Azimuthal Spin Asymmetries in Light-Cone Quark model

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**Outline**

- Drell-Yan at COMPASS with pion beam and transversely polarized target
  - asymmetry with convolution of proton pretzelosity and pion Boer-Mulders
- TMDs from Light-Cone Quark Model for proton and pion
  - transverse-spin structure of quarks in the pion
  - physical content of proton pretzelosity
- Model results for Drell-Yan asymmetry
- Conclusions
H\textsubscript{b}: pion beam

H\textsubscript{a}: proton target

\* DY cross section at leading-twist for transversely polarized target:

\[
\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha^2}{F_q^2} \hat{\sigma}_U \left\{ \left( 1 + D_{[\sin^2 \theta]} A_U^{\cos 2\phi} \cos 2\phi \right) \\
+ |\vec{S}_T| \left[ D_{[\sin^2 \theta]} \left( A_T^{\sin(2\phi+\phi_S)} \sin(2\phi + \phi_S) + A_T^{\sin(2\phi-\phi_S)} \sin(2\phi - \phi_S) \right) \\
+ A_T^{\sin \phi_S} \sin \phi_S \right] \right\}
\]

- \(A_U^{\cos 2\phi}\): Boer-Mulders functions of incoming hadrons
- \(A_T^{\sin(2\phi+\phi_S)}\): Boer-Mulders functions of pion beam and pretzelosity of nucleon target
- \(A_T^{\sin(2\phi-\phi_S)}\): Boer-Mulders functions of pion beam and transversity of nucleon target
- \(A_T^{\sin \phi_S}\): unpolarized TMD of pion beam and Sivers function of nucleon target

⇒ talk of C. Quintans
DY cross section at leading-twist for transversely polarized target:

\[
\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha^2}{F q^2} \hat{\sigma}_U \left\{ \left( 1 + D_{[\sin^2 \theta]} A^\cos 2\phi \right) \cos 2\phi + |S_T| D_{[\sin^2 \theta]} \left( A^{\sin(2\phi + \phi_S)} + A^{\sin(2\phi - \phi_S)} \sin(2\phi - \phi_S) \right) \right\}
\]

- \(A^\cos 2\phi\): Boer-Mulders functions of incoming hadrons
- \(A^{\sin(2\phi + \phi_S)}\): Boer-Mulders functions of pion beam and pretzelosity of nucleon target
- \(A^{\sin(2\phi - \phi_S)}\): Boer-Mulders functions of pion beam and transversity of nucleon target
- \(A^{\sin \phi_S}\): unpolarized TMD of pion beam and Sivers function of nucleon target

\(\hat{\sigma}_U\) is the parton momenta and typically the observables are presented as functions of kinematics is.

\(H_b: pion\ beam\)

\(H_a: proton\ target\)
Table 9: Expected statistical errors for various asymmetries assuming two years of data taking and a beam momentum of $v_{9u} \text{GeV}/c$.

<table>
<thead>
<tr>
<th>Asymmetry</th>
<th>Dimuon mass $\text{GeV}/c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta A \cos 2\phi$</td>
<td>$&lt;M_{\mu\mu}&lt;$ 9 $\text{GeV}/c$</td>
</tr>
<tr>
<td>$\delta A \sin \phi$</td>
<td>$&lt;M_{\mu\mu}&lt;$ 9 $\text{GeV}/c$</td>
</tr>
<tr>
<td>$\delta A \sin(2\phi + \phi_S)$</td>
<td>$&lt;M_{\mu\mu}&lt;$ 9 $\text{GeV}/c$</td>
</tr>
<tr>
<td>$\delta A \sin(2\phi - \phi_S)$</td>
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</tr>
</tbody>
</table>

Figure: Theoretical predictions and expected statistical errors on the Sivers Boer–Mulders $\sin \phi_o$ and $\sin \phi - \phi_S$ asymmetries for a DY measurement $\pi^- p \rightarrow \mu^+ \mu^- X$ with a $v_{9u} \text{GeV}/c$ beam in the high mass region $y \text{ GeV}/c^2$. In case of the Sivers asymmetry also the systematic error is shown (smaller error bar).
Boer-Mulders function of the pion
Light-Cone Wave Function of the Pion

