n_{\Delta\Sigma}} NN_{\Delta\Sigma}(x)$$

$$\Delta g(x,Q_0^2) = (1-x)^{m_{\Delta g}} x^{-n_{\Delta g}} NN_{\Delta g}(x)$$

$$\Delta T_3(x,Q_0^2) = A_{\Delta T_3}(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)$$

$$\Delta T_8(X,Q_0^2) = A_{\Delta T_8}(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)$$

1. Each polarized PDF parametrized with a multi-layer feed-forward NN (2-5-3-1)
2. Parametrization supplemented with a preprocessing polynomial:
   → exponents m and n randomly chosen in fixed intervals;
   → intervals must be sufficient large not to introduce a bias on the fit
   → check *a posteriori* by studying asymptotic exponents
3. Overall normalization constant factored out for triplet and octet.

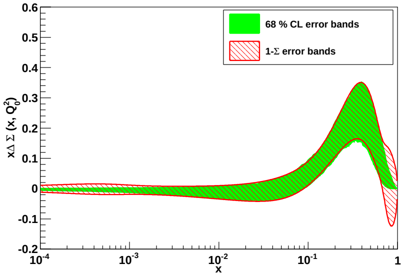
$$A_{\Delta T_3} = \frac{a_3}{\int_0^1 dx[(1-x)^{m_{\Delta T_3}} x^{-n_{\Delta T_3}} NN_{\Delta T_3}(x)]}$$

$$A_{\Delta T_8} = \frac{a_8}{\int_0^1 dx[(1-x)^{m_{\Delta T_8}} x^{-n_{\Delta T_8}} NN_{\Delta T_8}(x)]}$$

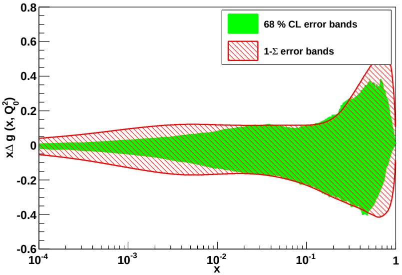
NNPDFpol1.0: results

Comparison between 1-Σ and 68% confidence level error bands

$x\Delta\Sigma(x, Q_0^2)$



$x\Delta g(x, Q_0^2)$



test for non-Gaussian behaviour
sizable deviations from Gaussian behaviour in the extrapolation region

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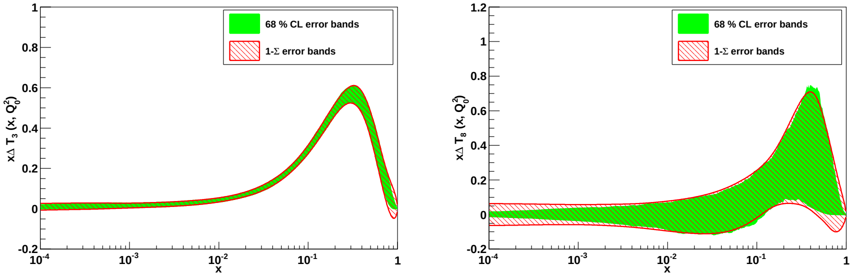
$x\Delta T_3(x, Q_0^2)$ $x\Delta T_8(x, Q_0^2)$

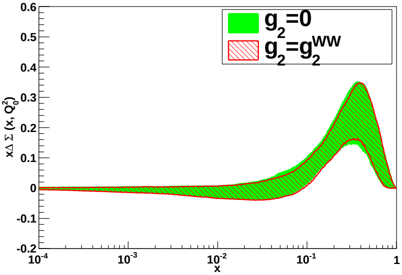


test for non-Gaussian behaviour
sizable deviations from Gaussian behaviour in the extrapolation region

Comparison between different inclusion of Target Mass Corrections

Target mass corrections are implemented iteratively during the minimization procedure
Two possible assumptions on $g_2(x, Q^2)$
$g_2 = 0$ OR $g_2 = g_2^{WW}$ (relate g_2 to g_1 by means of the Wandzura-Wilczek relation)

$x\Delta\Sigma(x, Q_0^2)$



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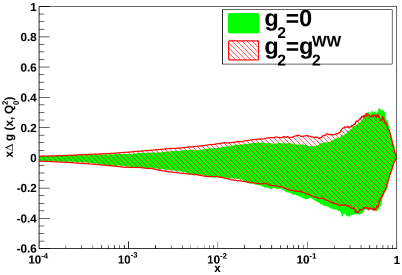

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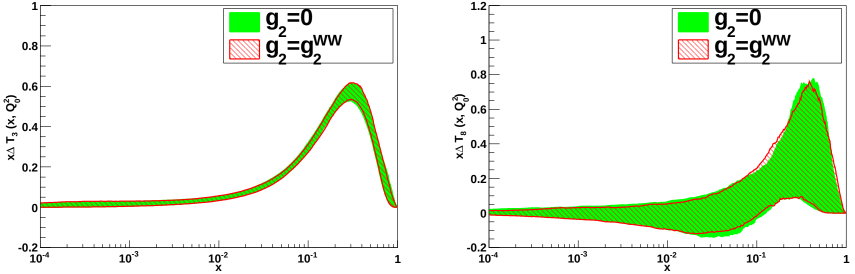
$x\Delta T_3(x, Q_0^2)$ $x\Delta T_8(x, Q_0^2)$



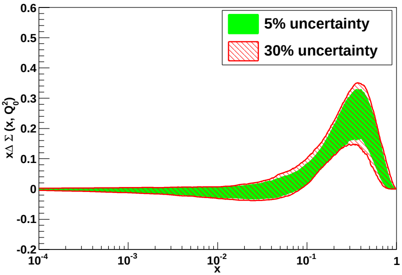
NNPDFpol1.0: results

Impact of uncertainty on a_8 coupling

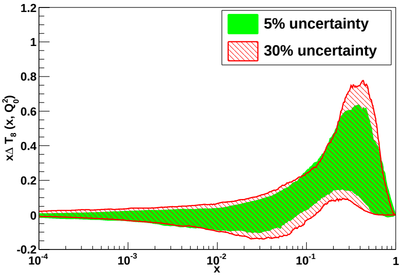
Fits done with

$a_8 = 0.585 \pm 0.025$ $\qquad$ $a_8 = 0.585 \pm 0.176$

$x\Delta\Sigma(x, Q_0^2)$



$x\Delta T_8(x, Q_0^2)$

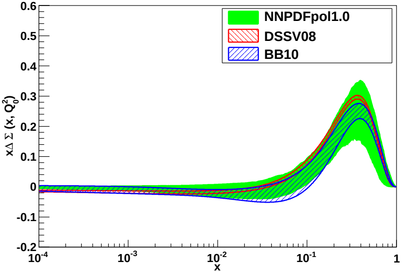


NNPDFpol1.0: results

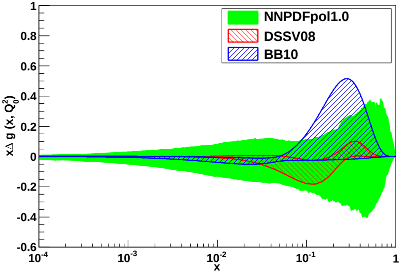
Comparison between `NNPDFpol1.0` and other parton sets (parametrization basis)

DSSV09: D. de Florian et al., Phys.Rev. D80 (2009) 034030
BB10: J. Blumlein and H. Bottcher, Nucl.Phys. B841 (2010) 205