\( \Psi^{LC}_{\pi\beta}(x_i, \vec{k}_{\perp,i}) \)

- invariant under boost, independent of \( P^\mu \)
- internal variables: \( x_i = \frac{p_i^+}{P^+}, \sum_{i=1}^{N} x_i = 1, \sum_{i=1}^{N} \vec{k}_{\perp,i} = \vec{0}_{\perp} \)

\[ |P, \pi\rangle = \sum_{\beta} \int \frac{d[1][2]}{\sqrt{3}} \frac{\delta_{ij}}{q_i\lambda_i} (1) \bar{q}_{j\lambda_2} |0\rangle \]

LCWF: eigenstate of total orbital angular momentum

\[ |\pi\rangle = |\pi\rangle^{L_z=1}_{-1} + |\pi\rangle^{L_z=0}_{0} + |\pi\rangle^{L_z=-1}_{1} \]

\[ \begin{align*}
L_z^q &= -1 \\
J_z^q &= -L_z^q \\
(\uparrow\uparrow)^{LC} \\
(\uparrow\downarrow)^{LC} \\
(\downarrow\downarrow)^{LC} \\
\end{align*} \]

- parity
- time reversal
- isospin symmetry

2 independent wave function amplitudes
Phenomenological Light-Cone Wave Function of the Pion

\[ \Psi^{LC}_{\pi\beta}(x_i, \vec{k}_{\perp,i}, \lambda_i) = \Psi^{Mom}(x_i, \vec{k}_{\perp,i}^2) \otimes \Psi^{Spin}(x_i, \vec{k}_{\perp,i}, \lambda_i) \]

\* \( \Psi^{Mom}(x_i, \vec{k}_{\perp,i}^2) \): momentum-space component
  - gaussian shape
  - two parameters: \( m_q \) and gaussian width fitted to exp. charge radius and pion decay constant

\* \( \Psi^{Spin}(x_i, \vec{k}_{\perp,i}, \lambda_i) \): spin-dependent part
  - eigenstate of the total angular momentum operator in light-front dynamics (\( L_z = 0, |L_z| = 1 \))
Phenomenological Light-Cone Wave Function of the Pion

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Electromagnetic Form Factor

\[ F_\pi(Q^2) \]

\[ Q^2 F_\pi(Q^2) \]
Model calculation of pion Boer-Mulders

\[ h_{1T}^\perp \]

one-gluon exchange approximation

overlap of LCWFs in S and P waves

Mismatch of helicity between initial and final state \( \Delta L_z = \pm 1 \)
Model calculation of pion Boer-Mulders

\[ h_{1T}^\perp \]

- one-gluon exchange approximation

\[ + h.c. \]

overlap of LCWFs in S and P waves

Mismatch of helicity between initial and final state \[ \Delta L_z = \pm 1 \]

SIDIS final state interactions

\[ h_{1T}^\perp [\text{SIDIS}] = -h_{1T}^\perp [\text{DY}] \]

DY initial state interactions

crossing
Boer-Mulders functions in LCCQM

\[ h_1^\perp u (\pi^+) = h_1^\perp d (\pi^+) = h_1^\perp d (\pi^-) = h_1^\perp \bar{u} (\pi^-) \]
Boer-Mulders functions in LCCQM

\[ h_1^\perp u(\pi^+) = h_1^\perp d(\pi^+) = h_1^\perp d(\pi^-) = h_1^\perp \bar{u}(\pi^-) \]

\[ h_1^{\perp(1)} u = \int d^2\vec{k}_\perp \frac{k_\perp^2}{2m_\pi^2} h_1^\perp u \]

\[ h_1^{\perp(1)} q = \int d^2\vec{k}_\perp \frac{k_\perp^2}{2M_p^2} h_1^\perp q \]