$x\Delta\Sigma(x, Q_0^2)$



$x\Delta g(x, Q_0^2)$



Much larger error bands for Singlet and Gluon

NNPDFpol1.0: results

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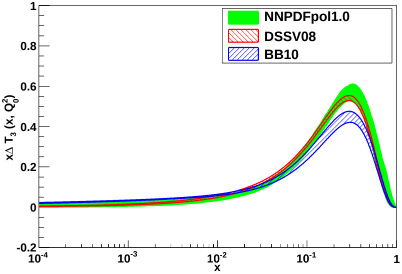
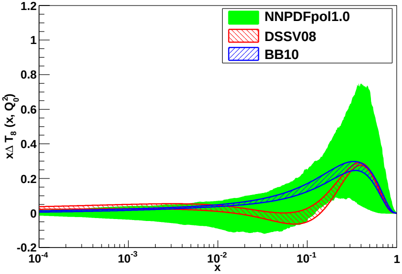
$x\Delta T_3(x, Q_0^2)$



$x\Delta T_8(x, Q_0^2)$

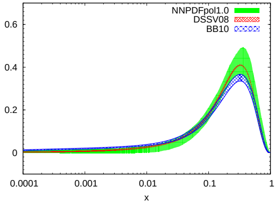


Triplet discriminates between DSSV08 and BB10 fits
Much larger error band for Octet

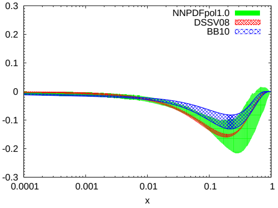
NNPDFpol1.0: results

Comparison between NNPDFpol1.0 and other parton sets (flavor basis)

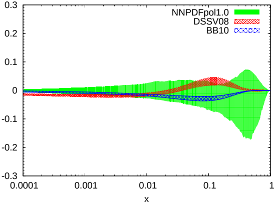
$x\Delta(u+\bar{u})(x, Q_0^2)$



$x\Delta(d+\bar{d})(x, Q_0^2)$



$x\Delta(s+\bar{s})(x, Q_0^2)$



- Much larger error bands for all light quark combinations
- NNPDFpol1.0 does not agree with BB10 for the $x\Delta(u+\bar{u})$
- discrepancy on strangeness between BB10 and DSSV08 reconciled within NNPDF uncertainties

Singlet and Gluon first moments in $\overline{\text{MS}}$ scheme at $Q_0^2 = 4$ GeV^2

$$[\Delta\Sigma] \equiv \int_0^1 \Delta\Sigma(x, Q_0^2)dx \qquad [\Delta g] \equiv \int_0^1 \Delta g(x, Q_0^2)dx$$

	NNPDFpol1.0	DSSV08	AAC08	BB10	LSS10
$[\Delta\Sigma]$	0.32 ± 0.11	0.26 ± 0.03	0.26 ± 0.06	0.19 ± 0.08	0.21 ± 0.03
$[\Delta g]$	-0.2 ± 1.1	-0.12 ± 0.12	0.40 ± 0.28	0.46 ± 0.43	0.32 ± 0.19

Notice the large uncertainty on the first moments:
Singlet between two and four times
Gluon almost one order of magnitude

DSSV09: D. de Florian et al., Phys.Rev. D80 (2009) 034030
AAC08: M. Hirai and S. Kumano, Nucl.Phys. B813 (2009) 106
BB10: J. Blumlein and H. Bottcher, Nucl.Phys. B841 (2010) 205
LSS10: E. Leader, A.V. Sidorov and D.B. Stamenov, Phys.Rev. D82 (2010) 114018

The Bjorken sum rule

1. Recall the Bjorken sum rule

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6}\Delta C_{\rm NS}^{\overline{\rm MS}}[\alpha_s(Q^2)]a_3$$

$$\Gamma_1^{p,n} \equiv \int_0^1 dx g_1^{p,n}(x,Q^2) \qquad \Delta C_{\rm NS}^{\overline{\rm MS}} = 1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots \text{(up to three loops)}$$

2. Fit without imposing a_3 sum rule

$$a_3 = 1.21 \pm 0.10 \quad \text{(fitted)} \qquad a_3 = 1.2670 \pm 0.0035 \quad \text{(measured)}$$

3. Consistency check of the fitting procedure
4. Suggest polarized fit to determine $\alpha_s(Q^2)$ in analogy with the unpolarized case

4. Conclusions

Final remarks

Summary

1. The NNPDF methodology provides a statistically sound procedure for PDF fitting
2. `NNPDFpol1.0` is the first polarized parton determination using the NNPDF approach. The analysis from inclusive DIS data leads to
   - Triplet discrimination (agreement with DSSV08, not with BB10)
   - large uncertainties on Singlet and Octet and very large on Gluon
   - uncertainty on Singlet first moment between two and four times bigger
   - uncertainty on Gluon first moment almost one order of magnitude bigger

Outlook

1. Include data sets from other processes (open charm and jet production with fixed target, inclusive jet production, W boson production at RHIC, ...)
2. Determine the strong-coupling constant from polarized DIS data

Final remarks

Summary

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Outlook

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Thank you for your attention!

5. Backup

PDF fitting: state of the art

1. First stage: first moments of polarized PDFs and polarized sum rules (last 25 years)
   $\longrightarrow$ "historical" experimental collaborations (at CERN, SLAC, DESY, JLAB):
   EMC, SMC, E142, E143, E154, E155, COMPASS, HERMES, CLAS, ...
2. Second stage: polarized PDF fits from global NLO QCD analysis (last ~15 years)
   $\longrightarrow$ different choice of datasets, parton parametrization, treatement of higher twists, ...
   ABFR (arXiv:hep-ph/9803237, 1998), BB (arXiv:1005.3113, 2010) (DIS only);
   AAC (arXiv:0808.0413, 2008), LSS (arXiv:1010.0574, 2010) (DIS+SIDIS);
   DSSV (arXiv:0904.3821, 2009) (DIS+SIDIS+pp)
3. Third stage: provide uncertainties on polarized PDFs (last ~10 years)
   $\longrightarrow$ Gaussian error propagation, Lagrange multiplier + Hessian method; fit with orthogonal polynomials (arXiv:1011.4873, 2010)

Polarized PDF evolution

In Mellin space the DGLAP equations

$$\begin{array}{rcl} \mu^2 \frac{\partial}{\partial \mu^2} \Delta q_{NS}^{\pm,\nu}(N,\mu^2) & = & \Delta\gamma_{NS}^{\pm,\nu} q_{NS}^{\pm,\nu}(N,\mu^2) \\ \mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} (N,\mu^2) & = & \begin{pmatrix} \Delta\gamma_{qq}(N,\alpha_s(Q^2)) & \Delta\gamma_{qg}(N,\alpha_s(Q^2)) \\ \Delta\gamma_{gq}(N,\alpha_s(Q^2)) & \Delta\gamma_{gg}(N,\alpha_s(Q^2)) \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} \end{array}$$

can be solved analitically

$$\Delta q_{NS}^{\pm,\nu}(N,Q^2) = \Gamma_{NS}^{\pm,\nu}(N,a_s,a_0)\Delta q_{NS}^{\pm,\nu}(N,Q_0^2) \;, a_s \equiv \alpha_s/2\pi$$

where, at NLO,

$$\Gamma_{NS,NLO}^{\pm,\nu}(N,a_s,a_0) = \exp\left\{ \frac{U_1^{\pm,\nu}}{b_1} \ln\left(\frac{1+b_1 a_s}{1+b_1 a_0}\right)\right\} \left(\frac{a_s}{a_0}\right)^{-R_0^{NS}}$$