B.P., Schweitzer, Efremov, in preparation

B.P., F. Yuan, PRD81(2010)
Chromodynamic lensing

Burkardt, PRD66 (02)

unpolarized quarks
Chromodynamic lensing

Burkardt, PRD66 (02)

transversely pol. quark

Distortion in impact parameter (related to GPD $E_T$)
Chromodynamic lensing

Burkardt, PRD66 (02)

transversely pol. quark

Distortion in transverse momentum (related to Boer Mulders function)

Final-state interaction (lensing function)
Chromodynamic lensing

Burkardt, PRD66 (02)

transversely pol. quark

Distortion in transverse momentum
(related to Boer Mulders function)

Final-state interaction
(lensing function)

model-dependent relation

$$2m_\pi^2 h_1^{(1)}(x) \approx \int d^2 b_T \vec{b}_T \cdot \vec{I}(x, \vec{b}_T) \frac{\partial}{\partial \vec{b}_T^2} \mathcal{E}_T(x, \vec{b}_T^2)$$

Boer-Mulders function
lensing function
FT of chiral-odd GPD
Pion GPDs in impact parameter space

**Lattice**

\(<b_y> = (0.151 \pm 0.024) \text{ fm}\)

Broemmel et al., PRL101, 2008

**Light-Cone CQM**

\(<b_y> = 0.197 \text{ fm}\)

Frederico, Pace, BP, Salme', PRD80 (2009)
Model calculations of pion Boer-Mulders function

Diquark spectator model and LCCQM with one gluon exchange approximation
BM funct. proportional to $\alpha_s$ ⇒ overall normalization constant depending on the model scale

Zhun Lu, B.Q. Ma
PRD70 (2011)

Drell-Yan process, pion BM

$pion\ BM = m_\pi \leftrightarrow M_p$

proton BM

fixed $k_\perp = 0.3 \text{ GeV}$

fixed $x=0.15$
Model calculations of pion Boer-Mulders function

Diquark spectator model and LCCQM with one gluon exchange approximation
BM funct. proportional to $\alpha_s \rightarrow$ overall normalization constant depending on the model scale

Beyond one-gluon exchange approx.

TMD-GPD relation + lensing function from eikonal methods

Zhun Lu, B.Q. Ma
PRD70 (2011)

Gamberg, Schlegel
PLB685 (2010)
Proton Pretzelosity
Light-Cone Wave Function of the Proton

- classification of LCWFs in orbital angular momentum components

\[ | P, \lambda \rangle = \sum_{\beta} \int [1] [2] [3] \Psi_{\lambda, \beta}^{\frac{f}{\text{Mom}}} (x_i, \vec{k}_{\perp}, \lambda_i) \frac{\varepsilon_{ijk}}{\sqrt{6}} u_{i\lambda_1}^\dagger (1) u_{j\lambda_2}^\dagger (2) d_{k\lambda_3}^\dagger (3) | 0 \rangle \]

**total quark helicity** \( J_{qz} \)

\[ J_z = J_{zq} + L_{zq} \]

\( J_{zq} \rightarrow (\uparrow\uparrow\uparrow)_{LC} \hspace{1cm} (\uparrow\uparrow\downarrow)_{LC} \hspace{1cm} (\uparrow\downarrow\downarrow)_{LC} \hspace{1cm} (\downarrow\downarrow\downarrow)_{LC} \]

\( L_{zq} = -1 \hspace{1cm} L_{zq} = 0 \hspace{1cm} L_{zq} = 1 \hspace{1cm} L_{zq} = 2 \)

**MODEL**

\[ \Psi_{\lambda\beta}^f (x_i, \vec{k}_{\perp}, \lambda_i) = \Psi_{\text{Mom}}^{\frac{f}{\lambda}} (x_i, \vec{k}_{\perp}^2) \otimes \Psi_{\text{Spin}}^{\lambda} (x_i, \vec{k}_{\perp}, \lambda_i) \]

- momentum-space component: spherically symmetric

  two parameters fitted to anomalous magnetic moments of proton and neutron

- spin-dependent part eigenstate of the total angular momentum operator (S, P, D waves)

[B.P., Cazzaniga, Boffi, PRD78 (2008)]
Pretzelosity

\[ h_{1T}^\perp = \begin{array}{c}
\uparrow \\
- \downarrow
\end{array} \]

|\Delta L_z| = 2

from the initial to the final nucleon state

\[ h_{1T}^\perp \]

up

\[ k_x [\text{GeV}] \]

\[ k_y [\text{GeV}] \]

down

\[ k_x [\text{GeV}] \]

P-P int.