Polarized PDF evolution

NNPDF NLO polarized PDF evolution (**Fast Kernel method**) benchmarked with the Les Houches PDF benchmarks (G. Salam and a. Vogt, hep-ph/0511119)

x	$\epsilon_{\rm rel}\,(\Delta u_V)$	$\epsilon_{\rm rel}\,(\Delta d_V)$	$\epsilon_{\rm rel}\,(\Delta\Sigma)$	$\epsilon_{\rm rel}\,(\Delta g)$
10^{-3}	$1.1\,10^{-4}$	$9.2\,10^{-5}$	$9.9\,10^{-5}$	$1.1\,10^{-4}$
10^{-2}	$1.4\,10^{-4}$	$1.9\,10^{-4}$	$3.5\,10^{-4}$	$9.3\,10^{-5}$
0.1	$1.2\,10^{-4}$	$1.6\,10^{-4}$	$5.4\,10^{-6}$	$1.7\,10^{-4}$
0.3	$2.3\,10^{-6}$	$1.1\,10^{-5}$	$7.5\,10^{-6}$	$1.7\,10^{-5}$
0.5	$5.6\,10^{-6}$	$9.6\,10^{-6}$	$1.6\,10^{-5}$	$2.5\,10^{-5}$
0.7	$1.2\,10^{-4}$	$9.2\,10^{-7}$	$1.6\,10^{-4}$	$7.8\,10^{-5}$
0.9	$3.5\,10^{-3}$	$1.1\,10^{-2}$	$4.1\,10^{-3}$	$7.8\,10^{-3}$

Very accurate evolution!

Target mass corrections

- Extracting both structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ from data requires measuring longitudinal and transverse spin asymmetries $A_{\parallel}$ and $A_{\perp}$
- Experimental information on $A_{\perp}$ is rather poor (in most cases only $A_{\parallel}$ is measured), thus $g_1(x, Q^2)$ and $g_2(x, Q^2)$ are related

$$g_1(x, Q^2) = \frac{F_1(x, Q^2)}{1 + \gamma\eta} \frac{A_{\parallel}}{D} + \frac{\gamma(\gamma - \eta)}{\gamma\eta + 1} g_2(x, Q^2)$$

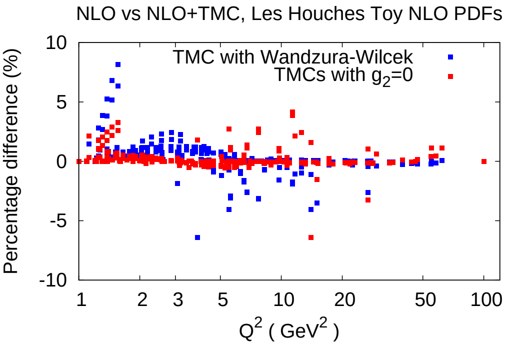
$$\gamma = \frac{2m_N x}{Q} \; ; \quad \eta = \frac{\epsilon\gamma y}{1 - \epsilon(1 - y)} \; ; \quad D = \frac{1 - (1 - y)\epsilon}{1 + \epsilon R(x, Q^2)} \; ; \quad \epsilon = \frac{4(1 - y) - \gamma^2 y^2}{2y^2 + 4(1 - y) + \gamma^2 y^2} \; ; \quad y = 1 - \frac{E'}{E}$$

MUST MAKE SOME ASSUMPTION ON $g_2(x, Q^2)$

$g_2 = 0$ OR $g_2 = g_2^{WW}$ (relate g_2 to g_1 by means of the Wandzura-Wilczek relation)

- Target mass corrections implemented iteratively during the minimization procedure

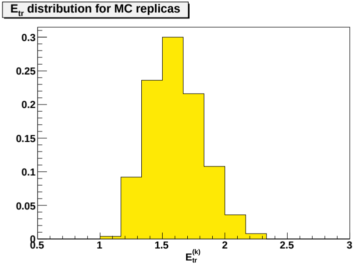
Target mass corrections

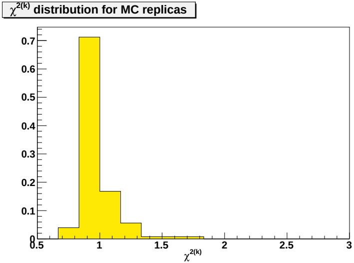


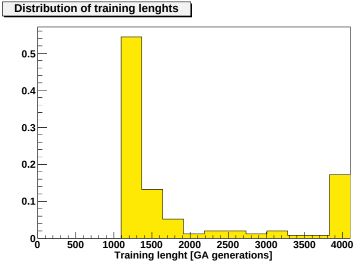
Kinematical cuts exclude the largest-x and smallest-Q^2 data region, where the TMC effects are most important

Moderate impact of TMC corrections (few percent at small Q^2)

NNPDFpol1.0: global χ^2

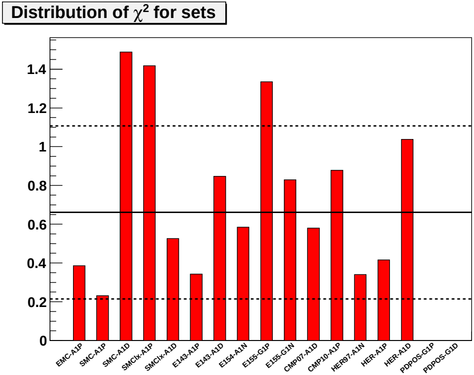




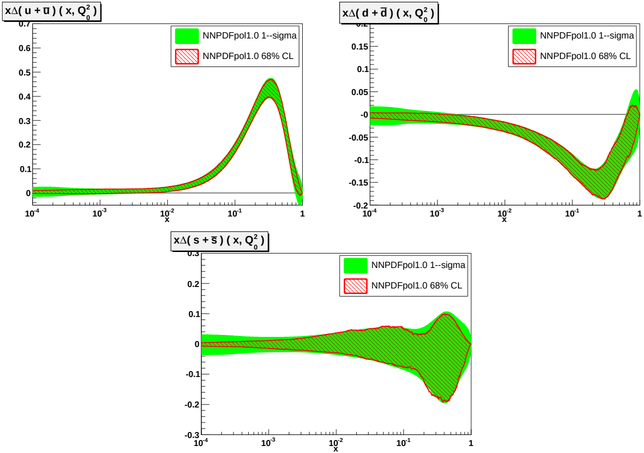


$\chi^2_{\rm tot}$	0.75
$\langle E\rangle \pm \sigma_E$ $\langle E_{\rm tr}\rangle \pm \sigma_{E_{\rm tr}}$ $\langle E_{\rm val}\rangle \pm \sigma_{E_{\rm val}}$	1.85 ± 0.19 1.60 ± 0.21 2.10 ± 0.31
$\langle \chi^{2(k)}\rangle \pm \sigma_{\chi^2}$	0.97 ± 0.14
$\left\langle \delta g_1^{(\rm exp)}\right\rangle_{\rm dat}$ $\left\langle \delta g_1^{(\rm net)}\right\rangle_{\rm dat}$	0.18 0.11
$\langle TL\rangle$	1921

NNPDFpol1.0: individual experiments χ^2



- No evidence of any specific dataset being inconsistet with each other
- Distribution of individual χ^2 values broadly consistent with statistical expectations