S-D int.

\[ h_{1T}^{\perp (1)u} \]

P-P int.

S-D int.
OAM and Pretzelosity

model-dependent relation

\[ \mathcal{L}_z = -\int \, dx \, d^2 \vec{k}_\perp \, \frac{k_\perp^2}{2M^2} \, h_{1T}^+ (x, k_\perp^2) \]

first derived in LC-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

\[ \Delta L_z = 0 \]

\[ |\Delta L_z| = 2 \]

no operator identity
relation at level of matrix elements of operators

valid in all quark models with spherical symmetry in the rest frame

[Lorce', BP, PLB710 (2012)]
Application to Observables
Fixing the scale of the LCCQM

- There exists a scale at which there are no sea and gluons.

- The valence quarks carry the whole momentum of the hadron.

\[ \langle x \rangle_v = 1 \quad \langle x \rangle_g = \langle x \rangle_{sea} = 0 \]

Roberts, “The structure of the proton”

Fixing the scale of the LCCQM

\[ x_v = 1 \quad \quad x_g = x_{sea} = 0 \]

\[ Q^2 = 4 \text{ GeV}^2 \quad \quad \langle x \rangle_v = 0.47 \]

\[ Q^2 = 10 \text{ GeV}^2 \quad \quad \langle x \rangle_v = 0.36 \]

The BM-Pretzelosity Asymmetry in πp Drell Yan

\[ A_{TU}^{\text{sin}(2\phi + \phi_p)} = \frac{F_{TU}^{\text{sin}(2\phi + \phi_p)}}{F_{UU}^1} \]

\( \phi = \) lepton angle in the CS frame  \( \phi_p = \) proton spin angle in the CM frame

**Numerator**

\[ F_{TU}^{\text{sin}(2\phi + \phi_p)} = C \left[ w(k_{T\pi}, \bar{k}_{T\pi}) h_{1T}^\perp h_{1\pi}^\perp \right] \]

\[ w(k_{T\pi}, \bar{k}_{T\pi}) = \frac{2(\bar{h} \cdot \bar{k}_{T\pi})(2(\bar{h} \cdot k_{T\pi})(\bar{h} \cdot \bar{k}_{T\pi}) - k_{T\pi}^2)(\bar{h} \cdot \bar{k}_{T\pi})}{2M_p^2m_{\pi}} \]

**Denominator**

\[ F_{UU}^1 = C \left[ 1 f_1^p f_1^\pi \right] \]

with \( \bar{h} = \frac{\bar{q}_T}{q_T} \)
The BM-Pretzelosity Asymmetry in πp Drell Yan

[Arnold, Metz, Schlegel, PRD79, (2008)]

\[ A_{TU}^{\sin(2\phi + \phi_p)} = \frac{F_{TU}^{\sin(2\phi + \phi_p)}}{F_{UU}^1} \]

\( \phi = \) lepton angle in the CS frame

\( \phi_p = \) proton spin angle in the CM frame

Numerator

\[ F_{TU}^{\sin(2\phi + \phi_p)} = C \left[ w(\vec{k}_{Tp}, \vec{K}_{T\pi}) h_{1T}^\perp h_{1T}^\perp \pi \right] \]

\[ w(\vec{k}_{Tp}, \vec{k}_{T\pi}) = \frac{2(\vec{h} \cdot \vec{k}_{Tp})(2(\vec{h} \cdot \vec{k}_{Tp})(\vec{h} \cdot \vec{k}_{T\pi}) - \vec{k}_{Tp}^2(\vec{h} \cdot \vec{k}_{T\pi}))}{2M_p^2m_\pi} \]