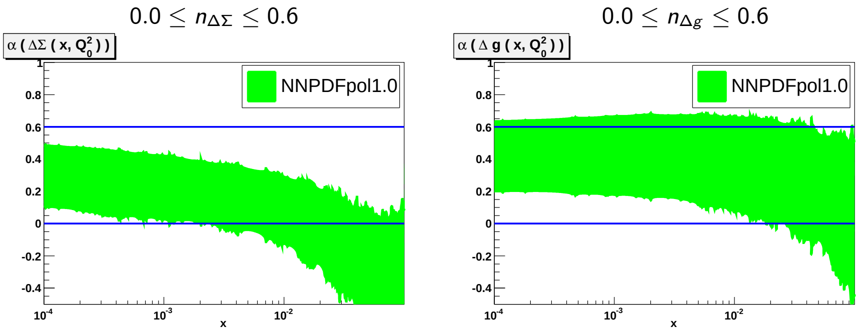


comparison between 1σ error bands and 68% confidence level

test for non-Gaussian behaviour

sizable deviations from Gaussian behaviour in the extrapolation region

Preprocessing: effective asymptotic exponents

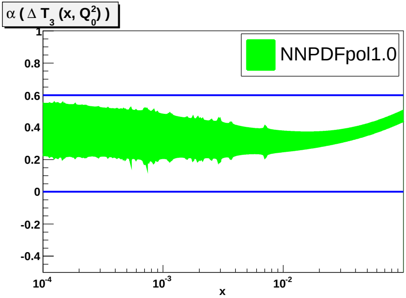


$$\alpha_{\rm eff}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

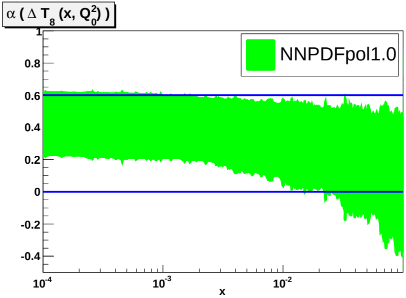
Effective exponents always contained in the preprocessing exponents range
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

$0.0 \leq n_{\Delta\Sigma} \leq 0.6$



$0.0 \leq n_{\Delta g} \leq 0.6$



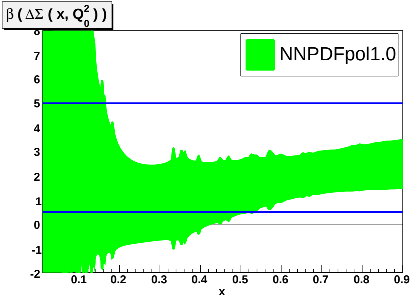
$$\alpha_{\rm eff}(x,Q^2) \equiv \ln(\Delta q(x,Q^2))/\ln(1/x) \quad \text{at } Q^2 = Q_0^2 = 1\,GeV^2$$

Effective exponents always contained in the preprocessing exponents range

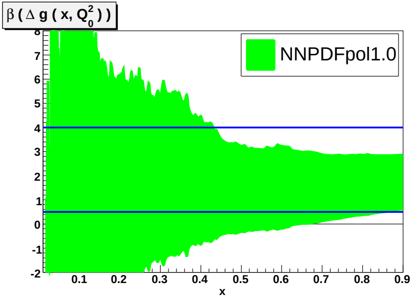
The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents

$0.5 \leq m_{\Delta\Sigma} \leq 5.0$



$0.5 \leq m_{\Delta g} \leq 4.0$

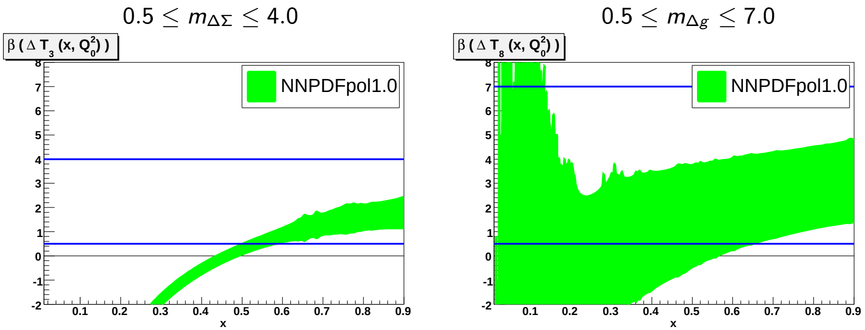


$$\beta_{\text{eff}}(x, Q^2) \equiv \ln(\Delta q(x, Q^2))/\ln(1-x) \quad \text{at } Q^2 = Q_0^2 = 1GeV^2$$

Effective exponents always contained in the preprocessing exponents range

The polarized PDF is driven only by experimental data

Preprocessing: effective asymptotic exponents



$$\beta_{\rm eff}(x, Q^2) \equiv \ln(\Delta q(x, Q^2)) / \ln(1 - x) \quad \text{at } Q^2 = Q_0^2 = 1 GeV^2$$

Effective exponents always contained in the preprocessing exponents range

The polarized PDF is driven only by experimental data

Distances

Compare two sets of $N_{\rm rep}^{(1)}$ and $N_{\rm rep}^{(1)}$ replicas coming from different fits
Do they have belong to the same underlying probability distribution?

MEAN VALUE

$$d^2\left(\langle q^{(k)}\rangle_{(1)},\langle q^{(k)}\rangle_{(2)}\right)=\frac{\left(\langle q^{(k)}\rangle_{(1)}-\langle q^{(k)}\rangle_{(2)}\right)^2}{\sigma^2\left[\langle q^{(k)}\rangle_{(1)}\right]+\sigma^2\left[\langle q^{(k)}\rangle_{(2)}\right]}$$

$$\langle q^{(k)}\rangle_{(i)}=\frac{1}{N_{{\rm rep}(i)}}\sum_{l=1}^{N_{{\rm rep}(i)}}q_l^{(k)}$$

$$\sigma^2\left[\langle q^{(k)}\rangle_{(i)}\right]=\frac{1}{N_{{\rm rep}(i)}}\sigma^2\left[q_{(i)}^{(k)}\right]=\frac{1}{N_{{\rm rep}(i)}-1}\sum_{l=1}^{N_{{\rm rep}(i)}}\left(q_{l,(i)}-\langle q\rangle_{(i)}\right)^2$$

UNCERTAINTY

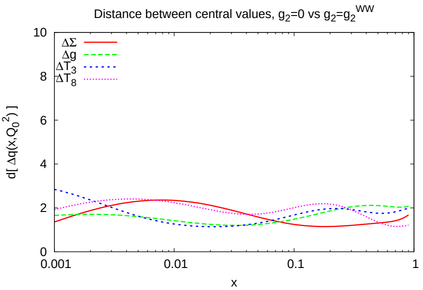
$$d^2\left(\sigma^2_{(1)},\sigma^2_{(2)}\right)=\frac{\left(\bar\sigma^2_{(1)}-\bar\sigma^2_{(2)}\right)}{\sigma^2\left[\sigma^2_{(1)}\right]+\sigma^2\left[\sigma^2_{(2)}\right]}$$

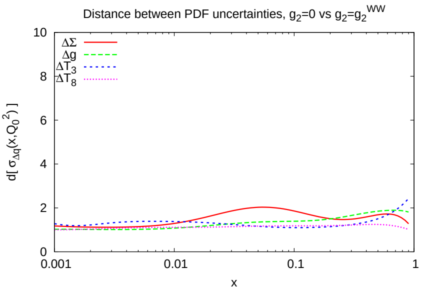
$$\bar\sigma^2_{(i)}\equiv\sigma^2\left[q_{(i)}^{(k)}\right]$$

$$\sigma^2\left[\sigma^2_{(i)}\right]=\frac{1}{N_{{\rm rep}(i)}}\left[\frac{1}{N_{{\rm rep}(i)}}\sum_{l=1}^{N_{{\rm rep}(i)}}\left(q_{l,(i)}-\langle q\rangle_{(i)}\right)^4-\frac{N_{{\rm rep}(i)}-3}{N_{{\rm rep}(i)}-1}\left(\bar\sigma^2_{(i)}\right)^2\right]$$

By definition, the distances have a χ^2 probability distribution with one degree of freedom
mean $\langle d\rangle=1$ and $d\lesssim 2.3$ at 90% confidence level

Comparison between different inclusion of Target Mass Corrections





Distances

Impact of uncertainty on a8 coupling