Denominator

\[ F_{UU}^1 = C [1 f_1^p f_1^\pi] \]

with

\[ \vec{h} = \frac{\vec{q}_T}{q_T} \]

Gaussian Ansatz

\[ F_{TU}^{\sin(2\phi + \phi_p)} = B_{\text{Gauss}} e_u^2 h_{1T}^\perp (1/2)u/p(x_p)h_{1T}^\perp (1)\bar{u}/\pi^- (x_\pi) \]

\[ F_{UU}^1 = e_u^2 f_1^u/p(x_p)f_1^{\bar{u}/\pi^-} (x_\pi) \]

\[ h_{1T}^\perp (1/2)u/p = \int d\vec{k}_{\perp} \frac{k_{\perp}}{2M_p} h_{1T}^\perp u/p(x, k_{\perp}^2) \]

\[ h_{1T}^\perp (1)\bar{u}/\pi^- = \int d\vec{k}_{\perp} \frac{k_{\perp}^2}{2m_\pi^2} h_{1T}^\perp \bar{u}/\pi^- (x, k_{\perp}^2) \]

\[ B_{\text{Gauss}} = \frac{3}{2} \frac{m_\pi}{M_p} \frac{1}{\left[1 + \frac{(\langle p_{T\pi}^2 \rangle)}{(p_{Tp}^2)}\right]^{3/2}} \]
Results from the LCCQM

\[ A_{TU}^{\sin(2\phi + \phi_p)} \]

hadronic scale

approximate evolution

pretzelosity and BM function evolved like transversity

\[ x_F = x_\pi - x_p \]

\[ \pi^- p \]

1/2 moment of pretzelosity

hadronic scale

evolution to \( Q^2 = 20 \text{ GeV}^2 \)

first moment of pion BM

B.P., Schweitzer, Efremov, in preparation
Model dependence from $f_1^p$ and $f_1^{\pi}$

$LCCQM$

$GRV$

$LCCQM$

Aicher, Schaefer, Vogelsang, PRL 105 (2010)

$SMRS, PRD 45 (1992)$

B.P., Schweitzer, Efremov, in preparation
Light-cone quark spectator model

Zhun Lu, B.-Q. Ma, PLB696(2011)

Predictions from other models

talk of Zhun Lu

cut in 1.0 GeV < q_T < 2.0 GeV

A sin(3Φ − φ_s)

0.06

0.04

0.02

0

−0.02

−0.04

−0.06

0.2 0.4 0.6 0.8

x_F

0 4 5 6 7 8 9

M/GeV

1 2 3 4

q_T/GeV

π^+

π^−

π^+

π^−

π^+

π^−
$A^\sin(3\phi_h-\phi_s)_{UT} \propto \sum_q e_q^2 h_{1T}^q \otimes H_1^q$

\checkmark leads to $\sin(3\Phi-\Phi_s)$ modulation in $A_{UT}$

\checkmark proton (HERMES) and deuteron (COMPASS) data consistent with zero

\checkmark pretzelosity equal to zero? or just the suppression by third power of $1/P_{h\perp}$?

\checkmark experiment planned at CLAS12 (H. Avakian at al., LOI 12-06-108)
Summary

- **Polarized Drell-Yan at COMPASS with pion beam: BM^{pretzelosity}_{proton}**
  
  complementary to SIDIS to extract information on proton pretzelosity
  
  new tool to learn about the transverse spin structure of quark in the pion

- **Predictions in a Light-Cone Quark model**
  
  small asymmetry of the order of 1-2%, with larger contribution at large $x_F$
  
  small model dependence from $f_1$ of proton
  
  larger uncertainties from $f_1$ of pion
  
  effects of evolution are important and need further studies

- **Future plans: extend the analysis within LCCQM to other spin asymmetries of (un)polarized DY**